Evaluating and improving international assistance programmes: Examples from Mongolia's transition experience

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Appendix B4.
Deriving a function for TFP with application to Mongolia

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B4.1 Introduction

This appendix is a corollary of chapter 4. In chapter 4 it was demonstrated that the quality of data is crucial to the calculation and use of TFP as indicator of efficiency. As reliable and qualitatively good data may be difficult to obtain, we present in this appendix a new method that allows us to construct an analytic function that describes the development of TFP in time. A method is constructed for quantifying the relative growth of technology, based on time series data for aggregate output, capital and labour. The focus of the application of the method is on situations where estimates of the input shares of capital and labour are unreliable or difficult to obtain.

Approaches for calculating or estimating economic growth can be subdivided into two categories: (1) growth accounting, and (2) econometric or statistical approaches. Both approaches use a production function for modelling the relation between output or GDP and inputs, amongst which capital stock and labour. The objective of growth accounting is to break down the relative growth of output in time into the relative growth of capital, labour, and of technology or total factor productivity (TFP). By calculating, for instance, the value shares of labour (e.g. by using data on salaries) and of capital in the total value of input factors, TFP change can be calculated for every year, which is used as an indicator for the efficiency of production of a country at macroeconomic level (see Barro and Sala-i-Martin (1995) for more details).

The second type of approach uses statistical procedures in order to estimate the shares of capital and labour, which are subsequently used to calculate TFP. An example of this approach is given by Cheng (2003), who applies linear regression to a logarithmic transformation of a Cobb-Douglas production function for estimating the input shares.

The validity of growth accounting rests upon the assumption that factor markets are competitive. The marginal product of each input then equals its factor price. If accurate data about either the share of rental payments to capital or the share of wage payments to labour can be collected, then growth accounting can be applied. If such data are difficult to obtain, or if these data are not very reliable, then one could combine growth accounting with a sensitivity or uncertainty analysis about the estimated input shares in order to quantify the effect of variations around data estimates of these shares on the conclusions about TFP-growth (see Chessa and Schouwstra (chapter 4 and 2005) for an application to Mongolia's economy).[117]

An alternative approach could be to apply a regression method in order to estimate the input shares from time series data on GDP, capital and labour. However, this approach implicitly makes the assumption that TFP behaves according to an exponential function in time. The analysis of TFP for Mongolia's economy since 1980, as carried out by Chessa and Schouwstra (in Chapter 4 and in 2005), shows that TFP does not follow a monotonic behaviour in time, which consequently rules out such a method for estimating input shares and TFP. Therefore, a new method should be developed which does not make this implicit assumption.

The point of departure of this new and relatively simple method is to quantify the relative growth of technology for the full range of values that the input shares can take (i.e., between 0 and 1) at each available point in time based on time series data of GDP, capital and labour. The identified analytic function of TFP can be used in a second step to estimate the input shares of capital and labour, and to investigate their variability in time. We will describe this approach in the next section, which will be applied to Mongolia's economic situation in the period 1985-2000. The results obtained will be discussed in Section B4.3.

B4.2 Constructing a function for TFP

B4.2.1 Production function

The starting point is a production function of the form

$$Y(t) = A(t)F(K(t), L(t)), \tag{43}$$

where $K(t)$, $L(t)$ and $Y(t)$ denote capital stock, labour and aggregate output at time t respectively, and $A(t)$ is an index of the level of technology at time t. In the literature, $A(t)$ is usually called *total factor productivity* or TFP. In this production function, technology is said to be Hicks neutral or output augmenting. In order to facilitate the analysis in the sequel, the present study assumes that t represents continuous time and that output Y is differentiable in t. Furthermore, F is a function of capital and labour, which is differentiable in both K and L.

Some well-known production functions that belong to the class described by (43) are the Cobb-Douglas production function and the CES-function (constant elasticity of substitution). In Chessa and Schouwstra (2005) it is proved that Y is uniquely described by the generalised form $Y = AK^aL^b$ of the Cobb-Douglas production function, if, and only if, K and L represent ratio scale measurements with different scale units. However, the methods and results of this study are not restricted to specific forms of (43), which also holds for the measurement scales of the inputs. For instance, in applications labour may be weighted by the number of years of schooling as in Cheng (2003), which gives rise to ratio scale values. When labour is quantified

[117] For an overview of methods for sensitivity and uncertainty analysis, see Granger Morgan and Henrion (1990).

as the number of persons employed, such values represent measurements on an absolute scale.[118]

