Monetary and fiscal policy under bounded rationality and heterogeneous expectations

Lustenhouver, J.E.

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The goal of this thesis is to use plausible and intuitive models of bounded rationality to give new insights in monetary and fiscal policy. Particular focus is put on the zero lower bound on the nominal interest rate, forward guidance, and fiscal consolidations. The thesis considers different forms of boundedly rational expectation formation and boundedly rational decision making by households and firms in the economy. Such bounded rationality effects the propagation of monetary and fiscal policy. The research in this thesis complements the macroeconomic literature that assumes rational expectations by providing new policy implications as well as by providing robustness to existing results.

Joep Lustenhouwer completed a master in econometrics at the university of Amsterdam in 2013 and graduated from the research master of the Tinbergen Institute in 2014. He subsequently started working on his PhD thesis at the Center for Nonlinear Dynamics in Economics and Finance at the University of Amsterdam. Currently he is a postdoctoral researcher at the Otto-Friedrich-University Bamberg. His research interests include monetary policy and fiscal policy, and behavioral and experimental macroeconomics.
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under Bounded Rationality
and Heterogeneous Expectations

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**Promotor:**  Prof. dr. C.H. Hommes  Universiteit van Amsterdam  
**Copromotor:**  Prof. dr. J. Tuinstra  Universiteit van Amsterdam  

**Overige leden:**  
- Prof. dr. H.M. Amman  Universiteit van Amsterdam  
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- Prof. dr. P. Levine  University of Surrey  
- dr. K. Mavromatis  Universiteit van Amsterdam  
- Prof. dr. C.R. Proaño  Otto Friedrich University Bamberg

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Chapter 1

Introduction

Introduction

In the last decade, countries all over the world have been suffering from high unemployment and below target inflation. Central banks have tried to fight this by reducing the nominal interest rate, which should stimulate investment and increase production and inflation. In doing so, many central banks have come close to the zero lower bound on nominal interest rates, preventing them to cut rates further and hence limiting their ability to stimulate the economy. This has led central banks to resort to unconventional policy measures such as forward guidance, where they make announcements about future interest rates and provide economic projections.

Additionally, years of low economic growth have led to lower tax incomes for governments, as well as higher spending on e.g. unemployment benefits. This has led to an increase in government debt in many countries. Debt levels have risen further because inflation has been falling more than nominal interest rates (due to the zero lower bound) causing governments to pay high real interest rates on existing government debt. Finally, some countries have tried to stimulate aggregate demand by further increasing government spending, which also resulted in a worsening of their fiscal position. All in all, this has led to possibly unsustainable levels of government debt, especially in some European countries. These countries now need to implement consolidations to reduce their debt levels, either by increasing taxes or by reducing government spending.

These economic developments have led to a large increase in interest in research topics such as the zero lower bound on the nominal interest rate, forward guidance, and fiscal consolidations. The majority of new studies in these topics have been performed within the paradigm of fully rational expectations. Agents are then assumed to have perfect knowledge and use perfect model consistent expectations to forecast future
variables, such as inflation and output.

While it can be a useful assumption, many people believe that the perfect rationality assumption may be too strong, "especially when considering relatively short-run responses to disturbances, or the consequences of newly adopted policies that have not been followed in the past" (Woodford, 2013). In the current macroeconomic conditions where governments and central banks try to manage liquidity traps and perform forward guidance and fiscal consolidations, short-run responses to disturbances and newly adopted policies are highly relevant. It is therefore crucial to also consider alternative assumptions about expectations. Results of models with alternative expectational assumptions can then, in combination with studies that assume rational expectations, lead to a deeper understanding of the problems that we face, and to more robust policy conclusions.

The goal of this thesis is to use plausible and intuitive models of bounded rationality to give new insights in monetary policy at the zero lower bound, forward guidance and fiscal consolidations. The aim is to complement existing research that assumes rational expectations, and to address in detail the consequences that boundedly rational expectations may have on the propagation of policy measures.

The importance of bounded rationality and a new behavioral approach to macroeconomics has recently been stressed in the books Animal Spirits (Akerlof and Shiller, 2010) and Lectures on behavioral macroeconomics (De Grauwe, 2012). These books point out that the financial crisis of 2007 has made it all to clear that the standard rational expectations models cannot explain everything. Instead, cognitive limitations and sentiments play an important role in emerging macro dynamics.

Surveys of consumers and professional forecasters show furthermore that there is considerable heterogeneity in the forecasts of macroeconomic variables (see e.g. Mankiw et al., 2004). Moreover, laboratory experiments with human subjects show that typically subjects do not coordinate on the rational expectations equilibrium. Instead, the expectations of subjects can more accurately be modeled by a behavioral heuristics switching model, where individuals switch between simple heterogeneous forecasting heuristics based upon their relative success (e.g. Assenza et al., 2014). Heterogeneous expectations will therefore play a large role throughout this thesis.

When deviating from the assumption of fully rational expectations, the question arises how exactly this deviation should take place. Different assumptions about expectation formation are likely to lead to different results and different policy conclusions. Therefore, instead of taking on the arguably impossible task of building a model that can perfectly explain real world expectation formation, I choose to introduce simple, intuitive and empirically relevant mechanisms of expectation formation that can shed
new light on issues such as anchoring of expectations and credibility of the central bank and of the fiscal authority. Care is however taken that a disciplinary measure on expectation formation is in place, so that agents in the models will only continue to use expectation heuristics that are sensible and have performed well in the past (Brock and Hommes, 1997). Additionally, I have performed a laboratory experiment where no assumptions need to be made about how expectations are formed, but where instead real human subjects are asked to submit their expectations, in a controlled environment.

Overview

In Chapters 2 and 3, we consider monetary policy and the zero lower bound on the nominal interest rate. In both chapters expectations are assumed to be heterogeneous and boundedly rational in such a way that new insights about the transmission of monetary policy can be obtained. Chapter 2 presents a framework where the credibility of the central bank can be explicitly modeled in an intuitive way, and where this credibility furthermore evolves endogenously with the past performance of the central bank. This is done by introducing two types of agents, one type of which forms expectations by trusting in the ability of the central bank to achieve its targets, while another type forms expectations by looking at past data. As the central bank manages to bring inflation and output closer to its targets, more agents trust that the central bank will be able to achieve its targets also in the future, and the credibility of the central bank increases. In this framework, it turns out that a high credibility of the central bank is crucial in preventing long lasting liquidity traps that could arise because of the zero lower bound on the nominal interest rate.

In Chapter 3 the focus is on the anchoring of expectations. Here, there is an endogenously evolving distribution of heterogeneous expectations around the targets of the central bank. When this distribution has a small standard deviation we can say that expectations are anchored to the targets, while a large standard deviation of this distribution implies unanchored expectations. It turns out that when expectations are strongly anchored, the central bank can easily achieve its targets and liquidity traps can not arise. For more unanchored expectations however, monetary policy needs to be aggressive enough to stabilize the economy, and the zero lower bound on the nominal interest rate can lead to persistent liquidity traps.

Chapter 4 looks at a different way the monetary authority could potentially stabilize

\footnote{Both chapters are based on joint work with Cars Hommes, available online as respectively CeNDEF working paper 15-03 and CeNDEF working paper 16-01.}
the economy: with forward guidance.\textsuperscript{2} The methodology is also different in this chapter. Instead of building a theoretical model, where inevitably some assumptions about expectation formation need to be made, the analysis is now conducted with a laboratory experiment where expectations about inflation are formed by real human subjects. In this setting, we investigate to what extent real human beings that try to forecast inflation can be influenced by forward guidance in the form of economic projections published by the central bank. This is done both in normal times and in times of severe economic stress, where the zero lower bound on the nominal interest rate is binding and forward guidance may be most direly needed. We find that forward guidance can stabilize the economy in normal times, reduce forecast errors of all participants in the economy, and prevent long lasting liquidity traps during times of severe economic stress.

Chapters 5 and 6 deal with fiscal consolidations under boundedly rational expectations.\textsuperscript{3} Chapter 5 takes a model similar to that of Chapter 2, with some agents forming expectations by believing in the commitments of the government and other agents forming expectations in a backward-looking manner. More specifically, some agents trust the commitment of the government to adjust either taxes or government spending when government debt becomes too high, while the other agents do not believe in this commitment and hence do not anticipate consolidations. Furthermore, the agents that do anticipate consolidations, may be wrong about the type of consolidation. That is, it might be that they expect tax increases while actually spending cuts will be implemented and vice versa. This setup allows us to study how the effectiveness of the two types of consolidations is affected by the extent to which agents anticipate upcoming consolidations and the extent to which they have correct expectations about the type of consolidations that will be implemented. Our results under bounded rationality mainly favor tax based consolidation over spending based ones.

Chapter 6 studies the issue of fiscal consolidations in a different framework and gives complementary insights to those obtained in Chapter 5. Instead of assuming heterogeneous expectations we now stay closer to the homogeneous rational expectations benchmark, but assume that agents base their decision on expectations about only a finite number of future periods. They then make consumption decisions by optimizing over a finite planning horizon of corresponding length, instead of the infinite planning horizon that is assumed under fully rational expectations. By varying the length of agent’s planning horizon we can investigate how the effects of fiscal consolidations

\textsuperscript{2}This chapter is based on joint work with Steffen Ahrens and Michele Tettamanzi.
\textsuperscript{3}Chapter 5 is based on joint work with Cars Hommes and Kostas Mavromatis; Chapter 6 is based on joint work with Kostas Mavromatis.
change if agents consider less what will happen in the future and put more weight on their current wealth, when making decisions. In this framework, we again consider the effectiveness of spending cuts and tax increases, and find that tax cuts are in general better able to reduce debt, just as in Chapter 5.
Chapter 2

Inflation Targeting and Liquidity Traps under Endogenous Credibility

2.1 Introduction

Many central banks (CB’s) have recently adopted some form of inflation targeting. Some CB’s do this by claiming to set the interest rate such that, if the interest rate were to be kept constant, the inflation in some specified future period is expected to equal the target value. Other CB’s\(^4\) include predictions about the future path of the interest rate in when forming the inflation projections. Both forms of inflation targeting can be described as "inflation forecast targeting".

Besides setting the interest rate optimally, an important aspects of inflation targeting is managing expectations. This is for example stressed by Woodford (2004). For the inflation targeting to be effective it is important that the the CB has enough credibility. If the private sector does not believe the CB when it announces an inflation target, the realized value of inflation will likely not be equal to this target. Whether the CB is likely to be believed furthermore will typically depend on whether it was able to achieve its targets in the past.

Inflation targeting is usually modeled in a New Keynesian setting under the assumption that agents have fully rational expectations. Under this assumption all agents form the same perfectly model consistent expectations, which, in the absence of shocks, implies that they have perfect foresight. When rational expectations are assumed, there is no longer a clear role for the credibility of an inflation and output gap target inside

\(^4\)E.g. the Central Bank of New Zealand.
the model. Either expectations about inflation and output coincide with the targets of the central bank and the CB has full credibility\(^5\), or expectations are not in line with the targets and the announcements of the CB are not credible.

Rational expectations are furthermore an unrealistically strong assumption when inflation and output forecasts by price setters (i.e. the private sector) are concerned. Both surveys of consumers and professional forecasters and laboratory experiments with human subjects show that there is considerable heterogeneity in inflation forecasts (e.g. Mankiw et al., 2004, and Pfajfar and Zakelj, 2016). Assenza et al. (2014) furthermore show that in their laboratory experiments, expectations of subjects can quite accurately (both qualitatively and quantitatively) be described as switching between simple heterogeneous forecasting heuristics based on their relative past performance.

In this chapter we investigate monetary policy in a standard New Keynesian model (in line with Woodford, 2003 and Galí, 2002) where the assumptions of homogeneous and rational expectations are relaxed. Instead, a Heuristic Switching Model with heterogeneous expectations is used, that allows for endogenous credibility. Heuristic Switching Models were introduced by Brock and Hommes (1997), and have since successfully been used to model heterogeneous expectations in finance and macroeconomics (Hommes, 2013). In our model agents switch between two intuitive forecasting heuristics based on relative performance. Branch and McGough (2010) and Cornea et al. (2013) use combinations of forecasting heuristics that are similar to ours. Other works with Heuristic Switching Models in a macroeconomic setting include De Grauwe (2011), Anufriev et al. (2013), Deak et al. (2015), Agliari et al. (2014), Agliari et al. (2015) and Pecora and Spelta (2017).

The most important forecasting heuristic in our model can be described as ”Trust the central bank”. Followers of this heuristic are called fundamentalists, and expect future inflation and output gap to be equal to the targets of the central bank. The fraction of fundamentalists can be interpreted as the credibility of the central bank. In contrast with rational expectations models, our model therefore involves endogenous credibility. We assume that this fundamentalist belief competes with naive expectations, which uses the last observation as a best guess for future realizations of inflation and output. The naive heuristic coincides with rational expectations when inflation or output follows a random walk. If inflation or output follows a near unit root process, the naive forecast is nearly rational. Naive agents furthermore adds persistence in inflation and output gap to our model in a very simple and intuitive manner, without the need to assume heavily serially correlated shocks. Similar to the naive heuristic, Milani

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\(^5\)Note that we refer to the credibility of the CB’s targets, and not to the credibility of its future policy actions. See Section 2.3 for details.
(2007) has stressed that homogeneous adaptive learning also generates high persistence in inflation and output dynamics, especially under constant gain learning (Evans and Honkapohja, 2009).

Cornea et al. (2013) estimate a New Keynesian Phillips curve assuming expectations are formed by a Heuristic Switching Model with fundamentalists and naive agents. Fundamentalists here make use of the forward looking relation between inflation and marginal cost and use a VAR approach to make inflation forecasts. Cornea et al. (2013) find that their model fits the data quite nicely and that the endogenous mechanism of switching between the two heuristics based on past performance is supported by the data. Branch (2004, 2007) fits a Heuristic Switching Model with amongst others a naive heuristic and a fundamentalistic VAR heuristic to data from Michigan Survey of Consumer Attitudes an Behavior. Both these papers find clear evidence of switching between heuristics based on past performance. Branch (2004) furthermore finds that both our heuristics are present in the survey data, and Branch (2007) finds that the Heuristic Switching Model better fits the survey data than a static sticky information model. A case study of the Volcker disinflation by Mankiw et al. (2004) furthermore nicely illustrates the presence of our two heuristics in survey data. In their Figure 12 (Mankiw et al., 2004, p. 46) the evolution of inflation expectations as measured by the Michigan Survey from 1979 up to and including 1982 is plotted. They show that at the start of 1979 expectations were centered around a high inflation value. Over the next eight quarters (during which Paul Volcker was appointed chairman of the Board of Governors of the Federal Reserve Board) the distribution of expectations clearly becomes bimodal, with a fraction of agents still expecting the same high values of inflation and another fraction expecting lower inflation. In terms of our model we can interpret this as follows. Before Volcker was appointed the FED had very little credibility and most agents expected inflation to remain at the high values that it had been in the recent past (they used the naive heuristic). In the following quarters the FED gained more credibility and an increasing fraction of agents started to believe that Volcker would be able to drive down inflation towards its target level (more agents started to follow the fundamentalistic heuristic). Furthermore, when in 1982 actual inflation started to decline, the mass on high inflation expectations slowly started to move towards lower inflation. We can interpret this as backward looking, naive agents believing that lower observed inflation would also mean lower inflation in the future.

Monetary policy is often modeled in the literature with a Taylor type interest rate rule. With such an interest rate rule, the CB adjusts the interest rate in response to inflation and output gap in order to steer inflation towards a long term target. There is however little consideration for the optimal paths of inflation and output gap, and the
time at which the long term target should be reached. Inflation forecast targeting is also modeled in the literature, but this is a form of strict inflation targeting, where no output considerations are allowed. While CB’s claim to set the interest rate to target inflation in some specified future period, in practice they will also take the consequences on output into account. A central bank would furthermore not only like inflation to be equal to its target in the specified period, but rather in every period.

For this reason we use an interest rate rule that is derived from a loss function consisting of the expectations of all future deviations of inflation from its target and all future output gaps. By choosing its policy rate to minimize this loss function the central bank can optimize the paths of future inflation and output gap. It turns out that, in our model of boundedly rational expectations, this optimal path can be achieved with an expectation based Taylor rule, where the interest rate depends on expectations of inflation and output gap instead of contemporaneous values. Such a rule is, amongst others, used by Bullard and Mitra (2002), and the optimal policy benchmark of a similar rule is derived by Evans and Honkapohja (2003).

In the derivation of this optimal rule, no restrictions are placed on the values that can be taken by the nominal interest rate. However, in practice this instrument will never be set negative. While ignoring this zero lower bound on the interest rate may not lead to problems when analyzing monetary policy in normal times, the recent financial crisis has shown the importance of the restrictions placed by this lower bound. Due to these restrictions the CB may not be able to stimulate the economy enough after a negative shock. This may then lead to a liquidity trap, as experienced by Japan since the 1990s.

When a central bank is constrained by the zero lower bound on the nominal interest rate, it can no longer use its main instrument to conduct monetary policy, but must instead rely on forward guidance and open-mouth operations. This implies that here it is even more crucial to realistically model expectations than during economically healthy times. Our model of endogenous credibility can provide new insights in liquidity traps that can not be obtained under rational expectations.

Closely related to our investigation of liquidity traps under bounded rationality is a series of papers by Evans et al. (2005, 2008, 2014). These authors study monetary and fiscal policy under adaptive learning in various macroeconomic models, ranging from a simple endowment economy (Evans and Honkapohja, 2005) to a more elaborate New Keynesian framework (Benhabib et al., 2014; Evans et al., 2008). They show the existence of multiple equilibria: the target equilibrium, and an equilibrium with low inflation. The existence of this second equilibrium when a zero lower bound is introduced to the model has been initially highlighted in Benhabib et al. (2001a b).
Evans et al. (2008) furthermore show that in their model a liquidity trap arises in the form of a deflationary spiral with ever decreasing inflation and output gap. A drawback of these models is their focus on a representative agent with adaptive learning. In our model, we extend the analysis to allow for heterogeneity in expectations and endogenous credibility.

We first analyze our Heuristic Switching Model without the zero lower bound on the interest rate. The main research question here is what policy parameters lead to desirable dynamics when expectations are heterogeneous and boundedly rational. It is shown that the region of policy parameters that leads to a locally stable fundamental steady state (with zero output gap and inflation equal to its target) is strictly larger than the region of policy parameters that gives a locally determinate rational expectations equilibrium. Even when the Taylor principle is not satisfied there could very well be convergence to the fundamental steady state in our model. Furthermore, under the set of policy parameters that minimize the central banks loss function, the fundamental steady state is unique and globally stable for any calibration of model parameters. Without the zero lower bound, the policy that minimizes the loss function can therefore indeed be considered desirable under heterogeneous expectations.

Next, we introduce the zero lower bound (ZLB) on the nominal interest rate to the above heterogeneous expectations framework, and investigate its effect on inflation and output gap dynamics. It turns out that with the zero lower bound, expectation driven liquidity traps can arise. In rational expectations models shocks to the fundamentals of the economy can lead to a temporary liquidity trap. However, as soon as the sequence of bad shocks is over, the liquidity trap is immediately resolved. In our model a one period shock to economic fundamentals can lead to a prolonged liquidity trap due to a loss in credibility of the central bank and low, self-fulfilling expectations. Mertens and Ravn (2014) highlight the distinction between expectation driven liquidity traps and fundamental liquidity traps. Depending on the magnitude of the shock and the loss in credibility, our expectation driven liquidity traps can be temporary or take the form of a deflationary spiral with ever decreasing inflation and output gap. Deflationary spirals have recently been observed in laboratory experiments by Assenza et al. (2014), Hommes et al. (2015), and Arifovic and Petersen (2015). We show that even under optimal monetary policy deflationary spirals can occur. When the zero lower bound is accounted for, the fundamental steady state can therefore no longer be globally stable, but only locally stable, coexisting with a deflationary spiral region.

Finally, we conduct simulations to investigate policies to prevent or recover from liquidity traps. We show three deviations of theoretically optimal monetary policy that are successful in preventing liquidity traps and deflationary spirals. First of all
the central bank can prevent deflationary spirals by letting the interest rate respond more aggressively to inflation than specified by optimal policy. Alternatively, the central bank can make liquidity traps less likely by increasing the inflation target and conducting aggressive monetary easing as soon as a liquidity trap is imminent.

The chapter is organized as follows. In Section 2.2 the New Keynesian model and interest rate rule are presented. Section 2.3 introduces the Heuristic Switching Model and conducts the analysis without the zero lower bound. In Section 2.4 we add the zero lower bound to the model and analyze liquidity traps. Simulations with policy interventions are presented in Section 2.5, and Section 2.6 concludes. In the appendix we present a micro-foundation of our heterogeneous expectations framework and proofs of the results.

2.2 Inflation targeting model

In order to facilitate comparison with the rational expectations benchmark we use a standard New Keynesian model in line with Galí (2002) and Woodford (2003). Micro foundations of this model under heterogeneous expectations are derived in Appendix 2.A. This derivation is closely related to Kurz et al. (2013), but additionally makes use of the properties of our Heuristic Switching Model, defined in Section 2.3. The New Keynesian Phillips curve and IS curve, describing inflation and output gap respectively, are given by

\begin{equation}
\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + e_t,
\end{equation}

\begin{equation}
x_t = E_t x_{t+1} - \frac{1}{\sigma} (i_t - E_t \pi_{t+1} - \bar{r}) + u_t.
\end{equation}

Here \(\beta\) is the discount factor, and

\begin{equation}
\kappa = \frac{(\sigma + \eta)(1 - \omega)(1 - \beta \omega)}{\omega},
\end{equation}

with \(\sigma\) and \(\eta\) the inverses of respectively the elasticity of intertemporal substitution and the elasticity of labor supply. \((1 - \omega)\) is the fraction of firms that can adjust their price in a given period, \(i_t\) is the nominal interest rate, which can be freely chosen by the central bank, and \(\bar{r} = \frac{1}{\beta} - 1\) is the steady state real interest rate. \(e_t\) and \(u_t\) are shocks to respectively inflation and output gap, which can be interpreted as cost-push shocks and productivity or preference shocks. Throughout the chapter, shocks will be
white noise and hence have no autocorrelation.

We assume the central bank wants to reach its inflation and output gap target both now and in the future. More specifically, the CB wants to minimize a loss function with the discounted sums of all squared deviations from these targets:

\[
E_t \sum_{i=0}^{\infty} \beta^i \left[ (\pi_{t+i} - \pi^T)^2 + \mu (x_{t+i} - x^T)^2 \right].
\]

Here \(\mu \geq 0\) is the relative weight that the central bank assigns to the minimization of the squared output gap compared to the squared deviation of inflation from target.

Rotemberg and Woodford (1999) and Woodford (2002) and others have shown that such a loss function can be derived from optimization of a second order approximation of the utility of a representative consumer. The optimal inflation and output gap targets are then however implied to be 0. In this chapter we analyze whether monetary policy aimed at minimizing the above loss function results in desirable dynamics under heterogeneous expectations, both with and without the restriction of \(\pi^T = x^T = 0\).

There are two ways the CB could minimize the loss function. If the CB optimizes under discretion, it chooses \(\pi_t\) and \(x_t\) to minimize the loss function in every period with the current information. If the CB optimizes under commitment it commits to a policy rule now, and does not reconsider this rule in future periods. This way it can influence future private sector expectations and will therefore ultimately be better off. The main problem with this approach is that the central bank will be tempted to re-optimize in every period. Commitment is only better for the CB because of the effect on private sector expectations. When those expectations have been formed the CB would be better off to renge on its commitment. However, the CB would then lose its credibility, so that we would be back in the discretion case.\(^6\) In this chapter we assume the CB optimizes under discretion. In that case, the first order conditions that are obtained from minimizing (2.4) result in the following optimal trade-off between inflation and the output gap:

\[
\pi_t - \pi^T = -\frac{\mu}{\kappa} (x_t - x^T).
\]

The optimal policy rule that does not assume rational expectations and implements the above condition is derived by Evans and Honkapohja (2003). The same rule in slightly different settings is derived by Berardi and Duffy (2007), and Gomes (2006). Evans and Honkapohja (2003) study optimal monetary policy under learning,

\(^6\)See e.g. Clarida et al. (1999) for further details.
with non-rational, but homogeneous expectations. They find that this rule leads to convergence to the optimum under discretion even if expectations are not rational. Branch and Evans (2011) find that the rule also performs well in their model with heterogeneous expectations. The rule is given by

\[ i - \bar{r} = \psi_0 + \psi_1 E_t \pi_{t+1} + \psi_2 E_t x_{t+1} + \psi_3 u_t + \psi_4 e_t, \]

\[ \psi_0 = -\frac{\sigma}{\mu + \kappa^2} (\kappa \pi^T + \mu x^T) \quad \psi_1 = 1 + \frac{\sigma \kappa \beta}{\mu + \kappa^2} \quad \psi_2 = \psi_3 = \sigma \quad \psi_4 = \frac{\sigma \kappa}{\mu + \kappa^2} \]

We assume that the central bank can perfectly observe private sector expectations, which is also done by the authors mentioned above, as well as by Branch and McGough (2010). Although the CB can respond to current period expectations, those expectations are assumed to be based on past information (as is standard in the learning literature). It is furthermore assumed that the central bank cannot respond to current period shocks. This way agents are not able to influence current period variables. Svensson (2003) strongly argues in favor of this.

In this chapter we assume a more general interest rate rule and consider the optimal policy benchmark as a special case. We assume a forward looking Taylor rule that replaces contemporaneous values of inflation and output gap by expectations in the rule proposed by Taylor (1993). The forward looking Taylor rule can be written as

\[ i_t = \bar{r} + \pi^T + \phi_1 (E_t \pi_{t+1} - \pi^T) + \phi_2 (E_t x_{t+1} - x^T). \]

Since shocks are white noise and not observed contemporaneously, the final two terms of (2.6) drop out. We further follow Rotemberg and Woodford (1999) and Woodford (2002) and assume an optimal inflation and output gap target of zero. Optimal policy can then be achieved with (2.7) with the following coefficients.

\[ \pi^T = x^T = 0, \quad \phi_1^{opt} = 1 + \frac{\sigma \kappa \beta}{\mu + \kappa^2}, \quad \phi_2^{opt} = \sigma \]

In order to get an idea of the magnitude of the optimal policy parameters, the model needs to be calibrated. The first two columns of Table 2.1 give the calibrations of \( \sigma \) and \( \kappa \) of Woodford (1999), Clarida et al. (2000), and McCallum and Nelson (1999). Under all calibrations the discount factor is set to \( \beta = 0.99 \). Column 3 of Table 2.1 states the corresponding optimal values of \( \phi_1 \). Here the weight on output gap, \( \mu \), is set to 0.25, as is done by McCallum and Nelson (2004), and Walsh (2003). Since under optimal policy \( \phi_2 = \sigma \), the different optimal values of \( \phi_2 \) can be read from Column 1 of Table 2.1. Column 4 and 5 are discussed in Section 3.
2.3. Analysis with Heuristic Switching Model

<table>
<thead>
<tr>
<th>Author(s)</th>
<th>(\sigma = \phi_2^{opt})</th>
<th>(\kappa)</th>
<th>(\phi_1^{opt})</th>
<th>(\phi_1^{PF})</th>
<th>(\phi_1^{PD})</th>
</tr>
</thead>
<tbody>
<tr>
<td>W</td>
<td>0.157</td>
<td>0.024</td>
<td>1.015</td>
<td>-5.607</td>
<td>20.56</td>
</tr>
<tr>
<td>CGG</td>
<td>1</td>
<td>0.3</td>
<td>1.874</td>
<td>-2.367</td>
<td>10.97</td>
</tr>
<tr>
<td>MN</td>
<td>1/0.164</td>
<td>0.3</td>
<td>6.326</td>
<td>-19.53</td>
<td>61.77</td>
</tr>
</tbody>
</table>

Table 2.1: Calibrations and corresponding bifurcation and optimal policy coefficients

The calibrations of Woodford (1999) and Clarida et al. (2000) result in reasonable optimal policy parameters that are close to empirical estimates of Taylor rules, both with contemporaneous values and with expectations of inflation and output gap.\(^7\) The McCallum and Nelson (1999) calibration on the other hand gives rise to what seems to be extremely aggressive monetary policy.

Abstracting from shocks and plugging (2.7) into (2.2), gives the following model

\[
x_t = (1 - \phi_2)E_t x_{t+1} - \frac{\phi_1 - 1}{\sigma} (E_t \pi_{t+1} - \pi^T) + \frac{\phi_2}{\sigma} x^T,
\]

(2.9)

\[
\pi_t = \beta E_t \pi_{t+1} + \kappa x_t.
\]

(2.10)

We assume here that the model parameters are positive, and that the policy parameters of the central bank are nonnegative.

Assumption 2.1. \(\kappa, \sigma > 0, \ \phi_1, \phi_2, \pi^T \geq 0\).

2.3 Analysis with Heuristic Switching Model

In this Section a Heuristic Switching Model is used to analyze the dynamics of output gap and inflation when expectations are non-rational and heterogeneous. In a Heuristic Switching Model as in Brock and Hommes (1997), beliefs are formed by a set of simple rules of thumb, or heuristics. The population consists of agents that can switch between those heuristics. As a heuristic preforms better in the recent past, the fraction of the population that follows that prediction rule increases. Agents are therefore learning over time by evolutionary selection based upon relative performance. The fractions of agents following the different heuristics evolve according to the following discrete choice model with multinomial logit probabilities (see Manski et al. (1981)):

\[
n_{h,t} = \frac{e^{U_{h,t-1}}}{\sum_{h=1}^{H} e^{U_{h,t-1}}}.
\]

(2.11)

\(^7\)See e.g Taylor (1999), Clarida et al. (2000) and Orphanides (2004).
Here $n_{h,t}$ is the fraction of agents that follows heuristic $h$ in period $t$, and $U_{h,t}$ is the fitness measure of heuristic $h$ in period $t$, i.e., a measure of how well the heuristic performed in the past. Finally, $b$ is the intensity of choice. The higher the intensity of choice, the more sensitive agents become with respect to relative performance of the heuristics.

We assume private sector beliefs are formed by two simple, but plausible heuristics: fundamental and naive. Followers of the naive heuristic make use of the high persistence in inflation and output gap dynamics, and believe future inflation or output gap to be equal to their last observed values. Note that the naive forecast is optimal when inflation and output follow a random walk, and close to optimal when the system contains a near unit root.

Followers of the fundamentalist heuristic on the other hand believe inflation or output gap to be equal to the fundamental values that would arise under rational expectations. Fundamentalists thus act as if all agents are rational. They do not take into account that there are other agents in the economy, as they lack the cognitive ability to know exactly the beliefs of other agents or the number of agents with different expectations. However, as long as other agents make the same predictions as the fundamentalists (not necessarily by using the same heuristic), fundamentalists will have perfect foresight in the absence of shocks.

In order to be able to assess the credibility of the central bank, we will only consider specifications of our model in which rational expectations (and thus the expectations of fundamentalists) coincide with the targets of the central bank. Fundamentalists expectations are then equal to $x_t = x^T$ and $\pi_t = \pi^T$, and their expectations could alternatively be interpreted as having been formed by trusting the central bank. Whichever value of inflation the central bank targets, fundamentalist believe that the central bank will be able to achieve it, so that they expect future inflation to be equal to this target.

Note that we talk about credibility of the central bank’s targets values of inflation and output gap, and not about the credibility of its future policy actions (as credibility is often referred to in the literature). This is in line with the fact that inflation targeting can be seen as a commitment to goals rather than a commitment to the CB’s future actions and details of its operations. Credibility over targets furthermore implicitly captures both the CB’s willingness to take actions to achieve its targets and its ability to do so, where the latter is not straightforward in an economy with boundedly rational agents.

Since, in our model, it is possible in any given period that one heuristic performs better in forecasting output gap while the other performs better in forecasting inflation, we do not impose any ex ante constraints on the relation between output gap and
inflation that agents expectations must satisfy. Instead, we allow the fraction of fundamentalists, denoted \( n^z_t \), to differ between inflation \((z = \pi)\) and output gap \((z = x)\).\(^8\) Agents will then learn to use the best heuristic for each variable. This can be the same heuristic in times where the time series of inflation and output gap have similar features. However, in periods of hyperinflation with an output gap close to target, agents will learn to be fundamentalistic about the output gap, but to use past inflation as a best predictor of future inflation. Furthermore, this set up of the model also allows for periods where the CB is perfectly credible in its inflation fighting policy, but where agents do not believe it will be able to keep the output gap at its target at the same time.

Finally, let the fitness measure for both variables be a weighted sum of the negative of the last observed squared prediction error, and the previous value of the fitness measure.

\[
U^z_{t-1} = -(1 - \rho)(z_{t-1} - E_{t-2}z_{t-1})^2 + \rho U^z_{t-2}, \quad z = \pi, x,
\]

where \(0 \leq \rho < 1\), is the memory parameter. For analytical tractability we set \(\rho = 0\) for now, and reintroduce the parameter in the simulations in Section 2.5.

To simplify calculations and presentation we introduce a new variable which is defined as the difference between the fraction of fundamentalistic agents \((n^z_t)\) and the fraction of naive agents \((1-n^z_t)\).

\[
m^z_t = n^z_t - (1 - n^z_t) = 2n^z_t - 1, \quad z = \pi, x.
\]

When all agents are fundamentalistic the difference in fractions equals 1, and when all agents are naive, the difference in fractions equals \(-1\). Henceforth we will refer to these differences in fractions simply as fractions. We can interpret these fractions as *endogenous credibility*. When \(m^\pi_t = m^x_t = 1\) the central bank has full credibility, and when \(m^\pi_t = m^x_t = -1\) the CB has lost all its credibility. This credibility measure will turn out to be very important in determining the effectiveness of monetary policy.

Using (2.13), average expectations about inflation and output gap can be written as

\[
E_{t} \pi_{t+1} = \frac{(1 + m^\pi_t)}{2} \pi_T + \frac{(1 - m^\pi_t)}{2} \pi_{t-1},
\]

\(^8\)All results presented in this section continue to hold when we impose that \(n^\pi_t\) and \(n^x_t\) should evolve together. Results presented in Section 2.4 and 2.5 would also remain valid qualitatively.
(2.15) \[ E_t x_{t+1} = \frac{(1 + m^x_t)}{2} x^T + \frac{(1 - m^x_t)}{2} x_{t-1}. \]

Using these expectations, (2.9) and (2.10) can be written as

(2.16) \[ x_t = x^T + (1 - \frac{\phi_2}{\sigma}) \frac{(1 - m^x_t)}{2} (x_{t-1} - x^T) - \frac{\phi_1 - 1(1 - m^\pi_t)}{\sigma} (\pi_{t-1} - \pi^T), \]

(2.17) \[ \pi_t = \beta \frac{(1 + m^\pi_t)}{2} \pi^T + \beta \frac{(1 - m^\pi_t)}{2} \pi_{t-1} + \kappa x_t. \]

To complete the model we specify \( m^x_{t+1} \) and \( m^\pi_{t+1} \) by combining (2.11), (2.12) and (2.13). This gives

(2.18) \[ m^x_{t+1} = \text{Tanh} \left( \frac{b}{2} \left( x^2_{t+2} - (x^T)^2 - 2(x_{t+2} - x^T)x_t \right) \right), \]

(2.19) \[ m^\pi_{t+1} = \text{Tanh} \left( \frac{b}{2} \left( \pi^2_{t+2} - (\pi^T)^2 - 2(\pi_{t+2} - \pi^T)\pi_t \right) \right). \]

The system defined by Equations (2.16) through (2.19) is six dimensional. First of all, next periods inflation and output gap \( (\pi_{t+1} \text{ and } x_{t+1}) \) are determined by the current values of these variables \((\pi_t \text{ and } x_t)\), and by next periods fractions \( (m^x_{t+1} \text{ and } m^\pi_{t+1}) \). These four variables are however not enough to determine the future dynamics of the system since \( m^x_{t+2} \) and \( m^\pi_{t+2} \), which determine \( \pi_{t+2} \) and \( x_{t+2} \), depend on \( \pi_{t-1} \) and \( x_{t-1} \), and are therefore not determined by the above mentioned variables. It follows that the system must be six dimensional and that the state vector can be written as

\[
\begin{pmatrix}
    x_t & \pi_t & x_{t-1} & \pi_{t-1} & m^x_{t+1} & m^\pi_{t+1}
\end{pmatrix},
\]

or as

(2.20) \[
\begin{pmatrix}
    x_t & \pi_t & m^x_{t+2} & m^\pi_{t+2} & m^x_{t+1} & m^\pi_{t+1}
\end{pmatrix}.
\]

### 2.3.1 Steady states and stability

The central bank aims to keep inflation at its target level. It would therefore be desirable for our dynamical system to have a steady state with \( \pi = \pi^T \). Proposition 2.1 states that such a steady state indeed exists, as long as the central bank chooses an output gap target corresponding to the desired inflation level. The proof of Proposition
Proposition 2.1. When the central bank sets $x^T = \frac{1-\beta}{\kappa} \pi^T$, then a steady state with $\pi^* = \pi^T$, $x^* = x^T$, $m^x = 0$, $m^\pi = 0$ exists for any value of $\pi^T$.

From now on we will assume that the central bank always chooses an output gap target consistent with its inflation target so that the steady state where $\pi^* = \pi^T$ and $x^* = x^T$ exists. Since this steady state coincides with the rational expectations equilibrium values of our model, we call this steady state the fundamental steady state. Even though in this steady state convergence to rational expectation values has taken place, it is not the case that all agents use the fundamentalist heuristic. This is so because the naive heuristic also gives perfect steady state predictions, so that both fundamentalists and naive agents have perfect foresight at the fundamental steady state. The difference in fractions therefore equals zero for both variables ($m^x = m^\pi = 0$).

The central bank would like to achieve convergence to the fundamental steady state from as wide a range of initial conditions as possible. This requires first of all that the fundamental steady state is locally stable. The central bank can try to achieve stability of the fundamental steady state by choosing the right values of the parameters in its monetary policy rule, $\phi_1$ and $\phi_2$. The inflation target $\pi^T$ turns out not to matter for stability of the fundamental steady state.

In order for the steady state to be locally stable, it is required that all six eigenvalues are inside the unit circle at the steady state. In Appendix 2.B.2 it is shown that in the fundamental steady state four eigenvalues are equal to zero and that the other two eigenvalues are real and given by

(2.21) \[ \lambda_1 = \frac{1}{4} \left( \left( 1 + \frac{\phi_2}{\sigma} - \kappa \frac{\phi_1 - 1}{\sigma} \right) + \sqrt{\left( 1 + \frac{\phi_2}{\sigma} - \kappa \frac{\phi_1 - 1}{\sigma} \right)^2 - 4\beta \left( 1 - \frac{\phi_2}{\sigma} \right)} \right), \]

and

(2.22) \[ \lambda_2 = \frac{1}{4} \left( \left( 1 + \frac{\phi_2}{\sigma} - \kappa \frac{\phi_1 - 1}{\sigma} \right) - \sqrt{\left( 1 + \frac{\phi_2}{\sigma} - \kappa \frac{\phi_1 - 1}{\sigma} \right)^2 - 4\beta \left( 1 - \frac{\phi_2}{\sigma} \right)} \right). \]

When $\lambda_1 = 1$ or $\lambda_2 = -1$ a bifurcation occurs, and the fundamental steady state loses its stability. Proposition 2.2 and 2.3 describe when this occurs. These results are illustrated in Figure 2.1 and will be discussed below. The proofs of the propositions are given in Appendix 2.B.3 and 2.B.4 respectively.
Proposition 2.2. (See Figure 2.1) A subcritical pitchfork bifurcation (with $\lambda_1 = +1$) occurs when the CB chooses $\phi_1$ to be equal to

$$\phi_1^{PF} = 1 - (2 - \beta)\frac{\sigma + \phi_2}{2\kappa}.$$  

For values of $\phi_1$ below $\phi_1^{PF}$, the fundamental steady state is unstable. For values of $\phi_1$ not too far above $\phi_1^{PF}$, the fundamental steady state is locally stable, and two unstable, non-fundamental steady states exist.

Proposition 2.3. (See Figure 2.1) A period doubling bifurcation (with $\lambda_2 = -1$) occurs when the CB chooses $\phi_1$ to be equal to

$$\phi_1^{PD} = 1 + (2 + \beta)\frac{3\sigma - \phi_2}{2\kappa}.$$  

For values of $\phi_1$ above $\phi_1^{PD}$, the fundamental steady state is unstable, and for values of $\phi_1$ not too far below $\phi_1^{PD}$, the fundamental steady state is locally stable. The bifurcation is subcritical (with an unstable 2-cycle below the bifurcation value) if $\phi_2 < 3\sigma$ and supercritical (with a 2-cycle above the bifurcation value) if $\phi_2 > 3\sigma$.

It follows from Proposition 2.2 and 2.3 that the fundamental steady state can be unstable either because the central bank responds too weakly or because the CB responds too strongly to inflation and output gap expectations.

The intuition of instability of the fundamental steady state due to monetary policy that reacts too weakly is the following. If period $t$ expectations of period $t + 1$ inflation and/or output gap are high, and the central bank does not respond with a large enough increase in the interest rate, these high expectations will lead to period $t$ realizations of inflation and output gap that are even higher. This will lead expectations about period $t + 2$, formed in period $t + 1$, to be higher than expectations about period $t + 1$, leading to even higher period $t + 1$ realizations. This will lead to a loss of credibility for the central bank (more agents become naive), leading to more instability. What follows is a continued rise of both inflation and output gap, together with declining credibility and rising expectations: an inflationary spiral. Analogously, for low initial conditions a deflationary spiral will occur under weak monetary policy.

If the central bank responds too strongly to expectations, high inflation and/or output gap expectations about period $t + 1$ are countered in period $t$ by a very high interest rate. This results in very low inflation and output gap realizations in period $t$, leading to very low expectations about period $t + 2$. The CB then sets the interest rate very low in period $t + 1$, leading to even higher inflation and output gap realizations in
period $t + 1$ than agents had expected. This causes a loss in credibility, and high naive expectations about period $t + 3$. The following increase in the interest rate leads to even lower realizations in period $t + 2$ than agents expected, again leading to a loss in credibility and more extreme expectations. These cyclical dynamics continue, leading inflation and output gap to jump up and down between ever higher and lower values: explosive overshooting.

The results of Proposition 2.1, Proposition 2.2 and Proposition 2.3 can be combined in a bifurcation diagram in $\phi_1$. This is done in Figure 2.1, with $\phi_1$ on the horizontal axis and $\pi_t$ on the vertical axis. The fundamental steady state is located at $\pi_t = \pi^T$, and the black line between $\phi_1^{PF}$ and $\phi_1^{PD}$ indicates the range of $\phi_1$ values for which this steady state is (locally) stable. To the left of $\phi_1^{PF}$ and to the right of $\phi_1^{PD}$ the fundamental steady state is unstable, which is indicated by blue dashed lines. In this picture it is assumed that $\phi_2 = \sigma$, so that the period doubling bifurcation is subcritical. This implies the existence of an unstable 2-cycle to the left of $\phi_1^{PD}$, which is depicted by the red dashed curves. The blue dashed curves between $\phi_1^{PF}$ and 1, represent the non-fundamental unstable steady states from Proposition 2.1 that are created in the subcritical pitchfork bifurcation. As discussed above, explosive cyclical dynamics, due to overshooting, occur to the right of $\phi_1^{PD}$. To the left of $\phi_1^{PF}$ inflation either monotonically increase or decrease, depending on initial conditions. In Figure 2.1 it is assumed that initial output gap is at its target, so that the inflation target is the boundary between inflationary and deflationary spirals.

The result of Proposition 2.2 and Proposition 2.3 are similar to the conditions required for local determinacy under rational expectations found by Bullard and Mitra (2002). These authors show that, when the central bank responds to inflation and output gap expectations, determinacy of the rational expectations equilibrium requires both that the well known Taylor principle is satisfied, and that the central bank does not to respond too strongly to expectations. More specifically, the authors find that first of all $\phi_1 > 1 - (1 - \beta) \frac{\sigma}{\kappa}$ must hold. However, the condition for local stability that follows from Proposition 2.2 requires $\phi_1$ to be larger than $\phi_1^{PF}$, which is (under Assumption 2.1) strictly smaller than the value found by Bullard and Mitra (2002). We can therefore have local stability even if the Taylor principle is not satisfied. The second condition for determinacy given by Bullard and Mitra (2002) reduces to $\phi_1 < 1 + (1 + \beta) \frac{2\sigma - \phi_2}{\kappa}$. This condition is again strictly stronger than our condition for local stability, which requires $\phi_1 < \phi_1^{PD}$.\footnote{We also analyzed local stability under a more traditional Taylor rule where the central bank responds to contemporaneous values of inflation and output. Here we find that local stability of the fundamental steady state requires that $\phi_1 > \frac{1}{2}(1 - (2 - \beta) \frac{\sigma + \phi_2}{2\kappa})$, which is a strictly weaker condition}
Figure 2.1: Bifurcation diagram of $\phi_1$ in case of $\phi_2 = \sigma$. The locally (globally) stable area of the fundamental steady state is indicated with a solid (thick) black line. Unstable steady states are indicated with blue dashed curves, while the red dashed curves represent an unstable 2-cycle. When the fundamental steady state is not globally stable, explosive overshooting can occur if monetary policy is too aggressive. When monetary policy is too weak, either a inflationary or a deflationary spiral will occur. In this picture of the dynamics it is assumed that initial output gap is at its steady state level.

We can conclude that with heterogeneous expectations the range of policy parameters that are allowed in order to have a locally stable fundamental steady state is strictly larger (in both directions) than the range of parameters allowed under rational expectations in order to have a locally determinate equilibrium. However, the conditions for a locally determinate rational expectations equilibrium coincide with the conditions for stability when all agents are naive. Since all naive expectations is the most unstable case in our model, these conditions imply global stability of the funda-

than the one found in Proposition 2.2. Furthermore, just as under rational expectations, there is no upper bound on the monetary policy parameters under a contemporaneous Taylor rule.
2.3. Analysis with Heuristic Switching Model

mental steady state. This is stated in Proposition 2.4, the proof of which is given in Appendix 2.B.5.

**Proposition 2.4.** The fundamental steady state is globally stable when the central bank chooses

\[
1 - (1 - \beta) \frac{\phi_2}{\kappa} = \phi_1^{GL} < \phi_1 < \phi_1^{GU} = 1 + (1 + \beta) \frac{2\sigma - \phi_2}{\kappa},
\]

The global stability region of the fundamental steady state is indicated by the thick black line in Figure 2.1. For this region of policy parameters no unstable steady states or 2-cycles exist.

### 2.3.2 Policy implications

The difference between the local and the global stability results of the previous section highlight the importance of the credibility of the central bank in stabilizing the economy. When the central bank is able to retain a substantial amount of credibility, even after a sequence of bad shocks, its conditions on monetary policy will not be very restrictive and lie close to those given in Proposition 2.2 and 2.3. In our model, this situation would e.g. occur if the intensity of choice is not too high. If, on the other hand, the central bank is likely to lose all its credibility after a sequence of bad shocks, the restrictions on monetary policy lie close to those given in Proposition 2.4. This is in line with the results of the agent based model of Salle et al. (2013), who find that under low (exogenous) credibility of the central banks inflation target, conditions on policy parameters are much more restrictive than under high credibility. As in our model, the Taylor principle is furthermore not a necessary condition in the latter case.

An important question now is whether the fundamental steady state is locally, or perhaps even globally stable under the theoretically optimal benchmark that is implemented by choosing parameter values as in (2.8). Proposition 2.5 states that this is indeed the case. Its proof is given in Appendix 2.B.6

**Proposition 2.5.** When the central bank implements optimal policy by choosing the values for \(\pi^T, x^T, \phi_1\) and \(\phi_2\) from (2.8), the fundamental steady state is globally stable.

In Figure 2.1 it is indicated that \(\phi_1^{opt}\) lies in the globally stable area where the non-fundamental steady states and 2-cycle do not exist. It follows that the optimal policy parameters are indeed a desirable choice for the central bank in our model with heterogeneous expectations.
It is however also of interest to know whether small deviations from optimal policy can lead to instability. That is, whether or not the bifurcations values from Proposition 2.2 and 2.3 lie close to the optimal parameter setting, or the bifurcations occur for parameter values that a central bank would never choose in practice. To investigate this we look at the calibrations discussed in Section 2. Column 4 and 5 of Table 2.1 state the values of the pitchfork bifurcation ($\phi_{PF}^1$) and the period doubling bifurcation ($\phi_{PD}^1$), given that $\phi_2$ is chosen optimally.

From Column 4 it follows that the pitchfork bifurcation occurs at negative values for all calibrations. This means that, in contrast to the Taylor principle, under these calibrations the fundamental steady state is locally stable for monetary policy that reacts weakly to inflation ($0 < \phi_1 < 1$) as long as $\phi_2$ is chosen optimally. This result furthermore turns out to hold for any nonnegative choice of $\phi_2$.

The values of the period doubling bifurcation ($\phi_{PD}^1$) given in Column 5 of Table 2.1 are all unrealistically high. This means that when $\phi_2$ is chosen optimally, reacting too strongly to inflation will not be a problem for any reasonable value of $\phi_1$. The dependence of this result on the optimality of the output gap coefficient however drastically differs over calibrations. Under the Clarida et al. (2000) calibration a very strong output gap coefficient of $\phi_2 = 2$ results in instability for $\phi_1 > 6$, implying that responding too aggressively will not be a problem. However, under the Woodford (1999) calibration, an output gap parameter of $\phi_2 > 0.49$ implies that the period doubling bifurcation occurs at a negative value of $\phi_1$, so that the fundamental steady state will be unstable for any positive inflation parameter.

### 2.4 Zero lower bound on the interest rate

In the previous section no restrictions were placed on the values that can be taken by the nominal interest rate. In practice, the nominal interest rate will however never be set negative. We will show that if the zero lower bound (ZLB) is accounted for, the global stability result of Proposition 2.4 no longer holds, but that with the ZLB deflationary spirals with ever decreasing inflation and output gap can always arise, even under optimal policy. We show this in a sequence of propositions for the limiting case of infinite intensity of choice, i.e., when all agents immediately switch to the best predictor, in Section 2.4.1. In Section 2.4.2 we argue that for finite intensity of choice qualitatively similar dynamics occur.

First we show that the introduction of the ZLB can lead to the existence of an additional steady state (Proposition 2.6). The appearance of this additional steady
state (or equilibrium) was first highlighted by Benhabib et al. (2001a,b) under rational expectations. The presence of this steady state causes divergence to minus infinity for low inflation and output gap in our model, just as in Evans et al. (2008) and Benhabib et al. (2014). Whether such a deflationary spiral occurs however not only depends on initial inflation and output gap, but also on the credibility of the central bank (i.e., the fractions of fundamentalists). We argue that a liquidity trap can never arise as long as the CB retains full credibility (Proposition 2.7), and that a self-fulfilling deflationary spiral only arises when naive agents perform better than fundamentalists about both variables (Proposition 2.8). However, even in a liquidity trap where the CB has lost all its credibility, recovery is still possible if inflation and output gap are not too low. The deflationary spiral and recovery regions are illustrated in Figure 2.2. The corresponding sufficient conditions for recovery or a deflationary spiral to occur will be presented in Proposition 2.9. This proposition also shows that initial conditions for which a deflationary spiral occurs always exist.

Our model is capable of describing expectation driven liquidity traps. We can interpret low initial inflation and output gap as having been caused by a negative shock to the fundamentals of the economy. Under rational expectations the economy would immediately recover from such a shock if there are no new negative shocks in the next period. This is not the case in our model of heterogeneous expectations. Here it is likely that the low realizations of inflation and output gap caused by the shock, lead to a loss of credibility of the central bank, i.e., a higher fraction of naive agents. These naive agents expect the low realizations of inflation and output gap, and therefore the liquidity trap, to continue. These low expectations then lead to low realizations of inflation and output gap in the next period, so that the liquidity trap indeed continues, and expectations become self-fulfilling. If the shocks to inflation and output gap were not too large, or if the central bank retained enough credibility, both variables will start to rise again, and recovery to the positive interest rate region occurs. However, if expectations are too low, inflation and output gap start to decline, resulting in more loss of credibility. The economy then ends up in a self-fulfilling deflationary spiral with no credibility for the central bank.

2.4.1 Analysis for infinite intensity of choice

When we introduce the zero lower bound, the interest rate becomes piecewise linear. In normal times the interest rate is still given by Equation (2.7) and the model is described by Equation (2.16) through (2.19). However, when (2.7) implies that the nominal interest rate is negative, it is instead set equal to zero. We will refer to combinations
of expectations where \( i_t > 0 \) as the "positive interest rate region". Combinations of expectations that imply a binding zero lower bound will be referred to as the "zero lower bound region", or simply the "ZLB region". In the ZLB region the model is described by

\[
(2.26) \quad x_t = \left(1 + m_t^x\right) x_{t-1} + \left(1 - m_t^x\right) \frac{\pi T}{2\sigma} + \left(1 - m_t^\pi\right) \frac{\sigma x_{t-1}}{\sigma} + \bar{r},
\]

\[
(2.27) \quad \pi_t = \beta \frac{1 + m_t^\pi}{2} \pi_T + \beta \frac{1 - m_t^\pi}{2} \pi_{t-1} + \kappa x_t,
\]

with fractions given by (2.18) and (2.19). The steady states of this nonlinear system depend on the fractions of agents following the different heuristics and therefore are, in general, quite difficult to analyze. For this reason we first consider the limiting case where the intensity of choice equals infinity, and all agents immediately switch to the best performing heuristic. The (6-dimensional) system then becomes piecewise linear.

Proposition 2.6 states that, when the intensity of choice equals infinity, at most one (unstable) steady state exists in the ZLB region. In this steady state fundamentalists make persistent prediction errors, implying that all agents have switched to naive expectations about both variables. Naive agents do not make prediction errors, so in this steady state expectations are perfectly self-fulfilling. The proof of Proposition 2.6 is given in Appendix 2.C.1.

**Proposition 2.6.** For \( b = +\infty \) there exists exactly one steady state in the ZLB region when the Taylor principle is adhered to \((\phi_1 > 1 - (1 - \beta) \frac{\phi_2}{\kappa})\). This steady state is given by \( \pi = -\bar{r} \), \( x = -\frac{1-\beta}{\kappa} \bar{r} \) and \( m^\pi = m^x = -1 \), and always is an unstable saddle point. When the Taylor principle is not adhered to \((\phi_1 < 1 - (1 - \beta) \frac{\phi_2}{\kappa})\) no steady states exist in the ZLB region.

Initial conditions in the ZLB region typically will not lead to convergence to the steady state of Proposition 2.6 since it is unstable. Two generic possibilities that can occur for initial conditions in the ZLB region are the following. First of all, it is possible that inflation and output gap start increasing, and at some point cause the system to cross the zero lower bound and enter the positive interest rate region. From then on the dynamics will be as in Section 3. That is, for monetary policy that satisfies the conditions of Proposition 2.4 convergence to the fundamental steady state occurs. The other possibility for dynamics in the ZLB region is that inflation and output gap decline towards minus infinity. Such a deflationary spiral can be interpreted as an inescapable liquidity trap. We will refer to the first case as "recovery", and to the second case as
2.4. Zero lower bound on the interest rate

Table 2.2: Conditions for recovery for different initial conditions when \( b = +\infty \)

"divergence", or as a "deflationary spiral".

When the intensity of choice equals infinity all fractions are either \(-1, 1,\) or \(0\).
Fractions of 0 will typically only occur in a steady state and are not relevant for
out-of-steady-state dynamics. This possibility will therefore not be considered below.
It is convenient to use (2.20) as state vector because four of the six variables will take
only two different values. These four state variables can be represented by a table of 16
different combinations. This is done in Table 2.2, illustrating which initial conditions
lead to recovery (rather than divergence).

In Proposition 2.7 it is stated that for the four cases in the first column of Table
2.2 initial fractions are such that the system already is in the positive interest rate
region from the very first period onwards. For the cases in the first row of Table 2.2
the system trivially is in the positive interest rate region after one period. Therefore it
is indicated in Table 2.2 that for these cases recovery "always" occurs. The intuition
behind this result is that the economy can never be in a liquidity trap when the central
bank has full credibility.

**Proposition 2.7.** If at any point in time all agents are fundamentalistic about both
inflation and output gap (\( m_t^\pi = 1 \) and \( m_t^x = 1 \); full credibility), recovery has occurred.

**Proof.** When expectations about both variables are fundamentalistic we have
\( E_t \pi_{t+1} = \pi^T \) and \( E_t x_{t+1} = x^T \), and (2.7) implies that \( i_t = \bar{r} + \pi^T > 0 \). Therefore, the system is
in the positive interest rate region by definition.

For the remaining nine cases of Table 2.2 recovery or divergence occurs conditional
on the initial conditions of the other two state variables: \( \pi_t \) and \( x_t \). For most cases
it is not straightforward to define exactly for which values of \( \pi_t \) and \( x_t \) recovery and
divergence occur. However, if a deflationary spiral occurs this must be because all
agents have negative naive expectations about both variables after a few periods. That
is, as long as the CB retains some credibility the economy has not (yet) entered a
deflationary spiral. This is stated more formally in Proposition 2.8, the proof of which
is given in Appendix 2.C.2.
**Proposition 2.8.** A necessary condition for a deflationary spiral to occur, is that at some point in time, $s \geq t$, all agents are naive with respect to both inflation and output gap for the next two periods ($m_{s+1}^\pi = -1$, $m_{s+1}^x = -1$, $m_{s+2}^\pi = -1$ and $m_{s+2}^x = -1$)

From Proposition 2.8 it follows that a necessary condition for a deflationary spiral is that the system at some point has moved to the bottom right entry of Table 2.2, with all naive expectations. This entry is therefore the most interesting case when a deflationary spiral is concerned, and hence is labeled the "deflationary spiral case".

