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Resolving structural errors in a spatially distributed hydrologic model using ensemble Kalman filter state updates

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Abstract. In hydrological modeling, model structures are developed in an iterative cycle as more and different types of measurements become available and our understanding of the hillslope or watershed improves. However, with increasing complexity of the model, it becomes more and more difficult to detect which parts of the model are deficient, or which processes should also be incorporated into the model during the next development step. In this study, we first compare two methods (the Shuffled Complex Evolution Metropolis algorithm (SCEM-UA) and the Simultaneous parameter Optimization and Data Assimilation algorithm (SODA)) to calibrate a purposely deficient 3-D hillslope-scale model to error-free, artificially generated measurements. We use a multi-objective approach based on distributed pressure head at the soil–bedrock interface and hillslope-scale discharge and water balance. For these idealized circumstances, SODA’s usefulness as a diagnostic methodology is demonstrated by its ability to identify the timing and location of processes that are missing in the model. We show that SODA’s state updates provide information that could readily be incorporated into an improved model structure, and that this type of information cannot be gained from parameter estimation methods such as SCEM-UA. We then expand on the SODA result by performing yet another calibration, in which we investigate whether SODA’s state updating patterns are still capable of providing insight into model structure deficiencies when there are fewer measurements, which are moreover subject to measurement noise. We conclude that SODA can help guide the discussion between experimentalists and modelers by providing accurate and detailed information on how to improve spatially distributed hydrologic models.

1 Introduction

Our understanding of hillslope and watershed hydrology is typically summarized in numerical models. Ideally, such models are the result of an iterative process that involves modeling, experimental design, data collection, and analysis of the model–data mismatch (e.g. Box and Tiao, 1973, Sect. 1.1.1 “The role of statistical methods in scientific investigation” and Popper, 2009, Sect. 1.1.3 “Deductive testing of theories”). Especially when combined with laboratory experiments, this iterative research cycle has proven to be a useful method for theory development. Its usefulness stems from the fact that in laboratory experiments, the state of the system under study as well as its parameters and the forcings/disturbances to which the system is subjected can usually be measured more or less accurately. This allows the investigation to focus on the one remaining uncertain factor, namely the hypothesis/model structure. There is, however, a stark contrast between experiments carried out in the laboratory and those carried out in the field. As hydrologists, we are often dealing with open systems (e.g. von Bertalanffy, 1950), meaning that flows such as precipitation, groundwater recharge, and evapotranspiration cross the system’s boundary. Unfortunately, we often lack the necessary technology to observe these flows (or how they affect the state of the system) at the scale triplet (e.g. Blöschl and Sivapalan, 1995; Western and Blöschl, 1999) of interest, and manipulation experiments are generally impossible (e.g. Young, 1983). Furthermore, many hydrological models have parameters that cannot be measured directly, either because of practical considerations or because the parameters are conceptual. The uncertainty associated with the parameters, state, forcings, and
output makes theory development at the scale of watersheds and hillslopes much more difficult than for small-scale experiments in the laboratory.

So, it is certainly not straightforward to collect enough data of sufficient quality in field experiments. This is not the only challenge though: making sense of the data (i.e. analysis) has proven just as difficult. In the remainder of this paper, we will focus on the latter problem. When discussing the analysis stage of the iterative research cycle, it is useful to distinguish between two possible scenarios. In the first scenario, the modeling is performed because a prediction is needed (for instance in support of estimating the chance of a flood of a certain magnitude). In this context, a good predictive model is one that is capable of estimating the variable of interest with little bias and small uncertainty, which can be demonstrated by performing a traditional split-sample test (e.g. Klemeš, 1986). In this scenario, the mechanisms underpinning the model structure need not concern the modeler too much – the important thing is that the model gives the right answer, even when it does so for the wrong reasons (e.g. Kirchner, 2006).

Being right for the wrong reasons is not acceptable in the second scenario, in which the purpose of the modeling is to test and improve our understanding of how hillslopes and watersheds function. Since it is axiomatic that for complex systems the initial model structure is at least partly incorrect, the challenge that we are facing in the analysis stage of the iterative research cycle is how to diagnose the current, incorrect model structure, such that we can make an informed decision on what needs to be changed for the next, hopefully more realistic model structure (e.g. Gupta et al., 2008).

A common way of diagnosing how a given model can be improved is through an analysis of model-observation residuals. It is important to note, though, that such an analysis is only possible after the model has been parameterized. In case the model parameters cannot be measured directly, the parameter values need to be determined by means of parameter estimation methods. In recent years, various authors have discussed the pitfalls associated with parameter estimation, specifically when applied to cases in which data error and model structural error cannot be neglected (e.g. Kirchner, 2006; Ajami et al., 2007). For example, it has been demonstrated how model parameters can compensate for model structural errors by assuming unrealistic values during parameter estimation (e.g. Clark and Vrugt, 2006). Without the right parameter values, interpretation of the residual patterns – and therefore model improvement – becomes much more difficult. To overcome these difficulties, various lines of research have been proposed that attempt to increase the diagnostic power of the analysis by extending the traditional parameter estimation paradigm in various ways.

For example, one line of research has argued that a multi-objective approach can provide more insight into how a model structure may be deficient (Yapo et al., 1998; Gupta et al., 1998). In the multi-objective approach, the performance of each model run is evaluated using not just one but multiple objectives. Individual objectives can vary in the function used (RMSE, HMLE, mean absolute error, Nash–Sutcliffe efficiency, etc.; e.g. Gupta et al., 1998), in the variable that the objective function operates on (streamflow, groundwater tables, isotope composition, major ion concentrations, etc.; e.g. Mroczkowski et al., 1997; Franks et al., 1998; Kuczera and Mroczkowski, 1998; Dunn, 1999; Seibert, 2000), or in the transformation, selection, or weighting that is used (e.g. Vrugt et al., 2003a; Tang et al., 2006). After a number of model runs have been executed, the population of model runs is divided into a “good” set and a “bad” set. The good set consists of points that are non-dominated, meaning that any point in this set represents in some way a best point. Together, the non-dominated points make up the Pareto front (Goldberg, 1989; Yapo et al., 1998). The multi-objective approach is useful for model improvement because it enables analyzing the trade-offs that occur between various objectives in the Pareto front. If the various objectives have been formulated such that individual objectives predominantly reflect specific aspects of the system under consideration, then inferences can be made about the appropriateness of these aspects (Gupta et al., 1998; Yapo et al., 1998; Boyle et al., 2000, 2001; Wagener et al., 2001; Bastidas et al., 2006). For a recent review of the multi-objective approach, see Efstratiadis and Koutsoyiannis (2010).

