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Appendix C

Appendix

C.1 Proofs

C.1.1 Proof of Lemma 4.1

Proof. First, consider that the maximum fine $\bar{f}$ is large enough, that is $r(1, V) < \bar{f}$. Since $\frac{r(x,V)}{x} > 0$, a larger fine is required to incentivize the platform to delete content created by higher types. Hence, a fine that results in deleting the content of types $x > x_s$ also yields deleting the content of type $x < x_s$. Therefore the problem of the regulator has a corner solution, and it is either optimal to impose the maximum fine or no fine at all. Second, consider that the maximum fine is not enough to incentivize the platform to delete content of the top users. The above argument still holds why an interior solution cannot be optimal. ■

C.1.2 Proof of Lemma 4.2

Proof. The profit of the platform for type x_s is

$$\pi(x_s, V, R, f_p) = r(x_s, V) - pf_p \tag{C.1}$$

Since the regulator's objective is to deter violations for types $x < x_s$, the optimal fine is

$$f_p^* = \frac{r(x_s, V) + h_s(x_s)}{p} \tag{C.2}$$

Under f_p^* the platform suspends only users with types lower than x_s. ■

C.1.3 Proof of Proposition 4.1

Proof. First, note that since the platform's profit increases in the type of the user while the regulator does not condition the fine on the type of the user, the profit of the platform can be rewritten as

$$\pi(x, f_p) = \int_R r(x, V)g(x)dx + \int_V r(x, V)g(x)dx - \int_V f_p g(x)d(x) \tag{C.3}$$

Where R denotes the region where the firm suspends the user. In that region, the revenue is 0.

The platform deletes the content of the users at the bottom of the type distribution while allowing violations at the top. Denote with x_p the largest type the platform deletes the content of. The platform's problem becomes

$$\max_{x_p} \pi(x, f_p, x_p) = \int_0^{x_p} 0dx + \int_{x_p}^1 r(x, V)g(x)dx - \int_{x_p}^1 f_p g(x)d(x) \tag{C.4}$$

The first order condition yields

$$\frac{\partial \pi(x, f_p, x_p)}{\partial x_p} = -r(x_p, V)g(x_p) + f_p g(x_p) = 0 \tag{C.5}$$

which can be rewritten as

$$r(x_p, V) = f_p. \tag{C.6}$$

The second order condition is also satisfied,

$$\frac{\partial^2 \pi(x, f_p, x_p)}{\partial^2 x_p} = g'(x_p)(f_p - r(x_p, V)) - g(x_p)\frac{\partial r(x_p, V)}{\partial x_p} < 0. \tag{C.7}$$

The interpretation of equation (C.6) is straightforward. The platform deletes the content of types whose contribution to the profit is less than 0, that is, for whom the fine is larger than the revenue from the content. That also means that an increase in fine decreases retention. Since the platform retains the content of the top users, cherry-picking is also increased with the larger fine.

$$x_p^* = r^{-1}(f_p, V) \tag{C.8}$$

■

C.1.4 Proof of Proposition 4.2 and 4.3

Proof. The problem of the regulator writes as

$$\max_{f_p} W(x, f_p) = \int_0^{x_p(f_p)} 0dx + \int_{x_p(f_p)}^1 [r(x, V) + h_s(x)]g(x)dx \tag{C.9}$$

The first order condition reads

$$\frac{\partial W(x_p^* f_p)}{\partial f_p} = g(x_p^*)\left[r(x_p^*, V) + h_p(x_s^*)\right]\frac{\partial x_p^*}{\partial f_p} = 0 \tag{C.10}$$

with $\frac{\partial x_p^*}{\partial f_p} = \frac{1}{r'(r^-1(f_p,V),V)}$. Hence to satisfy the first order condition for interior maximum,

$$r(x_p^*, V) = -h_s(x_p^*) \tag{C.11}$$

Combining (C.11) and (C.6) yields

$$f_p^* = r(x_s, V) \tag{C.12}$$

Note that the type for which the benefit and harms are equal is denoted by x_s. Hence it follows that the fine, f_p is set such that $x_p^* = x_s$