From any other entry in Table 2.2 either recovery occurs, or the system moves to the deflationary spiral case, after which the occurrence of recovery or divergence depends on the conditions of that case. We therefore do not present individual conditions for all these cases, but instead focus on the deflationary spiral case.

If, in the deflationary spiral case, initial inflation and output gap are too low, expectations will remain naive and output gap and inflation will keep decreasing without bound. If, however, initial inflation and output gap are high enough, recovery occurs, either because of positive naive expectations, or because at some point expectations become fundamentalistic.

In Proposition 2.9 sufficient conditions for both recovery and divergence for the deflationary spiral case are presented. This proposition thereby also proves that it is always possible to find initial conditions that lead to a deflationary spiral in our model. The proof of Proposition 2.9 is presented in Appendix 2.C.3, and the corresponding deflationary spiral and recovery regions will be presented in Figure 2.2.

**Proposition 2.9.** (See Figure 2.2) If all agents’ expectations about both inflation and output gap are naive for two consecutive periods ($m_{t+1}^\pi = m_{t+2}^\pi = m_{t+1}^x = m_{t+2}^x = -1$) a sufficient condition for recovery is that

\begin{equation}
    x_t > \max(x_{\text{ev}}^{\text{out}}, x_{\text{out}}) = x_{\text{ev}}^{\text{out}},
\end{equation}

and sufficient condition for a deflationary spiral (i.e. divergence) is that

\begin{equation}
    x_t < \min(\bar{x}_{\text{ev}}^{\text{out}}, \bar{x}_{\text{ev}}^{\text{inf}}),
\end{equation}

where $x_{\text{ev}}^{\text{out}}$, $x_{\text{out}}^{\text{out}}$, $x_{\text{ev}}^{\text{ev}}$, $x_{\text{out}}^{\text{out}}$ and $\bar{x}_{\text{inf}}$ are defined in Appendix 2.C.3

The above implies that for infinite intensity of choice a deflationary spiral can always occur if initial conditions are low enough.

The intuition for the conditions that need to be satisfied to provide a sufficient condition for a deflationary spiral is the following. First of all, $x_t < \bar{x}^{\text{ev}}$ guarantees
2.4. Zero lower bound on the interest rate

that inflation and output gap will keep decreasing as long as expectations about both variables remain naive (initial conditions must be below the stable eigenvector of the saddle point steady state). Secondly, $x_t < \bar{x}^{\text{out}}$ and $x_t < \bar{x}^{\text{inf}}$ guarantee that expectations do not become fundamentalistic about respectively output gap and inflation. Similar intuitions hold for the sufficient condition for recovery (see Appendix 2.C.3 for details).

Figure 2.2 plots the conditions from Proposition 2.9 in the $(\pi, x)$-plane for the Woodford (1999) calibration. The thick red line indicates the naive expectations zero lower bound for optimal policy with a weight on output gap of $\mu = 0.25$ and an annualized inflation target of 2%. This line separates the positive interest rate region from the ZLB region. Under the above calibration $x_t < \bar{x}^{\text{inf}}$ is always satisfied when $x_t < \bar{x}^{\text{ev}}$ and $x_t < \bar{x}^{\text{out}}$ hold. This condition is therefore not plotted in Figure 2.2. $\bar{x}^{\text{ev}}$ and $\bar{x}^{\text{out}}$ are plotted in respectively solid green and dashed black. For initial conditions.
to the left of these two lines the sufficient conditions for a deflationary spiral are satisfied and inflation and output gap will keep decreasing. For initial conditions to the right of these two lines (at least for the range of values considered in the figure) the sufficient conditions for recovery are satisfied, and inflation and output gap will increase towards the positive interest rate region.\footnote{In Appendix 2.C.3 it can be seen that as long as $\pi_t < \pi^T$ (which is the case for the green and the dashed line in Figure 2.2) it holds that $\bar{x}^{ev} = \bar{x}^{ev}$ and $\bar{x}^{out} = \bar{x}^{out}$.}

From the steepness of the green and dashed lines, and from the difference in scale on the axes of Figure 2.2, we can conclude that under this calibration inflation expectations are considerably more important than output gap expectations in determining whether recovery or divergence occurs. We find similar results under the Clarida et al. (2000) calibration.

2.4.2 Finite intensity of choice

Now we turn to the more general case of finite intensity of choice, where most, but not all, agents switch to the best performing rule. Because the system is linear in fractions it is of interest to look first at the other limiting case where the intensity of choice is zero. Here always half of the agents are naive, and half are fundamentalistic about each variable. Proposition 2.10 describes the dynamics of this system. Its proof is given in Appendix 2.C.4.

**Proposition 2.10.** Assume $b = 0$. If $\kappa \leq \frac{(2-\beta)\sigma}{2}$, the system described by (2.26), (2.27), (2.18) and (2.19) has a unique, stable steady state with inflation and output gap above their targets. If $\kappa > \frac{(2-\beta)\sigma}{2}$ the system has an unstable saddle steady state with inflation and output gap below their targets. Furthermore, the stable eigenvector of the system then has the same slope as that of the system with $b = +\infty$ and all naive expectations.

In all calibrations we consider it holds that $\kappa < \frac{(2-\beta)\sigma}{2}$. It then follows from Proposition 2.10 that with $b = 0$ all initial conditions under the zero lower bound lead to recovery (they are attracted to a steady state in the positive interest rate region). When the naive heuristic is best performing, the set of initial conditions that lead to recovery is therefore strictly larger when $b = 0$ (where half of the agents remain fundamentalists) than when $b = +\infty$.

For finite intensity of choice we must distinguish between two cases: periods where naive agents always perform best, and periods where fundamentalistic agents sometimes perform best. In line with Proposition 2.8, the naive heuristic must necessary be best.
2.5 Monetary policy and liquidity traps

Shocks to inflation and output gap can push the economy into a liquidity trap by triggering low, self-fulfilling expectations. How can monetary policy prevent these self-fulfilling liquidity traps? In this section we address this question with stochastic simulations. These simulations serve two purposes. First, they illustrate in an intuitive way how stochastic shocks can push our economy with heterogeneous expectations and a zero lower bound on the interest rate into an expectation driven liquidity trap (Section 2.5.2). Secondly, we study the effectiveness of an increased inflation target, aggressive monetary easing, and aggressive inflation targeting in preventing liquidity traps (Section 2.5.3).
2.5.1 Calibration

Unless stated otherwise, the following calibration is used. For $\kappa$ and $\sigma$ we use the Woodford (1999) calibration with $\kappa = 0.024$ and $\sigma = 0.157$, and we set $\beta = 0.99$. We further use optimal policy (as defined by (2.7) and (2.8)). In this policy rule we set the inflation target equal to an annualized 2%, and, following McCallum and Nelson (2004) and Walsh (2003), the weight on output gap is given by $\mu = 0.25$. The optimal monetary policy coefficients therefore are as given in the first row of Table 2.1: $\phi_1 = 1.015$ and $\phi_2 = 0.157$.

The shocks to inflation ($e_t$) and to output gap ($u_t$), presented in Equations (2.1) and (2.2) are reintroduced in this section. Both $e_t$ and $u_t$ are defined as Gaussian white noise and are calibrated to have an annualized standard deviation of 1.1%. The calibration of the standard deviations of shocks determines the likelihood of the occurrence of periods where the zero lower bound binds, as well as their severity. With the chosen calibration liquidity traps do arise, but they are not so severe that no reasonable police measure can prevent them. The same random seed will be used throughout this section.

Finally, the parameters of the Heuristic Switching Model need to be calibrated. We set the memory parameter in the fitness measure, (2.12), to $\rho = 0.5$, allowing agents to update their evaluation of the heuristics significantly when new information arises, but also to put considerable weight on the past.\footnote{We also ran all simulations in this section with $\rho = 0$. This only changes result quantitatively. The policy measures presented in Section 2.5.3 still work to prevent liquidity traps with a lower memory parameter, but the magnitude of the policy change needed to achieve this is larger in that case.} The intensity of choice is set to $b = 40.000$, so that it is possible that almost all agents switch to the same heuristic, but that typically both fundamentalists and naive agents will be present.\footnote{Note that the calibration of the intensity of choice depends on the unit of measurement of the fitness measure. Since a 1% deviation of quarterly inflation from steady state is measured as 0.01, and results in a squared forecast error of 0.0001, an intensity of choice of 40.000 should not be considered particularly large.}

2.5.2 The effect of the zero lower bound

Because of the presence of shocks in the model, inflation and output gap no longer exactly converge to a steady state, but instead keep fluctuating around it. First we simulate the model for 100 periods, assuming there is no zero lower bound on the nominal interest rate. In Figure 2.3 the time series of annualized inflation (upper left panel) and annualized output gap (upper right panel) are plotted in blue, together with the fractions of fundamentalists (Credibility) for both inflation (middle left panel) and output gap (middle right panel). The bottom panel depicts the annualized nominal...
interest rate. In the upper two panels the horizontal red lines indicate the inflation and output gap targets, and the green curves depict inflation and output gap time series that would have occurred under rational expectations.

Figure 2.3 illustrates that there are periods where inflation fluctuates around the target, and periods where inflation drifts away. The intuition behind downward drifts in inflation is the following. When shocks lead to inflation below target for a few consecutive periods, the central bank loses credibility and most agents become naive with respect to inflation (as can be seen in the middle left panel of Figure 2.3, where the fraction of fundamentalists about inflation moves towards 0). The low expectations of naive agents put downward pressure on inflation and become self-fulfilling. Meanwhile, the central bank tries to control inflation by decreasing the interest rate, but does not
immediately succeed. This is so because the CB also cares about output gap, and does not want to react too strongly to inflation expectations in order to limit variations in the output gap. Indeed, we see that output gap stays very close to its target during all periods, and eventually inflation returns to its target as well. The intuition for upwards drifts in inflation is similar.

Comparing the blue curves in the top panels with the green curves, we can conclude that the drifts in inflation indeed arise because of expectations and a loss in credibility and not just because of shocks. With the same shock sequence rational expectations would lead both inflation and output gap the always stay close to their targets.

Note that during the downward drift of inflation the central bank sets a negative
interest rate, which in practice cannot happen. In Figure 2.4 the zero lower bound on
the nominal interest rate is accounted for. Now the interest rate is set to zero when it
would otherwise have been set negative. All variables evolve in exactly the same way
as in Figure 2.3 until the point where the interest rate should be set negative. The blue
curve in the bottom panel again depicts the annualized nominal interest rate. The red
curve depicts the value the interest rate would have taken if it could be set negative.
When the ZLB is binding, the interest rate is set higher than optimal. In the liquidity
trap around period 55 the low inflation expectations and a higher than optimal nominal
interest rate imply a low real interest rate, which depresses output gap. Therefore, in
contrast with Figure 2.3, the economy now enters a recession. Furthermore, the CB
loses credibility with respect to output gap as well as inflation, implying that the econ-
omy is in the deflationary spiral case of Table 2.2 in Section 2.4.1. Moreover, inflation
and output gap have become so low, that the economy is in the deflationary spiral
region of Figure 2.2, and inflation and output gap keep decreasing while expectations
remain naive. Hence, the system has entered a self-fulfilling deflationary spiral.

2.5.3 Preventing deflationary spirals

How can the central bank prevent such a deflationary spiral? One possible solution
would be to respond with aggressive monetary easing as soon as a liquidity trap is
imminent. The central bank could set the interest rate as low as possible (i.e. zero)
as soon as it would otherwise have set the interest rate below some threshold. This
threshold indicates a danger zone of a low interest rates that threaten to fall below
zero. Another possibility is increasing the inflation target, which would make it less
likely that the zero lower bound becomes binding and might limit how low inflation
and output gap become when it does become binding. This could guarantee that the
system remains in the recovery region of Figure 2.2 and a deflationary spiral does not
occur.

It turns out that both an increased inflation target and aggressive monetary eas-
ing can indeed prevent deflationary spirals in our simulations. Furthermore, the two
measures are complements. When there is aggressive monetary easing in place, the
inflation target needs to be raised by less to prevent a particular deflationary spiral,
and vice versa.

Figure 2.5 illustrates how a combination of an increased inflation target and aggres-
sive monetary easing can prevent the deflationary spiral that occurred in Figure 2.4.
The inflation target is here set to an annualized 2.5%, and the central bank conducts
aggressive monetary easing as soon as the annualized interest rate would have fallen
Figure 2.5: Simulated time series of model with increased inflation target of 2.5% and aggressive monetary easing: the interest rate is set to 0 when it falls below the threshold of 1.5%. In the upper panels, the blue lines depict inflation and output gap time series of our model, the green lines depict time series that would have occurred under rational expectations, and the horizontal red lines depict the inflation and output gap targets. The bottom panel depicts both the actual interest rate (blue) and the rate prescribed by (2.7) (red).

During the period of low inflation from period 50 onwards, the higher inflation target and the aggressive monetary easing work together to prevent a deflationary spiral. Inflation still falls and inflation expectations still become naive, but due to the increased inflation target, there is more room for inflation to fall before the zero lower bound is hit. However, inflation does fall enough for interest rates to drop below 1.5%. This induces the central bank to start with aggressive monetary easing by setting the interest rate to 0% which leads to a lower real interest rate than would otherwise have occurred. As a consequence, output gap turns positive which puts upward pressure on
inflation, limiting the decline of inflation. When, in period 58, the interest rate finally reaches the zero lower bound, inflation and output gap are now high enough for the system to remain in the recovery region of Figure 2.2 and a deflationary spiral does not arise. Even though inflation expectations remain naive, inflation starts to rise slowly. During the subsequent recovery phase where the zero lower bound is no longer binding, the central bank keeps regularly applying aggressive monetary easing to let inflation increase towards its target more rapidly. Eventually, both inflation and output gap are brought back to their targets and the central bank regains its credibility.

Since the danger of a liquidity trap and a deflationary spiral arises when the central bank loses credibility and when low self-fulfilling expectations allow inflation to drift downward, instead of the above measures, the central bank could also try to prevent these drifts altogether and always maintain a measure of credibility. It could do so by responding more aggressively to any deviation of inflation form target. That is, by increasing its coefficient on inflation in the interest rate rule.

Figure 2.6 plots the case where $\phi_1 = 1.5$. This can either be interpreted as reacting more strongly to inflation than would have been optimal without the zero lower bound, or as optimal policy with a weight on output gap of approximately $\mu = 0.007$. This reflects that in light of the liquidity trap analysis above, it is much more important to stabilize inflation than it is to stabilize output gap. This point was also made in the analytical analysis in Section 2.4, where we concluded that inflation expectations play a larger role than output gap expectations in determining whether or not the economy can recover from a liquidity trap.

As a result of the higher inflation coefficient, the interest rate in Figure 2.6 tracks short term inflation fluctuations more closely than in the previous figures. Any negative shock to inflation is immediately countered by a low interest rate in the next period, which increases inflation again. As a result, drifts in inflation are less severe and credibility is never fully lost. While inflation still goes down somewhat between period 50 and 60, the zero lower bound never becomes binding.

Note that the cost of the increased inflation coefficient in the Taylor rule arise in the form of stronger output gap fluctuations (also in times where no liquidity trap is imminent) than in Figure 2.3, which is consistent with the fact that we are now considering optimal monetary policy with a very low weight on output gap. These large output gap fluctuations imply a welfare loss. However aggressive monetary easing also leads to large output gap fluctuations (although less frequent), and an increased inflation target implies welfare losses as well. We have illustrated that some of these costs need to be payed to prevent deflationary spirals.
2.6 Conclusion

In this chapter we use a New Keynesian model to study optimal inflation targeting and liquidity traps. Instead of assuming rational expectations, we allow expectations to be formed heterogeneously by using a model where agents switch between heuristics based on relative performance. In our model, fundamentalists, who trust the central bank, compete with naive agents, who base their forecast on past information. We therefore can interpret the fraction of fundamentalistic agents as the credibility of the central bank. Unlike in rational expectations models, this allows us to endogenously model the central banks’ credibility, which is of crucial importance in understanding...
liquidity traps.

Our first finding is that a nominal interest rate that responds too weakly or too strongly to output gap expectations leads to instability of the fundamental steady state. In this steady state both inflation and output gap are equal to the targets set by the central bank. The region of policy parameters that lead to local stability of the fundamental steady state is however strictly larger than the region of policy parameters that result in a locally determinate equilibrium when rational expectations are assumed. In fact, we find that the well known Taylor principle is not a necessary condition for local stability in our heterogeneous expectations model.

We furthermore show that the fundamental steady state is always globally stable and unique when a theoretically optimal interest rate rule is used that is derived from a loss function with all future output gaps and all future deviations of inflation from its target. This policy specification is however only optimal when the central bank is not hindered by the zero lower bound on the nominal interest rate.

When the zero lower bound is introduced to our model, we find that expectation driven liquidity traps can occur, even under optimal policy. In such a liquidity trap, the central bank has lost some, or all, of its credibility, and low, naive expectations make the zero lower bound constraint binding. Whether the economy can recover form such a liquidity trap, or whether a deflationary spiral with ever decreasing inflation and output gap occurs, depends critically both on how low inflation and output gap have become, and on how much credibility the central bank is able to retain. If the central bank has lost too much credibility, and inflation and output gap are too low, more and more agents start to coordinate on low, naive expectations, resulting in a self-fulfilling deflationary spiral. Coordination on naive expectations or on some other form of adaptive expectations is an empirically relevant and plausible situation that is encountered e.g. in Assenza et al. (2014) and other laboratory experiments.

In stochastic simulations with optimal monetary policy we find that small shocks to the economy can lead to coordination on low naive expectations, and that this can result in both transient liquidity traps and in deflationary spirals. We furthermore show that a central bank can prevent deflationary spirals by increasing the inflation target, applying aggressive monetary easing when the interest rate becomes too low, or by responding more aggressively to inflation than optimal without a zero lower bound. All these policy measures come however, with their own disadvantages and costs to the economy. Therefore, a well balanced combination of all three measures may be the best way to proceed.
Appendix 2.A  Microfoundations

The following derivation largely follows the steps of Kurz et al. (2013). We make however use of the properties of our Heuristic Switching Model, which allows us to fully aggregate, and to obtain, under heterogeneous Euler equation learning, the same equations that arise under a representative household with rational expectations.

There is a continuum \((i)\) of households who differ in the way they form expectations about inflation and about output gap. Households with the same expectations have the same preferences and will make the same decisions. The intratemporal problem of each household \(i\), consists of choosing consumption over a continuum of differentiated goods \((j)\) to minimize expenditure. This implies

\[
C^i_t(j) = \left( \frac{P_t(j)}{P_t} \right)^{-\theta} C^i_t,
\]

with \(C^i_t\) and \(P_t\) total consumption of the household and the aggregate price level, defined by

\[
C^i_t = \left( \int_0^1 C^i_t(j)^{\frac{\theta-1}{\theta}} dj \right)^{\frac{\theta}{\theta-1}},
\]

\[
P_t = \left( \int_0^1 P_t(j)^{1-\theta} dj \right)^{\frac{1}{1-\theta}},
\]

where \(\theta\) is the elasticity of substitution between the different goods.

The household \(i\) then chooses consumption \((C^i_t)\), labor \((H^i_t)\), and real bond holdings \((b^i_t)\) to maximize

\[
\tilde{E}_t^i \sum_{s=t}^{\infty} \beta^{s-t} \left[ \frac{(C^i_s)^{1-\sigma}}{1-\sigma} - \frac{(H^i_s)^{1+\eta}}{1+\eta} \right],
\]

subject to its budget constraint

\[
C^i_t + b^i_t \leq w_t H^i_t + \frac{b^i_{t-1}(1 + i_{t-1})}{1 + \pi_t} + T^i_t,
\]

where \(\beta_t\) is the discount factor, \(w_t\) the real wage rate, \(i_t\) the nominal interest rate, \(\pi_t = \frac{P_t}{P_{t-1}} - 1\) is the inflation rate, and \(T^i_t\) real lump sum transfers to household \(i\), including profits from firms. \(\tilde{E}_t^i\) represents the subjective expectation operator that differs over the households.
2.A. Microfoundations

The first order conditions with respect to $C^i_t$, $H^i_t$ and $b^i_t$ give

\[
(C^i_t)^{-\sigma} = \lambda^i_t, \\
(H^i_t)^{\eta} = \lambda^i_t(1 - \tau_H)w_t, \\
\lambda^i_t = \beta \tilde{E}^i_t \left( \frac{1 + i^i_{t+1}}{1 + \pi_{t+1}} \right),
\]

with $\lambda$ the Lagrange multiplier. Solving for this multiplier, we can rewrite these conditions to the Euler equation and an expression for the real wage rate, which, together with the budget constraint (2.34), must hold in equilibrium

\[
(C^i_t)^{-\sigma} = \beta \tilde{E}^i_t \left[ \frac{(C^i_{t+1})^{-\sigma}(1 + i^i_t)}{1 + \pi_{t+1}} \right],
\]

\[
w_t = (H^i_t)^{\eta}(C^i_t)^{\sigma}.
\]

The Euler equation, (2.35), can be log linearized around a zero inflation steady state to get

\[
\hat{C}^i_t = \tilde{E}^i_t[\hat{C}^i_{t+1}] - \frac{1}{\sigma}(i^i_t - \tilde{E}^i_t[\pi_{t+1}] - \tilde{r}),
\]

where $\hat{C} = \frac{C - \bar{C}}{\bar{C}}$, with $\bar{C}$ the steady state value of consumption, and $\tilde{r} = 1 - \frac{1}{\beta}$ is the steady state real (and due to a zero steady state inflation also nominal) interest rate.

We assume that our boundedly rational agents use Euler equation learning (see Honkapohja et al., 2012), implying that they use the two period trade-off of (2.37) to make optimal decisions given their subjective forecasts of next period. Microfoundations with heterogeneous expectations under infinite horizon learning are derived by Massaro (2013).

Next, we deviate from Kurz et al. (2013), and use a property of the discrete choice model (Equation (2.11)), which determines the fractions of agents in each period as in Brock and Hommes (1997). Under this model it is implicitly assumed that the probability to follow a particular heuristic next period is the same across agents, i.e., independent of the heuristic they followed in the past. This reflects the fact that our agents are not inherently different, but that each of them faces the same trade-off between becoming naive or fundamentalist each period. We assume agents know (have learned) that all agents have the same probability to follow a particular heuristic in the future, and that they know that consumption decisions only differ between households.
CHAPTER 2. INFLATION TARGETING ENDOGENOUS CREDIBILITY

in so far as their expectations are different. In that case households’ expectations about their own future consumption coincide with their expectations about the future consumption of any other agent, and therefore with aggregate consumption. That is, $\tilde{E}_t[\hat{C}_{t+1}] = \tilde{E}_t[\hat{\hat{C}}_{t+1}]$, with $\hat{C}_{t+1} = \int_0^t \hat{C}_{t+1}^i di$. Agents therefore realize they should base their current period consumption decision on expectations about future aggregate consumption. The Euler equation can then be written as

\begin{equation}
\hat{C}_t^i = \tilde{E}_t[\hat{C}_{t+1}] - \frac{1}{\sigma} (i_t - \tilde{E}_t[\pi_{t+1}] - \bar{r}),
\end{equation}

Market clearing in each good $j$ market imposes that

\begin{equation}
Y_t(j) = C_t(j),
\end{equation}

where $C_t(j) = \int C_t^i(j) di$ is aggregate consumption of good $j$. If we aggregate over all varieties of goods, we end up with the aggregate goods market clearing condition

\begin{equation}
Y_t = C_t.
\end{equation}

We assume that agents have learned about market clearing, so that their forecasts satisfy $\tilde{E}_t[\hat{C}_{t+1}] = \tilde{E}_t[\hat{Y}_{t+1}]$. Therefore, (2.38) can be written as

\begin{equation}
\hat{C}_t^i = \tilde{E}_t[\hat{Y}_{t+1}] - \frac{1}{\sigma} (i_t - \tilde{E}_t[\pi_{t+1}] - \bar{r}).
\end{equation}

Aggregating this equation over all agents, and using the period $t$ market clearing condition then gives

\begin{equation}
\hat{Y}_t = \tilde{E}_t[\hat{Y}_{t+1}] - \frac{1}{\sigma} (i_t - \tilde{E}_t[\pi_{t+1}] - \bar{r}).
\end{equation}

Here $\tilde{E}_t$ is the aggregate expectation operator defined as $\tilde{E}_t[Z_{t+1}] = n_t^F \tilde{E}_t^F[Z_{t+1}] + (1 - n_t^F) \tilde{E}_t^N[Z_{t+1}]$, with $\tilde{E}_t^F$ the fundamentalist expectation operator, $\tilde{E}_t^N$ the naive expectation operator, and $n_t^N$ the fraction of agents that are fundamentalist with respect to variable $Z$.

Next we turn to the supply side of the economy. There is a continuum $(j)$ of firms producing the differentiated goods. Each firm is run by a household and follows the same heuristics for prediction of future variables as that household in each period. Each firm has a linear technology with labor as its only input

\begin{equation}
Y_t(j) = A_t H_t(j),
\end{equation}

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where \(A_t\) is aggregate productivity in period \(t\). We assume that in each period a fraction \((1 - \omega)\) firms can change their price, as in Calvo (1983). Firms want to choose the price \(p(j)\) that maximizes their expected discounted profits

\[
(2.43) \quad \hat{E}_t^j \sum_{s=0}^{\infty} \omega^s Q_{t,t+s}^j \left[ \frac{p_t(j)Y_{t+s}(j) - P_{t+s}mc_{t+s}Y_{t+s}(j)}{P_{t+s}} \right],
\]

where

\[
(2.44) \quad Q_{t,t+s}^j = \beta^s \left( \frac{C_{t+s}^j}{C_t^j} \right)^{-\sigma} \frac{P_t}{P_{t+s}},
\]

is the stochastic discount factor of the household \((j)\) that runs firm \(j\).

\[
(2.45) \quad mc_t = \frac{w_t(1 - \nu)}{A_t},
\]

are real marginal cost incurred by firms, with \(\nu\) a production subsidy. Using the demand for good \(j\), the firm’s profits maximization problem writes as follows

\[
\max \hat{E}_t^j \sum_{s=0}^{\infty} \omega^s \beta^s \left( \frac{C_{t+s}^j}{C_t^j} \right)^{-\sigma} \frac{P_t}{P_{t+s}} \left[ \left( \frac{p_t(j)}{P_{t+s}} \right)^{1-\theta} Y_{t+s} - mc_{t+s} \left( \frac{p_t(j)}{P_{t+s}} \right)^{-\theta} Y_{t+s} \right].
\]

The first order condition for \(p_t(j)\) is

\[
(2.46) \quad \hat{E}_t^j \sum_{s=0}^{\infty} \omega^s \beta^s \left( \frac{C_{t+s}^j}{C_t^j} \right)^{-\sigma} \frac{P_t}{P_{t+s}} \left( \frac{p_t^*(j)}{P_{t+s}} \right)^{-1-\theta} Y_{t+s} \left[ \frac{p_t^*(j)}{P_{t+s}} - \frac{\theta}{\theta - 1} mc_{t+s} \right] = 0,
\]

where \(p_t^*(j)\) is the optimal price for firm \(j\) if it can re-optimize in period \(t\).

This can be written as

\[
q_t^*(j) \hat{E}_t^j \sum_{s=0}^{\infty} \omega^s \beta^s \left( \frac{C_{t+s}^j}{C_t^j} \right)^{-\sigma} \left( \frac{P_{t+s}}{P_t} \right)^{\theta-1} Y_{t+s}
\]

\[
= \frac{\theta}{\theta - 1} \hat{E}_t^j \sum_{s=0}^{\infty} \omega^s \beta^s \left( \frac{C_{t+s}^j}{C_t^j} \right)^{-\sigma} \left( \frac{P_{t+s}}{P_t} \right)^{\theta} Y_{t+s}mc_{t+s},
\]

with \(q_t^*(j) = \frac{p_t^*(j)}{P_t}\).

Log linearizing gives

\[
\frac{\hat{q}_t^*(j)}{1 - \omega \beta} = \hat{E}_t^j \sum_{s=0}^{\infty} \omega^s \beta^s (mc_{t+s} + \hat{p}_{t+s}) - \frac{1}{1 - \omega \beta} \hat{p}_t,
\]
which can be written as

\[ \hat{q}^*_t(j) + \hat{p}_t = (1 - \omega \beta)(\hat{m}c_t + \hat{p}_t) + \omega \beta (1 - \omega \beta) \hat{E}_t^{\beta} \sum_{s=0}^{\infty} \omega^s \beta^s (\hat{m}c_{t+s+1} + \hat{p}_{t+s+1}), \]

or recursively as

\[ \hat{q}^*_t(j) + \hat{p}_t = (1 - \omega \beta)(\hat{m}c_t + \hat{p}_t) + \omega \beta \hat{E}_t^{\beta}[\hat{q}^*_t(j) + \hat{p}_{t+1}], \]

\[ \hat{q}^*_t(j) = (1 - \omega \beta)\hat{m}c_t + \omega \beta \hat{E}_t^{\beta}[\hat{q}^*_t(j) + \pi_{t+1}]. \]

(2.47)

Just as in the case of consumption, it follows from the discrete choice model that \( \hat{E}_t^{\beta}[\hat{q}^*_t(j)] = \hat{E}_t^{\beta}[\hat{q}^*_t(j+1)] \). Therefore, agents base their pricing decisions on their expectations of future aggregate variables, and we can write

\[ \hat{q}^*_t(j) = (1 - \omega \beta)\hat{m}c_t + \omega \beta \hat{E}_t^{\beta}[\hat{q}^*_t(j+1) + \pi_{t+1}]. \]

(2.48)

Next we turn to the evolution of the aggregate price level. We assume that the set of firms that can change their price in a period is chosen independently of the types of the households running the firm, so that the distribution of expectations of firms that can change their price is identical to the distribution of expectations of all firms. Since decisions of firms only differ in so far their expectations differ, it follows that the aggregate price level evolves as

\[ P_t = [\omega P_{t-1}^{1-\theta} + (1 - \omega) \int_0^{\hat{q}^*_t(j)} \hat{p}_t(j)^{1-\theta} dj]^{\frac{1}{1-\theta}}. \]

(2.49)

This can be log linearized to

\[ \hat{p}_t = \omega \hat{p}_{t-1} + (1 - \omega) \int_0^{\hat{q}^*_t(j)} \hat{p}_t(j) dj, \]

(2.50)

from which it follows that

\[ \frac{\omega}{1 - \omega} \pi_t = \int_0^{\hat{q}^*_t(j)} \hat{q}_t(j) dj = \hat{q}^*_t. \]

(2.51)

Plugging this into (2.48) gives

\[ \hat{q}^*_t(j) = (1 - \omega \beta)\hat{m}c_t + \omega \beta \hat{E}_t^{\beta}[\pi_{t+1}]. \]

(2.52)
Aggregating over all firms and again using (2.51) gives

\[(2.53) \quad \pi_t = \beta \bar{E}_t[\pi_{t+1}] + \kappa \tilde{m}_t, \]

with

\[(2.54) \quad \tilde{\kappa} = \frac{(1 - \omega)(1 - \beta \omega)}{\omega}. \]

Log linearizing (2.45), (2.36) and (2.42), and combining with market clearing gives

\[(2.55) \quad \tilde{m}_t = \hat{w}_t - \hat{A}_t = \eta \hat{H}_t + \sigma \hat{C}_t - \hat{A}_t = (\sigma + \eta) \hat{Y}_t - (1 + \eta) \hat{A}_t, \]

Inserting this in (2.53) results in

\[(2.56) \quad \pi_t = \beta \bar{E}_t[\pi_{t+1}] + \tilde{\kappa}(\sigma + \eta) \hat{Y}_t - \tilde{\kappa}(1 + \eta) \hat{A}_t. \]

Finally we write (2.41) and (2.56) in terms of output gap. Here we assume that the subsidy to firms offsets the distortions due to monopolistic competition, so that the flexible price equilibrium is efficient.

\begin{equation}
\hat{Y}_{t}^{\text{pot}} = \frac{(1 + \eta)}{\sigma + \eta} \hat{A}_t.
\end{equation}

Plugging in \( x_t = \hat{Y}_t - \hat{Y}_{t}^{\text{pot}} \) in (2.41) and (2.56) gives

\[(2.58) \quad x_t = \bar{E}_t[x_{t+1}] - \frac{1}{\sigma}(i_t - \bar{E}_t[\pi_{t+1}] - \tilde{r}) + u_t, \]

\[(2.59) \quad \pi_t = \beta \bar{E}_t[\pi_{t+1}] + \kappa x_t, \]

with \( \kappa = \hat{\kappa}(\sigma + \eta) \), and \( u_t = \frac{(1 + \eta)}{\sigma + \eta} (\hat{A}_{t+1} - \hat{A}_t) \). If we introduce a cost push shock \( (e_t) \) in the Phillips curve the model of (2.1) and (2.2) is obtained.
Appendix 2.B  Monetary policy without the ZLB

2.B.1  Proof Proposition 2.1

In a steady state, (2.18) and (2.19) reduce to

\[(2.60)\]

\[m^x = \tanh\left(-\frac{b}{2}(x - x^T)^2\right)\]

\[(2.61)\]

\[m^\pi = \tanh\left(-\frac{b}{2}(\pi - \pi^T)^2\right)\]

This implies that when \(x = x^T\) and \(\pi = \pi^T\) fractions are given by \(m^x = m^\pi = 0\). Next, we need to check that these steady state values are also consistent with (2.16) and (2.17). It can immediately seen that when \(x_t = x_{t-1} = x^T\) and \(\pi_{t-1} = \pi^T\), then (2.16) is satisfied. Plugging in \(x_t = x, \pi_t = \pi_{t-1} = \pi^T = \pi\) and \(m^x = m^\pi = 0\) in (2.17) we can rewrite the steady state Phillips curve as

\[(2.62)\]

\[x = \frac{1 - \beta}{\kappa \pi}\]

This implies that as long as \(x^T = \frac{1 - \beta}{\kappa \pi}^T\), a steady state with \(x = x^T, \pi = \pi^T\) and \(m^x = m^\pi = 0\) exists.
2.B. Monetary policy without the ZLB

2.B.2 Jacobian and eigenvalues

In this section, first the Jacobian of the system given by by (2.16) through (2.19) is presented. Next this Jacobian is evaluated at the fundamental steady state, and eigenvectors are derived.

The Jacobian is given by

\[
\begin{pmatrix}
(1 - \frac{\phi_2}{\sigma})(1-m^*_t) & 0 & 0 & b_{11} & b_{12} \\
\kappa(1 - \frac{\phi_2}{\sigma})(1-m^*_t) & \beta(1-m^*_t) - \kappa\frac{\phi_1-1}{\sigma}(1-m^*_t) & 0 & 0 & b_{21} & b_{22} \\
1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
c_{11}s_A & c_{12}s_A & d_{11}s_A & 0 & e_{11}s_A & e_{12}s_A \\
c_{21}s_B & c_{22}s_B & 0 & d_{22}s_B & e_{21}s_B & e_{22}s_B
\end{pmatrix}
\]

with

\[
B = \begin{pmatrix}
-\frac{1}{2}(1 - \frac{\phi_2}{\sigma})\tilde{x}_{t-1} & \frac{\phi_1-1}{2\sigma} \tilde{\pi}_{t-1} \\
-\frac{\phi_2}{2}(1 - \frac{\phi_2}{\sigma})\tilde{x}_{t-1} & (\kappa\frac{\phi_1-1}{\sigma} - \frac{\beta}{2}) \tilde{\pi}_{t-1}
\end{pmatrix},
\]

\[
C = \begin{pmatrix}
-(1 - \frac{\phi_2}{\sigma})(1-m^*_t)\tilde{x}_{t-2} & \frac{\phi_1-1}{\sigma}(1-m^*_t)\tilde{x}_{t-2} \\
-\kappa(1 - \frac{\phi_2}{\sigma})(1-m^*_t)\tilde{\pi}_{t-2} & (\kappa\frac{\phi_1-1}{\sigma} - \beta)(1-m^*_t)\tilde{\pi}_{t-2}
\end{pmatrix},
\]

\[
E = \begin{pmatrix}
(1 - \frac{\phi_2}{\sigma})\tilde{x}_{t-1}\tilde{x}_{t-2} & -\frac{\phi_1-1}{\sigma}\tilde{x}_{t-1}\tilde{\pi}_{t-2} \\
\kappa(1 - \frac{\phi_2}{\sigma})\tilde{x}_{t-1}\tilde{\pi}_{t-2} & -(\kappa\frac{\phi_1-1}{\sigma} - \beta)\tilde{\pi}_{t-1}\tilde{\pi}_{t-2}
\end{pmatrix},
\]

\[
d_{11} = 2x_{t-2} - 2x_t, \quad d_{22} = 2\pi_{t-2} - 2\pi_t,
\]

\[
\tilde{x}_{t-j} = x_{t-j} - x^T, \quad \tilde{\pi}_{t-j} = \pi_{t-j} - \pi^T, \quad j = 1, 2,
\]

\[
s_A = \frac{b}{2} \text{sech} \left( \frac{b}{2}(x^T_{t-2} - (x^T)^2 - 2(x_{t-2} - x^T)x_t) \right),
\]

\[
s_B = \frac{b}{2} \text{sech} \left( \frac{b}{2}(\pi^T_{t-2} - (\pi^T)^2 - 2(\pi_{t-2} - \pi^T)\pi_t) \right),
\]

and with \(x_t\) and \(\pi_t\) given by (2.16) and (2.17).

In the fundamental steady state where \(\pi_t = \pi^T, x_t = x^T\), and \(m^*_t = m^*_t = 0\) for all
t, the Jacobian reduces to
\[
\begin{pmatrix}
\frac{1}{2}(1 - \frac{\phi_2}{\sigma}) & -\frac{1}{2} \frac{\phi_1 - 1}{\sigma} & 0 & 0 & 0 & 0 \\
\frac{1}{2}(1 - \frac{\phi_2}{\sigma}) & \frac{\beta}{2} - \frac{1}{2} \frac{\phi_1 - 1}{\sigma} & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}.
\]

This is a lower triangular block matrix, with four eigenvalues equal to 0. The other eigenvalues are the eigenvalues of the upper left 2x2 block. These two eigenvalues are equal to the values given in (2.21) and (2.22).

### 2.B.3 Proof Proposition 2.2

From (2.21) it follows that \(\lambda_1 > 1\) if and only if
\[
(1 + \beta - \frac{\phi_2}{\sigma} - \kappa \frac{\phi_1 - 1}{\sigma})^2 - 4\beta(1 - \frac{\phi_2}{\sigma}) > 16 + (1 + \beta - \frac{\phi_2}{\sigma} - \kappa \frac{\phi_1 - 1}{\sigma})^2 - 8(1 + \beta - \frac{\phi_2}{\sigma} - \kappa \frac{\phi_1 - 1}{\sigma}),
\]
or
\[
-(2 - \beta)(1 + \frac{\phi_2}{\sigma}) > 2\kappa \frac{\phi_1 - 1}{\sigma},
\]
which in terms of \(\phi_1\) gives
\[
(2.63) \quad \phi_1 < \phi_1^{PF} = 1 - (2 - \beta) \frac{\phi_2 + \sigma}{2\kappa}.
\]

Below we show that a pitchfork bifurcation occurs at this value of \(\phi_1\) by showing that two non-fundamental symmetric steady states are created here. In steady state (where \(x^T = \frac{1-\beta}{\pi} \pi^T\)) (2.16) and (2.17) can be combined to
\[
(2.64) \quad \pi = \left(1 - \beta \frac{(1 - m^\pi)}{2} + \kappa \frac{\phi_1 - 1}{\sigma} \frac{(1 - m^\pi)}{(1 - (1 - \frac{\phi_2}{\sigma}) \frac{(1 - m^\pi)}{2})} \right) \pi^T.
\]

Non-fundamental steady states (where \(\pi \neq \pi^T\)) could exist as solutions of (2.64) if
they satisfy
\[1 - \beta \left(1 - m^\pi\right) + \kappa \frac{\phi_1 - 1}{\sigma} \left(1 - \frac{\phi_2}{\sigma} \right) \frac{1}{2} \left(1 - m^\pi\right) = 0.\]

Writing this in terms of the inflation fraction gives
\[
\tag{2.65}
m^\pi = \frac{(2 - \beta)(1 - (1 - \frac{\phi_2}{\sigma}) \left(1 - m^x\right)) + \kappa \frac{\phi_1 - 1}{\sigma}}{-\beta(1 - (1 - \frac{\phi_2}{\sigma}) \left(1 - m^x\right)) + \kappa \frac{\phi_1 - 1}{\sigma}}.
\]

The steady state values of \(\pi\) then are
\[
\tag{2.66}
\pi^* = \pi^T \pm \sqrt{-\frac{2}{b} \tanh^{-1} \left( \frac{(2 - \beta)(1 - (1 - \frac{\phi_2}{\sigma}) \left(1 - m^x\right)) + \kappa \frac{\phi_1 - 1}{\sigma}}{-\beta(1 - (1 - \frac{\phi_2}{\sigma}) \left(1 - m^x\right)) + \kappa \frac{\phi_1 - 1}{\sigma}} \right)}.
\]

When non-fundamental steady states exist, there thus are two non-fundamental steady states, symmetric around the fundamental value \(\pi^T\).

Because in a non-fundamental steady states naive predictors perform better than fundamentalists, non-fundamental steady states can only exist with
\[
\tag{2.67}
-1 \leq m^\pi < 0.
\]

Since it is assumed that both \(\sigma\) and \(\phi_1\) are non-negative we must have
\[
\tag{2.68}
X^{\text{net}} \equiv (1 - (1 - \frac{\phi_2}{\sigma}) \left(1 - m^x\right)) > 0.
\]

Using this and (2.65), the inequalities in (2.67) reduce to
\[
\beta X^{\text{net}} - \kappa \frac{\phi_1 - 1}{\sigma} \geq (2 - \beta) X^{\text{net}} + \kappa \frac{\phi_1 - 1}{\sigma} > 0,
\]
or equivalently
\[
\tag{2.69}
1 - \frac{\sigma}{\kappa}(1 - \beta) X^{\text{net}} \geq \phi_1 > 1 - \frac{\sigma}{\kappa}(2 - \beta) X^{\text{net}}.
\]

From the equivalence of (2.67) and (2.69), it can be concluded that as \(\phi_1\) gets close to its right-hand limit, \(m^\pi\) gets close to zero. This implies that \(m^x\), \(x\) and \(\pi\) also go to their fundamental values as this happens. Using that \(m^x\) goes to zero in the limit, we see from Equation (2.69) that the limiting value of \(\phi_1\) for which the non-fundamental
steady state exist is

\[ \phi_{1}^{PF} = 1 - \frac{\sigma}{\kappa}(2 - \beta)(1 - \left(1 - \frac{\phi_{2}}{\sigma}\right)^{2}) = 1 - (2 - \beta) \frac{\phi_{2} + \sigma}{2\kappa}. \]

At this point both steady states coincide with the fundamental steady state. We can conclude that at the bifurcation value indeed two non-fundamental steady states are created, which exists for values of \( \phi_{1} \) larger than \( \phi_{1}^{PF} \). The bifurcation therefore is a subcritical pitchfork bifurcation and the non-fundamental steady states must be unstable.

### 2.B.4 Proof Proposition 2.3

From (2.22) it follows that \( \lambda_{2} \) is smaller than \(-1\) when

\[
\sqrt{\left(1 + \beta - \frac{\phi_{2}}{\sigma} - \kappa \frac{\phi_{1} - 1}{\sigma}\right)^{2} - 4\beta(1 - \frac{\phi_{2}}{\sigma})} > 4 + (1 + \beta - \frac{\phi_{2}}{\sigma} - \kappa \frac{\phi_{1} - 1}{\sigma}),
\]

the condition reduces to

\[(2 + \beta) \frac{\phi_{2}}{\sigma} > 3(2 + \beta) - 2\kappa \frac{\phi_{1} - 1}{\sigma},\]

which can be rewritten as

\[(2.70) \quad \phi_{1} > 1 + (2 + \beta) \frac{3\sigma - \phi_{2}}{2\kappa}.\]

When one eigenvalue becomes \(-1\), a 2-cycle must exists either below or above the bifurcation value. This makes the period doubling bifurcation either subcritical or supercritical. In what follows, \( \phi_{1} \) is treated as the bifurcation parameter. The value of \( \phi_{2} \) then turns out to determine if the bifurcation is subcritical or supercritical.

The 2-cycle in question is symmetric around the fundamental steady state. We thus have \((x_{1} - x^{T}) = -(x_{2} - x^{T})\) and \((\pi_{1} - \pi^{T}) = -(\pi_{2} - \pi^{T})\). Using this, (2.5) and (2.17) can be written as

\[
x = x_{1} = x^{T} + (1 - \frac{\phi_{2}}{\sigma}) \frac{(1 - m^{x})(1 - m^{z})}{2}(x_{2} - x^{T}) - \frac{\phi_{1} - 1}{\sigma} \frac{(1 - m^{\pi})}{2} \frac{(\pi_{2} - \pi^{T})}{(x_{2} - x^{T})}
\]

\[
(1 + (1 - \frac{\phi_{2}}{\sigma}) \frac{(1 - m^{x})}{2})x = -\frac{\phi_{1} - 1}{\sigma} \frac{(1 - m^{\pi})}{2} \frac{(\pi_{2} - \pi^{T})}{(x_{2} - x^{T})} + (1 + (1 - \frac{\phi_{2}}{\sigma}) \frac{(1 - m^{x})}{2})x^{T}
\]

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Plugging this in in the Phillips curve, and using \( x^T = \frac{1-\beta}{\kappa} \pi^T \) gives

\[
\pi = \pi_1 = \beta \left( \frac{1 + m^*}{2} \right) \pi^T + \beta \left( \frac{1 - m^*}{2} \right) \pi_2 - \frac{\phi_1 - 1}{\sigma} \frac{(1 - m^*)}{2} \left( \pi_2 - \pi^T \right) + (1 - \beta) \pi^T,
\]

(2.71) \[
\pi - \pi^T = -\left( \pi - \pi^T \right) \left( \beta \frac{1 - m^*}{2} - \kappa \frac{\phi_1 - 1}{\sigma} \frac{(1 - m^*)}{2} \right).
\]

So that a 2 cycle (where \( \pi \neq \pi^T \)) must satisfy

(2.72) \[
(1 + \beta \frac{1 - m^*}{2} - \kappa \frac{\phi_1 - 1}{\sigma} \frac{(1 - m^*)}{2}) = 0.
\]

In terms of the inflation fraction, this gives

(2.73) \[
m^* = \frac{(2 + \beta)(1 + \phi_2 \frac{(1 - m^*)}{2}) - \kappa \frac{\phi_1 - 1}{\sigma}}{\beta(1 + \phi_2 \frac{(1 - m^*)}{2}) - \kappa \frac{\phi_1 - 1}{\sigma}}.
\]

In a 2-cycle around the fundamental steady state naive agents use the observation from period \( t - 1 \) to give a prediction about period \( t + 1 \). Therefore, in a 2-cycle they make no prediction errors, while fundamentalists do make prediction errors. In a 2-cycle we must therefore have

(2.74) \[
-1 \leq m^*_t < 0.
\]

Now, if

(2.75) \[
(1 + \phi_2 \frac{(1 - m^*_t)}{2}) > 0,
\]

the inequalities of (2.74) reduce to

(2.76) \[
1 + (1 + \phi_2) \frac{(1 - m^*_t)}{2} \leq \phi_1 < 1 + (2 + \phi_2) \frac{(1 - m^*_t)}{2}.
\]

If

(1 + \phi_2 \frac{(1 - m^*_t)}{2}) = 0,
then (2.74) can never hold, and if
\[
1 + \left(1 - \frac{\phi_2}{\sigma}\right) \left(\frac{1 - m_t^\pi}{2}\right) < 0.
\]
then (2.74) reduces to
\[
1 + (2 + \beta) \frac{\sigma}{\kappa} \left(1 + \left(1 - \frac{\phi_2}{\sigma}\right) \left(\frac{1 - m_t^\pi}{2}\right)\right) < \phi_1 \leq 1 + (1 + \beta) \frac{\sigma}{\kappa} \left(1 + \left(1 - \frac{\phi_2}{\sigma}\right) \left(\frac{1 - m_t^\pi}{2}\right)\right).
\]
As \(\phi_1\) comes close to making the right hand side of (2.76) or the left hand side of (2.77) binding, the system comes close to the fundamental steady state. In the limit we therefore have \(m^\pi = 0\). The limiting value of these restrictions therefore reduces to the bifurcation value
\[
\phi_1^{PD} = 1 + (2 + \beta) \frac{\sigma}{\kappa} \left(1 + \left(1 - \frac{\phi_2}{\sigma}\right) \left(\frac{1}{2}\right)\right) = 1 + (2 + \beta) \frac{(3\sigma - \phi_2)}{2\kappa}.
\]
Finally, we can conclude that the bifurcation is subcritical (with a 2-cycle below the bifurcation) value if (2.75) holds for \(m^\pi = 0\), which is the case if and only if
\[
\phi_2 < 3\sigma,
\]
The bifurcation is supercritical (with a 2-cycle below the bifurcation) if
\[
\phi_2 > 3\sigma,
\]
and if \(\phi_2 = 3\sigma\) the bifurcation occurs at \(\phi_1 = 1\), and no 2-cycle is created.

### 2.B.5 Proof Proposition 2.4

The dynamical system given by (2.16) and (2.17) is linear in expectation fractions \((m_t^\pi\) and \(m_t^\pi\)). Furthermore, since the system with all fundamentalists is degenerate (a steady state is reached in every period), the Jacobian and eigenvalues for any given set of expectation fractions is scaled by the fraction of naive agents. It follows that if the linear system given by (2.16) and (2.17) is stable for all naive fractions, it is stable for any set of expectations fractions, which implies global stability of the fundamental steady state in our non-linear dynamical system. When \(m_t^\pi = m_t^\pi = -1\) the system reduces to
\[
x_t = (1 - \frac{\phi_2}{\sigma})x_{t-1} - \frac{\phi_1 - 1}{\sigma} (\pi_{t-1} - \pi^T),
\]
2. C. Zero lower bound on the nominal interest rate

(2.80) \[ \pi_t = \beta \pi_{t-1} + \kappa x_t. \]

The eigenvalues of this system are two times the eigenvalues given by (2.21) and (2.22). Now, replacing \( \frac{1}{4} \) with \( \frac{1}{2} \) and performing the same calculations as done in Appendix 2.B.3 and 2.B.4 gives the conditions given in the proposition.

2.B.6 Proof Proposition 2.5

Since \( \phi_1^{opt} > 1 \), the first condition from Proposition 2.4 is satisfied. The other condition for global stability reduces for \( \phi_2 = \phi_2^{opt} = \sigma \) to

\[ \phi_1^{opt} = 1 + \frac{\sigma \kappa}{\mu + \kappa^2} < 1 + (1 + \beta) \frac{\sigma}{\kappa}, \]

which is always satisfied for nonnegative \( \mu \).

Appendix 2.C Zero lower bound on the nominal interest rate

2.C.1 Proof Proposition 2.6

From (2.26) and (2.27) it follows that in steady state, we must have

\[ (1 + m^\pi)x = \frac{(1 + m^\pi)\pi^T + (1 - m^\pi)\pi + 2\bar{r}}{\sigma} + (1 + m^x)x^T, \]

and

\[ \kappa x = \left(1 - \frac{1 - m^\pi}{2}\beta\right)\pi - \frac{1 + m^\pi}{2} \beta \pi^T. \]

Combining these equations and using \( x^T = \frac{1 - \beta}{\kappa} \pi^T \) gives

(2.81) \[ \left( (1 + m^\pi) \left(1 - \frac{1 - m^\pi}{2}\beta\right) - \frac{\kappa}{\sigma} (1 - m^\pi) \right) \pi = \left( (1 + m^\pi) \left(1 - \beta + \frac{1 + m^\pi}{2}\beta\right) + \frac{\kappa}{\sigma} (1 + m^\pi) \right) \pi^T + \frac{\kappa}{\sigma} 2\bar{r}. \]

In a steady state in the ZLB region with infinite intensity of choice, differences in fractions can either be 0 or \(-1\). Differences in fractions of 1 are not possible since in a steady state naive agents never make prediction errors. Moreover, \( m^\pi = 0 \) if and only if \( \pi = \pi^T \). But in that case (2.81) reduces to \( \pi^T = -\bar{r} < 0 \), which contradicts the assumption of \( \pi^T \geq 0 \). Therefore \( m^\pi = -1 \) and \( \pi \neq \pi^T \) must hold. Furthermore,
if \( \pi \neq \pi^T \) then it follows from the Phillips curve that \( x \neq x^T \), so that it must be that \( m_x = -1 \) as well. The unique solution of (2.81) then is \( \pi = -\frac{1-\beta}{\kappa} \bar{r} \). Corresponding output gap is given by \( x = -\frac{1-\beta}{\kappa} \bar{r} \). This steady state only exists if it lies inside the ZLB region and hence implies a binding zero lower bound. It follows from (2.7) that this is the case if and only if

\[
(\bar{r} + \pi^T)(1 - \phi_1 - (1 - \beta)\frac{\phi_2}{\kappa}) < 0.
\]

The first term in brackets is always positive. Therefore it follows that the steady state exists if and only if the Taylor principle is satisfied.

Next, we turn to the stability of this steady state. The Jacobian evaluated at the steady state is

\[
\begin{pmatrix}
1 & \frac{1}{\sigma} & 0 & 0 & \frac{x^T}{2\sigma} & \frac{\pi^T}{2\sigma} \\
\kappa & (\beta + \frac{\kappa}{\sigma}) & 0 & 0 & \kappa \frac{x^T}{2} & (\beta + \frac{\kappa}{\sigma}) \frac{x^T}{2} \\
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
c_{11} s_A & c_{12} s_A & 0 & 0 & 0 & e_{12} s_A \\
c_{21} s_B & c_{22} s_B & 0 & 0 & 0 & e_{22} s_B \\
\end{pmatrix},
\]

where, \( c_{11}, c_{21}, e_{12}, e_{22}, A \) and \( B \) are finite nonzero terms. If we let the intensity of choice, \( b \), go to infinity, \( s_A \) and \( s_B \) go to zero. This means that the system has 4 eigenvalues equal to zero at the steady state. The other two follow from

\[
\begin{pmatrix}
1 & \frac{1}{\sigma} \\
\kappa & (\beta + \frac{\kappa}{\sigma}) \\
\end{pmatrix},
\]

and are given by

\[
\lambda_1 = \frac{1}{2}[1 + \beta + \frac{\kappa}{\sigma} - \sqrt{(1 + \beta + \frac{\kappa}{\sigma})^2 - 4\beta}],
\]

\[
\lambda_2 = \frac{1}{2}[1 + \beta + \frac{\kappa}{\sigma} + \sqrt{(1 + \beta + \frac{\kappa}{\sigma})^2 - 4\beta}],
\]

This means that the liquidity trap steady state is an unstable saddle point for all positive values of \( \beta, \kappa \) and \( \sigma \) (\( \lambda_2 > 1 \) and \( |\lambda_1| < 1 \) always hold).
2.C.2 Proof Proposition 2.8

A deflationary spiral (or divergence) is defined as a situations with ever decreasing inflation and output gap. If from any set of initial conditions in period \( t \) a deflationary spiral occurs, we must therefore at some future period \( s \geq t \) have that \( x_{s+1} < x_s < x_{s-1} < x_{s-2} < 0 \) and \( \pi_{s+1} < \pi_s < \pi_{s-1} < \pi_{s-2} < 0 \). From this it follows that naive agents turned out to perform better in their predictions about period \( s \) and \( s+1 \) than fundamentalists (who predicted \( x_t > 0 \) and \( \pi^T > 0 \)), so that, for infinite intensity of choice, we get \( m^x_{s+1} = -1, m^x_{s+1} = -1, m^\pi_{s+2} = -1 \) and \( m^\pi_{s+2} = -1 \).

