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WORKING PAPER RCLR-2023-03

# Decomposing Mortality Improvement Rates into Cause-Specific Contributions

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September 2023

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# Decomposing Mortality Improvement Rates into Cause-Specific Contributions

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The aim of our research is to explain changes in all-cause mortality by identifying the contributions of changes in the mortality from specific causes. We measure changes in mortality using the commonly applied improvement rate and present a simple and intuitive method to identify cause-specific contributions. In that way, we can attribute observed mortality improvements to specific causes of death.

The contribution of individual causes to changes in all-cause mortality and life expectancy has been studied by others. For example, Yiu et al. (2022), consider a collection of log-linear models to identify changes in mortality and life expectancy caused by trend changes for specific causes, and Villegas et al. (2023) apply a period-cohort improvement model to identify key drivers affecting developments in US mortality.

In this paper, we present an approach that allows us to decompose all-cause improvements at a specific age (or age group) that does not rely on a specific mortality model and ensures that cause-specific contributions add up to all-cause improvement rates. Empirical results, both for pre-pandemic and Covid-19 years are presented, and we discuss the consequences of this decomposition for log-linear mortality models.

## 1 General decomposition of realised mortality and improvement rates

We begin with the obvious statement that the total (random) number of deaths  $D_t$  occurring in a single calendar year  $t$  (or any time interval  $t$  to  $t + \Delta t$ ) is the sum over death counts  $D_t^{(i)}$  from each individual cause  $i$ . We denote by  $\mathcal{C}_t$  the set of causes of death that are present in year  $t$ . We then have that

$$D_t = \sum_{i \in \mathcal{C}_t} D_t^{(i)}.$$

To obtain observed mortality (or death) rates we divide the death counts by the exposure to risk  $E_t$ , and it follows immediately that the same relationship holds for the death rate  $\mu_t$  from all causes and the cause-specific rates  $\mu_t^{(i)}$ :

$$\mu_t := \frac{D_t}{E_t} = \sum_{i \in \mathcal{C}_t} \frac{D_t^{(i)}}{E_t} = \sum_{i \in \mathcal{C}_t} \mu_t^{(i)} \quad \text{with } \mu_t^{(i)} := \frac{D_t^{(i)}}{E_t} \quad (1)$$

## 1.1 Annual Improvement Rates

Changes in all-cause mortality and cause-specific mortality from calendar year  $t$  to  $t + 1$  are often reported using the mortality improvement rates, which we denote by  $\rho$  and  $\rho^{(i)}$ , and define through the relationships

$$\mu_{t+1} = (1 - \rho_{t+1})\mu_t \quad \text{and} \quad \mu_{t+1}^{(i)} = (1 - \rho_{t+1}^{(i)})\mu_t^{(i)} \text{ for } i \in \mathcal{C}_t.$$

Note that in these equations a positive improvement rate  $\rho_{t+1}$  is indeed a reduction (an improvement) in mortality.

We now assume that there is no change in the sets of causes  $\mathcal{C}_t$  from one year to the next. Of course, this assumption does not always hold, and we will therefore relax it in Section 4, but for now we assume  $\mathcal{C}_{t+1} = \mathcal{C}_t$ . With the above relationships we immediately obtain

$$1 - \rho_{t+1} = \frac{\mu_{t+1}}{\mu_t} = \sum_{i \in \mathcal{C}_t} \frac{\mu_{t+1}^{(i)}}{\mu_t} = \sum_{i \in \mathcal{C}_t} \frac{\mu_t^{(i)}}{\mu_t} \frac{\mu_{t+1}^{(i)}}{\mu_t^{(i)}} \quad (2)$$

and, defining

$$w_t^{(i)} := \frac{\mu_t^{(i)}}{\mu_t} = \frac{D_t^{(i)}}{D_t}$$

and using the above definition of improvement rates we have

$$1 - \rho_{t+1} = \sum_{i \in \mathcal{C}_t} w_t^{(i)} (1 - \rho_{t+1}^{(i)}). \quad (3)$$

Since  $\sum_i w_t^{(i)} = 1$  we find that all-cause mortality improvement rates are a weighted average of cause-specific improvement rates:

$$\rho_{t+1} = \sum_{i \in \mathcal{C}_t} w_t^{(i)} \rho_{t+1}^{(i)} \text{ with weights } w_t^{(i)} = \frac{D_t^{(i)}}{D_t}. \quad (4)$$

When reporting improvement factors for year  $t + 1$ , we think of the weights  $w_t^{(i)}$  as the relative importance of cause  $i$  for the mortality during the previous year  $t$ .

## 1.2 Small Time steps - Short Term Improvements

For a small time step  $\Delta t$  we can use the approximation

$$\log(\mu_{t+\Delta t}) - \log(\mu_t) \approx \frac{\mu_{t+\Delta t}}{\mu_t} - 1$$

to obtain a result equivalent to (4) (note that  $\sum_i w_t^{(i)} = 1$ )

$$\log(\mu_{t+\Delta t}) - \log(\mu_t) \approx \frac{\mu_{t+\Delta t}}{\mu_t} - 1 = \sum_{i \in \mathcal{C}_t} w_t^{(i)} \left[ \frac{\mu_{t+\Delta t}^{(i)}}{\mu_t^{(i)}} - 1 \right] \quad (5)$$

$$\approx \sum_{i \in \mathcal{C}_t} w_t^{(i)} \left[ \log(\mu_{t+\Delta t}^{(i)}) - \log(\mu_t^{(i)}) \right] \quad (6)$$

We will use this comment when considering log linear mortality rates in Section 3.