Let $\dot{Y}$ denote the derivative of Y with respect to t and let the time derivatives of A, K and L be denoted in the same way. Then the following expression holds for production function (43) (Barro and Sala-i-Martin, 1995):

$$\frac{\dot{Y}}{Y} = \frac{\dot{A}}{A} + \left(\frac{\partial Y/\partial K}{Y/K}\right)\frac{\dot{K}}{K} + \left(\frac{\partial Y/\partial L}{Y/L}\right)\frac{\dot{L}}{L}. \tag{44}$$

The two terms within brackets in this expression are the output elasticities with respect to capital and labour, which will be shortened to $\alpha_K(t)$ and $\alpha_L(t)$ respectively. The following equality holds under constant returns to scale, for every t:

$$\alpha_K(t) + \alpha_L(t) = 1 \tag{45}$$

(see Chiang (1984) and Barro and Sala-i-Martin (1995) for more details).

Expression (44) will be used in the next subsection in order to quantify $\dot{A}/A$ for different values of $\alpha_K(t)$ and $\alpha_L(t)$ ranging from 0 to 1. When time series data for Y, K and L are available, expression (44) yields a value for the relative growth of technology $\dot{A}(t)/A(t)$, for any $\alpha_K(t)$, $\alpha_L(t) \in [0, 1]$, at every time t in the time series.

The method presented in this appendix, which will be illustrated for Mongolia's economy in the following sections, is applicable to situations where it is difficult to obtain accurate data about the share of rental payments to capital and the share of wage payments to labour in total income. When these data are available, the values of these input shares can be set equal to the corresponding output elasticities, which is valid under the assumption of competitive factor markets. The methodology presented here, however, does not require this situation.

B4.2.2 A TFP function for Mongolia

Cheng (2003, Appendix Table 2) published aggregate yearly values of GDP, capital stock and labour for Mongolia's macroeconomic situation from 1980 until 2001. Cheng (2003, p. 9) expresses labour as the product of total employment and the number of years of schooling of the Mongolian population aged 15 or above, which he uses as a human capital index, due to the lack of data on earnings by educational status. Data are available for two years only: the average number of years of schooling in Mongolia is equal to 7.5 in 1989 and is equal to 8.5 in 1998. Cheng derived human capital index values by assuming education to be a linear function of time. If $t = 0$ is used to denote the year 1980 (the first year in the time series), then human capital q_t in year t can be expressed in terms of the two data points as follows:

[118] See Roberts (1979) for a rigorous treatment of measurement scales and their implications for the meaningfulness of statements in their regard.

$$q_t = (q_{18} - q_9)t/9 + 2q_9 - q_{18}, \tag{46}$$

where q_9 and q_{18} are the human capital values for 1989 and 1998, respectively. Labour L_t in year t is then calculated as

$$L_t = q_t E_t, \tag{47}$$

where E_t denotes the total number of persons employed in year t.

Capital stock is calculated with the conventional perpetual inventory method:

$$K_{t+1} = I_{t+1} + (1-\delta)K_t, \tag{48}$$

where I_t is the level of investment in year t and δ is the rate of depreciation of the capital stock. Usually, capital stock in each successive year is calculated recursively according to (21) by making use of time series data $\{I_t\}$ for investment. As a starting point for calculations, capital stock is assumed to be zero in a certain year (if data are not available such as is the case for Mongolia). The depreciation rate is often assumed to have a value between 0.04 and 0.10. Cheng (2003) assumed that $\delta = 0.06$, which will also be used in this study.[119]

The data of Mongolia's economy will now be used in order to derive an analytic function of TFP in time. For this purpose, the ratio $\dot{Y}(t)/Y(t)$ in (44) – the derivative of GDP divided by GDP $Y(t)$ in year t–will be approximated by the 'central difference' $(Y_{t+1} - Y_{t-1})/2Y_t$, where $Y_t = Y(t)$ for every year t. The same will be done for capital K and labour L. The central difference scheme was chosen because of its higher accuracy compared to the forward and backward schemes (Fried, 1979). Next, the constant returns to scale property will be assumed, so that (45) holds. The output elasticity α_K with respect to capital will be written as α in the sequel. The ratio $\dot{A}/A$ will be approximated by the following quantity $a_\alpha(t)$, for any $\alpha \in [0, 1]$:

$$a_\alpha(t) = \frac{Y_{t+1} - Y_{t-1}}{2Y_t} - \alpha(t)\frac{K_{t+1} - K_{t-1}}{2K_t} - (1-\alpha(t))\frac{L_{t+1} - L_{t-1}}{2L_t}, \tag{49}$$

where t ranges from the year 1981 until the year 2000. By letting α run from 0 to 1, the full range of values of $a_\alpha(t)$ will be obtained for every year t, given time series data on GDP, capital and labour. The result for Mongolia is shown in Figure B4-1.

