



UvA-DARE (Digital Academic Repository)

Higher-twist quark-mass contributions to deep-inelastic scattering

Boer, D.; Tangerman, R.D.

Published in:
Physics Letters B

DOI:
[10.1016/0370-2693\(96\)00541-2](https://doi.org/10.1016/0370-2693(96)00541-2)

[Link to publication](#)

Citation for published version (APA):

Boer, D., & Tangerman, R. D. (1996). Higher-twist quark-mass contributions to deep-inelastic scattering. *Physics Letters B*, 381, 305. DOI: 10.1016/0370-2693(96)00541-2

General rights

It is not permitted to download or to forward/distribute the text or part of it without the consent of the author(s) and/or copyright holder(s), other than for strictly personal, individual use, unless the work is under an open content license (like Creative Commons).

Disclaimer/Complaints regulations

If you believe that digital publication of certain material infringes any of your rights or (privacy) interests, please let the Library know, stating your reasons. In case of a legitimate complaint, the Library will make the material inaccessible and/or remove it from the website. Please Ask the Library: <http://uba.uva.nl/en/contact>, or a letter to: Library of the University of Amsterdam, Secretariat, Singel 425, 1012 WP Amsterdam, The Netherlands. You will be contacted as soon as possible.



ELSEVIER

18 July 1996

PHYSICS LETTERS B

Physics Letters B 381 (1996) 305–310

Higher-twist quark-mass contributions to deep-inelastic scattering

D. Boer, R.D. Tangerman

National Institute for Nuclear Physics and High Energy Physics (NIKHEF), P.O. Box 41882, NL-1009 DB Amsterdam, The Netherlands

Received 27 March 1996

Editor: P.V. Landshoff

Abstract

In this letter we extend the factorization procedure of the deep-inelastic hadron tensor, proposed by Qiu, to include non-zero quark masses. The manifest gauge invariance of both soft and hard parts is preserved. Using a so-called spurion to generate the quark-mass terms, the simple parton-model interpretation is also kept. The calculation of the deep-inelastic transverse-spin structure function g_2 is used to illustrate the algorithm.

The masses of the light quarks are small as compared to the typical hadronic mass scales. Therefore, in most calculations of deep-inelastic scattering (DIS) structure functions they are neglected. In some cases, however, this is not allowed. A notorious example is g_2 of a free quark target at tree-level [1]. In that case the twist-three quark-mass term exactly cancels the twist-two contribution, yielding $g_2 = 0$. In general, if the ratio m/M is not negligible, one is forced to keep the quark-mass contributions.

Let us recapitulate some historic facts. In Ref. [2] Politzer suggests a diagrammatical approach to unravel the power corrections in $1/Q$ for a wide class of hard scattering processes. This method was fully employed for the deep-inelastic scattering process by Ellis, Furmański, and Petronzio (EFP) in Ref. [3], in which they calculate the twist-four corrections to the unpolarized structure functions. Thereto they factorize the diagrams in hard, i.e. perturbative, and soft, i.e. non-perturbative, parts. An essential ingredient in their calculation is the expansion of the parton momenta around the target momentum direction, called

the *collinear expansion*. Using low-energy Ward identities they show that in all matrix elements the gluon field combines with a derivative to precisely the covariant derivative. Color gauge invariance is then manifest upon inclusion of so-called link operators. Electromagnetic gauge invariance of the hard scattering parts is not manifest in their approach. Matrix elements with a definite parton number contribute at different orders in $1/Q$.

It was Qiu in Ref. [4] who showed that, if one uses the concept of the *special propagator*, one can factorize such that a complete separation of the hard parts between different orders in $1/Q$ can be obtained. Hence, the electromagnetic gauge invariance of these hard parts is manifest. Moreover, only matrix elements with a fixed number of partons contribute to a particular order in $1/Q$. This fact also enables a clear parton-model interpretation of the higher-twist terms.