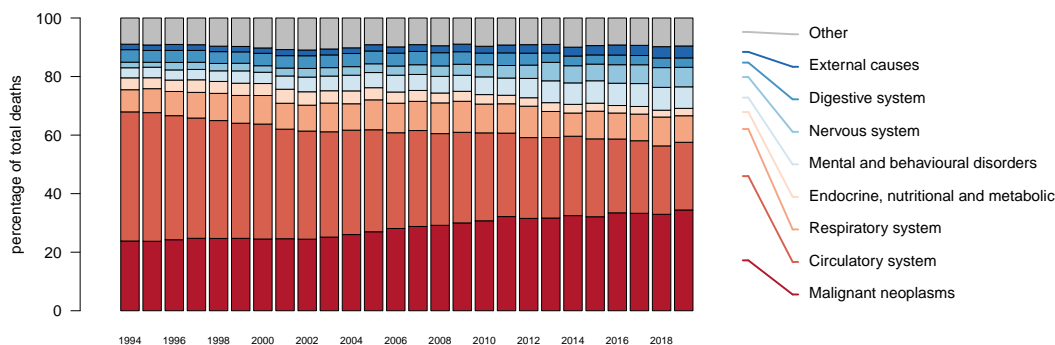


Figure 1: Weights as percentages,  $100w_t^{(i)}$  for calendar years  $t$  from 1994 to 2019 for Dutch women aged 70 to 84.

## 2 Empirical Results - Constant Set of Causes

Using data from the Human Mortality Database<sup>1</sup> for exposures and data from Eurostat for cause-specific death counts<sup>2</sup> we illustrate the decomposition in (4).

We illustrate this approach using data from the Netherlands; Figure 1 shows the weights  $w_t^{(i)}$  as percentages for the years 1994 to 2019 for the selected groups of causes.

In Figure 1 we observe that in 1994 diseases of the circulatory system accounted for more than 40% of all deaths among Dutch women aged 70 to 84. The weight for that cause of death was reduced over the years, and only about 20% of Dutch Women in that age group died of such diseases in 2019. In contrast we also find that the weights for cancers, diseases of the nervous system, and mental disorders have been increasing. From the decomposition in (4) we conclude that all-cause mortality improvements are now less related to improvement rates for deaths from diseases of the circulatory system, and that the more common causes mentioned above have a greater impact now as the weights  $w^{(i)}$  for those causes have increased over time.

The second ingredient to explain all-cause mortality improvements are the cause-specific mortality rates which are shown in Figure 2 for the same population - Dutch women aged 70 to 84. The developments in Figures 1 and 2 are, of course, not independent: causes with the strongest improvement rates have decreasing weights and vice versa. However, the two graphs show that, assuming past trends continue, we can expect that future all-cause mortality improvements will depend more on causes that have seen little or no improvements in recent years, and less on further strong improvements in mortality from diseases of the circulatory system.

Combining the two components, weights and mortality improvement rates, we can also show the contributions  $w_{t-1}^{(i)}\rho_t^{(i)}$  of each group of causes to all-cause improvements. Those contributions are shown in Figure 3. In this figure each

<sup>1</sup>[www.mortality.org](http://www.mortality.org)

<sup>2</sup>[ec.europa.eu/eurostat/web/health/database](http://ec.europa.eu/eurostat/web/health/database)

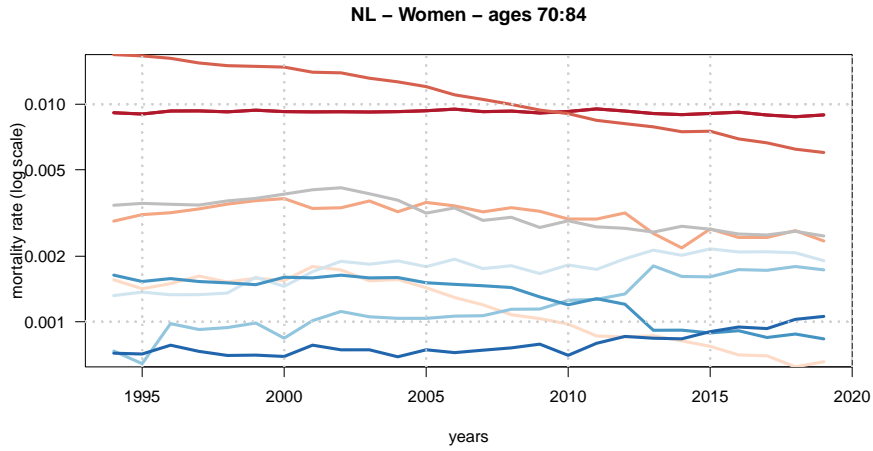


Figure 2: Observed cause-specific death rates  $\mu_t^{(i)}$  for calendar years  $t$  from 1994 to 2019 for Dutch women aged 70 to 84 (colour coding as in Figure 1).

