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Chapter 4

Do the Dynamics of Prior Information Depend on Task Context? An Analysis of Optimal Performance and an Empirical Test

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Abstract

In speeded two-choice tasks, optimal performance is prescribed by the drift diffusion model. In this model, prior information or advance knowledge about the correct response can manifest itself as a shift in starting point or as a shift in drift rate criterion. These two mechanisms lead to qualitatively different choice behavior. Analyses of optimal performance (i.e., Bogacz et al. 2006, Hanks et al. 2011) have suggested that bias should manifest itself in starting point when difficulty is fixed over trials, whereas bias should (additionally) manifest itself in drift rate criterion when difficulty is variable over trials. In this article, we challenge the claim that a shift in drift criterion is necessary to perform optimally in a biased decision environment with variable stimulus difficulty. This paper consists of two parts. Firstly, we demonstrate that optimal behavior for biased decision problems is prescribed by a shift in starting point, irrespective of variability in stimulus difficulty. Secondly, we present empirical data which show that decision makers do not adopt different strategies when dealing with bias in conditions of fixed or variable across-trial stimulus difficulty. We also perform a test of specific influence for drift rate variability.
In real–life decision making, people often have a priori preferences for and against certain choice alternatives. For instance, some people may prefer Audi to Mercedes, Mac to PC, or Gillette to Wilkinson, even before seeing the product specification. For product preferences, people are influenced by prior experiences and advertising. Here we study the effects of prior information in the context of perceptual decision making, where participants have to decide quickly whether a cloud of dots is moving to the left or to the right. Crucially, participants are given advance information about the likely direction of the dots. In the experiment reported below, for instance, participants were sometimes told that 80% of the stimuli will be moving to the right. How does this advance information influence decision making?

In general, advance information that favors one choice alternative over the other biases the decision process: people will prefer the choice alternative that has a higher prior probability of being correct. This bias usually manifests itself as a shorter response time (RT) and a higher proportion correct when compared to a decision process with equal prior probabilities. Because bias expresses itself in two dependent variables simultaneously (i.e., RT and proportion correct), and because people only have control over a specific subset of the decision environment (e.g., the participant cannot control task difficulty) an analysis of optimal adjustments is traditionally carried out in the context of a sequential sampling model. Prototypical sequential sampling models such as the Sequential Probability Ratio Test (SPRT; e.g., Wald & Wolfowitz, 1948; Laming, 1968) or the drift diffusion model (DDM; Ratcliff, 1978) are based on the assumption that the decision maker gradually accumulates noisy information until an evidence threshold is reached.

In such sequential sampling models, the biasing influence of prior knowledge can manifest itself in two ways (e.g., Diederich & Busemeyer, 2006; Ratcliff & McKoon, 2008; Mulder, Wagenmakers, Ratcliff, Boekel, & Forstmann, in press). The first manifestation, which we call prior bias, is that a decision maker decides in advance of the information accumulation process to lower the evidence threshold for the biased alternative. The second manifestation, which we call dynamic bias (cf. Hanks et al., 2011), is that a decision maker weighs more heavily the evidence accumulated in favor of the biased alternative. There is both theoretical and empirical evidence that shows that both types of bias manifest itself in different situations.

The situation that has been studied most often is one in which task difficulty is fixed across trials. For this case, W. Edwards (1965) has shown that optimal performance can be achieved by prior bias alone. In empirical support for this theoretical analysis, Bogacz et al. (2006) demonstrated that in three experiments the performance of participants approximated the optimality criterion from W. Edwards (1965).

Another situation is one in which task difficulty varies across trials. For this case, Hanks et al. (2011) reasoned that people should tend ever more strongly towards the biased response option as the information accumulation process continues (see also Yang et al., 2005; Bogacz et al., 2006, p. 730). The intuitive argument is that a lengthy decision process indicates that the decision is difficult (i.e., the stimulus does not possess much diagnostic information), and this makes it adaptive to attach more importance to the advance information. In the extreme case, a particular stimulus can be so difficult that it is better to simply go with the advance information and guess that the biased response option is the correct answer.

Thus, in an environment of constant difficulty an optimal decision maker accommodates advance information by prior bias alone, such that a choice for the likely choice alternative requires less evidence than that for the unlikely choice alternative. The conjecture of Hanks and colleagues is that, in an environment of variable difficulty, an optimal
4.1 The Drift Diffusion Model (DDM)

In the Drift Diffusion Model (DDM), a decision process with two response alternatives is conceptualized as the accumulation of noisy evidence over time. Evidence is represented by a single accumulator, so that evidence in favor of one alternative is evidence against the other alternative. A response is initiated when the accumulated evidence reaches one of two predefined thresholds. For instance, in a lexical decision task, participants have to decide whether a letter string is an English word, such as TANGO, or a nonword, such as TANAG (Figure 4.1).