Both EFP and Qiu neglect quark masses. Our goal is to include them in such a fashion that both color and electromagnetic gauge invariance remain manifest. The basics of the method are as follows: in addi-

tion to the collinear expansion, employed by EFP, we include a quark-mass expansion. Qiu's factorization can be extended as well. In order to preserve the parton interpretation we employ a *spurion* particle [5], which couples only to the fermions. The spurions will generate the mass contributions. One maintains the three advantages of combining EFP and Qiu's techniques, namely, (1) the use of color gauge invariant matrix elements only, (2) manifest electromagnetic gauge invariance of the hard scattering parts, and (3) a clear parton interpretation at all times, also beyond leading order, i.e., each order of power suppression means taking into account correlation functions with one parton more, where only the *good* fields [6] contribute.

The algorithm is best explained by applying it to a particular case, therefore we look at polarized DIS up to and including order $1/Q$ (see Ref. [7] and references therein). The EFP approach in this context was first applied by Efremov and Teryaev [8].

The starting point is the familiar diagrammatic expansion of the forward scattering amplitude $T^{\mu\nu}$ (we consider only quarks of one flavor for the moment)

$$\begin{aligned} T^{\mu\nu} = & \int d^4k \operatorname{Tr} [S^{\mu\nu}(k; m)\Gamma(k)] \\ & + \int d^4k_1 d^4k_2 \operatorname{Tr} [S_\alpha^{\mu\nu}(k_1, k_2; m)\Gamma_\alpha^\alpha(k_1, k_2)] \\ & + \dots, \end{aligned} \quad (1)$$

keeping only the terms contributing up to order $1/Q$. Here, S and Γ are the hard and soft scattering parts, respectively. We will not consider QCD corrections, so the hard parts consist of the forward parton-photon scattering tree graphs which are one-particle-irreducible after shrinking the two photon vertices to a point. We indicate explicitly their dependence on the parton momenta and masses, but not on the photon momentum q ($q^2 = -Q^2$). The soft parts are defined as

$$\Gamma_{ij}(k) = \int \frac{d^4z}{(2\pi)^4} e^{ik \cdot z} \langle P, S | T [\bar{\psi}_j(0)\psi_i(z)] | P, S \rangle, \quad (2)$$

$$\begin{aligned} \Gamma_{Aij}^\alpha(k_1, k_2) = & \int \frac{d^4z}{(2\pi)^4} \frac{d^4z'}{(2\pi)^4} e^{ik_1 \cdot z} e^{i(k_2 - k_1) \cdot z'} \\ & \times \langle P, S | T [\bar{\psi}_j(0)gA^\alpha(z')\psi_i(z)] | P, S \rangle. \end{aligned} \quad (3)$$

Note that we have included a color identity and g times t^a , respectively, from the hard into the soft parts. Thereby the hard parts effectively become QED graphs with unit charge [3].

The nucleon is characterized by its momentum, satisfying $P^2 = M^2$, and its spin vector, satisfying $S \cdot P = 0$ and $S^2 = -1$. We make a Sudakov decomposition of the relevant vectors with respect to two light-like vectors p and n , satisfying $p \cdot n = 1$,

$$q = -x_B p + \frac{Q^2}{2x_B} n, \quad (4)$$

$$P = p + \frac{M^2}{2} n, \quad (5)$$

$$S = \frac{\lambda}{M} \left(p - \frac{M^2}{2} n \right) + S_T, \quad (6)$$

$$k = xp + \frac{k^2 - k_T^2}{2x} n + k_T. \quad (7)$$

Here, $x_B = Q^2/(2P \cdot q)$ is the Bjorken variable, λ is the nucleon's helicity, S_T its transverse spin. Also, x is the quark's longitudinal momentum, k_T its transverse momentum. Since target mass corrections are $1/Q^2$ suppressed, we ignore them.