To check the sufficient condition the second derivative yields

$$\begin{aligned}\frac{\partial^2 W(x_p^*, f_p)}{\partial^2 f_p} &= \frac{\partial g(x_p^*)}{\partial x_p^*}\frac{\partial x_p^*}{\partial f_p}\left[r(x_p^*, V) + h_s(x_p^*)\right]\frac{\partial x_p^*}{\partial f_p} \\ &+ g(x_p^*)\left[\frac{\partial r(x_p^*, V)}{\partial x_p^*} + \frac{\partial h_s(x_p^*)}{\partial x_p^*}\right]\frac{\partial x_p^*}{\partial f_p}\frac{\partial x_p^*}{\partial f_p} \\ &+ g(x_p^*)\left[r(x_p^*, V) + h_s(x_p^*)\right]\frac{\partial^2 x_p^*}{\partial^2 f_p} \leq 0\end{aligned} \tag{C.13}$$

With $r(x_p^*, V) = h_s(x_p^*)$ the second order condition also becomes 0. However, inspecting the objective function yields a clear conclusion. If the violation is efficient for high types, $f_p = r(x_s, V)$ is the maximum of the objective. Low types for whom violation is inefficient are deterred, while high types are not. However, if violations of high types are inefficient, the above fine yields a minimum. Hence, the optimal fine has a corner solution, either allowing all users to violate or none of them. It depends on whether the platform generates social value if all users violate on the platform

$$W(x, f_p) = \int_0^1 r(x, V) + h_s(x)g(x)dx \geq 0 \tag{C.14}$$

■

C.1.5 Proof of Lemma 4.3

Proof. Given the information structure, the users can calculate the threshold on types, $x_p(f_p)$ the platform applies for retaining content. Hence, they correctly anticipate that if their type is lower than $r^{-1}(f_p, V)$ the firm would punish them. Therefore, in equilibrium, users self-select to be compliant if $x < r^{-1}(f_p, V)$ and to be violators if $x \geq r^{-1}(f_p, V)$. ■

C.1.6 Proof of Lemma 4.4

Proof.

$$\pi(x, f_p) = \int_V r(x, V)g(x)dx - n\int_V g(x)f_p d(x). \tag{C.15}$$

The first order condition reads

$$r(x_p, V) = nf_p \tag{C.16}$$

It follows that the larger n is, the higher the choice of the platform x_p is.

■

C.1.7 Proof of Lemma 4.5

Proof. Consider that $f_2 \geq f_1$. Take any choice of x_1 and x_2. By setting $V_p' = V_1 + V_2$, the platform's profit is always larger under full cherry picking. Hence, it follows that the regulator chooses $f_1 > f_2$. ■

C.1.8 Proof of Proposition 4.5

Proof. To simplify notation, I use $r(x, V) = r(x)$. Take a threshold y such that the revenue of the platform is equal under full cherry picking and decreased cherry-picking, that is

$$\int_0^{x_1} r(x)g(x)dx + \int_{x_2}^1 r(x)g(x)dx = \int_{x_2-y}^1 r(x)g(x)dx. \tag{C.17}$$

Hence y can be calculated from

$$\int_0^{x_1} r(x)g(x)dx = \int_{x_2-y}^{x_2} r(x)g(x)dx \tag{C.18}$$

The right handside is decreasing in y. The larger $r(x_2)$ is, that is equivalent the steeper the function $r(x)$ is, the larger y becomes. In other words, the more incentive the platform has to cherry pick, the smaller $x_2 - y$ is.

Intuitively, the larger the difference in the platform's revenue between high and low types, the larger y is, meaning a smaller increase in the violating area. Mathematically, since $r'(x) > 0$, $\Pr(x < x_2 - y) > \Pr(x_1 < x < x_2)$ that is

$$G(x_2 - y) > G(x_2) - G(x_1). \tag{C.19}$$

Moreover, $\Pr(x > x_2 - y) > \Pr(x > x_2)$, that is

$$G(x_2) > G(x_2 - y) \tag{C.20}$$

The probability of being fined with f_1 is $\Pr(f_1|\text{Fully cherry-picking}) = 2G(x_2 - y)(1 - G(x_2 - y))$ and $\Pr(f_1|\text{Decreased cherry-picking}) = 2(G(x_2) - G(x_1))(1 - G(x_2))$ under full-cherry picking and decreased cherry picking, respectively.

It follows from inequality (C.19) (C.20) that

$$\Pr(f_1|\text{Fully cherry-picking}) > \Pr(f_1|\text{Decreased cherry-picking}). \tag{C.21}$$

Similarly, $\Pr(f_3|\text{Fully cherry-picking}) = (1-G(x_2-y))^2$ and $\Pr(f_3|\text{Decreased cherry-picking}) = (G(x_1) + 1 - G(x_2))^2$. It follows form above that

$$\Pr(f_3|\text{Fully cherry-picking}) < \Pr(f_3|\text{Decreased cherry-picking}) \tag{C.22}$$

The platform may pay f_2 only under decreased cherry-picking.

