Drops and jets of complex fluids

Javadi, A.

Citation for published version (APA):
Javadi, A. (2013). Drops and jets of complex fluids
6.

Coiling of Yield Stress Fluids

One of the simplest experiments in all of fluid mechanics is to let a thin stream of liquid fall from a height onto a rigid surface. Yet from this elementary situation, remarkably rich and complex behavior arises that is still not fully understood. A stream a few millimeters in diameter with low viscosity and high volumetric flow rate impacting on a solid surface produces a hydraulic jump, in which the thickness of the fluid layer spreading away from the point of impact suddenly increases (81; 87; 88). At the other extreme, at low flow rates a thin thread of a very viscous liquid, such as honey, winds itself into a rotating helical ‘coil’ as it falls onto a surface. The liquid coiling behavior is remarkable since it corresponds to a solid-like behavior of a material that is a simple liquid; in fact, the equations describing solid rope coiling are very similar to those for the liquid (89). In this chapter we ask the question whether the same thing holds for the coiling of a filament of yield stress fluid falling from an orifice on a solid surface since yield stress fluids can behave both as a solid and as a liquid, depending on the stress; the question is then how they coil.

6.1 Introduction

A thin stream of honey poured from a sufficient height onto toast forms a regular coil. A similar phenomenon happens for a falling viscous sheet: it folds. Why does this happen, and what determines the frequency of coiling or folding? When pouring a viscous liquid on a solid surface, we encounter instabilities. A high viscosity can allow instabilities like buckling which normally happens only for solids, to happen for a liquid. In solid mechanics, the concept of buckling is an important and well-understood phenomenon. Buckling, which means the transition from a straight to a bent configuration due to the application of a load, occurs because the straight configuration is not stable. This instability arises as a result of the competition between axial compression and bending in slender objects (90). Within the realm of fluid mechanics, similar phenomena can be observed. An example is
Coiling of a thin stream of honey as it falls onto a flat plate. The spontaneous transition from a steady, stable flow to oscillations of parts of the jet column is called fluid buckling, in analogy with its counterpart in solid mechanics (Fig. 6.1).

Figure 6.1: A viscous jet of silicon oil falling onto a plate. a) stable, unbuckled jet, b) buckled jet at critical height, c) coiling jet. The scale shown is 1 mm.

When a vertical thin flexible rope falls on a horizontal surface such as a floor a similar phenomenon to liquid coiling is observed. This familiar phenomenon can also be reproduced at the lunch table when a spaghetti falls down into one’s plate, (Fig. 6.2). When the rope reaches the surface it buckles and then starts to coil regularly.

Figure 6.2: Coiling of elastic rope on solid surface. a) Spaghetti b) Cotton rope.

If the rope is fed continuously towards the surface from a fixed height its motion quickly settles down into a steady state in which the rope is laid out in a circular coil of uniform radius. The radius of the coil which depends on the height, stiffness and feeding velocity, determines the frequency of coiling.
6.1.1 Coiling of Newtonian fluids

The coiling instability of liquid filaments was called liquid rope coiling by Barnes and Woodcock (1958) (91), whose pioneering work was the first in a series of experimental studies spanning nearly 50 years (Barnes et al. 1958, 1959 (91; 92); Cruickshank 1980 (93); Cruickshank and Munson 1981 (94); Huppert 1986 (95); Griffiths and Turner 1988 (96); Mahadevan et al. 1998 (97)). The first theoretical study of liquid rope coiling was undertaken by Taylor (98), who suggested that the instability is similar to the buckling instability of an elastic rod (or solid rope) under an applied compressive stress. Subsequent theoretical studies based on linear stability analysis determined the critical fall height and frequency of incipient coiling (99; 94). They showed that the instability takes place for low Reynolds numbers and heights larger than a threshold height, which depends on the properties of the liquid (viscosity and surface tension).