The next step is performing a collinear and mass expansion of the hard parts (keeping only relevant terms):

$$\begin{aligned} S^{\mu\nu}(k; m) = & S^{\mu\nu}(xp; 0) \\ & + (k - xp)^\alpha \frac{\partial S^{\mu\nu}(k; m)}{\partial k^\alpha} \Big|_{k=xp, m=0} + m \frac{\partial S^{\mu\nu}(k; m)}{\partial m} \Big|_{k=xp, m=0} \\ & + \dots, \end{aligned} \quad (8)$$

$$S_\alpha^{\mu\nu}(k_1, k_2; m) = S_\alpha^{\mu\nu}(x_1 p, x_2 p; 0) + \dots \quad (9)$$

We use the following Ward identities to rewrite the partial derivatives:

$$\frac{\partial S^{\mu\nu}(k; m)}{\partial k^\alpha} = S_\alpha^{\mu\nu}(k, k; m), \quad (10)$$

$$\frac{\partial S^{\mu\nu}(k; m)}{\partial m} = S_{\text{spur}}^{\mu\nu}(k, k; m), \quad (11)$$

where the right-hand side of Eq. (10) is effectively obtained from $S^{\mu\nu}(k; m)$ by insertion of a zero-momentum gluon, with coupling $i\gamma_{ij}^a$ to the fermions. The right-hand side of Eq. (11) follows from the insertion of a zero-momentum (scalar) spurion [5]

(denoted by a dashed line) which couples to the fermions through the vertex $-i\mathbf{1}_{ij}$. Inserting the Ward identities, one arrives at

$$\begin{aligned}
 T^{\mu\nu} &= \int dx \text{Tr} [S^{\mu\nu}(xp;0)\Gamma(x)] \\
 &+ \int dx_1 dx_2 \text{Tr} [S_{\alpha}^{\mu\nu}(x_1p, x_2p;0)\omega^{\alpha\beta}\Gamma_D^{\beta}(x_1, x_2)] \\
 &+ \int dx_1 dx_2 \text{Tr} [S_{\text{spur}}^{\mu\nu}(x_1p, x_2p;0)\Gamma_m(x_1, x_2)] \\
 &+ \dots, \tag{12}
 \end{aligned}$$

where the projector $\omega^{\alpha\beta} = g^{\alpha\beta} - p^{\alpha}n^{\beta}$. Note that $A^{\alpha} = \omega^{\alpha\beta}A^{\beta}$ if one uses the light-cone gauge $n \cdot A = 0$. The soft parts read $(iD = i\partial + gA)$

$$\Gamma_{ij}(x) = \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P, S | T [\bar{\psi}_j(0)\psi_i(\lambda n)] | P, S \rangle, \tag{13}$$

$$\begin{aligned}
 \Gamma_{Dij}^{\alpha}(x_1, x_2) &= \int \frac{d\lambda}{2\pi} \frac{d\eta}{2\pi} e^{i\lambda x_1} e^{i\eta(x_2-x_1)} \\
 &\times \langle P, S | T [\bar{\psi}_j(0)iD^{\alpha}(\eta n)\psi_i(\lambda n)] | P, S \rangle, \tag{14}
 \end{aligned}$$

$$\begin{aligned}
 \Gamma_{mij}(x_1, x_2) &= \int \frac{d\lambda}{2\pi} \frac{d\eta}{2\pi} e^{i\lambda x_1} e^{i\eta(x_2-x_1)} \\
 &\times \langle P, S | T [\bar{\psi}_j(0)m\psi_i(\lambda n)] | P, S \rangle \\
 &= m \delta(x_2 - x_1) \Gamma_{ij}(x_1). \tag{15}
 \end{aligned}$$

One can restore color gauge invariance of these definitions by introducing path-ordered exponentials called link operators. In the gauge $n \cdot A = 0$, these links become unity, if the paths are chosen along the n -direction. In Ref. [9] it is shown that the gluons in the p -direction, i.e., of the form $(n \cdot A)p$, can be pulled into the two-parton amplitude, generating a link operator, showing that the gauge $n \cdot A = 0$ is inessential and color gauge invariance can always be achieved.

