Sliding friction
*From microscopic contacts to Amontons’ law*
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3.1 Abstract

In this chapter\(^1\) we show experimentally that the sliding friction on sand is greatly reduced by the addition of some -but not too much- water. The formation of capillary water bridges increases the shear modulus of the sand, which facilitates the sliding. Too much water on the other hand makes the capillary bridges coalesce, entailing a decrease of the modulus; in this case we observe that the friction coefficient increases again. Our results therefore show that the friction coefficient is directly related to the shear modulus, which has large repercussions for the transport of granular materials. In addition, the polydispersity of the sand is shown to also have a large effect on the friction coefficient.

3.2 Introduction

Sliding friction over and between sand layers is relevant for many problems ranging from civil engineering to earthquake dynamics. In many practical situations, small amounts of water may be present. Already ancient Egyptian tomb drawings suggest that wetting the sand with water may influence the friction between a sled and the sand (Figure 3.1), although the significance of the person wetting the sand has been much disputed\(^{2, 3, 4, 5, 6}\). If adding water to sand has an effect on friction, this should have consequential repercussions for e.g. sand transport through pipes\(^7\). This is an important

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\(^1\)This chapter is adapted from [1].
issue, since the transport and handling of granular materials is responsible for around 10% of the world energy consumption[8], and optimizing granular transport ultimately relies on understanding the friction between the granular system and the walls[9, 10, 11].

The effect of notably the air humidity on sliding friction of sand has been much discussed, the general consensus being[12, 11, 13, 14] that humidity leads to the condensation of water between the grains. The breaking up of the water bridges during sliding is then believed to significantly increase the friction coefficient. Consequently, sliding over dry sand should be easier than over sand with a bit of water[14]. If this were true for all water contents, the transport of granular materials would become very costly, and the Egyptians would have needed more workers to pull the sled through the desert if the sand was wetted.

### 3.3 Experiment

In this chapter we investigate the effect of the addition of small amounts of water on the sliding friction on sand, and find that the addition of small amounts of water can decrease the friction coefficient by almost a factor of two. To perform the experiment, we measure the force necessary to pull a sled (on which different weights could be placed) with a constant low
speed over three different sand types mixed with different amounts of water (Figure 3.2). The sand is first dried in the oven and cooled down to room temperature. Subsequently, water is mixed thoroughly with the sand, after which the system is compacted by repeated tapping. Measurements of the frictional force were done on a Zwick/Roell Z2.5 tensile tester which moves a force transducer at a constant speed (for details see chapter 2). The PVC sled had rounded edges; the front edge was attached to the tensile tester by a perfectly horizontal pulling cord. Sandpaper with a grain size of 35 µm was glued to the bottom of the sled.

![Figure 3.2: Force-displacement curves for wet and dry Iranian sand. Inset: picture of the setup: the picture on the left was taken while sliding over dry normalized sand. The picture on the right was taken while sliding over normalized sand wetted with 5% water. In the dry sand, a heap clearly builds up in front of the sled. The 11 cm × 7.5 cm sled is made out of PVC with rounded edges (as the Egyptian sled) and sandpaper with a grain size of 35 µm is glued to its bottom; the results were qualitatively similar but less reproducible with a smooth bottom.](image)

In the three-phase sand-water-air system, the water forms capillary bridges. The curvature of the liquid interface in the water bridges leads to a capillary pressure causing an attraction between the grains; the presence of these capillary bridges between the grains then causes the stiffness of wet sand, as in a sand castle[15]. However, different amounts of liquid lead to different distributions of the liquid between the grains, and this in turn leads to a dif-
ferent stiffness (shear modulus). Our X-ray tomography images (Figure 3.3) show that for 1% liquid (3.3a), liquid bridges are formed at the contact points of grains, this is the ‘pendular regime’. For 5% liquid (3.3b), liquid bridges around the contact points and liquid-filled pores coexist. Both give rise to cohesion between particles: this is usually referred to as the ‘funicular regime’. Finally, for 10% liquid (3.3c), more pores are filled with the liquid. The liquid surface forms large pockets within the material; this is the ‘capillary regime’.[16, 17, 7].