When a jet of a viscous liquid like honey is falling on a horizontal plate from a small height, it will smoothly connect to the horizontal surface (Fig. 6.1 (a)). In this case, the jet is stable. For a given flow rate and diameter, notably if the height exceeds a critical value \( H_c \) (94), the jet becomes unstable and will buckle (Fig. 6.1 (b)). Buckled jet is unstable and cannot remain falling onto the same spot. It bends to the right or left and this causes a torque, which makes the jet continue to move on a circle and form a coil (Fig. 6.1 (c)).

A viscous jet can buckle, because it may be either in tension or compression, depending on the velocity gradient along its axis. If the diameter of the jet increases in the downstream direction, the viscous normal stress along its axis is one of compression. If this viscous compressive component of the normal stress is large enough, the net axial stress in the jet (including surface tension) may be compressive. Thus, near the flat plate, a sufficiently large axial compressive stress for a sufficiently slender jet can cause buckling (94; 100).

Regimes of coiling

The motion of a coiling jet is controlled by the balance between viscous forces, gravity and inertia. Viscous forces arise from internal deformation of the jet by stretching (localized mainly in the tail) and by bending and twisting (mainly in the coil). Inertia includes the usual centrifugal and Coriolis accelerations, as well as terms proportional to the along-axis rate of change of the magnitude and direction of the axial velocity. The dynamical regime in which coiling takes place is determined by the magnitudes of the viscous \( F_V \), gravitational \( F_G \) and inertial \( F_I \) forces per unit rope length within the coil. These are (Mahadevan et al. (101), (102))

\[
F_V \sim \rho \nu a_1^4 U_1 R^{-4}, \quad F_G = \rho g a_1^2, \quad F_I \sim \rho a_1^2 U_1^2 R^{-1},
\]

(6.1)

where \( a_1 \) is the radius of the rope within the coil and \( U_1 \equiv Q/\pi a_1^2 \) is the corresponding axial velocity of the fluid. Since the rope radius is nearly constant in the coil, we define \( a_1 \) to be the radius at the point of contact with the plate. In
Figure 6.3: Coiling of a jet of viscous corn syrup (photograph by Neil Ribe), showing the parameters of a typical laboratory experiment.
6.1. Introduction

Fig. 6.3 the parameters that will be used in this section are shown. Each of the forces (6.1) depends strongly on \( a_1 \), which in turn is determined by the amount of gravity-induced stretching that occurs in the tail. Because this stretching increases strongly with the height \( H \), the relative magnitudes of the forces \( F_V \), \( F_G \) and \( F_I \) are themselves functions of \( H \). As \( H \) increases, the coiling traverses a series of distinct dynamical regimes characterized by different force balances in the coil. Fig. 6.6 shows how these regimes show up in curves of \( \Omega(H) \), the frequency of coiling and \( a_1(H) \) the radius of the rope, for one set of experimental parameters. These curves were determined by solving numerically the thin-rope equations of Ribe (102). We observe that different modes of coiling are possible, depending on how the three forces in the coil are balanced. For small dimensionless heights \( H(g/\nu^2)^{1/3} < 0.08 \), coiling occurs in the viscous (V) regime, in which both gravity and inertia are negligible and the net viscous force on each fluid element is zero. Coiling is here driven entirely by the injection of the fluid, like toothpaste squeezed from a tube. Because the jet deforms by bending and twisting with negligible stretching, its radius is nearly constant. Therefore, \( a_1 \approx a_0 \) and \( U_1 \approx U_0 \) (Fig. 6.4(a) and (e)). We observe that for very small height the rope is slightly compressed against the fluid pile as shown in Fig. 6.5.