2.C.3 Proof Proposition 2.9

At least for the first two periods dynamics are given by the all naive expectations system of

\[
(2.85) \quad x_{t+1} = x_t + \frac{\pi_t}{\sigma} + \frac{\bar{r}}{\sigma},
\]

\[
(2.86) \quad \pi_{t+1} = (\beta + \frac{\kappa}{\sigma}) \pi_t + \kappa x_t + \frac{\kappa \bar{r}}{\sigma}.
\]

Iterating one more period gives

\[
(2.87) \quad x_{t+2} = \left( 1 + \frac{\kappa}{\sigma} \right) x_t + \left( \frac{1 + \beta}{\sigma} + \frac{\kappa}{\sigma^2} \right) \pi_t + \left( \frac{2}{\sigma} + \frac{\kappa}{\sigma^2} \right) \bar{r},
\]

\[
(2.88) \quad \pi_{t+2} = \left( \left( \beta + \frac{\kappa}{\sigma} \right)^2 + \frac{\kappa}{\sigma} \right) \pi_t + \left( (1 + \beta) \kappa + \frac{\kappa^2}{\sigma} \right) x_t + \left( (1 + \beta) \frac{\kappa}{\sigma} + \frac{\kappa^2}{\sigma^2} \right) \bar{r}.
\]

In the all naive system, initial conditions above the stable eigenvector through the \( \pi = -\bar{r}, \ x = -\frac{1-\beta}{\kappa} \bar{r} \) steady state lead output and inflation to move to the positive interest rate region, while lower initial conditions lead to divergence to minus infinity for both variables. Therefore, as long as expectations always remain naive for both variables, recovery occurs if and only if

\[
(2.89) \quad x_t > - \frac{1 - \beta}{\kappa} \bar{r} - \frac{1 - (1 - \beta) \frac{\kappa}{\sigma} + \sqrt{1 + \frac{\sigma^2}{\kappa^2} ((1 + \beta)^2 - 4\beta) + 2(1 + \beta) \frac{\sigma}{\kappa}}}{2\sigma} (\pi_t + \bar{r}) \equiv x_{ev}.
\]

We must however also consider the possibility that agents become fundamentalists in period \( t + 3 \) about output gap or inflation. Naive expectations about output gap
are excluded when

$$x_{t+2} = \left(1 + \frac{\kappa}{\sigma}\right)x_t + \left(1 + \frac{\beta}{\sigma} + \frac{\kappa}{\sigma^2}\right)x_t - \left(\frac{2}{\sigma} + \frac{\kappa}{\sigma^2}\right)\bar{r} > \frac{x_t + x^T}{2}$$

(2.90)

$$x_t > -\left(2(1 + \beta) + 2\frac{\sigma}{\sigma}\right)\pi_t - (4 + 2\frac{\sigma}{\sigma})\bar{r} + \sigma x^T \equiv \bar{x}^{\text{out}}.$$

When inflation expectations remain naive, a sufficient condition for recovery therefore is that (2.89) and (2.90) both hold.

If $\pi_t < \pi^T$, fundamentalistic expectations about inflation only increase both variables for all subsequent periods, implying that above conditions that assume naive inflation expectations are still sufficient for recovery when inflation expectations could become fundamentalistic. If $\pi_t > \pi^T$, inflation expectations are at least as large as $\pi^T$, so that sufficient conditions for recovery can be obtained by replacing $\pi_t$ with $\pi^T$ in (2.89) and (2.90). Therefore, a sufficient condition for recovery is

$$x_t > \max(\bar{x}^{\text{ev}}, \bar{x}^{\text{out}}),$$

with

(2.91)

$$\bar{x}^{\text{ev}} = -\frac{1 - \beta}{\kappa} \bar{r}$$

$$- \frac{1 - (1 - \beta)\frac{\sigma}{\kappa} + \sqrt{1 + \frac{\sigma^2}{\kappa^2} ((1 + \beta)^2 - 4\beta) + 2(1 + \beta)\frac{\sigma}{\kappa}}}{2\sigma} \left(\min(\pi_t, \pi^T) + \bar{r}\right),$$

(2.92)

$$\bar{x}^{\text{out}} = -\frac{(2(1 + \beta) + 2\frac{\sigma}{\sigma}) \min(\pi_t, \pi^T) - (4 + 2\frac{\sigma}{\sigma})\bar{r} + \sigma x^T}{2\kappa + \sigma}.$$}

Next, we turn to sufficient conditions for divergence. From the above it follows that as long as inflation expectations remain naive, a sufficient condition for divergence is that (2.89) and (2.90) both do not hold. Inflation expectations remaining naive is guaranteed when $\pi_{t+2} < \frac{\pi_t + \pi^T}{2}$, which reduces to

(2.93)

$$x_t < -\frac{((\beta + \frac{\sigma}{\sigma})^2 + \frac{\sigma}{\sigma} - \frac{1}{2})\pi_t + \frac{1}{2}\pi^T}{(1 + \beta)\kappa + \frac{\sigma^2}{\kappa}} - \frac{r}{\sigma} \equiv \bar{x}^{\text{inf}}.$$}

A sufficient condition for a deflationary spiral therefore is

$$x_t < \min(\bar{x}^{\text{ev}}, \bar{x}^{\text{out}}, \bar{x}^{\text{inf}}).$$
2.C. Zero lower bound on the nominal interest rate

2.C.4 Proof Proposition 2.10

For $b = 0$ the system under the zero lower bound reduces to

\[ x_{t+1} = \frac{x_t}{2} + \frac{x_T}{2} + \frac{\pi_t}{2\sigma} + \frac{\pi_T}{2\sigma} + \frac{\bar{r}}{\sigma}, \]

(2.94)

\[ \pi_{t+1} = (\beta + \frac{\kappa}{\sigma})(\frac{\pi_t}{2} + \frac{\pi_T}{2}) + \frac{\kappa}{2} x_t + \frac{\kappa}{2} x_T + \frac{\kappa}{\sigma} \bar{r}. \]

(2.95)

The unique steady state of this system is

\[ \pi = \frac{((2 - \beta)x + 2\kappa)\pi_T + 4\kappa\bar{r}}{(2 - \beta)x - 2\kappa}, \]

(2.96)

\[ x = x_T + \frac{2(2 - \beta)}{(2 - \beta)x - 2\kappa}(\pi_T + \bar{r}). \]

(2.97)

It follows that $x$ and $\pi$ lie above the target if and only if the numerator in the fractions in (2.96) and (2.97) is positive. This is the case if and only if $\kappa < \frac{(2 - \beta)x}{2}$.

The Jacobian of this linear system does not depend on the values of $x$ and $\pi$ and is given by

\[ \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{\kappa}{2} & \frac{\beta}{2} + \frac{\kappa}{\sigma} \end{pmatrix}. \]

Since the Jacobian is $\frac{1}{2}$ times the Jacobian of the system with all naive expectations, the eigenvalues are given by $\frac{1}{2}$ times (2.83) and (2.84).

Both eigenvalues lie in the unit circle if and only if $\kappa < \frac{(2 - \beta)x}{2}$, otherwise the steady state is a saddle point. In that case, the slope of the stable eigenvector is the same as in (2.89).
Chapter 3

Managing Heterogeneous and Unanchored Expectations: A Monetary Policy Analysis

3.1 Introduction

Traditionally, monetary policy is modeled under the assumption of homogeneous rational expectations (e.g. Woodford (2003)). All agents are then assumed to have perfect knowledge and use perfect model consistent expectations to forecast future variables, such as inflation and output. Surveys of consumers and professional forecasters (e.g. Carroll (2003); Mankiw et al. (2004)) as well as laboratory experiments with human subjects (e.g. Assenza et al. (2014); Pfajfar and Zakelj (2016)) show however, that there is considerable heterogeneity in the forecasts of these macroeconomic variables. This raises the question whether policy implications that follow from models with a representative agent having rational expectations are accurate, or whether this assumption is so restrictive that rational expectations models do not reflect reality and may lead to other, perhaps sometimes misleading policy recommendations. In this chapter we investigate monetary policy in macroeconomic models under the alternative paradigm of bounded rationality and heterogeneous expectations (Brock and Hommes, 1997). In particular, we study how monetary policy can manage a continuum of heterogeneous expectation rules by applying the Large Type Limit (LTL) concept of Brock et al. (2005) to the New Keynesian framework. The LTL concept also allows us to give a precise conceptualization of the idea of ”strongly anchored” expectations in an analytically tractable way.

The importance of bounded rationality and a behavioral approach to macroeco-
nomics has been stressed in the books Akerlof and Shiller (2010) and De Grauwe (2012). Assenza et al. (2017) model animal spirits through heterogeneous expectations and show the emergence and amplification of boom and bust cycles in a macroeconomic model where lenders have heterogeneous expectations about the default probability of firms. The theory of bounded rationality and heterogeneous expectations has recently also been applied to the New Keynesian Dynamic Stochastic General Equilibrium (DSGE) setting in Branch and McGough (2010), De Grauwe (2011), Massaro (2013), Agliari et al. (2014), and in Deak et al. (2015). These models nicely match with empirical stylized facts of output and inflation (De Grauwe, 2012).

We study policy implications for an inflation targeting central bank (CB) under this new paradigm and these empirically relevant heterogeneous expectations in an otherwise standard New Keynesian framework. In addition, we study the effect of the zero lower bound on the nominal interest rate when expectations are heterogeneous. Expectations are assumed to be anchored around fundamental values of the model, i.e., the rational expectation equilibrium values that would arise if all agents were rational. However, not all agents expect exactly these fundamental values. Instead, some agents have slightly higher expectations, while other agents expect values that are somewhat lower. This heterogeneity can be interpreted in two different ways. First of all, it could be that agents make small mistakes in their otherwise adequate predictions. Alternatively, agents base their expectations on publicly available information, but also take their own personal views about the economy and about animal spirits into account.

Our agents furthermore realize that their expectations may not be perfect, and that other agents (e.g. professional forecasters) might be better at predicting the future. For this reason agents will adjust their expectations upwards if agents with higher expectations turned out to be right in the past. They do this to correct for their apparent mistakes, and to benefit from other agents that might have better information or prediction skills. The heterogeneity in expectations will however always be present. Our benchmark model specification assumes a continuum of prediction values, normally distributed around the fundamental values. We study inflation-output dynamics under heterogeneous expectations, using the large type limit concept, initially introduced by Brock et al. (2005) and used in other macroeconomic settings by Anufriev et al. (2013), Agliari et al. (2014), and Pecora and Spelta (2017). We also consider the implications of discrete expectation values, by studying a stylized example with 3 different expectation values: optimists, pessimists, and fundamentalists; and a richer model where agents only form expectations with a precision of 0.5%. This model may be especially relevant because in practice people do not report expectations with infinitely many decimals,
but instead prefer round numbers.\footnote{A phenomenon labeled “digit preference”. See Curtin (2010).}

Under these heterogeneous expectations we study monetary policy under three different interest rate rules. In the first rule, the central bank responds to expected future deviations of inflation and output gap from their targets; in the second rule, the CB can respond to contemporaneous deviations from target; and in the third rule, the CB responds to the deviations from targets that occurred in the previous period. These three Taylor type interest rate rules are also studied by Bullard and Mitra (2002), who compare rational expectation results with results obtained under adaptive learning. These authors, however, assume homogeneous agents. In contrast, our focus is on monetary policy under heterogeneous expectations.

We find that whether the economy can be stabilized depends both on monetary policy and on the anchoring of aggregate expectations. Only when expectations are unanchored, the Taylor principle is a necessary condition for stability. When aggregate expectations are somewhat anchored to the fundamental values of the economy, stability can be achieved with weaker monetary policy. The forward-looking and contemporaneous Taylor rules then work very well. If however the CB cannot observe contemporaneous values of inflation and output gap and must instead rely on lagged values, monetary policy can easily destabilize the economy by being too aggressive when expectations are strongly anchored.

In our benchmark model with a continuum of prediction values the fundamental target steady state is typically unique, and local stability implies global stability. However, if expectations are somewhat unanchored and monetary policy is relatively weak (i.e. the Taylor principle is just satisfied), the system has a near unit root, and optimistic and pessimistic expectations can be almost self-fulfilling. Convergence to the fundamental steady state will then occur only very slowly. In that case, a single shock can lead to persistently high or low expectations, and it may take a long time for the economy to recover and mean revert back to the fundamental equilibrium. Furthermore, when expectations are discrete, almost self-fulfilling optimistic and pessimistic expectations imply the existence of additional steady states. This multiplicity of equilibria disappears as monetary policy becomes more aggressive. Time series simulations of the benchmark model and the model where only multiples of 0.5\% are allowed look almost indistinguishable, and both models show high persistence when monetary policy is not very strong.

In the final part of the chapter, we turn to the implications of the zero lower bound of the nominal interest rate. We find that due to this lower bound, low initial inflation
and output gap can initiate a fall in both expectations and realizations, that either ends when the lowest possible expectations are reached, or goes on for ever if no lower limit on expectations exists. In the latter case, the system has entered a deflationary spiral. This change in dynamics occurs due to the appearance of an additional steady state. Benhabib et al. (2001a, b) first highlighted the appearance of an extra equilibrium when the ZLB is introduced in a rational expectations New Keynesian framework. Evans et al. (2008) and Benhabib et al. (2014) furthermore find that the additional steady state leads to deflationary spirals for low initial conditions in their models with boundedly rational agents.

The recent financial crises has highlighted the importance of both a lower bound on the interest rate, and its relation with low, self-fulfilling expectations. In order to fully understand liquidity traps and to come up with policy recommendations it is of crucial importance to realistically model expectations. Mertens and Ravn (2014) discuss the distinction between a liquidity trap that is driven by low economic fundamentals, and a liquidity trap that is driven by expectations. In Evans et al. (2008) and Benhabib et al. (2014) liquidity traps are driven by expectations, but these expectations are formed by homogeneous agents. In Chapter 2 of this thesis we study liquidity traps under heterogeneous expectations in a model with endogenous credibility of the central bank. In the current chapter, we construct a different heterogeneous agent model that allows for more heterogeneity. Furthermore, instead of analyzing the role of endogenously evolving credibility, we now assume that the expectations of all agents are to some extent anchored to the fundamental values of the economy, and that the magnitude of this anchoring is exogenously given. This allows us to directly study how expectation driven liquidity traps are affected by a combination of policy parameters and the anchoring of, and heterogeneity in, expectations.

We find that the central bank can prevent prolonged liquidity traps with a high enough inflation target. This lowers the values of inflation and output gap from which deflationary spirals occur, and may exclude the possibility of self-fulfilling liquidity trap steady states. Alternatively, liquidity traps can be prevented if expectations are strongly enough anchored to fundamental values of the economy.

The chapter is organized as follows. In Section 3.2 the model, interest rate rules, and expectation formation mechanisms are presented. In Section 3.3 the local stability of the fundamental steady state is analyzed. Section 3.4 considers uniqueness and global stability and the implications of discrete expectations. Section 3.5 focuses on the zero lower bound and liquidity traps, and Section 3.6 concludes.
3.2 Model Specification

3.2.1 Economic model and interest rate rules

We use a log linearized New Keynesian model in line with Woodford (2003). Microfoundations for this model for the case where expectations are heterogeneous can be found in Chapter 2 of this thesis. The derivations in that chapter largely follow Kurz et al. (2013).\footnote{In the derivations in Chapter 2 of this thesis, the aggregation of individual decisions in terms of aggregates only (which is not the case in Kurz et al. (2013)) hinges on a specific property of the way we model heterogeneous expectations. With the Heuristic Switching Model of Brock and Hommes (1997), in any period there is heterogeneity in expectations, with different fractions of agents using different heuristics. However, these fractions are updated in each period according to a probability distribution that depends on the relative past performance of each expectation formation heuristic. With this updating process each agent has in each period the same probability of following a particular heuristic as all other agents, independent of the heuristic the particular agent followed in the previous period. By assuming that agents are aware of this, their expectations about their own future consumption coincide with their expectation about the future consumption of any other agents, and therefore with their expectations about aggregate consumption, which is crucial for aggregation. See Chapter 2 of this thesis for details.}

The model is given by a New Keynesian Phillips curve describing inflation, an IS curve describing output gap, and a rule for the nominal interest rate. Log linearized output gap ($x_t$) and inflation ($\pi_t$) are given by

\begin{align}
  x_t &= E_t x_{t+1} - \frac{1}{\sigma} (i_t - E_t \pi_{t+1} - \bar{r}) + u_t, \\
  \pi_t &= \beta E_t \pi_{t+1} + \kappa x_t + e_t,
\end{align}

where $\kappa$, $\sigma$ and $\beta$ are model parameters, and $\bar{r} = \frac{1}{\beta} - 1$ is the steady state real interest rate. $e_t$ and $u_t$ are shocks to the economy, which we assume to be white noise.

Finally, $i_t$ is the nominal interest rate. We consider three different interest rate rules, where the central bank responds respectively to the expectations, the contemporaneous values and the lagged values of inflation and output gap.

The first interest rate rule we consider is a forward-looking Taylor type rule given by

\begin{align}
  i_t &= \bar{r} + \pi^T + \phi_1 (E_t \pi_{t+1} - \pi^T) + \phi_2 (E_t x_{t+1} - x^T).
\end{align}

We assume here that the central bank can observe private sector expectations. Expectations $E_t \pi_{t+1}$ and $E_t x_{t+1}$ are based on period $t - 1$ information, and are formed at the
end of period $t - 1$. It is therefore not unreasonable to assume that the CB can base its period $t$ decision for the interest rate, $i_t$, on period $t$ private sector expectations.

Following Evans and Honkapohja (2003), we show in Chapter 2 of this thesis that a forward-looking rule of the form (3.3), can be used to minimize the following loss function under discretion

$$E_t \sum_{i=0}^{\infty} \beta^i \left[ (\pi_{t+i})^2 + \mu (x_{t+i})^2 \right],$$

were $\mu \geq 0$ is the relative weight that the central bank assigns to the minimization of the squared output gap compared to the squared inflation. Rotemberg and Woodford (1999) show that this loss function can be derived from a second order approximation of the utility function of a representative agents. The coefficients that minimize (3.4) are given by

$$\phi_1 = 1 + \frac{\sigma \kappa \beta}{\mu + \kappa^2}, \quad \phi_2 = \sigma$$

We will consider these coefficients as a benchmark case.

Abstracting from shocks and plugging (3.3) into (3.1), gives the following model

$$x_t = (1 - \phi_2)E_t x_{t+1} - \frac{\phi_1 - 1}{\sigma} (E_t \pi_{t+1} - \pi^T) + \frac{\phi_2}{\sigma} x^T,$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t.$$  

Secondly, we look at a more traditional contemporaneous Taylor rule:

$$i_t = \bar{r} + \pi^T + \phi_1 (\pi_t - \pi^T) + \phi_2 (x_t - x^T).$$

With this interest rate rule the model is given by (3.7) and

$$(1 + \frac{\phi_2}{\sigma}) x_t = E_t x_{t+1} + \frac{1}{\sigma} (E_t \pi_{t+1} - \pi^T - \phi_1 (\pi_t - \pi^T) + \phi_2 x^T).$$

Substituting for inflation, the output gap equation can be written as

$$x_t = \frac{1}{\sigma + \phi_2 + \kappa \phi_1} \left( \sigma E_t x_{t+1} - (\beta \phi_1 - 1) E_t \pi_{t+1} + (\phi_1 - 1) \pi^T + \phi_2 x^T \right).$$

Since it is perhaps not very realistic that the CB can respond to contemporaneous values of inflation and output gap, the final interest rate rule we consider is a Taylor
rule with lagged values of inflation and output gap

\begin{equation}
    i_t = \bar{r} + \pi^T + \phi_1(\pi_{t-1} - \pi^T) + \phi_2(x_{t-1} - x^T).
\end{equation}

The model is then given by (3.7) and

\begin{equation}
    x_t = E_t x_{t+1} - \frac{\phi_2}{\sigma} (x_{t-1} - x^T) + \frac{1}{\sigma} (E_t \pi_{t+1} - \pi^T) - \frac{\phi_1}{\sigma} (\pi_{t-1} - \pi^T).
\end{equation}

### 3.2.2 Heuristic Switching Model

We deviate from the rational expectations hypothesis, and do not assume all agents always exactly expect the same outcome of future variables. Instead, we assume that some heterogeneity is present, with some agents expecting values that are a bit higher and some agents expecting values that are a bit lower. This heterogeneity could be caused by agents making small mistakes. Alternatively, the heterogeneity can arise because some agents think they have reasons to be more optimistic or pessimistic in their predictions than is warranted by the publicly available information.

More specifically, expectations are distributed around the targets of the central bank, \(\pi^T\) and \(x^T\). This can be interpreted as some agents trusting the central bank and expecting inflation and output gap to be equal to their targets, while other agents expect that the central bank will not be able to exactly achieve its goals, but that inflation and output gap will be somewhat higher or lower. An alternative interpretation is that expectations are heterogeneously distributed around the minimum state variable (MSV) solution of the model. Since we assume no autocorrelation in shocks, for the contemporaneous and forward looking Taylor rule, the MSV solution coincides with the targets of the central bank in any period.

We furthermore assume that when agents that were more optimistic or pessimistic in their prediction turn out to be right, then other agents will learn from this and adjust their expectations in the direction of the better performing agents. Agents may, for example, think that the correct agents had additional information available to them, or just had more skills in analyzing the economic environment.

We implement this way of expectation formation with a Heuristic Switching Model as in Brock and Hommes (1997), where agents switch between simple prediction rules, or heuristics. The heuristics in our model consists of deviations from the fundamental values of the economy. The fraction of agents using the heuristic with deviation, or bias, \(b_h\) in period \(t\) is updated according to the discrete choice model with multinomial
logit probabilities (see Manski et al. (1981)) given by

\[ n_t^{z,h} = \frac{e^{\omega U_t^{z,h}}}{\sum_{h=1}^{H} e^{\omega U_t^{z,h}}}, \quad z = \pi, x. \] (3.13)

Here \( H \) is the total number of prediction values, and \( U_t^{z,h} \) is the fitness measure of predictor \( h \) for variable \( z \) in period \( t \). We will assume fitness measures to equal minus a weighted sum of the negative of the last observed squared prediction error, and the previous value of the fitness measure.

\[ U_t^{z,h} = -(1 - \rho)(z_{t-1} - E_t^{h} z_{t-1})^2 + \rho U_{t-2}^{z,h}, \quad z = \pi, x, \] (3.14)

where \( 0 \leq \rho < 1 \), is the memory parameter, which we will set to 0 for analytical tractability.

\( \omega \) (in Equation (3.13)) is the intensity of choice. The higher the intensity of choice, the more sensitive agents become with respect to relative performance of prediction values, and the more agents will coordinate their forecasts.

Inflation expectations are now given by a weighted average of the predictions of all types

\[ E_t^{\pi_{t+1}} = \pi^T + \sum_{h=1}^{H} b_h n_t^{\pi,h} = \pi^T + \sum_{h=1}^{H} b_h \frac{e^{-\omega(\pi_{t-1} - b_h - \pi^T)^2}}{\sum_{h=1}^{H} e^{-\omega(\pi_{t-1} - b_h - \pi^T)^2}}. \] (3.15)

This equation can be written as

\[ E_t^{\pi_{t+1}} = \pi^T + \frac{1}{H} \sum_{h=1}^{H} b_h e^{-\omega(\pi_{t-1} - b_h - \pi^T)^2} \frac{1}{\sum_{h=1}^{H} e^{-\omega(\pi_{t-1} - b_h - \pi^T)^2}}. \] (3.16)

Below we will first consider the limit of \( H \) going to infinity. Brock et al. (2005) show that in this case the dynamics can be closely approximated with the so called large type limit (LTL), where there is a continuum of prediction biases. Stated differently, the LTL, is an accurate description of the evolutionary selection, (3.13), of \( H \) forecasting rules when \( H \) is large. We assume that the prediction biases \( b_h \) are normally distributed around zero (so that expectations are distributed around \( \pi^T \)), with variance \( s^2 \).

We can now approximate (3.16) by the LTL obtained by replacing sample means by population means:

\[ E_t^{\pi_{t+1}} = \pi^T + \int_{-\infty}^{\infty} b e^{-\omega(\pi_{t-1} - b - \pi^T)^2} e^{-\frac{b^2}{2s^2}} dB \frac{1}{\int_{-\infty}^{\infty} e^{-\omega(\pi_{t-1} - b - \pi^T)^2} e^{-\frac{b^2}{2s^2}} DB}. \] (3.17)
3.2. Model Specification

The large type limit can also be interpreted in terms of Bayesian updating. Agents then try to learn in each period about the correct value of \( b \), with \( N(0, s^2) \) as their prior. The likelihood function (the distribution of \( \pi_{t-1} \) given the true value of \( b \)) is normal, with a variance inversely related to the intensity of choice (\( \omega \)). This means that the intensity of choice (\( \omega \)) is inversely related to the perceived noise with which agents can observe the correct value of \( b \). This interpretation is in line with the random utility model underlying the multinomial logit probabilities given in (3.13) (see Anderson et al. (1992)). A more detailed comparison between the LTL and Bayesian updating is presented in Appendix 3.A.

It can be shown that (3.17) reduces to

\[
E_t \pi_{t+1} = \frac{\pi^T}{2\omega s^2 + 1} + \frac{2\omega s^2}{2\omega s^2 + 1} \pi_{t-1}.
\]

Similarly, for output gap we can write

\[
E_t x_{t+1} = \frac{x^T}{2\omega s^2 + 1} + \frac{2\omega s^2}{2\omega s^2 + 1} x_{t-1},
\]

In the LTL model, expectations thus are a linear combination of the fundamental values of the economy and of past realizations. The weights on these values determine to what extent aggregate expectations are anchored to the fundamentals of the economy. If the weight on the fundamental values is high (near 1), aggregate expectations are always close to the fundamentals of the economy. We refer to this situation as "strongly anchored expectations". If the weight on the fundamental values is low (near 0), aggregate expectations jump around considerably in response to shocks. We refer to this situation as "unanchored expectations".

In Equation (3.18) and (3.19) it can be seen that the weight on the fundamental values is strictly decreasing in the intensity of choice (\( \omega \)) and the variance of the distribution of types (\( s^2 \)). The intuition for this is that a higher intensity of choice allows more and faster changes of expectations and a higher variance of the distribution of types makes it more likely that these changes move expectations towards values far away from the fundamentals of the economy. Therefore, a higher \( \omega \) and \( s^2 \) imply that aggregate expectations move more in response to shocks and thus become less anchored.
CHAPTER 3. MANAGING UNANCHORED EXPECTATIONS

3.3 Stability

Throughout the chapter we will assume that the central bank always picks an output gap target that is consistent with its inflation target and satisfies $x_T = \frac{1-\beta}{\kappa} \pi_T$. It then holds that aggregate expectations of $E_t \pi_{t+1} = \pi_T$ and $E_t x_{t+1} = x_T$ imply realized inflation and output gap that are equal to their fundamental values as well. Moreover, it then follows from (3.18) and (3.19) that expectations will again be equal to their targets in the next period. Hence, the fundamental values comprise a steady state in the two dimensional dynamical system defined by (3.19), (3.18), (3.7) and either (3.6), (3.10) or (3.12), depending on the interest rate rule.

We find that in our economy the core task of monetary policy should be providing a feedback mechanism that prevents optimistic or pessimistic expectations from becoming self-fulfilling. We define self-fulfilling optimistic expectations as high aggregate expectations that lead to realizations that are as high as (or perhaps even higher than) these expectations. If monetary policy fails to provide mean reversion of expectations, explosive drifts of inflation and output gap, away from their fundamental values $\pi_T$ and $x_T$ may arise. The policy feedback mechanism must however not be so strong that it overreacts to so small fluctuations in the economy and thereby causes larger fluctuations in the opposite direction.

More technically speaking, monetary policy must aim at making the above mentioned steady state locally (and preferably also globally) stable. The specific restrictions on monetary policy that result in stability depend on both the monetary policy rule that is used, and on how strongly expectations are anchored to the fundamental values of the economy. This is discussed in detail below.

3.3.1 Forward-looking Taylor rule

The first feedback mechanism we consider is responding directly to observed expectations with the forward-looking Taylor rule given by (3.3). By responding strongly enough to expectations, the CB can make sure that high (low) expectations do not lead to too high (low) realizations of inflation and output gap, and therefore cannot become self-fulfilling.

However, if the CB responds too strongly, high inflation expectations lead to very low inflation realizations, which (depending on how strongly expectations are anchored), can lead to very low expectations in the subsequent period. These low expectations then again induce a strong policy response that leads to very high realizations of inflation, and high expectations in the subsequent period. This process of explo-
3.3. Stability

Figure 3.1: Stability region of fundamental steady state under the forward-looking Taylor rule. For policy parameters ($\phi_1$, $\phi_2$) and expectation parameters ($\omega$, $s^2$) between the two surfaces, the fundamental steady state is locally stable.

Proposition 3.1 formally states the conditions for stability of the fundamental steady state under the forward-looking Taylor rule. As expected, there is both a lower and an upper bound on how aggressive monetary policy responses can be. The conditions are presented in terms of the inflation policy coefficient, $\phi_1$, and are a function of the output gap policy coefficient, $\phi_2$, and the parameters $\omega$ and $s^2$ (which together determine how strongly expectations are anchored). The first condition ensures that the first eigenvalue of the dynamical system, $\lambda_1$, is smaller than $+1$; and the second condition ensures that the other eigenvalue, $\lambda_2$, is larger than $-1$. Proof of Proposition 3.1 is given in Appendix 3.B.1.

**Proposition 3.1.** (See Figure 3.1) When the CB adheres to the forward-looking Taylor rule given by (3.3), the fundamental steady state is locally stable if and only if

\( \phi_1 > 1 - \frac{1 + (1 - \beta)2\omega s^2}{2\omega s^2 + 1} \left( \frac{\sigma}{2\omega s^2 \kappa} + \frac{\phi_2}{\kappa} \right), \quad (\lambda_1 < +1), \) (3.20)

and

\( \phi_1 < 1 + \frac{(1 + \beta)2\omega s^2 + 1}{2\omega s^2 + 1} \left( \frac{(4\omega s^2 + 1)\sigma}{2\omega s^2 \kappa} - \frac{\phi_2}{\kappa} \right), \quad (\lambda_2 > -1). \) (3.21)

Figure 3.1 plots the two conditions for stability from Proposition 3.1 as a function of $\omega s^2$ and $\phi_2$. The lower surface represents Condition (3.20), and the upper surface...
depicts Condition (3.21). In the figure, it can be seen that for low values of $\omega s^2$ (where aggregate expectations are strongly anchored to the fundamental values), weak inflation policy ($0 < \phi_1 < 1$) does not lead to instability. For higher values of $\omega s^2$ (unanchored expectations) however, the central bank must respond strongly enough to inflation in order to satisfy (3.20). If we let $\omega s^2$ go to infinity, Condition (3.20) reduces to $\phi_1 > 1 - (1 - \beta) \frac{\sigma^2}{\kappa}$, which is the well known Taylor principle that must be satisfied under rational expectations in order to obtain local determinacy.

Condition (3.21) also becomes more stringent as expectations become unanchored (higher $\omega s^2$). For low values of $\omega s^2$ the upper limit on $\phi_1$ goes to infinity; and as $\omega s^2$ goes to infinity it is required that $\phi_1 < 1 + (1 + \beta) \left( \frac{2\sigma}{\kappa} - \frac{\phi_2}{\kappa} \right)$. This condition coincides with the upper bound for local determinacy with a forward-looking Taylor rule under rational expectations (Bullard and Mitra, 2002). We can conclude that with a forward-looking Taylor rule, unanchored expectations (high values of $\omega$ and $s^2$) require the same restrictions on policy parameters as under RE, while when expectations are anchored the region of stable policy parameters becomes larger.

It is further of interest whether the fundamental steady state is locally stable under the policy coefficients that minimize the loss function (3.4). Proposition 3.2 states that this always is the case. Its proof is given in Appendix 3.B.2

**Proposition 3.2.** When the CB adheres to the forward-looking Taylor rule given by (3.3), with $\phi_1$ and $\phi_2$ chosen as in (3.5) the fundamental steady state is always locally stable.

### 3.3.2 Contemporaneous Taylor rule

Next, we consider the feedback mechanism of letting the interest rate respond to contemporaneous values of inflation and output (equation (3.8)). By responding strongly enough to these values any potential deviations from the fundamental values can be eliminated, including deviations caused by expectations. In that case, self-fulfilling drifts away from the fundamental steady state cannot arise.

Furthermore no matter how strongly the central bank responds, the overshooting mechanism discussed in the previous section can never occur with a contemporaneous Taylor rule. That is, a strong policy response to high inflation will never lead to low realized inflation. Indeed, we find that there is no upper bound on the policy coefficients for local stability of the fundamental steady state. This result is in line with rational expectations findings of Bullard and Mitra (2002).

Proposition 3.3 states that when the CB uses (3.8), all that is required is that it responds strongly enough to inflation or output, no matter how expectations are...
formed.\textsuperscript{15} The proof is provided in Appendix 3.B.3.

**Proposition 3.3.** \textit{(See Figure 3.2)} When the CB adheres to the contemporaneous Taylor rule given by (3.11), the fundamental steady state is locally stable if and only if

\[
\phi_1 > \frac{2\omega s^2}{2\omega s^2 + 1} - \frac{1 + (1 - \beta)2\omega s^2}{2\omega s^2 + 1} \left( \frac{\sigma}{(2\omega s^2 + 1)\kappa} + \frac{\phi_2}{\kappa} \right), \quad (\lambda_1 < +1).
\]

The bottom surface of Figure 3.2 plots Condition (3.22). This surface is (both qualitatively and quantitatively) very similar to condition (3.20), and again reduces to the Taylor principle when $\omega s^2 \to \infty$.

### 3.3.3 Lagged Taylor rule

Finally, we consider the case where the CB cannot observe contemporaneous values of inflation and output gap and instead responds to lagged values, by using Equation (3.11). The feedback mechanism to expectations then is an indirect one. If expectations are unanchored, there is a strong correlation between lagged values and expectations of inflation and output gap. In this case, responding to lagged values results in almost the same interest rate as would have been obtained by responding directly to observed expectations.

If, however, expectations are strongly anchored to the fundamental values, this correlation between expectations and lagged values disappears. On the one hand, this is not a problem since strongly anchored expectations also imply that expectations are already stable, so that there is no need for a feedback mechanism. However, when expectations are always fairly stable and uncorrelated with lagged values, there is a danger of destabilizing the economy by responding too strongly to these lagged values. This happens through a similar overshooting mechanism as described in Section 3.3.1: high inflation is, in the subsequent period, followed by a strong policy response, resulting in very low inflation. This again induces a strong policy response in the period after that, resulting in very high inflation, and so on.

Proposition 3.4 describes the conditions for local stability under the lagged Taylor rule. These conditions consist of a lower and an upper bound on the CB’s policy response. The proof of the proposition is provided in 3.B.4.

**Proposition 3.4.** \textit{(See Figure 3.2)} When the CB adheres to the lagged Taylor rule given by (3.8), the fundamental steady state is locally stable if and only if

\textsuperscript{15}A similar proposition can be found in Pecora and Spelta (2017)
CHAPTER 3. MANAGING UNANCHORED EXPECTATIONS

Figure 3.2: Stability region of fundamental steady state under the contemporaneous and lagged Taylor rule. For policy parameters \((\phi_1, \phi_2)\) and expectation parameters \((\omega, s^2)\) between the two surfaces, the fundamental steady state is locally stable under the lagged Taylor rule. For the contemporaneous Taylor rule it is stable everywhere above the bottom surface.

\[
(3.23) \quad \phi_1 > \frac{2\omega s^2}{2\omega s^2 + 1} - \frac{1 + (1 - \beta)2\omega s^2}{2\omega s^2 + 1} \left( \frac{\sigma}{(2\omega s^2 + 1)\kappa} + \frac{\phi_2}{\kappa} \right), \quad (\lambda_1 < +1),
\]

and

\[
(3.24) \quad \phi_1 < \frac{2\omega s^2}{2\omega s^2 + 1} + \frac{(1 + \beta)2\omega s^2 + 1}{2\omega s^2 + 1} \left( \frac{(4\omega s^2 + 1)\sigma}{(2\omega s^2 + 1)\kappa} - \frac{\phi_2}{\kappa} \right), \quad (\lambda_2 > -1).
\]

Note that Condition (3.23), is exactly the same as Condition (3.22), so that when it comes to reacting strongly enough to inflation, it does not matter whether a contemporaneous rule or a Taylor rule with lagged values is used.

Condition (3.23) and (3.24) are both plotted in Figure 3.2. The upper limit on \(\phi_1\) clearly differs from the one in Figure 3.1, by becoming more stringent instead of less stringent when expectations become more strongly anchored. The reason for this is, as described above, that when expectations become more strongly anchored the correlation between expectations and lagged values decreases. Responding aggressively to lagged values then no longer serves the function of providing a feedback mechanism to deviations in expectations, but instead only destabilizes the economy by amplifying any fluctuations in the economy.

For the limiting case of no anchoring in expectations \((\omega s^2 \to \infty)\) Condition (3.24)
3.4 Multiple steady states and self-fulfilling expectations

In the previous Section we analyzed local stability of the fundamental steady state. Here we found that for unanchored expectations, local stability can be achieved under all three interest rate rules by satisfying the conditions for rational expectations local determinacy. That is, the Taylor principle and the same upper bounds that are presented by Bullard and Mitra (2002). When expectations are anchored to the fundamental values of the economy, the Taylor principle is no longer a necessary condition for local stability of the fundamental steady state. This does however not necessarily mean that convergence to this steady state will occur within a reasonable number of periods, or that convergence to the fundamental steady state occurs from all initial conditions.

In this section we investigate whether almost self-fulfilling expectations (i.e., expectations that lead to realizations that are close to the expected values) can hinder convergence to the fundamental steady state and how almost self-fulfilling expectations can be ruled out by adequately chosen monetary policy. In Section 3.4.1 we consider the benchmark LTL model with a continuum of prediction values. Here, almost self-fulfilling expectations take the form of a near unit root. In Sections 3.4.2 and 3.4.3 we look at almost self-fulfilling steady states that can arise when expectations are discrete.

3.4.1 Steady states in LTL model

Proposition 3.5 states that in the LTL model specified above, the fundamental steady state is typically unique. The proof of Proposition 3.5 is given in Appendix 3.B.5.

Proposition 3.5. The fundamental steady state in the LTL model is the unique steady state under the forward-looking Taylor rule unless \( \phi_1 \) is exactly equal to the value of (3.20). For the lagged and contemporaneous Taylor rule the fundamental steady state is unique unless \( \phi_1 \) exactly equals the value of (3.22). At these knife edge cases where one eigenvalue equals +1, there is a continuum of steady states. Steady state output gap corresponding to inflation level \( \tilde{\pi} \) is then given by

\[
\hat{x} = \frac{1}{\kappa} \left( 1 - \frac{2\omega s^2 \beta}{2\omega s^2 + 1} \right) \tilde{\pi} - \frac{\beta \pi^T}{2\omega s^2 + 1}.
\]
CHAPTER 3. MANAGING UNANCHORED EXPECTATIONS

From Proposition 3.5 it follows that if one eigenvalue equals +1, expectations are perfectly self-fulfilling for a continuum of inflation and output gap values (all comprising a steady state). When monetary policy is slightly more aggressive this eigenvalue is slightly smaller than +1 and the continuum of steady states disappears. Convergence to the fundamental steady state then occurs from all initial inflation and output levels. However, as long as the eigenvalue is close to +1 (near unit root), expectations are still almost self-fulfilling for a continuum of non-fundamental values, and convergence to the fundamental steady state will be very slow. This implies near random walks in inflation and output when shocks are added to the model.

Figure 3.3 plots the largest eigenvalue of the model with the forward-looking Taylor rule as a function of $\phi_1$ for different values of $\phi_2$, and for $\omega s^2 = 13.8$.

Figure 3.3 illustrates that the largest eigenvalue can be close to 1 for a considerable range of values of $\phi_1$, especially when $\phi_2$ is relatively high or relatively low. A near unit root therefore occurs quite generally in our model. For higher values of $\omega s^2$ all curves are shifted upward, so that the largest eigenvalue becomes even higher, and for lower values of $\omega s^2$ all curves of Figure 3.3 are shifted downward. The largest eigenvalues of the models with the contemporaneous and lagged Taylor rule are similar and therefore not shown.

Furthermore, the fact that the non-fundamental steady states disappear when the eigenvalue no longer exactly equals +1 (and hence optimistic and pessimistic expectations can no longer be perfectly self-fulfilling) is heavily dependent on the assumption of a continuum of prediction values. When expectations are discrete, almost self-fulfilling expectations can still lead to the existence of multiple (stable) non-fundamental steady
3.4. Multiple steady states and self-fulfilling expectations

states. We illustrate this below, first with a stylized model with 3 prediction values, and then with a quantitatively more realistic model with 41 prediction values.

3.4.2 Steady states in the 3-type model

First consider a stylized example with three different prediction biases. Fundamentalists have no bias and believe in the targets of the central bank (or alternatively in the MSV solution of the model). Their expectations are therefore given by $E_t^{\text{fun}} x_{t+1} = x^T$ and $E_t^{\text{fun}} \pi_{t+1} = \pi^T$. Then there are optimists and pessimists who have a bias of $b$ and $-b$ respectively. Their expectations are given by $E_t^{\text{opt}} x_{t+1} = x^T + b$, $E_t^{\text{opt}} \pi_{t+1} = \pi^T + b$, $E_t^{\text{pes}} x_{t+1} = x^T - b$, and $E_t^{\text{pes}} \pi_{t+1} = \pi^T - b$. The above specification implies that the distribution of belief types is discrete uniform with mean zero and variance $\frac{2}{3}b^2$.

Note that we allow an agent to be optimistic about one variable, while being pessimistic or fundamentalistic about the other, so that the fractions of agents that are optimistic and pessimistic (denoted respectively $n_t^{z,\text{opt}}$ and $n_t^{z,\text{pes}}$) may differ between the two variables ($z = x, \pi$). The fraction of fundamentalists of a variable equals $n_t^{z,\text{fun}} = 1 - n_t^{z,\text{opt}} - n_t^{z,\text{pes}}$. Aggregate expectations are given by

$$E_t x_{t+1} = x^T + b(n_t^{x,\text{opt}} - n_t^{x,\text{pes}}),$$

$$E_t \pi_{t+1} = \pi^T + b(n_t^{\pi,\text{opt}} - n_t^{\pi,\text{pes}}).$$

Finally, fractions are given by (3.13), with $h = \text{opt}, \text{pes}, \text{fun}$.

The fundamental steady state with $\pi_t = \pi^T$ and $x_t = x^T$ always exist in the 3-type model, no matter what interest rate rule is chosen. The fractions of optimists and pessimists in the fundamental steady state are for both variables equal to

$$n_t^{\text{opt}} - \bar{n}^{\text{pes}} = \frac{1}{2 + \omega b^2}.$$  

Because of the heterogeneity of our agents, these fractions will typically never be zero, and the highest fraction of agents that can have fundamentalistic expectations at any time is given by

$$1 - n_t^{\text{opt}} - \bar{n}^{\text{pes}} = 1 - \frac{2}{2 + \omega b^2}.$$  

This quantity crucially depends on the intensity of choice, $\omega$, which can be seen as a measure of coordination of agents. When the intensity of choice equals zero there can
never be any coordination of expectations. All expectations fractions are then always equal to $\frac{1}{3}$, and the model reduces to $x_t = x^T$ and $\pi_t = \pi^T$. That is, the system will always be in the fundamental steady state, and global stability is always achieved for any specification of monetary policy.

When $\omega$ goes to infinity agents coordinate perfectly, which implies that the fraction of fundamentalists in the fundamental steady state goes to 1. Under the forward looking and contemporaneous Taylor rules, the expectations of all agents then equal those of a fully rational representative agent. However, infinite intensity of choice also facilitates the possibility of coordination on non-fundamental steady states.

Since our dynamical system is linear in expectation fractions, the system for an arbitrary positive but finite value of the intensity of choice, is a convex combination of the system with zero intensity of choice, and the system with infinite intensity of choice. For this reason we first analyze this second limiting case below.

**Steady states for infinite intensity of choice**

In what follows it will be convenient to make the following assumptions about the model parameters: $0 < \kappa < 2\beta - 1$. This is not unreasonable since $\beta$ is usually calibrated around 0.99, and most calibrations of $\kappa$ are much lower than 1.\(^{16}\)

In Proposition 3.6 it is stated that, for $\omega = +\infty$, nine different stable steady states can coexist. Proof of Proposition 3.6 is given in Appendix 3.C.1.

**Proposition 3.6.** (See Table 3.1) When $\omega = +\infty$ there are nine different locally stable steady states that each exist for some range of values of the policy parameters $\phi_1$ and $\phi_2$. The fundamental steady state is the only steady state that exists for all parameter settings.

The intuition behind the multiplicity of steady states is that there can be (almost) self-fulfilling coordination on optimism (Opt.), on fundamentalism (Fun.), or on pessimism (Pes.). Since this can happen both for inflation and for output gap there are nine combinations of heuristics on which coordination can occur. This gives nine candidate steady states. Whether or not these steady states actually exist depends on whether expectations are sufficiently self-fulfilling to ensure that the heuristics where agents coordinate upon are indeed the best performing heuristics. This depends on the interest rate rule and policy parameters chosen by the central bank.

The first column of Table 3.1 summarizes the nine steady states that can exist. The first term indicates which heuristic is best performing with respect to inflation, and the

\(^{16}\)See e.g. Schorfheide (2008)
3.4. Multiple steady states and self-fulfilling expectations

Table 3.1: Steady States of 3-type model with \( \omega = +\infty \) together with conditions for existence.

<table>
<thead>
<tr>
<th>Belief ( \pi.x )</th>
<th>Forward Looking</th>
<th>Contemporaneous/Lagged</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fun.Fun.</td>
<td>Always</td>
<td>Always</td>
</tr>
<tr>
<td>Opt.Opt.</td>
<td>( \phi_1 &lt; 1 + \frac{\sigma}{\kappa} - \phi_2 )</td>
<td>( \phi_1 &lt; \frac{2 + \sigma - \phi_2}{2\beta - \kappa} )</td>
</tr>
<tr>
<td>Pes.Pes.</td>
<td>( \phi_1 &lt; 1 + \frac{\sigma}{\kappa} - \phi_2 )</td>
<td>( \phi_1 &lt; \frac{2 + \sigma - \phi_2}{2\beta - \kappa} )</td>
</tr>
<tr>
<td>Pes.Opt.</td>
<td>( 1 - \frac{\sigma}{\kappa} + \phi_2 &lt; \frac{\sigma}{\kappa} - \phi_1 + 1 + \frac{\sigma}{\kappa} (\beta - \frac{1}{2}) - \sigma + \phi_2 )</td>
<td>( \frac{2 - \sigma + \phi_2}{2\beta + \kappa} &lt; \phi_1 &lt; 2 - 2\sigma + \frac{\sigma + \phi_2}{\kappa}(2\beta - 1) )</td>
</tr>
<tr>
<td>Opt.Pes.</td>
<td>( 1 - \frac{\sigma}{\kappa} + \phi_2 &lt; \frac{\sigma}{\kappa} - \phi_1 + 1 + \frac{\sigma}{\kappa} (\beta - \frac{1}{2}) - \sigma + \phi_2 )</td>
<td>( \frac{2 - \sigma + \phi_2}{2\beta + \kappa} &lt; \phi_1 &lt; 2 - 2\sigma + \frac{\sigma + \phi_2}{\kappa}(2\beta - 1) )</td>
</tr>
<tr>
<td>Opt.Fun.</td>
<td>( 1 - \frac{\sigma}{\kappa} &lt; \phi_1 &lt; 1 + \frac{\sigma}{\kappa} )</td>
<td>( \frac{2 - \sigma - \phi_1}{2\beta + \kappa} &lt; \phi_1 &lt; \frac{2 + \sigma + \phi_1}{2\beta - \kappa} )</td>
</tr>
<tr>
<td>Pes.Fun.</td>
<td>( 1 - \frac{\sigma}{\kappa} &lt; \phi_1 &lt; 1 + \frac{\sigma}{\kappa} )</td>
<td>( \frac{2 - \sigma - \phi_1}{2\beta + \kappa} &lt; \phi_1 &lt; \frac{2 + \sigma + \phi_1}{2\beta - \kappa} )</td>
</tr>
<tr>
<td>Fun.Opt.</td>
<td>( \sigma(1 - \frac{1}{2\kappa}) &lt; \phi_2 &lt; \frac{\sigma}{\kappa} )</td>
<td>( 2\kappa - 1)\sigma - \kappa\phi_1 &lt; \phi_2 &lt; \sigma - \kappa\phi_1 )</td>
</tr>
<tr>
<td>Fun.Pes.</td>
<td>( \sigma(1 - \frac{1}{2\kappa}) &lt; \phi_2 &lt; \frac{\sigma}{\kappa} )</td>
<td>( 2\kappa - 1)\sigma - \kappa\phi_1 &lt; \phi_2 &lt; \sigma - \kappa\phi_1 )</td>
</tr>
</tbody>
</table>

second with respect to output gap. The second column of Table 3.1 states the conditions on the monetary policy parameters \( \phi_1 \) and \( \phi_2 \), for which the corresponding steady state exist under the forward-looking Taylor rule. The final column gives the conditions for existence under the contemporaneous and lagged Taylor rule. In Appendix 3.C.1 it is illustrated how these conditions are derived.

Figure 3.4 plots the inflation value of the first 7 steady states for different values of the parameter \( \phi_1 \) in case \( \phi_2 = \sigma \). The final two steady states of Table 3.1 do not exist in this case. We again use the Woodford (1999) calibration, and we set \( \pi_T = 0 \) and \( b = 0.035/4 \). For this value of the bias, optimists expect annualized inflation and output gap to be 3.5% above their fundamental values. The first panel of Figure 3.4 corresponds to the model with the forward-looking Taylor rule and the second panel to the models with the contemporaneous and lagged Taylor rules. Even though the fundamental steady state is always locally stable, it is not the only attractor. It can be seen that if the central banks wants to achieve uniqueness under the forward-looking rule it needs to respond with \( \phi_1 > 4.5 \) under this calibration while under the contemporaneous and lagged Taylor rule even more aggressive monetary policy is required.

Steady states for finite intensity of choice

As mentioned above, for strictly positive but finite intensity of choice the system is a convex combination of the systems with zero and infinite intensity of choice. This implies that the steady states presented in Table 3.1 still can exist for finite intensity of choice, but that they will exist for a smaller region of policy parameters. It is in this case therefore only a sufficient and no longer a necessary condition for uniqueness that the inequalities of Table 3.1 do not hold.
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Figure 3.4: Steady states of 3-type model from Table 3.1 as a function of $\phi_1$ for $\phi_2 = \sigma$. The black lines indicate the fundamental steady state and colored lines represent the non fundamental states with optimism or pessimism in inflation.

In Figure 3.5, a bifurcation diagram is plotted, with $\phi_1$ as bifurcation parameter.\textsuperscript{17} The intensity of choice is calibrated such that $\bar{n}_{\text{opt}} = \bar{n}_{\text{pes}} = 0.075$, so that in the fundamental steady state 85% of the agents are fundamentalists.\textsuperscript{18} It can be seen that the steady states of the previous section still exist under this lower calibration of the intensity of choice, but that policy implications are now less extreme than in the limiting case of infinite intensity of choice. It can further be seen in the figure that, as in the LTL model, the CB can also respond too strongly under the forward-looking Taylor rule. This leads to the existence of a two cycle, and causes the fundamental steady state to become unstable for high values of $\phi_1$.

3.4.3 41 types

The above stylized 3-type model is not very realistic in the sense that it does not allow for different gradations in optimism and pessimism. Agents are forced to expect either exactly fundamental inflation or a fairly high or fairly low number. Some discreteness in

\textsuperscript{17}This bifurcation diagram was obtained by simulating the model from a range of initial conditions, for different values of $\phi_1$.

\textsuperscript{18}This requires $\omega = 31709$. 

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3.4. Multiple steady states and self-fulfilling expectations

Expectations is however an empirically relevant phenomenon, since people do not form expectations with infinitely many decimals. Instead, humans prefer round numbers when reporting their expectations (a phenomenon labeled *digit preference*). It may therefore be desirable to construct a model where e.g. only multiples of 0.5% are allowed as expectations. In such a model, parameter settings that lead to almost self-fulfilling expectations will result in multiple steady states in the same manner as in the 3-type model. While in the 3-type model conditions on self-fulfillingness are relatively weak, steady states must be very close to self-fulfilling in a model with a large number of different biases. This model will therefore contain characteristics of both the 3-type model and the LTL model specification.

Figure 3.6 plots a bifurcation diagram of a model with 41 types uniformly distributed around the fundamental values. These types are located at all multiples of 0.5% between 10% below and 10% above the fundamental value. The intensity of choice is chosen relatively high (2 million) to facilitate the existence of almost self-filling non-fundamental steady states. It can be seen that for each of the 41 inflation

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19 Curtin (2010).
20 Measured in annualized values.
Figure 3.6: Bifurcation diagram of 41-type model in $\phi_1$ for large intensity of choice (2 million). The blue lines represent steady states at the 41 different levels of inflation expectations.

prediction values there is a range of $\phi_1$-values where this prediction comprises an almost self-fulfilling steady state. For lower intensity of choice this range becomes smaller, just as in the 3-type model, and at some point non-fundamental steady states only exist at a single value of $\phi_1$, just as in the LTL model.

Figure 3.7 shows a simulated time series of inflation and output gap of the LTL model, the 3-type model, and the model with 41 types. We again use the calibration of Woodford (1999) and set $s = 0.059/4$, so that the variance of the distribution of types in the LTL model, matches that of 41-type model. We calibrate the intensity of choice at $\omega = 63500$ to let the 41-type model match expectations from survey data.\(^{21}\) At this calibration, the interquartile range of the expectations distribution in the fundamental steady state is 1% (in annualized terms). Outside of this steady state the interquartile range then typically is 1.5%, and less for realized values close to the highest or lowest possible prediction value. This is in line with the findings of Mankiw et al. (2004), who show that the interquartile range of the Livingston Survey and the Survey of Professional Forecasters is around 1%.

Shocks to inflation and output gap are white noise, and have an annualized standard

\(^{21}\)Note that the magnitude of the intensity of choice depends on the units of measurement of the data. 63500 should therefore not necessarily be seen as a high number. If we interpret the LTL model in terms of Bayesian updating, then $\omega = 63500$ implies that the perceived noise agents encounter in observing true values has a standard deviation of 1.12% of annualized inflation (see Appendix 3.A).
deviation of 1.5%. The interest rate rule that is used is the forward-looking Taylor rule, with $\phi_1 = 1.5$ and $\phi_2 = \sigma = 0.157$. This specification minimizes (3.4) when the weight on output gap is $\mu = 0.007$.

In the top panels of Figure 3.7 it can be seen that there are large drifts in inflation and output gap in the LTL model, even though there is no autocorrelation in shocks. This is due to the fact that, even though monetary policy is considerably more aggressive than required by the Taylor principle, the largest eigenvalue (0.882) is still relatively close to +1.

Turning to the the 3-type model in the bottom panels of Figure 3.7, it can be seen that this model nicely captures the general sentiments in terms of optimism and pessimism that appear in the LTL specification, but that the 3-type model is not rich enough to correspond closely to the LTL model quantitatively. It can furthermore be
seen that periods of optimism about inflation arise together with periods of pessimism about output gap. This is consistent with Table 3.1 from which it follows that for the current policy parameter setting there are three different steady states in the 3-type model: the Fun.Fun., the Opt.Pes., and the Pes.Opt. steady state. This tells us that combinations of high output and low inflation expectations or vice versa are almost self-fulfilling. This implies that, in the presence of shock, in the 3-type model there is switching between coordination on these two levels and on the fundamental steady state, but also that in the LTL model temporary coordination on the same three levels arises.

Finally, the dynamics of the 41-type model (middle panels) are almost indistinguishable from the LTL dynamics. So even though this model consists of discrete expectations, its simulated time series are almost the same as those of the continuous expectations LTL model.\textsuperscript{22} We conclude that both the LTL model and the 3-type model show features that are also present in the 41-type model.

### 3.5 Zero lower bound on the interest rate

In this section the effect of a zero lower bound (ZLB) on the nominal interest rate is investigated. This lower bound turns out to have important consequences for the dynamics of our models. The global stability results of the previous section no longer hold when the ZLB is accounted for. Instead, prolonged liquidity traps can arise in the form of pessimistic steady states or even deflationary spirals with ever decreasing inflation and output gap.

In normal times the interest rate is still given by one of the interest rate rules of Section 3.2.1. However, when this rule implies that $i_t < 0$, then $i_t$ is instead set equal to 0. When the zero lower bound on the interest rate is a binding constraint we say that the system is in the "ZLB region". Otherwise the system is in the "positive interest rate region".

In the ZLB region the model is described by

$$x_t = E_t x_{t+1} + \frac{1}{\sigma} E_t \pi_{t+1} + \frac{\bar{r}}{\sigma},$$

\textsuperscript{22}As a robustness check, we also simulated the 41-type model with persistence in the fitness measure. That is, we set a positive value of $\rho$ in (3.14). Agents then not only look at the last observation when choosing their forecast, but consider all (discounted) past realizations. We find that more persistence in fitness measures leads to slightly less autocorrelation in inflation and output gap dynamics. Very high or very low values of inflation and output gap are then furthermore less likely to be reached. However, for typical values of the parameter $\rho$ dynamics still look similar to those in Figure 3.7.
3.5. Zero lower bound on the interest rate

\[ \pi_t = \beta E_t \pi_{t+1} + \kappa x_t. \]  

3.5.1 LTL under the Zero Lower Bound

First, consider the benchmark LTL model with a continuum of prediction values. When the zero lower bound is binding, the policy coefficients \( \phi_1 \) and \( \phi_2 \) can no longer provide a feedback mechanism that prevents optimistic or pessimistic expectations from becoming self-fulfilling. Low inflation and output gap can then induce a self-fulfilling deflationary spiral with ever decreasing inflation and output gap. Whether this will occur or not for a given level of inflation and output gap depends on how strongly expectations are anchored to the fundamentals of the economy, and on the inflation target. Technically speaking, it depends on the existence and position of a non-fundamental steady state in the ZLB region. This is discussed in Proposition 3.7. The proof of the proposition is given in Appendix 3.D.1.