![Figure 3.3: Sections through 3D X-ray micro-tomograms of 500 µm polystyrene spheres mixed with (a) 1%, (b) 5% and (c) 10% liquid.](image)

The mechanical behavior of the sand upon addition of small amounts of water is fully understood, and has been tested for different grain materials and different liquids[16]. The basic physics is that the modulus starts to increase when capillary bridges form between the grains. However for too much liquid, the capillary bridges start to merge (as is shown in Figure 3.3) and eventually disappear altogether when the sand is fully saturated. Therefore, there must be an optimum strength at a finite amount of added water.

### 3.4 Results

The mechanical behavior of the sand upon addition of small amounts of water turns out to have large repercussions for the friction coefficient. The force as a function of the sled displacement (Figure 3.2) shows that especially
for the dry sand, a high peak force has to be exceeded before a steady state can be reached. In steady state, we find that the pulling force is independent of pulling speed \(v\) over the range of our measurements \((10 < v < 800 \text{ mm/s})\), but depends roughly linearly on the weight that is on the sled (Figure 3.4a). Defining an overall dynamic friction coefficient \(\mu_d\) as the plateau value of the tangential force divided by the normal (gravitational) force given by the total weight of the sled, the friction coefficient is found to decrease if a small amount of water is added to the sand (Figure 3.2). One of the reasons for this is rather simple - and hence perhaps also observed by the Egyptians: in the dry case a heap of sand forms in front of the sled, before it can really start to move. This is also the reason for the peak in the force-displacement curve observed for the dry sand (Figure 3.2), which shows that the static friction coefficient is significantly higher for the dry sand. The peak, and hence the static friction, progressively decreases in amplitude when more water is added to the system; visual observation confirms that indeed the amount of sand that heaps up in front of the sled decreases also with increasing water content. We checked that our conclusion is not affected by the roughness of the bottom of the sled: with and without sandpaper glued to the bottom, similar results were obtained.

Surprisingly, we find that for water contents in excess of 5%, the pulling becomes more difficult again: the friction coefficient increases (Figures 3.2 and 3.4b). We also verified this conclusion for two other types of sand: more polydisperse (ISO 679 standard) and more monodisperse (Nemours) sand (Figures 3.5 and 3.6). On all three sand types, there is a minimum in the friction vs. water content curve. The reason for this behavior follows from our measurement of the shear modulus (Figure 3.4b); for too high water contents the stiffness of wet sand decreases again. In reference [16], a detailed description of the behaviour of the shear modulus of wet granular material is given. We use the model from reference [16] to successfully predict (without adjustable parameters) the correct order of magnitude of the maximal shear modulus of the wet sand (Blue horizontal line in Figure 3.4b). The non-monotonic behaviour of the shear modulus with water content is also known from building sandcastles[7]: for too large water contents the capillary bridges start to merge[17], the capillary pressure in the bridges decreases and so does the elastic modulus. The measurement of the shear elastic modulus vs. volume fraction of water shows in fact a trend that is exactly opposite to that of the friction coefficient, showing that there is an inverse relation between the two; the ‘softer’ the sand, the higher the friction coefficient (Figure 3.4b).
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Figure 3.4: (a), Macroscopic dynamic friction coefficient for different water contents (Iranian sand). (b), Friction coefficient and shear elastic modulus (right axis) as a function of the water content in Iranian sand. The blue horizontal line indicates the optimum shear modulus according to the model in reference [16] using a grain radius of 100 µm, a Young’s modulus of the grains of 60 GPa and a water surface tension of 70 mN/m. The latter measurements were done on a commercial rheometer using a plate in cup geometry where the bottom of the cup, as well as the plate, was covered with sandpaper and the sand compacted as for the sled experiments.

We further investigate this relation by plotting the friction coefficient as a function of shear modulus for the three different sand types. Figure 3.7 not
only shows that the friction coefficient goes down as the sand becomes more rigid, but also that the decrease in friction coefficient is proportional to the increase in modulus. In fact, the data for three different sand types collapse onto a single line, indicating that all three frictional systems follow the exact same relation between shear modulus and friction coefficient.