It is important to note that this result holds irrespective of any relation between α-values for different years. Figure B4-1 is thus the same both for a constant α over time and for any time-dependent function $\alpha(t)$. This graph is a discrete-time representation of the space containing all possible functions $a_\alpha(t)$ that approximate $\dot{A}(t)/A(t)$. A second noteworthy feature of this figure is that TFP decreases from 1985

[119] Cheng assumed that $\delta = 0.06$ but did not give any arguments for this choice. In his calculations of capital stock he used the time series data for investment $\{I_t\}$ since 1980 and furthermore assumed capital stock to be zero in 1959. He performed a sensitivity analysis to test the robustness of his '1959-assumption'.

until 1992, while TFP increases between 1993 and 1997, for every $\alpha \in [0, 1]$. In other words, the increase or decrease of TFP in the period 1985-1997 can be established without knowledge about α (see also Chessa and Schouwstra (2005)).

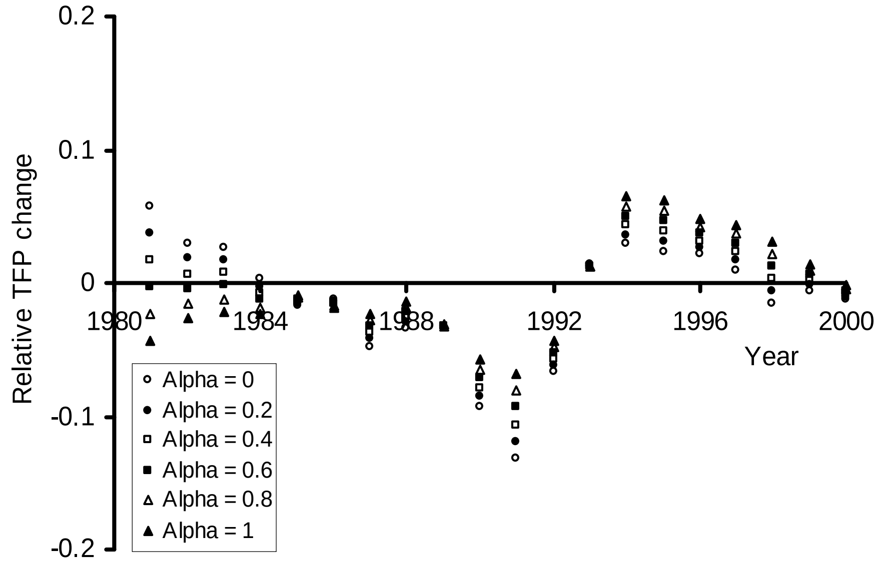

Figure B4-1. Values of $a_\alpha(t)$ for Mongolia, calculated with expression (49), which are used as approximations of $\dot{A}(t)/A(t)$. Values are shown for six values of the input share of capital α.

A third characteristic is that the behaviour of $a_\alpha(t)$ can be subdivided into two parts. A 'fan-like' behaviour seems to emerge until 1984, which, however, will be difficult to analyse because this pattern can only be observed during about four years. This part of the data will be ignored in the rest of this appendix, also because the main interest of this study is focused on the economic growth in Mongolia in the years that precede the transition from a planned to a market economy and on the development in the subsequent years. The economic growth during the period 1985-2000 will therefore be studied. The behaviour of $a_\alpha(t)$ in these years suggests a sinusoidal shape of $\dot{A}(t)/A(t)$ for every α. The following function therefore seems a reasonable candidate for the relative growth of technology in this period:

$$\frac{\dot{A}(t)}{A(t)} = a\sin(c(t-\tau))+b, \quad a,c \geq 0, \tag{50}$$

where τ is a reference year, a is the amplitude of the sine function, c controls the wavelength, while b denotes a general trend, which is taken to be time-independent here. The sine function thus describes oscillations around the trend b. In the next section, a user-interactive method will be presented for estimating the parameters of function (50), which is very simple to use. The method will be illustrated for Mongolia's economy.

B4.2.3 Calculation of TFP-parameters

With an explicit function for $\dot{A}(t)/A(t)$ at hand, classical statistical methods might be proposed for estimating the parameters of this function from time series data, together with input share α. This section proposes a different method, which emphasises a more direct involvement of users of the method and its results. A method for calculating parameter values of function (50) will be outlined, which gives economists possibilities to control the computational process, as it tries to avoid the use of 'black-box' methods. It will turn out to be possible to fully specify function (50) on the basis of $a_{\alpha}(t)$ in Figure B4-1. The method requires a limited amount of input data by users, which have a simple economic interpretation.