Dimensional considerations and the general relation \( \Omega \sim U_1/R \) then imply

\[
R \sim H, \quad \Omega_V = \frac{Q}{Ha_1^2}.
\]

(6.2)

After the viscous coiling regime, \( 0.08 \leq H(g/\nu^2)^{1/3} \leq 0.4 \), when inertia is negligible, viscous forces in the coil are balanced by gravity \( (F_G \approx F_V \gg F_I) \), giving rise to gravitational (G) coiling (Fig. 6.4(b) and (f)). The scaling laws for this mode are

\[
\rho \nu a_1^4 U_1 R^{-4} \sim \rho g a^2
\]

(6.3)

\[
R \sim g^{-1/4} \nu^{1/4} Q^{1/4} \equiv R_G, \quad \Omega \sim g^{1/4} \nu^{1/4} a_1^{-2} Q^{3/4} \equiv \Omega_G,
\]

(6.4)

which is identical to the typical frequency for the folding of a rope confined to a plane (101). The rope’s radius is nearly constant \( (a_1 \approx a_0) \) at the lower end \( (0.08 \leq H(g/\nu^2)^{1/3} \leq 0.15) \) of the gravitational regime, implying the seemingly paradoxical conclusion that gravitational stretching in the tail can be negligible in ‘gravitational’ coiling. This apparent paradox is resolved by noting that for a given strain rate, the viscous forces associated with bending and twisting of a slender rope are much smaller than those that accompany stretching. The influence of gravity is therefore felt first in the (bending/twisting) coil and only later in the (stretching) tail, and thus can be simultaneously dominant in the former and negligible in the latter. When the height gradually increases to \( H(g/\nu^2)^{1/3} \approx 1.2 \), a third mode, inertial coiling is observed. (Fig. 6.4(c) and (g)). Viscous forces in the coil are now balanced almost entirely by inertia \( (F_I \approx F_V \gg F_G) \), giving rise to inertial (I) coiling with this scaling law:

\[
\rho \nu a_1^4 U_1 R^{-4} \sim \rho a_1^2 U_1^2 R^{-1}.
\]

(6.5)
Chapter 6. Coiling of Yield Stress Fluids

Figure 6.4: Different coiling regimes. (a) Viscous regime: coiling of silicone oil with $\nu = 1000 \text{ cm}^2/\text{s}$, injected from an orifice (top of image) of radius $a_0 = 0.034 \text{ cm}$ at a volumetric rate $Q = 0.0044 \text{ cm}^3/\text{s}$. Effective fall height $H = 0.36 \text{ cm}$. (b) Gravitational regime: coiling of silicone oil with $\nu = 300 \text{ cm}^2/\text{s}$, falling from an orifice of radius $a_0 = 0.25 \text{ cm}$ at a flow rate $Q = 0.093 \text{ cm}^3/\text{s}$. Fall height is 5 cm. The radius of the portion of the rope shown is 0.076 cm. (c) Inertial regime: coiling of silicone oil with $\nu = 125 \text{ cm}^2/\text{s}$, $a_0 = 0.1 \text{ cm}$, $Q = 0.213 \text{ cm}^3/\text{s}$ and $H = 10 \text{ cm}$. The radius $a_1$ is 0.04 cm. (e)-(g) Jet shapes calculated using Auto97 (Doedel et al. 2002) for three modes of fluid coiling (102). (e) Viscous coiling. (f) Gravitational coiling. (g) Inertial coiling.
6.2. Coiling of elastic ropes

Figure 6.5: Rope of silicon oil with \( \nu = 1000 \text{ cm}^2/\text{s} \), injected from an orifice of radius \( a_0 = 0.034 \text{ cm} \) at a volumetric rate \( Q = 0.0044 \text{ cm}^3/\text{s} \) and effective fall height \( H = 0.30 \text{ cm} \) is slightly compressed against the plate, so the diameter at the bottom is larger than the diameter of the filament near the nozzle.

The radius and frequency for this mode are proportional to

\[
R \sim \equiv R_I, \quad \Omega \sim \equiv \Omega_I
\]

For \( 0.4 < H (g/\nu^2)^{1/3} < 1.2 \), viscous forces in the coil are balanced by both gravity and inertia, giving rise to a complex transitional regime ‘inertio-gravitational’ (IG). The curve of frequency vs. height is now multivalued, comprising a series of roughly horizontal ‘steps’ connected by ‘switchbacks’ with strong negative slopes (Fig. 6.6). The curve exhibits five points where it folds back on itself.