The first term in Eq. (12) contributes to order $(1/Q)^0$ and higher, whereas the latter terms contribute to order $(1/Q)^1$ and higher. The new ingredient due to Qiu [4] is basically the splitting of the different terms into parts which contribute at a specific order in $1/Q$. Let us illustrate how this can be done. The Dirac trace in the first term in Eq. (12) can be Fierz decomposed according to

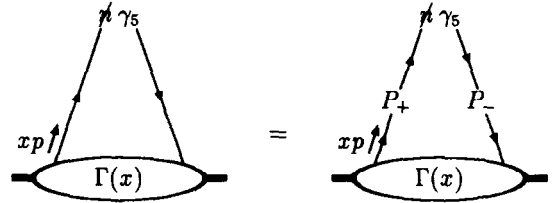


Fig. 1. The leading axial-vector projection of $\Gamma(x)$.

$$\begin{aligned}
 &\text{Tr} [S^{\mu\nu}(xp;0)\Gamma(x)] \\
 &= \frac{1}{4} \text{Tr} [S^{\mu\nu}(xp;0)\gamma_{\rho}] \text{Tr} [\gamma^{\rho}\Gamma(x)] \\
 &+ \frac{1}{4} \text{Tr} [S^{\mu\nu}(xp;0)\gamma_5\gamma_{\rho}] \text{Tr} [\gamma^{\rho}\gamma_5\Gamma(x)], \tag{16}
 \end{aligned}$$

where we used the fact that in the hard part $m = 0$, such that chirality is conserved. The vector term contributes to the unpolarized scattering only, so we discard it. We make a Sudakov decomposition of the axial-vector projection of the soft part,

$$\begin{aligned}
 \text{Tr} [\gamma^{\rho}\gamma_5\Gamma(x)] &= p^{\rho} \text{Tr} [\not{n}\gamma_5\Gamma(x)] \\
 &+ \text{Tr} [\gamma_T^{\rho}\gamma_5\Gamma(x)] + n^{\rho} \text{Tr} [\not{p}\gamma_5\Gamma(x)], \tag{17}
 \end{aligned}$$

where $\gamma_T^{\rho} = g_T^{\rho\sigma}\gamma_{\sigma} \equiv (g^{\rho\sigma} - p^{\rho}n^{\sigma} - n^{\rho}p^{\sigma})\gamma_{\sigma}$. The only dimensionful quantity in the hard part is Q , whereas the soft parts are assumed not to contain large scales. This, in combination with the fact that p has dimension 1, and n dimension -1 , leads to the observation that the first term is the leading one, while the second and third are $1/Q$ and $1/Q^2$ suppressed, respectively, so we discard the last one.

Consider the leading $\not{n}\gamma_5$ trace first. Introducing the projectors $P_+ = \not{p}\not{n}/2$ and $P_- = \not{n}\not{p}/2$, which project on ‘good’ and ‘bad’ quark fields, respectively [6], and using the relations

$$\not{n} = P_- \not{n} = \not{n} P_+, \tag{18}$$

it can be diagrammatically represented as in Fig. 1. Consider next the trace of $\Gamma(x)$ with $\gamma_T\gamma_5$, which we write as $(\not{p}\gamma_T\gamma_5\not{n} + \not{n}\gamma_T\gamma_5\not{p})/2$. Again, we can multiply the \not{n} with a P_+ on the right, or with a P_- on the left. For the \not{p} we use the following relations, which follow from the equations of motion $i\not{p}\psi = m\psi$ [2],

$$\begin{aligned}
 \Gamma(x)\not{p} &= \int dx_2 [\Gamma_D^{\beta}(x, x_2)\omega^{\alpha\beta}(i\gamma_{\alpha}) \\
 &+ \Gamma_m(x, x_2)(-i\mathbf{1})] \frac{i\not{p}}{2x}, \tag{19}
 \end{aligned}$$