An effective and transparent way for calculating the four parameters of (50) is by establishing three roots of $\dot{A}(t)/A(t)$, that is, three years t for which $\dot{A}(t)/A(t) = 0$. The method will first be outlined in general terms and will be illustrated afterwards for Mongolia. For reasons of simplicity, the method is based on the assumption of time-independent input shares, so that $\alpha(t) = \alpha$ for all t. The method consists of the following steps.

1. Identify three roots $t_1 < t_2 < t_3$ of $\dot{A}/A$, such that $\dot{A}(t)/A(t) \neq 0$ for all $t \in (t_1, t_2)$ and for all $t \in (t_2, t_3)$.
2. The difference between the largest and the smallest root is equal to the wavelength of the sinusoidal function, so that $c(t_3 - t_1) = 2\pi$. This yields:

$$c = \frac{2\pi}{t_3 - t_1}.$$

3. If $t_3 - t_2 = t_2 - t_1$, then there is no trend, that is, $b = 0$. Otherwise, for any root t_i, where $i = 1, 2, 3$, the trend parameter satisfies the equality:

$$a \sin(c(t_i - \tau)) + b = 0.$$

4. If $t_3 - t_2 = t_2 - t_1$, then set $\tau = t_1$ when for any $z_1 \in (t_1, t_2)$ and for any $z_2 \in (t_2, t_3)$ one finds $\dot{A}(z_1)/A(z_1) > 0$ and $\dot{A}(z_2)/A(z_2) < 0$. Otherwise, if $\dot{A}(z_1)/A(z_1) < 0$ and $\dot{A}(z_2)/A(z_2) > 0$, then set $\tau = t_2$. If $t_3 - t_2 \neq t_2 - t_1$, then the value of τ will be shifted, which becomes equal to

$$\tau = t_1 - \tfrac{1}{4}\left((t_3 - t_2) - (t_2 - t_1)\right)$$

when $\dot{A}(z_1)/A(z_1) > 0$ and $\dot{A}(z_2)/A(z_2) < 0$, and equal to

$$\tau = t_2 + \tfrac{1}{4}\left((t_3 - t_2) - (t_2 - t_1)\right)$$

when $\dot{A}(z_1)/A(z_1) < 0$ and $\dot{A}(z_2)/A(z_2) > 0$ for any $z_1 \in (t_1, t_2)$ and $z_2 \in (t_2, t_3)$. The values of τ are roots of the sine component in function (50).
5. According to step 3, write b as follows for one of the three roots t_i:

$$b = -a\sin(c(t_i - \tau)).$$

Substituting this expression in function (50) gives:

$$a\{\sin(c(t-\tau)) - \sin(c(t_i - \tau))\}. \tag{51}$$

For uniformly chosen values of α in [0, 1] or a subset of [0, 1], set the sum of the absolute values of (51) over the interval (t_1, t_3) equal to an equivalent version for the values of $a_\alpha(t)$, where t runs through the same interval. For every α, a value a_α for the parameter a results from the following expression:

$$a_\alpha = \frac{\left|\sum_{t\in(t_1,t_2)} a_\alpha(t) - \sum_{t\in(t_2,t_3)} a_\alpha(t)\right|}{\sum_{t\in(t_1,t_3)} \left|\sin(c(t-\tau)) - \sin(c(t_i-\tau))\right|}.$$

6. For every α, substitute a_α in the expression for b in step 5 to obtain a value b_α:

$$b_\alpha = -a_\alpha \sin(c(t_i - \tau)).$$

The amount of variability in the a_α values depends on the behaviour of the difference between $\dot{K}(t)/K(t)$ and $\dot{L}(t)/L(t)$ over t, as can be deduced from (49). If this difference does not indicate a time-dependent behaviour, then there will be little variation in a_α for different values of α. If this is the case, then a can be set equal to the average of the a_α values. Before proceeding with this discussion when the opposite holds (Section 3), steps 1-6 of the above procedure will first be illustrated for Mongolia.

The values of $a_\alpha(t)$ in Figure B4-1 will be used for applying the computational procedure to Mongolia's economy. First, the input share α will be restricted to values between 0.3 and 1. Studies concerning growth in western economies often use $\alpha = ⅓$ (Aiyar and Dalgaard, 2005). Cheng (2003) finds a value of $\alpha = 0.69$ for Mongolia. Heytens and Zebregs (2000) estimated that α was equal to 0.63 in China, an Asian transition economy that may share a number of characteristics with Mongolia. Steps 1-6 are worked out below.