As shown in Fig. 6.7, the observed frequencies in ‘inertio-gravitational’ regime are concentrated along the roughly horizontal ‘steps’ of the \( \Omega(H) \) curve, leaving the steeper portions with negative slope (switchbacks) empty. The absence of observed steady coiling states along the switchbacks suggests that such states are indeed unstable to small perturbations (103). The dashed part of the curve in Fig. 6.7 show the theoretically obtained unstable parts.

6.2 Coiling of elastic ropes

All mountaineers know that a rope held vertically with its lower end in contact with a surface will coil spontaneously when it is dropped. The initial stage of the coiling is just the buckling of the rope under its own weight. In general, when a solid material buckles the subsequent non-linear evolution of the instability can occur in two ways. If the material is very stiff, it will break: it is for this reason that the resistance of structures to buckling and breaking is a key parameter in architecture and construction engineering. If on the other hand the material is sufficiently flexible, the structure remains intact but undergoes a large finite-amplitude deformation whose dynamics are essentially nonlinear. In many cases,
Figure 6.6: Regimes of liquid rope coiling. The solid line is the numerically predicted curve of frequency vs. height for the same parameters. Portions of the curve representing the different coiling regimes are labeled: viscous (V), gravitational (G), inertio-gravitational (IG), and inertial (I). The symbols show experimental observations of the coiling frequency as a function of the fall height $H$ for an experiment performed using viscous silicone oil ($\rho = 0.97$ g cm$^{-3}$, $\nu = 1000$ cm$^2$ s$^{-1}$, $\gamma = 21.5$ dyne cm$^{-1}$) with $d = 0.068$ cm and $Q = 0.00215$ cm$^3$ s$^{-1}$ (103).
Figure 6.7: Stability of steady coiling with $\Pi_1 = 3690$, $\Pi_2 = 2.19$, and $\Pi_3 = 0.044$. The continuous curve shows the numerically calculated frequency of steady coiling as a function of height. The solid and dashed portions of the curve indicate stable and unstable steady states, respectively, as predicted using the numerical stability analysis described in the text. Symbols indicate experimental measurements (103) obtained in series with $H$ increasing (squares), decreasing (circles), and varied randomly (triangles).
the cause of the nonlinearity is the breakdown at large strain of an initially linear relation between stress and displacement, either because the nonlinear (quadratic) terms in the elastic strain tensor (104) become significant or because the material no longer satisfies Hookes law. Much progress has been made recently in understanding these sorts of nonlinear behavior in structures such as crumpled sheets of paper (105; 106) and crumpled wires (107). The other principal cause of nonlinearity is purely geometrical: the fact that the final (deformed) shape of the structure is unknown because it is far from that of the initial state. This sort of nonlinearity can be obtained even if the materials elasticity is perfectly linear and Hookean, the classic example being the large deformation of elastic rods and filaments described by the so-called Kirchhoff equations (for a review, see (108)). Manifestations of the geometrically nonlinear dynamics of elastic rods include the kinking of telephone cables on the ocean floor (109), handedness reversal in the coiled tendrils of climbing plants (110), the supercoiling of DNA strands (111), and the steady coiling of elastic ropes (89).