A second line of research abandons the idea of using just one model structure for describing system behavior but instead uses an ensemble of model structures. The ensemble may be composed of multiple existing model structures that are run using the same initial state and forcings (e.g. Georgakakos et al., 2004). Alternatively, the ensemble may be made up of model structures that are assembled from a limited set of model structure components using a combinatorial approach (e.g. Clark et al., 2008). The predictions generated by members of the ensemble may further be combined in order to maximize the predictive capabilities of the ensemble, for example by using Bayesian model averaging (e.g. Hoeting et al., 1999; Raftery et al., 2003, 2005; Neuman, 2003). Regardless of how the ensemble was constructed, differences between members of the ensemble can be exploited to make inferences about the appropriateness of specific model components.

The idea underpinning the third line of research originates with calibration attempts in which it was found that the optimal values of a given model’s parameters tend to change depending on what part of the empirical record is used in calibration (see for example Fig. 2b in Gupta et al., 1998). This is generally taken as an indication that the model is structurally deficient, because it is unable to reproduce the entire empirical record with a single set of parameters (Gupta et al., 1998; Yapo et al., 1998; Wagener et al., 2001; Lin and Beck, 2007). Due to the deficiency, the model does not extract all of the information that is present in the observations, which in turn means that the residuals contain “information
... errors (Doherty and Welter, 2010). Over the last few decades, various mechanisms have been proposed with which such misplaced information can be accommodated. For example, the time-varying parameter (TVP) approach (Young, 1978) and the related state-dependent parameter (SDP) approach (Young, 2001) relax the assumption that the model parameters are constant during the entire empirical record. Somewhat related to TVP is the DYNIA approach of Wagener et al. (2003). DYNIA attempts to isolate the effects of individual model parameters. To do so, it uses elements of the well-known generalized sensitivity analysis (GSA) and generalized likelihood uncertainty estimation (GLUE) methods (Spear and Hornberger, 1980; Beven and Binley, 1992). DYNIA facilitates making inferences about model structure by analyzing how the probability distribution of the parameter values changes over simulated time, and by analyzing how the distribution is affected by certain response modes, such as periods of high discharge.

A similar approach was taken in Sieber and Uhlenbrook (2005), who used linear regression to analyze how parameter sensitivity varied over simulated time and in relation to additional variables. This allowed them to make inferences about the appropriateness of the model structure and provided insight into when certain model parameters were relevant and when they were not. Reusser and Zehe (2011) combined a time-variable parameter sensitivity analysis with multiple objectives. In their approach, the objective scores are aggregated using a clustering algorithm. Individual cluster members thus represent a certain type of deviation between the simulated behavior and the observed behavior (for instance, the simulation lags behind the observations). Subsequent analysis of when certain cluster members were dominant, combined with the (in)sensitivity of the parameters at that time, proved useful in determining the appropriateness or otherwise of certain model components, as well as in distinguishing between data error and model structure error.

Relaxing the assumption that the parameters are constant with time is not the only mechanism with which misplaced information may be accommodated though; some authors have advocated the introduction of auxiliary parameters, whose primary purpose is to absorb the misplaced information, such that the actual model parameters can adopt physically meaningful values during parameter estimation (e.g. Kavetski et al., 2006a,b; Doherty and Welter, 2010; Schoups and Vrugt, 2010).

In contrast to parameter-oriented methods described above, state-oriented methods let the misplaced information be absorbed into the model states. The most widespread of the state-oriented methods is the Kalman filter (KF; Kalman, 1960) and its derivatives, notably the extended KF (EKF; e.g. Jazwinski, 1970) and the ensemble KF (EnKF; Evensen, 1994, 2003). The family of KFs has further been extended with that of particle filters (PFs), which have become popular due to their ability to cope with complex probability distributions. Both KFs and PFs use a sequential scheme to propagate the model states through simulated time. In this sequential scheme, the simulation continues until the next measurement becomes available. At that point in simulated time, the simulation is temporarily halted and control is passed to the filter. The filter compares the state value suggested by the model (i.e. the prior model state) with the state value suggested by the measurement, and calculates the value of the posterior model state. The specific way in which the calculation is done depends on the type of filter but generally takes into account the uncertainties associated with the simulated and measured state values. For example, if more confidence is placed on, say, the measured state value than on the simulated state value, the posterior state value will generally be closer to the measured value than to the simulated value. The simulation is then resumed, starting from the posterior state value (as opposed to the prior state value). The process of halting the simulation, calculating the posterior, and resuming the simulation is continued until all measurements have been assimilated.

The sequential nature of this process allows for retaining information about when and where simulated behavior deviates from what was observed. This is a particularly attractive property when the objective is to evaluate and improve a given model. Nonetheless, filtering methods have hitherto been used mostly to improve the accuracy and precision of either the parameter values themselves or the predictions made with those parameters (Eigbe et al., 1998). That is, the focus has been on the a posteriori estimates. In contrast, we argue that an analysis of how the a priori estimates are updated may yield valuable information about the appropriateness of the model structure: if there are no apparent patterns in the updating, the model structure is as good as the data allow. On the other hand, if there are patterns present in the updating, an alternative model formulation exists that better captures the observed dynamics. Analysis of state updating patterns could thus provide a much-needed diagnostic tool for improving model structures.

The aim of our study is to demonstrate that, when a model does not have the correct structure given the data,

1. parameter estimation may yield residual patterns in which the origin of the error is obscured due to compensation effects;
2. combining parameter estimation with ensemble Kalman filtering provides accurate and specific information that can readily be applied to improve the model structure.