**Proposition 3.7.** The unique ZLB steady state of the LTL model is given by

\begin{align}
\pi^{zlb} &= \frac{(\sigma(1 + (1 - \beta)2\omega^2) + \kappa(2\omega^2 + 1)) \pi^T + \kappa(2\omega^2 + 1)^2 \bar{r}}{\sigma(1 + (1 - \beta)2\omega^2) - \kappa(2\omega^2 + 1)2\omega^2}, \\
x^{zlb} &= \frac{(1 - \beta)2(1 + (1 - \beta)2\omega^2) + (2\omega^2 + 1)) \pi^T + (1 + (1 - \beta)2\omega^2)(2\omega^2 + 1)\bar{r}}{\sigma(1 + (1 - \beta)2\omega^2) - \kappa(2\omega^2 + 1)2\omega^2}.
\end{align}

This steady state is an unstable saddle-point, and exists if and only if

\[
\omega^2 > \frac{1}{4} \left( \sqrt{(1 - (1 - \beta)\frac{\sigma}{\kappa})^2 + 4 \frac{\sigma}{\kappa}} - (1 - (1 - \beta)\frac{\sigma}{\kappa}) \right).
\]

When the steady state exists, all initial conditions above the stable eigenvector through \((\pi^{zlb}, x^{zlb})\) imply recovery to the positive interest rate region, while all initial conditions below it result in a deflationary spiral. This eigenvector has slope

\[
-1 - (1 - \beta)\frac{\sigma}{\kappa} + \sqrt{(1 - (1 - \beta)\frac{\sigma}{\kappa})^2 + 4 \frac{\sigma}{\kappa}}.
\]

Figure 3.8 illustrates the "recovery region" and the "deflationary spiral region" in the \((\pi, x)\)-plane. Here we set \(\pi^T = 2\%\), and otherwise use the calibration of the previous section, so that \(\omega^2 \approx 13.8\). The red line in Figure 3.8 depicts the zero lower bound. For values of inflation and output gap above this line the nominal interest rate is positive and convergence to the fundamental steady state \((\pi^T, x^T)\) can occur.
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Figure 3.8: Regions of recovery and a deflationary spiral in the annualized \((\pi, x)\)-plane. The thick red line indicates the ZLB. The black dot at 2% inflation indicates the target steady state, while the black dot at the bottom of the figure depicts the unstable ZLB saddle steady state from Proposition 3.7. The unstable eigenvector through this saddle is depicted by the dashed line, while the stable eigenvector is depicted by the solid black line. For initial conditions to the left of this black line a deflationary spiral occurs, and for initial conditions to the right of this line inflation and output gap will recover and increase to the positive interest rate region.

For combinations of inflation and output gap below this line the zero lower bound is a binding constraint. The black line indicates the stable eigenvector through the unstable saddle steady state of Proposition 3.7. Combinations of inflation and output gap above (i.e. to the right of) this line are not too low, so that recovery to the positive interest region occurs. Inflation and output gap combinations below the black line lead to declining inflation and output gap and hence a deflationary spiral.

The position of the black line depends on the position of the ZLB steady state, which in turn depends on the anchoring of expectations and the inflation target. As can be seen in Equation (3.32), the inflation level of the ZLB steady state depends linearly on the inflation target. This implies that increasing the inflation target linearly moves the black line in Figure 3.8 down (to the left) and thereby linearly increases the recovery
3.5. Zero lower bound on the interest rate

region and decreases the deflationary spiral region. So even when the ZLB is binding and the CB loses its control over the interest rate, it can still affect the economy with its inflation target.

The relation between the anchoring of expectations and the size of the deflationary spiral region is slightly more complex. Figure 3.9 shows how exactly the position of the ZLB steady state depends on the anchoring of expectations, with $\omega s^2$ on the horizontal axis and inflation on the vertical axis. The dashed curve represents the ZLB saddle steady state, while the solid line depicts the stable target steady state.

It can be seen in Figure 3.9 that for low anchoring of expectations (high $\omega s^2$) the ZLB saddle steady state lies relatively close to the fundamental steady state. As $\omega s^2$ increase towards infinity (unanchored expectations) the ZLB steady state approaches its limiting value of $(\pi^z, x^z) = (-\bar{r}, -\frac{1-\beta}{\alpha} \bar{r})$, and a deflationary spiral becomes more likely. Decreasing $\omega s^2$ initially only slowly changes the position of the steady state. However, as expectation become more strongly anchored (low $\omega s^2$) the inflation level of the steady state is rapidly decreased. This implies a large movement of the black line in Figure 3.8, and a considerable decrease in the deflationary spiral region. Now, very low levels of inflation and output gap are needed for a deflationary spiral to occur. When expectations become even more strongly anchored, the ZLB saddle steady state disappears altogether, and the fundamental steady state becomes globally stable.

3.5.2 ZLB with finitely many expectation values

Above, we found that in the benchmark LTL model deflationary spirals can arise when the zero lower bound on the interest rate is introduced, and that these deflationary
spiral can be prevented by strongly anchored expectations, or by a high inflation target. But how dependent are these results to the assumption of a continuum of prediction values?

Consider the 3-type model with fundamentalists, optimists and pessimists, and the model with 41-expectation values, discussed in Sections 3.4.2 and 3.4.3. In these models the possible values that can be taken by expectations is limited, and expectations cannot become unboundedly negative. In contrast with the LTL model, a deflationary spiral can therefore not occur. The coordination on pessimistic expectations can however occur, in the form of almost self-fulfilling pessimistic steady states. We refer to such a steady state where the ZLB is binding as a “liquidity trap steady state”.

The most interesting case of a pessimistic steady state arises when pessimistic expectations about at least one variable are more than self-fulfilling. That is, a steady state can exist with pessimistic expectations, where the realized values of inflation (and/or output gap) are even lower than the expectations. If the model then would allow agents to decrease their expectations (for example by allowing for a larger number of different biases to choose from), this steady state would no longer be a steady state. Instead agents would choose lower expectations for the next period. This would decrease both inflation and output gap, and lead agents to choose even lower expectations for both variables in the period after that, again reducing inflation and output gap. This process would go on until the lowest possible expectations are chosen about at least one variable. In this sense reaching the steady state with the most pessimistic expectations in a model with bounded expectation values represents a liquidity trap similar to a deflationary spiral in the LTL model.

Below, we first consider such a steady state in the 3-type model and analyze its properties. We find that the anchoring of expectations and the level of the inflation target affect the possibility of a liquidity trap in a similar manner as in the LTL model. Next, we turn to the richer model discussed in Section 3.4.3 (that has the same qualitative features as the 3-type model), to investigate how shocks in the economy can trigger self-fulfilling pessimistic coordination and how policy can prevent this.

**Pessimistic steady states in 3-type model**

When the ZLB is introduced to the model 3-type model of Section 3.4.2 the interest rate will be constrained by its lower bound when pessimistic expectations dominate.\(^{23}\) Conditions on existence of pessimistic steady states then change compared to Section

\(^{23}\)Here it is assumed that the calibration is such that it is possible that the ZLB can become binding in the 3-type model. In the uninteresting case that this cannot happen, the model always behaves exactly as in Section 3.4.2.
3.5. Zero lower bound on the interest rate

3.4.2 The most interesting liquidity trap steady state is the steady state where most agents have pessimistic expectations about both inflation and output gap (Pes.Pes.). Proposition 3.8 states the conditions for existence of this pessimistic steady state under the zero lower bound. Its proof is given in Appendix 3.E.1

**Proposition 3.8.** When the zero lower bound is binding, the liquidity trap steady state where pessimism is the best performing heuristic for both variables exists if and only if

\[(3.36) \quad \pi^T + \bar{r} < b \left( \sigma m^p_t + \min \left( m^p_t - \frac{\sigma}{2}, (1 + \beta \frac{\sigma}{\kappa}) m^p_t - \frac{\sigma}{2\kappa} \right) \right).\]

Here \(m^p_t = n^\pi,pes_t - n^\pi,opt_t\) and \(m^x_t = n^\pi,pes_t - n^\pi,opt_t\) lie between 0 and 1 and are increasing in the intensity of choice.

When the intensity of choice is infinite all agents become pessimistic in the liquidity trap steady state so that \(m^p_t = m^x_t = 1\). In this case the condition for existence from Proposition 3.8 is least stringent and reduces to \(\pi^T + \bar{r} < b (1 + \frac{\sigma}{2})\). As the intensity of choice is decreased, the condition for existence of the liquidity trap steady state becomes more stringent. In the other limiting case of \(\omega = 0\) there are equal fractions of pessimists and optimists so that \(m^p_t = m^x_t = 0\). In this case Condition 3.36 can never be satisfied, so that the liquidity trap steady state does not exist.

It can further be seen in Equation (3.36) that the condition on existence becomes more stringent as \(b\), and thereby the variance of the distribution of types, decreases. Finally, it follows from Proposition 3.8 that the pessimistic steady state can also be made to disappear by increasing the inflation target. We can therefore conclude that, as in the benchmark LTL model, a liquidity trap can be prevented by expectations that are strongly anchored around the fundamental values, or by a high inflation target.24

**Preventing liquidity traps in the 41-type model**

We now turn to the question of whether relatively small shocks to the economy can trigger self-fulfilling pessimistic expectations, and how this can be prevented with appropriate policy measures. To study these questions we turn to the quantitatively more realistic model from section 3.4.3, with 41 multiples of 0.5% as possible biases.

In this model, liquidity trap steady states can also exist and simulations show that the conditions on existence of these steady states follow the same qualitative features as those in the 3-type model. That is, the possibility of a liquidity trap disappears if expectations are strongly anchored or if the inflation target is high enough.

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24 Other liquidity trap steady states show these same qualitative features for conditions on existence. Results are available on request.
Figure 3.10: Bifurcation diagram of 41-type model in $\pi^T$. The upper blue curve represents the fundamental steady state, and the lower blue curve the liquidity trap steady state. The green curve depicts the unstable steady state that separates their basins of attraction.

Figure 3.11: Bifurcation diagram of 41-type model in $\omega$ for $\pi^T = 0$. The upper blue curve represents the fundamental steady state, and the lower blue curve the liquidity trap steady state. The green curve depicts the unstable steady state that separates their basins of attraction.

Figure 3.10 and 3.11 present bifurcation diagrams of the 41-type model with the zero lower bound, with respectively $\pi^T$ and $\omega$ as bifurcation parameters. The same calibration as in Section 3.4.3 is used. It can be seen that for low values of $\pi^T$ and for high values of $\omega$ (weak anchoring of expectations), there exists two stable steady states (blue): the fundamental steady state at $\pi = \pi^T$, and a liquidity trap steady state with low inflation, where pessimistic expectations dominate. The basin of attraction of these two steady states are separated by an unstable steady state (green). As $\pi^T$ is increased or $\omega$ is decreased, the liquidity trap steady state comes closer to the basin of attraction of the fundamental steady state, and eventually seizes to exist. This implies
3.5. Zero lower bound on the interest rate

that the fundamental steady state can be made globally stable with a high enough inflation target or with strongly anchored expectations.

Figure 3.12 illustrates how the 41-type model is affected by the zero lower bound, and how a raised inflation target can be used to prevent self-fulfilling coordination on pessimism. A similar figure could be made for a decreased intensity of choice. In the simulated time series the same random seed is used as in Figure 3.7.

The first column of Figure 3.12 shows the time series of inflation, output gap and the nominal interest rate for the case of \( \pi^T = 0 \). The first part of the dynamics (where the nominal interest rate is positive) are exactly as in the top panels of Figure 3.7. However, in the bottom left panel of Figure 3.12 it can be seen that the wave of pessimism around period 100 results in a desired interest rate (blue) that is below its lower bound, so that the actual interest rate (green) is set to 0. The combination of low inflation and a nominal interest rate bounded by its lower bound implies a high real interest rate. This reinforces the wave of pessimism, and facilitates a self-fulfilling
decline in inflation and output gap expectations that comes to a halt only when the lowest possible expectations about both variables (and thereby the liquidity trap steady state from Figure 3.10) are reached. This pessimistic steady state lies quite far from the basin of attraction of the fundamental steady state, so that even when series of positive shocks occur (around periods 120 and 160), the economy keeps moving back to the liquidity trap steady state.

In the middle panels of Figure 3.12 the annualized inflation target is increased to 2%. From Figure 3.10 we know that the pessimistic steady state then still exists, and we indeed observe that this steady state is reached around period 100. However, the steady state now lies closer to the basin of attraction of the fundamental steady state, so that recovery to the fundamental steady state occurs after a sequence of positive shocks. A new wave of pessimism then leads to a new liquidity trap around period 150, but after some time recovery again occurs.

Finally, the right column of Figure 3.12 depicts the case where the inflation target is increased to 5%. As can be seen in Figure 3.10, the deflationary spiral steady state does not exist for this value of the policy parameter. In the bottom right panel of Figure 3.12 it can be seen that waves of pessimism still lead the zero lower bound to become binding. However, in the absence of shocks inflation and output gap now would always start to increase towards the fundamental steady state. Consequently, periods of low inflation and low output gap are less severe, and never last very long.

3.6 Conclusion

We study a New Keynesian macroeconomic model with heterogeneity in expectations. In this setup we compare three different interest rate rules and obtain a number of policy recommendations. To achieve local stability of the fundamental steady state, the central bank must prevent self-fulfilling coordination on optimistic or pessimistic expectations by responding strongly enough to (lagged) inflation and output gap, or their expectations. The Taylor principle is however only a necessary condition for local stability when expectations are unanchored. When aggregate expectations are strongly anchored to the fundamentals of the economy (because there is not much heterogeneity in expectations, and agents only slowly change their predictions) the CB is able to stabilize the economy with fairly weak monetary policy.

However, even when the fundamental steady state is locally stable, convergence to it may be quite slow due to almost self-fulfilling expectations and corresponding near unit root behavior. When expectations are discrete, these almost self-fulfilling expectations
may furthermore lead to the existence of non-fundamental steady states where agents coordinate on optimism or pessimism. The Central bank can mitigate these problems with more aggressive policy than otherwise required. If the CB responds to lagged values of inflation and output gap (e.g. because it cannot observe contemporaneous values) it must however take care not to destabilize the economy with policy that is too aggressive.

When the zero lower bound on the nominal interest rate is taken into account, convergence to the targets of the CB cannot be guaranteed just by the reaction coefficients of the monetary policy rule. Negative shocks can now drive the economy to a liquidity trap with a zero interest rate and low inflation and output gap (expectations). If there is no lower limit on expectation values that agents may consider, a liquidity trap can take the form of a self-fulfilling deflationary spiral with ever decreasing inflation and output gap.

We find that prolonged liquidity traps can be prevented by increasing the inflation target, or by increasing the anchoring of expectations. While the latter cannot be directly controlled by the CB, this does not mean that in the real world the anchoring of expectation is not affected by the actions of the central bank. Expectations might for example become more strongly anchored around the fundamental values after a decade of stable inflation and output gap. After such a time of stability agents would not be inclined to expect very high or very low inflation, even after a shock. The variance of the expectation values considered by agents, as well as the amount of switching between expectation values (intensity of choice) would then have been reduced by the performance of the central bank.\footnote{As a robustness check, we simulated versions of our models where the intensity of choice or the variance of distribution of types change endogenously to the variance of inflation or output gap (in line with De Grauwe (2011)). This does not change any of the results or dynamics in this chapter qualitatively.} On the other hand, if, during some years of economic turmoil, inflation and output are very volatile and stray far from their fundamental values, expectations will become more unanchored, which makes it more likely that the economy locks into a liquidity trap. It may then take a long time before the economy can recover from such an (almost) self-fulfilling equilibrium. Endogenizing the anchoring of expectations in the model is a subject of further research.
Appendix 3.A  LTL and Bayesian updating

Equation (3.17) can be written as

\[ E_{\pi_{t+1}} - \pi^T = \frac{\int_{-\infty}^{\infty} b e^{-\omega(\pi_{t-1} - b - \pi^T)^2/2\sigma^2} db}{\int_{-\infty}^{\infty} e^{-\omega(\pi_{t-1} - b - \pi^T)^2} e^{-b^2/2\sigma^2} db} = \int_{-\infty}^{\infty} b \psi(b) db. \]

Here, \( \psi(b) = \frac{e^{-\omega(\pi_{t-1} - b - \pi^T)^2/2\sigma^2} e^{-b^2/2\sigma^2}}{\int_{-\infty}^{\infty} e^{-\omega(\pi_{t-1} - b - \pi^T)^2} e^{-b^2/2\sigma^2} db} \) can be interpreted as agents’ (posterior) probability density function of choosing bias \( b \). This facilitates a comparison between the large type limit and Bayesian updating.

To make the relation between LTL and Bayesian updating more apparent, we multiply the numerator and denominator of \( \psi(b) \) by \( \frac{\sqrt{2\omega}}{\sqrt{2\pi}} \) and by \( \frac{1}{s\sqrt{2\pi}} \) to get

\[ \psi(b) = \frac{\sqrt{2\omega}}{\sqrt{2\pi}} e^{-\omega(\pi_{t-1} - b - \pi^T)^2/2\sigma^2} \frac{1}{s\sqrt{2\pi}} e^{-b^2/2s^2} \]

We now obtain a perfect mapping to the posterior of a Bayesian updating process, given by

\[ P(b|\pi_{t-1}) = \frac{P(\pi_{t-1}|b)P(b)}{\int_{-\infty}^{\infty} P(\pi_{t-1}|b)P(b)db} \]

First of all, the posterior \( P(b|\pi_{t-1}) = \psi(b) \) gives the probability density of choosing bias \( b \), given the observed value of \( \pi_{t-1} \). The prior distribution of \( b \) is given by \( P(b) = \frac{1}{s\sqrt{2\pi}} e^{-b^2/2s^2} \), which is the assumed distribution of the possible biases over the real line. The likelihood of observing realization \( \pi_{t-1} \), given a value of the bias \( b \), is now given by \( P(\pi_{t-1}|b) = \frac{\sqrt{2\omega}}{\sqrt{2\pi}} e^{-\omega(\pi_{t-1} - b - \pi^T)^2} \). That is, agents believe that the data generating process (DGP) of \( \pi_{t-1} \) is equal to a constant, \( \pi^T + b \), plus a normally distributed error term with variance \( \frac{1}{2\omega} \). They then try to learn the true constant of the DGP by observing the noisy time series and applying Bayesian updating. Note that the true constant value in principle is allowed to change over time. What the agents try to infer in the above two equations is what the true value was likely to be at the moment that \( \pi_{t-1} \) was generated.

When \( \omega \) goes to zero, the variance of the noise in the perceived DGP goes to infinity, so that the ”likelihood” \( \sqrt{2\omega/\pi} e^{-\omega(x_{t-1} - b)^2} \) goes to zero for all values of \( \pi_{t-1} \). This means
that agents believe that no useful information about \( b \) can be inferred from observing \( \pi_{t-1} \). As a consequence the posterior should just equal the prior, which is indeed the case according to Equation (3.38) (all types get equal weight which means that the distribution of mass over values on the real line is fully determined by the distribution of types over the real line).

When \( \omega = +\infty \) the variance of the noise in the perceived DGP goes to zero and the likelihood \( e^{-\omega (\pi_{t-1} - b)^2} \) becomes degenerate: it equals 1 when the observation \( \pi_{t-1} \) equals the true value \( b \), and 0 for all other possible values of \( \pi_{t-1} \). This implies that observing \( \pi_{t-1} \) is perceived as being fully informative about the true value of the constant. After observing \( \pi_{t-1} \) agents immediately know what the true constant value was, and the posterior distribution should be degenerate as well, with all its mass on the true value \( b \). This is indeed what Equation (3.38) tells us, i.e., for infinite intensity of choice it holds that \( \psi(b) = 1 \) for \( b = \pi_{t-1} \) and \( \psi(b) = 0 \) everywhere else (all agents switch with probability 1 to the last observed value \( \pi_{t-1} \)).

We can conclude that if we interpret the LTL as Bayesian updating, then the intensity of choice parameter determines how informative observations of \( \pi_t \) are about the biases that agents should choose. The larger the intensity of choice, the less ”noise” agents think they encounter, and the more clear it is to them, what prediction value they should choose in the next period.

**Appendix 3.B  LTL without the ZLB**

**3.B.1 Proof Proposition 3.1**

Under the forward-looking Taylor rule, the Jacobian in the fundamental steady state equals

\[
B \left( \begin{array}{cc}
1 - \frac{\phi_2}{\sigma} & -\frac{\phi_1 - 1}{\sigma} \\
\kappa(1 - \frac{\phi_2}{\sigma}) & \beta - \kappa \frac{\phi_1 - 1}{\sigma}
\end{array} \right),
\]

with

\[
B = \frac{2\omega s^2}{2\omega s^2 + 1}.
\]

The eigenvalues therefore are given by

\[
\lambda_{1,2} = \frac{B}{2} \left( (1 + \beta - \frac{\phi_2}{\sigma} - \kappa \frac{\phi_1 - 1}{\sigma}) \pm \sqrt{(1 + \beta - \frac{\phi_2}{\sigma} - \kappa \frac{\phi_1 - 1}{\sigma})^2 - 4\beta(1 - \frac{\phi_2}{\sigma})} \right).
\]

Local stability requires \( \lambda_1 < 1 \) and \( \lambda_2 > -1 \). By keeping only the square root on one
side of the equation and taking squares, \( \lambda_1 < 1 \) can be written as

\[
(3.41) \quad \phi_1 > 1 + (-\beta B - \frac{1}{B} + 1 + \beta) \frac{\sigma}{\kappa} + (\beta B - 1) \frac{\phi_2}{\kappa},
\]

Filling in \( B \) from (3.40) results in Condition (3.20).

Similarly, \( \lambda_2 > -1 \) can be written as

\[
(3.42) \quad \phi_1 < 1 + (1 + \beta + \beta B + \frac{1}{B}) \frac{\sigma}{\kappa} - (1 + \beta B) \frac{\phi_2}{\kappa},
\]

from which Condition (3.21) can be obtained.

### 3.B.2 Proof Proposition 3.2

When \( \phi_1 \) and \( \phi_2 \) equal the values given in (3.5), the eigenvalues reduce to

\[
(3.43) \quad \lambda_{1,2} = \frac{B}{2} \left( \beta - \frac{\kappa^2}{\mu + \kappa^2} \right) \pm \sqrt{\left( \beta - \frac{\kappa^2}{\mu + \kappa^2} \right)^2 - 4 \frac{\beta}{\sigma + \phi_2 + \kappa \phi_1} \left( \frac{\beta \phi_1 - 1}{\sigma + \phi_2 + \kappa \phi_1} \right)}.
\]

Since \( \mu \) is the relative weight on output gap in the loss function and therefore is nonnegative, \( \lambda_2 \) reduces to zero and \( \lambda_1 \) becomes

\[
(3.44) \quad \lambda_1 = B \left( \beta \mu - (1 - \beta) \frac{\kappa^2}{\mu + \kappa^2} \right).
\]

Both eigenvalues are therefore inside the unit circle as long as \( \lambda_1 < 1 \), which is satisfied since \( 0 < \beta < 1 \), and \( B < 1 \).

### 3.B.3 Proof Proposition 3.3

Under the contemporaneous Taylor rule, the Jacobian in the fundamental steady state equals

\[
B \left( \frac{\sigma}{\sigma + \phi_2 + \kappa \phi_1} - \frac{\beta \phi_1 - 1}{\sigma + \phi_2 + \kappa \phi_1} \right).
\]

The eigenvalues therefore are given by

\[
\lambda_{1,2} = \frac{B}{2} \left( \frac{(1 + \beta) \sigma + \beta \phi_2 + \kappa}{\sigma + \phi_2 + \kappa \phi_1} \right) \pm \sqrt{\left( \frac{(1 + \beta) \sigma + \beta \phi_2 + \kappa}{\sigma + \phi_2 + \kappa \phi_1} \right)^2 - 4 \beta \left( \frac{\sigma}{\sigma + \phi_2 + \kappa \phi_1} \right)}.
\]

\( \lambda_1 > \lambda_2 \) always holds, so local stability requires \( \lambda_1 < 1 \) and \( \lambda_2 > -1 \). The first
condition can be written as

\[ \phi_1 > B + (-\beta B^2 + (1 + \beta)B - 1)\frac{\sigma}{\kappa} + (\beta B - 1)\frac{\phi_2}{\kappa}. \]  

(3.45)

Filling in \( B \) from (3.40) gives Condition (3.22).

\[ \lambda_2 > -1 \] can be written as

\[ -4\beta \left( \frac{\sigma}{\sigma + \phi_2 + \kappa\phi_1} \right) < \frac{4}{B^2} + \frac{4(1 + \beta)\sigma + \beta\phi_2 + \kappa}{\sigma + \phi_2 + \kappa\phi_1}, \]  

(3.46)

which is always satisfied for \( \phi_1, \phi_2 > 0 \).  

### 3.B.4 Proof Proposition 3.4

Under the lagged Taylor rule, the Jacobian in the fundamental steady state equals

\[
\begin{pmatrix}
B - \frac{\phi_2}{\sigma} & \frac{B}{\sigma} - \frac{\phi_1}{\sigma} \\
\kappa(B - \frac{\phi_2}{\sigma}) & B(\beta + \frac{\kappa}{\sigma}) - \frac{\kappa\phi_1}{\sigma}
\end{pmatrix}.
\]

The eigenvalues are given by

\[
\lambda_{1,2} = \frac{1}{2} \left( T \pm \sqrt{T^2 - 4\beta B(B - \frac{\phi_2}{\sigma})} \right),
\]

with

\[ T = B(1 + \beta + \frac{\kappa}{\sigma}) - \frac{\phi_2 + \kappa\phi_1}{\sigma} \]  

(3.47)

Local stability again requires \( \lambda_1 < 1 \) and \( \lambda_2 > -1 \). The first condition can be written as

\[ \phi_1 > B + (-\beta B^2 + (1 + \beta)B - 1)\frac{\sigma}{\kappa} + (\beta B - 1)\frac{\phi_2}{\kappa}. \]  

(3.48)

\[ \lambda_2 > -1 \] can be written as

\[ \phi_1 < B + (\beta B^2 + (1 + \beta)B + 1)\frac{\sigma}{\kappa} - (\beta B + 1)\frac{\phi_2}{\kappa}. \]  

(3.49)

Filling in \( B \) from (3.40) in (3.48) and (3.49) results in Conditions (3.23) and (3.24) respectively.
3.B.5 Proof Proposition 3.5

It follows from Equation (3.6) that in a steady state the model under the forward-looking Taylor rule satisfies

\[
x = \frac{x^T (1 - \frac{\phi_2}{\sigma}) + \frac{\phi_2}{\sigma} x^T - \frac{\phi_1 - 1}{\sigma} (2\omega s^2 + 1) \pi + \frac{\pi^T}{2\omega s^2 + 1} - \pi^T}{(1 - (1 - \frac{\phi_2}{\sigma}) \frac{2\omega s^2}{2\omega s^2 + 1})}.
\]

Plugging this in in (3.7), using \(x^T = 1 - \beta \frac{\kappa}{\delta} \pi^T\) and rearranging results in

\[
\pi (1 - \beta \frac{2\omega s^2}{2\omega s^2 + 1} + \kappa (\phi_1 - 1) 2\omega s^2) = \pi^T \left(1 - \beta \frac{2\omega s^2}{2\omega s^2 + 1} + \kappa (\phi_1 - 1) 2\omega s^2 \right).
\]

This has as a solution \(\pi = \pi^T\), so that the fundamental steady state where average expectations equal the rational expectations equilibrium values always exists.

Alternatively, if the part in brackets in (3.51) is 0, any inflation level is a steady state. This is the case if and only if

\[
\phi_1 = 1 - \frac{1 + (1 - \beta) 2\omega s^2}{2\omega s^2 + 1} \left(\frac{\sigma}{2\omega s^2 \kappa} + \frac{\phi_2}{\kappa}\right),
\]

which is exactly the value where the fundamental steady state loses stability and one eigenvalue equals +1 (see Proposition 3.1).

For any inflation level \(\tilde{\pi}\), corresponding steady state output gap \(\tilde{x}\) then follows from (3.7) and is given by (3.25).

Under the contemporaneous and lagged Taylor rules we can derive in the same way from (3.10) (or (3.12)) and (3.7) that the fundamental steady state \((\pi = \pi^T)\) always exists and that any inflation level can comprise a steady state if and only if

\[
\phi_1 = \frac{2\omega s^2}{2\omega s^2 + 1} - \frac{1 + (1 - \beta) 2\omega s^2}{2\omega s^2 + 1} \left(\frac{\sigma}{(2\omega s^2 + 1) \kappa} + \frac{\phi_2}{\kappa}\right).
\]

which is again exactly the bifurcation value where the fundamental steady states loses stability and one eigenvalue equals +1 (see Proposition 3.1). Steady state output gap corresponding to inflation \(\tilde{\pi}\) is again equal to (3.25).
3.C. 3-type model without the ZLB

Appendix 3.C 3-type model without the ZLB

3.C.1 Proof Proposition 3.6

Each of the nine combinations of optimism, pessimism and fundamentalism about inflation and output gap comprises a steady state if and only if under that particular combination of expectations, realized inflation and output gap are such that these expectations have the highest fitness measure. We will first illustrate how the existence conditions of these steady states can be derived when $\omega \rightarrow \infty$. Then we will show that all nine steady states are locally stable when they exist.

Consider the case where all agents are pessimists about both inflation and output gap. Under the forward-looking Taylor rule the model (given by (3.6) and (3.7)) then reduces to

\begin{align}
\frac{dx_t}{dt} &= (1 - \phi_2)(x^T - b) - \frac{\phi_1 - 1}{\sigma}(\pi^T - b - \pi^T) + \frac{\phi_2}{\sigma}x^T, \\
\pi_t &= \beta E_t(\pi^T - b) + \kappa x_t.
\end{align}

The steady state exists if and only if both output gap and inflation are such that in the next period again all agents are pessimists about both variables. This requires that both $x_t < x^T - \frac{b}{2}$ and $\pi_t < \pi^T - \frac{b}{2}$. Rewriting (3.54), the condition for output gap reduces to

\begin{align}
x_t = x^T + b\left(\frac{\phi_2}{\sigma} - 1 + \frac{\phi_1 - 1}{\sigma}\right) < x^T - \frac{b}{2},
\end{align}

which gives

\begin{align}
\phi_1 < 1 + \frac{\sigma}{2} - \phi_2.
\end{align}

If this condition is satisfied (and thereby $x_t < x^T - \frac{b}{2}$), then it follows from (3.55) that

\begin{align}
\pi_t < \beta E_t(\pi^T - b) + \kappa(x^T - \frac{b}{2}).
\end{align}

This implies (since $2\beta + \kappa > 1$) that $\pi_t < \pi^T - \frac{b}{2}$ is then satisfied as well.

If (3.55) does not hold, then output gap expectations (and possibly also inflation expectations) will not stay pessimistic, so that the pessimistic steady state does not exist.
The conditions for the contemporaneous lagged Taylor rule, and the conditions for the other steady states presented in Table 3.1, can be derived in the same way.

The local stability properties of the 3-type model depend on the derivative of expectations with respect to lagged values of inflation and output gap. The derivative of $n_t^{x,\text{opt}}$ with respect to $x_{t-1}$ is given by

$$
\frac{\partial n_t^{x,\text{opt}}}{\partial x_{t-1}} = 2\omega b n_t^{x,\text{opt}}(1 - n_t^{x,\text{opt}} + n_t^{x,\text{pes}}).
$$

This can be written as

$$
\frac{\partial n_t^{x,\text{opt}}}{\partial x_{t-1}} = 2\omega b n_t^{x,\text{opt}}(1 - n_t^{x,\text{opt}} + n_t^{x,\text{pes}}).
$$

Similarly, the derivative of $n_t^{x,\text{pes}}$ with respect to $x_{t-1}$ can be written as

$$
\frac{\partial n_t^{x,\text{pes}}}{\partial x_{t-1}} = -2\omega b n_t^{x,\text{pes}}(1 - n_t^{x,\text{pes}} + n_t^{x,\text{opt}}).
$$

The derivatives of $n_t^{\pi,\text{opt}}$ and $n_t^{\pi,\text{pes}}$ with respect to $\pi_t$ are obtained by replacing $x_{t-1}$ by $\pi_{t-1}$ and $x^T$ by $\pi^T$ in (3.59) and by replacing $n_t^{x,\text{opt}}$ and $n_t^{x,\text{pes}}$ by $n_t^{\pi,\text{opt}}$ and $n_t^{\pi,\text{pes}}$ in (3.60) and (3.61). In the fundamental steady state the above reduces to

$$
\frac{\partial n_t^{x,\text{opt}}}{\partial x_{t-1}} = \frac{\partial n_t^{\pi,\text{opt}}}{\partial \pi_{t-1}} = 2\omega b n_t^{\pi,\text{opt}} = 2\omega b \frac{1}{2 + e^{\omega b^2}},
$$

$$
\frac{\partial n_t^{x,\text{pes}}}{\partial x_{t-1}} = \frac{\partial n_t^{\pi,\text{pes}}}{\partial \pi_{t-1}} = -2\omega b n_t^{\pi,\text{pes}} = -2\omega b \frac{1}{2 + e^{\omega b^2}}.
$$

In the fundamental steady state the derivative of output (inflation) expectations with respect to lagged output gap (inflation), is therefore given by

$$
B^{3t} = b(2\omega b \frac{1}{2 + e^{\omega b^2}}) - b(-2\omega b \frac{1}{2 + e^{\omega b^2}}) = 4\omega b^2 \frac{1}{2 + e^{\omega b^2}}.
$$

Plugging this in in Conditions (B.2), (B.3), (B.6), (B.8) and (B.9) gives the conditions for local stability under the 3-type model for the three different interest rate rules.
When the intensity of choice goes to infinity, there are two possibilities for the expectations of each variable in steady state. Either all agents adhere to one heuristic, because this heuristic performs best in the steady state, or half of the agents adheres to one heuristic, while the other half adheres to another heuristic. In the second case the two heuristics must perform equally well in steady state. It follows from (3.60) and (3.61) that in a steady state where all agents adhere to one heuristic for both inflation and output gap (not necessarily the same heuristic), we have

\[
\frac{\partial n_{x,\text{opt}}}{\partial x_{t-1}} = \frac{\partial n_{\pi,\text{opt}}}{\partial \pi_{t-1}} = \frac{\partial n_{x,pes}}{\partial x_{t-1}} = \frac{\partial n_{\pi,pes}}{\partial \pi_{t-1}} = 0.
\]

The derivative of expectations with respect to lagged values then equals \( B = 0 \). It then follows from Appendix 3.B.1, 3.B.3 and 3.B.4 that both eigenvalues equal 0 under all three interest rate rules, so that these steady states are always locally stable.

For steady states where two heuristic about one variable perform equally well, at least one eigenvalue goes to infinity in absolute value so that these steady states are always unstable.

**Appendix 3.D  Zero lower bound LTL**

### 3.D.1 Proof Proposition 3.7

When the zero lower bound is binding the LTL model becomes

\[
x_t = \frac{x^T}{2\omega s^2 + 1} + \frac{2\omega s^2}{2\omega s^2 + 1} x_{t-1} + \frac{1}{\sigma} \frac{\pi^T}{2\omega s^2 + 1} + \frac{1}{\sigma} \frac{2\omega s^2}{2\omega s^2 + 1} \pi_{t-1} + \frac{\bar{r}}{\sigma},
\]

\[
\pi_t = \beta \frac{\pi^T}{2\omega s^2 + 1} + \beta \frac{2\omega s^2}{2\omega s^2 + 1} \pi_{t-1} + \kappa x_t.
\]

Solving for the steady state of this model results in (3.32) and (3.33). Steady state output gap and inflation both are negative if and only if

\[
\sigma (1 + (1 - \beta)2\omega s^2) - \kappa (2\omega s^2 + 1)2\omega s^2 < 0,
\]

which can be rewritten as (3.34).
CHAPTER 3. MANAGING UNANCHORED EXPECTATIONS

The eigenvalues of the system defined by (3.66) and (3.67) are given by

\[(3.69) \quad \lambda_{1,2} = \frac{\omega s^2}{2\omega s^2 + 1} \left( (1 + \beta + \frac{\kappa}{\sigma}) \pm \sqrt{(1 + \beta + \frac{\kappa}{\sigma})^2 - 4\beta} \right).\]

This implies that the steady state is an unstable saddle if and only if

\[(3.70) \quad \omega s^2 > \frac{1}{\beta - 1 + \frac{\kappa}{\sigma} + \sqrt{(1 + \beta + \frac{\kappa}{\sigma})^2 - 4\beta}},\]

which, after some algebraic manipulation, reduces to (3.34). Therefore, when (3.34) does not hold the system has a unique attractor that lies outside the ZLB region. This implies that from all initial conditions inflation and output gap will go towards this attractor and cross the zero lower bound. Recovery then always occurs.

When (3.34) holds, initial conditions below the stable eigenvector through the steady state given by (3.32) and (3.33) lead to ever decreasing inflation and output gap, while initial conditions above it lead to increasing inflation and output gap, and thereby to recovery. The slope of this eigenvector is given by (3.35).

Appendix 3.E Zero lower bound 3-type model

3.E.1 Proof Proposition 3.8

In the ZLB region inflation and output gap are given by

\[(3.71) \quad x_t = x^T + (n_i^{x,opt} - n_i^{x,pes})b + \frac{\pi^T + n_i^{\pi,opt}b - n_i^{\pi,pes}b + \bar{r}}{\sigma},\]

\[(3.72) \quad \pi_t = \beta(\pi^T + n_i^{\pi,opt}b - n_i^{\pi,pes}b) + \kappa x_t.\]

It therefore follows that pessimism remains the best performing heuristic if and only if

\[(3.73) \quad x^T + (n_i^{x,opt} - n_i^{x,pes})b + \frac{\pi^T + n_i^{\pi,opt}b - n_i^{\pi,pes}b + \bar{r}}{\sigma} < x^T - \frac{b}{2},\]

and

\[(3.74) \quad (\pi^T + n_i^{\pi,opt}b - n_i^{\pi,pes}b)(\beta + \frac{\kappa}{\sigma}) + \kappa(x^T + b(n_i^{x,opt} - n_i^{x,pes})) + \frac{\kappa}{\sigma} \bar{r} < \pi^T - \frac{b}{2}.\]

These conditions together reduce to Equation (3.36).
Chapter 4

The Stabilizing Role of Forward Guidance: A Macro Experiment

4.1 Introduction

In economics, expectations can be seen as one of the main driving forces for economic dynamics. As Hommes (2011) puts it, “individual expectations about future aggregate outcomes are the key feature that distinguishes social sciences and economics from the natural sciences. Daily weather forecasts, either by the public or by experts, do not affect the probability of rain.” Therefore, managing market expectations has become a primary objective for economic policy makers seeking to actively influence the economic development.26

For monetary policy makers, market expectations not only determine the effectiveness of their main conventional monetary policy instrument (i.e., the short-term nominal interest rate) in normal times; they are also central to the transmission of unconventional monetary policy (e.g. quantitative easing and forward guidance) when the short-term nominal interest rate is restricted by the zero lower bound (as we currently witness in many of the leading industrialized economies). So it has come that central banks worldwide have become increasingly communicative, providing the public with detailed information about their views of monetary policy and the fundamental factors driving their monetary policy decisions (Blinder et al., 2008).

A pivotal aspect in this regard is the central bank practice to publish inflation projections. This practice, which qualifies as a tool of forward guidance.27 intends to

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26See, for instance, the speech held by Janet Yellen at “The Elusive ‘Great’ Recovery: Causes and Implications for Future Business Cycle Dynamics” (60th annual economic conference) on October 14, 2016.

27There is no official definition of forward guidance. Commonly, forward guidance is understood
CHAPTER 4. THE STABILIZING ROLE OF FORWARD GUIDANCE

provide superior information about future macroeconomic developments to the private sector and thereby to reduce private-sector uncertainty (Campbell et al., 2012). But central banks may also use this tool to strategically influence private-sector expectations by intentionally over- or underreporting the projected level of inflation (Charemza and Ladley, 2016; Gomez-Barrero and Parra-Polania, 2014; Jensen, 2016). Independent of the central banks’ motive to publish inflation projections, ample empirical evidence reveals that this practice considerably impacts on private-sector expectations (Hubert, 2014, 2015a, b).

While the publication of central bank inflation projections might be a powerful tool for private-sector expectations management, the central bank must consider its effects on the endogenous credibility of future central bank inflation projection28 (Blinder, 2000). Publishing accurate inflation projections strengthens the central bank’s reputation as a good, accurate forecaster but deters the central bank from the ability to steer private-sector expectations into a different direction if preferred. Conversely, strategic inflation projections might allow the central bank to steer private-sector expectations, but may be damaging to credibility if they systematically over- or underestimate inflation. Thus, the central bank faces the risk of diminishing its ability to strategically influence private-sector expectations in the future. This trade-off between short term gains and potential long term losses raises the question how the central bank’s ability to manage expectations via publishing inflation projections depends on the credibility of the central bank’s past forecasting performance.

In this chapter, we study to what extent the central bank can influence economic activity through market-expectations management via the publication of (strategic) official central bank inflation projections29, i.e., by using forward guidance, both in

as a (commitment to a) projected path of the nominal interest rate. This understanding of forward guidance is, however, very restrictive. In this chapter, the term “forward guidance” refers to the rather vague concept of Delphic forward guidance as defined by Campbell et al. (2012). According to Campbell et al. (2012), (Delphic) forward guidance publicly states a forecast of macroeconomic fundamentals and the likely future course of monetary policy. As we show below, in our experiment, this form of forward guidance can be approximated by public central bank inflation projections, as they convey information also about the expected future interest rate.

28 Throughout this chapter, the term credibility refers exclusively to the central bank’s inflation projections, and not to the central bank as the monetary authority.

29 The focus on the publication of inflation projections rather than interest rate projections is motivated by the work of Ferrero and Secchi (2010), who study the effect of different central bank communication strategies in a standard New Keynesian model when agents are learning. They find that the communication of interest rate projections can be destabilizing, while the communication of inflation projections is stabilizing. Although, the model attributes a stabilizing role also to output gap projections, we choose to abstract from output gap projections entirely based on institutional and empirical grounds. Institutionally, it is inflation stabilization which has traditionally been the core mandate of many central banks. Empirically, the relationship between output gap predictions and
normal times and in times of severe economic stress (i.e. periods where there is a high probability of the zero lower bound on the nominal interest rate becoming binding), and how this influence depends on the endogenous degree of credibility given to the central bank projections by the private sector.

The analysis is conducted by means of a laboratory experiment. For the question at hand, a laboratory experiment has several advantages over traditional empirical or theoretical approaches. First, it allows us to study the expectation formation process of the subjects and its interaction with monetary policy design, without having to rely on prescribed expectations formation processes, as e.g., rational or adaptive expectations. Second, we are able - in a very natural way - to depart from the representative agent hypothesis commonly put forth in macroeconomics and to allow for substantial heterogeneity. Finally, we can control the subjects’ incentives and information sets in the laboratory.

The underlying economic environment of the experiment is given by a standard forward-looking New Keynesian model. The experimental task for the subjects is a learning-to-forecast experiment as pioneered by Marimon and Sunder (1993). All but one subject are “professional forecasters” in the private sector who are asked repeatedly to form one-period ahead expectations about future inflation, having only a limited understanding of the true data generating process. The remaining subject is assigned the role of the “central bank forecaster.” Apart from the control treatment, at the beginning of each period the central bank publishes an official one-period ahead central bank inflation projection. Depending on the treatment, this public central bank inflation projection is produced either by the central bank forecaster or by a computerized algorithm. In any case, the central bank is provided with superior information that can be used in the forecasting process. Professional forecasters are presented with the public central bank projection before they submit their own inflation forecasts.

The novelty of the proposed experiment is that, through a series of different treatments, we can study the impact of (strategic) forward guidance on the subjects’ expectation formation process and the resulting dynamic evolution of the underlying theoretical economy. We are mainly interested in answering the questions: if a central bank can influence or even manage private-sector expectations via the publication of (strategic) central bank inflation projections in such a laboratory environment, if such private-sector expectations is rather vague. E.g., in the United States, the FOMC’s central bank output gap projections neither have an informational advantage over private-sector output gap forecasts (Romer and Romer, 2000), nor do they significantly influence private-sector output gap expectations (Hubert 2014).

30 For a thorough discussion about the potential advantages of laboratory experiments for the conduct of monetary policy analysis, see Cornand and Heinemann (2014).
expectations management can successfully be applied as an additional monetary policy instrument to stabilize the economy in normal times and in times of severe economic stress, and how effectiveness of forward guidance depends on the endogenous degree of central bank credibility.

For normal times, we find that the publication of inflation projections strongly affects private-sector expectations. Instead of simply following trends, under forward guidance subjects put a large weight on the public inflation projection when forming their expectations about future inflation. We show that the macroeconomic consequences of this influence depend on the quality of the published projection. Reasonable, informative public projections act as focal points which decrease the dispersion among individual professional forecasters and increases their forecasting performance. They stabilize the economy, i.e., they unambiguously mitigate the mean squared errors of the fundamentals by bringing the economies faster and closer towards the steady state and by reducing the volatility of inflation, output, and the interest rate.

In times of severe economic stress, the publication of optimistic central bank projections reduces the risk of deflationary spirals. Noisy inflation projections, by contrast, are generally harmful to the economy as they unleash disturbing forces which give rise to more dispersed and less precise individual private-sector forecasts. Finally, credibility of central bank inflation projections, at least in times of severe economic stress, seems to be of minor importance for the stabilizing role of forward guidance.

The chapter is organized as follows. Section 4.2 reviews the relevant literature. Section 4.3 describes our experimental design. In Section 4.4 we study the influence of forward guidance on economic stability and the dynamic performance of inflation expectations. Section 4.5 analyzes the expectation formation processes of the subjects, and Section 4.6 briefly discusses central bank projection credibility and its interaction with forward guidance. Finally, Section 4.7 concludes.

4.2 Related Literature

Laboratory experiments on monetary policy have become increasingly popular in recent years (see Cornand and Heinemann (2014) for a survey). A considerable fraction of this newly developed literature deals with learning-to-forecast experiments in New Keynesian models. Adam (2007) shows that in such an environment subjects expectation formation process generally fails to be rational, but can rather be described by simple forecasting rules based on lagged inflation. Pfajfar and Zakelj (2014) and Assenza et al. (2014) study the expectation formation process of the subjects and its in-
4.2. Related Literature

They find a stronger mandate for price stability to better stabilize private-sector expectations and thereby the economy. Kryvtsov and Petersen (2015) show that much of this stabilizing power is through the effect on private-sector expectations. Close to the zero lower bound, however, Hommes et al. (2015) find that conventional monetary policy is generally not very effective in stabilizing the economy and insulating it from the risk of falling into an expectation driven liquidity trap.

The effects of forward guidance on economic stability in New Keynesian learning-to-forecast experiments are mixed. While Cornand and M’Baye (2016a,b) find that the communication of the central bank’s inflation target can reduce the volatility of the economy in normal times, Arifovic and Petersen (2015) find that it does not provide a stabilizing anchor in crisis times, e.g. in a liquidity trap. Mokhtarzadeh and Petersen (2016) find that providing the economy with the central bank’s projections for inflation and the output gap stabilizes the economy, while Kryvtsov and Petersen (2015) find that providing the expected future interest rate path diminishes the effectiveness of monetary policy in stabilizing the economy.

The contribution of this project to the literature is twofold. First, we analyze the stabilizing role of central bank forward guidance in the form of inflation projections in normal times and in times of severe economic stress. Publishing inflation projections is common practice for central banks, but has yet received very little attention in the context of New Keynesian learning-to-forecast experiments. To the best of our knowledge, the only exception is Mokhtarzadeh and Petersen (2016). Second, we investigate how the central bank’s effectiveness to influence expectations depends on its endogenous degree of credibility.

The papers closest to ours are Mokhtarzadeh and Petersen (2016) and Goy et al. (2016). Mokhtarzadeh and Petersen (2016) also study the effect of public central bank projections on expectation formation, future credibility and the stabilizing role of forward guidance in a New Keynesian learning-to-forecast experiment. Yet there are substantial differences in the methodology. Mokhtarzadeh and Petersen (2016) study the publication of a larger set of macroeconomic projections comprising five-period ahead projections of inflation, the output gap, and the nominal interest rate. The projections are generated by a computerized central bank that assumes agents to form expectations rationally or adaptively. The current chapter focuses on the publication of one-period ahead inflation projections, i.e., we abstract from output gap and interest rate projections. The inflation projections in this chapter are generated either by a student subject who does not have to follow any specific expectation formation mechanism, or by a computerized algorithm that assumes that agents form expectations...
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according to a Heuristic Switching Model as presented in Assenza et al. (2014). The perhaps largest difference between the two studies arises from the motive for forward guidance. Mokhtarzadeh and Petersen (2016) study the stabilizing role of informative forward guidance, whereas the present chapter studies the stabilizing role of strategic forward guidance in normal times and at the zero lower bound. Goy et al. (2016) present a theoretical analysis under learning - instead of a laboratory experiment - of central bank forward guidance in a New Keynesian model with zero lower bound and boundedly rational and heterogeneous agents. Despite the differences in methodology, the general conclusions of Mokhtarzadeh and Petersen (2016), Goy et al. (2016), and this chapter are reconfirming. All three contributions find that the publication of inflation forecasts is a helpful tool to anchor private sector expectations and to stabilize the economy.

The experimental setup mainly follows the work by Assenza et al. (2014), with two major differences: (i) subjects face a public central bank projection which they can utilize in forming their own expectations and (ii) output gap expectation are not subject-based but model-based. The latter assumption is made in order to keep the experimental task for the subjects simple and to focus this study entirely on inflation expectations. Output gap expectations are endogenously determined by the model following a Heuristic Switching Model, which has proven to fit well learning-to-forecast experiments in New Keynesian frameworks (e.g. Assenza et al., 2014).

4.3 Experimental Design

Subjects interact with the economy through expectations of inflation, which affect the outcome of the economy through a positive feedback of the form:

$$\pi_t = f(\bar{E}_t\pi_{t+1})$$

where $\pi_t$ and $\bar{E}_t\pi_{t+1}$ denote inflation and aggregate private-sector expected future inflation, respectively, and $f$ is a functional form, which is specified below. Note that subjects do not yet know the realization of $\pi_t$ when they form their expectation about $\pi_{t+1}$. We follow Arifovic and Petersen (2015) and Kryvtsov and Petersen (2015)

31Positive feedback means that the derivative of the function $f(\cdot)$ is positive. Note that although the nominal interest rate rule (4.3) adds some negative feedback to the economy, the overall feedback of inflation expectations on current inflation remains positive, independent of the coefficients in this interest rate rule.
and define aggregate inflation expectations as the median\(^{32}\) of the individual inflation expectations, i.e., \(\tilde{E}_t \pi_{t+1} = \text{median}(E_t \pi_{t+1})\), where \(E_t \pi_{t+1}\) is a vector collecting all \(j = 1, \ldots, J\) professional forecasters’ individual inflation expectations \(E_{t}^{f_{c-j}} \pi_{t+1}\) of period \(t\) for period \(t+1\).

### 4.3.1 The New Keynesian Economy

The underlying economy evolves according to a New-Keynesian model under heterogeneous expectations.\(^{33}\)

\[
(4.1) \quad x_t = \tilde{E}_t x_{t+1} - \frac{1}{\sigma} \left( i_t - \tilde{E}_t \pi_{t+1} - \bar{r} \right) + \varepsilon_t, \\
(4.2) \quad \pi_t = \beta \tilde{E}_t \pi_{t+1} + \kappa x_t + \eta_t, \\
(4.3) \quad i_t = \max \left[ 0, \bar{r} + \pi^T + \phi_x (\pi_t - \pi^T) + \phi_y x_t \right],
\]

where \(x_t\) is the aggregate output gap, \(i_t\) is the nominal interest rate, \(\bar{r} = \frac{1}{\beta} - 1\) is the steady state interest rate, and \(\tilde{E}_t x_{t+1}\) is the aggregate expected future output gap. The parameter \(\pi^T\) denotes the central bank’s target value for inflation. Finally, the economy is perturbed by stochastic i.i.d demand and supply shocks, denoted \(\eta_t\) and \(\varepsilon_t\), respectively.\(^{34}\)

The calibration of the constant model parameters follows Clarida et al. (2000). I.e., we set the quarterly discount factor \(\beta = 0.99\), implying an annual risk-free interest rate of four percent. The coefficient of relative risk aversion is set to \(\sigma = 1\) and the output elasticity of inflation is \(\kappa = 0.3\). The quarterly inflation target is set to \(\pi^T = 0.00045\), implying an annual inflation rate of 0.18 per cent.\(^{35}\) The Taylor rule coefficients are

\(^{32}\)When the aggregate is determined as the mean of all forecasts, an individual could cast an extreme forecast, in order to obtain an extreme aggregate, which would then feed back into the economy. Such individual strategic power that does not reflect the real world is eliminated when the aggregate is instead determined by the median of all forecasts.

\(^{33}\)Microfoundations for this model under heterogeneous expectations can be found, Chapter 2 of this thesis.

\(^{34}\)There are six economies (groups) in each treatment. Therefore, there are six random shock processes each for \(\eta_t\) and \(\varepsilon_t\). These are applied to all treatments so that each shock sequence is applied once in each treatment. In particular, the following pairings arise: E1-E7-E13-E19, E2-E8-E14-E20, E3-E9-E15-E21, E4-E10-E16-E22, E5-E11-E17-E23, E6-E12-E18-E24.

\(^{35}\)We choose a value of the inflation target near zero to be in line with the zero inflation steady state that is assumed when log-linearizing the macro economic model to obtain equations (4.1) and (4.2). We choose however a value slightly different from zero in order not to present subjects with a
chosen to be $\phi_\pi = 1.25$ and $\phi_y = 0.3$, which is well within the range of values that are common in related experiments.$^{36}$

Equation (4.1) refers to an optimized IS curve, equation (4.2) is the New Keynesian Phillips curve and equation (4.3) is the rule for the nominal interest rate set by the central bank. We assume the central bank follows a Taylor (1993) type interest rate rule, where it adjusts the interest rate in response to inflation and output gap. Furthermore, equation (4.3) also shows that the nominal interest rate is subject to a zero lower bound.$^{37}$ Under rational expectations this model has two equilibria. A determinate equilibrium equal to the target steady state$^{38}$ that has values of inflation and output (close to) $\pi_t = x_t = 0$ given that $\pi^T$ is (close to) zero, and an indeterminate equilibrium where the zero lower bound on the nominal interest rate is binding and $(\pi_t, x_t) = (-\bar{r}, -\frac{1-\beta}{\kappa}\bar{r})$ (Benhabib et al., 2001a). Under adaptive learning the target steady state is locally stable (if the Taylor principle is satisfied), while the zero lower bound steady state is an unstable saddle-point. Therefore, depending on initial conditions, either convergence to the target steady state occurs or the economy falls into a deflationary spiral (Evans et al., 2008).

Since we focus on how the central bank can stabilize the economy by publishing inflation projections, we do not make any assumptions on the way inflation expectations are formed, but ask the subjects in the lab for their inflation expectations. $\tilde{E}_t \pi_{t+1}$ is therefore an aggregation of elicited expectations. In contrast, $\tilde{E}_t x_{t+1}$ is endogenously determined by the model. $\tilde{E}(y)$ follows a Heuristic Switching Model$^{39}$ that was originally developed to fit a learning-to-forecast experiment in an asset price setting (Anufriev and Hommes 2012), but has proven its robustness to fit also learning-to-forecast experiments in New Keynesian frameworks (e.g. Assenza et al., 2014).

The Heuristic Switching Model can be summarized by the following equations:

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36 Standard values for comparable experiments range from $\phi_\pi \in (1,2)$ and $\phi_y \in (0,0.5)$, e.g., Cornand and M’Baye (2016b) and Arifovic and Petersen (2015) among others.

37 Note that under commitment to a Taylor rule, setting the nominal interest rate is not part of the task attributed to the subject with the role as central bank forecaster. Rather the nominal interest rate is influenced implicitly, through the effects of forward guidance on private-sector expectations and their feedback on the economy. Information about likely feedback effects and the corresponding prescribed reaction of future interest rates are provided to the central bank (described in detail in Section 4.3.3) as input for the inflation projection. Thereby, forward guidance and the nominal interest rate are in practice not chosen independent of each other.