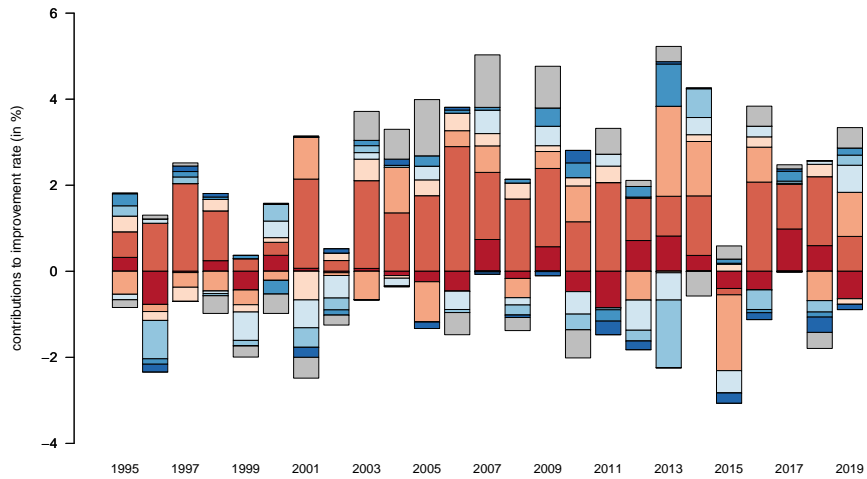


Figure 3: Cause-specific contributions to all-cause improvement rates,  $w_{t-1}^{(i)} \rho_t^{(i)}$  for calendar years  $t$  from 1995 to 2019 for Dutch women aged 70 to 84 (colour coding as in Figure 1). All-cause improvements are given by the total height of the positive bar reduced by the height of the negative bar. The height of the bar for each individual colour represents the all-cause improvement rate that would have been observed if all other causes had seen no improvements.

single colour bar represents the term  $w_{t-1}^{(i)}\rho_t^{(i)}$  for the specific cause  $i$ . If a single colour bar is shown on the positive axis, the corresponding cause has contributed to a reduction in mortality. Therefore, the combined bars on the positive axis show the positive contributions to improvement rates, and the combined bars on the negative axis show the contributions that have reduced all-cause improvements.

All-cause improvement rates  $\rho_t$  are given by the total height of the positive bar reduced by the height of the negative bar. An alternative interpretation is that the height of the bar for each individual colour represents the all-cause improvement rate that would have been observed if all other causes had seen no improvements.

Figure 3 shows that there is considerable variation in the improvement rates. More importantly in the context of this paper, the figure also shows that we observed strong variations in the contributions that each cause has made to all-cause improvements.

### 3 Implications for Log-Linear Mortality Models

In Section 1 we considered realised death rates,  $\mu_t = D_t/E_t$ . With a slight abuse of notation we will now consider the relationships found above for the actual underlying mortality rates (for which the death rates are an estimate, Pitacco et al. (2009) and Macdonald et al. (2018)). This allows us to discuss implications of the above relationships for mortality models.

A commonly made assumption is that log mortality rates at specific ages have a linear time trend. For example, assuming a Lee-Carter model with a period effect modelled by a random walk with constant drift implies a linear time trend of log mortality rates (in expectation).

Considering the empirical results in Section 2 we argue that this assumption would also be reasonable for many, if not all, causes of deaths. To investigate the implication of such log-linear models, we assume that for each cause  $\log \mu_t^{(i)}$  is linear in  $t$  with cause-specific slope  $a_i$  and intercept  $b_i$ , that is,

$$\mu_t^{(i)} = \exp(a_i t + b_i) \quad (7)$$

We then find

$$\log(\mu_{t+\Delta t}) - \log(\mu_t) \approx \left( \sum_i w_t^{(i)} a_i \right) \Delta t \quad (8)$$

$$= \left( \sum_i \frac{\mu_t^{(i)}}{\sum_i \mu_t^{(i)}} a_i \right) \Delta t \quad (9)$$

$$= \left( \frac{\sum_i \mu_t^{(i)} a_i}{\mu_t} \right) \Delta t \quad (10)$$

Therefore, the drift of all-cause log mortality,  $\log \mu_t$ , will, in general, not be constant over time if the drift terms  $a_i$  of the log mortality rates,  $\log \mu^{(i)}$  from individual causes are all constant but different from each other.