The model assumes that the decision process commences at the starting point $z$, from which point evidence is accumulated with a signal–to–noise ratio that is governed by mean drift rate $v$ and Wiener noise. Without trial–to–trial variability in drift rate, the change in evidence $x$ is described by the following stochastic differential equation

$$dx(t) = v \cdot dt + s \cdot dW(t),$$

where $W$ represents the Wiener noise process (i.e., idealized Brownian motion). Parameter $s$ represents the standard deviation of $dW(t)$. Values of $v$ near zero produce long RTs and high error rates.

Evidence accumulation stops and a decision is initiated once the evidence accumulator hits one of two response boundaries. The difference between these boundaries, boundary separation $a$, determines the speed–accuracy tradeoff; lowering $a$ leads to faster RTs at the cost of a higher error rate. When the starting point, $z$, is set at $a/2$, bias in the decision process is not manifested in the starting point. Together, these parameters generate a distribution of decision times (DTs). The observed RT, however, also consists of stimulus–nonspecific components such as response preparation and motor execution, which together make up non–decision time $T_{er}$. The model assumes that $T_{er}$ simply shifts the distribution of DT, such that $RT = DT + T_{er}$ (Luce 1986).

Thus, the four key parameters of the DDM are (1) speed of information processing, quantified by mean drift rate $v$; (2) response caution, quantified by boundary separation $a$; (3) evidence criterion, quantified by starting point $z$; and (4) non–decision time, quantified by $T_{er}$. In addition to these four parameters, the full DDM also includes parameters that decision maker accommodates advance information not just by prior bias, but also by dynamic bias.

This paper has two main goals. The first goal is to extend the analytical work of [W. Edwards 1965] and show that prior bias accounts for optimal performance regardless of whether stimulus difficulty is fixed or variable across trials. The second goal is to examine empirically which kind of bias decision makers implement when stimulus difficulty is fixed or variable.

The organization of the paper is as follows. In the first section we briefly introduce the drift diffusion model (DDM), the prototypical sequential sampling model that can be used to model both prior bias and dynamic bias (Bogacz et al. 2006; van Ravenzwaaij, van der Maas, & Wagenmakers in press). Next we present analytical work that shows how optimal performance in biased decision environments may be achieved with prior bias alone, regardless of whether stimulus difficulty is fixed or variable. We then present an empirical study showing how people accommodate bias for fixed and variable stimulus difficulty in similar fashion.
4. Bias

Recall from the introduction that decision makers may implement bias in two ways. A decision maker may decide prior to the start of the decision process that less evidence is required for a response in favor of the biased alternative than for the non-biased alternative. This type of bias, which we call prior bias, is manifested in the DDM as a shift in starting point (see the top panel of Figure 4.2, see also Ratcliff, 1985, Ratcliff & McKoon, 2008, Mulder et al., in press). Prior bias is most pronounced at the onset of the decision process, but dissipates over time due to the effects of the diffusion noise $s$. W. Edwards (1965) showed that when across-trial stimulus difficulty is fixed, it is optimal to shift the starting point an amount proportional to the odds of the prior probabilities of each response alternative.

Alternatively, a decision maker may weigh evidence in favor of the biased response alternative more heavily than evidence in favor of the non-biased response alternative.
4.3 Bias in Theory

Figure 4.2: Schematic representation of bias due to a shift in starting point $z$ (top panel) or a shift in drift rate criterion $v_c$ (bottom panel). The grey lines represent neutral stimuli for comparison.

This type of bias, which we call dynamic bias, is manifested in the DDM as a shift in drift rate criterion (see the bottom panel of Figure 4.2, see also Ratcliff, 1985; Ratcliff & McKoon, 2008; Mulder et al., in press). With a shift in the drift rate criterion, drift rate for the likely choice alternative is enhanced by a bias component, such that the cumulative effect of dynamic bias grows stronger over time (compare the difference between the biased and neutral lines for shift in $z$ and shift in $v_c$).

4.3 Bias in Theory

Below, we examine analytically which DDM parameter shifts in starting point and drift rate criterion produce optimal performance. We differentiate between fixed and variable difficulty. Two criteria of optimality are discussed: highest mean proportion correct when there is a fixed response deadline (i.e., the interrogation paradigm) and lowest mean RT (MRT) for a fixed mean proportion correct (see also Bogacz et al., 2006). In the next section, we discuss highest mean proportion correct for the interrogation paradigm.

Optimality Analysis I: The Interrogation Paradigm

In the interrogation paradigm, participants are presented with a stimulus for a fixed period of time. Once the response deadline $T$ is reached, participants are required to immediately
make a response (see Figure 4.3). In this section, we will look at optimal DDM parameter settings for a biased decision in the interrogation paradigm. The performance criterion is the mean proportion correct. First, we discuss fixed stimulus difficulty across trials, or $\eta = 0$. Second, we discuss variable stimulus difficulty across trials, or $\eta > 0$.

![Figure 4.3: The interrogation paradigm. At deadline $T$, decision makers choose a response alternative depending on the sign of the evidence accumulator.](image)

### Fixed Difficulty

In order to find the maximum mean proportion correct for the interrogation paradigm, we assume that participants base their response depending on whether the evidence accumulator is above or below zero when the accumulation process is interrupted (see e.g., van Ravenzwaaij, Brown, & Wagenmakers, 2011, Fig. 9). We choose parameter settings such that if the accumulator is above zero at time $T$, the biased response is given, whereas if the accumulator is below zero at time $T$, the non–biased response is given.