6.2.1 Regimes of coiling

Non-dimensionalization of the governing equations shows that the dimensionless coiling frequency $\hat{\Omega} \equiv \Omega (d^2 E/\rho g^4)^{1/6}$ depends only on the dimensionless fall height $\hat{H} \equiv H (\rho g/d^2 E)^{1/3}$ and the dimensionless feed rate $\hat{\dot{U}} \equiv U (\rho/d^2 g^2 E)^{1/6}$. Fig. 6.8 shows numerically calculated curves of $\hat{\Omega}(\hat{H})$ for for several values of $\hat{\dot{U}}$. Coiling can occur in either of three regimes, depending on how the elastic forces that resist the bending of the ‘coil’ portion of the rope are balanced. Per unit rope length, the magnitudes of the elastic ($E$), gravitational ($G$), and inertial ($I$) forces in the coil are

$$F_E \approx Ed^4 R^{-3}, \quad F_G \approx \rho gd^2, \quad F_I \approx \rho d^2 U^2 R^{-1}. \quad (6.7)$$

In the first regime, which we call ‘elastic’ coiling, both gravity and inertia are negligible ($F_G, F_I \ll F_E$) and the net elastic force acting on every element of the rope is zero. A second ‘gravitational’ regime occurs when inertia is negligible and the elastic forces are balanced by gravity ($F_G \approx F_V \gg F_I$). Finally, ‘inertial’ coiling occurs when gravity is negligible and the elastic forces are balanced by inertia ($F_I \approx F_V \gg F_G$). The corresponding coiling frequencies $\Omega_E$, $\Omega_G$ and $\Omega_I$ can be found by estimating the coil radius $R$ and then using the relation $\Omega = U/R$ for conservation of volume flux at the moving contact point. For elastic coiling, $R \sim H$ For gravitational and inertial coiling, $R$ is obtained from the force balances $F_G \approx F_E$ and $F_I \approx F_E$, respectively. The results are

$$\Omega_E \sim U H^{-1}, \quad \Omega \sim U (\rho g/d^2 E)^{1/3}, \quad \Omega_I \sim U^2 (\rho/d^2 E)^{1/2}. \quad (6.8)$$

The above expression for $\Omega_E$ corresponds to the portions of the curves with slope $= -1$ (labeled $E$) on the left side of Fig. 6.8 The scaling law for $\Omega_G$ (equivalent to equation (2.4) of (113)) corresponds to the nearly horizontal portions of the curves.
6.2. Coiling of elastic ropes

Figure 6.8: Main portion: Dimensionless coiling frequency $\hat{\Omega} \equiv \Omega(d^2E/\rho g^4)^{1/6}$ as a function of dimensionless fall height $\hat{H} \equiv H(\rho g/d^2E)^{1/3}$ for several values of the dimensionless feed rate $\hat{U} \equiv U(\rho/d^2g^2E)^{1/6}$. The curves for $\hat{U} \geq 1.0$ continue indefinitely to the right (continuations not shown for clarity.) Thick horizontal bars correspond to the inertial frequency $\Omega_I$ defined by (6.8). The first six ‘whirling string’ and ‘whirling shaft’ eigenfrequencies are indicated by dotted and dashed lines, respectively. Inset: Phase diagram for elastic coiling as a function of $\hat{H}$ and $\hat{U}$. The coiling frequency $\hat{\Omega}(\hat{H}, \hat{U})$ is multivalued everywhere above the solid line. The vertical dashed line indicates the approximate location of the smooth transition between elastic ($E$) and gravitational ($G$) coiling. The dashed line in the inertial ($I$) portion of the diagram indicates a smooth transition between ‘whirling string’ and ‘whirling shaft’ resonant modes (112).
labeled $G$ at the bottom right of Fig. 6.8 (the numerics show that $\Omega_G$ also depends on $H$, but in a way that is too weak to be determined by scaling analysis). Finally, the scaling law for $\Omega_I$ corresponds to the horizontal lines labeled $I$ at the upper right of Fig. 6.8.

### 6.3 Yield stress fluids

Yield stress materials can be either fluid or solid, depending on the imposed stress. For an imposed shear stress above the yield point the material shows fluid like behavior, while below this point it is solid like. The characteristic stress $\sigma_y$ is the yield stress above which the material behaves as a solid while below that stress it behaves as a liquid (114). It should be noted that for the materials chosen in this chapter $\sigma_y$ is the same for both the liquid to solid and for the solid to liquid transitions (115), which is not necessarily the case for all yield stress materials (114; 115).