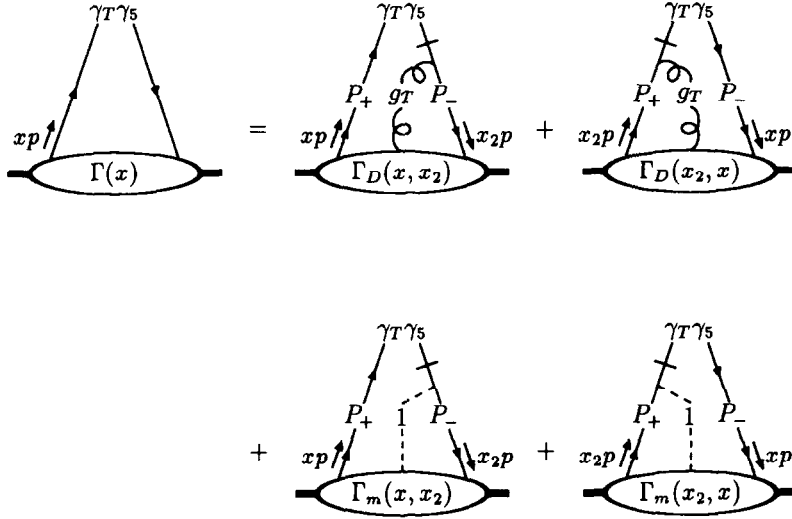


Fig. 2. The sub-leading axial-vector projection of $\Gamma(x)$.

$$\not{p} \Gamma(x) = \not{p} \frac{i\not{h}}{2x} \int dx_2 [(i\gamma_\alpha) \omega^\alpha_\beta \Gamma_D^\beta(x_2, x) + (-i\mathbf{1}) \Gamma_m(x_2, x)]. \quad (20)$$

Finally, we use

$$\omega^\alpha_\beta \gamma_\alpha \not{h} = P_- g_T^\alpha \gamma_\alpha \not{h}, \quad (21)$$

$$\not{h} \gamma_\alpha \omega^\alpha_\beta = \not{h} \gamma_\alpha g_T^\alpha P_+, \quad (22)$$

along with Eq. (18), to project out the good quark and good (transverse) gluon fields. Putting the pieces together, we can write the trace diagrammatically as a sum of four diagrams; two gluon and two spurion diagrams. They are depicted in Fig. 2. We have used the special propagator [6,4]

$$j \xrightarrow{k} \text{---} i = \frac{i\not{h}_{ij}}{2k \cdot n}. \quad (23)$$

The essence of the above derivation is that an \not{h} pulls out a good quark (or anti-quark) from the soft part, whereas a \not{p} pulls out a special propagator, a good quark, along with a good gluon or (for non-zero quark mass) spurion. Note that for the last trace in Eq. (17) this implies the inclusion of four-parton amplitudes, leading to $1/Q^2$ suppression.

In order to undo the Fierz decomposition, Eq. (16), we use the following relations:

$$P_- \not{p} \not{h} P_+ = P_- \gamma^\rho P_+, \quad (24)$$

$$P_- \not{h} \gamma_T^\rho P_+ = P_- \not{h} \gamma^\rho P_+, \quad (25)$$

$$P_- \gamma_T^\rho \not{h} P_+ = P_- \gamma^\rho \not{h} P_+. \quad (26)$$

Returning to the second and third terms in Eq. (12), from dimensional arguments one can again infer that only good fields contribute at leading order. So we may include projectors in between the hard and soft parts. The crucial step in the Qiu factorization is to absorb everything above the projectors into the upper, hard, part (see Figs. 1 and 2). For the leading order this has no consequences. At sub-leading order, however, the modified hard gluonic and spurionic parts read

$$H_\alpha^{\mu\nu}(x_1 p, x_2 p; 0) = S_\alpha^{\mu\nu}(x_1 p, x_2 p; 0) + i\gamma_\alpha \frac{i\not{h}}{2x_1} S^{\mu\nu}(x_1 p; 0) + S^{\mu\nu}(x_2 p; 0) \frac{i\not{h}}{2x_2} i\gamma_\alpha, \quad (27)$$

$$H_{\text{spur}}^{\mu\nu}(x_1 p, x_2 p; 0) = S_{\text{spur}}^{\mu\nu}(x_1 p, x_2 p; 0) + (-i\mathbf{1}) \frac{i\not{h}}{2x_1} S^{\mu\nu}(x_1 p; 0) + S^{\mu\nu}(x_2 p; 0) \frac{i\not{h}}{2x_2} (-i\mathbf{1}). \quad (28)$$