Across trials, the final point of the evidence accumulator will be normally distributed with mean $vT + z$ and standard deviation $s\sqrt{T}$ (e.g., Bogacz et al., 2006; van Ravenzwaaij, Brown, & Wagenmakers, 2011). In what follows, we assume that stimuli corresponding to either of the two response alternatives are equally difficult.$^1$ For fixed difficulty, the analytical expression for mean proportion correct for an unbiased decision in the interro-

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$^1$This is commonly the case in perceptual decision tasks, such as the random dot motion task (Newsome et al., 1989), which we will be using in the experiment reported below.
4.3. Bias in Theory

gation paradigm, denoted by $P_{CF,U}(I)$, is given by

$$P_{CF,U}(I) = \Phi \left[ \frac{vT + z}{s\sqrt{T}} \right],$$  \hspace{1cm} (4.2)

where $\Phi$ denotes the standard normal cumulative distribution. For a biased decision, this expression becomes:

$$P_{CF,B}(I) = \beta \Phi \left[ \frac{(v + v_c)T + z}{s\sqrt{T}} \right] + (1 - \beta) \Phi \left[ \frac{(v - v_c)T - z}{s\sqrt{T}} \right].$$  \hspace{1cm} (4.3)

where $\beta$ denotes the proportion of stimuli that are consistent with the prior information (i.e., in the experiment below, $\beta = .80$) and $v_c$ denotes the shift in drift rate criterion. Equation (4.3) is derived from Equation (4.2) as follows. To incorporate a shift in starting point $z$ towards the biased alternative, the left–hand side of Equation (4.3) contains $z$ (cf. Equation 4.2). In the right–hand side, $z$ is replaced by $-z$ to account for the fact that the correct answer lies at the non–biased threshold. To incorporate a shift in drift rate criterion, the left–hand side of Equation (4.3) adds a shift $v_c$ to mean drift rate $v$. In the right–hand side, the shift $v_c$ is subtracted from mean drift rate $v$.

In the absence of a criterion shift in drift rate, or $v_c = 0$, a maximum for $P_{CF,B}(I)$ occurs for:

$$z_{max} = \frac{s^2 \log \left( \frac{\beta}{1-\beta} \right)}{2v},$$  \hspace{1cm} (4.4)

where $z_{max}$ denotes the value of the starting point that leads to the highest $P_{CF,B}(I)$ (e.g., [W. Edwards, 1965] [Bogacz et al., 2006]).

Inspection of both numerators in Equation (4.3) shows that there is a trade–off between the starting point $z$ and the shift in drift rate criterion $v_c$, such that $\Delta z = \Delta v_c \cdot t$. Therefore, the maximum value of $P_{CF,B}(I)$ does not belong to a unique set of parameters, but exists along an infinite combination of values for $z$ and $v_c$. This trade–off is graphically displayed for different sets of parameter values in Figure 4.4.

Figure 4.4 shows that for a drift rate $v$ of 0.2 or 0.3, combined with a deadline $T$ of 300 ms. or 500 ms., every maximum that occurs for a particular value of $z$ with $v_c = 0$ may also be reached for different combinations of $z$ and $v_c$ (bias $\beta$ is set to .8). Results for different sets of parameters look qualitatively similar.

Taken at face value, this result challenges the claim that optimal behavior in biased decision problems is exclusively accomplished by a shift in starting point $z$: the same level of accuracy may be accomplished by, for instance, setting $z = 0$ and $v_c = z_{max}/T$ or any other combination of values that is consistent with the parameter trade–off $\Delta z = \Delta v_c \cdot T$. Importantly, however, participants have to be aware of the exact moment of the deadline $T$ to be able to utilize the trade–off between starting point and drift rate criterion. As such, the safest and least demanding way to perform optimally is to accommodate bias by only shifting the starting point. We will return to this point in the conclusion.

Next, we examine optimal decision making in the interrogation paradigm when participants have no a priori knowledge of the difficulty of the decision problem. In other words, stimulus difficulty varies across trials.

**Variable Difficulty**

To obtain an expression for the mean proportion correct in the interrogation paradigm when there is across–trial variability in stimulus difficulty, it is necessary to include across–trial variability in drift rate, or $\eta$. The mean proportion correct for this situation, or
4. Bias

\[ \text{Figure 4.4: Mean percentage correct for fixed difficulty in the interrogation paradigm for different combinations of starting point } z \text{ and shift in mean drift rate } v_c. \text{ Due to the trade–off } z = -v_c \cdot T, \text{ no unique maximum exists.} \]

\[ P_{CV,B}(I) \text{, is derived by multiplying Equation 4.3 by a Gaussian distribution of drift rates with mean } v \text{ and standard deviation } \eta. \text{ The resulting expression needs to be integrated over the interval } (-\infty, \infty) \text{ with respect to drift rate } \xi: \]