By using artificially generated measurements, we avoid any issues related to accuracy and precision of field measurements, as well as any issues related to incommensurability of field measurements and their model counterparts.
2 Methods

This study consists of four parts: (1) generation of an idealized data set; (2) calibration with a parameter estimation algorithm, the Shuffled Complex Evolution Metropolis algorithm (SCEM-UA; Vrugt et al., 2003b); (3) calibration with a combined parameter and state estimation algorithm, the Simultaneous parameter Optimization and Data Assimilation (SODA; Vrugt et al., 2005); and (4) calibration with SODA using a reduced data set which is furthermore subject to measurement noise.

In the first part, we generated artificial measurements by simulating the hydrodynamics of a small, hypothetical hillslope with a relatively shallow soil, using the SWMS_3D model for variably saturated flow (Šimůnek, 1994; Šimůnek et al., 1995). We then introduced a model structural error by making some small simplifications to the model structure. Hereafter, we use the terms “reference model” and “simplified model” to differentiate between these two model structures. Due to the simplifications, the simplified model is structurally deficient: it does not fully capture the complexity apparent in the artificial measurements. In the second and third part of this study, the simplified model was calibrated to the idealized data set using SCEM-UA and SODA, respectively. We analyzed the model output associated with the optimal parameter combination(s) for both methods, and we evaluated how useful each result was for identifying the structural deficiency in the simplified model. In the fourth part of this study, we expand on the SODA results from part 3 by performing another SODA calibration, but this time using a reduced set of measurements, which are moreover subject to measurement noise.

2.1 Generation of the artificial measurements

We used the SWMS_3D model (Šimůnek, 1994; Šimůnek et al., 1995) to generate artificial measurements. SWMS_3D implements the Richards equation for variably saturated flow through porous media (Richards, 1931):

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial s} \left[ K(h) \frac{\partial (h + z)}{\partial s} \right] - B,$$

in which \( \theta \) is the volumetric water content, \( t \) is time, \( s \) is distance over which the flow occurs, \( K \) is hydraulic conductivity, \( h \) is pressure head, \( z \) is gravitational head, and \( B \) is a sink term. While \( B \) is normally used for simulating water extraction by roots, we instead used it to simulate downward vertical loss of water from the soil domain to the underlying bedrock. The SWMS_3D model solves the Richards equation using the Mualem–van Genuchten functions (van Genuchten, 1980):

$$\theta(h) = \left\{ \begin{array}{ll}
\theta_s + \left( \frac{\theta_i - \theta_s}{\left[1 + n \cdot \left(\frac{h}{\theta_s}\right)^m\right]^{1/n}} \right) & h < 0 \\
\theta_s & h \geq 0
\end{array} \right.$$

The soil domain is represented by a grid of 15 rows, 7 columns and 5 layers of nodes. The soil depth is spatially variable, ranging from 0.16 to 1.47 m (Fig. 2). Horizontally, the nodes are regularly spaced at 3 m intervals. Vertically, the nodes are distributed uniformly over the local soil depth (Fig. 1). In what follows, we use a shorthand notation for the horizontal location of a node: e.g. X03Y12 refers to a location 3 m from the left of the hillslope and 12 m from the seepage face at the bottom. Unless specifically stated otherwise, this notation always refers to the lowest of 5 nodes at a given XY location. The top of the domain represents the atmosphere–soil interface. It is a more-or-less planar surface with an incline of approximately 13°. The bottom of the domain represents the soil–bedrock interface. The model exclusively simulates the hydrodynamics of the soil domain: neither the atmosphere nor the bedrock is explicitly included in the model. Instead, the interface between atmosphere and soil is treated as a source of soil water, whereas the soil–bedrock interface is treated as a sink. In order to mimic typical field situations, the sink mechanism is set up as a spatially heterogeneous process. Using this configuration, we represent bedrock material that is somewhat permeable in most places, but that also has small areas where the bedrock material has disintegrated. In these areas, transient saturation infiltrates the underlying bedrock more quickly.

To simulate the vertical loss of water from the soil domain, we let the sink term \( B \) in Eq. (1) operate on all nodes at the soil–bedrock interface except those that were part of the seepage face (Fig. 1). We used a spatially heterogeneous pattern for the sink rate (Fig. 2). Nodes X00Y18, X06Y30, X09Y15, X15Y39 and X18Y09 were assigned a relatively high sink rate; hereafter, they are referred to as “hotspots”. The other nodes for which we enabled the sink term were assigned a relatively low value. The magnitude of the sink term is determined according to

$$B = \left\{ \begin{array}{ll}
0 & h < 0 \\
\rho_{\text{sink}}(\text{high}) \cdot h & h \geq 0 \text{ hotspots} \\
\rho_{\text{sink}}(\text{low}) \cdot h & h \geq 0 \text{ not a hotspot}
\end{array} \right.$$

in which \( \rho_{\text{sink}}(\text{high}) \) and \( \rho_{\text{sink}}(\text{low}) \) are sink efficiency parameters. \( \rho_{\text{sink}}(\text{high}) \) was set to 0.30 h⁻¹ while \( \rho_{\text{sink}}(\text{low}) \) was set to

$$K(h) = \left\{ \begin{array}{ll}
K_s \cdot S_e^{1/2} \left[ 1 - \left( 1 - S_e^{1/2} \right)^m \right]^2 & h < 0 \\
K_s & h \geq 0
\end{array} \right.$$
We took the soil–bedrock interface as the reference level for pressure head. For what follows, it is useful to define \( S_{t=\infty} \) as the model state, i.e. the 3-D pressure head pattern at time \( x \). In this paragraph we describe how we determined the initial state \( S_{t=0} \) for the warm-up, as well as \( S_{t=96} \), which served as a starting state for all simulations presented hereafter. \( S_{t=0} \) was determined by setting the pressure head to zero at the soil–bedrock interface, while nodes in the other 4 layers were assigned a pressure head of \(-z\), in which \( z \) is the absolute vertical distance from a given node to the soil–bedrock interface at a particular \( XY \) location. Starting from \( S_{t=0} \), the reference model was then run until \( t = 96 \, h \). During this period, soil water was redistributed due to hydraulic head differences. The slope of the domain, convergence of flow due to varying soil depth, as well as water removal from the domain at the sink hotspots were the driving factors in this redistribution. No precipitation was applied during the warm-up period. The resulting pressure head pattern \( S_{t=96} \) then served as the initial state for the reference model (during generation of the artificial measurements) as well as for the simplified model (during calibration with SCEM-UA and SODA).