1. Inspection of Figure B4-1 shows that a root of $\dot{A}(t)/A(t)$ is located between 1992 and 1993. With the restriction on the values of α, a second root appears between 1999 and 2000. A third root is more difficult to identify, since the values of $a_\alpha(t)$ before 1985 show a different behaviour than in the period 1985-2000. The following three roots will be set as initial values:

$$t_1 = 1984.5,\ t_2 = 1992.5,\ t_3 = 1999.5.$$

2. These roots imply for the wavelength parameter:

$$c = 2\pi/15.$$

3. Since $t_3 - t_2 \neq t_2 - t_1$, there is a nonzero trend b, which will be obtained from the expression:

$$b = -a \sin\left(\tfrac{2\pi}{15}(t_2 - \tau)\right),$$

where t_2 is arbitrarily chosen from the three roots identified in step 1. Notice that $t_3 - t_2 < t_2 - t_1$ and that $a_\alpha(t) < 0$ for all α, for every $t \in (t_1, t_2)$, so that $b < 0$.
4. Since $a_\alpha(t) < 0$ for all α, for every $t \in (t_1, t_2)$, this implies that τ is equal to

$$\tau = t_2 + \tfrac{1}{4}\left((t_3 - t_2) - (t_2 - t_1)\right) = 1992.25.$$

5. The values for c and τ, and the expression for b in step 3 can now be used to calculate a value a_α for the parameter a, for values of α between 0.3 and 1. We obtain that $0.054 < a_\alpha < 0.056$ for every α, which thus shows little variation. Therefore, we set a equal to the central value of this interval:

$a = 0.055.$

6. Substitution of the values a_α in the expression for b in step 3 shows that b varies only by $2 \cdot 10^{-4}$ as α varies between 0.3 and 1. A similar value as for a in step 5 is computed for b, which yields:

$b = -0.0057.$

The function for $\dot{A}(t)/A(t)$ thus becomes:

$$\frac{\dot{A}(t)}{A(t)} = 0.055 \sin\left(\tfrac{2\pi}{15}(t - 1992.25)\right) - 0.0057. \qquad (52)$$

This function is shown in Figure B4-2. Based on the three roots specified in step 1, the function for relative TFP-growth $\dot{A}(t)/A(t)$ that is derived from the $a_\alpha(t)$ values thus shows a general downward trend of –0.57% per year (step 6). The sine function shows a superposed effect, which first strengthens the downward trend after 1984, but turns positive after 1992, reaching a maximum relative growth of 5.50% – 0.57% = 4.93% in 1996. The sine function captures periodic effects of TFP-development in Mongolia, which are related to the political changes in the Soviet Union. The two major effects captured by this function are the maximum decline of TFP at the end of the 1980s, which coincides with the disintegration of the Soviet Union, and the growth after 1992, showing the effects of reforms during the transition from a planned to a market economy in Mongolia.

The influence of α on the parameter values of function (52) is negligible. From step 5 it follows that a can be fixed quite easily at one value. The small variation in the a_α values is due to the fact that the sum of the variations in the negative $a_\alpha(t)$ values (between 1985 and 1992) and in the positive $a_\alpha(t)$ values (between 1993 and 1999) are approximately the same as α varies, which thus compensate each other, so that the calculated amplitude a in step 5 remains about the same. This may not be the case in other applications. If, consequently, the amplitude values a_α show considerable variation for different values of α, then a should be estimated in a different way. This will be discussed further in Section 3.

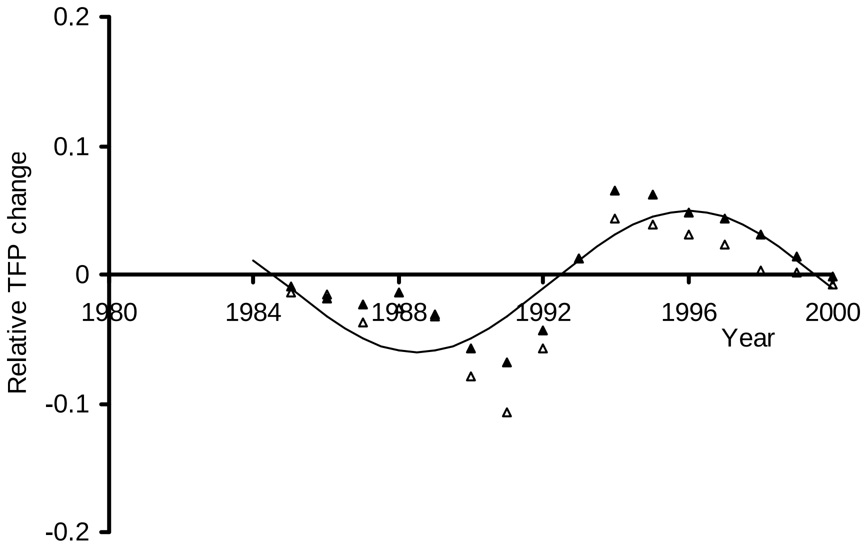

Figure B4-2. Function (52) for $\dot{A}(t)/A(t)$ derived with the computational procedure of Section B4.2.3 (solid line). Also shown are the values of $a_\alpha(t)$ for α = 0.4 (open triangles) and for α = 1 (closed triangles).