\[ P_{CV,B}(I) = \int_{-\infty}^{\infty} \left( \beta \Phi \left[ \frac{(\xi + v_c)T + z}{s\sqrt{T}} \right] + (1 - \beta) \Phi \left[ \frac{(\xi - v_c)T - z}{s\sqrt{T}} \right] \right) \times \frac{1}{\sqrt{2\pi\eta^2}} \exp \left( -\frac{(\xi - v)^2}{2\eta^2} \right) d\xi, \tag{4.5} \]

which simplifies to

\[ P_{CF,B}(I) = \beta \Phi \left[ \frac{(v + v_c)T + z}{s\sqrt{T + T^2\eta^2/s^2}} \right] + (1 - \beta) \Phi \left[ \frac{(v - v_c)T - z}{s\sqrt{T + T^2\eta^2/s^2}} \right] \tag{4.6} \]

(see section B.1 “Derivation of Equation 4.6” of the appendix for the derivation).

By integrating out drift rate \( \xi \) in a way analogous to the integration of the diffusion noise \( dW \), a maximum value of \( P_{CV,B}(I) \), in the absence of a criterion shift in drift rate, or \( v_c = 0 \), is found for:

\[ z_{\text{max}} = \frac{(s^2 + \eta^2T) \log \left( \frac{\beta}{1 - \beta} \right)}{2v}, \tag{4.7} \]
where $z_{max}$ denotes the value of the starting point that leads to the highest $PC_{v,B}(I)$.

Inspection of both numerators in Equation 4.5 shows that once again, there is a trade–off present between the starting point $z$ and the shift in drift rate criterion $v_c$, such that $\Delta z = \Delta v_c \cdot t$. Figure 4.5 graphically displays the parameter trade–off for the same sets of parameter values that were used in Figure 4.4 ($\eta$ is set to 0.1). Results for different sets of parameters look qualitatively similar.

In sum, our derivations and figures show that in the interrogation paradigm with variable across–trial stimulus difficulty, optimal decisions may again be reached by shifting either starting point $z$, drift rate criterion $v_c$, or a combination of the two. Contrary to the fixed difficulty situation, however, participants need to know the value of deadline $T$ to optimize performance even when just shifting starting point.

As noted in the introduction, Hanks et al. (2011) suggested that bias should manifest itself as shifts in both starting point and the drift rate criterion when stimulus difficulty varies over trials. The authors reasoned that decision makers should decide in favor of the biased alternative when the decision process is lengthy, because slow decisions are likely to be difficult decisions. However, in the interrogation paradigm, all decisions take equally long. In order to more thoroughly investigate the claim of Hanks et al. (2011), it is necessary to eliminate the deadline $T$ and examine a different criterion of optimality: minimum MRT for a fixed percentage correct.
Optimality Analysis II: Minimum MRT for Fixed Accuracy

In this section we consider the minimum mean RT for fixed accuracy. First, we discuss the situation in which across–trial stimulus difficulty is fixed.

Fixed Difficulty

In order to find the minimum MRT for a given level of accuracy, we need expressions for both MRT and accuracy in the DDM for a biased decision. We can then calculate combinations of starting point $z$ and drift criterion shift $v_c$ that yield a given level of accuracy for a given set of parameter values for drift rate $v$, boundary separation $a$, and diffusion noise $s$. The last step is to calculate the MRT for each combination of starting point $z$ and drift criterion shift $v_c$.

For fixed difficulty, the analytical expression for mean proportion correct for an unbiased decision without a deadline, denoted by $P_{CF,U}(D)$, is given by

$$P_{CF,U}(D) = \frac{\exp \left( \frac{2av}{s^2} \right) - \exp \left( \frac{2(a-z)v}{s^2} \right)}{\exp \left( \frac{2av}{s^2} \right) - 1}, \tag{4.8}$$

(see e.g., Wagenmakers et al., 2007, Eqn. 2). Transforming Equation 4.8 to an expression for a biased decision occurs in a similar fashion as the derivation of Equation 4.3 from Equation 4.2:

$$P_{CF,B}(D) = \beta P_{+F,B}(D) + (1 - \beta) P_{-F,B}(D), \tag{4.9}$$

where

$$P_{+F,B}(D) = \left[ \frac{\exp \left( \frac{2a(v+v_c)}{s^2} \right) - \exp \left( \frac{2(a-z)(v+v_c)}{s^2} \right)}{\exp \left( \frac{2a(v+v_c)}{s^2} \right) - 1} \right], \tag{4.10}$$

$$P_{-F,B}(D) = \left[ \frac{\exp \left( \frac{2a(v-v_c)}{s^2} \right) - \exp \left( \frac{2z(v-v_c)}{s^2} \right)}{\exp \left( \frac{2a(v-v_c)}{s^2} \right) - 1} \right].$$

To incorporate a shift in starting point $z$ towards the biased alternative, $P_{+F,B}(D)$ contains $z$ as in Equation 4.8. For $P_{-F,B}(D)$, $z$ is replaced by $a - z$ to account for the fact that the correct answer lies at the non–biased threshold. To incorporate a shift in drift rate criterion, $P_{+F,B}(D)$ adds a shift $v_c$ to mean drift rate $v$. For $P_{-F,B}(D)$, the shift $v_c$ is subtracted from mean drift rate $v$.