In order to determine the yield stress of our working fluids we measured rheological properties of the fluids with a Haake RS 150 Rheometer, cone-plate geometry ($35\,mm - 1^o$), at a frequency of 1 Hz. We do measurements in oscillation, in which an oscillatory stress is applied to the sample and its deformation is recorded by the rheometer. The real (in phase) and imaginary part (out of phase) of the response are the storage (elastic) modulus $G'$ and the loss (viscous) modulus $G''$ (116). Fig. 6.9 shows $G'$ and $G''$ vs. the applied stress for the gel and the foam. The point where $G'$ and $G''$ cross is marked by $\sigma_y$ on the plots. Before this point $G'$ is larger than $G''$ and the material is more solid like. Beyond the crossing point $G''$ becomes larger than $G'$ and we have more liquid like behavior. So the stress at this point gives us an estimate of the yield stress of the material. The rheological data then show a yield stress of about 100 Pa for the hair gel and about 10 Pa for the shaving foam (Fig. 6.9).

### 6.4 Coiling of yield stress fluids

In this section we will consider the coiling of non-Newtonian yield stress fluids experimentally. We will use shaving foam and hair gel as our test fluids. We use two typical yield stress fluids, commercial shaving foam and commercial hair gel, for this investigation. The non-Newtonian behavior will show up as a viscoelastic behavior: the gel coils like a viscous liquid, whereas the foam coils according to the predictions for a solid rope, in spite of the fact that their rheology is very similar. The reason for this turns out to be that, for the foam, the yield stress is not exceeded during the coiling, whereas for the gel it is.
6.4. Coiling of yield stress fluids

Figure 6.9: $G'$ and $G''$, storage and loss moduli, vs. stress for: a) hair gel b) shaving foam. The yield stress ($\sigma_y$) has been shown on plots. Rheological measurements were done with a Haake RS 150 Rheometer, cone-plate geometry (35 mm – 1°), at frequency of 1 Hz.

6.4.1 Experimental procedure and observations

In our experiments, a filament of yield stress fluid was extruded from a syringe with a constant flow rate ($Q$) by a piston driven by a stepper motor (Fig. 6.10) and after a distance ($H$) from an orifice of diameter ($d^*$) falls onto a solid surface and starts to coil with radius ($R$) and frequency ($\Omega$). We measured the radius and frequency of coiling for different fall heights for a set of fixed diameter of orifice, density and flow rate. The coiling frequency was measured by frame counting on movies taken with a high speed camera operating at 50-1000 frames s$^{-1}$, depending on the resolution required. The flow rate was measured to within 5 – 10% accuracy by measuring the change in the volume of gel or foam within the syringe as a function of time during the experiment. For foam at high flow rates, this method was not very accurate because of the compressibility of the fluid. We therefore estimated the flow rate by measuring the velocity of the filament at the nozzle using frame counting on the movies that we took with the rapid camera: the diameter $d$ of the rope just above the coil was measured from these movies.

For both systems (the foam and the gel) we did not observe stretching of the filament due to gravity before break up. This means that the diameter of the filament remains constant as a function of height to a good approximation. We therefore assume in the following that the diameter of the filament near the coil ($d$) is the same as $d_0$ the diameter of the filament at the nozzle. The largest deviation of $d$ from $d_0$ is about 10% in our experiments. For foam at high flow rates because
Figure 6.10: Experimental setup for the coiling of yield stress fluids.
of the die swell effect (116), the diameter of the filament near the nozzle $d_0$ is somewhat larger than the diameter of the nozzle $d^*$. As we increased the height, the weight of the filament becomes comparable to the yield stress of the fluid, and at some critical height the fluid within the filament started to flow; the filament becomes narrower and consequently breaks up (5). This break up is visually completely different from the Rayleigh-Plateau instability of a viscous thread. In the Rayleigh-Plateau problem the breakup is surface tension driven and the Radius of the filament at breakup point decreases smoothly to zero. In our experiments the filament becomes so thin that it can no longer sustain the weight of the material below it, and it tears apart quite suddenly. For the gel that has a much higher density than the foam, the critical height for which breakup happens was considerably smaller than for the foam.