B4.2.4 Estimation of input shares

Each of the functions $a_\alpha(t)$ gives an approximation of $\dot{A}(t)/A(t)$ for every α. The sinusoidal structure revealed by these functions made it possible to derive one function for $\dot{A}(t)/A(t)$ based on the identification of three roots of this function, without having to estimate α. An estimate of α can thus be derived separately. In order to do this, a framework is needed, in which the function derived for $\dot{A}(t)/A(t)$ is assigned a precise mathematical meaning.

The central difference schemes used for approximating the derivatives of GDP, capital and labour in time have a theoretical error. Another source of error in these quantities may be inaccuracy of the time series data. Data may be unreliable for different reasons, such as the data acquisition methods used, which may be dated, uncertainty about the exact values of parameters (e.g., the depreciation rate δ for capital in (48)) and the exclusion of the informal sector from the official statistics. In Chessa and Schouwstra (2005), a sensitivity analysis was used to quantify the effects of variations in parameters, such as δ, and in the data values of GDP, capital and labour on TFP-growth in Mongolia. The results showed that variations in GDP have the greatest effect on TFP, while the effects of variations in δ, capital and labour are relatively small.

These findings could be used to propose a statistical framework, in which GDP is considered as a stochastic process that has an error term as its only source of randomness. An estimate of α could be found by minimising the sum of squared errors (SSQ) defined by

$$\sum_{t=1985}^{2000}\left(\frac{Y_{t+1}-Y_{t-1}}{2Y_t}-\frac{\dot{A}(t)}{A(t)}-\alpha\frac{K_{t+1}-K_{t-1}}{2K_t}-(1-\alpha)\frac{L_{t+1}-L_{t-1}}{2L_t}\right)^2, \qquad (53)$$

where $\dot{A}(t)/A(t)$ is function (52).[120] The resulting estimate is α = 0.990 (SSQ = 0.0074). As the parameter values of the function for $\dot{A}(t)/A(t)$ depend on the roots given as input for the computational procedure in Section 2.3, it is worth considering the effect of variations in these roots on the parameter values and α.

As was stated in Section 2.3, there is uncertainty in the location of the smallest root t_1 = 1984.5. If t_1 is shifted backwards by one year, then we obtain α = 0.919 (SSQ = 0.0101). If t_1 is set at the year 1985, then α = 1, which improves the fit of the yearly relative GDP-change considerably with respect to a model with t_1 = 1983.5 (SSQ = 0.0063). The fit with function (52) is slightly worse. Variations in the other two roots do not improve the fits. This brief sensitivity analysis for the location of the roots confirms that α is close to 1.

The fit of the yearly relative GDP-growth data with function (52) for $\dot{A}(t)/A(t)$ is shown in Figure B4-3. This figure shows that the economic development in Mongolia can be subdivided in two distinct periods. The period 1985-1992 shows that the contribution of TFP to GDP, especially during the first five years of this period, is very small. In contrast, the period 1993-2000 shows that TFP-growth largely determines GDP. These observations can be used to gain more understanding in the high value of α. Figure B4-4 shows that capital stock declines during the period 1990-1992, while labour increases. The yearly relative change in GDP is smaller than that of TFP (Figure B4-3); the behaviour of GDP can thus only be explained by a value of α equal to 1 for this period. In the period 1996-2000, the level of capital stock remains about the same, while labour increases. The observation in Figure B4-3 that the relative changes in TFP and GDP are about the same can therefore only be explained by an equivalent value of the input share of capital α. The relative changes in capital and labour show about the same behaviour during the period 1985-1989, which therefore is expected to put little weight in the value of α. Having discussed almost the entire period 1985-2000, these observations lead to the conclusion that α should have a very high value.

[120] It should be noted that an estimate of α can only be obtained when the central differences for the derivatives of capital and labour differ for one or more years. If this is not the case, then $a_\alpha(t)$ is the same for all α, which then is the only characteristic that can be derived from the data.