Now that we have an expression for mean proportion correct for a biased decision without a response deadline, we need an expression for MRT. For an unbiased decision without deadline, MRT is given by

$$MRT_U = \left( \frac{a}{2v} \right) \frac{1 - \exp \left( \frac{-av}{s^2} \right)}{1 + \exp \left( \frac{-av}{s^2} \right)}, \tag{4.11}$$

where $z$ is fixed on $a/2$ (e.g. Link & Heath, 1975). For a biased decision, this expression becomes

$$MRT_B = \beta \cdot \left( \frac{a-z}{v+v_c} \right) \frac{1 - \exp \left( \frac{-2z(v+v_c)}{s^2} \right)}{1 + \exp \left( \frac{-2(a-z)(v+v_c)}{s^2} \right)} + (1 - \beta) \cdot \left( \frac{z}{v-v_c} \right) \frac{1 - \exp \left( \frac{-2(a-z)(v-v_c)}{s^2} \right)}{1 + \exp \left( \frac{-2(a-z)(v-v_c)}{s^2} \right)}, \tag{4.12}$$
(e.g. Link & Heath 1975).

In what follows, we will examine which shifts in starting point $z$ and drift rate criterion $v_c$ lead to a predetermined level of mean proportion correct for two sets of parameter values. We will then calculate the MRT for each combination of starting point $z$ and drift rate criterion $v_c$ and determine which combination is optimal in the sense that it results in the lowest MRT.

Firstly we considered the following set of parameters: mean drift rate $v = 0.2$, boundary separation $a = 0.12$, and bias $\beta = .8$. The top–left panel of Figure 4.6 shows that an accuracy level of 90% may be achieved by a combination of shifts for starting point $z$ and shifts of drift rate criterion $v_c$. Again, both parameters exist in a trade–off relationship, such that higher values of $z$ combined with lower values of $v_c$ produce the same mean proportion correct.

The bottom–left panel of Figure 4.6 shows MRT for each calculated combination of starting point $z$ and drift rate criterion $v_c$. The $x$–axis shows the value for starting point $z$ (as a proportion of boundary separation $a$), the associated value for drift rate criterion $v_c$ can be found in the top–left panel. The MRT results show that the lowest value for MRT is reached when all bias is accounted for by a shift in starting point $z$.

Secondly we considered a mean drift rate $v = 0.3$ and an accuracy level of 95%. The results, shown in the right two panels of Figure 4.6 are qualitatively similar.

In sum, when there is no response deadline and across–trial difficulty is fixed, the optimal way to deal with bias is by shifting the starting point towards the biased response alternative, without shifting the drift rate criterion. In the next subsection, we examine optimal performance for variable stimulus difficulty across trials.

Variable Difficulty

In this subsection we examine what combinations of starting point $z$ and drift criterion shift $v_c$ lead to the minimum MRT for a given level of accuracy for variable across–trial stimulus difficulty. In order to do so, we first find the sets of parameter values for starting point $z$ and drift criterion shift $v_c$ that yield a predetermined mean proportion correct. Then, we find the corresponding values of MRT for these parameter combinations.

Unfortunately, for variable difficulty there are no expressions for mean percentage correct. Thus, we have to find these values by simulation. The top–left panel of Figure 4.7 shows the set of parameter values of starting point $z$ and drift rate shift $v_c$ that lead to a percentage correct of 90% with a mean drift rate $v = 0.2$. The top–right panel shows the set of parameter values of starting point $z$ and drift rate shift $v_c$ that lead to a percentage correct of 95% with a mean drift rate $v = 0.3$. For both panels, boundary separation $a = 0.12$, bias $\beta = .8$, and standard deviation of drift rate $\eta = 0.1$. The bottom panels of Figure 4.7 show corresponding values of MRT. The MRT results show that as for fixed difficulty, the lowest value for MRT is reached when all bias is accounted for by a shift in starting point $z$.

In sum, in paradigms with variable across–trial difficulty, but no response deadline, the optimal way to deal with bias is by shifting the starting point towards the biased response alternative, without shifting the drift rate criterion. This result mirrors the result for fixed difficulty.

2For the remainder of this article we set non–decision time $T_{cr} = 0$. 65
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Interim Conclusion

Hanks et al. (2011) claimed that for biased decisions with variable across–trial stimulus difficulty, optimal performance requires not just a shift in starting point but also a shift in drift rate criterion (i.e., dynamic bias). Our results challenge this claim: regardless of whether stimulus difficulty is fixed or variable, optimal performance is obtained by having bias only shift the starting point, and not the drift rate criterion. In the next section, we will investigate how people perform in practice.