The treatment of the spurionic part is the new result of this paper. The leading and sub-leading forward scattering amplitudes become

$$T_{\text{twist-2}}^{\mu\nu} = \int dx \text{Tr} [P_- S^{\mu\nu}(xp; 0) P_+ \Gamma(x)], \quad (29)$$

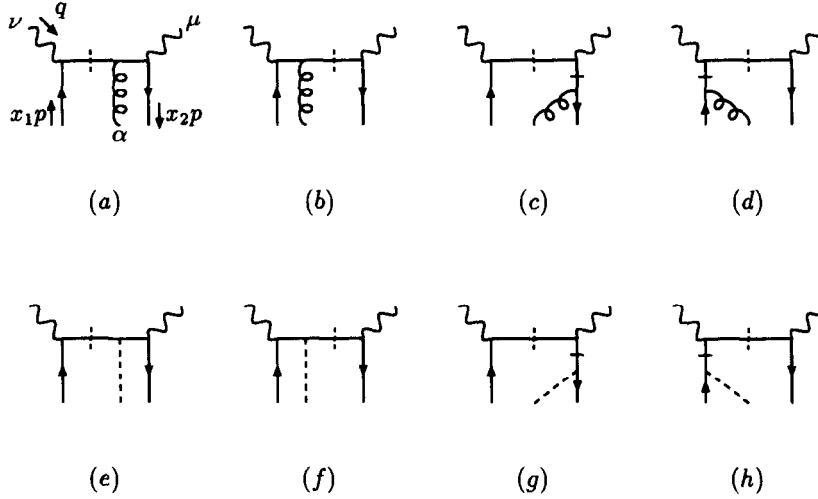


Fig. 3. Uncrossed twist-three gluonic [(a)–(d)] and spurionic [(e)–(h)] diagrams.

$$\begin{aligned}
 T_{\text{twist-3}}^{\mu\nu} = & \int dx_1 dx_2 \text{Tr} [P_- H_\alpha^{\mu\nu}(x_1 p, x_2 p; 0) P_+ g_T^\alpha{}_\beta \\
 & \times \Gamma_D^\beta(x_1, x_2)] \\
 & + \int dx_1 dx_2 \text{Tr} [P_- H_{\text{spur}}^{\mu\nu}(x_1 p, x_2 p; 0) P_+ \\
 & \times \Gamma_m(x_1, x_2)], \quad (30)
 \end{aligned}$$

which are of a specific order in $1/Q$. That is, ‘twist- t ’ contributes only at order Q^{2-t} . Also, t always equals the number of partons connecting the hard and soft parts. To arrive at the DIS hadronic tensor, $W^{\mu\nu} = \text{Disc}[T^{\mu\nu}]/(4M\pi i)$, one has to cut the hard diagrams. The uncrossed twist-three cut diagrams are depicted in Fig. 3. Explicitly, one finds for the (projected) discontinuities (displaying only the antisymmetric parts in $\mu \leftrightarrow \nu$):

$$\text{Disc}[P_- S^{\mu\nu}(xp; 0) P_+] = \pi i e^2 \delta(x - x_B) i \epsilon_T^{\mu\nu} \not{x} \gamma_5, \quad (31)$$