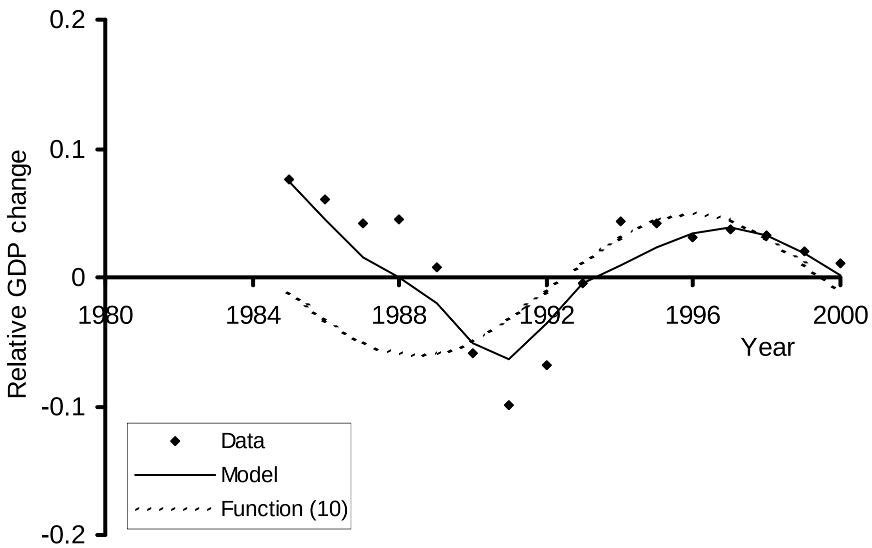

Figure B4-3. Fit to the yearly relative GDP-growth data of Mongolia. The fitted model is the sum of function (52) for $\dot{A}(t)/A(t)$ and of the relative growth contributions from capital and labour, which are shown in Figure B4-4 with $\alpha = 0.990$.

Figure B4-4. Yearly relative growth $\dot{K}(t)/K(t)$ and $\dot{L}(t)/L(t)$ of capital K and labour L for the period 1985-2000 in Mongolia. The derivatives are approximated with central differences.

B4.3 Discussion

The objective of this study was to find a method for quantifying the relative growth of technology, based on time series data for aggregate output, capital and labour. The focus of the application of the method was on situations where estimates of the input shares of capital and labour are unreliable or difficult to obtain.

The main results of this study are threefold: (1) based on a general form of the production function (Hicks neutral), it was shown how a straightforward application of expression (44), between relative output growth and the relative contributions from

TFP and inputs, gives a quantification of the space, or bandwidth, of relative TFP-growth (Figure B4-1); (2) based on this representation of TFP, it was possible to identify an analytic function for relative TFP-growth, which suggests a sinusoidal form for Mongolia; (3) a method has been proposed for calculating the parameters of the TFP-function. The strengths and limitations of these results will now be discussed.

The appeal of result (1) is the simplicity by which it is obtained and its validity. The latter means that Figure B4-1 is obtained with a general form for the production function, with output-augmenting technology, but without any assumption on the relation between output and inputs. Beside Hicks neutrality, the only assumption made was that of constant returns to scale. Within this assumption, the output elasticities or input shares may have an arbitrary behaviour in time.

The inferences about TFP-growth that can be drawn from the functions $a_\alpha(t)$ in Figure B4-1 are interesting in their own right, as they are directly related to a central issue in economic growth studies. In the case of Mongolia, for instance, TFP decreases between 1985 and 1992, while TFP increases between 1993 and 1997, which holds for every value of α between 0 and 1. This means that the $a_\alpha(t)$ values allow strong conclusions to be made about TFP, which is possible prior to any modelling or statistical analysis and estimation. Statements about TFP-growth could be made even stronger by restricting the space of functions $a_\alpha(t)$ to a subset, for example, by adding 'expert knowledge' about α (see Section 2.3). By setting a lower bound for α at 0.4, so that $\alpha \in [0.4, 1]$, the corresponding values of $a_\alpha(t)$ reveal that TFP decreases between 1985 and 1992, that TFP increases between 1993 and 1999, and that TFP decreases in the year 2000, for every α in the restricted interval (Figure B4-2). We thus believe that the derivation of the functions $a_\alpha(t)$ should be the first step in studies of economic growth. Because of the importance of the statements that can be made about TFP, it may be worth, as a topic of future study, to investigate the effects on the $a_\alpha(t)$ functions by considering other types of production functions (e.g., with Harrod neutral or labour augmenting technology) and without the assumption of constant returns to scale.

The final remark is also important in the light of result (2), the analytic form of TFP as a function of time. The functions $a_\alpha(t)$ reveal a common, sinusoidal form for the relative growth of technology in Mongolia during the period 1985-2000. It would be interesting to investigate whether this form also applies to other transition economies. It does not apply to China, however, since the form of the production function estimated in Bairam (1996) for different regions in China shows an exponential growth of technology during the period 1973-1989.