4.4 Bias in Practice

We have demonstrated that optimal performance in decision conditions with a biased response alternative is achieved by shifting only the starting point criterion. However, people may not accommodate bias in an optimal manner. For instance, Hanks et al. (2011) demonstrated that in a decision environment with variable across–trial stimulus difficulty, participants accommodate advance information by dynamic bias. The authors did not, however, directly compare the performance of participants in conditions with fixed and
variable across–trial stimulus difficulty. In this section, we perform such a comparison and address the question whether the inclusion of variability in stimulus difficulty alters the way in which people accommodate advance information. We also examine if the performance of people in practice corresponds to the theoretical optimality indicated in the previous sections.

Our experiment also allows us to test a prediction from the DDM, namely that increasing the variability in across–trial stimulus difficulty results in a higher estimate of across–trial drift rate variability $\eta$. The experiment used a random dot motion task (Newsome et al., 1989) with advance information about the upcoming direction of movement. In a within–subjects design, each participant was administered a condition with fixed stimulus difficulty (i.e., identical coherence of movement from trial to trial) and a condition with variable stimulus difficulty (i.e., variable coherence).

**Participants**

Eleven healthy participants (8 female) performed a random-dots motion (RDM) paradigm in exchange for course credit or a monetary reward of 28 euros. Participants were recruited...
through the University of Amsterdam and had normal or corrected-to-normal vision. The procedure was approved by the ethical review board at the University of Amsterdam and informed consent was obtained from each participant. According to self-report, no participant had a history of neurological, major medical, or psychiatric disorder.

Materials

Participants performed an RT version of an RDM task. Participants were instructed to maintain fixation on a cross at the middle of the screen and decide the direction of motion of a cloud of partially randomly moving white dots on a black background. The decision was made at any time during motion viewing with a left or right button press. The stimulus remained on screen until a choice was made. The motion stimuli were similar to those used elsewhere (e.g., Newsome & Paré, 1988; Britten, Shadlen, Newsome, & Movshon, 1992; Gold & Shadlen, 2003; Palmer, Huk, & Shadlen, 2005; Ratcliff & McKoon, 2008; Mulder et al., 2010): white dots, with a size of 3 x 3 pixels, moved within a circle with diameter of 5 deg with a speed of 5 deg/s and a density of 16.7 dots/deg²/s on a black background. On the first three frames of the motion stimulus, the dots were located in random positions. For each of these frames the dots were repositioned after two subsequent frames (the dots in frame one were repositioned in frame four, the dots in frame two were repositioned in frame five, etc.). For each dot, the new location was either random or in line with the motion direction. The probability that a dot moved coherent with the motion direction is defined as coherence. For example, at a coherence of 50%, each dot had a probability of 50% to participate in the motion stimulus, every third frame (see also Britten et al., 1992; Gold & Shadlen, 2003; Palmer et al., 2005).

Visual stimuli were generated on a personal computer (Intel Core2 Quad 2.66 GHz processor, 3GB RAM, two graphical cards: nvidia GeForce 8400 GS and a nvidia GeForce 9500 GT, running MS Windows XP SP3) using custom software and the Psychophysics Toolbox Version 3.0.8 (Brainard, 1997) for Matlab (version 2007b, Mathworks, 1984).

Difficulty

To get acquainted with the task, each participant performed a practice block of 40 easy trials (60% coherence). To match the difficulty level of the motion stimuli across participants, each participant performed an additional block of 400 trials of randomly interleaved stimuli with different motion strengths (resp. 0%, 10%, 20%, 40%, and 80% coherence, 80 trials each). We fitted the proportional–rate diffusion model to the mean response times and accuracy data of this block using a maximum likelihood procedure (Palmer et al., 2005). For each participant, the motion strength at 75% accuracy was then interpolated from the psychometric curve (predicted by the proportional–rate diffusion model) and used in the experimental blocks with fixed coherence across trials. For blocks with mixed coherences, we randomly sampled performance levels from a uniform distribution (range: 51-99), and interpolated for each randomly chosen performance level the associated coherence from the psychometric curve.

Design

Each participant performed four sessions of the RDM task. In each session, participants performed six blocks of 100 trials: three blocks with the coherence fixed across trials, and three blocks with coherence varied across trials. For each condition (fixed and variable coherence) there were two biased and one neutral block. Prior information was given at
4.4. Bias in Practice

the start of each experimental block. In the first experimental block, participants were told that there was a larger probability that the dots will move to the left (left–bias). In the second block, participants were told that there was an equal probability that the dots will move to the left or to the right (neutral). In a third block, the instructions indicated that there was a larger probability that the dots will move to the right (right–bias). For the biased blocks, prior information was consistent with the stimulus direction in 80% of the trials. The sequence of conditions was counterbalanced across participants.

Analyses

In order to quantify bias in the starting point and the drift rate criterion, we fit the diffusion model to the data with the DMAT toolbox. We estimated the following parameters: mean drift rate \( v \), boundary separation \( a \), non–decision time \( T_{er} \), starting point \( z \), standard deviation of drift rate \( \eta \), range of starting point \( s_z \), and range of non–decision time \( s_t \).