We used the following boundary conditions: all of the nodes located at \( y = 0 \) were assigned a seepage face boundary condition (Fig. 1), meaning that they shed water only if the pressure head is positive. At the atmosphere–soil interface, nodes which were not part of the seepage face were assigned an atmospheric boundary condition, across which infiltration occurs. Precipitation was applied at a rate of \( 6 \, \text{mm}h^{-1} \) for a period of \( 6 \, h (t = 96 - 102 \, h) \), followed by a period of no rain for \( 114 \, h (t = 102 - 216 \, h) \) until the end of the simulation. Evaporation and transpiration were not included in this study. Nodes located on the outside of the domain that were not part of the seepage face, the atmosphere–soil interface, or the soil–bedrock interface were assigned a no-flow boundary condition.

Fig. 1. Transect of lower section of the domain with boundary node types shown. Infiltration enters the domain at the atmospheric boundary nodes. Excess water is removed from the domain either vertically at the sink nodes or laterally as seepage.

Fig. 2. Soil depth distribution. The locations of sink hotspots are indicated with asterisks – these are the locations where the reference model applies \( r_{\text{sink (high)}} \). At the complementary locations, the reference model applies \( r_{\text{sink (low)}} \).

0.01 h\(^{-1}\). Note that we do not mean to claim that this concept is necessarily realistic – its sole purpose here is to introduce a structural difference between the reference model and the simplified model (described below). We use spatially homogeneous soil properties and hydraulic conductivity. Table 1 provides additional details about how we configured SMWS\(_{3D}\). The soil hydraulic parameters, as well as the geometry of the model domain and numerical integration settings are loosely based on Hopp and McDonnell (2009).
Table 1. Overview of the most relevant parameters in the SWMS_3D model.

<table>
<thead>
<tr>
<th>parameter description</th>
<th>SWMS_3D parameter name</th>
<th>value</th>
<th>units</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>geometry</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>total number of nodes</td>
<td>NumNP</td>
<td>525</td>
<td>–</td>
</tr>
<tr>
<td>total number of boundary nodes</td>
<td>NumBP</td>
<td>133</td>
<td>–</td>
</tr>
<tr>
<td>total number of elements</td>
<td>NumEl</td>
<td>2016</td>
<td>–</td>
</tr>
<tr>
<td><strong>soil hydraulic parameters</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>residual water content</td>
<td>θ_r</td>
<td>0.28</td>
<td>–</td>
</tr>
<tr>
<td>saturated water content</td>
<td>θ_s</td>
<td>0.475</td>
<td>–</td>
</tr>
<tr>
<td>air entry value</td>
<td>α</td>
<td>4.00  m</td>
<td>m^-1</td>
</tr>
<tr>
<td>pore tortuosity</td>
<td>n</td>
<td>2.0</td>
<td>–</td>
</tr>
<tr>
<td>saturated conductivity</td>
<td>K_s</td>
<td>0.35  m h</td>
<td>m h^-1</td>
</tr>
<tr>
<td><strong>numerical integration settings</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>maximum number of iterations</td>
<td>MaxIt</td>
<td>31</td>
<td>–</td>
</tr>
<tr>
<td>tolerance on theta</td>
<td>TolTh</td>
<td>1 × 10^-6</td>
<td>–</td>
</tr>
<tr>
<td>tolerance on head</td>
<td>Tolh</td>
<td>1 × 10^-6</td>
<td>m</td>
</tr>
<tr>
<td>initial integration time step</td>
<td>dt</td>
<td>6     min</td>
<td>min</td>
</tr>
<tr>
<td>minimum integration time step</td>
<td>dtMin</td>
<td>1     min</td>
<td>min</td>
</tr>
<tr>
<td>maximum integration time step</td>
<td>dtMax</td>
<td>20    min</td>
<td>min</td>
</tr>
<tr>
<td>time step decrease factor</td>
<td>DMul</td>
<td>0.7</td>
<td>–</td>
</tr>
<tr>
<td>time step increase factor</td>
<td>DMul2</td>
<td>1.2</td>
<td>–</td>
</tr>
</tbody>
</table>

With the settings described above, we ran the reference model in order to generate artificial measurements for \( t = 97 \) through \( t = 216 \) h at hourly intervals. During the simulation, approximately 26.3 m\(^3\) of precipitation was applied. After the simulation ended, a total volume of about 15.7 m\(^3\) had been extracted vertically from the soil domain according to Eq. (5), while a total of about 12.1 m\(^3\) was removed from the domain as seepage from the seepage face. At the end of the simulation period at \( t = 216 \) h, 186.5 m\(^3\) of water was present in the soil, slightly less than the 188.0 m\(^3\) of water that had been present at \( t = 96 \) h. Figure 3 shows how pressure head developed at the soil–bedrock interface for all nodes during the simulation period. Transient saturation occurred in about 60% of the nodes, but dissipated relatively quickly for most nodes once precipitation stopped.

Upon completion of the run, we saved 3 variables: (1) the total volume of soil water that had been extracted according to Eq. (5); (2) the time series of discharge from the seepage face at the bottom of the hillslope; and (3) the space–time distributed pattern of pressure head at the soil–bedrock interface. That is, the first two variables describe integrated hydrologic responses, whereas the third is spatially distributed. In parts 2 and 3 of this study (see Sects. 2.2 and 2.3, respectively), these 3 variables were then used as error-free artificial measurements with which to calibrate the simplified model; in part 4 (see Sect. 2.4), the SODA calibration from part 3 is repeated, but this time using only a subset of the artificial measurements, which were moreover perturbed in order to mimic the effect of measurement noise.