Comparing the profit of the platform under the two strategies, (4.21) and (4.22) the platform prefers decreased-cherry picking if

$$\begin{aligned}
&(1 - G(x_2 - y))^2 f_3 + 2G(x_2 - y)(1 - G(x_2 - y)) f_1 \\
&\qquad\qquad \geq \\
&(G(x_1) + 1 - G(x_2))^2 f_3 + 2(G(x_2) - G(x_1))(1 - G(x_2)) f_1 - 2G(x_1)(G(x_2) - G(x_1)) f_2
\end{aligned} \tag{C.23}$$

.

Combining (C.23) with inequalities (C.21) and (C.22) yields that the regulator sets $f_1 > f_3$.

■

C.1.9 Proof of Proposition 4.6

Proof. The regulator's problem can be written as

$$\max_{f_1,f_2,f_3} W = \int_0^{x_p^*} r(x)g(x)dx + \int_0^{x_p^*} h_s g(x)dx \tag{C.24}$$

s.t.

$$\pi_1(x_p^*) \geq \pi_2(x_p'^*) \tag{C.25}$$

$$f_1, f_2, f_3 \leq \bar{f} \tag{C.26}$$

Under $\pi_1(x_p)$, the first order condition for the optimal cutoff point of the platform is

$$\frac{\partial \pi_1(x_p)}{\partial x_p} = r(x_p)g(x_p) - 2f_3 G(x_p)g(x_p) - 2f_2(1-2G(x_p))g(x_p) = 0 \tag{C.27}$$

and second order condition

$$\begin{aligned}\frac{\partial^2 \pi_1(x_p)}{\partial x_p^2} &= g'(x_p)[r(x_p) - 2f_3 G(x_p)g(x_p) - 2f_2(1-2G(x_p))g(x_p)] \\ &\quad + g(x_p)[r'(x_p) - 2g(x_p)f_3 + 2g(x_p)f_2] < 0\end{aligned} \tag{C.28}$$

Under $\pi_2(x_p')$, the first order condition for the optimal cutoff point is

$$\frac{\partial \pi_2(x_p')}{\partial x_p'} = -r(x_p')g(x_p') - 2f_1(1-2G(x_p'))g(x_p') + 2f_3(1-G(x_p'))g(x_p') = 0 \tag{C.29}$$

Increasing f_1 decreases the optimal choice $x_p'^*$, while increasing f_3 decreases it.

The second order condition is

$$\begin{aligned}\frac{\partial^2 \pi_2(x_p')}{\partial^2 x_p'^2} &= g'(x_p')[-r(x_p) - 2f_1(1-2G(x_p')) + 2f_3(1-G(x_p'))] \\ &\quad + g(x_p')[-r'(x_p') + 2g(x_p)f_1 - 2g(x_p)f_2] < 0\end{aligned} \tag{C.30}$$

The Lagrangian of the regulator is

$$\mathcal{L} = \int_0^{x_p^*} r(x)g(x)dx + \int_0^{x_p^*} h_s g(x)dx - \mu_0(\pi_2(x_p'^*) - \pi_1(x_p^*)) - \mu_1(f_1 - \bar{f}) - \mu_2(f_2 - \bar{f}) - \mu_3(f_3 - \bar{f}) \tag{C.31}$$

The first order conditions for an interior solution with $i = \{1, 2, 3\}$

$$\frac{\partial \mathcal{L}}{\partial f_i} = g(x_p^*)[r(x_p^*) + h(x_p^*)]\frac{\partial x_p^*}{\partial f_i} - \mu_0 \left(\frac{\partial \pi_2(x_p'^*)}{\partial x_p'^*}\frac{\partial x_p'^*}{\partial f_i} - \frac{\partial \pi_1(x_p^*)}{\partial x_p^*}\frac{\partial x_p^*}{\partial f_i} \right) - \mu_i = 0 \tag{C.32}$$

with complementary slackness conditions

$$\mu_0(\pi_2(x_p'^*) - \pi_1(x_p^*)) = 0 \tag{C.33}$$

$$\mu_i(f_i - \bar{f}) = 0 \tag{C.34}$$

First, note that f_1 only decreases the profit under π_2 but does not decrease the objective function W. Hence, by setting f_1 high the regulator can shift the incentives of the platform from cherry-picking to the higher social welfare option with profit denoted by π_1. f_2 only decreases the platform's profit under π_1, therefore it is optimal to set $f_2 = 0$. Substituting $x_s = x_p^*$ into (C.27) yields