$$\begin{aligned}
 \text{Disc}[P_- H_\alpha^{\mu\nu}(x_1 p, x_2 p; 0) P_+ g_T^\alpha{}_\beta] = & \frac{\pi i e^2}{Q^2} i \epsilon^{\mu\nu\rho\sigma} \\
 & \times q_\rho \{ i \epsilon_{T\sigma\beta} \not{x} [\delta(x_1 - x_B) - \delta(x_2 - x_B)] \\
 & + g_{T\sigma\beta} \not{x} \gamma_5 [\delta(x_1 - x_B) + \delta(x_2 - x_B)] \}, \quad (32)
 \end{aligned}$$

$$\begin{aligned}
 \text{Disc}[P_- H_{\text{spur}}^{\mu\nu}(x_1 p, x_2 p; 0) P_+] = & \frac{\pi i e^2}{Q^2} i \epsilon^{\mu\nu\rho\sigma} \\
 & \times q_\rho \not{x} \gamma_{T\sigma} \gamma_5 [\delta(x_1 - x_B) + \delta(x_2 - x_B)]. \quad (33)
 \end{aligned}$$

where $\epsilon_T^{\mu\nu} = \epsilon^{\mu\nu\rho\sigma} p_\rho n_\sigma$. The calculation of these discontinuities is especially simple since the partons are massless, the effects of non-zero m being already taken into account. At this point the electromagnetic gauge invariance is manifest. That is, contraction of the modified hard parts with q gives zero identically. Compare this with the much more elaborate demonstration of gauge invariance in Ref. [7, App. C].

For the soft parts we use the parametrizations [10]

$$\begin{aligned}
 \Gamma(x) = & \frac{1}{2} g_1(x) \lambda \gamma_5 \not{x} + \frac{1}{2} h_1(x) \gamma_5 \not{x} \not{p} \\
 & + \frac{1}{2} M g_T(x) \gamma_5 \not{x} \not{p} + \dots, \quad (34)
 \end{aligned}$$

$$\begin{aligned}
 \Gamma_D^\alpha(x_1, x_2) = & \frac{1}{2} M G(x_1, x_2) i \epsilon_T^{\alpha\beta} S_{T\beta} \not{x} \\
 & + \frac{1}{2} M \tilde{G}(x_1, x_2) S_T^\alpha \gamma_5 \not{x} + \dots, \quad (35)
 \end{aligned}$$

$$\Gamma_m(x_1, x_2) = \frac{1}{2} M H_1(x_1, x_2) \gamma_5 \not{x} \not{p} + \dots, \quad (36)$$

where we listed only the distribution functions relevant for the process under consideration. The functions \tilde{G} and H_1 are symmetric, G is antisymmetric under exchange of the two arguments. The functions are not all independent. One has the relations

$$\begin{aligned}
 2x_1 g_T(x_1) = & \int dx_2 [G(x_1, x_2) - G(x_2, x_1) \\
 & + \tilde{G}(x_1, x_2) + \tilde{G}(x_2, x_1) \\
 & + H_1(x_1, x_2) + H_1(x_2, x_1)], \quad (37)
 \end{aligned}$$

$$H_1(x_1, x_2) = \frac{m}{M} \delta(x_2 - x_1) h_1(x_1), \quad (38)$$

where the first equation follows from the equations of motion, the second from Eq. (15). Contracting the hard parts, Eqs. (31)–(33), with the soft parts, Eqs. (34)–(36), gives for the antisymmetric hadron tensor

$$W_{\text{twist-2}}^{A\mu\nu} = \frac{e^2}{2M} \lambda i \epsilon_T^{\mu\nu} g_1(x_B), \quad (39)$$

$$W_{\text{twist-3}}^{A\mu\nu} = \frac{e^2}{2P \cdot q} i \epsilon^{\mu\nu\rho\sigma} q_\rho S_{T\sigma} g_T(x_B), \quad (40)$$

where we used Eq. (37) to eliminate the two-argument functions. Comparing with the standard expression in terms of structure functions

$$W^{A\mu\nu} = \frac{1}{P \cdot q} i \epsilon^{\mu\nu\rho\sigma} q_\rho \left[S_\sigma g_1(x_B, Q^2) + \left(S_\sigma - \frac{S \cdot q}{P \cdot q} P_\sigma \right) g_2(x_B, Q^2) \right], \quad (41)$$