### 2.1.1 Simplified model and calibrated parameters

The simplified model differs only slightly from the reference model, in that it assumes that the sink is spatially homogeneous. Equation (5) thus simplifies to

\[
B = \begin{cases} 
0 & h < 0 \\
 r_{\text{sink}} \cdot h & h \geq 0
\end{cases}
\]  

For field studies, it is common to make this assumption, even in cases where soft data suggest the presence of preferential-flow features such as cracks in the bedrock. Even though its validity may often be questionable, the modeler’s hand is forced by the lack of direct observations.

We calibrated 2 parameters: \( K_s \), the saturated hydraulic conductivity (Eq. 3), and \( r_{\text{sink}} \), which controls the rate at which water is lost from the soil domain as it infiltrates the bedrock (Eq. 6).

### 2.2 SCEM-UA (idealized data set)

The Shuffled Complex Evolution Metropolis (SCEM-UA) algorithm has been discussed in detail elsewhere (e.g. Vrugt et al., 2003b); only a summary is presented here. SCEM-UA is a parameter estimation algorithm which was developed to better deal with uncertainty in parameter estimates, while improving the efficiency and effectiveness of searching the parameter space. The algorithm is based on the popular SCE parameter estimation algorithm (Duan
Fig. 3. Artificial measurements: simulated traces of pressure head in space and time. For each node at the soil–bedrock interface, the figure contains a subplot showing a time series of the pressure head (magenta line) for \( t = 97–216 \) h. Horizontal dashed lines represent zero pressure head. Each subplot's axes have been clipped vertically to \([-0.5, 0.25]\) in order to better show the pressure head dynamics during relatively wet conditions. To further ease interpretation, each subplot was assigned a background color depending on how long saturated conditions lasted at a given location. The locations of sink hotspots are indicated with asterisks.

Following the classical approach to inverse modeling, SCEM-UA assumes that the hydrological model structure \( f(\cdot) \) is a perfect description of the processes as they occur in reality, and that data errors are negligible. The model output vector \( Y \) is calculated using the model forcings \( U \) by propagating the model initial state \( X_0 \) using the model structure \( f(\cdot) \) and the model parameters \( \theta \):

\[
Y = f(X_0, U, \theta).
\]  
(7)

The model output is compared with observations \( Z \), and the goodness-of-fit is expressed in an objective function \( g(\cdot) \):

\[
OF = g(Y, Z),
\]  
(8)

where \( OF \) represents the objective score. Points in the parameter space thus become associated with an objective score. Note that the single-objective approach described here can be extended to include multiple objectives using the concept of Pareto optimality (Yapo et al., 1998; Gupta et al., 1998; Vrugt et al., 2003a).

The goal of parameter estimation is to find the parameter combinations(s) \( \theta \) whose associated output follows the observations as closely and consistently as possible (Vrugt et al., 2005). In order to find the part of the parameter space that yields the best parameter combinations, SCEM-UA proceeds as follows:
1. Initialize a population of points in the parameter space, divide them over multiple complexes as per Duan et al. (1992);

2. When convergence has not been achieved, do
   a. sample new points from the feasible parameter space;
   b. determine the objective score OF for each point, by
      i. running the model
      ii. running the objective function(s)
   c. accept or reject each new point according to the Metropolis rule;
   d. shuffle complexes;
   e. calculate Gelman–Rubin convergence statistic (Gelman and Rubin, 1992).

SCEM-UA reliably finds the part of the parameter space that yields the best possible objective scores. It has been used successfully to identify model parameters in a variety of disciplines including hydrology, soil chemistry, and ecology (e.g. Vrugt et al., 2003b, 2007; Nierop et al., 2002).

2.2.1 Objective functions

We use the following 3 functions to evaluate the performance of individual parameter combinations:

\[ \text{OF}_1 = |\epsilon_{\text{obs}} - \epsilon_{\text{sim}}|, \]  
\[ \text{OF}_2 = \frac{1}{n_t} \sum_{t=1}^{n_t} (q_{\text{obs},t} - q_{\text{sim},t})^2, \]  
\[ \text{OF}_3 = \frac{1}{n_i} \sum_{i=1}^{n_i} (h_{\text{obs},i} - h_{\text{sim},i})^2, \]

in which \( \epsilon_{\text{obs}} \) and \( \epsilon_{\text{sim}} \) are the total observed and total simulated vertical water loss, respectively. \( \epsilon \) is calculated as \( \epsilon = V_{\text{init}} + V_{\text{in}} - V_q - V_{\text{end}}, \) in which \( V_{\text{init}} \) is the total volume of water that is present in the soil at \( t = 96 \) h, \( V_{\text{in}} \) is the total volume of infiltration, \( V_q \) is the total volume of discharge that is removed from the soil as seepage, and \( V_{\text{end}} \) is the total volume of water that is present in the soil at \( t = 216 \) h;

in which \( n_t \) is the number of time steps, and \( q_{\text{obs},t} \) and \( q_{\text{sim},t} \) are the observed and simulated hillslope-scale discharge for the \( t \)th time step, respectively; and

in which \( n_i \) is the product of the number of rows and columns in the grid times the number of time steps, and \( h_{\text{obs},i} \) and \( h_{\text{sim},i} \) are the observed and simulated pressure head at the soil–bedrock interface for the \( i \)th combination of row, column and time step, respectively.

The rationale behind this combination of objective functions is as follows. The simplified model simulates redistribution of the available water, i.e. initial storage and infiltration. The redistribution is subject to losses due to bedrock infiltration (vertically) and seepage (laterally). Successful calibration of the model requires that the volume of water that leaves the domain is accurate, which is achieved by minimizing the first two objectives (Eqs. 9 and 10). However, the first two objectives are not capable of extracting any spatial information. Using just the first two objectives could therefore lead to a proliferation of equally realistic solutions of pressure head patterns internal to the hillslope. To avoid that, the third objective (Eq. 11) attempts to use the spatial information in the pressure head data series, such that the solutions that were equally realistic based on just the first two objectives can now be differentiated.