$$r(x_s) = 2f_3 G(x_s) \tag{C.35}$$

or

$$f_3 = \frac{r(x_s)}{2G(x_s)} \tag{C.36}$$

Which is the first best value of f_3. However, this may not be achievable. Proposition 4.5 establishes that $f_1 > f_3$ and

$$\Pr(f_3|\text{Fully cherry-picking}) < \Pr(f_3|\text{Decreased cherry-picking}) \tag{C.37}$$

Since f_3 is constrained by f_1, the regulator may not be able to set f_3 high enough since that would countereffect the impact of f_1 on cherry-picking. From (C.27) it follows that a

lowering f_3 yields a higher threshold, x_p^* and hence under deterrence occurs. Overdeterence is not possible, since if f_3 is too high, the regulator would decrease it to the first best level, which must also satisfy (C.25).

■

C.1.10 Proof of Proposition 4.7

Proof. The platform's expected profit when all agents violate on the platform is

$$\pi(x, f_p, q_V) = \int_V q_V r(x, V) g(x) dx + \int_V (1 - q_V) r(x, C) g(x) dx - \int_{V_p} q_V g(x) f_p d(x) \quad \text{(C.38)}$$

As established in the previous section, the platform follows a threshold strategy, that is the platform retains the content only if the agent's type is high enough. Hence, the objective reads similar to (C.4).

$$\pi(x, f_p, q_V) = \int_{x_p(q_V)}^1 q_V r(x, V) g(x) dx + \int_{x_p(q_v)}^1 (1 - q_V) r(x, C) g(x) dx - \int_{x_p(q_V)}^1 q_V g(x) f_p d(x) \quad \text{(C.39)}$$

The first order condition writes

$$-q_V r(x_p, V) - (1 - q_V) r(x_p, C) + q_V f_p = 0 \quad \text{(C.40)}$$

Since $r(x, V) > r(x, C)$, the better the technology of the firm, represented by q_V, the higher the threshold, $x_p^*(q_V)$ becomes. The better technology means the firm is more certain than agent is violating therefore only contents of high types are retained.

Next, consider that all agents are compliant on the platform.

$$\pi(x, f_p, q_C) = \int_C q_C r(x, C) g(x) dx + \int_C (1 - q_C) r(x, V) g(x) dx - \int_C (1 - q_C) g(x) f_p d(x) \quad \text{(C.41)}$$

As established before, the firm follows a threshold strategy with compliants too. If the

the type of the agent is high enough, the platform retains her content.

$$\pi(x, f_p, q_C) = \int_{x_p(q_c)}^{1} q_C r(x, C) g(x) dx + \int_{x_p(q_c)}^{1} (1 - q_C) r(x, V) g(x) dx - \int_{x_p(q_c)}^{1} (1 - q_C) g(x) f_p d(x) \tag{C.42}$$

With first order condition

$$-q_C r(x_p, C) - (1 - q_C) r(x_p, V) + (1 - q_C) f_p = 0 \tag{C.43}$$

The threshold $x_p^*(q_C)$ becomes smaller the better the technology is. That is, the more certain the platform is that the users are complying with the rules, the more content the platform retains.

Finally, consider that with z probability all agents are violent, and with $1 - z$ probability all agents are compliant. [1] The platform's objective is then

$$\begin{aligned}
&\pi(x, f_p, q_V, q_C, z) = \\
&z \left[\int_{x_p(q_V)}^{1} q_V r(x, V) g(x) dx + \int_{x_p(q_v)}^{1} (1 - q_V) r(x, C) g(x) dx + - \int_{x_p(q_V)}^{1} q_V g(x) f_p d(x) \right] \\
&(1 - z) \left[\int_{x_p(q_c)}^{1} q_C r(x, C) g(x) dx + \int_{x_p(q_c)}^{1} (1 - q_C) r(x, V) g(x) dx - \int_{x_p(q_c)}^{1} (1 - q_C) g(x) f_p d(x) \right].
\end{aligned} \tag{C.44}$$

The objective yields identical first order conditions to (C.40) and (C.43), hence leads to the same result as previously. ■

[1] Similar is if z proportion of the agents are violent.