An alternative approach to derive the slope coefficient of all-cause log mortality is to consider  $\mu_t$  and  $\mu_t^{(i)}$  as continuous functions of time. We then obtain for the

change in all-cause mortality from  $s$  to  $t > s$ :

$$\mu_t - \mu_s = \sum_i \left[ \mu_t^{(i)} - \mu_s^{(i)} \right] \quad (11)$$

$$= \sum_i \left[ \exp(a_i t + b_i) - \exp(a_i s + b_i) \right] \quad (12)$$

$$= \sum_i \int_s^t a_i \exp(a_i u + b_i) du \quad (13)$$

$$= \sum_i a_i \int_s^t \mu_u^{(i)} du \quad (14)$$

Defining the average mortality rate from cause  $i$  during the period  $s$  to  $t$  as

$$\bar{\mu}_{[s,t]}^{(i)} = \frac{1}{t-s} \int_s^t \mu_u^{(i)} du$$

we, again, find that the change in all-cause mortality per time unit is a weighted average of the cause-specific slopes weighted with the importance of each cause now measured by their average mortality during the period  $[s, t]$ :

$$\mu_t = \mu_s + \left( \sum_i a_i \bar{\mu}_{[s,t]}^{(i)} \right) (t-s). \quad (15)$$

## 4 New Causes of Death - Covid-19

At the time of writing, mankind has just been reminded that mortality does not always change for the better, and that new causes can emerge at any time. The Covid-19 pandemic has been the subject of much research. In the field of mortality modelling, the focus has been on how to treat data from the Covid-19 years when fitting models, and what assumptions to make about future developments of this new cause of death and the possible emergence of others, see for example, Robben et al. (2022), Schnürch et al. (2023), van Berkum et al. (2022) and references therein.

We argue that considering models for the joint development of cause-specific rates is a good approach to incorporating a new cause of death into our models. In this section we will extend the results about observed death rates from Section 1 to include newly emerging causes of death.

As in Section 1 we define  $\mathcal{C}_t$  to be the set of causes that people have died of during year  $t$ , but we now drop the assumption that those sets are not changing over time. In particular, we allow for those sets to increase, that is, new causes of death can emerge. We do not consider the case of causes being extinct, as this can easily be dealt with in the setting above.

We assume that for a certain year  $t$  we have that  $\mathcal{C}_t \subset \mathcal{C}_{t+1}$ , that is, there exist new causes  $k \in \mathcal{C}_{t+1}$  that did not cause any deaths in year  $t$ . We note that it is not possible to proceed by augmenting  $\mathcal{C}_t$  to include a new cause  $k$  and then applying the results from Section 1 since that would imply that  $\mu_t^{(k)} = 0$  in (2) and the improvement rate  $\mu_{t+1}^{(k)}/\mu_t^{(k)}$  for cause  $k$  is not defined.

To obtain the contribution of all new causes  $k \in \mathcal{C}_{t+1}$  (that are not in  $\mathcal{C}_t$ ) to the all-cause improvement rate  $\rho_{t+1}$ , we return to (1) and split the sum over all causes into those causes that have been present in year  $t$  and new causes:

$$\mu_{t+1} = \sum_{i \in \mathcal{C}_{t+1}} \mu_{t+1}^{(i)} = \sum_{i \in \mathcal{C}_t} \mu_{t+1}^{(i)} + \sum_{k \notin \mathcal{C}_t} \mu_{t+1}^{(k)} \quad (16)$$

Turning our focus to the improvement rate  $\rho_{t+1}$ , we divide by the all-cause mortality in year  $t$  and find

$$\frac{\mu_{t+1}}{\mu_t} = \frac{\sum_{i \in \mathcal{C}_t} \mu_{t+1}^{(i)} + \sum_{k \notin \mathcal{C}_t} \mu_{t+1}^{(k)}}{\mu_t} \quad (17)$$

$$= \sum_{i \in \mathcal{C}_t} w_t^{(i)} \frac{\mu_{t+1}^{(i)}}{\mu_t^{(i)}} + \frac{1}{\mu_t} \sum_{k \notin \mathcal{C}_t} \mu_{t+1}^{(k)}, \quad w_t^{(i)} = \frac{D_t^{(i)}}{D_t} \quad (18)$$

With the same arguments as in Section 1 and using the fact that  $\sum_{i \in \mathcal{C}_t} w_t^{(i)} = 1$  we now find for the improvement rate from year  $t$  to  $t + 1$ :

$$\rho_{t+1} = \sum_{i \in \mathcal{C}_t} w_t^{(i)} \rho_{t+1}^{(i)} - \frac{1}{\mu_t} \sum_{k \notin \mathcal{C}_t} \mu_{t+1}^{(k)} \quad \text{with} \quad w_t^{(i)} = \frac{D_t^{(i)}}{D_t} \quad (19)$$

Therefore, all-cause improvement rates are now the weighted average of cause-specific improvements for those causes present in the previous period, reduced by the relative importance of mortality from all new causes in relation to all-cause mortality in the previous year.

## 5 Empirical Results for Covid-19

Combining all ICD10 codes for Covid-19 into one group, we find for the mortality improvement rate  $\rho_{2020}$  from 2019 to 2020:

$$\rho_{2020} = \sum_{i \in \mathcal{C}_t} w_{2019}^{(i)} \rho_{2020}^{(i)} - \frac{\mu_{2020}^{\text{Covid}}}{\mu_{2019}} \quad (20)$$

For the following years (assuming no further new causes of death) we can apply the result in (4).