2.3 SODA (idealized data set)

The Simultaneous parameter Optimization and Data Assimilation (SODA Vrugt et al., 2005) algorithm may be viewed as an extension of SCEM-UA. It combines SCEM-UA’s parameter estimation procedure with an ensemble Kalman filter (EnKF; Evensen, 1994, 2003) such that uncertainty in the model states can be accommodated. SODA’s general structure is therefore similar to the SCEM-UA structure outlined earlier, except that Step 2.b is different. Instead of running the model for all time steps at once, the EnKF generates an ensemble of model predictions, each of which having slightly different states. Each ensemble member is then propagated time step by time step, using the parameter combination suggested by the parameter estimation part of SODA. Step 2.b thus becomes

2.b. Determine the objective score OF for each point, by
   i. generating an ensemble of model states based on the last a posteriori state (or the initial state);
   ii. propagating each ensemble member one time step, using the model structure, the model forcings, and the parameter combination. This results in an a priori state estimate for each ensemble member. Use the same parameter combination for all ensemble members and time steps;
   iii. determining the magnitude and direction of the state updates by calculating the weighted average of the a priori state estimates and the observations (the weights are related to the degree of uncertainty in each component);
   iv. adding the state updates to the a priori state estimates to get the a posteriori state estimates.
   v. returning to (i) if the current time is less than the simulation end time; and
   vi. running the objective function(s).
shows the evolution of the parameter distribution. The pressure head information, and did indeed not occur at X15Y39 (see Fig. 3), node X15Y39 models (compare Eqs. 5 and 6). Since saturated conditions are introduced on the model states (either as a result of errors in the model initial state, errors in the model forcings, or by the use of an imperfect model structure), their propagation through time is limited by the EnKF’s intermediate updating.

### 2.3.1 Objective functions

For water balance and discharge, we let SODA use the same objective functions and measurements that were used for SCEM-UA (Eqs. 9 and 10). The pressure head information, however, is used in a different way: rather than aggregating all errors using the objective function defined by Eq. (11), the pressure head measurements are used to update the a priori model prediction in the EnKF. This is necessary in order to retain the timing and localization information pertaining to errors that may occur, and is therefore crucial for model improvement.

### 2.4 SODA (reduced and perturbed data set)

In part 4 of this study, we expand on the SODA results from part 3 by investigating whether SODA’s diagnostic capabilities are negatively affected when there are fewer measurements, and when those measurements are moreover subject to measurement noise. For this, we removed approximately half of the pressure head observation locations from the data set according to a random elimination pattern. Among the eliminated locations were two important sink hotspot locations (X09Y15 and X00Y18). We then mimicked the effect of measurement noise by perturbing the remaining measurements. For the pressure head observations, we used zero-mean, homoscedastic Gaussian noise of standard deviation 0.005 m. The discharge measurements were perturbed using a zero-mean, heteroscedastic Gaussian noise, with standard deviation equal to 10% of the true (unperturbed) discharge.

For water balance and discharge, we let SODA use the same objective functions that were used in part 3 (Eqs. 9 and 10). The pressure head information is applied in the same fashion as in part 3, although there are now fewer locations where measurements are available and the values that remain are now subject to measurement noise.

### 3 Results and discussion

#### 3.1 SCEM-UA (idealized data set)

Figure 4 shows the evolution of the parameter distribution during the SCEM-UA calibration. Once the distribution becomes stable after about 500 model evaluations, $K_s$ is accurately and precisely identified, but the $r_{sink}$ parameter has settled on a range of values that represents neither $r_{sink}(low)$ nor $r_{sink}(high)$. Because of the simplified model’s structural deficiency, any value for $r_{sink}$ leads to errors somewhere in the hillslope. For $r_{sink}$ values close to $r_{sink}(low)$, large errors would be introduced at the locations of the hotspots – specifically, they would be too wet. The excess water at these locations would subsequently lead to a plume of overestimated pressure heads downslope from each hotspot. However, the extent of the plumes is smaller for $r_{sink}$ values that are somewhat higher than $r_{sink}(low)$. This is due to the combination of two effects: first, slightly raising the value of $r_{sink}$ leads to a less severe overestimation of pressure head at the hotspot; and secondly, the excess water downslope from the hotspot infiltrates the underlying bedrock more quickly. Since the extent of the plume directly affects the objective score (Eq. 11), $\log_{10}(r_{sink})$ values from the $[-1.4, -1.3]$ range are associated with better objective scores than $\log_{10}(r_{sink})$ values.
Fig. 5. SCEM-UA (idealized data set): model-observation pressure head residuals in space and time, as associated with the Pareto-optimal parameter combination $\log_{10}(r_{\text{sink}}) \approx -1.30$; $\log_{10}(K_s) \approx -0.47$. For each node at the soil–bedrock interface, the figure contains a subplot showing a time series of model-observation residuals (black line) for $t = 97–216$ h. Horizontal dashed lines represent zero residual. To ease interpretation of the residual pattern, each subplot was assigned a background color depending on the cumulative positive and cumulative negative residual: magenta colors represent under-estimation of artificial measurements (simulated value is too dry) while cyan means over-estimation (too wet). Note the spatial coherence and error propagation. The locations of sink hotspots as used in the reference model (but not the simplified model) are indicated with asterisks.

Considering the abundance and high quality of the data (no measurement error, no incommensurability), an experienced hydrologist would probably be able to make an educated guess based on Fig. 5 about what goes wrong in the simplified model and how it could be improved – perhaps by focusing on where errors are first introduced, and relating them to the value of various model variables at that time and place. In the next section, we show that the SODA methodology bears some resemblance to this approach, albeit that SODA is a more formalized and fully automated methodology.
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Fig. 6. SODA (idealized data set): evolution of the parameters. Dashed lines represent value of $r_{\text{sink}}^{\text{low}}$ and $r_{\text{sink}}^{\text{high}}$ (upper plot) and $K_s$ (lower plot). Markov chains are color-coded; open symbols are samples that have been rejected by the Metropolis scheme, while solid colors have been accepted. Note that the vertical axes are logarithmic.