![Graph](image)

**Figure 3.5:** Dynamic friction coefficient as a function of water volume fraction for different types of sand.

Considering the three types of sand, we see that the drop in friction coefficient with the addition of small amounts of water becomes larger as the sand is more polydisperse; Nemours sand, which is the most monodisperse sand type, gives a 10% decrease, Iranian sand a 26% decrease and the polydisperse standard sand a 40% decrease in the dynamic friction coefficient (Figure 3.5). The Egyptians were pulling their sled through desert sand, which is very polydisperse[18] (Figure 3.6). On such polydisperse sand the addition of a small amount of water reduces the pulling force by almost a factor of two, according to our measurements.

Our measurements in fact span a similar range of stresses as the Egyptians; an estimate of the maximum load they pulled is one ton per square meter or 10,000 Pa. We put up to 20 N on roughly 80 cm², so we get to 2,500 Pa, of the same order of magnitude. As for the archeologists, some have interpreted the pouring of the water in front of the sled as being purely ceremonial[2, 3], which does not seem a correct interpretation, in view of the results presented here. There is also evidence that in some cases the Egyptians built roads for the sleds out of wooden sleepers[4, 5, 6]. The possibility of dragging the sled through desert sand is often precluded because it is believed to
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Figure 3.6: Grain size distribution for four sand types. The probability distribution function (PDF) gives the relative occurrence of different grain sizes. The data for the Egyptian desert sand was taken from reference [18]. Nemours sand and Iranian sand are similar; both containing mainly grains in the 150 – 300 µm range, while ISO 679 standard sand is much more polydisperse.

be too difficult[4, 6]. However, in view of our results it seems very well possible to drag the sleds over wet sand with the manpower available to the Egyptians[6]. In fact, the value of the friction coefficient of wood on wood is in the range of 0.25 < µd < 0.7[19]; especially for the polydisperse sand here that is closest to the Egyptian desert sand[18], we arrive at friction coefficients as low as 0.3, so that the dragging can be just as easy over sand as over the wooden sleepers. In addition, the ‘optimal’ friction coefficient of 0.3 we find here coincides remarkably well with estimates that have been made on the basis of the tomb drawings. A friction coefficient of 0.33 was estimated, on the basis of the maximum pulling strength that the ropes were able to sustain[20].

3.5 Conclusion

Summarizing, we find that there is a pronounced effect of the addition of small amounts of water to sand. The force necessary to move the sled at constant speed with a given weight on top of it can be reduced by as much as 40%, and the force necessary to get the sled to move by up to 70% on standard sand. This happens because the addition of water makes the sand more rigid
which prevents the heaping up of sand in front of the sled that makes the pulling difficult. This result strongly contrasts earlier experiments, where the pulling in fact became more difficult upon the formation of capillary bridges between the grains[14, 11]. Interestingly, the measured friction coefficients for the highest water contents measured here are again larger than that of dry sand; perhaps the proposed mechanism of friction increase due to breaking of capillary bridges applies here[14, 11].

One of the most striking results is that the friction coefficients measured for polydisperse sand are significantly lower than that for monodisperse sand. Perhaps the modulus of wet polydisperse sand can exceed that of wet monodisperse sand, because the grain size distribution allows for a denser packing which is more rigid. In view also of the large amount of energy consumed worldwide for the transport of granular materials, this merits a more detailed study. It has been suggested for dry sand that the polydisperse grains can form its own ballbearing system in which friction is minimized by a size segregation that allows the grains to roll over each other with little friction[21]; perhaps a similar mechanism is at play here. On the other hand, Fig.7 shows that in fact all the measured friction coefficients decrease roughly linearly with increasing modulus. The conclusion must be that the more polydisperse sand has a lower friction coefficient simply because it has a higher modulus. The reason for the higher modulus is likely
to be that the more polydisperse sand can be more densely packed, leading
to a larger number of capillary bridges per unit volume, and hence a higher
modulus. More generally, the frictional drag for transporting sand is still an
issue of debate[7], and our results show that the presence of even very small
quantities of water and polydispersity can change the friction and hence the
flow behavior profoundly.