As an example for the decomposition in (19) and of the impact of Covid-19 on all-cause mortality we consider data for women in Switzerland aged 70 to 84. Relevant data at the Human Mortality Database ([www.mortality.org](http://www.mortality.org)) and Eurostat are available for the years 1994 to 2021. As in Section 2, we plot the weights  $w^{(i)}$ , the cause-specific death rates  $\mu^{(i)}$  and the cause-specific contributions to all-cause improvement,  $w_{t-1}^{(i)} \rho_t^{(i)}$  in Figures 4 - 6.

The figures show that (for this specific population) the importance of Covid-19 (Fig. 4) as a cause of death in 2021 was similar to the importance of mental and behavioural disorders as cause of death, in 2020 Covid-19 was more important than mental disorders, and in both years substantially more women died of diseases of the circulatory system or cancers.

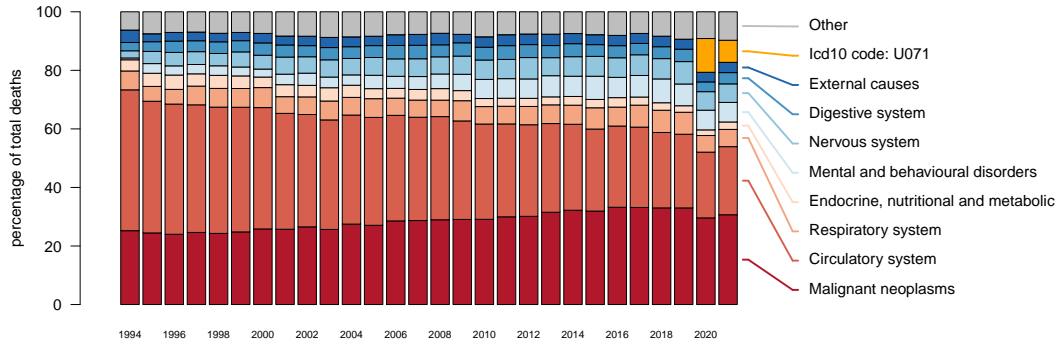


Figure 4: Weights as percentages,  $100w_t^{(i)}$  for calendar years  $t$  from 1994 to 2021 for women in Switzerland aged 70 to 84.

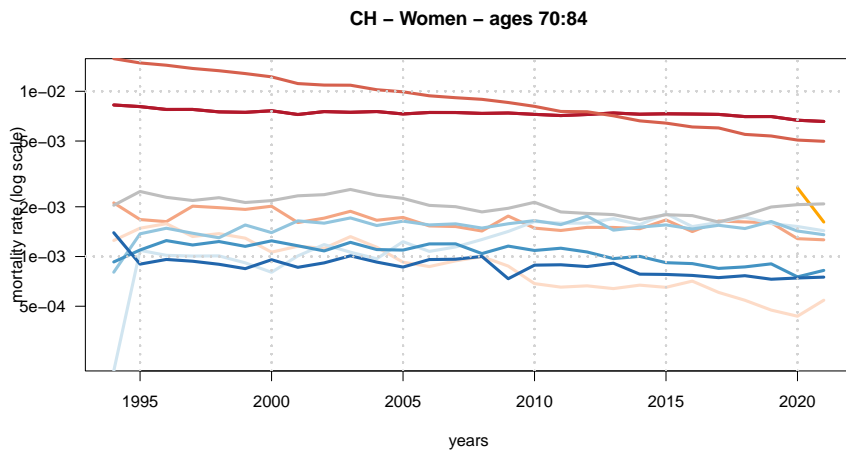


Figure 5: Observed cause-specific death rates  $\mu_t^{(i)}$  for calendar years  $t$  from 1994 to 2021 for women in Switzerland aged 70 to 84 (colour coding as in Figure 4).

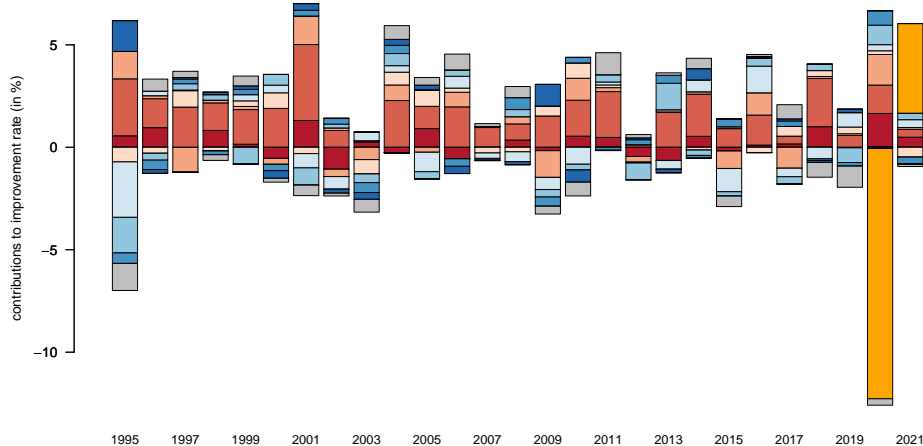


Figure 6: Cause-specific contributions to all-cause improvement rates,  $w_{t-1}^{(i)}\rho_t^{(i)}$  for calendar years  $t$  from 1994 to 2021 for women in Switzerland aged 70 to 84 (colour coding as in Figure 4). All-cause improvements are given by the total height of the positive bar reduced by the height of the negative bar. The height of the bar for each individual colour represents the all-cause improvement rate that would have been observed if all other causes had seen no improvements.