3.2 SODA (idealized data set)

Figure 6 shows the evolution of the parameters during the SODA calibration. The figure shows that, after about 600 model evaluations, SODA draws exclusively from the narrow range around $\log_{10}(K_s) \approx -0.45$ and $\log_{10}(r_{\text{sink}}) \approx \log_{10}(r_{\text{sink}}^{\text{low}}) = -2$. The simplified model applies $r_{\text{sink}}$ to all sink nodes at the soil–bedrock interface, leading to an overestimation of pressure heads at the hotspots. However, when this happens, the EnKF recognizes that the simulation is systematically deviating from the artificial measurements, and it updates the model states accordingly. Updating the model states (i.e. pressure head) downward equates to physically relevant processes. Figure 7 shows the state updating that was performed when SODA evaluated the simplified model using $\log_{10}(r_{\text{sink}}) \approx -1.97$ and $\log_{10}(K_s) \approx -0.46$. Two types of responses can be distinguished in the figure: on the one hand, most of the states do not need any updating. For those nodes, the simplified model provides an appropriate representation of the measurements, at least when the model is run with parameter values from the optimal range. On the other hand, there are other nodes that need substantial updating (e.g. X00Y18, X06Y30, X09Y15, and X18Y09). These nodes coincide with hotspots. The structural difference between the simplified model and the reference model leads to consistently deviating a priori estimates of pressure head in these areas, which are subsequently corrected by SODA. Further down, we explain why state updating is not limited to just the hotspots, but also occurs at nodes that are located close to hotspots.

Besides model evaluation, state adjustment patterns are also helpful in generating the inspiration and guidance for constructing new, improved hypotheses. Such guidance is necessary to avoid making ad hoc decisions with regard to model design. As an example of how SODA can help to formulate an improved model design, Fig. 8 shows the magnitude of the state updating as a function of pressure head for the nodes that have the strongest cumulative updates. The figure suggests that the simplified model structure could be improved by including an additional nonlinear term at the locations for which the strongest cumulative updating was performed. Figure 8 further shows that no state updating was needed for node X06Y30 when the pressure head was below 0. This is consistent with the difference between Eqs. (5) and (6). We argue that relations such as those visualized in Figs. 7 and 8 greatly stimulate the discussion between modeler and experimentalist about what process could explain the state updating patterns. At the same time, these relations also guide model improvement by setting constraints on the functional form of the relation.

Fig. 7. SODA (idealized data set): pressure head state updates in space and time, as associated with the Pareto-optimal parameter combination $\log_{10}(r_{\text{sink}}) \approx -1.97; \log_{10}(K_s) \approx -0.46$. For each node at the soil–bedrock interface, the figure contains a subplot showing a time series of state updates (black line) for $t = 97–216$ h. Horizontal dashed lines represent zero updates (no adjustment). To ease interpretation of the updating pattern, each subplot was assigned a background color that visually shows the magnitude of cumulative negative updating. Greener backgrounds mean stronger over-estimation (i.e. the model’s a priori value is too wet with respect to the artificial measurements). This figure uses a color scheme that is different from that of Fig. 5 to emphasize the difference in interpretation between model-observation residuals on the one hand and state updates on the other. The locations of sink hotspots as used in the reference model (but not the simplified model) are indicated with asterisks.

3.2.1 Limitations with respect to measurement interval

In this part, we are using an idealized data set without any measurement noise. As a result, the EnKF places much more confidence on the measurements than it does on the a priori estimate of the model state: the a posteriori model state is effectively determined by “resetting” the a priori model state to the value of the measurement. When the confidence balance is strongly in favor of the measurements, any errors that may have been introduced since the last measurement time are canceled almost completely after the states are updated at the time of the next measurement. However, if a lot of time passes in between measurement times (relative to the dynamics of the modeled process) the error can still significantly affect other model states. For example, we used 60 min measurement intervals, but the SWMS_3D model used integration time steps of 1–20 min (Table 1). Small overestimation errors introduced at the hotspots could thus spread to neighboring nodes, where the a priori estimate of model state was subsequently reset to the measured state during state updating. This explains the small state updates that the EnKF performed at the nodes adjacent to the hotspots at X00Y18, X06Y30, X09Y15, and X18Y09 (Fig. 7). Because the spreading of errors is stronger when the measurements
are taken at larger intervals, it is important that the measurement frequency is balanced with the timescale of the modeled process. Similarly, when individual measurements are associated with realistic measurement noise (as opposed to being error-free), the EnKF will not place as much confidence on the measurements. Consequentially, errors are not corrected as quickly, and the state updating pattern will be more difficult to interpret. This problem can be alleviated by measuring at smaller intervals, or by installing a more precise measurement device. When setting up future field experiments, it may thus be worthwhile to increase the measuring frequency, even at the cost of the experiment’s duration.

3.2.2 Limitations with respect to information content

Besides balancing the measurement interval with the timescale of the process of interest, it also remains important that the measurements contain all relevant behavioral modes. This is because information content is more important than sheer data volume (e.g. Sorooshian et al., 1983; Gupta and Sorooshian, 1985); even powerful methods of analysis cannot extract information that is not present. For example, the reference model includes 5 hotspots, but only 4 of these can be identified from the SODA analysis. The reason for this is that the difference between the reference model and the simplified model only becomes apparent when saturated conditions occur at the location of one of the hotspots. If saturated conditions do not occur, the simplified model’s behavior cannot be distinguished from the reference model’s behavior: the sink term \( B \) is equal to zero for both models (compare Eqs. 5 and 6). Since saturated conditions did indeed not occur at X15Y39 (see Fig. 3), node X15Y39 cannot be identified as a hotspot from these measurements, regardless of the method used.

3.3 SODA (reduced and perturbed data set)

Figure 9 shows the evolution of the parameters during calibration with SODA using the reduced and perturbed data set. Parameter \( K_s \) is identified with similar accuracy and precision as for the idealized data set, while parameter \( r_{sink} \) is slightly more uncertain and a very small bias has been introduced. The space–time pattern of state updating associated with one of the Pareto-optimal parameter combinations is visualized in Fig. 10. In terms of magnitude, timing, and location, it is quite similar to the state updating associated with the idealized data set (Fig. 7). There are, however, some differences as well.