we identify the tree-level structure functions $g_1(x_B, Q^2) = \sum_a (e_a^2/2) [g_1^a(x_B) + g_1^a(-x_B)]$ and $g_1(x_B, Q^2) + g_2(x_B, Q^2) = \sum_a (e_a^2/2) [g_T^a(x_B) + g_T^a(-x_B)]$, where we reinstated the flavor sum and added the crossed (anti-quark) diagrams. These results are in agreement with the standard EFP-type of calculations [8,11,7]. It is the way they are derived that makes the difference. The algorithm can be summarized as follows. To obtain the regular contributions to the order $Q^{2-\tau}$ DIS hadron tensor, following Qiu [4], one has to (1) write down all possible forward 2-photon t -parton cut diagrams, where partons can be quarks, antiquarks or gluons; (2) replace all propagators that do not lie between the photon vertices by special propagators; (3) project out the good fields¹ and couple them with t -parton soft matrix elements. The quark-mass terms follow in the same way, except that one (or more) of the gluon legs that connect hard and soft parts is replaced by a spurion. In the hard parts the partons are kept massless.

We have also applied the method to one-hadron-production in e^+e^- -annihilation, which essentially only differs from the above by the fact that the photon momentum is now time-like and the distribution functions have to be replaced by fragmentation functions (indicated with a hat, e.g., \hat{g}_T^a). We can again

express the sub-leading structure function (denoted by $\hat{g}_1 + \hat{g}_2$) in terms of \hat{g}_T^a only, that is, without explicit mass terms. This result differs from Eq. (38) of Ref. [12], in which Qiu's factorization is also applied, by quark-mass terms (accompanied by the fragmentation function \hat{h}_1^a). Our claim is that these terms should not be present.

In this letter we have extended the formalism by Qiu, leading to a factorization of hard and soft scattering parts in a manifestly gauge-invariant way, to include non-zero quark masses. We have illustrated the method for polarized DIS at sub-leading order. Another interesting application would be the calculation of the quark-mass contributions at twist-four in unpolarized DIS [13]. Also, one should investigate whether the formalism can be extended to include QCD corrections. Work in this direction is in progress.

We thank P.J. Mulders for useful comments. This work was supported by the Foundation for Fundamental Research on Matter (FOM) and the National Organization for Scientific Research (NWO).

References

- [1] R.L. Jaffe and X. Ji, Phys. Rev. D 43 (1991) 724.
- [2] H.D. Politzer, Nucl. Phys. B 172 (1980) 349.
- [3] R.K. Ellis, W. Furmański and R. Petronzio, Nucl. Phys. B 207 (1982) 1; 212 (1983) 29.
- [4] J. Qiu, Phys. Rev. D 42 (1990) 30.
- [5] C.H. Llewellyn Smith and J.P. de Vries, Nucl. Phys. B 296 (1988) 991.
- [6] J.B. Kogut and D.E. Soper, Phys. Rev. D 1 (1970) 2901.
- [7] M. Anselmino, A. Efremov and E. Leader, Phys. Rep. 261 (1995) 1.
- [8] A.V. Efremov and O.V. Teryaev, Sov. J. Nucl. Phys. 39 (1984) 962.
- [9] J. Qiu and G. Sterman, Nucl. Phys. B 353 (1991) 105.
- [10] R.L. Jaffe and X. Ji, Phys. Rev. Lett. 67 (1991) 552; Nucl. Phys. B 375 (1992) 527.
- [11] P.G. Ratcliffe, Nucl. Phys. B 264 (1986) 493.
- [12] W. Lu, X. Li and H. Hu, Phys. Rev. D 53 (1996) 131.
- [13] S.H. Lee, Phys. Rev. D 49 (1994) 2242.

¹ These projections pick out one specific order in $1/Q$. They were not used in Ref. [4].