On the other hand, the contribution of Covid-19 to overall improvements from 2019 to 2020 was huge, not just cancelling out the combined improvements of about 6% from all other causes, but leading to increased mortality in 2020 compared to the previous. It is worth noting, that improvements without Covid-19 would have been positive assuming that presence of Covid-19 has not altered the development in other causes - a strong assumption.

In the following year, 2021, mortality improved, and those improvements were mainly driven by the reduction of Covid-19 mortality combined with its very high weight in 2020. This is different for other populations, for example, Swiss women 65-69, Covid-19 had a negative impact on improvement rates from 2020 to 2021, see Figure 8 (top left plot) in the appendix. In the Czech Republic, Covid-19 had a very strong negative effect on all-cause improvement rates from 2019 to 2020 and in the following year for both sexes and three different age groups (65-69, 70-84, 85+), see Figure 9.

We also show the results for both men and women and three age groups for the Netherlands and for Spain, see figures in the appendix. For Spain, the Covid-19 effect is very strong, but additionally there are some strong negative contributions from other causes in 2020. This needs further investigation.

## 6 Conclusions

To further our understanding of changes in human mortality and to improve mortality model it helps to identify contributions from individual causes to changes in all-cause mortality. The decomposition of improvement rates in (4) (and (19))

is one way to achieve this. Compared to other approaches, it has the advantage that contributions for individual causes add up to the total improvement rate, and that it does not rely on a specific mortality model or assumptions about the dependencies between causes.

We note that the relationships between all-cause mortality and cause-specific mortality in Section 1 are relationships between random variables with all the usual implications for the expectation, variance and distribution. Of course, those relationships also apply to the observed death rates.

## Acknowledgement

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## A Further Empirical Results

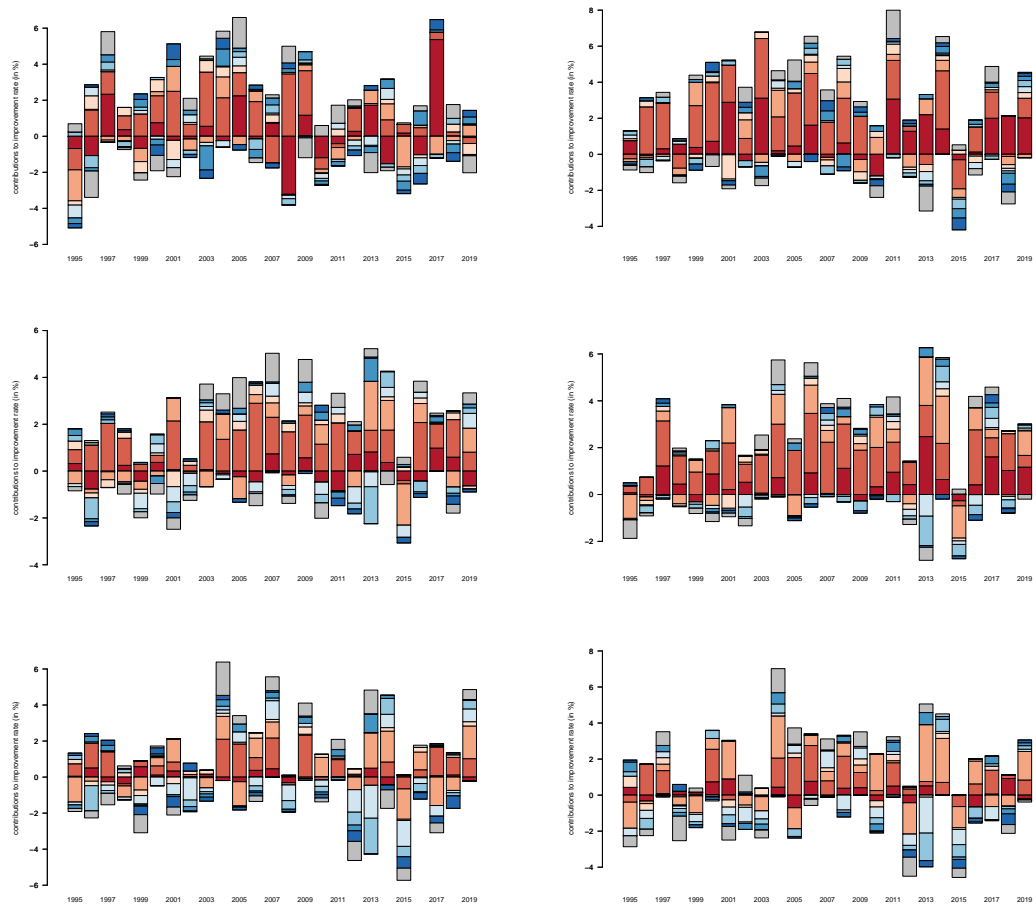


Figure 7: Cause-specific contributions to all-cause improvement rates,  $w_{t-1}^{(i)}\rho_t^{(i)}$ , in the Netherlands for age groups 65-69, 70-84 and 85+ (top to bottom), women (left) and men (right).