First there is the general appearance of state updating, which is much noisier here than for the idealized data set. This is because every state update is now partly a correction for the measurement noise that was introduced at the previous time step. Because the magnitude of measurement noise is only small in comparison to the model uncertainty, and because the measurement noise is not systematic, the noise does not negatively affect the state updating pattern as a whole.

The second difference relates to nodes for which state updating cannot be solely attributed to spurious measurement noise, but for which the updating is controlled by an additional systematic component (nodes with greenish background color in Fig. 10). These locations coincide with, or are in close proximity to, hotspots. If measurements of pressure head are available at the location where the model structural error is introduced, i.e. the hotspot, the nature of the model structure error is reflected in the state updates (left and middle subplots in Fig. 11). For these nodes, the relation between the state and the subsequent update is quite similar to the result of the idealized data set, although naturally the relation is somewhat masked by the effect of measurement noise. In any case, relations derived from state updating can directly be used to construct an improved model. On the other hand, if no measurements of pressure head are available at the hotspot, the update can only occur some distance away, and as a result the state updating is then only indirectly related to the source of the error (e.g. right subplot in Fig. 11). The separation between source of the error and the subsequent update means that the diagnostic information contained in the original error is scrambled beyond the point where it could be retrieved. Consequentially, additional measurements must be made before the model structure can be improved. Figure 10 may once again be used to infer that such additional measurements are most likely to yield new insights when placed at a previously unmeasured location near one of the nodes with systematic updating. Furthermore, the figure tells us which measurement device may be removed without affecting the diagnostic power of the pattern as a whole; any node with a light background color qualifies for this. Their lack of systematic updating means that the corresponding measurement devices have not provided any new insights, so they are better deployed at a new location.
In summary, the analysis of SODA’s state updating helps to identify model deficiencies and provides us with the information necessary to improve our understanding of hydrological processes in an iterative cycle of modeling, experimental design, data collection, and analysis.

4 Conclusions

As models get more complex, there is a growing need for better tools with which to evaluate them (e.g. Beck, 1987; Gupta et al., 1998; Kirchner, 2006). It has been argued in the literature that model evaluation should not be limited to ranking model runs (differentiating between the good and the bad representations of the real world), but should also provide some guidance on how to improve a given model structure (Beck, 1987; Lin and Beck, 2007; Gupta et al., 2008). In this study, we purposely used a deficient model structure, which we calibrated both by a parameter estimation approach (SCEM-UA) and by a combined parameter and state estimation approach (SODA). We then assessed how suitable each method was for providing aforementioned guidance.

Four main conclusions that can be drawn from this work are

1. State adjustment patterns generated by SODA are helpful in evaluating when and where model structural errors occur.

2. Relations can be constructed between SODA’s state adjustments and the model states themselves. Such relations can readily be adopted in an improved version of the model. Perhaps most importantly, they stimulate the discussion between modeler and experimentalist about what process could explain them, while at the same time guiding the discussion by setting constraints on the functional form of the relation.

3. For a reduced and perturbed data set, SODA proved useful in identifying the location of some hotspots, as well as the functional relation driving the model structure error. Due to gaps in the spatial distribution of measurements, some other hotspots could not be identified, although the space–time pattern of state updating did show the general area where model structure errors were introduced.

4. SCEM-UA cannot provide information that is as informative as that provided by SODA. Parameter estimation methods such as SCEM-UA lack a strategy with which the propagation of errors is reduced. Due to compensation effects, tuning the parameters of a structurally deficient model may therefore result in optimal parameter values without any physical relevance. This inhibits a straightforward interpretation of the model-observation residuals with regard to model improvement.

By using artificially generated measurements, we were able to focus strictly on the usefulness of the algorithms, while avoiding any issues relating to the quality of measurements.

5 Next steps

The usefulness of the SODA approach as part of the iterative research cycle must ultimately be demonstrated by its application to real-world problems. Only then, we can judge how much it can contribute to reducing the many uncertainties we
Fig. 10. SODA (reduced and perturbed data set): pressure head state updates in space and time. These state updates are associated with the Pareto-optimal parameter combination $\log_{10}(r_{sink}) \approx -2.13$; $\log_{10}(K_s) \approx -0.46$. Where pressure head observations were available, the figure contains a subplot showing a time series of state updates at the soil–bedrock interface (black line) for $t = 97–216$ h; the other nodes have been grayed out. Horizontal dashed lines represent zero updates (no adjustment). To ease interpretation of the updating pattern, each subplot was assigned a background color that visually shows the magnitude of cumulative negative updating. Greener backgrounds mean stronger over-estimation (i.e. the simplified model’s a priori value is too wet with respect to the artificial measurements). This figure uses a color scheme that is different from that of Fig. 5 to emphasize the difference in interpretation between model-observation residuals on the one hand and state updates on the other. The locations of sink hotspots as used in the reference model (but not the simplified model) are indicated with asterisks.

Fig. 11. SODA (reduced and perturbed data set): state updating as a function of state value for selected nodes. The data for this figure originate from SODA’s evaluation of $\log_{10}(r_{sink}) \approx -2.13$ and $\log_{10}(K_s) \approx -0.46$. 
are facing when we study hydrological dynamics of catchments. While this study has focused on improved analysis of simulation-measurement discrepancies, various other aspects also play a role in successfully learning from experimentation. For these aspects, continued experimentation with artificial measurements can be of great value, for instance when investigating the effect of spatially distributed soil properties or the effect of uncertainties in the model forcings. Potentially, there are many such effects influencing the model state, and we do not claim that the proposed analysis is the way to identify all of them. Nonetheless, SODA's state updating will certainly help to provide the information necessary to improve our understanding of hydrological processes in an iterative cycle of modeling, experimental design, data collection and analysis.

Supplementary material related to this article is available online at: http://www.hydrol-earth-syst-sci.net/17/3455/2013/hess-17-3455-2013-supplement.zip.

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References


Vrugt, J. A., Gupta, H. V., Bouten, W., and Sorooshian, S.: A shuffled complex evolution Metropolis algorithm for optimization and uncertainty assessment of hydrologic model parameters, Wa-