38 The rational expectations equilibrium coincides with the steady state because shocks are not autocorrelated.

39 Heuristic Switching Models were introduced by Brock and Hommes (1997).
4.3. Experimental Design

\[
\begin{align*}
\text{Adaptive Rule} \quad &\quad \implies E_{t}^{\text{ada}} x_{t+1} = 0.65 x_{t-1} + 0.35 E_{t-1}^{\text{ada}} x_{t} \\
\text{Weak Trend} \quad &\quad \implies E_{t}^{\text{wtr}} x_{t+1} = x_{t-1} + 0.4 (x_{t-1} - x_{t-2}) \\
\text{Strong Trend} \quad &\quad \implies E_{t}^{\text{str}} x_{t+1} = x_{t-1} + 1.3 (x_{t-1} - x_{t-2}) \\
\text{Learn and Anchor} \quad &\quad \implies E_{t}^{\text{laa}} x_{t+1} = \frac{(x_{t-1} - x_{t-2})}{2} + (x_{t-1} - x_{t-2})
\end{align*}
\]

Equation (4.4) lists the set of heuristics available to the agents when forming their expectations. The variable \(x_{t-1}^{av}\) denotes the average of past output gaps. Once heuristics are used, the agents weight their past performance following equation (4.5), with \(\eta\) denoting the parameter describing the preference for the past. Equation (4.6) updates the probability of using heuristic \(h\) when forecasting for period \(t + 1\). Notice that \(\gamma\) captures the sensitivity of agents to heuristic performances and \(\delta\) denotes the fraction of agents that in period \(t\) stick to the heuristic they used in period \(t - 1\). Then, using, (4.7) the expectation are aggregated and \(\hat{E}_{t} x_{t+1}\) is determined. The calibration of the Heuristic Switching Model follows Assenza et al. (2014), i.e., we set \(\eta = 0.7\), \(\delta = 0.9\), and \(\gamma = (0.4 \cdot 4^2) = 6.4\).

4.3.2 The Experiment

We apply a learning-to-forecast experiment following the approach of Assenza et al. (2014). The general setup is as follows: subjects in the laboratory are randomly divided in groups of 7. Subjects either take the role as a professional forecaster or as a central bank forecaster. Professional forecasters are employed at the forecasting department of a company which needs predictions about future inflation as input for the

\[n_{t}^{h} = \delta n_{t-1}^{h} + (1 - \delta) \frac{\exp (\gamma U_{t-1}^{h})}{\sum_{j=1}^{4} \exp (\gamma U_{t-1}^{j})}
\]

Equation (4.4) lists the set of heuristics available to the agents when forming their expectations. The variable \(x_{t-1}^{av}\) denotes the average of past output gaps. Once heuristics are used, the agents weight their past performance following equation (4.5), with \(\eta\) denoting the parameter describing the preference for the past. Equation (4.6) updates the probability of using heuristic \(h\) when forecasting for period \(t + 1\). Notice that \(\gamma\) captures the sensitivity of agents to heuristic performances and \(\delta\) denotes the fraction of agents that in period \(t\) stick to the heuristic they used in period \(t - 1\). Then, using, (4.7) the expectation are aggregated and \(\hat{E}_{t} x_{t+1}\) is determined. The calibration of the Heuristic Switching Model follows Assenza et al. (2014), i.e., we set \(\eta = 0.7\), \(\delta = 0.9\), and \(\gamma = (0.4 \cdot 4^2) = 6.4\).\(^{40}\)

\[\hat{E}_{t} x_{t+1} = E_{t}^{\text{ada}} x_{t+1} n_{t}^{\text{ada}} + E_{t}^{\text{wtr}} x_{t+1} n_{t}^{\text{wtr}} + E_{t}^{\text{str}} x_{t+1} n_{t}^{\text{str}} + E_{t}^{\text{laa}} x_{t+1} n_{t}^{\text{laa}}
\]

\(^{40}\)We multiply \(\gamma\) by \(4^2\) relative to the calibration of Assenza et al. (2014) because we use a Heuristic Switching Model with quarterly rather than annualized data.
management’s operative decisions. Professional forecasters’ job is to generate these inflation forecasts and to communicate them to the management. Professional forecasters are provided with some qualitative knowledge of the economy, the direction of the feedback on their expectations (i.e. positive feedback), and a public central bank projection. The professional forecasters’ payoffs are determined according to their forecasting performance, measured by the following payoff function from Assenza et al. (2014):

\[
\Pi_{fc,j} = \frac{100}{1 + |\pi_{t+1}^{f,c,j} - E_{t}^{f,c,j}_{\pi_{t+1}}|}.
\]

The central bank forecaster is employed at the forecasting department of the central bank and the central bank forecaster’s job, too, is to generate inflation forecasts, which we denote \( E_{t}^{cbf,\pi_{t+1}} \). However, this forecast does not enter the vector \( E_{t}\pi_{t+1} \) from which the aggregate inflation expectation is determined. The incentives for the central bank forecaster in determining her inflation forecasts, therefore, are different from the incentives of professional forecasters and also differ strongly between treatments, as explained below.

Whether a subject is assigned the role of a professional forecaster or a central bank forecaster is the outcome of a preliminary stage (henceforth: Stage I). Independent of the treatment, in Stage I, all subjects of a group play 8 initial rounds of the experiment as professional forecasters in the absence of any public central bank inflation projection. To level the playing field, all participating subjects are presented with an identical three-period history (for periods \( t = -2, t = -1, \) and \( t = 0 \)) for inflation, the output gap and the interest rate, which initializes the economy off the steady state.

Subjects are ranked according to their relative forecasting performance. The role of the central bank forecaster for the remaining rounds of the experiment (period 9-37) is assigned to the best ranked subject. This is common knowledge.

Apart from the control treatment (Treatment 1), as the economy enters period 9, at the beginning of each period the central bank publishes an official central bank inflation projection, denoted by \( E_{t}^{pub,\pi_{t+1}} \). Depending on the treatment, this official central bank inflation projection is produced either by the central bank forecaster so that \( E_{t}^{cbf,\pi_{t+1}} = E_{t}^{pub,\pi_{t+1}} \) (Treatment 2) or by a computer algorithm (Treatments 3 and 4). In any case, the central bank is provided with superior information that can

---

41This is a common assumption shared among all studies cited in Section 4.2 except for Adam (2007), who does not provide any information about the working of the economy. We abstract from providing the subjects with the fully quantified set of equations, as real world economists neither know the full set of specific equations nor their quantitative relations in the real world economy.

42The history is displayed in Figure 4.3 in Appendix 4.C. It comprises the first three observations.
be used in the forecasting process. Professional forecasters are subsequently presented with the official central bank projection before they submit their own forecasts.

Since we are interested in the expectations channel of monetary policy both in normal times and in times when the zero lower bound on the nominal interest rate may become binding, in the spirit of Arifovic and Petersen (2015), there is a series of negative fundamental shocks, which hit the economy. In this experiment, the series of fundamental shocks appears in a very late stage of the experiment, in particular it starts in period 29 and prevails for four periods. This series of fundamental shocks is chosen such that it is likely to induce a liquidity trap and therewith the possibility of a deflationary spiral.

With this subdivision, the economy is fairly stable in the first part of the actual experiment (periods 9-28; henceforth: Stage II) and it is investigated whether central bank forward guidance can influence private-sector expectations and actively stabilize the economy. In the latter part of the experiment (periods 29-37; henceforth: Stage III), on the other hand, it is investigated whether the central bank can prevent or reverse a deflationary spiral by means of forward guidance.

The timing of the experiment is as follows: In \( t = 1, \ldots, 8 \) (Stage I), all subjects submit their inflation forecast \( E^{fc,j}_{t} \pi_{t+1} \) simultaneously. In \( t = 9, \ldots, 37 \) (Stages II and III), first the central bank forecaster submits her forecast \( E^{cbf}_{t} \pi_{t+1} \). With the exception of Treatment 1, afterwards the official central bank projection \( E^{pub}_{t} \pi_{t+1} \) is published. Professional forecasters observe the public inflation projection of the central bank and subsequently submit their own inflation forecasts \( E^{fc}_{t} \pi_{t+1} \). After all professional forecasters have submit their forecast, the aggregate inflation forecast \( \bar{E}_{t} \pi_{t+1} \) is determined and the values for the variables in period \( t \) are computed. The economy proceeds to the next round.

While the objective of the professional forecasters remains the same in all treatments throughout the whole experiment, the objectives of the central bank forecaster differ across treatments. These differences are described in detail in the following subsection.

### 4.3.3 Treatments

We consider four treatments in this experiment.

**Treatment 1: Control treatment**

In this treatment, the control treatment, no central bank projections are published, i.e., there is no central bank forward guidance. The central bank forecaster produces
forecasts, but the forecasts of the central bank forecaster are not publicly shown. Therefore, the central bank forecaster has no ability to influence the professional forecasters’ expectations and thereby no incentive to produce strategic forecasts.

In each period, the central bank forecaster is provided with a data-driven forecast \( E_{t}^{ddf} \pi_{t+1} \), which she can choose to consider or to ignore when forming her own inflation forecasts. The data-driven forecast uses model equations (4.1) to (4.7) and data up to period \( t - 1 \) to predict what level of inflation is likely to prevail in period \( t + 1 \). However, when the data-driven forecast is made, it is not yet known what aggregate inflation expectations formed in periods \( t \) and \( t + 1 \) will be, which are important determinants of inflation in period \( t + 1 \). These expectations therefore need to be modeled. This is done by assuming a Heuristic Switching Model for inflation expectations analogous to equations (4.4) to (4.7).

In this treatment, all subjects (including the central bank forecaster) share the same incentives arising from equation (4.8); i.e., even the central bank forecaster’s goal is simply to predict inflation accurately.

Treatment 2: Forward Guidance from a Human Central Bank Forecaster

In this treatment, the central bank forecaster publishes official central bank inflation projections, i.e., \( E_{t}^{pub} \pi_{t+1} = E_{t}^{cbf} \pi_{t+1} \). Hence, there is central bank forward guidance. The other subjects of her group are informed (i) that there is a central bank forecaster publishing official central bank inflation projections in this economy, (ii) that the central bank forecaster is the subject that predicted inflation best in Stage I, (iii) that the central bank forecaster has additional information about the economy without specifying this any further, and (iv) that the central bank has an inflation target without quantifying this target.

In this setup, the central bank forecaster may have an ability to influence the professional forecasters’ expectations and thereby may have an incentive to produce strategic projections. Note that it is not a priori clear whether it is optimal for professional forecasters to use the published projection when forming their own forecasts or to ignore it. This depends on what a subject believes about how the central bank forms its projection and about how other subjects form their expectations.\(^{43}\)

\(^{43}\)For example, it is optimal for a subject to predict exactly the published forecast when she thinks that the central bank is able to foresee what the median forecast will be and that the central bank will use all its information to publish a truthful forecast. If, on the other hand, the subject believes that the central bank is not good in predicting the median forecast of the professional forecasters or if she believes that the central bank is more concerned with strategically trying to steer the economy rather than publishing accurate projections, then the subject is better of ignoring the published forecast.
4.3. Experimental Design

As in Treatment I, the central bank forecaster is provided with a data-driven forecast. The data-driven forecast algorithm is however somewhat different than in the control treatment. Now it must account for the potential self-fulfilling properties a published central bank projection can have on the economy. This works as follows: when the central bank publishes a projection, this is likely to affect, to some extent, the inflation expectations of the professional forecasters. Since the main determinant of current inflation is inflation expectations, aggregate expectations of professional forecasters in turn affect realized inflation. This implies that when the published projection is high, this is likely to also lead to somewhat higher aggregate inflation expectation, and therefore to a higher inflation realization. The task of the data-driven forecast in this treatment is to find, given this possibly self-fulfilling feedback mechanism, those expectation values that, when published, are most likely to come true. This is done by including a fifth heuristic to the Heuristic Switching Model that is used to model inflation expectations. This heuristic can be termed “Follow the Published projection” and is defined by

$$E_{t}^{fp} \pi_{t+1} = E_{t}^{pub} \pi_{t+1}.$$  

The fitness of this heuristic is calculated analogously to equation (4.5), and 5 heuristics are now considered when calculating fractions as in equation (4.6). This implies that the data-driven forecast assumes that aggregate inflation expectations will be more in line with the published projection when the past published projections have been relatively accurate. The data-driven forecast then performs a grid search to choose the forecast that is most likely to be accurate, taking account of the effects that such a forecast are likely to have on aggregate expectations.

Moreover, the central bank forecaster is provided with information about which aggregate inflation expectations for the following period need to prevail for inflation to be (in expectations) at target level $\pi^T$ already in the current period. This specific aggregate inflation expectation is calculated by doing a grid search on $\bar{E}_{t} \pi_{t+1}$ in the model defined by equations (4.1) to (4.7). This information tells the central bank forecaster in what direction she should steer aggregate expectations about next period in order to get closer to her inflation target in this period. We label this piece of information “required for target” and denote it by $E_{t}^{rf} \pi_{t+1}$.

Finally, the central bank forecaster is presented with a credibility index $I_{t}^{cred}$, given

\[44\text{Since the published forecast about } t + 1 \text{ affects realizations in period } t, \text{ and the published forecast about } t + 2 \text{ affects realizations in } t + 1, \text{ an assumption needs to be made about what the published forecast about } t + 2 \text{ will be, in order to evaluate whether the forecast made about } t + 1 \text{ is likely to come true. The data driven forecast simply assumes here that the published forecast about } t + 2 \text{ will be the same as the published forecast about } t + 1. \text{ Since both inflation and the published forecast turn out to be highly persistent, also in our experimental sessions, this is arguably not a very restrictive assumption.} \]
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by

\[ I_t^{cred} = \frac{1}{4} \sum_{i=1}^{4} \left[ \frac{1}{6} \sum_j \exp \left( -3 \cdot \left( E_{t-i}^{pub} - E_{t-i}^{f,c,j} \right)^2 \right) \right], \]

where, in the spirit of Cecchetti and Krause (2002), the central bank’s credibility towards professional forecaster \( j \) is defined by the distance between the central bank’s inflation projection and the inflation forecasts of professional forecaster \( j \). The credibility index is based on the distance between the projections of the central bank forecaster and of the individual professional forecasters in the last four periods. We take the exponent of the negative of each squared distance in order to make sure that the index is between 0 and 1. The scale parameter 3 is calibrated based on pilot data in such a way that the index is not too easily close to 1. When the index equals 1, every individual forecaster predicted exactly the same as the central bank forecaster in each of the last four periods. When all professional forecasters made forecasts that were quite far away from the projections of the central bank forecaster in the last four periods, the index is close to zero.

Having available these three sources of information, the central bank forecaster must decide to what extent she follows the data-driven forecast or to what extent she publishes a strategic projection based on the “required for target”, taking into account her credibility.

The central bank forecaster’s objective, in this treatment, is twofold; i.e., there are two payoff functions. On the one hand she has to stabilize inflation, i.e., minimize the deviations of inflation from her target values, while on the other hand her inflation projections have to remain maximally credible, as measured by the credibility index. We consider central bank credibility explicitly, as it is of utmost importance for the functioning of monetary policy and thereby enjoys a lot of attention of monetary policy makers (Blinder, 2000; Bordo and Siklos, 2014). In line with this strategy, Gomez-Barrero and Parra-Polania (2014) present a theoretical model of strategic central bank forecasting which explicitly considers reputational concerns of central bank credibility in the central bank’s loss function. The payoff functions of the central bank forecaster

\[ \text{In the econometric analysis, additionally we consider two alternative approaches to measure credibility. First, credibility as the weight of the information content from public announcements of the central bank (in our environment the public forecasting signals) attached by professional forecasters to their expectation formation process (Bomfin and Rudebusch 2000). Second, as subjectively elicited measure, asking them a Lickert question at the beginning, in the middle, and at the end of the actual experiment.} \]

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have the following form:

\[
\begin{align*}
\Pi_{\text{stability}}^{\text{chf}} &= \max \left( 0, 100 - 44 \left( \pi_t - \pi^T \right)^2 \right), \\
\Pi_{\text{credibility}}^{\text{chf}} &= \max \left( 0, 100 - 400 \left( 1 - I_t^{\text{cred}} \right)^2 \right).
\end{align*}
\]

Equation (4.10) is calibrated such that in each period the central bank forecaster receives a payoff of zero for stability if inflation deviates from target by more than 1.5 percentage points and receives a payoff of zero for credibility of the projection if the credibility index is below 0.5. At the end of the experiment, one of these two objectives is chosen randomly by the computer and the central bank forecaster is paid according to the total payoff of the chosen objective. The randomization eliminates any incentives to focus on only one of the two goals or to strategically play one goal of another in any other way.

Treatment 3: Forward Guidance from a “good” Computerized Central Bank Forecaster

In this treatment, the official central bank projection is published by a computer algorithm, i.e., again there is central bank forward guidance. The computer algorithm publishes strategic inflation projections, which intend to steer the economy back to target via the manipulation of private-sector expectations. The extent to which the projections try to manipulate rather than to purely inform private-sector expectations depends primarily on the current state of the economy (in particular, whether previous inflation was (i) close to, (ii) above or (iii) below its target value) and secondarily on the credibility of recent central bank inflation projections.

The computer algorithm works as follows: (i) If previous inflation was close to the target (within ±0.5 percentage points), the central bank tries to initiate long term coordination on its inflation target through projections equal to the inflation target. (ii) If previous inflation was far above target (for more than 0.5 percentage points), the algorithm makes a trade-off between building credibility and steering the economy. If past projections have been only little credible, the algorithm aims at building credibility through accurate inflation projections by publishing projections close to the data driven forecast (which is calculated in the same way as in Treatment 2). If projections have already been credible, the algorithm leans more towards the “required-for-target” information. (iii) If previous inflation was far below target (for more than 0.5 percentage points) the economy faces the risk of a binding zero lower bound and a deflationary
spiral. Now, building up credibility by following the data-driven forecast becomes dangerous as the data-driven forecast may predict a deflationary spiral. Therefore, the algorithm balances forecasting the target with forecasting the last observed inflation level, where the latter can improve on credibility without amplifying the downturn in inflation. The weight on the last observed value is relatively high when there is a downward trend in inflation, because then it might not be credible that inflation will suddenly go up by much. On the other hand, if there is an upward trend in inflation it might be more credible that inflation will go up more, so the computer algorithm can put more weight on the target.

The explicit algorithm is spelled out below:

“close to target”: \( E_t^{\text{pub}} \pi_{t+1} = \pi^T \)

“sufficiently above target”: \( E_t^{\text{pub}} \pi_{t+1} = I_t^{\text{cred}} \ast E_t^{\text{rft}} \pi_{t+1} + (1 - I_t^{\text{cred}})E_t^{\text{ddf}} \pi_{t+1} \)

“sufficiently below target”: if \( \pi_{t-1} < \pi_{t-2} \): \( E_t^{\text{pub}} \pi_{t+1} = 0.5 \pi^T + 0.5 \pi_{t-1} \)

if \( \pi_{t-1} > \pi_{t-2} \): \( E_t^{\text{pub}} \pi_{t+1} = 0.8 \pi^T + 0.2 \pi_{t-1} \)

For reasons of comparability, in this treatment, the central bank forecaster subject takes the same role as in Treatment 1, however, she is not provided with any additional information. This allows us to learn more about the expectation formation process of the central bank forecaster, especially the dependence on additional information for the forecasting performance and the expectation formation process of the central bank forecaster in case this subject cannot interact with the economy.

**Treatment 4: Forward Guidance from a “bad” Computerized Central Bank Forecaster**

This treatment is similar to Treatment 3, but with a different computer algorithm in Stage II. In Stage II of this treatment, the computer algorithm publishes inflation projections, which it randomly draws from a uniform distribution with support from -5 to 5, i.e., \( E_t^{\text{pub}} \pi_{t+1} \sim \text{Unif}(-5, 5) \). The support is chosen according to the support of realized inflation throughout the first three treatments of this experiment. In Stage III of this treatment, the computer algorithm is the same as in Treatment 3. This twist after Stage II allows us to analyze the influence of credibility on the central bank’s ability to stabilize the economy in times of severe economic stress.
4.3.4 Hypotheses

Our experimental design allows us to address several hypotheses. The following hypotheses distinguish between “informative” and “random” forward guidance. We consider forward guidance informative, if the central bank inflation projection lies systematically (i.e. most of the time) inside the interval between the data-driven forecast and the “required for target” information. Analogously, forward guidance is considered “random,” if the central bank inflation projection lies systematically (i.e. most of the time) outside the interval between the data-driven forecast and the “required for target” information. According to this criterion, forward guidance from a human central banker forecaster (Stages II and III of Treatments 2) and from the “good” computerized central bank forecaster (Stages II and III of Treatment 3 and Stage III of Treatment 4) are considered “informative” and forward guidance from the “bad” computerized central bank forecaster (Stage II of Treatment 4) is considered “random.”

Hypothesis 1: Informative forward guidance stabilizes the economy (a) in normal times and (b) in times of severe economic stress; random forward guidance does not.

Although from an empirical point of view published central bank inflation projections seem beneficial for macroeconomic stability (Chortareas et al., 2002), from a theoretical point of view, the effects of published central bank inflation projections on macroeconomic stability are generally ambiguous and depend on the quality of the projections (see Geraats (2002) for an extensive survey).

For instance, Gersbach (2003) and Jensen (2002) find that publishing inflation projections may be destabilizing, as it carries information about future shocks, which are internalized by private-sector expectations and therefore cannot be stabilized by the central bank anymore. Geraats (2002), Amato and Shin (2006), Walsh (2007) argue that central bank communication can be destabilizing as potentially noisy public information crowds out accurate private information. By contrast, Tarkka and Mayes (1999) find that publishing central bank projections conveys information about the central bank’s targets as well as about the central bank’s belief about private-sector expectations, which enhances the predictability of monetary policy actions and reduces output volatility. Along similar lines, Geraats (2005) finds that publishing inflation pro-

46For the central bank forecaster subjects, more than 85% of all public central bank projections lie within the required interval; for the “good” computer algorithm it is more than 80% (and above 90% if the predictions of the target inflation rate when the economy is “close to target” are considered as well). For the “bad” computer algorithm, less than 4% of all public central bank projections lie within the required interval.
 CHAPTER 4. THE STABILIZING ROLE OF FORWARD GUIDANCE

projections reduces the inflation bias. In a standard New Keynesian model with learning, Eusepi and Preston (2010) and Ferrero and Secchi (2010) find a stabilizing role of public central bank inflation projections through an anchoring effect on private-sector inflation expectations in normal times. Goy et al. (2016) reach similar conclusions in a New Keynesian model with boundedly rational heterogeneous agents, however, not only in normal times but also in times of severe economic stress. At the zero lower bound, the authors show that the publication of inflation projections can lower the likelihood of a deflationary spiral if central bank projections are sufficiently credible.

Hypothesis 2: Informative forward guidance anchors private-sector inflation expectation; random forward guidance does not.

In their seminal theoretical contribution, Morris and Shin (2002) show that public central bank information can act as a coordination device by anchoring private-sector expectations and thereby reduce the dispersion of private-sector expectations. Empirical support for such an anchoring effects for expectations (especially in the context of public central bank projections) is given by Hubert (2014) for the Federal Reserve, by Fujiwara (2005) for the Bank of Japan, and by Ehrmann et al. (2012) for 12 advanced economies (including the former two).

Hypothesis 3: Informative forward guidance increases the forecasting accuracy of all market participants; random forward guidance does not.

Dale et al. (2011) show in a stylized model of imperfect knowledge and learning that if the central bank has an informational advantage with respect of the functioning of the economy and if this informational advantage is perceived correctly by the private sector, publishing inflation projections can improve the accuracy of private-sector expectations. By contrast, if the central bank projections are imprecise and noisy, the publication of these projections might unleash distracting forces which deteriorate the accuracy of private-sector expectations.

Hypothesis 4: The degree of “strategic-ness” of a public central bank projection depends on its credibility.

Recent empirical evidence gives rise to the assumption that central bank projections are not just a purely informational tool but are also used as a strategic instrument to influence private-sector expectations, which manifests in biased projections. Indicative
evidence for such a claim is presented by Romer and Romer (2008), who find that the forecasting accuracy of the Federal Open Market Committee (FOMC) is systematically lower relative to the projection of their own research staff (the so-called Greenbook projections), even though these forecasts are available to the FOMC when publishing their projections. For a sample of ten inflation targeting central banks (Australia, Canada, Chile, Czech Republic, Korea, New Zealand, Mexico, Norway, Poland, and Sweden) Charemza and Ladley (2016) find that central bank projections are biased towards their inflation targets.

From a theoretical point of view, Jensen (2016) shows that - if credible - optimal inflation projections are indeed misleading, whereas non-misleading inflation projections are time-inconsistent in an augmented Barro-Gordon type game featuring a New Keynesian sticky price model. In a related augmented Barro-Gordon type game, Gomez-Barrero and Parra-Polania (2014) show that the degree of the inflation projection bias is endogenous, as it optimally solves the trade-off between the benefits (i.e. the enhanced stabilizing effects) and the costs (i.e. the loss of credibility for the inflation projection) of the strategic bias. Empirical evidence from six inflation targeting central banks (Brazil, Canada, England, Iceland, New Zealand, and Sweden), presented by Gomez-Barrero and Parra-Polania (2014), turns out to be consistent with their theoretical predictions.

In the context of this experiment, the degree of “strategic-ness” is measured as the relative weight given to the “required for target” over the data-driven forecast in the published inflation projection. This measure is consistent with the notion of an intentional over- or underreporting of the projected level of inflation.

**Hypothesis 5:** (a) The credibility of the central bank projections depends positively on their past performance and (b) the ability of the central bank to stabilize the economy by means of its projections depends positively on the past credibility of the central bank projections.

In a survey among 84 central bank presidents worldwide, Blinder (2000) finds that the most important matter for credibility is believed to be a consistent track record. With respect to inflation projections and projection of inflation in particular, such a consistent track record is established primarily by a sustained projection accuracy. Loss in credibility of the central bank’s projections can therefore be attributed to a (systematic) failure to produce accurate projections (Mishkin, 2004). Following this line of reasoning, also the two most closely related studies to this chapter, Goy et al. (2016) and Mokhtarzadeh and Petersen (2016), determine central bank credibility by
looking at past central bank forecasting performance.

A good deal of credibility, in turn, is necessary for forward guidance to be effective in stabilizing the economy (Filardo and Hofmann, 2014). A particularly illustrative example in this respect is provided by Svensson (2015) for the Swedish case (although with respect to interest path projections). While credible projections remarkably influenced market behavior towards stabilization in 2009, in 2011 non-credible projections left the market unimpressed and without any response in market behavior.

4.3.5 Experimental Procedure

Each treatment of this experiment consists of six economies with seven subjects each. Thus, the experiment has a total of $4 \times 6 \times 7 = 168$ subjects. Subjects were recruited from a variety of academic backgrounds using ORSEE (Greiner, 2015). The subject population comprised undergraduate students (64%), graduate students (34%), and non-students (2%). Subjects were mostly from the natural sciences (61%) and the social sciences (16%). Around two thirds of the subjects were male (62%) and one third were female (38%). During the experiment, subjects earned experimental currency units (ECU) according to their respective payoff functions. At the end of the experiment, subjects were paid €1 for every 85 ECU; that is, each ECU paid approximately €0.012. The average payment was €31.66. The experimental software was programmed in oTree (Chen et al., 2016). The experiment was conducted in May and June 2016 at the experimental lab of the Technische Universität Berlin.

4.4 Macroeconomic Results

In this section, we address Hypotheses 1, 2, and 3, i.e. we analyze the role of central bank forward guidance for the macroeconomy. To fix ideas, first we juxtapose the median economic dynamics arising from the actual experiment in each of the four treatments and their statistical properties.

Figure 4.1 shows the median evolution of inflation, the output gap, and the interest rate for all four treatments; Treatment 1 is depicted by the solid lines, Treatment 2 by the dashed lines, Treatment 3 by the dotted lines, and Treatment 4 by the dashed-dotted lines.47 The figure shows that all four treatments share a common pattern for the evolution of the macroeconomy over much of the 37 rounds of the experiment. First, there is slow convergence towards the steady state. Second, starting in period 29 (the second vertical, gray line), a deep recession takes place, which drives the economy.

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47Figures 4.4 to 4.7 in Appendix 4.C show all 6 individual economies for each treatment, respectively.
Figure 4.1: Median responses of inflation (upper panel), the output gap (middle panel), and the interest rate (lower panel) for all four treatments. For each treatment, median responses are generated by taking the median of each inflation, the output gap, and the interest rate from all six economies at each period $t = 1, \ldots, 37$.

Note that for Treatment 1 the median interest rate leaves the zero lower bound despite a deflationary recession. This abnormal artifact is a result from the aggregation procedure (median) as three economies of Treatment 1 remain at the zero lower bound, while three economies leave the zero lower bound (see Figure 4.4 in Appendix 4.C).
towards the zero lower bound at which it remains for an extended period of time. However, while median economies recover from the recession under central bank forward guidance (Treatments 2-4), the median economy produces a deflationary spiral in the absence of central bank forward guidance (Treatment 1).

Although, at first sight, the general pattern looks very similar across all four treatments (with the exception of Stage III), we find considerable effects of central bank forward guidance on the economy. Tables 4.9 to 4.12 in Appendix 4.C summarize descriptive statistics for all 24 economies in Treatments 1, 2, 3, and 4, respectively.

Comparing the descriptive statistics shows that in the preliminary stage (i.e., Stage I) medians and variances of all three macroeconomic variables inflation, the output gap, and the interest rate are very close across treatments. We test for equality of the medians and variances for pairwise comparison of treatments using the non-parametric Mann-Whitney-Wilcoxon-test and Siegel-Turkey-test, respectively. The results of these tests are presented in Tables 4.13 and 4.14 in Appendix 4.C. They show that the Null hypothesis of equality in medians and variance cannot be rejected for any pairwise comparison of treatments.

In Stage II, by contrast, median and variance of all three variables in Treatments 2 and 3 (i.e., under informative forward guidance) are considerably and statistically significantly closer to the (determinate) target steady state compared to Treatments 1 and 4 (i.e., without informative forward guidance). Hence, informative forward guidance has a significant influence on private-sector expectations which helps reduce the economy’s volatility and drives it closer to the steady state. Random forward guidance (Treatment 4), by contrast, has rather averse effects on the economy. We find a marginal but statistically significant increase in median inflation and the median interest rate compared to Treatment 1, while a slight reduction in the median output gap is statistically insignificant. Moreover, random forward guidance is without significant effect on the volatility of the economy.

During severe economic stress (Stage III), informative forward guidance (in Stage III this is Treatments 2-4) keeps the economy closer to the steady state and strongly reduces the volatility of the economy, as it strongly reduces the occurrence of deflationary spirals.

Taken together, these results point towards an important role of forward guidance for the stability and predictability of the macroeconomy, which we scrutinize more
4.4. Macroeconomic Results

In Section 4.4.1 we analyze the stabilizing role of forward guidance in normal times (Hypothesis 1(a)), in Section 4.4.2 we focus on the stabilizing role of forward guidance at the zero lower bound (Hypothesis 1(b)). In Section 4.4.3 we analyze the anchoring effect of forward guidance (Hypothesis 2). Subsequently, in Section 4.4.4 we study the influence of forward guidance on the predictability of the economy (Hypothesis 3).

4.4.1 Macroeconomic Stability in Normal Times

In this section, we focus on Hypothesis 1(a), i.e. we analyze in more detail the stabilizing role of central bank forward guidance for the economy in normal times, i.e., we focus entirely on Stage II. Macroeconomic stability is of utmost importance, as it can be directly linked to welfare in the economy. Woodford (2002) shows that minimizing the squared deviations of inflation and the output gap from zero, maximizes expected household utility and thereby welfare. Consequently, for each experimental economy \(i = 1, \ldots, 24\), we evaluate macroeconomic stability by the mean squared deviations of inflation and the output gap from zero

\[
S^\pi_i = \frac{1}{20} \sum_{t=9}^{28} \pi_t^2, \\
S^x_i = \frac{1}{20} \sum_{t=9}^{28} x_t^2.
\]

The lower \(S^\pi_i\) and \(S^x_i\) the more stable the economy \(i\). The results are summarized in Table 4.1. The last column of Table 4.1 shows the average mean squared error for each respective treatment. Informative forward guidance (Treatments 2 and 3) dramatically reduces the average mean squared error for inflation by two thirds and the output gap by one third. These differences are statistically significant: the \(p\)-values of the Mann-Whitney-Wilcoxon-test for pairwise comparisons of Treatment 1 with Treatments 2 and 3 are \(p(s^\pi_{T1}, s^\pi_{T2}) = 0.0152\) and \(p(s^\pi_{T1}, s^\pi_{T3}) = 0.0087\) for inflation and \(p(s^x_{T1}, s^x_{T2}) = 0.0931\) and \(p(s^x_{T1}, s^x_{T3}) = 0.0649\) for the output gap. Random forward guidance, by contrast, has no statistically significant effect on macroeconomic stability, i.e., \((p(s^\pi_{T1}, s^\pi_{T4}) = 0.2403\) and \(p(s^x_{T1}, s^x_{T4}) = 0.3095\). The stabilizing role of informative forward guidance manifests itself impressively through a much faster convergence of inflation towards the steady state of the economy. In Treatments 2 and 3, inflation reaches the close neighborhood of the steady state, say an interval of ±25 basis points around the steady state, on average within 5 periods. In Treatments 1 and 4, time
Table 4.1: Stage II mean squared deviations of inflation and the output gap. The table shows the stability measures given by equations (4.11) and (4.12) for all 24 economies, as well as their respective treatment averages.

to convergence triples, with a third of the economies not reaching convergence at all during Stage II. All these results carry over, if stability is measured by the squared deviation from target rather than from zero.

The stabilizing role of informative forward guidance becomes even more pronounced when taking into consideration the influence of the first stage developments on the starting point of Stage II. In Stage I, Treatment 1 economies are on average at least as stable (measured analogously to (4.11) and (4.12)) as Treatment 2 and 3 economies. Although not statistically significant, they hand over the economy to Stage II even at slightly lower mean and median levels of inflation with the consequence that, if at all, Treatments 2 and 3 face a marginally unfavorable situation upon entering Stage II.

To account for the influence of Stage I stability on Stage II stability, for each economy $i$ we normalize the Stage II mean squared error of inflation and the output gap by their respective mean squared errors from Stage I

\begin{align}
R_i^\pi &= \frac{S^\pi_i}{\frac{1}{8} \sum_{t=1}^{8} \pi^2_t}, \\
R_i^x &= \frac{S^x_i}{\frac{1}{8} \sum_{t=1}^{8} x^2_t}.
\end{align}

The results are presented in Table 4.15 in Appendix 4.C. First, the table implies that for
each of the 24 economies medians are lower for all variables in Stage II relative to Stage I, which manifests in values smaller than unity. This results, however, is not surprising as the Taylor rule slowly drives the economy towards the steady state. Second, the table generally confirms our results from above. Informative forward guidance strongly helps stabilize the economy with respect to inflation and output gap relative to the absence of forward guidance, albeit not statistically significantly for the output gap in the case for Treatment 3. In the latter case the p-value of the Mann-Whitney-Wilcoxon-test is \( p(R^\pi_{T1}, R^\pi_{T3}) = 0.1797 \). The remaining p-values are \( p(R^\pi_{T1}, R^\pi_{T2}) = 0.0411 \) and \( p(R^\pi_{T1}, R^\pi_{T2}) = 0.0043 \) for Treatment 2 and 3 inflation and \( p(R^\pi_{T1}, R^\pi_{T2}) = 0.0649 \) for the Treatment 2 output gap. Concerning random forward guidance, the picture changes somewhat. Relative to the Stage I development, random forward guidance stabilizes inflation slightly better compared to no forward guidance at all, but stabilizes the output gap less effectively. However, neither of these differences is statistically significant, i.e., \( p(R^\pi_{T1}, R^\pi_{T3}) = 0.4848 \) and \( p(R^x_{T1}, R^x_{T4}) = 0.5887 \). Again, all these results carry over, if stability is measured by the squared deviation from target rather than from zero.

The analysis above implies that informative forward guidance is an effective instrument to increase welfare through its stabilizing role in the economy. Random forward guidance, by contrast, remains without statistically significant effects on stabilization. As a result, the above analysis confirms Hypothesis 1(a).

4.4.2 Forward Guidance at the Zero Lower Bound

Now, we focus on Hypothesis 1(b), i.e. we analyze the impact of informative forward guidance in times of severe economic stress. To do so, we look at Stage III (periods 29-37) of the experiment.\(^{49}\) Between periods 29 and 32, a series of severe shocks to the output gap (\( \varepsilon_{t} \) takes a value of \(-2.5\% \) annually in periods \( t = 29, \ldots, 32 \)) hits all 24 economies alike. Figure 4.1 and Figures 4.4 to 4.7 in Appendix 4.C show the reaction of the macroeconomies to these shocks. In each case, a deflationary recession takes place, which drives the economy to the zero lower bound on the nominal interest rate. The severity of the economic downturn, however, can be mitigated when the central bank conducts informative forward guidance. This can be seen from Table 4.2 where we summarize important key indicators describing the median severity of the economic downturn in each of the four treatments. In the results description below, \( p \)-values of Mann-Whitney-Wilcoxon-tests are reported only if differences in medians are statistically significant.

\(^{49}\) Be reminded that in Stage III of Treatment 4 the public inflation projection is produced by the “good” computer algorithm instead of the random number generator.
Table 4.2: Important key indicators for Stage III. The table shows treatment medians of key indicators describing the severity of the recession and the accompanying liquidity trap in Stage III.

Table 4.2 shows that informative forward guidance on average halves the median time spent at the zero lower bound, from 5 periods in Treatment 1 to less than 2.5 periods on average in Treatments 2-4. Secondly, the length of the recession is significantly reduced from 8 periods in Treatment 1 to less than 4 periods on average in Treatments 2-4. Also, the depth of the recession radically reduces in the presence of informative forward guidance. We measure the depth of the recession by comparing the latest pre-crisis output gap with the largest negative output gap during the crisis. In Treatment 1, the median depth is a loss in output gap of approximately -240 percent, whereas this loss is around -4 percent on average for Treatments 2-4. Prices, in all economies, fall, i.e., there is deflation. However, with 6.5 periods on average in Treatments 2-4 median deflation episodes are reduced by 1.5 periods relative to Treatment 1. All qualitative results carry over for pairwise comparisons of Treatment 1 to Treatments 2, 3, and 4.

Despite binding zero lower bounds and prolonged deflationary episodes, deflationary spirals are rare. However, they occur much more often in the absence of informative forward guidance than in the presence of informative forward guidance. In Treatment 1 three out of six economies result in a deflationary spiral after a series of severe fundamental shocks. While deflationary spirals can be avoided successfully in all six economies of Treatment 3, in both Treatments 2 and 4 one out of six economies result in a deflationary spiral. Therefore, forward guidance significantly reduces the occurrence of deflationary spirals.

In the following, we examine the deflationary spiral in economy E11 of Treatment 2 in more detail. We believe that it provides an informative counterfactual that help understand the stabilizing role of forward guidance at the zero lower bound.

We argue that in this particular case the central bank forecaster not only failed to prevent the deflationary spiral, but to a large part powered the deflationary spiral.

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50 According to the NBER, a recession is a drop in economic activity between peak and trough.
Figure 4.2: Time series for data-driven forecast (black solid line), public central bank inflation projection (dashed line), aggregate inflation (dotted line), and the “required for target” information (gray solid line) for all six experimental economies of Treatment 2.

through inadequate central bank forward guidance. Indicative evidence for this assertion can be found in Figure 4.2. The figure shows the time series for the data-driven forecast (black solid line), the required-for-target value (gray solid line), the published central bank projection (dashed line), realized inflation (dotted line), and the inflation
target (horizontal line) for all six economies of Treatment 2. As is apparent from the lower left panel of Figure 4.2, rather than trying to strategically stabilize the economy through publishing over-optimistic inflation projections - that are inflation projections moving into the direction of the inflation target or even the necessary aggregate forecast as prescribed by the “required for target” criterion -, the central bank forecaster followed the advice of the data-driven forecast and thereby publicly predicted the deflationary spiral. The central bank forecasters of the other five economies, by contrast, do not publicly predict a deflationary spiral, but resort to over-optimistic inflation projections and pull the economy out of the slump.

To quantify the above argument, we define a measure of “strategic-ness” of the published central bank projection, denoted by \( SP_t \) below. The measure illustrates in any given period to what extent the published inflation projection follows the data-driven forecast (non-strategic behavior) and to what extend it tries to strategically influence inflation expectations in the direction consistent with the economy’s determinate target steady state. The latter can be measured by how much the central bank uses either the “required for target” tool or the inflation target. In the following, we report results applying the “required for target” tool. All results are robust to applying the inflation target instead. The explicit “strategic-ness” measure takes the following form

\[
SP_t = \frac{E_{t}^{\text{pub}} \pi_{t+1}}{E_{t}^{\text{rft}} \pi_{t+1}} - E_{t}^{\text{ddf}} \pi_{t+1}.
\]

If \( SP_t = 1 \) the published projection coincides with the “required for target” forecast, whereas if \( SP_t = 0 \) the published projection coincides with the data-driven forecast. To quantify the above argument, we define a measure of “strategic-ness” of the published central bank projection, denoted by \( SP_t \) below. The measure illustrates in any given period to what extent the published inflation projection follows the data-driven forecast (non-strategic behavior) and to what extent it tries to strategically influence inflation expectations in the direction consistent with the economy’s determinate target steady state. The latter can be measured by how much the central bank uses either the “required for target” tool or the inflation target. In the following, we report results applying the “required for target” tool. All results are robust to applying the inflation target instead. The explicit “strategic-ness” measure takes the following form

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SP_t = \frac{E_{t}^{\text{pub}} \pi_{t+1}}{E_{t}^{\text{rft}} \pi_{t+1}} - E_{t}^{\text{ddf}} \pi_{t+1}.
\]

If \( SP_t = 1 \) the published projection coincides with the “required for target” forecast, whereas if \( SP_t = 0 \) the published projection coincides with the data-driven forecast. To quantify the above argument, we define a measure of “strategic-ness” of the published central bank projection, denoted by \( SP_t \) below. The measure illustrates in any given period to what extent the published inflation projection follows the data-driven forecast (non-strategic behavior) and to what extent it tries to strategically influence inflation expectations in the direction consistent with the economy’s determinate target steady state. The latter can be measured by how much the central bank uses either the “required for target” tool or the inflation target. In the following, we report results applying the “required for target” tool. All results are robust to applying the inflation target instead. The explicit “strategic-ness” measure takes the following form

\[
SP_t = \frac{E_{t}^{\text{pub}} \pi_{t+1}}{E_{t}^{\text{rft}} \pi_{t+1}} - E_{t}^{\text{ddf}} \pi_{t+1}.
\]

If \( SP_t = 1 \) the published projection coincides with the “required for target” forecast, whereas if \( SP_t = 0 \) the published projection coincides with the data-driven forecast. To quantify the above argument, we define a measure of “strategic-ness” of the published central bank projection, denoted by \( SP_t \) below. The measure illustrates in any given period to what extent the published inflation projection follows the data-driven forecast (non-strategic behavior) and to what extent it tries to strategically influence inflation expectations in the direction consistent with the economy’s determinate target steady state. The latter can be measured by how much the central bank uses either the “required for target” tool or the inflation target. In the following, we report results applying the “required for target” tool. All results are robust to applying the inflation target instead. The explicit “strategic-ness” measure takes the following form

\[
SP_t = \frac{E_{t}^{\text{pub}} \pi_{t+1}}{E_{t}^{\text{rft}} \pi_{t+1}} - E_{t}^{\text{ddf}} \pi_{t+1}.
\]

If \( SP_t = 1 \) the published projection coincides with the “required for target” forecast, whereas if \( SP_t = 0 \) the published projection coincides with the data-driven forecast.

Table 4.3: “Strategic-ness” measure of Equation (4.15) for periods \( t = 31, ..., 37 \) of Stage III (as well as their respective medians and standard deviations) for each of the six economies of Treatment 2

<table>
<thead>
<tr>
<th>Economy</th>
<th>Period</th>
<th>31</th>
<th>32</th>
<th>33</th>
<th>34</th>
<th>35</th>
<th>36</th>
<th>37</th>
<th>Median</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>E7</td>
<td></td>
<td>0.62</td>
<td>0.44</td>
<td>0.58</td>
<td>0.81</td>
<td>0.78</td>
<td>0.73</td>
<td>0.83</td>
<td>0.73</td>
<td>0.14</td>
</tr>
<tr>
<td>E8</td>
<td></td>
<td>0.27</td>
<td>0.89</td>
<td>0.56</td>
<td>0.81</td>
<td>0.93</td>
<td>0.99</td>
<td>1.00</td>
<td>0.89</td>
<td>0.27</td>
</tr>
<tr>
<td>E9</td>
<td></td>
<td>0.41</td>
<td>0.19</td>
<td>0.63</td>
<td>0.80</td>
<td>0.36</td>
<td>0.45</td>
<td>0.31</td>
<td>0.41</td>
<td>0.21</td>
</tr>
<tr>
<td>E10</td>
<td></td>
<td>0.61</td>
<td>0.02</td>
<td>0.09</td>
<td>0.70</td>
<td>1.00</td>
<td>0.72</td>
<td>1.00</td>
<td>0.70</td>
<td>0.40</td>
</tr>
<tr>
<td>E11</td>
<td></td>
<td>−0.05</td>
<td>−0.01</td>
<td>−0.01</td>
<td>−0.00</td>
<td>0.31</td>
<td>−0.24</td>
<td>−0.30</td>
<td>−0.01</td>
<td>0.20</td>
</tr>
<tr>
<td>E12</td>
<td></td>
<td>0.49</td>
<td>0.51</td>
<td>0.39</td>
<td>0.62</td>
<td>0.00</td>
<td>0.10</td>
<td>0.71</td>
<td>0.49</td>
<td>0.26</td>
</tr>
</tbody>
</table>

\( ^{51} \)The index can also take values above unity and below zero. If \( SP_t > 1 \) the published projection lies outside the interval of the data driven forecast and the “required for target” information, on the side of the “required for target”. This implies that the central bank is trying to steer more than necessary to achieve the target. If \( SP_t < 0 \), the published projection lies outside the band of the data driven forecast and the “required for target” information, on the side of the data driven forecast. This

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Table 4.3 presents the measure of “strategic-ness” for the 7 periods after the shocks have died out in all six economies of Treatment 2 along with their median and standard deviation. For all five economies without a deflationary spiral $SP_t$ for $t = 31, \ldots, 37$ is mostly substantially above zero, with median values in the interval $[0.41, 0.89]$, which are always significantly different from zero given the Wilcoxon signed rank test ($p \leq 0.02$ in each case). This implies that in these economies the central bank forecaster does not just follow trends or the data-driven forecast, but that she considerably tries to steer the economy towards target. By contrast, in the economy with deflationary spiral $SP_t$ is mostly very close to zero with a median value of $-0.01$. A median of zero cannot be rejected by the Wilcoxon signed rank test ($p = 0.4$) in this case. This implies that the central bank forecaster is not concerned with steering the economy back to the target, but solely with giving “accurate” projections. Moreover, the negative sign of the “strategic-ness” measure indicates that the central bank forecaster predicts an even more extreme deflationary spiral relative to what is predicted by the data-driven forecast (see Appendix 4.B).

The stabilizing role of forward guidance at the zero lower bound is particularly surprising, since at the zero lower bound an overoptimistic (or strategic) inflation projection must be considered cheap talk. At the zero lower bound, the central bank has no means to actively support the public projection using the interest rate. We believe that the evidence presented in this section is a confirmation of Hypothesis 1(b). Furthermore, our results support the finding by Duffy and Heinemann (2014), that cheap talk can be a very successful strategy for the central bank to achieve its stabilization goals.

### 4.4.3 Anchoring Effect of Forward Guidance

In the following we address Hypothesis 2, i.e. we analyze the anchoring effect of forward guidance. Eusepi and Preston (2010) argue that in economies with potentially self-fulfilling expectations and learning, such as the economy in the present experiment, central bank projections generate macroeconomic stabilization through the anchoring of private-sector expectations on a path consistent with monetary policy. In the absence of inflation projections, expectation may be unanchored and even be inconsistent with monetary policy. Therefore, unlike under rational expectations, the Taylor principle alone does not guarantee macroeconomic stability.

The anchoring effect manifests itself in a lower disagreement among individual pro-
fessional forecasters in the presence of forward guidance relative to the absence of forward guidance. We measure disagreement among individual professional forecasters by the cross-sectional dispersion of individual professional forecasts in each period $t$, using three alternative dispersion measures which are commonly found in the literature; the variance as in Fujiwara (2005), the distance between the highest and the lowest forecast (henceforth: range) as proposed by the FED, and the inter-quartile range of forecasts in any given period as in Ehrmann et al. (2012); Hubert (2014). Table 4.4 presents the median values of all three measures for each of the single economies and the respective averages for each of the four treatments.\footnote{The results are the same for the mean in stead of the median of the dispersion measures.} The table shows that for all three measures considered informative forward guidance (Treatments 2 and 3) reduces the disagreement among individuals roughly by one third. Not surprisingly, the random forward guidance (Treatment 4) increases disagreement considerably, almost doubling the dispersion of individual forecasts. According to the Mann-Whitney-Wilcoxon test the above mentioned differences are highly statistically significant at the 1\% significance level. To account for Stage I influences, analogously to the previous stability analysis, Table 4.16 in Appendix 4.C presents the dispersion measures in Stage II relative to their counterparts in Stage I. The numbers in Table 4.16 confirm the previous results that informative central bank forward guidance more successfully reduces the disagreement among the individual professional forecasters.

To quantify the anchoring effect of public inflation projections, in the spirit of Ehrmann et al. (2012) and Hubert (2014), we resort to a simple regression analysis of the form

$$\sigma_{fc,t} = \text{constant} + \beta_1 PP_t + \beta_2 \sigma_{fc,t-1} + \beta_3 X_{t-1} + \varepsilon_t,$$

where $\sigma_{fc,t}$ is the cross-sectional dispersion of the professional forecasters in period $t$, $PP_t$ is a dummy variable which takes value 1 when a public inflation projection is present and $X_t$ is a vector of macroeconomic controls. The macroeconomic controls $X_{t-1}$ comprise the lagged interest rate, the lagged output gap, and lagged inflation uncertainty defined by $IU_{t-1} = |\pi_{t-1} - \pi_{t-2}|$, which is the absolute error of a random walk forecast (Ahrens and Hartmann, 2015). We expect a positive relationship between lagged inflation uncertainty and cross-sectoral dispersion. The higher lagged inflation uncertainty, the harder the prediction of inflation and thereby the greater the cross-sectional dispersion (Capistrán and Timmermann, 2009; Dovern and Hartmann, 2016). Concerning the remaining control variables, first, we expect cross-sectoral dispersion to be positively influenced by the lagged interest rate. According to the Taylor rule
4.4. Macroeconomic Results

<table>
<thead>
<tr>
<th>Economy</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>Avg</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatment 1</td>
<td>0.06</td>
<td>0.07</td>
<td>0.22</td>
<td>0.07</td>
<td>0.40</td>
<td>0.27</td>
<td>0.18</td>
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<tr>
<td>Treatment 2</td>
<td>0.09</td>
<td>0.01</td>
<td>0.09</td>
<td>0.24</td>
<td>0.04</td>
<td>0.11</td>
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<tr>
<td>Treatment 3</td>
<td>0.15</td>
<td>0.06</td>
<td>0.04</td>
<td>0.02</td>
<td>0.07</td>
<td>0.19</td>
<td>0.09</td>
</tr>
<tr>
<td>Treatment 4</td>
<td>0.45</td>
<td>0.05</td>
<td>0.12</td>
<td>0.41</td>
<td>0.41</td>
<td>2.02</td>
<td>0.58</td>
</tr>
</tbody>
</table>

Dispersion Measure: Range

<table>
<thead>
<tr>
<th>Economy</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>Avg</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatment 1</td>
<td>0.61</td>
<td>0.75</td>
<td>1.30</td>
<td>0.66</td>
<td>1.76</td>
<td>1.42</td>
<td>1.08</td>
</tr>
<tr>
<td>Treatment 2</td>
<td>0.82</td>
<td>0.27</td>
<td>0.65</td>
<td>1.25</td>
<td>0.54</td>
<td>0.84</td>
<td>0.73</td>
</tr>
<tr>
<td>Treatment 3</td>
<td>0.94</td>
<td>0.70</td>
<td>0.55</td>
<td>0.40</td>
<td>0.62</td>
<td>1.13</td>
<td>0.73</td>
</tr>
<tr>
<td>Treatment 4</td>
<td>1.68</td>
<td>0.62</td>
<td>0.85</td>
<td>1.64</td>
<td>1.89</td>
<td>3.56</td>
<td>1.71</td>
</tr>
</tbody>
</table>

Dispersion Measure: Interquartile Range

<table>
<thead>
<tr>
<th>Economy</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>Avg</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatment 1</td>
<td>0.25</td>
<td>0.24</td>
<td>0.39</td>
<td>0.19</td>
<td>0.43</td>
<td>0.60</td>
<td>0.35</td>
</tr>
<tr>
<td>Treatment 2</td>
<td>0.29</td>
<td>0.12</td>
<td>0.30</td>
<td>0.56</td>
<td>0.19</td>
<td>0.30</td>
<td>0.29</td>
</tr>
<tr>
<td>Treatment 3</td>
<td>0.26</td>
<td>0.16</td>
<td>0.24</td>
<td>0.10</td>
<td>0.29</td>
<td>0.44</td>
<td>0.25</td>
</tr>
<tr>
<td>Treatment 4</td>
<td>0.69</td>
<td>0.21</td>
<td>0.47</td>
<td>0.81</td>
<td>0.61</td>
<td>1.23</td>
<td>0.67</td>
</tr>
</tbody>
</table>

| Table 4.4: Medians of Stage II Dispersion Measures. The table shows the medians of three dispersion measures from periods $t = 9, ..., 28$ for all six economies in each treatment and the respective treatment averages. Dispersion in economy $i$ in period $t$ is either the variance, range, or interquartile range of the period $t$ inflation forecasts for period $t + 1$ from all six individual professional forecasters in economy $i$. |
parameter values generally have the expected sign. More importantly, informative forward guidance unambiguously reduces the cross-sectional dispersion of individual expectations. The reduction is statistically significant. The influence of inflation uncertainty on the cross-sectional dispersion remains statistically significant. The interest rate and the output gap coefficients again are negligible and statistically insignificant. Finally, Column [5] shows the parameter estimates of (4.16) using data from Treatments 1 and 4 only. Now, the effect of publishing inflation projections is positive and statistically significant. Random forward guidance increases the cross-sectional dispersion by approximately 29%. The results are similar, if we consider contemporaneous macroeconomic controls \( X_t \), as applied in the original studies by Ehrmann et al. (2012) and Hubert (2014).

Taken together, the above results give rise to the notion that informative forward guidance acts as an anchor for private-sector inflation expectations, while random forward guidance unleashes disturbing forces driving private-sector expectations apart. Therefore, our evidence confirms Hypothesis 2. Furthermore, the evidence documents a substantial influence of central bank forward guidance on private-sector expectation formation. Section 4.5 analyzes this influence in more detail.
4.4. Macroeconomic Results

4.4.4 Forecasting Performance

In this section, we address Hypothesis 3, i.e. we analyze the relative forecasting performance of the subjects, the data-driven forecast, and the computer-algorithms. We evaluate the relative forecasting performance by means of the mean squared error in Stage II of the experiment.

Table 4.17 in Appendix 4.C presents the mean squared errors of the published central bank projection, the central bank forecaster subject’s forecast, the data-driven forecast, and the aggregate forecast of the professional forecasters for each economy of all four treatments. Analogously, Table 4.18 in Appendix 4.C presents the mean square errors of the individual professional forecasters. Following Romer and Romer (2000) and Hubert (2015c), p-values for pairwise comparisons of the mean squared errors are calculated by estimating

\[
\left(\pi_{t+1} - E_t^a \pi_{t+1}\right)^2 - \left(\pi_{t+1} - E_t^b \pi_{t+1}\right)^2 = c + u_{t+1},
\]

where \(E_t^a, E_t^b \in \{E_t^{pub} \pi_{t+1}, E_t^{cbf} \pi_{t+1}, \bar{E}_t \pi_{t+1}, E_t^{ddf} \pi_{t+1}\}\). The p-values test the null hypothesis that \(c = 0\). The standard errors are corrected for autocorrelation and heteroskedasticity using the Newey-West HAC method (Newey and West, 1987).

The tables show that for Treatment 1, the central bank forecaster, the aggregate inflation forecast, and the data-driven forecast seem to do equally well in most cases, as well as on average. In four out of six economies, there cannot be found a significant ranking of the forecasts. For the remaining two economies, there can be found the worst forecasting entity, but not the best forecasting entity. On average, in Treatment 1 there is no (or only very little) evidence for the superiority of one forecasting entity over the other. If at all, the central bank forecaster subject does worst. Given the fact that the central bank has more and better information about potential future inflation than the professional forecasters, it is somewhat surprising that the central bank forecaster performs no better (rather slightly worse) than the aggregate forecast of the professional forecasters. Two explanations come to mind. First, being rather persistent, the aggregate forecast becomes highly self-fulfilling and thereby accurate by construction. A second potential explanation can be found in the “wisdom of the crowd,” which describes the phenomenon that groups can achieve higher forecast accuracy by taking the group average or median compared to their individual forecasts, as the mean or median filters out idiosyncratic noise (Surowiecki, 2005). The latter argument is also supported by the fact that (in all four treatments) the mean squared error of the aggregate forecast of the professional forecasters for a treatment is always
below the average mean squared error of all individual forecasters within that treatment. Moreover, in 21 out of 24 economies at most one individual forecaster performs better individually (has a lower mean squared error) than the aggregate forecast in that respective economy.\footnote{In 9 out of these 21, no individual performs better than the aggregate forecast.} The remaining three economies feature at least 2 and at most 3 individual forecasters who perform better individually than the aggregate forecast.