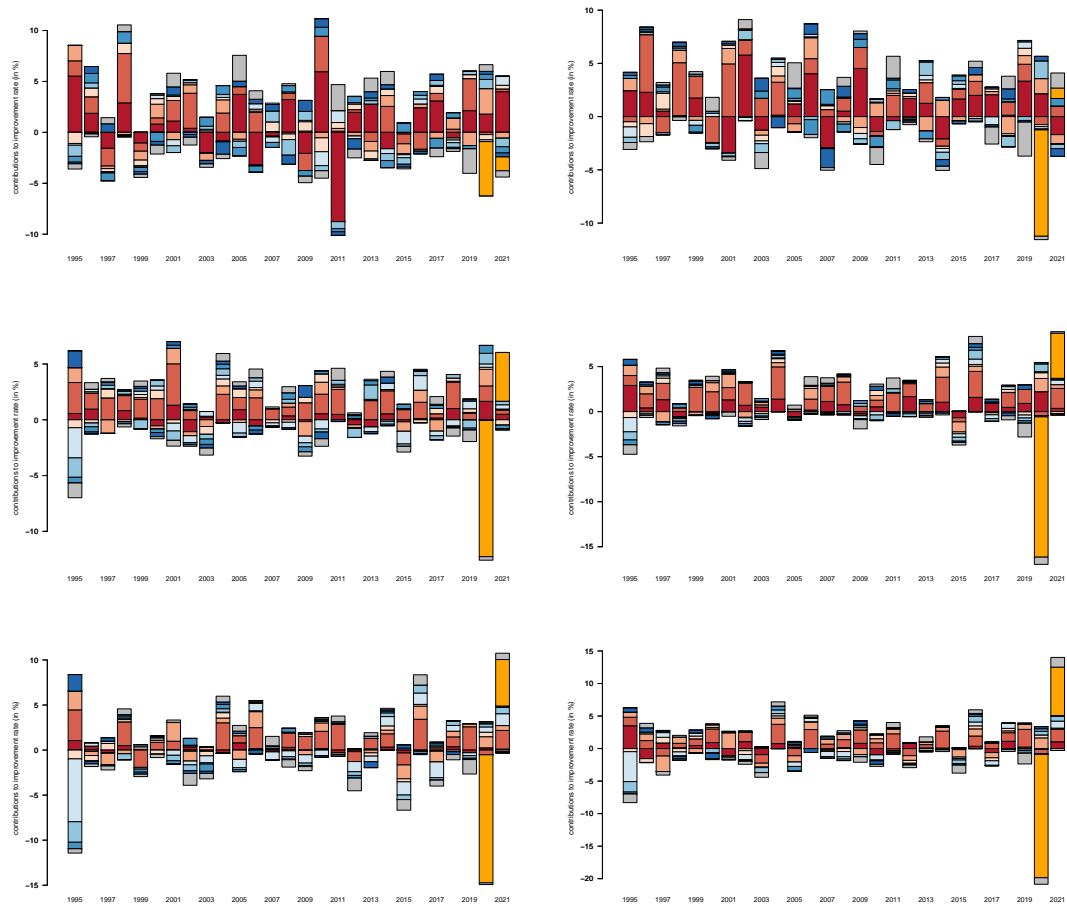


Figure 8: Cause-specific contributions to all-cause improvement rates,  $w_{t-1}^{(i)}\rho_t^{(i)}$ , in Switzerland for age groups 65-69, 70-84 and 85+ (top to bottom), women (left) and men (right).