For both the fixed difficulty condition and the variable difficulty condition we estimated a mean drift rate \( v \) for congruent, neutral, and incongruent stimuli. This resulted in six different estimates for mean drift rate \( v \). In addition, for both the fixed difficulty condition and the variable difficulty condition we estimated a starting point \( z \) for left–biased, neutral, and right–biased stimuli. This resulted in six different estimates for starting point \( z \). Furthermore, for the fixed difficulty condition and the variable difficulty condition we estimated a boundary separation \( a \) and a standard deviation of drift rate \( \eta \), resulting in two different estimates for these parameter. Finally, we constrained non–decision time \( T_{er} \), range of starting point \( s_z \), and range of non–decision time \( s_t \) to be equal over stimulus type, conditions, and sessions, resulting in a single estimate for each of those parameters.

Starting point bias was calculated as half of the difference between the starting point \( z \) for left–biased stimuli and the starting point \( z \) for right–biased stimuli, scaled by boundary separation \( a \). The maximum bias in starting point was therefore 50%. Drift rate criterion bias was calculated as half of the difference between mean drift rate \( v \) for congruent stimuli and mean drift rate \( v \) for incongruent stimuli, scaled by the sum of mean drift rate \( v \) for valid and invalid stimuli. The maximum bias in drift rate criterion was therefore 50% as well.3

Results

For the presented analyses, we report Bayesian posterior probabilities in addition to conventional \( p \)–values. When we assume, for fairness, that the null hypothesis and the alternative hypothesis are equally plausible \textit{a priori}, a default Bayesian \( t \)–test allows one to determine the posterior plausibility of the null hypothesis and the alternative hypothesis. We denote the posterior probability for the null hypothesis as \( P_{H_0}^{\text{Bayes}} \). When, for example, \( P_{H_0}^{\text{Bayes}} = .9 \), this means that the plausibility for the null–hypothesis has increased from .5 to .9. Posterior probabilities avoid the problems that plague \( p \)–values, allow one to directly quantify evidence in favor of the null–hypothesis, and arguably relate more closely to what researchers want to know (e.g., Wagenmakers, 2007).

3In theory, negative drift rates could lead to a larger bias. However, none of our participants had negative drift rates for any of the stimulus types.
In order to assess whether our manipulation of across–trial parameter difficulty was successful, we compared the estimate of across–trial drift rate variability $\eta$ between the fixed and variable difficulty conditions. The results show that across–trial drift rate variability $\eta$ was larger in the variable difficulty condition (mean 0.23) than in the fixed difficulty condition (mean 0.14; $t(20) = 2.25, p < .05, p_{Bayes}^{H0} = .34$), suggesting that the across–trial difficulty manipulation was successful.\footnote{Note that the alternative hypothesis is roughly twice as likely as the null–hypothesis according to $p_{Bayes}^{H0}$.}

The left panel of Figure 4.8 shows both starting point bias and drift rate criterion bias for the fixed difficulty condition and the variable difficulty condition. In the fixed difficulty condition, both the bias in starting point and the bias in drift rate are larger than zero ($t(10) = 5.85, p < .01, p_{Bayes}^{H0} < .01$, and $t(10) = 3.43, p < .01, p_{Bayes}^{H0} = .10$, respectively). In the variable difficulty condition, both the bias in starting point and the bias in drift rate also are larger than zero ($t(10) = 6.89, p < .01, p_{Bayes}^{H0} < .01$, and $t(10) = 4.14, p < .01, p_{Bayes}^{H0} = .04$, respectively). The right panel of Figure 4.8 shows MRT and proportion correct for consistent, neutral, and inconsistent stimuli for the fixed and variable difficulty conditions.

Since both types of bias are measured on different scales, they cannot be compared
directly. The important claim by Hanks et al. (2011) is that bias in the drift rate criterion is larger in the variable difficulty condition than in the fixed difficulty condition. In order to assess this claim we compared both types of bias directly between the two conditions. Consistent with the visual impression from the left panel of Figure 4.8, the statistical analysis reveal no differences for starting point bias and drift rate criterion bias ($t(20) = 0.61, p > .05, p_{Bayes} = .74$, and $t(20) = 0.75, p > .05, p_{Bayes} = .73$, respectively).

In sum, both in the fixed difficulty condition and in the variable difficulty condition, participants exhibit bias in starting point and drift rate criterion. The results suggest there is no difference between the two conditions in the amount of each type of bias, challenging the claim by Hanks et al. (2011) that bias in the drift rate criterion should be more pronounced when stimulus difficulty is variable than when it is fixed.

Model Predictives

In cognitive modeling, model fit can be assessed by means of model predictives. Model predictives are simulated data generated from the cognitive model, based on the parameter estimates for the real data. If the generated data closely resemble the real empirical data, then the model fit is deemed adequate (e.g., Gelman & Hill, 2007).

For this experiment, we drew 100 samples from the real data set and generated diffusion model parameter estimates for each participant and each condition separately using a bootstrap procedure. For each of these samples, we generated synthetic data, for which we calculated the 0.1, 0.3, 0.5, 0.7, and 0.9 RT quantiles for both the real data set and the synthetic data set. The real and synthetic RT quantiles are shown in Figure 4.9.