We performed different series of experiments for different diameters of the orifice, flow rates and falling heights of the hair gel and shaving foam.

### 6.4.2 Results and discussion

**Coiling of shaving foam:**

Previous research on coiling of Newtonian fluid and elastic ropes show at low heights in both systems inertial and gravitational forces are negligible and the system is in a forced coiling regime (117; 102; 118; 103; 119; 89). Coiling in this regime is driven entirely by the injection of the fluid or, equivalently, the velocity of the rope. We observe a similar behavior for foam coiling at low heights. Figs 6.11(a) and (b) show a pile of coils at low height; similar to the previous systems we have $R \sim H$ and the characteristic frequency of this regime is $\Omega \sim U/H$, where $U$ is the velocity of the filament (89).

Fig. 6.12 shows the non-dimensional coiling frequency of the foam filament ($\Omega d/U$) vs. $(d/H)$ on a logarithmic scale. At low heights ($d/H \geq 0.1$) data points collapse onto a line with slope 1, this is the forced coiling regime for which ($\Omega \sim U/H$). For larger heights (smaller $d/H$) most of the experimental data show that $\Omega$ does not change much with increasing height. Therefore for $d/H < 0.1$, we have a transition to a second regime; here the gravitational term becomes important. In the gravitational regime the data points collapse roughly on a horizontal line. In Fig. 6.11(c) coiling pattern of the foam filament in the gravitational regime is shown. By increasing the flow rate further patterns of “figure of 8”, can be observed, (Fig. 6.11(d)). “Figure of 8” patterns were reported in previous works for both liquid and solid ropes (117; 102; 118; 103; 119; 89) and are thus also observed here.

Increasing the the flow rate leads to an increase in the velocity of the filament, and one would expect an inertial regime to set in. However for shaving foam at high flow rates, as can be seen in Fig. 6.11(e) and Fig. 6.13, the filament becomes unstable just after going out of the nozzle and buckles very randomly before reaching the solid surface. Therefore we do not have any regular coiling pattern and well defined frequency in this regime.
Figure 6.11: Different coiling patterns of the shaving foam in different regimes: a,b) Elastic regimes, the diameter of the coil is proportional to the distance from the nozzle. c) Gravitational regime. d) Figure of “8” can be observed at high flow rates. e) Instability at high flow rate.
6.4. Coiling of yield stress fluids

Figure 6.12: Non-dimensional frequency of coiling of the foam filament ($\Omega d/U$) vs. $(d/H)$ in a logarithmic scale. At low heights ($d/H \geq 0.1$) data points collapse on line with slope 1, and it represents the elastic regime (E). With increasing height where $d/H < 0.1$ we have a transition to the gravitational regime (G). In this regime data points collapse on a horizontal line.
Chapter 6. Coiling of Yield Stress Fluids

To see whether the behavior corresponds to the theoretical predictions for either solid or liquid-type coiling, we shall estimate an order of magnitude for the effective elastic modulus and effective viscosity of the foam from the measured coiling frequencies. The coiling frequency for the gravitational regime of solid rope coiling is $\Omega_{G, Solid} \sim U\left(\frac{\rho g}{(a_1)^2 E}\right)^{1/3}$, while for liquid rope coiling it can be written as $\Omega_{G, Liquid} \sim \left(\frac{g Q^3}{\nu a_8}\right)^{1/4}$. Thus, by comparison we can extract respectively an elastic modulus and a kinematic viscosity (89; 117). By using data of a typical experiment in these relations, we find respectively $E_{eff} \sim 3$ Pa for the effective elastic modulus, $\eta_{eff} \sim 10^{17}$ Pa.s and $\nu_{eff} \sim 2 \times 10^{19}$ St for the effective shear and kinematic viscosity. The inferred elastic constant gives us an order of magnitude of the stress in the filament during the experiments. The magnitude of this stress is smaller than the yield stress of foam which is about 10 Pa (Fig. 6.9). The effective viscosity on the other hand is so high that we can assume that we have no flow for this viscosity. According to these estimates we can conclude that the shaving foam is more solid like in our experiments. It is for this reason that we called the first regime at low height “elastic” regime in the captions of Fig. 6.11 and Fig. 6.12. The observation of the two regimes (E and G) and their scaling is also in agreement with the previous experiments on solid rope coiling (89).