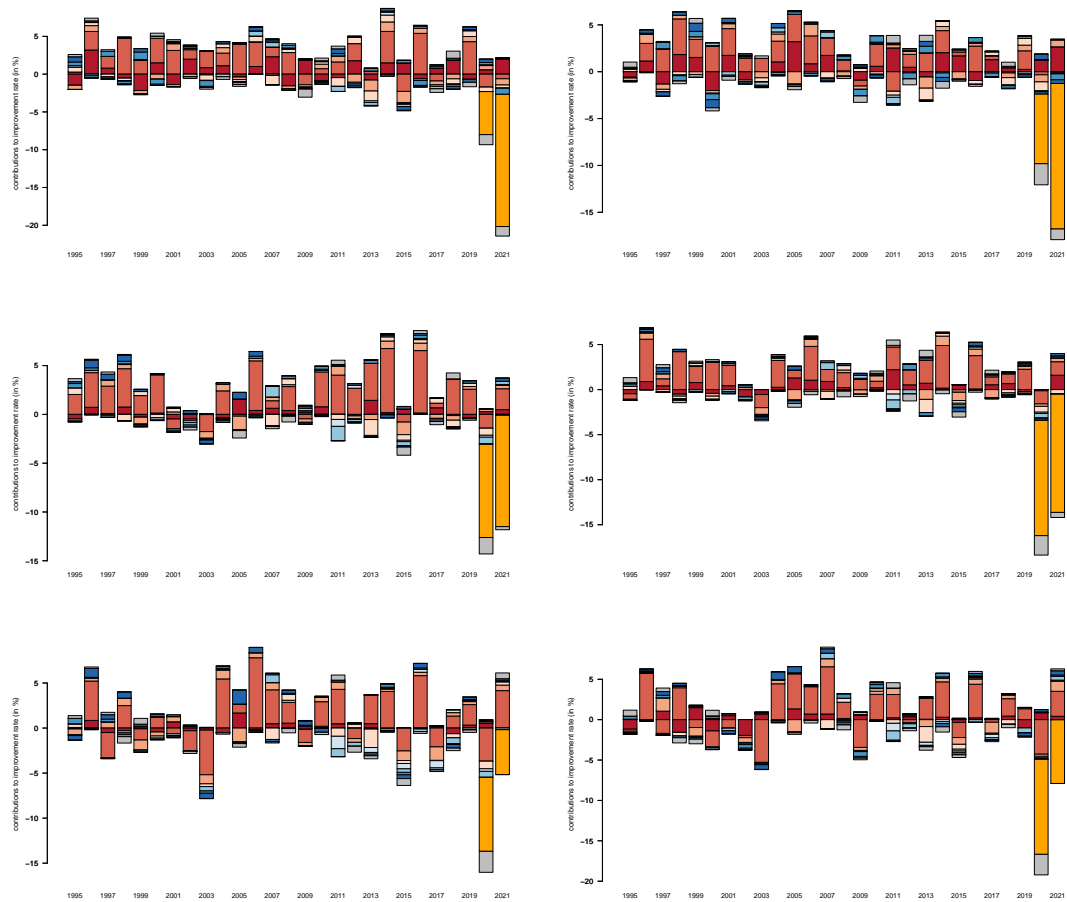


Figure 9: Cause-specific contributions to all-cause improvement rates,  $w_{t-1}^{(i)}\rho_t^{(i)}$ , in the Czech Republic for age groups 65-69, 70-84 and 85+ (top to bottom), women (left) and men (right).

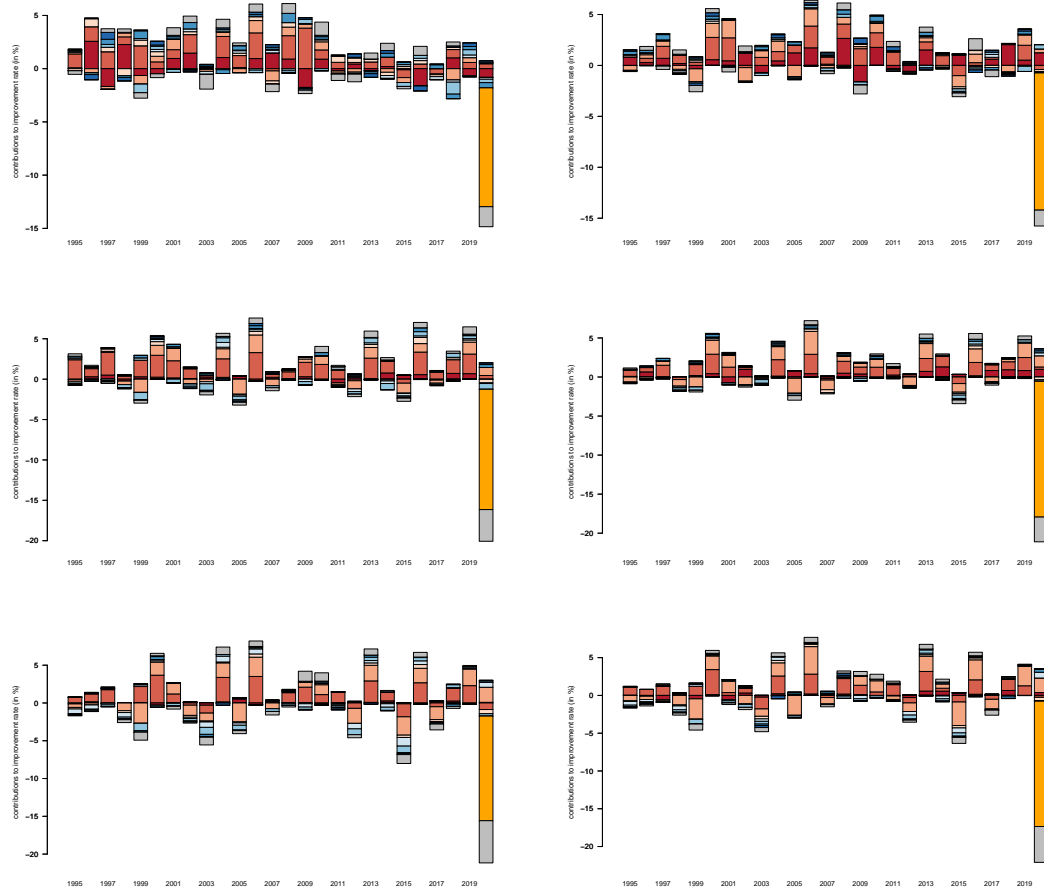


Figure 10: Cause-specific contributions to all-cause improvement rates,  $w_{t-1}^{(i)}\rho_t^{(i)}$ , in Spain for age groups 65-69, 70-84 and 85+ (top to bottom), women (left) and men (right).

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