A similar insignificant pattern amongst the central bank forecaster, the aggregate inflation forecast, and the data-driven forecast arises in Treatment 4. The computerized random published projection in Treatment 4, not surprisingly, is considerably less accurate compared to all other forecasting entities with these differences being statistically significant.

By contrast to Treatment 4, for Treatments 2 and 3 the published inflation projection improves substantially and performs significantly better than any other forecasting entity, both on average and for most of the individual economies. The performance of the data-driven forecast remains basically unchanged (for pairwise comparisons the p-value is never below 0.10). The aggregate inflation forecast of the professional forecasters improves significantly only when a good computerized central bank projection is provided ($p_{afcT1,afcT3} = 0.0579$), whereas the improvement is not significant when the human central bank provides the projections ($p_{afcT1,afcT2} = 0.5542$). For the individual professional forecasters, the average reduction of the mean forecast error under informative forward guidance is substantial (approximately one third) and statistically significant independent of whether the central bank is computerized or not, i.e. $p_{fcT1,fcT2} = 0.000$ and $p_{fcT1,fcT3} = 0.000$. The central bank forecaster improves her forecasting performance considerably (by more than 50%) and significantly ($p_{cbfT1,cbfT2} = 0.0745$) when her forecast is published. Summing up, the above evidence confirms Hypothesis 3.

The result that the central bank forecasting performance is improved when its forecast is published is particularly interesting in the light of analyzing the potential source of the well-documented superiority of central bank projections over private-sector expectations. Romer and Romer (2000) put forth the hypothesis that the FOMC is able to produce superior inflation projections from publicly available information projections simply by committing far more resources to forecasting compared to the private sector. The results from Treatments 1 and 2 in our experiment reject this hypothesis. In both treatments, an equal amount of resources is invested to process the publicly available information and to provide it to the central bank forecaster in the form of the data-driven forecast.\footnote{Note that the “required for target” information supplied to the central bank forecaster in Treat-}
caster differs substantially. Three potential explanations may be put forward for this difference: the quality of the information supplied to the central bank forecaster, the different incentive structure for the central bank forecaster across both treatments, and the publication of the inflation projection. Since the predictive power of the data-driven forecast is similar across Treatments 1 and 2, information quality cannot explain the difference. Also, the central bank forecaster’s incentive cannot explain this result. The central bank forecaster’s incentive to predict inflation accurately is stronger in Treatment 1 relative to Treatment 2 (as in Treatment 2 there is a trade-off between predicting correctly and strategically). Consequently, a better forecasting performance should be expected in Treatment 1 and not the other way around. By contrast, the publication of the inflation projection is a potential explanation for the improved forecasting performance. If credible, central bank projections are self-fulfilling. That is, when the central bank publishes a high (low) projection and professional forecasters respond by giving a forecast in the same direction, then a high (low) rate of inflation realizes. Consequently, even ex-ante incorrect projections, if believed by the professional forecasters, are evaluated as quite accurate projections ex-post. Whether or not central bank projections indeed positively affect private sector expectations and hence are (at least in part) self-fulfilling, we explore in more detail in the next section.

4.5 Expectation Formation

In this section we analyze how individual subjects form their expectations. We consider the professional forecasters (Section 4.5.1) and the central bank forecasters (Sections 4.5.2) separately. In the latter section, we also address Hypothesis 4.

4.5.1 Professional Forecaster Expectation Formation

In the four treatments of the experiment we can distinguish two types of professional forecasters: professional forecasters that were not exposed to a published projection, and professional forecasters that did see a published projection prior to submitting their own forecast. In this section we investigate (i) whether these two groups of forecasters formed expectations in a qualitatively different way and (ii) to what extent the expectation formation of the forecasters that did see a central bank projection depended on the quality of this projection. Since we treat Stage 1 as learning stage in
all treatments and Stage III presents subjects with an inherently unstable environment, we focus this analysis on Stage II only.

First, consider professional forecasters that did not see a published projection when making their own forecast. This group consists of the professional forecasters in Treatment 1 (the control treatment). We follow Assenza et al. (2014) and Pfajfar and Zakelj (2014) and regress each subject’s inflation forecast on a general linear forecasting rule of the form

\[
E_{t}^{fc,j} \pi_{t+1} = c^{j} + \sum_{i=1}^{2} \alpha^{j}_{i} E_{t-i}^{fc,j} \pi_{t+1-i} + \sum_{i=1}^{2} \beta^{j}_{i} \pi_{t-i} + \gamma^{j} x_{t-1} + \delta^{j} E_{t}^{pub} \pi_{t+1} + \varepsilon^{j},
\]

where \(\varepsilon^{j}\) is the error term of each individual regression. The results are summarized in the second column of Table 4.6, which show the percentage of individually significant regressors and the median estimated parameter values for each treatment, respectively.\(^{55}\)

The second column of Table 4.6 shows that 92% of subjects consider the first lag of inflation when forming their expectation about future inflation. 36% of subjects also consider the second lag of inflation. Given that the sign of the coefficient on the first lag is generally positive with a median of 1.11, while the sign on the second lag of inflation is generally negative with median of -1.14 it appears that the professional forecasters engaged in trend following behavior when forecasting inflation. In line with early evidence from Adam (2007) only few subjects consider past realizations of the output gap to predict future inflation.

Next, we consider all subjects which were shown a public central bank projection prior to submitting their own forecast. This group consists of all subjects in Treatment 2, 3, and 4. We follow the same procedure as above, but with one difference. Now, we include the published central bank inflation projection as an additional regressors in the set of possible regressors. Thus, the subject’s new general linear forecasting rule has the form

\[
E_{t}^{fc,j} \pi_{t+1} = c^{j} + \sum_{i=1}^{2} \alpha^{j}_{i} E_{t-i}^{fc,j} \pi_{t+1-i} + \sum_{i=1}^{2} \beta^{j}_{i} \pi_{t-i} + \gamma^{j} x_{t-1} + \delta^{j} E_{t}^{pub} \pi_{t+1} + \varepsilon^{j},
\]

Column 3 of Table 4.6 shows the results of the regression on the subjects with a published projection provided by a human central banker. It can be seen that for 69% of the subjects the published projection has a statistically significant effect on their expectations. This is more than for the first lag of inflation which is now statisti-
4.5. Expectation Formation

<table>
<thead>
<tr>
<th>Treatment</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>constant</td>
<td>39%</td>
<td>36%</td>
<td>56%</td>
<td>50%</td>
</tr>
<tr>
<td>(E_{t-1}^{fc} \pi_t)</td>
<td>14%</td>
<td>19%</td>
<td>14%</td>
<td>19%</td>
</tr>
<tr>
<td>(E_{t-2}^{fc} \pi_{t-1})</td>
<td>3%</td>
<td>11%</td>
<td>17%</td>
<td>8%</td>
</tr>
<tr>
<td>(\pi_{t-2})</td>
<td>92%</td>
<td>47%</td>
<td>56%</td>
<td>42%</td>
</tr>
<tr>
<td>(\pi_{t-1})</td>
<td>36%</td>
<td>25%</td>
<td>17%</td>
<td>11%</td>
</tr>
<tr>
<td>(x_{t-1})</td>
<td>14%</td>
<td>11%</td>
<td>14%</td>
<td>25%</td>
</tr>
<tr>
<td>(E_{t}^{pub} \pi_{t+1})</td>
<td>69%</td>
<td>31%</td>
<td>31%</td>
<td>31%</td>
</tr>
<tr>
<td>avg. (R^2)</td>
<td>0.76</td>
<td>0.72</td>
<td>0.66</td>
<td>0.46</td>
</tr>
<tr>
<td>#Sign.Coeff</td>
<td>1.97</td>
<td>2.19</td>
<td>2.03</td>
<td>1.86</td>
</tr>
</tbody>
</table>

Table 4.6: Percentages of significant regressors and the median regression coefficients (in parentheses) estimating equations (4.18) and (4.19) for all professional forecasters per treatment. Additionally, the table shows the average \(R^2\) and the average number of significant coefficients per forecaster for each treatment.

cally significant for less than half of the subjects. The significance of the second lag of inflation is also reduced considerably. Surprisingly, when the public central bank projection is given by a computer algorithm, it is statistically significant only for 31% of the subjects (Columns 4 and 5). Nevertheless, the first and second lag of past inflation lose significance compared to the control treatment. We conclude from this that when subjects are presented with a published central bank projection, many subjects let their own forecast be affected by the public projection. In this case, subjects put less weight on past inflation and trend behavior in inflation in particular.

The bottom row of Table 4.6 presents the average number of significant regressors used in the expectation formation process in each of the four treatments. Interestingly, this number is around two for all of the four treatments. This leads to the conclusion that subjects rather substitute the public central bank inflation projection for another source of information than complement their information set in the expectation formation process.

Another interesting observation is that the fraction of subjects using the published inflation projection is the same, independent of whether the computer algorithm is sophisticated and good or bad (Treatment 3 versus 4).\(^{56}\) However, even though the

\(^{56}\)Interestingly, when we split the sample between subjects with high cognitive ability and subjects with low cognitive ability this is no longer the case (see Table 4.19 in Appendix 4.C). We measure cognitive ability with the three-item “cognitive reflection test” of Frederick (2005). Subjects with a
fraction of subjects considering the public projection in Treatments 3 and 4 are the same, we find that the weight subjects put on this projection when forming their expectations is much higher for Treatment 3 compared to Treatment 4. The median of the significant coefficients on the published projection in Treatment 3 is 1.44, whereas it is 0.22 in Treatment 4. The coefficient in Treatment 3 is furthermore also bigger than in Treatment 2, where the median coefficient is equal to 0.818.

4.5.2 Central Bank Forecaster Expectation Formation

Now, consider the expectations of the central bank forecaster subjects. Analogous to the previous analysis, for each central bank forecaster subject $j$ we estimate a general linear forecasting rule. To account for the additional information supplied to the central bank forecaster subject, the general forecasting rule takes the following form:

\[
E_{t}^{cbf,j} \pi_{t+1} = \alpha_{j}^{2} E_{t-1}^{cbf,j} \pi_{t+1-1} + \sum_{i=1}^{2} \beta_{i}^{j} \pi_{t-i} + \gamma^{j} x_{t-1} \\
+ \delta_{1}^{j} E_{t}^{ddf} \pi_{t+1} + \delta_{2}^{j} E_{t}^{rf} \pi_{t+1} + \epsilon_{t}^{j}.
\]

Note that for subjects in Treatments 1 $\delta_{2}^{j} = 0$ and that for subjects in Treatments 3 and 4 $\delta_{1}^{j} = \delta_{2}^{j} = 0.57$. Table 4.7 summarizes the percentages of significance, which is now based on 6 observations per treatment.

The central bank forecasters of Treatments 3 and 4 face the same decision as the professional forecasters in Treatment 1, with the only difference that the central bank forecasters have no influence whatsoever on the economy. Since the influence of an individual professional forecaster is relatively small, we should expect to see a similar expectation formation process. Indeed, from columns 4 and 5 of Table 4.7, it can be seen that most subjects have a significant (positive) coefficient (with medians of 1.01 in Treatment 3 and a median of 0.93 in Treatment 4) on the first lag of inflation and that many subjects also have a negative coefficient (with medians of -0.73 in Treatment 3 and a median of -0.68 in Treatment 4) on the second lag of inflation.

When facing the same decision as in Treatments 3 and 4, but when presented with the data-driven forecast (Treatment 1), the central bank forecaster resorted strongly to this source of information in their expectation formation process. This is in line with higher CRT use the published projection more often when it is informative (Treatments 2 and 3), while they use it less often when it is random (Treatment 4). Therefore, subjects with a high CRT score can be seen as more “rational.”

57The estimation procedure again follows the procedure described in the Appendix. The order of removal for equation (4.20) is: $\alpha_{2}, \gamma_{1}, \beta_{2}, \delta_{2}, \delta_{1}, \beta_{1}, \alpha_{1}, c$. 

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4.5. Expectation Formation

Table 4.7: Percentage of significant regressors and the median regression coefficients (in parentheses) from estimating equations (4.18) and (4.19) for all central bank forecasters per treatment. Additionally, the table shows the average $R^2$ and the average number of significant coefficients per forecaster for each treatment.

<table>
<thead>
<tr>
<th>Treatment</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>constant</td>
<td>17%</td>
<td>33%</td>
<td>17%</td>
<td>67%</td>
</tr>
<tr>
<td>$(E_{t-1}^{cbf})_t ^{\pi_t}$</td>
<td>(0.338)</td>
<td>(0.200)</td>
<td>(0.123)</td>
<td>(0.682)</td>
</tr>
<tr>
<td>$(E_{t-2}^{cbf})<em>t ^{\pi</em>{t-1}}$</td>
<td>33%</td>
<td>33%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>$(E_{t-2}^{cbf})<em>t ^{\pi</em>{t-1}}$</td>
<td>(0.597)</td>
<td>(0.323)</td>
<td>(0.000)</td>
<td>(0.000)</td>
</tr>
<tr>
<td>$\pi_{t-1}$</td>
<td>0%</td>
<td>0%</td>
<td>3%</td>
<td>17%</td>
</tr>
<tr>
<td>$\pi_{t-2}$</td>
<td>50%</td>
<td>0%</td>
<td>100%</td>
<td>83%</td>
</tr>
<tr>
<td>$\pi_{t-2}$</td>
<td>(0.792)</td>
<td>(0.000)</td>
<td>(1.014)</td>
<td>(0.927)</td>
</tr>
<tr>
<td>$x_{t-1}$</td>
<td>33%</td>
<td>0%</td>
<td>50%</td>
<td>33%</td>
</tr>
<tr>
<td>$x_{t-1}$</td>
<td>(-0.887)</td>
<td>(0.000)</td>
<td>(-0.678)</td>
<td>(-0.773)</td>
</tr>
<tr>
<td>$E_{t-1}^{ddf}$</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>$E_{t-1}^{ddf}$</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
</tr>
<tr>
<td>avg. $R^2$</td>
<td>0.66</td>
<td>0.63</td>
<td>0.64</td>
<td>0.52</td>
</tr>
<tr>
<td>#Sign.Coeff</td>
<td>1.83</td>
<td>1.5</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

The result of the previous section that professional forecasters that got a published central bank projection in each period partly substituted this forecast for their own trend extrapolations. This results seems even stronger for the central bank forecaster in Treatment 2. Five out of the six subjects get a significant coefficient on the data-driven forecast and past inflation is never significant.\(^{58}\)

It is further noteworthy that not a single significant coefficient on the “required for target” is obtained. To investigate in more detail to what extent the central bank forecaster in Treatment 2 made use of the data-driven forecast rather than the “required for target”, we pool the observations of all 6 subjects together and estimate the following regression.\(^{59}\)

\[
E_t^{cbf} \pi_{t+1} = b_1 E_t^{rft} \pi_{t+1} + (1 - b_1) E_t^{ddf} \pi_{t+1}
\]

\(^{58}\)For the sixth subject we do not obtain a significant coefficient on any of the regressors.

\(^{59}\)We also estimate regressions where we add past inflation and a constant as extra regressors. However, both of them do not obtain coefficients that are significant at a 5% level, while the data-driven forecast and the required for target both are significant at a 1% level. It therefore seems that it is not necessary to add these controls.
This regression can give insight in the conditions under which subjects expectations where more in line with the data-driven forecast or more with the “required for target”, and sheds light on Hypothesis 4. Specifically, we look at how this depends on credibility, as measured by the credibility index, and on how close inflation was to the target in the previous period. We do this by splitting the sample of observations according to the credibility index and according to the distance from target.

Rows 2 and 3 show the results for estimating equation (4.21) split in two subsamples. The subsample for Row 2 collects all observations for which the credibility index is larger than its sample median. Row 3 collects all observations for which the credibility index is lower than or equal to the sample median. As can be seen, the relative weights on the “required for target” and the data-driven forecast change only negligibly. Thus, credibility seems to have only a negligible influence on the central bank forecasters’ decision whether to follow the data-driven forecast or the “required for target.”

Rows 4 and 5 show the relative weights, when the sample is split according to the distance of inflation from the target. For observations close to the target, the estimated weight on the “required for target” (0.374) is considerably higher than for observations further away from the target (0.287). This may indicate that when inflation is far from the target, subjects are mainly occupied with predicting correctly, but that when the
target comes in sight they start aiming more to achieve this target. Due to a limited number of observations the difference is however not statistically significant.

Finally, for the last four rows of Table 4.8 the sample is split in four quadrants according to both the credibility index and the distance of inflation from target. This analysis seems to indicate that when inflation is close to the target credibility does not play much of a role, but when inflation is far from the target, less credibility implies less weight on “required for target” and more weight on the data-driven forecast. The differences are, yet again, not statistically significant.

The above above seems to indicate that there might be some truth to Hypothesis 4, at least when inflation is far away from its target. However, because of a small sample size, we are not able to confirm or reject the hypothesis.

4.6 Credibility of the Central Bank Projections

In the stability analysis in Section 4.4 we have not explicitly considered the role of the credibility of the published central bank projections. We now turn to this issue in more detail. In Section 4.6.1 we consider whether credibility of central bank projections depends on past performance of the projections (Hypothesis 5(a)), while in Section 4.6.2 we look at the influence of credibility on the central bank’s ability to stabilize the economy (Hypothesis 5(b)).

4.6.1 How Past Performance Shapes Future Credibility

First, we address Hypothesis 5(A) by studying whether past performance of public inflation projections determines the credibility of future inflation projections. To analyze this, we follow Mokhtarzadeh and Petersen (2016) and estimate a series of probit models, where the dependent variable $Utilize_t$ is binary taking value 1 if individual professional forecasters utilized the central bank projection and 0 if not. A central bank projection is said to be utilized if an individual professional forecasters forecast is within 5 basis points of the respective central bank projection.\(^\text{(60)}\) In accordance with Mokhtarzadeh and Petersen (2016), our explanatory variable is past forecasting per-

\(^{60}\)Mokhtarzadeh and Petersen (2016) choose a band of 2 basis points to identify utilization of the central bank projection, which yields approximately 20% of private forecasts to utilize the central bank projection in their experiment. In our experiment, a 2-basis-point band yields a utilization of only around 7.5%, whereas a 5-basis-point band yields around 17.5% utilization. The increased number of observations in the 5-basis-point case does not change the qualitative results of the estimation, but results in stronger statistical significance.
formance of the central bank projections, measured by the absolute forecast error from the previous period. As controls we employ the absolute deviation of previous inflation from the central bank’s inflation target, the professional forecasters previous absolute forecast error, period $t-2$ utilization of the central bank projection, and the interaction of the latter two. The interaction term measures the degree to which past shaken confidence in the central bank projection influences the willingness to utilize the central bank projection in the future. Additionally, we control for past aggregate credibility of the central bank projection measured by the period $t-1$ credibility index, and the subjects cognitive ability measured by the three-item “cognitive reflection test” of Frederick (2005). The estimation results for Stage II from Treatments 2, 3, and 4 are presented in Table 4.20 and for Stage III in Table 4.21. Both tables are located in Appendix 4.C.

The tables show that central bank projections are more likely to be adopted in the future, if they were accurate in the past, independent of whether the economy functions in normal times or in times of severe economic stress. Consequently, credibility increases in past forecasting performance, confirming Hypothesis 5(a). Additionally, credibility also seems to be a persistent phenomenon. If a professional forecaster adopted the central bank projection in the past or if it was credible in the past, the professional forecasters are more likely to adopt the central bank projection in the future. Even if the adoption of a past projection ex-post turns out to be a disappointment, i.e., it resulted in an own large forecast error, the willingness of the professional forecaster to adopt future central bank projections remains unchanged, which can be read from the insignificance of the interaction term in Tables 4.20 and 4.21. Subjects seem to pay more attention to the performance of the central bank projections than to reflect on their past behavior and its outcomes. Finally, in line with evidence presented in footnote 56, the probit regressions reveal that cognitive ability increases the likelihood to adopt central bank projection.

4.6.2 The Role of Credibility for Stabilizing Forward Guidance

Next, we analyze how credibility of central bank projections affects the central bank’s ability to stabilize the economy.

Comparing Treatments 3 and 4 when entering Stage III yields a natural test for Hypothesis 5(b), at least for times of severe economic stress. Right before entering
the recession in period 29, central bank projections in Treatment 3 are highly credible (credibility index is $I^{cred}_{28} = 0.89$ and the median subjective survey measure is 7), whereas they are not credible at all in Treatment 4 (credibility index is $I^{cred}_{28} = 0.125$ and the median subjective survey measure is 3). Therefore, credibility of central bank projections is highly different (and statistically significantly so $p^{survey}_{T_3,T_4} = 0.005$ and $p^{cred}_{T_3,T_4} = 0.005$) upon entering the deep recession initiated by the series of fundamental shocks. Despite this large difference in credibility, the behavior of the economy in the time of severe economic stress is basically identical, as documented by the important key indicators describing the median severity of the economic downturn in Treatments 3 and 4 from Table 4.2. Therefore, low credibility of the inflation projection upon entering the recession does not hamper the stabilizing role of informative forward guidance at the zero lower bound. Consequently, this evidence rejects Hypothesis 5(b) for times of severe economic stress. Whether or not credibility has an influence on the stabilizing power of forward guidance in normal times is subject to further research.

4.7 Conclusion

In this chapter, we study the stabilizing role of central bank forward guidance in a standard New Keynesian learning-to-forecast laboratory experiment. Subjects take the role of “professional forecasters” in the private sector who form one-period ahead inflation forecasts. Subjects are provided with a limited understanding of the true data generating process and a public central bank inflation projection, i.e. central bank forward guidance. We show that central banks can manage private-sector expectations via the publication of (strategic) central bank inflation projections and that such expectations management can successfully be applied as an additional monetary policy instrument to stabilize the economy.

In particular, we show that such central bank forward guidance considerably influences the subjects’ expectations formation process. In the absence of forward guidance, subjects expectation formation process is well characterized as mostly backward-looking with simple trend following. In the presence of forward guidance, by contrast, the public inflation projection becomes an influential piece of information which starkly diminishes the prevalence of backward-looking expectation formation. The utilization of the central bank projections, albeit very persistent, is not unconditional. Weak past performance, which manifests in large forecast errors, reduces future credibility of the public projection and therewith the subjects’ probability of future utilization.

The documented influence on expectations by means of forward guidance strongly
impacts on macroeconomic activity. We show that whether this impact is stabilizing or destabilizing yet again depends on the quality of the published forecast. During normal times and given reasonable and informative public projections, the economy quickly converges towards its steady state and subsequently fluctuates around it closely. Random inflation projections, by contrast, are generally harmful to the economy as they unleash disturbing forces which give rise to large fluctuations of the economy. In times of severe economic stress at the zero lower bound, the publication of overly optimistic central bank projections turns out to strongly reduce the risk of deflationary spirals. Pessimistic central bank projections, however, have the potential to fuel or even initiate a deflationary spiral.

Finally, the increase in economic stability due to informative forward guidance has positive effects on the predictability of the economy. Professional forecasters increase their forecasting performance significantly and the disagreement among forecasters diminishes. By contrast, the aversive effects of random forward guidance unleash disturbing forces which give rise to more dispersed and less precise individual private-sector forecasts.

Our results have important implications for central bank practice. We show that central bank forward guidance is a powerful tool for stabilization policy in normal times and at the zero lower bound. However, while a good track record of accurate forecasts is important for credibility, we find that some strategic-ness in the published forecasts greatly enhances the stabilizing power of forward guidance. Especially in times of severe economic stress, fully truthful projections may be harmful rather than beneficial.

Appendix 4.A Estimation procedure for equation (4.18)

First Formula (4.18) is estimated with OLS. Then the joint significance of all the coefficients that were found to be individually insignificant in the above regression is tested. If these coefficients are jointly insignificant, all of them are removed. If they are jointly significant, exactly 1 coefficient is removed. The coefficient that is removed is then the individually insignificant coefficient that ranks first in the following order of removal list: \( \alpha_2, \gamma_1, \beta_2, \delta, \beta_1, \alpha_1, c \).

After one or more coefficients are removed, Equation (4.18) is reestimated without this (these) coefficient(s). Then the joint significance of the coefficients found to be individually insignificant in the new regression is tested and coefficient(s) are removed.
Appendix 4.B  Proof of “strategic-ness” index

Below we proof the following three claims about the “strategic-ness” index, \( SP_t \), of equation (4.15):

(a) \( 0 < SP_t < 1 \) implies that the published forecast lies in the band between the data driven forecast and the “required for target” information:
\[
E_t^{\text{ft}} \pi_{t+1} < E_t^{\text{pub}} \pi_{t+1} < E_t^{\text{def}} \pi_{t+1} \quad \text{or} \quad E_t^{\text{def}} \pi_{t+1} < E_t^{\text{ft}} \pi_{t+1} < E_t^{\text{pub}} \pi_{t+1}
\]

(b) \( SP_t > 1 \) implies that the published forecast lies outside the band of the data driven forecast and the “required for target” information, on the side of the “required for target”:
\[
E_t^{\text{pub}} \pi_{t+1} < E_t^{\text{ft}} \pi_{t+1} < E_t^{\text{def}} \pi_{t+1} \quad \text{or} \quad E_t^{\text{def}} \pi_{t+1} < E_t^{\text{ft}} \pi_{t+1} < E_t^{\text{pub}} \pi_{t+1}
\]

(c) \( SP_t < 0 \) implies that the published forecast lies outside the band of the data driven forecast and the “required for target” information, on the side of the data driven forecast:
\[
E_t^{\text{pub}} \pi_{t+1} < E_t^{\text{def}} \pi_{t+1} \quad \text{or} \quad E_t^{\text{def}} \pi_{t+1} < E_t^{\text{pub}} \pi_{t+1}
\]

**Proof:**

(1) When \( 0 < SP_t \) either both numerator and denominator in Equation (4.15) are negative or both are positive:

(i) Consider that both numerator and denominator are negative. It must be that \( E_t^{\text{pub}} \pi_{t+1} < E_t^{\text{def}} \pi_{t+1} \) and \( E_t^{\text{ft}} \pi_{t+1} < E_t^{\text{def}} \pi_{t+1} \). Since the denominator is negative \( SP_t < 1 \) further implies that \( E_t^{\text{pub}} \pi_{t+1} - E_t^{\text{def}} \pi_{t+1} > E_t^{\text{ft}} \pi_{t+1} - E_t^{\text{def}} \pi_{t+1} \) so that \( E_t^{\text{ft}} \pi_{t+1} < E_t^{\text{def}} \pi_{t+1} \) and hence \( E_t^{\text{ft}} \pi_{t+1} < E_t^{\text{pub}} \pi_{t+1} < E_t^{\text{def}} \pi_{t+1} \). \( SP_t > 1 \), on the other hand, implies that \( E_t^{\text{ft}} \pi_{t+1} > E_t^{\text{pub}} \pi_{t+1} \) and hence \( E_t^{\text{pub}} \pi_{t+1} < E_t^{\text{ft}} \pi_{t+1} < E_t^{\text{def}} \pi_{t+1} \).

(ii) Consider that both numerator and denominator are positive. \( 0 < SP_t \) implies \( E_t^{\text{pub}} \pi_{t+1} > E_t^{\text{def}} \pi_{t+1} \) and \( E_t^{\text{ft}} \pi_{t+1} > E_t^{\text{def}} \pi_{t+1} \). \( SP_t < 1 \) then implies \( E_t^{\text{pub}} \pi_{t+1} - E_t^{\text{def}} \pi_{t+1} < E_t^{\text{ft}} \pi_{t+1} - E_t^{\text{def}} \pi_{t+1} \) so that \( E_t^{\text{def}} \pi_{t+1} < E_t^{\text{pub}} \pi_{t+1} < E_t^{\text{ft}} \pi_{t+1} \), while \( SP_t > 1 \) implies \( E_t^{\text{def}} \pi_{t+1} < E_t^{\text{ft}} \pi_{t+1} < E_t^{\text{pub}} \pi_{t+1} \). This completes the proof of (a) and (b).

(2) When \( SP_t < 0 \), either the numerator of Equation (4.15) is negative while the denominator is positive, or the numerator is positive while the denominator is negative: In the first case, it must hold that \( E_t^{\text{pub}} \pi_{t+1} < E_t^{\text{def}} \pi_{t+1} \) while \( E_t^{\text{ft}} \pi_{t+1} > E_t^{\text{def}} \pi_{t+1} \). In the second case, it must hold that \( E_t^{\text{pub}} \pi_{t+1} > E_t^{\text{def}} \pi_{t+1} \) while \( E_t^{\text{ft}} \pi_{t+1} < E_t^{\text{def}} \pi_{t+1} \). This proves (c).
Appendix 4.C Additional tables and figures
### Table 4.9: Descriptive Statistics of Treatment 1 (Control). The table summarizes mean, median, and variance in each of the three stages for each of the six economies of Treatment 1 as well as their corresponding averages over all six economies of Treatment 1.
### Table 4.10: Descriptive Statistics of Treatment 2

The table summarizes mean, median, and variance in each of the three stages for each of the six economies of Treatment 2 as well as their corresponding averages over all six economies of Treatment 2.
## Table 4.11: Descriptive Statistics of Treatment 3

The table summarizes mean, median, and variance in each of the three stages for each of the six economies of Treatment 3 as well as their corresponding averages over all six economies of Treatment 3.
<table>
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<th>E21</th>
<th>E22</th>
<th>E23</th>
<th>E24</th>
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<td></td>
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<td>2.6</td>
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<td>2.4</td>
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<td>0.6</td>
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<td>1.1</td>
<td>0.6</td>
<td>0.9</td>
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<td></td>
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<td>-1.0</td>
<td>-0.8</td>
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<td>0.1</td>
<td>0.3</td>
<td>0.2</td>
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<td></td>
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<td>6.7</td>
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<td></td>
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</tr>
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<td>2.5</td>
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<td>1.1</td>
<td>3.3</td>
<td>3.2</td>
<td>7.7</td>
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Table 4.12: Descriptive Statistics of Treatment 4. The table summarizes mean, median, and variance in each of the three stages for each of the six economies of Treatment 4 as well as their corresponding averages over all six economies of Treatment 4.
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<th>Periods 1-8</th>
<th>Periods 9-28</th>
<th>Periods 29-37</th>
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</thead>
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<td>no*** (0.0000) no*** (0.0001)</td>
<td>no** (0.0110)</td>
</tr>
<tr>
<td>Output</td>
<td>yes (0.7030) no** (0.0369) no** (0.0174)</td>
<td>no*** (0.0001) yes (0.1373)</td>
<td>yes (0.0174)</td>
</tr>
<tr>
<td>Interest</td>
<td>yes (0.5750) no*** (0.0001)</td>
<td>yes (0.0174)</td>
<td>yes (0.1373)</td>
</tr>
</tbody>
</table>

<table>
<thead>
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<th>Variable</th>
<th>Periods 1-8</th>
<th>Periods 9-28</th>
<th>Periods 29-37</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inflation</td>
<td>yes (0.9795) no*** (0.0000) no** (0.0110)</td>
<td>yes (0.1517) no*** (0.0085)</td>
<td>yes (0.0365)</td>
</tr>
<tr>
<td>Output</td>
<td>yes (0.9124) no*** (0.0052) no*** (0.0001)</td>
<td>no** (0.0174) no*** (0.0001)</td>
<td>yes (0.0365)</td>
</tr>
<tr>
<td>Interest</td>
<td>yes (0.9649) no*** (0.0000)</td>
<td>yes (0.0365)</td>
<td>yes (0.0365)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Variable</th>
<th>Periods 1-8</th>
<th>Periods 9-28</th>
<th>Periods 29-37</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inflation</td>
<td>yes (0.7499) no** (0.0401) no*** (0.0016)</td>
<td>yes (0.0540) no*** (0.0085)</td>
<td>yes (0.0109)</td>
</tr>
<tr>
<td>Output</td>
<td>yes (0.6207) yes (0.1517)</td>
<td>no*** (0.0085)</td>
<td>yes (0.0109)</td>
</tr>
<tr>
<td>Interest</td>
<td>yes (0.8922) no* (0.0509)</td>
<td>no*** (0.0085)</td>
<td>no*** (0.0016)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Variable</th>
<th>Periods 1-8</th>
<th>Periods 9-28</th>
<th>Periods 29-37</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inflation</td>
<td>yes (0.3444) no** (0.0354) yes (0.2100)</td>
<td>yes (0.5047) yes (0.8658)</td>
<td>yes (0.4092)</td>
</tr>
<tr>
<td>Output</td>
<td>yes (0.3671) yes (0.5047)</td>
<td>yes (0.8658)</td>
<td>yes (0.4092)</td>
</tr>
<tr>
<td>Interest</td>
<td>yes (0.3851) no** (0.0486)</td>
<td>yes (0.8658)</td>
<td>yes (0.4092)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Variable</th>
<th>Periods 1-8</th>
<th>Periods 9-28</th>
<th>Periods 29-37</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inflation</td>
<td>yes (0.6469) no*** (0.0002) yes (0.2740)</td>
<td>yes (0.1266) yes (0.1222)</td>
<td>yes (0.6487)</td>
</tr>
<tr>
<td>Output</td>
<td>yes (0.6336) no*** (0.0002) yes (0.2740)</td>
<td>yes (0.1266) yes (0.1222)</td>
<td>yes (0.6487)</td>
</tr>
<tr>
<td>Interest</td>
<td>yes (0.6735) no*** (0.0005) yes (0.2740)</td>
<td>yes (0.1266) yes (0.1222)</td>
<td>yes (0.6487)</td>
</tr>
</tbody>
</table>

**Table 4.13:** Mann-Whitney-Wilcoxon-test for equality in medians. The table shows the outcome and the p-values (in parentheses) of the Mann-Whitney-Wilcoxon-test for pairwise across-treatment comparisons of inflation, the output gap, and the interest rate for each of the three stages, respectively. The Null hypothesis is that samples have identical median. The outcome “yes” implies that we cannot reject that both samples have identical median, while the outcome “no” implies that we reject equality of the median.
### Table 4.14: Siegel-Turkey-test for equality in variance.

The table shows the outcome and the *p*-values (in parentheses) of the Siegel-Turkey-test for pairwise across-treatment-comparisons of inflation, the output gap, and the interest rate for each of the three stages, respectively. The Null hypothesis is that samples have identical variance. The outcome “yes” implies that we cannot reject that both samples have identical variance, while the outcome “no” implies that we reject equality of the variance.
4.C. Additional tables and figures

<table>
<thead>
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<tr>
<td>Economy</td>
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<tr>
<td>-------------</td>
</tr>
<tr>
<td>$R_\pi^T$</td>
</tr>
<tr>
<td>$R_x^T$</td>
</tr>
</tbody>
</table>

<table>
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<tbody>
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<td>-------------</td>
</tr>
<tr>
<td>$R_\pi^T$</td>
</tr>
<tr>
<td>$R_x^T$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Treatment 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Economy</td>
</tr>
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<td>-------------</td>
</tr>
<tr>
<td>$R_\pi^T$</td>
</tr>
<tr>
<td>$R_x^T$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Treatment 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Economy</td>
</tr>
<tr>
<td>-------------</td>
</tr>
<tr>
<td>$R_\pi^T$</td>
</tr>
<tr>
<td>$R_x^T$</td>
</tr>
</tbody>
</table>

**Table 4.15:** Relative stability of inflation and the output gap (Stage II/Stage I). The table shows the relative stability measures (stability measure in Stage II relative to the respective stability measure of Stage I) given by equations (4.13) and (4.14) for all 24 economies, as well as their respective treatment averages. If the measure takes a value below unity, the economy is more stable in Stage II relative to Stage I.
## Table 4.16: Relative dispersion of individual forecasts (stage II/stage I). The table shows the medians of three dispersion measures from periods $t = 9, ..., 28$ relative to the periods $t = 1, ..., 8$ medians for all six economies in each treatment and the respective treatment averages.
<table>
<thead>
<tr>
<th>MSE of...</th>
<th>E1</th>
<th>E2</th>
<th>E3</th>
<th>E4</th>
<th>E5</th>
<th>E6</th>
<th>Avg</th>
</tr>
</thead>
<tbody>
<tr>
<td>pub</td>
<td>0.3526</td>
<td>0.1285</td>
<td>0.6639</td>
<td>0.1735</td>
<td>3.1312</td>
<td>1.0832</td>
<td>0.9221</td>
</tr>
<tr>
<td>cbf</td>
<td>0.4433</td>
<td>0.1557</td>
<td>0.5825</td>
<td>0.1823</td>
<td>1.4843</td>
<td>0.8233</td>
<td>0.6119</td>
</tr>
<tr>
<td>ddf</td>
<td>0.2716</td>
<td>0.1999</td>
<td>0.3963</td>
<td>0.1975</td>
<td>1.7205</td>
<td>0.8746</td>
<td>0.6101</td>
</tr>
<tr>
<td>afc</td>
<td>0.3107</td>
<td>0.3466</td>
<td>0.0265</td>
<td>0.8480</td>
<td>0.8128</td>
<td>0.9177</td>
<td>0.9926</td>
</tr>
</tbody>
</table>

Table 4.17: Stage II mean squared forecast errors of public central bank projection (pub), the central bank forecaster (cbf), the data-driven forecast (ddf), and the aggregate inflation forecast (afc). The p-values test (pairwise) the null hypothesis of equal forecast performance from estimation of equation (4.17).
### Table 4.18: Mean squared forecast errors of individual forecasters. The table shows Stage II mean squared forecast errors, i.e. \( \pi_{t+1} - E_t \pi_{t+1} \) for each individual professional forecaster in all 24 economies, as well as their respective treatment averages.
<table>
<thead>
<tr>
<th>Treatment CRT score</th>
<th>Treatment 2</th>
<th>Treatment 3</th>
<th>Treatment 4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>High (Constant)</td>
<td>Low (Efc,j t-1)</td>
<td>High (πt-1)</td>
</tr>
<tr>
<td># of subj.</td>
<td>27</td>
<td>9</td>
<td>22</td>
</tr>
<tr>
<td>constant</td>
<td>37%</td>
<td>33%</td>
<td>59%</td>
</tr>
<tr>
<td>(0.325)</td>
<td>(0.115)</td>
<td>(0.292)</td>
<td>(0.602)</td>
</tr>
<tr>
<td>Efc,j t-1 πt</td>
<td>22%</td>
<td>11%</td>
<td>13%</td>
</tr>
<tr>
<td>(-0.479)</td>
<td>(0.395)</td>
<td>(0.475)</td>
<td>(0.613)</td>
</tr>
<tr>
<td>πt-1</td>
<td>40%</td>
<td>66%</td>
<td>50%</td>
</tr>
<tr>
<td>(-0.479)</td>
<td>(-0.247)</td>
<td>(-0.536)</td>
<td>(0.658)</td>
</tr>
<tr>
<td>πt-2</td>
<td>22%</td>
<td>33%</td>
<td>22%</td>
</tr>
<tr>
<td>(-0.358)</td>
<td>(-0.969)</td>
<td>(-0.444)</td>
<td>(-0.811)</td>
</tr>
<tr>
<td>xt-1</td>
<td>11%</td>
<td>11%</td>
<td>13%</td>
</tr>
<tr>
<td>(-1.242)</td>
<td>(-1.317)</td>
<td>(-0.202)</td>
<td>(0.898)</td>
</tr>
<tr>
<td>Epub πt+1</td>
<td>77%</td>
<td>44%</td>
<td>36%</td>
</tr>
<tr>
<td>(0.816)</td>
<td>(1.732)</td>
<td>(1.418)</td>
<td>(0.120)</td>
</tr>
<tr>
<td>avg. R²</td>
<td>0.78</td>
<td>0.78</td>
<td>0.77</td>
</tr>
<tr>
<td>#Sign.Coeff</td>
<td>2.18</td>
<td>2.22</td>
<td>2.13</td>
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</table>

**Table 4.19:** Percentages of significant regressors and the median regression coefficients (in parentheses) from estimating equations (4.18) and (4.19) for all professional forecasters per treatment, when samples are split according to the subjects’ CRT score. A CRT score is considered high, if a subject answered at least two out of three questions correctly. Additionally, the table shows the average $R^2$ and the average number of significant coefficients per forecaster for each treatment.
Table 4.20: Determinants of the utilization of central bank projections in Stage II. This table summarizes the results of a series of probit models from Section 4.6.1, where the dependent variable Utilize is binary taking value 1 if individual professional forecasters utilized the central bank projection and 0 if not. A central bank projection is said to be utilized if an individual professional forecasters forecast is within 5 basis points of the respective central bank projection. The data used for estimation of the models from Section 4.6.1, where the dependent variable Utilize is binary taking value 1 if individual professional forecasters forecast is within 5 basis points of the respective central bank projection. The data used for estimation of the series of probit models stems from Stage II of Treatments 2, 3, and 4.

<table>
<thead>
<tr>
<th>Constant</th>
<th>$\mu_0$</th>
<th>$\mu_1$</th>
<th>$\mu_2$</th>
<th>$\mu_3$</th>
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<tbody>
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<td>0.035</td>
<td>0.110</td>
<td>0.090</td>
<td>0.185</td>
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<tr>
<td>0.190</td>
<td>0.090</td>
<td>0.185</td>
<td>0.090</td>
<td>0.185</td>
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<td>0.185</td>
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<tr>
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<td>0.185</td>
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</table>
Table 4.21: Determinants of the utilization of central bank projections in Stage III. This table summarizes the results of a series of probit models from Section 4.6.2, where the dependent variable \( Utilize_t \) is binary taking value 1 if individual professional forecasters utilized the central bank projection and 0 if not. A central bank projection is said to be utilized if an individual professional forecasters forecast is within 5 basis points of the respective central bank projection. The data used for estimation of the series of probit models stems from Stage III of Treatments 2, 3, and 4.

<table>
<thead>
<tr>
<th></th>
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<tbody>
<tr>
<td>constant</td>
<td>-0.798***</td>
<td>-0.481***</td>
<td>-0.923***</td>
<td>-0.983***</td>
<td>-1.012***</td>
<td>-0.892***</td>
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<tr>
<td></td>
<td>[0.070]</td>
<td>[0.082]</td>
<td>[0.141]</td>
<td>[0.152]</td>
<td>[0.130]</td>
<td>[0.136]</td>
</tr>
<tr>
<td>(</td>
<td>E_{t-2}^{pub} \pi_{t-1} - \pi_{t-1}</td>
<td>)</td>
<td>-0.314***</td>
<td>-0.258***</td>
<td>-0.273***</td>
<td>-0.277***</td>
</tr>
<tr>
<td></td>
<td>[0.052]</td>
<td>[0.058]</td>
<td>[0.061]</td>
<td>[0.061]</td>
<td>[0.056]</td>
<td>[0.061]</td>
</tr>
<tr>
<td>( Utilize_{t-2} )</td>
<td>0.826***</td>
<td>0.616***</td>
<td>0.521***</td>
<td>0.509***</td>
<td>0.498***</td>
<td>0.411***</td>
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<tr>
<td></td>
<td>[0.160]</td>
<td>[0.167]</td>
<td>[0.169]</td>
<td>[0.170]</td>
<td>[0.170]</td>
<td>[0.098]</td>
</tr>
<tr>
<td>(</td>
<td>E_{t-2}^{fc,i} \pi_{t-1} - \pi_{t-1}</td>
<td>)</td>
<td>0.059</td>
<td>0.114**</td>
<td>0.136**</td>
<td>0.139**</td>
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<tr>
<td></td>
<td>[0.050]</td>
<td>[0.055]</td>
<td>[0.057]</td>
<td>[0.057]</td>
<td>[0.056]</td>
<td>[0.056]</td>
</tr>
<tr>
<td>(</td>
<td>E_{t-2}^{fc,i} \pi_{t-1} - \pi_{t-1}</td>
<td>* Utilize_{t-2} )</td>
<td>-0.171*</td>
<td>-0.057</td>
<td>-0.085</td>
<td>-0.082</td>
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<tr>
<td></td>
<td>[0.102]</td>
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<td>[0.107]</td>
<td>[0.108]</td>
<td></td>
</tr>
<tr>
<td>(</td>
<td>\pi_{t-1} - \pi^T</td>
<td>)</td>
<td>-0.334***</td>
<td>-0.310***</td>
<td>-0.310***</td>
<td>-0.358***</td>
</tr>
<tr>
<td></td>
<td>[0.040]</td>
<td>[0.040]</td>
<td>[0.040]</td>
<td>[0.040]</td>
<td>[0.040]</td>
<td></td>
</tr>
<tr>
<td>( I_{t-1}^{cred} )</td>
<td></td>
<td>0.703***</td>
<td>0.698***</td>
<td>0.694***</td>
<td></td>
<td></td>
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<td>[0.182]</td>
<td>[0.182]</td>
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<td>( CRT )</td>
<td>0.034</td>
<td>0.026</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>[0.034]</td>
<td>[0.034]</td>
<td></td>
<td></td>
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</tbody>
</table>

| \( Cred_{Reported} \) | 0.099***     |               |               |               |               |               |
|                     | [0.015]      |               |               |               |               |               |

Note: *** denotes statistical significance at the 1% level, ** at the 5% level, and * at the 10% level.
Figure 4.3: Computer interface as seen by the subjects. The figure shows the graphical and tabular representation of the complete history of the economy as well as the timer and the input box. The exemplary subject is currently in period 3 and she is asked to provide a forecast for period 4.
Figure 4.4: Resulting aggregate time series for inflation (solid line), the output gap (dashed line), and the interest rate (dotted line) for all six experimental economies of Treatment 1 (control treatment).
**Figure 4.5:** Resulting aggregate time series for inflation (solid line), the output gap (dashed line), and the interest rate (dotted line) for all six experimental economies of Treatment 2.
Figure 4.6: Resulting aggregate time series for inflation (solid line), the output gap (dashed line), and the interest rate (dotted line) for all six experimental economies of Treatment 3.
Figure 4.7: Resulting aggregate time series for inflation (solid line), the output gap (dashed line), and the interest rate (dotted line) for all six experimental economies of Treatment 4.
Chapter 5

Fiscal Consolidations and Heterogeneous Expectations

5.1 Introduction

The recent financial crisis gave rise to various types of government interventions. These interventions led to rising government debt levels in most advanced economies (see IMF (2011)). Consequently, the majority of governments of those economies started the implementation of consolidation policies. In the Eurozone, such policies are still pursued in an effort to overcome the debt crisis in the countries of the Periphery. This crisis reshaped the way governments, within and outside the Monetary Union, think in terms of fiscal policy. In particular, the necessity for fiscal sustainability has arisen. There is a vast empirical and theoretical literature focusing on fiscal consolidations. However, little attention has been paid to the analysis of fiscal consolidations when agents are boundedly rational and heterogeneous in the way they form expectations. In this chapter, we build a framework that allows for these features and, given those, we provide further insights as regards the effects of fiscal consolidations in the economy.

Most of the theoretical literature on consolidations, so far, has assumed that agents are fully rational. The failure of traditional rational expectations models to capture some key facts in the data, especially after the recent financial crisis, raises the need for a richer modeling of economic behavior. In fact, there is very little analysis of the effects of fiscal consolidations when agents are not behaving fully rational at all times. In this chapter, we try to bridge this gap by building a closed economy New Keynesian model with distortionary taxes, where agents are boundedly rational and heterogeneous in the way they form expectations, in the spirit of Brock and Hommes (1997). Our target is to provide a framework where we can analyze the anticipation
effects of fiscal consolidations, be it tax-based or spending-based, and how heterogeneity in expectations can alter the way a given type of consolidation affects the economy both ex-ante and ex-post. In fact, we show that heterogeneity in expectations and uncertainty regarding the type of the upcoming consolidation are crucial in determining the effectiveness of consolidations in stabilizing debt and its effects on economic activity, both before and after implementation.

We assume that there are two types of agents in the economy, namely, the Fundamentalists and the Naive agents. The first type uses announced policy (e.g. monetary and fiscal) rules when it forms its expectations about inflation and output. In particular, Fundamentalists take into account the commitment of the central bank to price stability. Moreover, when forming their expectations, they take into account the commitment of the fiscal authority to stabilize debt-to-GDP when the latter exceeds a certain threshold. On the contrary, Naive agents ignore the commitments of the two authorities when forming their expectations, and use the last observations of economic variables as their forecast for the future. Notice that the Naive forecast would be optimal if inflation, output and other variables followed a random walk. The Naive forecast will therefore be nearly optimal when economic variables are highly persistent and can be described by a near unit root process. Agents can switch between the two forecasting types according to an endogenous fitness measure. Agents choose the type with the higher fitness measure (i.e. lower past forecast error). Moreover, for analytical tractability within the heterogeneous expectations framework, we assume Euler equation learning (see Honkapohja et al. (2012)), so that both types of agents need to form one period ahead expectations only.

Following Bi et al. (2013), we introduce uncertainty about the nature of fiscal consolidations. In particular, agents are uncertain about whether the consolidation will be tax-based or spending-based once debt exceeds a known debt threshold. As a result, they assign a probability to the occurrence of each type of consolidation. Given that the fiscal authority implements consolidations with a certain lag, this type of uncertainty affects expectations of Fundamentalists. This is due to the assumption that those agents are forward looking and take into account the future monetary/fiscal policy stance.

We find that tax-based consolidations lead to a quick drop in debt-to-GDP ratio in the short-run, as opposed to spending-based consolidations, when Fundamentalists anticipate spending-based consolidations. Interestingly, in the period before consolidation, output expands due to the expected spending cuts. We show that, in this case, once the tax-based consolidation is implemented, Fundamentalists realize that they were wrong when forming their expectations. This leads to a rise in the fraction
of Naive agents the next period, who expect the expansion in output to continue.\textsuperscript{62} This is equivalent to an increasing optimism in the economy in the spirit of De Grauwe (2012) and De Grauwe and Ji (2016). Consequently, they consume more, boosting output further. On the contrary, when consolidation is spending-based, output contracts abruptly upon consolidation and continues to contract throughout the consolidation. These events are also illustrated in the output multipliers. In fact, the output multiplier under tax-based consolidation shows that the latter is expansionary, owing to the increasing optimism in the economy that offsets the contractionary effects on output. The multiplier under spending-based consolidations instead points towards a persistent contraction. Interestingly, when we increase the strength of consolidations, spending-based consolidations become even more contractionary while optimism is further boosted under tax-based consolidations.

Tax-based consolidations continue to be more successful in stabilizing debt in the short-run when Fundamentalists believe that consolidations are more likely to be tax-based. We show that, in this case, output contracts in the period before implementation. As output multipliers indicate, both types of consolidation are now contractionary, with spending-based consolidations causing deeper and more persistent contractions. This is due to the increasing pessimism in the economy. Once a spending-based consolidation is implemented, Fundamentalists realize that they have been wrong in their expectations about the upcoming consolidation and hence they switch to become Naive. Given the initial contraction in output prior to implementation, this switch amplifies the contraction in output further as more and more agents now expect output to stay low. Consequently, the deep and long lasting recession delays the drop in the debt-ratio and protracts the consolidation until the ratio falls below the new threshold. Under tax-based consolidations instead, the contraction in output comes from a different channel. The rise in distortionary taxes increases marginal costs and, hence, inflation. Since monetary policy satisfies the Taylor principle, the rise in inflation leads to an increase in real interest rates which intensifies the initial contraction in output. However, Fundamentalists have anticipated correctly the type of consolidation, which does not allow pessimism in the economy to dominate. Furthermore, a tax increase today reduces the need for further tax increases. Given that the fraction of Fundamentalists stays high in this case, this affects aggregate expectations accordingly.

\textsuperscript{62}In fact, Blanchard (1990) argues that the conclusion about the contractionary effects of tax increases in Keynesian models can theoretically be overturned. For instance, a small increase in taxes today may reduce the need for further tax increases in the future. Moreover, they may signal possible tax cuts in the future which will raise households’ expected disposable income and investors’ confidence. In this case, a fiscal consolidation could stimulate private consumption and investment even in the short-run.
dampening thus the induced contraction in output. This allows the debt ratio to fall faster than under spending-based consolidations.

Our findings contribute to the existing literature in several ways. To the best of our knowledge, this is the first study to analyze fiscal consolidations when agents are boundedly rational and switch to better performing heuristics over time.63 We highlight the importance of anticipation effects, first, and, second, the degree of heterogeneity towards improving or deteriorating the performance of the consolidation, during and after implementation. We distinguish between the short and long-run effects of fiscal consolidations in terms of their performance in stabilizing debt. In line with the existing literature, we show that the magnitude, the duration, the composition and the likelihood of consolidation matter in determining the extent to which a specific type of consolidation is successful in stabilizing debt and/or is expansionary. Our major contribution, though, is that the assumption of boundedly rational agents leads to much richer dynamics and policy implications that may differ to those under rational expectations substantially.

The next section provides a brief overview of the existing empirical and theoretical literature on fiscal consolidations. Section 5.3 outlines the model and the fiscal consolidations that may occur, as well as the heterogeneity in the way agents form expectations. Sections 5.4 and 5.5 analyze the performance of the two types of consolidations under different private sector expectations, by means of theoretical results and impulse response functions, respectively. Section 5.6 concludes.

5.2 Related literature

In the literature, there is substantial research on the effects of fiscal consolidations. In particular, there has been much research on the effects of different types of fiscal consolidations (e.g. spending-based and tax-based). A large empirical literature provides evidence supporting the expansionary fiscal consolidations hypothesis (see Alesina and Perotti (1995), Perotti (1996), Alesina and Ardagna (1998, 2010), Ardagna (2004)). In particular, the key finding is that fiscal consolidations are sometimes correlated with rapid economic growth, especially when implemented by spending cuts rather than tax

---

63Erceg and Linde (2013) analyze fiscal consolidations in a currency union model with boundedly rational and heterogeneous agents. However, in their model a fixed fraction of agents consume all their income in every period and do not solve any inter-temporal optimization problem. In our model on the other hand, all agents solve an inter-temporal optimization problem given their expectations, and switch to a different expectation formation heuristic, if that heuristic has better past performance. Another distinctive feature of our model compared to theirs is the short-sightedness of Fundamentalists who are not fully rational and do not form expectations over the infinite future.
5.2. Related literature

Increases. On the other hand, another strand of the empirical literature using narrative data to identify consolidations, initially introduced by Romer and Romer (2010), finds that output drops following both types of consolidations and that recessions are deeper after tax hikes (Guajardo et al. (2014)) \(^{64}\) Similarly, Alesina et al. (2015), using a richer structure for modeling fiscal consolidations, find that spending-based consolidations are less costly, in terms of output losses than tax-based ones. However, as Guajardo et al. (2014) argue, a drawback of contemporaneous estimates is that planned impacts on budgets may tend to be over-optimistic relative to the ex-post outcomes. Consequently, the negative effects of consolidations on output may be understated due to the induced bias. This is the case with spending cuts in many instances, where the announced cuts were stronger than those actually implemented (Beetsma et al. (2016)).

As Guajardo et al. (2014) note, the alternative ways to identify fiscal consolidations used so far ignore the role of anticipation effects \(^{65}\). In the current chapter, we mainly explore this channel. We use a framework of bounded rationality and expectations heterogeneity to analyze how a specific consolidation may affect the economy upon expectation and upon implementation.

Beetsma et al. (2015) analyze the confidence effects of fiscal consolidations, showing that consumer confidence deteriorates after revenue-based consolidation announcements, in real-time, particularly in European countries. Our model also captures this fact. When agents expect tax-based consolidations to be more likely to happen, output contracts even before implementation owing to pessimism due to the drop in expected future disposable income. On the other hand, Beetsma et al. (2015) find that spending-based consolidation announcements are less harmful in terms of consumer confidence before implementation. Our model captures this finding. In particular, we show that for a high enough probability of an upcoming spending-based consolidation, output can even expand prior to implementation.

In the theoretical literature, Bertola and Drazen (1993) develop a model where the government satisfies its intertemporal budget constraint by periodically cutting spending, where the latter is inherently unsustainable. A worsening of the fiscal conditions

\(^{64}\) Earlier papers using the conventional approach to identify fiscal consolidations argued in favor of the expansionary effects of spending-based consolidations ("expansionary fiscal austerity"), see Alesina and Ardagna (2010), Alesina et al. (2002), Alesina and Perotti (1996) and Giavazzi and Pagano (1990) among others. However, their measure of identifying consolidations (i.e. the CAPB) suffers from problems like reverse causality or changes in fiscal variables due to non-policy changes correlated with other developments in output. Finally, as Romer and Romer (2010) point out another approach, followed by Blanchard and Perotti (2002) using SVAR analysis and institutional information to identify consolidations, suffers from problems similar to those of the studies above.

\(^{65}\) Alesina et al. (2015) account for anticipated consolidations. However, they do not focus on the effects of those specifically, nor do they account for heterogeneity in expectations regarding the upcoming consolidation.
can increase the probability of a beneficial fiscal consolidation which can thus be expansionary. Bertola and Drazen (1993) consider the importance of expectations in the analysis of fiscal consolidations. However, the issue of the uncertainty regarding the composition of the upcoming consolidation is not addressed, nor the importance of heterogeneity in expectations.

Bi et al. (2013) augment the model of Bertola and Drazen (1993) with distortionary taxation and analyze the effects of different types of fiscal consolidations. Moreover, they look at the effects of persistence in those, as well as of the uncertainty of economic agents over the composition of the upcoming fiscal consolidation. Accounting for the monetary policy stance as well, they find that spending and tax-based consolidations can be equally successful in stabilizing government debt at low debt levels. Nevertheless, at high debt levels, spending-based consolidations are expected to be expansionary and more successful in stabilizing debt, especially when agents anticipate a tax-based consolidation. Although the analysis in Bi et al. (2013) is richer than in Bertola and Drazen (1993), heterogeneity in expectations and its potential consequences on the effects of consolidations, before and after implementation, are not addressed here either.