Figure 4.9 shows a quantile probability plot (e.g., Ratcliff, 2002), where the left-hand side represents error RTs for the five quantiles, and the right-hand side represents correct RTs for those same quantiles. The circles represent stimuli that were consistent with the biased response direction, the squares represent stimuli that were neutral, and the triangles represent stimuli that were inconsistent with the biased response direction. The filled symbols in the figure show the empirical data, the open symbols show the simulated data that were generated using the DMAT parameter estimates.

For response accuracy, the correspondence between the empirical data and the synthetic data can be judged by the horizontal disparity between the data points and the model points. Figure 4.9 shows that the diffusion model captures the error rate reasonably well for most of the stimulus types, as indicated by the horizontal disparity between the filled and open symbols. The diffusion model does capture the RTs well, as can be judged from the vertical disparity between the filled and open symbols. The quantile probability plot nicely shows how correct responses are fastest for consistent stimuli and slowest for inconsistent stimuli, whereas error responses are fastest for inconsistent stimuli and slowest for consistent stimuli, which is consistent with prior bias (see e.g., Mulder et al., in press).

4.5 Conclusion

It is not straightforward to perform optimally in response time tasks in which one response option is more likely than the other. When stimulus difficulty is fixed across trials, W. Edwards (1965) has demonstrated that optimal performance requires advance information to be accommodated solely by a shift in starting point. Recently, Hanks et al. (2011)
claimed that when stimulus difficulty varies from trial to trial, optimal performance also requires a shift in drift rate criterion (i.e., dynamic bias).

The contribution of this paper is twofold. Firstly, we demonstrated that in theory, optimal performance is achieved by a shift in starting point only. This result holds regardless of whether stimulus difficulty is fixed or variable from trial to trial. Secondly, we presented empirical data showing that people accommodate bias similarly for conditions of fixed and variable across–trial difficulty. Specifically, decision makers incorporate both prior and dynamic bias, and no evidence suggested that the presence of variability in stimulus difficulty made participants rely more on shifts in the drift rate criterion.

In the theoretical part, we first considered the interrogation paradigm. We demonstrated that optimal performance can be achieved by entertaining a bias in starting point, by entertaining a bias in the drift rate criterion, or by a combination of the two. There was no qualitative difference between conditions of fixed and variable across–trial stimulus difficulty.

It could be argued that the theoretical results of the interrogation paradigm are unlikely to apply in practice. Specifically, participants need to know the exact moment of the deadline $T$ to be able to utilize the trade–off between starting point and drift rate criterion. Also, the argument of Hanks et al. (2011) depends on the time course of the decision process: long decisions are most likely difficult decisions, so the longer a decision takes, the more adaptive it becomes to select the biased response alternative. In

Figure 4.9: Model predictives indicate that the parameter estimates of DMAT describe the data well. Filled symbols: empirical data. Open symbols: bootstrapped synthetic data, based on the model parameter estimates.
the interrogation paradigm, however, each decision process takes an identical amount of

time.

Another complication when interpreting results for the interrogation paradigm is that
for variable difficulty, the optimal setting of the starting point depends on the time
deadline (see Equation 4.7). Because the decision maker does not know the exact value
of the time deadline, a more realistic expression should integrate over some unknown
distribution of time deadlines that describes the uncertainty in $T$ on the part of the
decision maker.

In a task setting without response deadline, our results show that optimal performance
is achieved by shifting only the starting point; additionally shifting the drift rate criterion
only serves to deteriorate performance. Crucially, we found that this result is true for
both fixed and variable across–trial stimulus difficulty. As such, our results conflict with
the claim by Hanks et al. (2011) that optimal decision makers should entertain a shift in
drift rate criterion to accommodate bias under conditions of variable stimulus difficulty.

In the empirical part, we conducted an experiment in which we manipulated across–
trial stimulus difficulty in order to investigate if performance of decision makers is optimal.
We also wanted to test whether decision makers accommodate bias differently in decision
environments with fixed and variable across–trial stimulus difficulty. We successfully
manipulated across–trial variability in difficulty, as evidenced by a higher value of across–
trial drift rate variability $\eta$ for variable across–trial difficulty then for fixed–trial difficulty.
Our results showed that performance of participants was not optimal: decision makers
implemented both a shift in starting point and a shift in drift rate criterion in order
to deal with bias in prior information about the decision alternatives. Contrary to the
theory of Hanks et al. (2011), there was no difference in the implementation of dynamic
bias between conditions of fixed and variable across–trial stimulus difficulty.

In sum, we conclude that dynamic bias is not needed for optimal performance, not
when stimulus difficulty is fixed and not when it is variable. From the perspective of
optimality, advance information should affect only prior bias (i.e., starting point), such
that the evidence threshold is lowered for the choice alternative that is most likely to
be correct a priori. In practice it appears that people use both prior bias and dynamic
bias; our experiment suggests that increasing the variability of stimulus difficulty does
not cause participants to accommodate advance information preferentially by shifting the
drift rate criterion.