Figure 6.13: Irregular patterns of the shaving foam at high flow rates where the inertial terms are important. The filament becomes unstable after going out of the nozzle and buckles very randomly before reaching the solid surface. There is not any regular coiling pattern or well defined coiling frequency.

Coiling of hair gel:

Fig. 6.14 shows the non-dimensional frequency of coiling vs. $d/H$. For $d/H < 0.1$ the data points collapse onto a horizontal line which again represents the gravitational regime. In this regime the frequency of coiling changes only very slightly with height. Here, it is only over a very small range of heights $(0.2 < d/H < 0.6)$ that we observe a tendency that goes perhaps towards the forced coiling regime.

Fig. 6.15(a) and (b) show typical patterns of coiling of the gel at low and intermediate height viscous regime and gravitational regime. Increasing the height up to
6.4. Coiling of yield stress fluids

Figure 6.14: (Color online) The non-dimensional frequency of coiling of the hair gel filament ($\Omega d/U$) vs. ($d/H$) in a logarithmic scale. In the gravitational regime (G) data points collapse onto a straight line. The dashed line with slope 1 indicates the behavior expected for the viscous regime (V).

In a similar way to what we did for the foam and by using the same relations we can find effective elasticity and effective viscosity for hair gel during the experiments. Using again the data from the gravitational regime, we now find values that are very different: $E_{eff} \sim 1000$ Pa and $\eta_{eff} \sim 10^6$ Pa s. The effective stress in the filament is now one order of magnitude larger than the yield stress of gel, which was about 100 Pa, therefore we can not assume the hair gel as an elastic material during our experiments. This implies that when the stress increases in the gel filament beyond the yield point, it flows like a viscous fluid. The estimated viscosity is nearly $10^5$ times as large as that of honey at room temperature. However, earlier experiments with Newtonian liquid filaments with a similar viscosity shows liquid rope coiling behavior (119). We therefore conclude that, contrary to the foam, the system of hair gel behaves as a very highly viscous coiling system.
Chapter 6. Coiling of Yield Stress Fluids

Figure 6.15: a) Coiling patterns of the hair gel in the viscous regimes. b) Coiling patterns of the hair gel in the gravitational regimes. c,d) Break up of the hair gel filament at large heights.

6.5 Conclusion

The coiling of liquids is surprising, since it implies a solidlike behavior of a simple Newtonian liquid: the liquid filament coils much like a rope does. However, the equations describing ‘liquid’ and ‘solid’ coiling are very different: the material properties of the liquid come in the equations as the viscosity, whereas the properties of the solid rope come in as an elastic modulus. What happens in the case of a visco-elastic liquid, such as the ones used here, was not clear so far. The surprising result we report here is that for the two different yield stress fluids investigated, one shows liquidlike behavior, and one solidlike. This means that in order to predict the coiling behavior, for the foam we need to know its elastic modulus, whereas for the gel we need its viscosity. In addition we find that the crossover between ‘solid’ and ‘liquid’ rope coiling behavior happens around the yield stress of the material, implying that if we know the latter from a rheology measurement, we can predict the coiling behavior. The coiling of yield stress materials is frequently observed, e.g., in filling food containers or extruding polymeric materials; our findings here provide a way to predict the way the material will coil.