Finally, Erceg and Linde (2013) examine the effects of tax-based and spending-based consolidations in a two country DSGE model for a currency union. They assume agent heterogeneity by introducing fixed fractions of forward looking and "hand-to-mouth" households. They find that tax-based consolidations have less adverse output costs than spending-based ones in the short to medium-run. Moreover, they show that large spending-based consolidations can be counterproductive in the short-run when the zero lower bound in interest rate binds, while they argue in favor of a "mixed strategy" combining both types of consolidations.

5.3 The model

5.3.1 Households

In our model time is infinite and there is a continuum of households that differ only in the way they form expectations. In particular, a household can be either Naive or Fundamentalist. Households of the same type make identical decisions. The intratemporal problem of each household $i$, consists of choosing consumption over a continuum of different goods to minimize expenditure. The elasticity of substitution between
the different goods is \( \theta \), so that households choose

\[ C_i^t(j) = \left( \frac{p_t(j)}{P_t} \right)^{-\theta} C_i^t, \]

with \( C_i^t \) and \( P_t \) being total consumption of the household and the aggregate price level, respectively, defined by

\[ C_i^t = \left( \int_0^1 C_i^t(j)^{\frac{\theta-1}{\theta}} dj \right)^{\frac{\theta}{\theta-1}}, \]

\[ P_t = \left( \int_0^1 P_t(j)^{1-\theta} dj \right)^{\frac{1}{1-\theta}}. \]

Household \( i \) chooses consumption \((C_i^t)\), labor \((H_i^t)\), and nominal bond holdings \((B_i^t)\) to maximize

\[ E_i^t \sum_{s=t}^{\infty} \beta^s \left[ \frac{(C_i^s)^{1-\sigma}}{1-\sigma} - \frac{(H_i^s)^{1+\eta}}{1+\eta} \right], \]

subject to its budget constraint

\[ P_i C_i^t + B_i^t \leq (1 - \tau_t) W_t H_i^t + (1 + i_{t-1}) B_{i-1}^t + \int_0^1 \Xi_j^t dj, \]

where \( W_t \) is the nominal wage, \( \tau_t \) is the labor tax rate, and \( i_t \) is the nominal interest rate. \( \Xi_j^t \) represents firm \( j \)'s profits while \( E_i^t \) is the type-specific expectation operator of household \( i \) (which can either be Naive or Fundamentalist).

The first order conditions with respect to \( C_i^t \), \( H_i^t \) and \( b_i^t \) lead to the usual Euler equation and to an expression for the real wage, which, together with the budget constraint (5.5), must hold in equilibrium:

\[ (C_i^t)^{-\sigma} = \beta \bar{E}_i^t \left[ \frac{(1 + i_t)(C_i^{t+1})^{-\sigma}}{\Pi_{t+1}} \right], \]

\[ w_t = \frac{(H_i^t)^{\eta}(C_i^t)^{\sigma}}{(1 - \tau_t)}, \]

where \( \Pi_t = \frac{P_t}{P_{t-1}} \) is the gross inflation rate, and \( w_t = \frac{W_t}{P_t} \) is the real wage rate.
5.3.2 Firms

There is a continuum of monopolistically competitive firms, producing the final differentiated goods. Each firm is run by a household and follows the same heuristic for prediction of future variables as the household it is run by. We assume Rotemberg pricing. Each monopolistic firm faces a quadratic cost of adjusting nominal prices, which can be measured in terms of the final good, and is given by

\[ Y_t(j) = A H_t(j), \]

where \( A \) is an aggregate productivity which we assume to be constant. The problem for firm \( j \) is then

\[ \max_{\{Y_t(j), P_t(j)\}_{t=0}^{\infty}} \sum_{s=0}^{\infty} Q_{t+s}^j \Xi_{t+s}(j), \]

subject to the demand for its product. In the expression above, the term \( Q_{t+s}^j \) represents the stochastic discount factor of the household that runs firm \( j \), while the term \( \Xi_t(j) \) denotes firm \( j \)'s aggregate nominal profits defined as

\[ \Xi_t(j) = P_t(j) Y_t(j) - mc_t Y_t(j) P_t - \frac{\phi}{2} \left( \frac{P_t(j)}{P_{t-1}(j)} - 1 \right)^2 Y_t P_t \]

\[ = P_t(j)^{1-\theta} P_t^\theta Y_t - mc_t P_t(j)^{-\theta} P_t^{1+\theta} Y_t - \frac{\phi}{2} \left( \frac{P_t(j)}{P_{t-1}(j)} - 1 \right)^2 Y_t P_t. \]

The first order condition of the maximization problem of the firm with respect to

\[^{66} \text{Since our focus is on the effects of fiscal consolidations following a drop in the debt threshold we decide to shut down all shocks in order to keep the analysis simple. However, our results hold when debt rises above an unchanged threshold due to shocks hitting the economy other than an unanticipated fall in the debt threshold.} \]
5.3. The model

\( P_t(j) \) is

\[
(1 - \theta)Y_t(j) + \theta m_c \frac{P_t(j)}{P_{t-1}(j)} Y_t(j) - \phi \left( \frac{P_t}{P_{t-1}(j)} \right) (\Pi_t(j) - 1) Y_t \\
+ \phi E_t^j \left[ Q_{t,t+1}^j Y_{t+1} \frac{P_{t+1}(j)}{P_t(j)} \Pi_{t+1}(j)(\Pi_{t+1}(j) - 1) \right] = 0,
\]

where \( \Pi_t(j) \) is the (gross) price inflation of the good produced by firm \( j \). \( m_c \) denotes the marginal cost of the firm and is equal to \( m_c = \frac{w_t}{A} \). Finally, multiplying (5.12) by \( \frac{P_t(j)}{P_y} \) and plugging in the stochastic discount factor gives

\[
(\theta - 1) \frac{P_t(j)Y_t(j)}{P_t Y_t} + \phi \Pi_t(j)(\Pi_t(j) - 1)
\]

\[
= \theta m_c \frac{Y_t(j)}{Y_t} + \phi \beta E_t^j \left[ \left( \frac{C_{t+1}^j}{C_t^j} \right)^{-\sigma} \frac{Y_{t+1}}{Y_t} \Pi_{t+1}(j)(\Pi_{t+1}(j) - 1) \right].
\]

### 5.3.3 Government and market clearing

The government issues one period bonds and levies labor taxes \( \tau \) to finance its spending \( G_t \). Its budget constraint is given by

\[
B_t = P_t G_t - \tau_t W_t H_t + (1 + i_{t-1}) B_{t-1},
\]

with \( H_t = \int H_t di \) and \( B_t = \int B_t di \) aggregate labor and aggregate bond holdings, respectively. Dividing by \( Y_t P_t \) gives

\[
b_t = g_t - \tau_t w_t \frac{H_t}{Y_t} + (1 + i_{t-1}) \frac{b_{t-1} Y_{t-1}}{\Pi_t} = g_t - \tau_t m_c + \frac{(1 + i_{t-1}) Y_{t-1}}{\Pi_t} Y_t,
\]

where \( b_t = \frac{B_t}{P_t Y_t} \) and \( g_t = \frac{G_t}{Y_t} \) are the real debt to GDP ratio and government expenditure to GDP, respectively.

When the government does not implement consolidations, we assume constant taxes \( \tau_t \) and constant government spending \( g_t \) that correspond to their steady state values. Instead, when the debt to GDP ratio rises above a certain threshold, which we denote by \( DT_t \), a consolidation is initiated and the government uses either taxes or spending in order to stabilize debt. This threshold is set by the government and ensures that the government will always have fiscal space. That is, the government wants to make sure that the debt to GDP ratio never rises to a level close to the fiscal limit of the economy. If the debt to GDP ratio is far from this fiscal limit the government is not worried about the debt to GDP ratio in the economy and does not respond to it with fiscal variables.
When the debt to GDP ratio rises above the threshold, the government starts to get concerned about its fiscal space, and implements consolidations proportional to how far the debt ratio has risen above the threshold. Fiscal policy is therefore defined by

\begin{equation}
\label{eq:5.16}
g_t = g_1 - \zeta \gamma_1 \max(0, b_{t-2} - DT_{t-2}),
\end{equation}

and

\begin{equation}
\label{eq:5.17}
\tau_t = \tau_1 + (1 - \zeta) \gamma_2 \max(0, b_{t-2} - DT_{t-2}).
\end{equation}

Here $\zeta$ is the fraction of the consolidation that is spending-based. For expository clarity, we will only consider the two extreme cases where either fully spending-based consolidations ($\zeta = 1$) or fully tax-based consolidations ($\zeta = 0$) are implemented.

The threshold ($DT_t$) may change over time for two reasons. First of all the government’s preference for its desired fiscal space may change for e.g. political reasons. It may therefore increase (lower) its threshold to start consolidations farther from (closer) to the economy’s fiscal limit. Secondly, the economy’s fiscal limit may change due to market pressures, and the government may adjust its threshold accordingly to keep the same amount of fiscal space. In this chapter we do not model these market pressures nor the fiscal limit. Instead, we consider exogenous changes in the debt threshold as a trigger for consolidations, without specifying for what reason the government decides to change this threshold.

The aggregate resource constraint of the economy is summarized as

\begin{equation}
\label{eq:5.18}
Y_t = C_t + G_t + \phi \left( \frac{P_t}{P_{t-1}} - 1 \right)^2 Y_t = C_t + g_t Y_t + \phi \left( \frac{P_t}{P_{t-1}} - 1 \right)^2 Y_t.
\end{equation}

### 5.3.4 Log-Linearization and Aggregation

We log-linearize the model around the zero inflation steady state. The Euler equation, (5.6), can be log-linearized to get

\begin{equation}
\label{eq:5.19}
\hat{\mathcal{C}}(i)_t = E_t^{\infty}[\hat{\mathcal{C}}(i)_{t+1}] - \frac{1}{\sigma}(i_t - E_t^{\infty}[\pi_{t+1}]).
\end{equation}

\footnote{We specify the zero inflation steady state in Appendix 5.A.1
\footnote{In what follows, for all the variables normalized with respect to GDP (debt, government purchases, federal expenditure, tax revenues) $\tilde{x}_t$ denotes a linear deviation ($\tilde{x}_t = X_t - \bar{X}$) from its steady state. Instead, for all other variables $\hat{x}_t$ denotes a percentage deviation ($\hat{x}_t = \log(X_t/\bar{X})$) from its steady state. This distinction avoids having the percentage change of a percentage. As regards inflation, we denote the log-linearized gross inflation $\Pi_t$ as $\pi_t$.}

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As in Chapter 2 of this thesis, we assume that our boundedly rational agents use Euler equation learning (see Honkapohja et al. (2012)), implying that they use the two period trade-off of (5.19) to make decisions given their subjective forecasts of next period.

**Assumption 5.1.** Agents realize they may switch to another heuristic in the future, and that other agents may do so as well. They furthermore assume that the probability to follow a particular heuristic next period is the same across agents.

Agents with the same expectations will make the same consumption decision. It therefore follows from Assumption 5.1 that, from the perspective of any agent, its own expected future consumption is the same as its expected aggregate future consumption, that is, $E^i_t[\hat{C}(i)_{t+1}] = E^i_t[\hat{C}_{t+1}]$, with $\hat{C}_{t+1} = \int_0^1 \hat{C}(i)_{t+1} di$. Agents therefore realize they should base their current period consumption decision on expectations about aggregate consumption. The Euler equation can then be written as

\[
(5.20) \quad \hat{C}(i)_t = E^i_t[\hat{C}_{t+1}] - \frac{1}{\sigma}(i_t - E^i_t[\pi_{t+1}]).
\]

Log linearizing the market clearing condition, (5.18), around the zero inflation steady state gives

\[
(5.21) \quad \hat{Y}_t = \hat{C}_t + \frac{\hat{g}_t}{1 - \bar{g}}.
\]

Agents know about market clearing and their forecasts satisfy $E^i_t[\hat{C}_{t+1}] = E^i_t[\hat{Y}_{t+1}] - \frac{1}{1 - \bar{g}} E^i_t[\hat{g}_{t+1}]$. Therefore, (5.20) can be written as

\[
(5.22) \quad \hat{C}(i)_t = E^i_t[\hat{Y}_{t+1}] - \frac{1}{\sigma}(i_t - E^i_t[\pi_{t+1}]) - \frac{1}{1 - \bar{g}} E^i_t[\hat{g}_{t+1}].
\]

Aggregating this equation over all agents, and using the period $t$ market clearing condition then gives

\[
(5.23) \quad \hat{Y}_t = \bar{E}_t[\hat{Y}_{t+1}] - \frac{1}{\sigma}(i_t - \bar{E}_t[\pi_{t+1}]) - \frac{1}{1 - \bar{g}} (\bar{E}_t[\hat{g}_{t+1}]) - \bar{g}_t).
\]

Here $\bar{E}_t$ is the aggregate expectation operator defined by $\bar{E}_t[X_{t+1}] = n_t^N \bar{E}^N_t[X_{t+1}] + (1 - n_t^N) E^F_t[X_{t+1}]$, with $n_t^N$ the fraction of Naive agents.

Log-linearizing the optimal pricing equation (5.13) and combining it with the market clearing condition and the marginal cost equations in their log-linearized form we
end up with the inflation equation

\begin{equation}
\pi_t = \beta \tilde{E}_t[\pi_{t+1}] + \kappa (\sigma + \eta) \tilde{Y}_t - \kappa \sigma \frac{\tilde{g}_t}{1 - \tilde{g}} + \kappa \frac{\tilde{\tau}_t}{1 - \tilde{\tau}},
\end{equation}

where $\kappa = \frac{\theta - 1}{\varphi}$.

Finally, the government budget constraint, (5.15), can be log-linearized to get an equation for the evolution of the government debt to GDP ratio

\begin{equation}
\tilde{b}_t = \tilde{g}_t - \frac{1}{\theta} \left( \tilde{\tau}_t + \tilde{\tau} \tilde{mc}_t \right) + \frac{1}{\beta} \tilde{b}_{t-1} + \frac{\tilde{b}}{\beta} (\tilde{i}_{t-1} - \pi_t - \tilde{Y}_t + \tilde{\tau}_t - \bar{\tau}),
\end{equation}

with $\tilde{mc} = (\sigma + \eta) \tilde{Y}_t - \sigma \frac{\tilde{g}_t}{1 - \tilde{g}} + \frac{\tilde{\tau}_t}{1 - \tilde{\tau}}$.

We assume the central bank targets only inflation and that the inflation target is zero (which is consistent with the assumption of a zero inflation steady state that was assumed in the log-linearization in the previous section). The log-linearized forward looking Taylor rule is given by

\begin{equation}
\tilde{i}_t = \phi_\pi \tilde{E}_t[\pi_{t+1}].
\end{equation}

Linearizing (5.16) and (5.17) gives

\begin{equation}
\tilde{g}_t = -\zeta \gamma_1 \max(0, \tilde{b}_{t-2} - \tilde{DT}_{t-2}),
\end{equation}

and

\begin{equation}
\tilde{\tau}_t = (1 - \zeta) \gamma_2 \max(0, \tilde{b}_{t-2} - \tilde{DT}_{t-2}).
\end{equation}

### 5.3.5 Expectations Formation

We assume private sector beliefs are formed by two heuristics: Fundamentalistic and Naive. Naive agents comprise a fraction $n_t^N$ of the population and believe future inflation, output and government spending to be equal to their last observed values: $E_t^N \tilde{Y}_{t+1} = \tilde{Y}_{t-1}$, $E_t^N \pi_{t+1} = \pi_{t-1}$, $E_t^N \tilde{g}_{t+1} = \tilde{g}_{t-1}$. Their expectations about future taxes do not show up in the equations of our model, and also do not influence Naive agents’ predictions about other variables. We therefore do not need to specify these expectations.

Fundamentalists comprise a fraction $1 - n_t^N$ of the population and know that future taxes and government spending will depend on the debt ratio. They therefore base
their expectations about these variables on the debt ratio. We assume that before a consolidation has taken place, they do not know whether the consolidation will be tax-based or spending-based, i.e., they do not know the value of $\zeta$. In any period $t$, they think it will be spending-based ($\zeta = 1$) with probability $\alpha_t$. Furthermore if it is spending-based, they expect the magnitude of the consolidation to be proportional to the excess debt, with coefficient $\gamma_1$. Government spending expectations are therefore given by

$$E_t g_{t+1} = -\alpha_t \gamma_1 \max(0, \hat{b}_{t-1} - \hat{D}T_{t-1}).$$

(5.29)

Similarly, they expect that consolidations are tax-based ($\zeta = 0$) with probability $1 - \alpha_t$. Taxes are then proportionally increased in response to excess debt with coefficient $\gamma_2$:

$$E_t \tilde{\tau}_{t+1} = (1 - \alpha_t) \gamma_2 \max(0, \hat{b}_{t-1} - \hat{D}T_{t-1}).$$

(5.30)

After they have observed a period of consolidations, Fundamentalists update their value of $\alpha_t$ using Bayesian updating. Since we have assumed that $\zeta$ only takes the values 1 or 0, this implies that after observing a period of spending-based consolidations Fundamentalists update their belief to $\alpha_t = 1$, and after observing a period of tax-based consolidations they update their belief to $\alpha_t = 0$.

Fundamentalists furthermore take account of their expectations about next period taxes and government spending when they form expectations about the other variables (they do this simultaneously). They do not use long horizon forecasts in their decision making process and they do not think about how the fiscal variables may change after two or more periods. Instead they calculate the perfect foresight fixed point values of inflation and output, corresponding to the fiscal regime they expect to be implemented next period. They know the model equations of inflation and output, and know the specification of the nominal interest rate rule.

Fundamentalists know that there are agents in the model that form expectations in a different manner. However, they do not know how many such agents there are in a given period and they also do not know what values other agents will predict. They can therefore not take these other agents into account in there expectation formation process. Instead Fundamentalists expect the values that would occur if all agents made correct (perfect foresight) predictions. That said, Fundamentalists’ expectations about inflation and output at date $t$ about $t + 1$ are as follows:\footnote{Full derivation of these expectations are shown in Appendix 5.A.2}
(5.31) \[ E_t^F \pi_{t+1} = 0, \]

(5.32) \[ E_t^F \hat{Y}_{t+1} = -\left( \frac{\sigma}{\eta + \sigma \gamma_{1}} \alpha_1 \gamma_{1} + \frac{1}{\eta + \sigma \gamma_{2}} (1 - \alpha_t) \gamma_{2} \right) \max(0, \hat{b}_{t-1} - \tilde{D}T_{t-1}). \]

Since Fundamentalists make a joint prediction about all variables, the fractions of agents following this heuristic must be based on the relative performance of all predictions. Since expectations about taxes do not influence agents’ decisions directly, it should not be included in the fitness measure. Similarly, the fraction of Naive agents is also based on the performance of all predictions. The most natural fitness measure then would be

(5.33) \[ U^i_{t-1} = -(\hat{g}_{t-1} - E^{i}_{t-2} \hat{g}_{t-1})^2 - (\pi_{t-1} - E^{i}_{t-2} \pi_{t-1})^2 - (\hat{Y}_{t-1} - E^{i}_{t-2} \hat{Y}_{t-1})^2, \]

where \( i = F, N \). Following the fitness measure above, the fraction of Naive agents evolves as in Brock and Hommes (1997), according to

(5.34) \[ n^N_t = \frac{e^{\omega U^N_{t-1}}}{e^{\omega U^N_{t-1}} + e^{\omega U^F_{t-1}}}, \]

with \( U^N_{t-1} \) and \( U^F_{t-1} \) given by (5.33) evaluated at the Naive predictions and Fundamentalistic predictions respectively. \( \omega \) is the intensity of choice parameter that determines how sensitive agents are to past performance of heuristics and how fast they switch between heuristics.

Above we assumed that agents know the model equations and are able to calculate the fixed point values that would arise if all agents made the ”correct” forecasts. This however does not mean that these theoretically correct forecasts are indeed good predictors of the future. Due to heterogeneity in expectations formation, the above fixed point will not necessarily be reached. Instead, it is possible that the presence of Naive predictors in combination with shocks to the economy causes completely different dynamics. Naive predictors may then perform better than Fundamentalists. This would cause more Fundamentalists to abandon their model, since this model turned out not to be good enough to make adequate predictions about the actual law of motion of the economy. The fraction of Naive agents would then increase and waves of optimism or pessimism could arise.
5.3.6 Complete model

Our system is piecewise linear. The equation for inflation and output depend on the level of the debt to GDP ratio in the last two periods, even though the debt ratio does not show up in the equations explicitly.

1. When debt is low \((\tilde{b}_t < \tilde{D}_T_t < \tilde{b}_{t-1} < \tilde{D}_T_{t-1})\) we obtain, by plugging in monetary and fiscal policy in (5.23) and (5.24), the following system for inflation and output

\[
\hat{Y}_t = \bar{E}_t[\hat{Y}_{t+1}] - \frac{\phi_\pi - 1}{\sigma} \bar{E}_t[\pi_{t+1}] - \frac{1}{1 - g} \bar{E}_t[\hat{g}_{t+1}],
\]

\[
\pi_t = \beta \bar{E}_t[\pi_{t+1}] + \kappa(\sigma + \eta) \hat{Y}_t.
\]

In this region of low debt, Fundamentalists expect all variables to be at their steady state value of 0, so that aggregate expectations are given by

\[
\bar{E}_t \tilde{g}_{t+1} = n_t^N \tilde{g}_{t-1},
\]

\[
\bar{E}_t \pi_{t+1} = n_t^N \pi_{t-1},
\]

\[
\bar{E}_t \hat{Y}_{t+1} = n_t^N \hat{Y}_{t-1}.
\]

2. When debt has just crossed the critical boundary, but consolidation is not yet implemented \((\tilde{b}_{t-2} < \tilde{D}_T_{t-2}, \tilde{b}_{t-1} > \tilde{D}_T_{t-1})\), then (5.35), (5.36) and (5.38) still hold, while for aggregate government spending expectations we then have

\[
\bar{E}_t \tilde{g}_{t+1} = n_t^N \tilde{g}_{t-1} - (1 - n_t^N) \alpha_t \gamma_1 (\tilde{b}_{t-1} - \tilde{D}_T_{t-1}).
\]

This is because Fundamentalists expect a consolidation to start in the next period. As regards aggregate expectations about output, they are formed as follows

\[
\bar{E}_t Y_{t+1} = n_t^N Y_{t-1} - (1 - n_t^N) \left( \frac{\sigma}{1 - \tilde{g}} \alpha_t \gamma_1 + \frac{1}{1 - \tau} (1 - \alpha_t) \gamma_2 \right) (\tilde{b}_{t-1} - \tilde{D}_T_{t-1}).
\]

3. When both \(\tilde{b}_{t-2} > \tilde{D}_T_{t-2}\) and \(\tilde{b}_{t-1} > \tilde{D}_T_{t-1}\) expectations are again given by
(5.38), (5.40) and (5.41), respectively. However, current output and inflation are now equal to

\[ (5.42) \quad \hat{Y}_t = \frac{\phi_x - 1}{\sigma} E_t[\hat{Y}_{t+1}] - \frac{1}{1 - \tilde{g}} \left( E_t[\hat{g}_{t+1}] + \zeta \gamma_1 (\tilde{b}_{t-2} - DT_{t-2}) \right), \]

\[ (5.43) \quad \pi_t = \beta E_t[\pi_{t+1}] + \kappa (\sigma + \eta) \hat{Y}_t + \kappa \left( \zeta \frac{\sigma \gamma_1}{1 - g} + (1 - \zeta) \frac{\gamma_2}{1 - \tau} \right) (\tilde{b}_{t-2} - DT_{t-2}). \]

4. At some point, the consolidation has worked through, and the debt to GDP ratio falls again below the critical threshold. One period later, a consolidation is no longer expected for the future, but still implemented in the current period (since \( \tilde{b}_{t-2} > DT_{t-2} \) but \( \tilde{b}_{t-1} < DT_{t-1} \)). In that case, expectations are given by (5.37), (5.38) and (5.39), while current output and inflation are given by (5.42) and (5.43).

## 5.4 Consolidations

When the debt ratio is above the debt threshold consolidations are first expected and then also implemented. That is, we are first in case 2 and then in case 3 of the previous section. Below, we analyze the dynamics that arise when consolidations are necessary, and how these dynamics depend on the type of consolidation (spending or tax-based), the strength of consolidations (\( \gamma_1 \) and \( \gamma_2 \)), and agents’ initial beliefs. The latter consists of the fraction of agents that are Naive (\( n_t^N \)) and the initial probability Fundamentalists place on consolidations being spending-based (which we call \( \alpha^* \)).

More specifically, the following quantities are affected: the levels of variables in the period where Fundamentalists expect a consolidation, but where it is not yet implemented (Section 5.4.1); the levels of variables during the implementation of consolidations (Section 5.4.2); and finally, the existence and stability (largest eigenvalue) of a fixed point above the debt threshold, as well as the debt ratio corresponding to that fixed point (Section 5.4.3). The first two subsections thus study short run dynamics, while the latter studies medium to long run dynamics. Short run dynamics are affected by initial beliefs (\( n_t^N \) and \( \alpha^* \)), while the effect of these initial conditions die out in the long run.
5.4. Consolidations

5.4.1 Effects of expected consolidations due to a shock to debt or the debt threshold

If, in period \( t \), debt is above the debt threshold for the first time, then in period \( t + 1 \), consolidations are expected by Fundamentalists, but not yet implemented. The actual type of consolidation then does not matter yet, but instead dynamics are driven by the type of consolidation that Fundamentalists expect. Depending on what type Fundamentalists expect, a consolidation can lead to either an expansion or a contraction in output. Furthermore, if the expansion is large enough, expected consolidations lead to a reduction in debt. We formalize this result in Proposition 5.1.

**Proposition 5.1.** Assume that a shock to debt or to the debt threshold has driven the debt to GDP ratio above its threshold in period \( t \) \( (\bar{b}_t > \bar{DT}_t) \). This affects next period’s output (and thereby also inflation and debt) through Fundamentalists’ expectations about future government spending and future output. The effect of a debt or debt threshold shock on next period’s output is given by

\[
\frac{\partial \hat{Y}_{t+1}}{\partial (\bar{b}_t - \bar{DT}_t)} = (1 - n_t^N) \left( \frac{\eta}{\sigma + \eta} \frac{\alpha^* \gamma_1}{1 - \bar{g}} - \frac{1}{\sigma + \eta} \frac{1}{1 - \bar{\tau}} (1 - \alpha^*) \gamma_2 \right).
\]

This implies that the effect of expected consolidations on output and inflation is positive, if and only if

\[
\alpha^* > \frac{\gamma_2(1 - \bar{g})}{\gamma_1(1 - \bar{\tau})\eta + \gamma_2(1 - \bar{g})}.
\]

When this condition does not hold and the expectations lead to a contraction, then expected consolidation results in an increase in debt. When Condition (5.45) holds, the expected consolidation leads to a reduction in debt if and only if the expansion in output is large enough, so that it satisfies

\[
\frac{\partial \hat{Y}_{t+1}}{\partial (\bar{b}_t - \bar{DT}_t)} > \frac{1 - \beta}{\beta \left( (\bar{\tau}(\sigma + \eta))^{\frac{\alpha^* - 1}{\sigma}} + \frac{\bar{b}}{\beta}(1 + \kappa(\sigma + \eta)) \right)}.
\]

**Proof.** In Appendix 5.B.1.

It follows from Proposition 5.1 that it is desirable that agents mainly expect spending-based consolidations. The higher the probability Fundamentalists place on spending-based consolidations, the larger the expansion of output, and the lower the debt to GDP ratio in the period before the consolidation is actually implemented. Furthermore, as
can be seen in equation (5.44), when Fundamentalists do mainly expect spending-based consolidations, the expansion in output and the reduction in the debt ratio are increasing in the fraction of Fundamentalists \((1 - n_t^N)\).

### 5.4.2 Effects of implemented consolidations

If, after one period, debt still is above the debt threshold, the expectational effects analyzed above are still present two periods after the shock. However, in addition there now are direct effects of implemented consolidations on output, inflation and debt. Below we analyze the total effects (including both the expectational and direct effects) of consolidations in the period that they are implemented for the first time, i.e., two periods after the shock. Proposition 5.2 states that in this period, a tax-based consolidation is always more effective than a spending-based consolidation.

**Proposition 5.2.** For tax-based and spending-based consolidations of equal direct impact on the government's budget deficit \((\gamma_1 = \frac{\theta - 1}{\theta} \gamma_2)\), a tax-based consolidation always results in lower debt than a spending-based consolidation in the first period of implementation. Moreover, the difference in the impact on debt \(\left(\frac{\partial b_{t+2}}{\partial (b_t - DT_t)}\right)\) between spending-based and tax-based consolidations is given by

\[
(5.47) \quad \gamma_1 \left( \frac{\theta - 1}{\theta} + \frac{\bar{\tau}^{\theta - 1} - \bar{g}}{1 - \beta} \kappa \right) \left( \frac{\theta}{(\theta - 1)(1 - \bar{\tau})} + \frac{\eta}{1 - \gamma} \right) + \frac{\bar{\tau}^{\theta - 1} - \bar{g}}{1 - \beta} \frac{1}{1 - \bar{g}}
\]

**Proof.** In Appendix 5.B.2.

Expression (5.47) is always positive, so that, in the period where it is implemented for the first time, the debt ratio falls faster following a tax-based consolidation than a spending-based one. Moreover, it can immediately be seen that the difference in the debt ratio is increasing in the magnitude of consolidations \((\gamma_1)\) and in steady state taxes \((\bar{\tau})\). Note furthermore, that the difference in reducing debt under the two types of consolidations does not depend on initial expectations \((n_t^N \text{ and } a^*)\). The reason for this is that these expectations affect the evolution of variables in the economy equally under both types of implemented consolidations.

### 5.4.3 Medium and long run dynamics

Above we explicitly analyzed the levels of variables in the first two periods after the debt or debt threshold shock. This gives clear insights in the short run dynamics that result form such a shock. Medium to long run dynamics will be determined by the existence
and stability of a fixed point in the high debt region of the model. In this section, we investigate under what conditions this fixed point exists and how its stability is affected by the strength of consolidation, $\gamma_1$ and $\gamma_2$. One period after the first implementation of consolidations, Fundamentalists have learned the type of consolidation and update their belief to $\alpha_t = 1$ or to $\alpha_t = 0$. For long run dynamics the value of $\alpha^*$ (their initial belief) therefore does not matter. Furthermore, the initial fraction of Naive agents ($n_i^N$) does not matter either for long run dynamics, because in a fixed point both types of agents will perform equally well, and the fractions will converge to 0.5 each.

Existence high debt fixed point

Proposition 5.3 states the condition for a fixed point with high debt to exist, in case of spending-based consolidations.

**Proposition 5.3. When consolidations are spending-based, a fixed point above the debt threshold exists if and only if**

$$\gamma_1 > \frac{1}{\beta} - 1. \tag{5.48}$$

**Proof.** In Appendix 5.B.3.

Proposition 5.4 states that in case of tax-based consolidations, the condition for existence of the high debt fixed point is the same as in case of spending-based consolidations, but with $\gamma_1$ replaced by $\gamma_2 \frac{\theta - 1}{\theta}$. This is intuitive because if we let $\gamma_1 = \gamma_2 \frac{\theta - 1}{\theta}$, tax-based and spending-based consolidation are of equal magnitude in their effect on the budget deficit.

**Proposition 5.4. When consolidations are tax-based, a fixed point above the debt threshold exist if and only if**

$$\gamma_2 \frac{\theta - 1}{\theta} > \frac{1}{\beta} - 1. \tag{5.49}$$

**Proof.** In Appendix 5.B.4.

We can conclude from Propositions 4 and 5 that when consolidations are strong enough (making fiscal policy "passive" above the debt threshold in the terminology of Leeper, 1991), an extra fixed point exists above the debt threshold, in addition to the fixed point below the debt threshold where all variables are at the steady state levels of Appendix 5.A.1.
CHAPTER 5. CONSOLIDATIONS AND HETEROGENEOUS EXPECTATIONS

Stability of the high debt fixed point

When the fixed point above the debt threshold exists, dynamics in this region of the model are determined by the stability properties of the fixed point. When it is locally stable, convergence to the fixed point may occur, while otherwise debt will move away from the fixed point. Both convergence and divergence can happen monotonically or in an oscillatory fashion, depending on whether the eigenvalues are real and positive, or complex/negative. In order to get insight in dynamics when the debt ratio is above the threshold, we calculate the eigenvalues in the fixed point numerically. For this, we need to calibrate the model. Unless otherwise stated, we use the parameter values given in Table 5.1.

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<th>Parameter</th>
<th>Name</th>
<th>Value</th>
</tr>
</thead>
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<td>Discount factor</td>
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<tr>
<td>$\sigma$</td>
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<td>Frisch elasticity of labor supply</td>
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<td>Elasticity of substitution</td>
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<tr>
<td>$\phi_\pi$</td>
<td>Coefficient on inflation in Taylor rule</td>
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<tr>
<td>$\omega$</td>
<td>Intensity of choice</td>
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</tr>
<tr>
<td>$\phi$</td>
<td>Price adjustment costs</td>
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<tr>
<td>$\bar{g}$</td>
<td>Steady state government spending</td>
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</tr>
<tr>
<td>$\bar{\tau}$</td>
<td>Steady state taxes</td>
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</tr>
<tr>
<td>$\bar{b}$</td>
<td>Steady state debt</td>
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</tr>
<tr>
<td>$DT_t$</td>
<td>Deviation of debt threshold from steady state</td>
<td>0.14</td>
</tr>
</tbody>
</table>

Table 5.1: Parameter values

In the top two panels of Figure 5.1 the absolute value of the largest eigenvalues in the high debt fixed point for spending-based (left panels) and tax-based (right panels) are plotted as a function of $\gamma_1 = \frac{\theta - 1}{\theta} \gamma_2$ (the strength of consolidations). In the dashed dotted part of the curves, the largest eigenvalue is real and positive. From the previous section we know that for very low values of $\gamma_1$ and $\gamma_2$, the fixed point above the debt threshold does not exist. For this reason we do not plot the largest eigenvalue for $\gamma_1 < \frac{1}{\beta} - 1$. It can be seen in Figure 5.1, that under both types of consolidations the largest eigenvalue is real and equal to unity for the lowest allowed value of $\gamma_1$. As $\gamma_1$ (and $\gamma_2$) go up, this real eigenvalue decreases and the fixed point becomes more and more stable, implying faster monotonic convergence to the fixed point. Around $\gamma_1 = 0.2$ the eigenvalues are quite low in absolute value under both spending-based and tax-based consolidations.

In the solid part of the eigenvalue curves, the largest eigenvalues (in absolute value) are complex. Here, the dotted line depicts the real part of these largest eigenvalues. When eigenvalues are complex, cyclical dynamics arise. This implies for our model that
when the debt ratio is above the debt threshold, it will first decrease towards the fixed point, and then overshoot this debt ratio level and possibly also the debt threshold. When this happens, consolidations have been successful and no further spending cuts or tax increases are necessary up until the point that the debt ratio increases above the debt threshold again.\textsuperscript{70} It is therefore of interest how many periods it takes before the overshooting occurs. This is proportional to how many periods it would take the dynamical system to complete a full cycle. The latter can be calculated based on the angle between the real and the imaginary part of the largest eigenvalues, and is plotted in the bottom two panels of Figure 5.1.

First, considering the case of spending-based consolidations in the bottom left panel, it can be seen that for relatively low values of $\gamma_1$ the cycle length is quite large. In this case, dynamics will initially look like monotonic convergence and only after a consid-

\textsuperscript{70}The time it takes before the debt ratio increases above the threshold again does not depend on the dynamical system analyzed in this section. Instead, it depends on the system in case 1 of section 5.3.6. Here fiscal policy is active (taxes and spending do not respond to debt) and debt will always monotonically increase as long as $b_t > 0$. 

---

**Figure 5.1:** Absolute values of largest eigenvalue (top panels) and lengths of a full cycle in case of oscillatory dynamics (bottom panels) in high debt fixed point for spending and tax-based consolidations. In the top panels a dashed-dotted segment depicts a real and positive eigenvalue, while a solid segment depicts complex eigenvalues. The real part of the complex eigenvalues is plotted as a dotted curve. In case of tax-based consolidations it holds that $\gamma_2 = \frac{\theta}{\sigma-1} \gamma_1$. 

---


erable amount of periods an overshooting of the debt threshold will occur. However, as $\gamma_1$ gets larger, the cycle length decreases, and consolidations will reduce the debt ratio to a level below its threshold relatively quickly. Additionally, it can be seen in the top left panel of Figure 5.1 that the largest eigenvalue increases in absolute value as $\gamma_1$ increases. This means that not only overshooting occurs faster when consolidations are more aggressive, but also that debt will overshoot the threshold by more, leading to a lower debt ratio when consolidations are over.

In the bottom right panel of Figure 5.1 it can be seen that the cycle length for tax-based consolidations is initially larger than for spending-based consolidations. However, when $\gamma_1$ is larger than approximately 0.3, the cycle length becomes considerably shorter. Furthermore, in the top right panel of Figure 5.1 it can be seen that, in this case, the largest eigenvalue is larger in absolute value under tax-based than under spending-based consolidations. This implies that tax-based consolidations are considerably more effective in reducing debt than spending-based consolidations when the government responds relatively aggressive to debt.

In Appendix 5.C we show that these qualitative results are robust to the specification of monetary policy, and to the parameterization of price adjustment costs ($\phi$) and relative risk aversion ($\sigma$).

**Debt ratio in the high debt fixed point**

When the high debt fixed point is stable, the level of the debt ratio in this fixed point is of crucial importance. When this debt ratio lies very close to the debt threshold, the government might be content with convergence to the fixed point. However, if the debt ratio lies considerably above the threshold, convergence to the fixed point is not desirable.

From the proofs of Proposition 5.3 and 5.4 it follows that, when the high debt fixed point exists and is stable, a more aggressive policy (higher $\gamma_1$, $\gamma_2$) leads to a lower debt ratio in the fixed point. Combining this with the results of the previous section, we can conclude that the government should respond strongly enough to debt. If it responds too weakly, slow monotonic convergence to a debt ratio significantly above the government’s threshold will occur.

**5.5 Impulse responses to a debt threshold shock**

In this section we analyze the effects of a one time permanent drop to the debt threshold, both in the short run and in the long run. We show the difference between
5.5. **Impulse responses to a debt threshold shock**

spending-based and tax-based consolidations and the difference between strong and weak responses to debt. We also look at how these results depend on agents’ expectations.

Since our model is linear in expectation fractions, impulse responses are only affected by the fractions of Naive agents in the initial two periods, and not by other initial conditions (like the level of output when the shock hits). We therefore let the initial fractions of Naive agents vary over a two dimensional grid to obtain both a median impulse response, and an upper and lower bound on the impulse responses.\(^\text{71}\)

In Figure 5.2, the impulse responses of output, inflation and the debt to GDP ratio are plotted for the case where the government implements spending-based consolidations, and Fundamentalists also expected mostly spending-based consolidations (\(\alpha^* = 0.8\)). Before period 0, the debt threshold is set at 80% of GDP which is higher than the initial debt ratio of 75%, while from period 0 onwards the debt threshold is lowered to 70% of GDP. The debt threshold is indicated by the blue lines in the bottom two panels. Black lines indicate median impulse responses, and upper and lower bounds are given by the dashed-dotted red curves.

First consider the left column of panels. Here, the reaction coefficient to debt is given by \(\gamma_1 = 0.2\). In the top left panel, we see that in period 1, the period that consolidation is expected but not yet implemented, an expansion in output occurs. This is in line with Proposition 5.1, since we are considering \(\alpha^* = 0.8\). As can be seen in Equation (5.44), the effect of the shock on output in period 1 is scaled by the fraction of Fundamentalists. When all agents are Naive in period 1, nobody expects consolidations to occur in period 2 and nothing happens in period 1. When all agents are Fundamentalists, they all expect consolidations, which results in an expansion in output. The middle left panel shows that the effect on inflation is similar to that of output, which also is in line with the theoretical results of Section 5.4.1. In the bottom left panel, it can be seen that the expansion in output and inflation causes a decline in debt. However, when all agents are Naive, there is no expansion, and debt slightly increases in the first period due to interest rate payments (top red dashed line).

In period 2, government spending is lowered, which leads to a sharp decline in output, and a (less severe) decline in inflation. This initially causes a slight increase in debt. However, in the long run debt decreases towards the high debt fixed point, which

\(^{71}\)In all impulse responses that we plot in this chapter, the bounds are constructed as follows. Given the sensitivity of the results to the initial fractions of agents and also given that the only shock in the economy is a one time drop in the debt threshold, we simulate the model 121 times (24 periods each), allowing for the fraction of Naive agents in the first and the second period to vary, independent of each other, from 0 to 1 with a grid interval of 0.1. We get the lower bound by extracting the minimum impulse response value in each period. The upper bound is created in the same way.
Figure 5.2: Impulse response to debt threshold shock of output, inflation and debt for spending-based consolidation when $\alpha^* = 0.8$. Impulse responses depend on initial fractions of Naive agents. The median is plotted in solid black and the maximum and minimum are plotted in dotted red. The debt threshold (after the shock) is plotted in blue in the bottom two panels. The left column depicts the case of a moderately weak fiscal response and the right column the case of a strong fiscal response. 

lies slightly above the debt threshold. Meanwhile, output and inflation converge to their fixed point values, close to 0. From Section 5.4.3 we know that for spending-based consolidations with low $\gamma_1$, the largest eigenvalue in the high debt fixed point is real and positive. In the bottom left panel of Figure 5.2 we indeed see that debt converges monotonically to its fixed point value, and that it does not cross the debt threshold. Therefore, even though debt is considerably reduced after 10 quarters, the government will keep responding to debt.

Turning to the right column of panels of Figure 5.2, where the policy coefficient is increased to $\gamma_1 = 0.5$, we see that debt overshoots both its fixed point value and the
5.5. Impulse responses to a debt threshold shock

debt threshold in period 5. This is in line with the finding of Section 5.4.3 that the largest eigenvalue now is complex. Even though the debt ratio is eventually reduced faster and to a lower level, it can be seen from the upper bound (red dashed line) that in the first two periods debt could increase considerably more than under weaker fiscal policy. On the other hand, looking at the lower bound, it is also possible that debt is already decreased by a lot in the first period of consolidation. The intuition for the wider bounds in the impulse responses is that the stronger consolidations magnify expectation effects caused by Naive and Fundamentalists, both prior and during the consolidations. For the same reason, the sign of the response of output to the shock is the same as in the case of $\gamma_1 = 0.2$, but the upper and lower bound now lie much further apart. Furthermore, the contraction in output is deeper due to the stronger spending cuts.

We can conclude that when Fundamentalists expect a spending-based consolidation to be more likely, a stronger response will typically be more effective in stabilizing debt than a weaker response. However, a stronger response can also lead to sharper fluctuations in economic aggregates (depending on the initial fractions of agents that are Naive), as well as a deeper recession.

In Figure 5.3, the impulse responses to a debt threshold shock are plotted for the case where the government implements a tax-based consolidation, while Fundamentalists mostly expected spending-based consolidations ($\alpha^* = 0.8$). In period 1, no consolidation is implemented yet, so dynamics are the same as in Figure 5.2 (note the difference in scale on the y-axis though). In period 2, when the consolidation is implemented, debt is immediately reduced. This is in line with Proposition 5.2, which says that, initially, tax-based consolidations always lead to lower debt than spending-based consolidations. Additionally, we see that there is no contraction in output now (neither initially nor in the long run), which is also more desirable. Overall, a tax-based consolidation is clearly preferable when agents expect a spending-based consolidation. This also holds for the case of stronger consolidations, as can be seen in the right column of Figure 5.3. Here, a larger increase in inflation and output leads to a very fast reduction of debt, and to considerable overshooting of the debt threshold.

In order to further explore the effects of the two types of consolidations both in short-run and the longer-run, we compute the impact and the present value output multipliers.\footnote{We compute the present value fiscal multipliers as in Mountford and Uhlig (2009) and Bi et al. (2013) where the multiplier $\Gamma_y^{\pi}$ = $\sum_{j=0}^{k} \left( \prod_{i=0}^{j} r_{t+i}^{-1} \right) \left( Y_{t+j}^{\text{shock}} - Y_{t+j}^{\text{no}} \right) / \sum_{j=0}^{k} \left( \prod_{i=0}^{j} r_{t+i}^{-1} \right) \left( x_{t+j}^{\text{shock}} \right)$, where $r_t$ is the real interest rate, and $x$ denotes the type of fiscal consolidation: $x_t = \tau_r \bar{H} \bar{w}$ (change in tax income due to tax rate change) for tax-based consolidations and $x_t =$}
CHAPTER 5. CONSOLIDATIONS AND HETEROGENEOUS EXPECTATIONS

Figure 5.3: Impulse response to debt threshold shock of output and debt for tax-based consolidation when $\alpha^* = 0.8$. Impulse responses depend on initial fractions of Naive agents. The median is plotted in solid black and the maximum and minimum are plotted in dotted red. The debt threshold (after the shock) is plotted in blue in the bottom two panels. The left column depicts the case of a moderately weak fiscal response and the right column the case of a strong fiscal response.

and tax-based consolidations for $\alpha^* = 0.8$. When agents anticipate spending-based consolidations, and the government in fact implements it, the spending cuts generate a persistent contraction in output. When the government implements consolidations through tax hikes instead, there is an expansionary effect on output, as the multiplier is positive both upon implementation and in the subsequent quarters. This is

$-G_t$ for spending-based ones. With our log-linear approximations these multipliers reduce to

$$\Gamma_t^{\pi} = \sum_{j=0}^k \left( \prod_{i=0}^{r_t^{-1}_t} \left( \hat{Y}_{t+j} \right) / \sum_{j=0}^k \left( \prod_{i=0}^{r_t^{-1}_t} \left( \hat{\pi}^\theta_{t+j} \right) \left( \hat{\pi}^{\theta}_{t+j} \right) \left( \hat{\pi}^{\theta}_{t+j} \right) \right) \right)$$

for tax-based and

$$\Gamma_t^{\pi} = \sum_{j=0}^k \left( \prod_{i=0}^{r_t^{-1}_t} \left( \hat{Y}_{t+j} \right) / \sum_{j=0}^k \left( \prod_{i=0}^{r_t^{-1}_t} \left( \hat{\pi}^\theta_{t+j} \right) \left( \hat{\pi}^{\theta}_{t+j} \right) \left( \hat{\pi}^{\theta}_{t+j} \right) \right) \right)$$

for spending-based consolidations.

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5.5. Impulse responses to a debt threshold shock

Table 5.2: Output Multipliers

<table>
<thead>
<tr>
<th>Panel a: $\alpha^* = 0.8$</th>
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<th>4qr</th>
<th>8qr</th>
<th>12qr</th>
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<tbody>
<tr>
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<td>-0.78</td>
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<table>
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because of the anticipation effects and a switch in the way the majority of agents form their expectations. In Proposition 5.1, we showed that for a high enough probability of a spending-based consolidation and a positive fraction of fundamentalists, output expands before implementation. When the consolidation is implemented through tax hikes instead, Fundamentalists realize that they have been wrong in the past regarding their expected type of consolidation. As a result, more agents switch now to using the Naive rule to form their expectations. But, with the previous expansion in output, such a switch is equivalent to an increased optimism in the economy, making the initial expansion in output more persistent and increasing the tax base further. This leads to a positive output multiplier following the tax cuts. Additionally, the expansionary effect on output due to agent heterogeneity is strengthened further by the fact that a tax increase today reduces the need for future tax increases. The expansionary effect of the consolidation in this case is in line with Blanchard (1990), who argues that the contractionary effects of consolidations in a Keynesian model can theoretically be overturned, if a tax increase today can generate confidence in the economy. This is because a small increase in taxes today may reduce the need for a larger fiscal adjustment later, which can stimulate consumption and investment in the short-run.

From Table 5.2, we observe that the expansionary effect of tax-based consolidations improves for stronger tax hikes (i.e. higher $\gamma_1$), while the contractionary effect of a spending-based consolidation is worsened. However, the effect on the output multiplier under tax-based consolidations is significantly more pronounced. In fact, bigger tax hikes increase the fraction of Fundamentalists that switch to the Naive rule. Given the initial increase in output prior to consolidation, even more agents now expect output to keep on expanding. This increasing optimism translates into higher output multipliers.
Figure 5.4: Impulse response to debt threshold shock of output and debt for spending-based consolidation when $\alpha^* = 0.2$. Impulse responses depend on initial fractions of Naive agents. The median is plotted in solid black and the maximum and minimum are plotted in dotted red. The debt threshold (after the shock) is plotted in blue in the bottom two panels. The left column depicts the case of a moderately weak fiscal response and the right column the case of a strong fiscal response.

for $\gamma_1 = 0.5$ under tax-based consolidations. Under spending-based consolidations instead a higher $\gamma_1$ increases pessimism in the economy. However, only few agents switch to the Naive rule and this switch is transitory. Therefore, the multiplier is not substantially affected.

But what if agents mostly expect tax-based consolidation? Figures 5.4 and 5.5 plot impulse responses with spending-based and tax-based consolidation respectively when $\alpha^* = 0.2$. First of all, as formalized in Proposition 5.1, there is now a contraction instead of an expansion in output in period 1. Secondly, in Period 2, the debt ratio increases in case of a spending-based consolidation, but decreases in case of a tax-based
5.5. Impulse responses to a debt threshold shock

Figure 5.5: Impulse response to debt threshold shock of output and debt for tax-based consolidation when $\alpha^* = 0.2$. Impulse responses depend on initial fractions of Naive agents. The median is plotted in solid black and the maximum and minimum are plotted in dotted red. The debt threshold (after the shock) is plotted in blue in the bottom two panels. The left column depicts the case of a moderately weak fiscal response and the right column the case of a strong fiscal response.

In panel (b) of Table 5.2, we display the output multipliers for $\alpha^* = 0.2$. When Fundamentalists expect tax-based consolidations to be more likely, the multipliers change significantly. When the consolidation turns out to be spending-based, the contraction in output is now deeper than before. Fundamentalists are wrong about the type of the consolidation. Consequently, the fraction of Naive agents increases, dampening output further and making the recession deeper. The most significant change in the multipliers though, is when consolidations turn out to be tax-based. Now the multipliers are negative, meaning that tax-based consolidations are contractionary, contrary to the
case where $\alpha^* = 0.8$. Fundamentalists are now correct in their expectations about both the type of the upcoming consolidation and also its contractionary effects on output. However, the contraction is milder compared to spending-based consolidations.

Turning to debt dynamics in Figures 5.4 and 5.5, we see that the debt ratio is reduced at a slower pace compared to the case where agents expect mainly spending-based consolidators. This holds both for spending- and taxed-based consolidations, and for $\gamma_1 = 0.2$ as well as for $\gamma_1 = 0.5$. This reflects the lower tax revenues caused by the lower output levels discussed above. If we compare spending-based and tax-based consolidations, it can clearly be seen that tax-based consolidations are still more effective in reducing debt, in the short run as well as the long run. Overall we can conclude that tax-based consolidations remain better than spending-based consolidations, also when agents mainly expect tax-based consolidations.

### 5.6 Conclusions

In this chapter we have explored the effects of fiscal consolidations when agents are heterogeneous and uncertain about the composition of the consolidations. We assumed heterogeneity in the way expectations are formed in the spirit of Brock and Hommes (1997). Agents can switch between two types, namely, the Fundamentalist and the Naive. The former type consisted of forward looking agents that trust the commitments of the government, whereas the latter consisted of backward looking agents.

The fiscal authority was assumed to engineer a consolidation once the debt ratio exceeds an announced debt threshold, with a lag. Consolidations were implemented either through spending cuts or tax increases. Prior to the consolidation agents were uncertain about the composition. Given the lag in implementation, such uncertainty affected only the way Fundamentalists formed their expectations, while leaving the expectations of Naive agents unaffected.

Our first finding was that tax-based consolidations outperform spending-based ones in the short-run, leading to an abrupt fall in the debt to GDP ratio. We showed that the type of consolidation that was anticipated was crucial in determining whether the consolidation would trigger expansions or abrupt contractions in output so long as it lasts. Moreover, whether the type of consolidation anticipated was correct or not, determined its duration. Consolidations last longer when agents wrongly anticipate them to be tax-based, but they turn out to be spending-based. This is due to the persistent contraction in output triggered by the implemented spending cuts and expected tax hikes. When agents anticipate spending-based consolidations, debt can also be more
effectively decreased by tax hikes than by spending cuts. This is because anticipated spending cuts followed by tax hikes can create a wave of optimism and a boom in output.

The model was complex in its dynamics and we kept the analysis as simple as possible. Cases like the effect of the zero lower bound on the potential of a consolidation to be expansionary and/or successful in stabilizing debt, in a heterogeneous agents model, deserve further research.
Appendix 5.A  Derivations

5.A.1  Zero inflation steady state

In this section, we derive the steady state of the non-linear model, where gross inflation equals 1, and where we normalize technology to $A = 1$.

Evaluating (5.13) at the zero inflation steady state gives

$$\bar{mc} = \frac{\theta - 1}{\theta}. \tag{5.50}$$

From (5.6) it follows that in this steady state we must have

$$1 + \bar{i} = \frac{1}{\beta}. \tag{5.51}$$

Furthermore, from (5.9) it follows that

$$\bar{H} = \bar{Y}. \tag{5.52}$$

Next, we solve the steady state aggregate resource constraint, (5.18), for consumption, and write

$$\bar{C} = \bar{Y}(1 - \bar{\eta}). \tag{5.53}$$

Plugging in these steady state labor and consumption levels in the steady state version of (5.7) gives

$$\bar{w} = \frac{\bar{Y}^\eta(\bar{Y}(1 - \bar{\eta}))^\sigma}{1 - \bar{\tau}} = \frac{\bar{Y}^{\eta+\sigma}(1 - \bar{\eta})^\sigma}{1 - \bar{\tau}} = \frac{\theta - 1}{\theta}, \tag{5.54}$$

where the last equality follows from the fact that $\bar{w} = \bar{mc}$ and from (5.50). We can thus write

$$\bar{Y} = \left(\frac{\theta - 1}{\theta} \cdot \frac{1 - \bar{\tau}}{(1 - \bar{\eta})^\sigma}\right)^{\frac{1}{\eta+\sigma}}. \tag{5.55}$$

Then we turn to the government budget constraint. In steady state (5.15) reduces to

$$\bar{b} = \bar{g} - \bar{\tau} \frac{\theta - 1}{\theta} + (1 + \bar{i})\bar{b}. \tag{5.56}$$
which gives

\[ \bar{b} = \beta \frac{(\bar{\tau} - 1 - \bar{g})}{1 - \beta}, \]  

where we used (5.51) to substitute for the interest rate.

Steady state government spending and taxes are given by

\[ \bar{g} = g_1 - \zeta_1 \max(0, \bar{b} - \bar{DT}), \]  

and

\[ \bar{\tau} = \tau_1 + (1 - \zeta) \gamma_2 \max(0, \bar{b} - \bar{DT}). \]  

Assuming that the steady state debt threshold equals steady state debt, this reduces to

\[ \bar{g} = g_1, \]  

and

\[ \bar{\tau} = \tau_1. \]  

5.A.2 Derivation of Fundamentalists’ expectations about output and inflation

From (5.26) it follows that in a state of the economy where all variables except debt are constant over time, we must have

\[ i = \phi \pi. \]  

Furthermore, (5.23) reduces in such a fixed point state to

\[ Y = Y - \frac{1}{\sigma}(i - \pi) - \frac{1}{1 - g}(g - g), \]  

from which it follows that

\[ \pi = i. \]
(5.62) and (5.64) can only both hold (assuming $\phi_\pi \neq 1$) if

\[
\pi = i = 0.
\]

Therefore, Fundamentalists inflation expectations satisfy $E_t^F \pi_{t+1} = 0$. Using this in (5.24), it follows that

\[
0 = \kappa(\sigma + \eta)Y - \sigma\kappa \frac{g}{1-g} + \kappa \frac{\tau}{1-\tau}.
\]

Output expectations of Fundamentalists therefore satisfy

\[
E_t^F \hat{Y}_{t+1} = \frac{\sigma}{\sigma + \eta} \frac{E_t^F \hat{y}_{t+1}}{1-g} - \frac{1}{\sigma + \eta} \frac{E_t^F \tilde{\tau}_{t+1}}{1-\tilde{\tau}}.
\]

Plugging in (5.29) and (5.30) gives

\[
E_t^F \hat{Y}_{t+1} = -(1 - n_{t+1}^N) \alpha^* \gamma_1.
\]

Similarly it follows from (5.41) that

\[
\frac{\partial E_{t+1} \hat{y}_{t+2}}{\partial (b_t - DT_t)} = -(1 - n_{t+1}^N) \frac{1}{\sigma} \left( \frac{\sigma}{1-g} \alpha^* \gamma_1 + \frac{1}{\eta} \frac{\sigma}{\eta} (1 - \alpha_t) \gamma_2 \right) < 0.
\]

Meanwhile, leading (5.35) one period, we obtain

\[
\frac{\partial \hat{y}_{t+1}}{\partial (b_t - DT_t)} = \left( \frac{\partial E_{t+1} \hat{y}_{t+2}}{\partial (b_t - DT_t)} - \frac{1}{\alpha} \frac{\partial E_{t+1} \tilde{\tau}_{t+2}}{\partial (b_t - DT_t)} \right)
\]

\[
= (1 - n_{t+1}^N) \left( \frac{\eta}{\sigma + \eta} \alpha^* \frac{\gamma_1}{1-g} - \frac{1}{\sigma + \eta} \frac{1}{1-\tau} (1 - \alpha^*) \gamma_2 \right).
\]

It follows that the effect of a debt (or debt threshold) shock on next periods output

\[
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\]
is positive, if and only if
\[ \eta \alpha^* \gamma_1 \frac{1}{1 - \bar{g}} > \frac{1}{1 - \bar{\tau}} (1 - \alpha^*) \gamma_2. \]

When \( \alpha^* = 1 \) this will always hold and when \( \alpha^* = 0 \) it will never hold. Solving for \( \alpha^* \) gives
\[ \alpha^* > \frac{\gamma_2 (1 - \bar{g})}{\gamma_1 (1 - \bar{\tau}) \eta + \gamma_2 (1 - \bar{g})}. \]

Next we turn the effect of a debt (threshold) shock on debt on the other variables of the model. Using (5.36), we have
\[ (5.72) \quad \frac{\partial \pi_{t+1}}{\partial (\tilde{b}_t - \tilde{DT}_t)} = \kappa (\sigma + \eta) \frac{\partial \hat{Y}_{t+1}}{\partial (\tilde{b}_t - \tilde{DT}_t)}. \]

Therefore, the effect on inflation is positive, if and only the effect of the shock on output is positive.