The identification of a common functional form of the relative technological growth in the functions $a_\alpha(t)$ opened possibilities to calculate the parameters of the sinusoidal function from the $a_\alpha(t)$ values. The computational procedure that has been developed for this purpose requires a limited amount of input data from users, which is very easy to interpret. In fact, users are requested to specify three time points at which the relative growth of technology is equal to zero, based on the behaviour of $a_\alpha(t)$. This information is sufficient to calculate exact values of the four parameters of the sinusoidal function by making use of some of its analytical properties.

Beside the properties just mentioned, the computational procedure has some other attractive properties that are worth emphasising. The method enhances the interaction with and the involvement of the user with respect to more conventional approaches, like statistical methods for estimating the parameters. The user has the possibility to control the computational process continuously: once the user sets three time points of zero growth, the consequences in terms of the calculated relative growth function and the minimised sum of squared errors – when α is estimated as well – are immediately obtained.

As the method requires only three zero-growth times as input, the method can be combined with a sensitivity or uncertainty analysis about these time points in a very simple and transparent way, as was illustrated in Section 2.4. Such an analysis enables the user to quantify the variability in the calculated parameters of the sinusoidal function and in α. In addition, insight will be gained in this way into the combination of parameter values that give the best fit with the observed times series of GDP, as was also shown in Section 2.4.

Other interesting characteristics of the method are that no statistical estimation methods are required for calculating the parameters of the sinusoidal technology growth function. In addition, the calculation of these parameters and of α can be carried out separately, when the values a_α for the amplitude a that are obtained for different values of α do not show a significant variation. This turned out to be the case for Mongolia (Section 2.3), but may not hold in other situations. As was noted earlier, the variation in a_α depends completely on the difference between the relative time derivatives of capital and labour. If, for instance, this difference increases or decreases as a function of time, then the contribution of α to $a_\alpha(t)$ in expression (49) becomes larger or smaller with time t. This implies that the values of a_α calculated in step 5 of the procedure in Section 2.3 will depend on α.

In the latter case, the standard procedure of Section 2.3 can be adapted in the following simple way. The user may set a tolerance level for the admissible variation in the a_α values, for instance, in terms of the difference between the maximum and minimum values of a_α. If the tolerance level is not exceeded, then the computational procedure can be applied in the way shown for Mongolia in Section 2.3, so that only α remains to be estimated as in Section 2.4. Otherwise, the standard procedure may be adapted as follows:

i. Perform steps 1-4 of the standard procedure of Section 2.3. This yields values of c and τ, and the relation $b = -a\sin(c(t_i - \tau))$, where t_i is one of the three roots.
ii. Estimate a and α by minimising the sum of least squares in Section 2.4, given the values of c and τ obtained in step i. The function for $\dot{A}(t)/A(t)$ to be used is:

$$a\left\{\sin(c(t-\tau)) - \sin(c(t_i-\tau))\right\}.$$

iii. Calculate $b = -a\sin(c(t_i - \tau))$.

This straightforward adaptation makes it possible to apply the above method in a wide range of situations. For example, it also becomes possible to investigate the effect of a time-dependent input share α on the fit to the yearly relative GDP-growth data in Figure B4-3. In order to illustrate this, let α_1 and α_2 denote the input shares for the periods 1985-1992 and 1993-2000 respectively. With the same three zero-growth time points as specified for the sinusoidal technology growth function in Figure B4-3, we obtain $\alpha_1 = 1$ and $\alpha_2 = 0.451$. This model gives marginal improvements over the model with a time-independent input share (SSQ = 0.00567).

The results in sections 2.3 and 2.4 show that technological growth in Mongolia can be adequately described by a sinusoidal function with a slightly negative trend b. Variations in the roots may give small improvements in the fitted yearly relative GDP-growth data, but the values estimated for α remain very high (greater than 0.9). The value of the input share found here for Mongolia falls in the upper part of the range of values for α found in other studies referred to by Cheng (2003). Cheng found a value of 0.69 based on linear regression, which, as was stated in the introduction of this appendix, makes the erroneous implicit assumption that TFP behaves according to an exponential function of time.

Of course, the applicability of the procedure in Section B4.2.3 and the generalised version above heavily depend on the form of the function for $\dot{A}(t)/A(t)$. Conventional statistical and econometric approaches may have a greater general applicability, but the method presented in this appendix was originally intended to exploit as much as possible the structure of the functions $a_\alpha(t)$ (Figure B4-1). The sinusoidal function that has been proposed for technological growth is valuable in its own right, as it gives more insight into structural and periodic components of technological development in time. For Mongolia, the structural growth component is captured by the trend parameter b and the periodic effects are described by a sine function. The four parameters give a summary description of technological growth. This opens possibilities for making transparent comparisons between the growth of technology in different economies, both at regional and at national level within and between countries.