For debt we have, using (5.25),
\[ (5.73) \quad \frac{\partial \tilde{b}_{t+1}}{\partial (\tilde{b}_t - \tilde{DT}_t)} = \frac{1}{\beta} - \left( (\bar{\tau}(\sigma + \eta)) \frac{\theta - 1}{\theta} + \frac{\bar{b}}{\beta} \right) (1 + \kappa(\sigma + \eta)) \frac{\partial \hat{Y}_{t+1}}{\partial (\tilde{b}_t - \tilde{DT}_t)}. \]

We can conclude that if (5.72) is not satisfied (and the debt (threshold) shock leads to a lower output level), then an increase in \( \tilde{b}_{t-1} \) implies a more than one for one increase in \( \tilde{b}_t \), so that \( \tilde{b}_t > \tilde{b}_{t-1} \) and consolidation expectations lead to an increase in debt. If, on the other hand, (5.72) is satisfied, then the expectations of consolidations may reduce period \( t+1 \) debt compared to period \( t \) debt. This happens if and only if
\[ (5.74) \quad \frac{\partial \tilde{b}_{t+1}}{\partial (\tilde{b}_t - \tilde{DT}_t)} = \frac{1}{\beta} - \left( (\bar{\tau}(\sigma + \eta)) \frac{\theta - 1}{\theta} + \frac{\bar{b}}{\beta} \right) (1 + \kappa(\sigma + \eta)) \frac{\partial \hat{Y}_{t+1}}{\partial (\tilde{b}_t - \tilde{DT}_t)} < 1, \]

or
\[ (5.75) \quad \frac{\partial \hat{Y}_{t+1}}{\partial (\tilde{b}_t - \tilde{DT}_t)} > \frac{1 - \beta}{\beta ((\bar{\tau}(\sigma + \eta)) \frac{\theta - 1}{\theta} + \frac{\bar{b}}{\beta} (1 + \kappa(\sigma + \eta)))}. \]
5.B.2 Proof Proposition 5.2

We first assume spending-based consolidation. Leading (5.35) and taking the derivative, we can write

\[
\frac{\partial \hat{Y}_{t+2}}{\partial (b_t - DT_t)} = \frac{\partial \hat{Y}_{t+2}}{\partial b_{t+1}} \frac{\partial \hat{b}_{t+1}}{\partial (b_t - DT_t)} + \frac{\sigma \kappa \gamma_1}{1 - \bar{g}} + \beta n^N_{t+2} \frac{\partial \hat{Y}_{t+1}}{\partial (b_t - DT_t)} + \frac{\phi_x - 1}{\sigma} n^N_{t+2} \frac{\partial \pi_{t+1}}{\partial (b_t - DT_t)}. \tag{5.76}
\]

Here, the first term represents the effect of the expectation of fundamentalists, taking account of the fact that these are only effected by the most recent value of the debt ratio. The second term embodies the direct effect of implemented consolidations. The third and fourth term contain the effect that the changes in output and inflation in the anticipation period have on Naive expectations one period later.

Similarly, we obtain for inflation

\[
\frac{\partial \pi_{t+2}}{\partial (b_t - DT_t)} = \frac{\partial \pi_{t+2}}{\partial b_{t+1}} \frac{\partial \dot{b}_{t+1}}{\partial (b_t - DT_t)} + \frac{\sigma \kappa \gamma_1}{1 - \bar{g}} + \beta n^N_{t+2} \frac{\partial \pi_{t+1}}{\partial (b_t - DT_t)} + \kappa (\sigma + \eta) \frac{\partial \hat{Y}_{t+2}}{\partial (b_t - DT_t)}. \tag{5.77}
\]

\[
\frac{\partial \hat{Y}_{t+2}}{\partial b_{t+1}} \frac{\partial \hat{b}_{t+1}}{\partial (b_t - DT_t)} \]

\[
\text{can be obtained by replacing } n^N_{t+1} \text{ by } n^N_{t+2} \text{ in (5.71). Updating (5.72) analogously gives }
\]

\[
\frac{\partial \hat{Y}_{t+2}}{\partial \hat{b}_{t+1}} \frac{\partial \hat{b}_{t+1}}{\partial (b_t - DT_t)} \frac{\partial \hat{b}_{t+1}}{\partial (b_t - DT_t)} \frac{\partial \hat{b}_{t+1}}{\partial (b_t - DT_t)} \]

Finally, we have

\[
\frac{\partial \dot{b}_{t+2}}{\partial (b_t - DT_t)} = -\gamma_1 + \frac{1}{\beta} \frac{\partial \dot{b}_{t+1}}{\partial (b_t - DT_t)} - \frac{\theta - 1}{\theta} \frac{\partial \hat{m}_{t+2}}{\partial (b_t - DT_t)} + \frac{\bar{b}}{\beta} \left( \frac{\hat{Y}_{t+1}}{\partial (b_t - DT_t)} - \frac{\hat{Y}_{t+2}}{\partial (b_t - DT_t)} - \frac{\partial \pi_{t+2}}{\partial (b_t - DT_t)} \right), \tag{5.78}
\]

with

\[
\frac{\partial \hat{m}_{t+2}}{\partial (b_t - DT_t)} = (\sigma + \eta) \frac{\hat{Y}_{t+2}}{\partial (b_t - DT_t)} \frac{\partial \dot{b}_{t+1}}{\partial (b_t - DT_t)} + \frac{\sigma \gamma_1}{1 - \bar{g}} + (\sigma + \eta) \frac{\hat{Y}_{t+2}}{\partial (b_t - DT_t)}. \tag{5.79}
\]
We can therefore write

\[
\frac{\partial \tilde{b}_{t+2}}{\partial (b_{t} - DT_{t})} = -\gamma_{1} + \frac{1}{\beta} \frac{\partial \tilde{b}_{t+1}}{\partial (b_{t} - DT_{t})} - \left( \frac{\theta - 1}{\theta} + \frac{b}{\beta} \kappa \right) \frac{\sigma \gamma_{1}}{1 - \tilde{g}} - \left( \frac{\tau (\sigma + \eta) + \tilde{b} \beta (1 + \kappa (\sigma + \eta))}{\beta} \right) \frac{\partial \tilde{Y}_{t+2}}{\partial (b_{t} - DT_{t})} \]

\[
- \left( \frac{\tau (\sigma + \eta)}{\theta} + \frac{b}{\beta} (1 + \kappa (\sigma + \eta)) \right) \frac{\partial \tilde{Y}_{t+2}}{\partial (b_{t} - DT_{t})} + \frac{b}{\beta} (1 - \beta n_{t+2} \kappa (\sigma + \eta)) \frac{\partial \hat{Y}_{t+1}}{\partial (b_{t} - DT_{t})}.
\]

In case of tax-based consolidation we get

\[
\frac{\partial \tilde{Y}_{t+2}}{\partial (b_{t} - DT_{t})} = \frac{\partial \tilde{Y}_{t+2}}{\partial b_{t+1}} \frac{\partial b_{t+1}}{\partial (b_{t} - DT_{t})} + n_{t+2} \frac{\partial \tilde{Y}_{t+1}}{\partial (b_{t} - DT_{t})} - \frac{\phi_{\pi} - 1}{\sigma} n_{t+2} \frac{\partial \pi_{t+1}}{\partial (b_{t} - DT_{t})},
\]

\[
\frac{\partial \pi_{t+2}}{\partial (b_{t} - DT_{t})} = \frac{\partial \pi_{t+2}}{\partial b_{t+1}} \frac{\partial b_{t+1}}{\partial (b_{t} - DT_{t})} + \frac{\kappa \gamma_{2}}{1 - \tilde{\tau}} + \beta n_{t+2} \frac{\partial \pi_{t+1}}{\partial (b_{t} - DT_{t})} + \kappa (\sigma + \eta) \frac{\partial \tilde{Y}_{t+2}}{\partial (b_{t} - DT_{t})},
\]

and

\[
\frac{\partial \tilde{b}_{t+2}}{\partial (b_{t} - DT_{t})} = -\theta - 1 \frac{\partial \tilde{b}_{t+1}}{\partial (b_{t} - DT_{t})} - \frac{\theta - 1}{\theta} \frac{\partial \hat{m}_{c_{t+2}}}{\partial (b_{t} - DT_{t})} + \frac{b}{\beta} \left( \frac{\partial \tilde{Y}_{t+1}}{\partial (b_{t} - DT_{t})} - \frac{\partial \tilde{Y}_{t+2}}{\partial (b_{t} - DT_{t})} - \frac{\partial \pi_{t+2}}{\partial (b_{t} - DT_{t})} \right),
\]

\[
\frac{\partial \hat{m}_{c_{t+2}}}{\partial (b_{t} - DT_{t})} = \frac{\partial \hat{m}_{c_{t+2}}}{\partial b_{t+1}} \frac{\partial b_{t+1}}{\partial (b_{t} - DT_{t})} + \frac{\gamma_{2}}{1 - \tilde{\tau}} + (\sigma + \eta) \frac{\partial \tilde{Y}_{t+2}}{\partial (b_{t} - DT_{t})}.
\]
So that we can write

\[
\frac{\partial \hat{b}_{t+2}}{\partial (b_t - DT_t)} = -\frac{\theta - 1}{\theta} \gamma_2 + \frac{1}{\beta} \frac{\partial \hat{b}_{t+1}}{\partial (b_t - DT_t)} - \left( \frac{\theta - 1}{\theta} + \frac{\bar{b}}{\beta} \right) \frac{\gamma_2}{1 - \bar{\tau}} - \left( \frac{\theta - 1}{\theta} \left( \sigma + \eta \right) + \frac{\bar{b}}{\beta} \kappa (\sigma + \eta) \right) \frac{\partial \bar{Y}_{t+2}}{\partial (b_t - DT_t)} \frac{\partial \hat{b}_{t+1}}{\partial (b_t - DT_t)} - \left( \frac{\theta - 1}{\theta} \left( \sigma + \eta \right) + \frac{\bar{b}}{\beta} (1 + \kappa (\sigma + \eta)) \right) \frac{\partial \bar{Y}_{t+2}}{\partial (b_t - DT_t)} \frac{\partial \hat{Y}_{t+1}}{\partial (b_t - DT_t)} + \frac{\bar{b}}{\beta} \left( 1 - \beta n_t^N \kappa (\sigma + \eta) \right) \frac{\partial \hat{Y}_{t+1}}{\partial (b_t - DT_t)}.
\]

Subtracting the tax-based debt derivative from the spending-based debt derivative results in

\[
- (\gamma_1 - \frac{\theta - 1}{\theta} \gamma_2) - (\bar{\tau} \frac{\theta - 1}{\theta} + \frac{\bar{b}}{\beta} \kappa) \left( \frac{\sigma \gamma_1}{1 - \bar{g}} - \frac{\gamma_2}{1 - \bar{\tau}} \right) + \left( \frac{\bar{\tau} (\sigma + \eta)}{\theta} + \frac{\bar{b}}{\beta} (1 + \kappa (\sigma + \eta)) \right) \gamma_1 \frac{1}{1 - \bar{g}}.
\]

Assuming consolidations of equal impact on the budget deficit \((\gamma_1 - \frac{\theta - 1}{\theta} \gamma_2)\), this reduces to

\[
\gamma_1 \left( \frac{\bar{\tau} \theta - 1}{\theta} + \frac{\bar{b}}{\beta} \kappa \right) \left( \frac{\theta}{(\theta - 1)(1 - \bar{\tau})} + \frac{\eta}{1 - \bar{g}} \right) + \frac{\bar{b}}{\beta} \frac{1}{1 - \bar{g}}.
\]

This implies that a spending-based consolidation leads to a higher debt than a tax-based consolidation after two periods. Substituting for steady state debt we get

\[
\gamma_1 \left( \frac{\bar{\tau} \theta - 1}{\theta} + \frac{\bar{\tau} \theta - 1}{\theta} - \bar{g} \right) \left( \frac{\theta}{(\theta - 1)(1 - \bar{\tau})} + \frac{\eta}{1 - \bar{g}} \right) + \frac{\bar{\tau} \theta - 1}{\theta} - \bar{g} \frac{1}{1 - \beta} \frac{1}{1 - \bar{g}}.
\]

5.B.3 Proof Proposition 5.3

We assume steady state levels and we assume a spending-based consolidation is implemented. In a fixed point where Fundamentalists have correctly updated their belief to \(a_t = 1\) inflation and output satisfy

\[
\pi(1 - \beta n^N) = \kappa (\sigma + \eta) Y + \kappa \sigma \frac{b - DT}{1 - \bar{g}}.
\]
5.B. Proofs of propositions

\[(5.89) \quad (1 - n_t^N)Y = -(1 - n_t^N)\frac{1}{\eta + \sigma} \gamma_1(b - DT) \frac{\phi_\pi - 1}{\sigma} - n_t^N \pi.\]

Solving this two equations shows that fixed point inflation and marginal cost are zero and fixed point output is given by the Fundamentalists expected value:

\[(5.90) \quad Y = \frac{1}{\eta + \sigma} \frac{\gamma_1(b - DT)}{1 - \bar{g}}.\]

For debt we have in the fixed point where marginal cost and inflation are zero:

\[(5.91) \quad b = -\gamma_1(b - DT) + \frac{1}{\beta} b.\]

The fixed point debt ratio therefore is

\[(5.92) \quad b = \frac{DT\gamma_1}{1 - \frac{1}{\beta} + \gamma_1}.\]

This is indeed a fixed point when the fixed point debt ratio lies above the debt threshold. This is the case if and only if

\[(5.93) \quad \gamma_1 > \frac{1}{\beta} - 1.\]

5.B.4 Proof Proposition 5.4

In tax case of tax-based consolidations that are fully expected by Fundamentalists \((\alpha_t = 0)\) we again have a fixed point where marginal cost and inflation are zero. Output is now given by

\[Y = -\frac{1}{\eta + \sigma} \frac{\gamma_2(b - DT)}{1 - \bar{\tau}}.\]

For debt we now have

\[(5.94) \quad b = \frac{DT\gamma_2 \frac{\theta - 1}{\theta}}{1 - \frac{1}{\beta} + \frac{\theta - 1}{\theta}}.\]

This is indeed a fixed point when this debt ratio lies above the debt threshold

The condition now becomes

\[(5.95) \quad \gamma_2 \frac{\theta - 1}{\theta} > \frac{1}{\beta} - 1.\]
Appendix 5.C  Robustness largest eigenvalues

In this section, we look at the robustness of the largest eigenvalue results of Section 5.4.3 to the chosen parametrization. In Figure 5.6, we plot the case of weaker monetary policy, where $\phi_\pi = 1.1$ instead of $\phi_\pi = 1.5$. This figure looks very similar to Figure 5.1, so monetary policy does not seem to have a big impact on dynamics. This is because agents are short-sighted, heterogeneous and form expectations for one period ahead only. As such, changes in the commitment of the central bank to a specific rule have a smaller effect on expectations and thereby on the dynamics of the model. In Bi et al. (2013) instead, agents forecast over an infinite horizon and there is no heterogeneity in expectations. There, a less active monetary policy makes tax increases more expansionary after the consolidation, while it makes spending cuts much more contractionary.

Figure 5.6: Absolute values of largest eigenvalue (top panels) and lengths of a full cycle in case of oscillatory dynamics (bottom panels) in high debt fixed point for spending and tax-based consolidations. In the top panels a dashed-dotted segment depicts a real and positive eigenvalue, while a solid segment depicts complex eigenvalues. The real part of the complex eigenvalues is plotted as a dotted curve. Finally, negative real eigenvalues are represented by a dashed segment. In case of tax-based consolidations it holds that $\gamma_2 = \frac{\theta}{g-\gamma_1} \gamma_1$. In the Figure, the benchmark calibration is used, but with $\phi_\pi = 1.1$ instead of $\phi_\pi = 1.5$. 

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In Figure 5.7, we plot the case where price adjustment costs are equal to $\phi = 10$ rather than $\phi = 100$. This leads to a much flatter Phillips curve. Similarly, Figure 5.8 plots the case where the relative risk aversion parameter is $\sigma = 0.157$ instead of $\sigma = 2$.

**Figure 5.7:** Absolute values of largest eigenvalue (top panels) and lengths of a full cycle in case of oscillatory dynamics (bottom panels) in high debt fixed point for spending and tax-based consolidations. In the top panels a dashed-dotted segment depicts a real and positive eigenvalue, while a solid segment depicts complex eigenvalues. The real part of the complex eigenvalues is plotted as a dotted curve. Finally, negative real eigenvalues are represented by a dashed segment. In case of tax-based consolidations it holds that $\gamma_2 = \frac{\theta}{\theta - 1} \gamma_1$. In the Figure, the benchmark calibration is used, but with $\phi = 10$ instead of $\phi = 100$. 
Figure 5.8: Absolute values of largest eigenvalue (top panels) and lengths of a full cycle in case of oscillatory dynamics (bottom panels) in high debt fixed point for spending and tax-based consolidations. In the top panels a dashed-dotted segment depicts a real and positive eigenvalue, while a solid segment depicts complex eigenvalues. The real part of the complex eigenvalues is plotted as a dotted curve. Finally, negative real eigenvalues are represented by a dashed segment. In case of tax-based consolidations it holds that $\gamma_2 = \theta - 1\gamma_1$. In the Figure, the benchmark calibration is used, but with $\sigma = 0.157$ instead of $\sigma = 2$.

In both Figures, qualitative results for reasonable values of $\gamma_1$ are the same as in Section 5.4.3. That is, for low values of $\gamma_1$ slow monotonic convergence occurs for both spending and tax-based consolidations due to a large positive eigenvalue. As $\gamma_1$ decreases, the eigenvalue becomes smaller and eventually the largest eigenvalues are complex. As in Figure 5.1, the cycle length is initially shorter under spending-based consolidations, but as $\gamma_1$ increases more, tax-based consolidations become more effective in reducing debt.

Finally, it turns out that as $\gamma_1$ becomes very large, the largest eigenvalue under spending-based consolidations is real and negative (denoted by the dashed line). This implies immediate overshooting of the debt ratio to a level below the debt threshold. Such an extreme drop in government debt may however not be very realistic.
Chapter 6

Fiscal Consolidations and Finite Planning Horizons

6.1 Introduction

The effects of fiscal consolidation on output and their effectiveness in stabilizing debt have attracted much attention in the empirical literature to date. The recent debt crisis in the Eurozone gave rise to further research as regards the effects of the composition of fiscal consolidations. It appears that a long standing debate is still active regarding this issue. The debate focuses on the distinct effects of tax-based and spending-based fiscal consolidations. The theoretical literature has also contributed to that debate, mostly arguing in favor of spending-based fiscal consolidations due to their expansionary effect and hence their effectiveness in stabilizing debt faster (Bi et al. (2013)). Apart from the contribution of Blanchard (1985), the theoretical literature has ignored, to a great extent, the implication of households’ finite planning horizons on the effects of consolidations. In this chapter, we try to fill this gap and take the analysis a step further by constructing a theoretical model where agents are boundedly rational with finite planning horizons, and by looking at the effects of such behavior on aggregate macro variables. Comparing the model with the rational expectations benchmark, we show that planning horizons are important for the behavior of output and inflation following a fiscal consolidation, with short planning horizons leading to completely different policy implications than the rational expectations paradigm.

Blanchard (1985) examines the dynamic effects of government deficit finance when the horizon of households is finite and is a parameter chosen arbitrarily. Our contribution is different in many respects. First, we assume that agents are infinitely lived but have finite planning horizons. However, they care about the level of their wealth at
the end of their planning horizon. This structure generates wealth effects that are not present in Blanchard’s approach. In particular, households derive utility from the level of their wealth at the end of their planning horizon. This implies that households aim to decide upon this level of wealth optimally. In Blanchard’s structure this channel is absent. Moreover, we are interested in the interactions between monetary and fiscal policy. Therefore, we incorporate our approach into a New-Keynesian framework. In our model, households are identical, they do not have different ages, they have the same levels of wealth and the same marginal propensities to consume. This facilitates aggregation in our framework. Our approach not only captures the finite horizon aspect, but also the changes in the behavior of households. We capture changes in consumption decisions - equivalent to time inconsistency of consumption plans - by assuming that households are uncertain about the state of the economy after their planning horizon and /or learn in a recursive manner about the true valuation of their end-of-horizon wealth. Finally, we consider both tax-based and spending-based fiscal consolidations and analyze their effects on output, inflation and the debt ratio. Moreover, instead of assuming lump-sum taxes as in Blanchard, we assume distortionary income taxes.

In our model, we assume that the government announces consolidations one period in advance. First, we look at the effects of consolidations for different horizons. We find that when agents are short-sighted - plan for three quarters ahead - consumption falls following spending cuts. This is in line with the empirical literature on the sign of consumption responses following spending cuts (see Blanchard and Perotti (2002), Fatas and Mihov (2001) and Gali et al. (2007) among others). The intuition is that they weigh more changes in their current wealth and they care less about their future disposable income and future interest rates. One of the major weaknesses of New-Keynesian models without labor market distortions and rule-of-thumb consumers as argued in Gali et al. (2007), is that they do not capture this positive co-movement.

73 Blanchard introduces a probability of death in order to induce equal horizons across individuals and equal marginal propensity to consume. Otherwise, the relation among different levels and compositions of wealth and different propensities to consume makes aggregation impossible.

74 In that respect our approach is closer to the life-cycle formulation by Modigliani than to that of permanent income by Milton Friedman. In Blanchard (1985) instead the opposite holds. Moreover, in Blanchard, the assumption of a constant probability of death implies that the objective function of households does not change over time. Therefore, there is no time inconsistency of initial optimal programs.

75 Our approach is closer to myopic individual consumption decision which is time inconsistent. That is, actual consumption at date $t$ may be different from what the household had previously planned to consume at that date. This is because at any date $t$, the household plans its consumption accounting for elements not considered in its plans computed previously. For a more detailed analysis of those issues see Lovo and Polemarchakis (2010) and the references therein.
between government spending and household consumption. Instead, in these models consumptions increases following spending cuts, which is also what we find for longer planning horizons. However, for short planning horizons, our model is able to capture the positive co-movement between private consumption and government spending observed in the data without introducing labor market frictions and/or rule-of-thumb households.

Following tax-based consolidations, private consumption drops, regardless of the planning horizon. However, the decline in consumption is smaller as the planning horizon increases. This is due to the positive wealth effect due to lower expected future taxes once the consolidation is over. For short horizons, though, the drop in current wealth due to the consolidation leads to a larger decline in consumption owing to a weaker wealth effect.

The responses of consumption have important implications for the output costs of both spending- and tax-based consolidations. In both cases, when agents have short planning horizons, consolidations can initiate considerably bigger recessions than would have occurred if agents had longer or infinite planning horizons. This is also reflected in the present value multipliers. The effects on inflation are sensitive to the length of planning horizons as well. For shorter horizons persistently lower demand leads to persistent deflationary pressures, under both types of consolidations. What is interesting though, is that the response of inflation under tax-based consolidations changes sign for very short horizons. Even though tax hikes increase marginal costs, which in any standard New-Keynesian model would push inflation up, inflation falls both upon anticipation and upon implementation. In this case, the demand effect is stronger than the supply side effect with the drop in household consumption more than offsetting the increase in marginal costs, leading to a persistent drop in inflation. Finally, the debt ratio is not very sensitive to households planning horizons. This is the result of the interactions of monetary and fiscal policy. Although shorter horizons are associated with deeper recessions, active monetary policy keeps real interest rates substantially lower which offsets (part of) the upward pressures on debt ratio due to the recession.

Comparing spending- and tax-based consolidations, we find that the former leads to a faster drop in the debt ratio in the short run due to the induced expansion upon anticipation of the imminent consolidation. Under tax-based consolidations, instead,

\footnote{In fact, when households have long to infinite planning horizons, they anticipate lower future taxes after the consolidation is over due to lower debt. This creates a positive wealth effect which is reflected in higher consumption. Moreover, households with longer planning horizons anticipate the lower future debt service costs which boosts their current consumption further.}

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output drops both upon anticipation and in the subsequent quarters. Moreover, inflation and the real interest rate increase which delay the fall in debt further. Once consolidations are implemented, under our benchmark calibration, output drops abruptly after spending cuts and the subsequent slower recovery deteriorates the performance of spending-based consolidations in the medium-run, compared to tax-based consolidations. We show however, that this result crucially depends on the degree of relative risk aversion of households. Lower degrees of risk aversion mitigate the adverse effects of spending-based consolidations while they amplify those of tax-based consolidations, leading to deeper recessions. In this case, spending consolidations perform better than tax-based consolidations, both in terms of reducing debt and in their associated output losses.

We find that the monetary policy stance is also important for the way consolidations affect the economy, and for the relative performance of spending- and tax-based consolidations. In general, as monetary policy becomes more aggressive, the adverse effects of spending-based consolidations are mitigated, while those of tax-based consolidations are amplified. Following spending cuts, a stronger monetary easing following the drop in consumption and inflation can partly offset the negative effects of the abrupt fall in aggregate demand.\footnote{Canova and Pappa (2011) show that fiscal multipliers are sensitive to the monetary policy stance. However, as Alesina et al. (2015) point out, monetary policy cannot be the main explanation behind the differences between the two types of consolidation.} Under tax-based consolidations instead, the upward pressures on inflation - due to higher marginal costs - do not allow for a monetary easing necessary to counteract the short-run effects of higher taxes. On the contrary, as monetary policy becomes more aggressive, real interest rates are even higher, which leads to deeper recessions, increasing the induced output losses.

All in all, we show that an otherwise standard New Keynesian model, but with households having a finite horizon and being uncertain about the future state of the economy, can capture some key facts observed in the data. Those facts concern the positive co-movement between private consumption and government spending, the fact that spending-based consolidations are milder in terms of output losses and their ability to stabilize debt faster. In that respect, we show that the monetary policy stance is crucial as well. The virtue of our approach, although computationally challenging, is that it can capture those key facts observed in the data without resting on further complications, like agent heterogeneity. The current Chapter therefore differs from Chapter 5, in its focus of the planning horizon of forward looking agents, rather than on agent heterogeneity and the interaction of forward and backward looking agents. Both chapters give different insights in how bounded rationality affects the implications...
of fiscal consolidations.

The chapter is organized as follows. In the next section, we briefly discuss the literature on fiscal consolidations along with that on the existence of non-Ricardian households. Also, we briefly discuss the literature dealing with the optimization problem of boundedly rational agents who form expectations over a finite number of periods, and explain how our approach differs from the existing literature. In Section 6.3, we present the New Keynesian model with distortionary taxes and finite planning horizons. In Section 6.4, determinacy and E-stability properties of our model are presented. Section 6.5 discusses the effects of fiscal consolidations for boundedly rational agents with varying planning horizons. Section 6.6 concludes.

6.2 Related literature

6.2.1 Fiscal Consolidations

In this section we review the literature on the empirical relevance of non-Ricardian households, first, and, second, on the effects of fiscal consolidations in general.

Gali et al. (2007) construct a New-Keynesian model with forward looking and rule-of-thumb households which decide upon their consumption on the basis of their current disposable income only. They test the empirical validity of this assumption and find supporting evidence. Such a structure (along with labor market frictions) is able to account for the positive co-movement between consumption and government spending as opposed to traditional neo-classical models. Along the same lines Parker (1999) finds evidence of a high sensitivity of consumption to variations in after-tax income due to anticipated changes in social security taxes, while Souleles (1999) finds evidence of excess sensitivity of households’ consumption to predictable tax refunds. Finally, Campbell and Mankiw (1989) reject the permanent income hypothesis in favor of a model with borrowing constraints or myopic behavior. This empirical evidence motivates our approach of households that have finite planning horizons, but care about their end of horizon wealth. The difference between our approach and that of Gali et al. (2007) is that our agents always have access to financial markets and thus make intertemporal decisions, whereas in their approach rule-of-thumb consumers do not have access to financial markets.

In the literature, there is substantial evidence on the effects of fiscal consolidations. In particular, there has been much research over the effects of different types

\footnote{In particular, the existence of rule-of-thumb households leads to an aggregate Euler equation where anticipated changes in taxes have predictive power over private consumption growth.}
of fiscal consolidations (e.g. spending-based and tax-based). A large empirical literature provides evidence supporting the expansionary fiscal consolidations hypothesis (see Alesina and Perotti (1995), Perotti (1996), Alesina and Ardagna (1998, 2010), Ardagna (2004)). In particular, the key finding is that fiscal consolidations are sometimes correlated with rapid economic growth, especially when implemented by spending cuts rather than tax increases. On the other hand, another strand of the empirical literature, using narrative data to identify consolidations, initially introduced by Romer and Romer (2010), finds that output drops following both types of consolidations and that recessions are deeper after tax hikes (Guajardo et al. (2014)). Along the same lines Alesina et al. (2015), using a richer structure for modeling fiscal consolidations, find that spending-based consolidations are less costly, in terms of output losses than tax-based ones. However, as Guajardo et al. (2014) argue, a drawback of contemporaneous estimates is that planned impacts on budgets may tend to be over-optimistic relative to the ex-post outcomes. Consequently, the negative effects of consolidations on output may be understated due to the induced bias. This is the case with spending cuts in many instances, where the announced cuts were stronger than those actually implemented (Beetsma et al. (2016)).

In the theoretical literature, Bertola and Drazen (1993) develop a model where the government satisfies its intertemporal budget constraint by periodically cutting spending, where the latter is inherently unsustainable. A worsening of the fiscal conditions can increase the probability of a beneficial fiscal consolidation which can thus be expansionary. Bertola and Drazen (1993) consider the importance of expectations in the analysis of fiscal consolidations.

Bi et al. (2013) augment the model of Bertola and Drazen (1993) with distortionary taxation and analyze the effects of different types of fiscal consolidations. Moreover, they look at the effects of persistence in those, as well as of the uncertainty of economic agents over the composition of the upcoming fiscal consolidation. Accounting for the monetary policy stance as well, they find that spending- and tax-based consolidations can be equally successful in stabilizing government debt at low debt levels. Nevertheless, at high debt levels, spending-based consolidations are expected to be expansionary and more successful in stabilizing debt, especially when agents anticipate a tax-based

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79 Earlier papers using the conventional approach to identify fiscal consolidations argued in favor of the expansionary effects of spending-based consolidations ("expansionary fiscal austerity") Alesina and Ardagna (2010), Alesina et al. (2002), Alesina and Perotti (1996) and Giavazzi and Pagano (1990) among others. However, their measure of identifying consolidations (i.e. the CAPB) suffers from problems like reverse causality or changes in fiscal variables due to non-policy changes correlated with other developments in output. Finally, as Romer and Romer (2010) point out another approach, followed by Blanchard and Perotti (2002) using SVAR analysis and institutional information to identify consolidations, suffers from problems similar to those of the studies above.
6.2. Related literature

Finally, Erceg and Linde (2013) examine the effects of tax-based and spending-based consolidations in a two country DSGE model for a currency union. They assume agent heterogeneity by introducing fixed fractions of forward looking and "hand-to-mouth" households. They find that tax-based consolidations have less adverse output costs than spending-based ones in the short to medium-run. Moreover, they show that large spending-based consolidations can be counterproductive in the short-run when the zero lower bound in interest rate binds, while they argue in favor of a "mixed strategy" combining both types of consolidations.

6.2.2 Bounded rationality an bounded optimality

Our model is related to a large literature on bounded rationality and bounded optimality. First of all, under Euler equation learning (see Honkapohja et al. (2012)) agents need only form expectations one period into the future. When these expectations are not fully rational (and hence do not implicitly take the infinite future into account through a recursive formulation), agents have a one period ahead planning horizon. Resting on this finite horizon learning approach, Evans and McGough (2015) build a framework that formulates the agents' optimization in a way that is consistent with their short sightedness in forecasting. In their approach, agents learn about the shadow price of their wealth in a recursive way. They show that such a problem can be cast in a dynamic programming setting, which is consistent with the time inconsistency of consumption plans.

Our approach differs from Evans and McGough (2015) in some crucial aspects. First, our agents are uncertain not only about the correct valuation of their end-of-horizon wealth, but also about the state of the economy at the end of their planning horizon. This means that even if our agents might know the true valuation of their wealth, they can still be time inconsistent in their consumption decisions if their beliefs about the state of the economy at the end of their horizon are wrong. As such, even though in a two period setting our approach is similar to theirs, the implied expectations path in our case is different. Moreover, we apply our approach to cases where agents optimize and form expectations for more than two (but a finite number of) periods. Consequently, our approach incorporates important wealth effects which are absent in Evans and McGough (2015).

An alternative to Euler equation learning and shadow price learning is infinite

\footnote{Essentially Evans and McGough (2015) highlight the difference between learning to forecast with learning to optimize and build an optimizing framework satisfying both kinds of learning.}
horizon learning (see Preston (2005) and Eusepi and Preston (2017)), where agents fully solve an infinite horizon optimization problem, given their (possibly boundedly rational) expectations that they have to form over an infinite horizon. We believe that both a horizon of one period and a planning horizon of infinitely many periods may be too extreme. Branch et al. (2010) consider the case of finite horizons. In their model it is however required that agents form expectations about their end of horizon wealth and optimize based on these expectations. In contrast, in our model choosing optimal end of horizon bond holdings is in every period part of the agents’ optimization problem.

6.3 The model

6.3.1 Households

Households want to maximize their discounted utility of consumption and leisure over their planning horizon ($T$ periods), and they also value the state they expect to end up in at the end of these $T$ periods (their state in period $T+1$). They are however not able to rationally induce (by solving the model forward until infinity), how exactly they should value their state in period $T+1$. Instead households use a rule of thumb to evaluate the value of their state. Since bond holdings are the only relevant state variable for households, their maximization problem becomes

$$\max_{\{C_s^i, H_s^i, B_s^i\}_{s=t}^{t+T}} \tilde{E}_t \sum_{s=t}^{t+T} \beta^{s-t} u(C_s^i, H_s^i) + \beta^{T+1} V_i(t) \left( \frac{B_{t+T+1}^i}{P_{t+T}} \right),$$

subject to

$$P_s C_s^i + \frac{B_{s+1}^i}{1+i_s} \leq (1-\tau_s) W_s H_s^i + B_s^i + P_s \Xi_s, \quad s = t, t+1, \ldots, t+T,$$

where $B_s^i$ are nominal bond holdings from household $i$ at the beginning of period $t$. $\Xi_t$ are real profits from firms which are equally distributed among households and $W_t$ is the nominal wage rate.

Furthermore, we assume that households have CRRA preferences with relative risk aversion $\sigma$. We also assume that the relative valuation of real bond holdings in period $t + T + 1$ compared to consumption in period $t + T$ as valued in period $t$ is given by
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The functional form of $V_{i,t}(.)$ therefore becomes

$$V_{i,t}(x) = \Lambda_{i,t}^{x^{1-\sigma}} \frac{1}{1-\sigma}. \tag{6.3}$$

The parameter $\Lambda_{i,t}$ is time varying, implying that households can change over time how they plan to trade off future wealth with future consumption. More specifically we assume that households use an adaptive updating mechanism to learn over time how they should optimally value future wealth.\textsuperscript{81} If households would always choose $\Lambda_{i,t}$ such that they value future wealth correctly then, under rational expectations, the above problem would be equivalent to an infinite horizon optimization problem. The bounded rationality in putting an exact value on wealth far in the future is what will drive many of our results.

Dividing the budget constraint by $P_s$ gives

$$C_i^s + \frac{B_{s+1}^i}{(1+i_s)P_s} \leq (1 - \tau_s)w_s H_s + \frac{B_s^i}{P_s} + \Xi_s, \quad s = t, t + 1, ... t + T. \tag{6.4}$$

The first order conditions of the maximization problem are

$$\left( H_s^i \right)^\eta = (C_s^i)^{-\sigma} (1 - \tau_s)w_s, \quad s = t, t + 1, ... t + T, \tag{6.5}$$

$$\left( C_s^i \right)^{-\sigma} = \beta \frac{(1 + i_s)(C_{s+1}^i)^{-\sigma}}{\Pi_{s+1}}, \quad s = t, t + 1, ... t + T - 1, \tag{6.6}$$

$$\left( C_{t+T}^i \right)^{-\sigma} = \beta (1 + i_{t+T}) \frac{B_{t+T}^i}{P_{t+T}} \Lambda_t, \tag{6.7}$$

where $\eta$ is the Frisch elasticity of labor supply.\textsuperscript{82}

Next we define a measure of real bond holdings scaled by steady state output: $b_t = \frac{B_t}{\bar{Y}_{t-1}Y}$. Substituting for this expression in (6.4) and (6.7) gives

$$C_s^i + \bar{Y} \frac{b_{s+1}^i}{1 + i_s} \leq (1 - \tau_s)w_s H_s + \frac{\bar{Y} b_s^i}{\Pi_s} + \Xi_s, \quad s = t, t + 1, ... t + T. \tag{6.8}$$

\textsuperscript{81}Details of the updating mechanism are presented in Section 6.3.5.
\textsuperscript{82}Our utility function takes the functional form $u(C_s^i, H_s^i) = \frac{(C_s^i)^{1-\sigma}}{1-\sigma} - \frac{(H_s^i)^{1+\eta}}{1+\eta}$. 

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and

\[(6.9) \quad (C^i_{t+T})^{-\sigma} = \beta(1 + i_{t+T})(\bar{Y}^i_{t+T+1})^{-\sigma}A^i_t.\]

### 6.3.2 Firms

There is a continuum of firms producing the final differentiated goods. There is monopolistic competition. Each firm has a linear technology with labor as its only input

\[(6.10) \quad Y_t(j) = AH_t(j),\]

where \(A\) is aggregate productivity, which is assumed to be constant.

Firms are run by households, and hence will have finite planning horizons. That is, they will form expectations about their marginal costs and the demand for their product for \(T\) periods ahead only. We assume that in each period a fraction \((1 - \omega)\) firms can change their price. The problem of firm \(j\) that can reset its price is then to maximize the discounted value of its profits for the next \(T\) periods.

\[(6.11) \quad \tilde{E}_t^j \sum_{s=0}^{T} \omega^s Q^j_{t,t+s} \left[ p_t(j)Y_{t+s}(j) - P_{t+s}mc_{t+s}Y_{t+s}(j) \right],\]

where

\[(6.12) \quad Q^j_{t,t+s} = \beta^s \left( \frac{C^j_{t+s}}{C^j_t} \right)^{-\sigma} \frac{P_t}{P_{t+s}},\]

is the stochastic discount factor of the household that runs firm \(j\).

Using the demand for good \(j\), the firm’s profits maximization problem writes as follows

\[(6.13) \quad \max \tilde{E}_t^j \sum_{s=0}^{T} \omega^s \beta^s \left( \frac{C^j_{t+s}}{C^j_t} \right)^{-\sigma} P_t \left[ \left( \frac{p_t(j)}{P_{t+s}} \right)^{1-\theta} Y_{t+s} - mc_{t+s} \left( \frac{p_t(j)}{P_{t+s}} \right)^{-\theta} Y_{t+s} \right].\]

The first order condition for \(p_t(j)\) is

\[(6.14) \quad \tilde{E}_t^j \sum_{s=0}^{T} \omega^s \beta^s \left( \frac{C^j_{t+s}}{C^j_t} \right)^{-\sigma} P_t \frac{p_t(j)}{P_{t+s}} Y_{t+s} \left[ (1 - \theta) \left( \frac{p_t(j)}{P_{t+s}} \right)^{-\theta} + \theta mc_{t+s} \left( \frac{p_t^*(j)}{P_{t+s}} \right)^{-1-\theta} \right] = 0,\]

where \(p_t^*(j)\) is the optimal price for firm \(j\) if it can re-optimize in period \(t\). Multiplying
by \( \frac{(C_t^j)}{P_t} \frac{(C_t^j)^{1+\theta}}{1-\theta} \) gives

\[
\tilde{E}_t^j \sum_{s=0}^{T} \omega^s \beta^s (C_{t+s}^j)^{-\sigma} Y_{t+s} \left[ p_t^*(j) P_{t+s}^\theta - \frac{\theta}{\theta - 1} m_{t+s} P_{t+s}^{1+\theta} \right] = 0.
\]

This can be written as

\[
\frac{p_t^*(j)}{P_t} \tilde{E}_t^j \sum_{s=0}^{T} \omega^s \beta^s (C_{t+s}^j)^{-\sigma} \left( \frac{P_{t+s}}{P_t} \right)^{\theta-1} Y_{t+s} = \frac{\theta}{\theta - 1} \tilde{E}_t^j \sum_{s=0}^{T} \omega^s \beta^s (C_{t+s}^j)^{-\sigma} \left( \frac{P_{t+s}}{P_t} \right)^{\theta} Y_{t+s} m_{t+s}.
\]

Finally, the aggregate price level evolves as

\[
P_t = \left[ \omega P_{t-1}^{1-\theta} + (1 - \omega) \int_0^1 p_t^*(j)^{1-\theta} dj \right]^{\frac{1}{1-\sigma}}.
\]

### 6.3.3 Government and market clearing

The government issues bonds and levies labor taxes (\( \tau \)) to finance its spending (\( G_t \)). Its budget constraint is given by

\[
B_{t+1} \frac{1}{1 + i_t} = P_t G_t - \tau_t W_t H_t + B_t,
\]

with \( H_t = \int H_t' di \) and \( B_t = \int B_t' di \) aggregate labor and aggregate bond holdings respectively. Dividing by \( \bar{Y} P_t \) gives

\[
b_{t+1} \frac{1}{1 + i_t} = g_t - \tau_t w_t \frac{H_t}{\bar{Y}} + b_t \frac{1}{\Pi_t},
\]

where \( b_t = \frac{B_t}{\bar{Y} P_t} \) and \( g_t = \frac{G_t}{\bar{Y}} \) are the ratios of debt to steady state GDP and government expenditure to steady state GDP, respectively.

Market clearing is given by

\[
Y_t = C_t + G_t = C_t + \bar{Y} g_t.
\]

Monetary policy is defined by a Taylor type interest rule where the government responds to current inflation and output.
We assume that taxes always respond to debt in order to keep the debt ratio close to a (possibly time varying) target set by the government. The coefficient with which taxes respond to debt \( \gamma_0^\tau \) is however not very large. If debt lies far above its target, e.g. because market pressures or political considerations have led the government to lower its debt target, the government may decide to additionally implement consolidations. These consolidations can be tax-based or spending-based, or a combination of both. In case of tax-based consolidation, the government temporarily increases the coefficient with which taxes respond to deviations of the debt ratio from its target by \( \gamma_{\text{cons}}^\tau \). If consolidations are spending-based the government temporarily lets spending respond to deviations of the debt ratio from its target with a coefficient \( \gamma_{\text{cons}}^g \). In particular, government spending and taxes, evolve as

\[
g_t = g_1 - \zeta \gamma_{\text{cons}}^g \mathbb{1}_{\text{cons}}(b_t - DT_{t-1}),
\]

and

\[
\tau_t = \tau_{\text{cons}}^{DT} + \gamma_0^\tau + (1 - \zeta) \gamma_{\text{cons}}^\tau \mathbb{1}_{\text{cons}}(b_t - DT_{t-1})
\]

Here, \( DT_{t-1} \) denotes the target for the debt ratio that the government sets and which we assume to be publicly known. \( g^1 \) and \( \tau_{\text{cons}}^{DT} \) determine the steady state levels of spending and taxes. \( \tau_{\text{cons}}^{DT} \) is made time varying, so that the government can make it depend on \( DT_{t-1} \) and thereby assure that any debt target can be reached in the long run. \( \mathbb{1}_{\text{cons}} \) is an indicator function that is 1 when the government is implementing consolidations, and 0 otherwise. \( \zeta \) is the fraction of the consolidation that is spending-based. Below we will only consider the two extreme cases of fully spending-based consolidation \( (\zeta = 1) \) and fully tax-based consolidation \( (\zeta = 0) \).

### 6.3.4 Log linearized model

The steady state of the non-linear model described in the previous section, assuming zero inflation, is given in Appendix 6.A. We proceed by log-linearizing all the above model equations around this steady state. Starting with the consumers, an optimal consumption decision can be derived by iterating the budget constraint from period \( t + \)

\[1 + i_t = \left( \frac{\Pi_t}{\Pi} \right)^{\phi_1} + \left( \frac{Y_t}{Y} \right)^{\phi_2}.
\]

\(^{83}\)Note that this specification of fiscal policy is different than the one in Chapter 5.
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$T$ backward, and substituting for future labor and future consumption and final period bond holdings, using the first order conditions of the household. This expression can then be aggregated across household to obtain an expression for aggregate consumption.

Combining the resulting aggregate consumption equation with the log-linearized supply side and government budget constraint results in the following system of equations. Details of the derivations can be found in Appendix 6.B.

(6.24)

\[(1 - \nu_y)\hat{Y}_t = \frac{1}{\rho} \hat{b}_t + g_t + \nu_\tau \sum_{s=0}^{T} \beta^s(\hat{E}_t \hat{\tau}_{t+s}) + \nu_y \sum_{s=0}^{T} \beta^s(\hat{E}_t \hat{g}_{t+s}) + \nu_\tau \sum_{s=0}^{T} \beta^s(\hat{E}_t \hat{\pi}_{t+s}) \]

(6.25)

\[\pi_t = \tilde{\kappa} (\eta + \frac{\sigma}{1 - \hat{g}}) \sum_{s=0}^{T} \omega^s \beta^s \hat{E}_t \hat{\pi}_{t+s} - \frac{\tilde{\kappa} \sigma}{1 - \hat{g}} \sum_{s=0}^{T} \omega^s \beta^s \hat{E}_t \hat{g}_{t+s} \]

(6.26)

\[\hat{b}_{t+1} = \frac{1}{\beta} \hat{g}_t - \frac{\tilde{\kappa}}{\beta} \left[ \tilde{\tau} \left( 1 + \eta + \frac{\sigma}{1 - \hat{g}} \right) \hat{Y}_t - \sigma \frac{\hat{g}_t}{1 - \hat{g}} + \frac{\hat{\tau}_t}{1 - \tilde{\tau}} \right] + \frac{1}{\beta} \hat{b}_t + \tilde{b}(\hat{\pi}_t) \]

The definition of composite parameters $\nu_y, \nu_g, \nu_\tau$ and $\tilde{\kappa}$ are given in Appendix 6.B.3.

The monetary and fiscal policy equations, obtained by log-linearizing (6.21), (6.22) and (6.23) are

(6.27)

\[\hat{\pi}_t = \phi_1 \pi_t + \phi_2 \hat{Y}_t,\]

(6.28)

\[\tilde{g}_t = -\zeta \gamma_{cons}^g (\tilde{b}_t - \tilde{D}_t),\]

(6.29)

\[\tilde{\tau}_t = \tilde{z}^{DT}_{t-1} + (\gamma_{\tau}^0 + (1 - \zeta) \gamma_{cons}^\tau) (\tilde{b}_t - \tilde{D}_t),\]
where we let

\[ (6.30) \] \[ \hat{\tau}^{DT}_{t-1} = \frac{1 - \beta}{\bar{w} \left( 1 - \frac{1}{\eta + \frac{1}{\tau}} \right)} \tilde{D}T_{t-1}. \]

The term \( \hat{\tau}^{DT}_{t-1} \) in the tax rule represents the tax rate consistent with a debt ratio equal to the debt target, \( \tilde{D}T_{t-1} \). When the debt target is equal to its value of the nonlinear steady state around which the model is log-linearized (so that \( \tilde{D}T_{t-1} = 0 \)), then the term \( \hat{\tau}^{DT}_{t-1} \) is zero. In that case, the tax rate converges to its nonlinear steady state value after the debt ratio has been stabilized. However, if the debt target is above this steady state (\( \tilde{D}T_{t-1} \) is positive), the tax rate needs to be raised proportionately in order to guarantee that the debt ratio will converge to the debt target. Otherwise, debt will converge to a level different from the target. In other words, the term \( \hat{\tau}^{DT}_{t-1} \) guarantees the existence of a (unique) steady state in our linearized model where the debt ratio is equal to its target, for any value of the debt target. Expression (6.30) is derived by using the linearized government budget constraint under the restriction that the debt ratio is equal to the debt target, and by using the fact that steady state inflation and marginal costs are not affected by the level of the debt target. We hence assume that the government believes that agents behave in a rational way. For agents instead we assume that they know the form and the parameters of the tax rule and that they are always aware of the current values of \( \tilde{D}T_{t-1} \) and \( \hat{\tau}^{DT}_{t-1} \), which are announced by the government. However, agents do not know how the term \( \hat{\tau}^{DT}_{t-1} \) is set by the government.

### 6.3.5 Valuation of future bond holdings

Finally, we turn to the evolution of \( \hat{\Lambda}_t \). This parameter determines how agents value bond holdings relative to consumption at the end of their horizon. It thereby determines what trade-off agents plan to make between saving and consuming at the end of their horizon. We assume that agents try to learn what trade-off to make in the future based on the actual trade-offs they have been making in the past. Since both the optimal trade-off between consumption and debt and the trade-off agents actually make in each period vary over time, due to changes in their wealth, we assume agents use constant gain learning to update their \( \hat{\Lambda}^i_t \).

\[ ^{84} \text{Note that agents could also try to learn how their valuation of future debt exactly depends on their current bond holdings. There are however 2 problems with such an approach. First of all, such a learning algorithm might not converge or be extremely slow to converge, leading to unintuitive dynamics. Secondly, if the algorithm would converge, agents would then know exactly how to value debt, even if their financial position changes. This may not be fully realistic and would bring us} \]
More specifically, agents use first order condition (6.9) to observe their actual saving/consumption trade-off in the past. In log-linearized form this gives

\[ \hat{\Lambda}_{t,\text{realized}}^i = \frac{\sigma}{\bar{b}} \hat{y}_{t+1} - \sigma \hat{C}_t - i_t, \]

where \( \hat{\Lambda}_{t,\text{realized}}^i \) is the realized relative valuation of debt used by the household as opposed to the relative valuation the households planned \( T \) periods in advance \( (\hat{\Lambda}_t^i) \).

Agents then use past realizations of \( \hat{\Lambda}_{t,\text{realized}}^i \) to determine how they plan to trade-off bond holdings and consumption in the future. This results in

\[ \hat{\Lambda}_t^i = \gamma \sum_{j=0}^{\infty} (1 - \gamma)^j \hat{\Lambda}_{t-1-j}^{i,\text{realized}}, \]

with \( \gamma \) the gain parameter. This can be written in the following adaptive form.

\[ \hat{\Lambda}_t^i = (1 - \gamma) \hat{\Lambda}_{t-1}^i + \gamma \hat{\Lambda}_{t-1}^{i,\text{realized}}, \]

Aggregating (6.31) and (6.33) and using market clearing gives a formula for the evolution of \( \hat{\Lambda}_t \):

\[ \hat{\Lambda}_t = (1 - \gamma) \hat{\Lambda}_{t-1} + \gamma \left( \frac{\sigma}{\bar{b}} \hat{y}_t - \frac{\sigma}{1 - \hat{g}} \hat{y}_{t-1} + \frac{\sigma}{1 - \hat{g}} \hat{y}_{t-1} - i_{t-1} \right). \]

6.4 Model properties

Before we turn to an analysis of fiscal consolidations, we briefly review the dynamic properties of our model. In particular, we consider different horizons and investigate under what conditions our model is locally determinate under rational expectations and E-stable under adaptive learning.

6.4.1 Determinacy

First, we assume that, given the boundedly optimal optimization problem with finite horizons, and given the law of motion of \( \hat{\Lambda}_t \) of Equation (6.34), expectations are fully close again to an infinite horizon optimization problem. With the approach chosen in this chapter, as debt varies, agents are constantly learning in their new environment how exactly they should value future debt. Therefore we capture the bounded rationality aspect that agents may dynamically make mistakes in how to value debt and therefore make suboptimal decisions. However, in the long run, as there bond holdings converge to a steady state, they learn how to value debt correctly.
rational. We check for determinacy by writing our system in the form \( Az_t = Bz_{t-1} \) with 
\[
\begin{bmatrix}
\hat{Y}_{t+T}, \hat{\pi}_{t+T}, \hat{b}_{t+T}, \hat{Y}_{t+T-1}, \hat{\pi}_{t+T-1}, \hat{b}_{t+T-1}, ..., \hat{Y}_t, \hat{\pi}_t, \hat{b}_t, \Lambda_t
\end{bmatrix}. \] 
The matrices \( A \) and \( B \) are chosen such that they represent equations (6.24) and (6.25) in the first 2 rows, and (6.26) forwarded \( T - 1 \) periods in the third row. (6.34) is written in the bottom row of \( A \) and \( B \), and all other rows of these matrices consist only of zeros and ones. Since \( z_{t-1} \) contains \( 2T \) forward looking variables and \( T + 3 \) predetermined ones, our system is determinate if and only if the matrix \( A^{-1}B \) has exactly \( 2T \) eigenvalues outside the unit circle.

Figure 6.1 plots the determinacy region in case where taxes respond sufficiently to debt (passive fiscal policy in the terminology of Leeper, 1991), and monetary policy does not respond to output gap \( (\phi_2 = 0) \). It can be seen in the figure, that in this case determinacy requires \( \phi_1 > 1 \), no matter how long or short agents’ planning horizons. When \( \phi_1 < 1 \) there are not enough eigenvalues outside the unit circle, and there is indeterminacy. We further find that when the central bank also responds to output gap \( (\phi_2 > 0) \) the condition for determinacy on \( \phi_1 \) is slightly relaxed. This occurs (for all horizons) in line with the Taylor principle.

Figure 6.2 plots the case where taxes respond to debt only very weakly (fiscal policy is active in the terminology of Leeper). In this case the condition for determinacy is reversed and now requires \( \phi_1 < 1 \) for all horizons. When \( \phi_1 > 1 \) there are too many eigenvalues outside the unit circle and our system is explosive (there does not exists a rational expectations equilibrium satisfying the transversality condition).

We can conclude that the conditions for determinacy in our model do not depend on the horizon, and are in line with those of models with infinite planning horizons.

### 6.4.2 E-stability

Next, we consider the properties of our model when agents do not form expectations rationally, but instead are learning. For this we turn to the concept of E-stability. When an equilibrium is E-stable, this implies that agents could learn the equilibrium by acting as econometricians and performing regressions on past data (recursive least squares learning).\(^{85}\)

We determine E-stability by numerically evaluating the Jacobian of our system at the minimum state variable solution (MSV) of our model. When all eigenvalues of this Jacobian are inside the unit circle then the MSV is E-stable, and when some eigenvalues

\(^{85}\)More specifically, agents are assumed the know the correct functional form of the minimum state variable solution, and try to learn the parameters of the MSV solution by means of OLS. See Evans and Honkapohja (2012) and references therein for details.
Determinate and E-stable
Indeterminate and E-unstable
Explosive

6.5 Fiscal Consolidations

In this section we solve the model and simulate two different experiments, namely tax-based and spending-based consolidations. First, we look at the effects of different planning horizons on the ability of each type of consolidation to stabilize debt (Sections 6.5.3 and 6.5.4). Second, we fix the forecast horizon $T$ of our agents and look at the differences between the two types of consolidations. Additionally, we analyze the relative importance of the fiscal policy stance, as measured by the reaction of taxes or spending to debt ratio fluctuations, as well as the effect of the monetary policy stance (Section 6.5.5). Finally, we consider the role of relative risk aversion in determining the relative performance of spending- and tax-based consolidations (Section 6.5.6).

---

Bi et al. (2013) show that the responsiveness of the interest rate to inflation fluctuations affects the ability of spending-based consolidations to stabilize debt while that of tax-based is unaffected.
Before this analysis can be conducted, we need to specify how agents form expectations (Section 6.5.1), and how we calibrate the model parameters (Section 6.5.2).

### 6.5.1 Expectations

In Section 6.4 we discussed how our model behaves under rational expectations and under adaptive learning. In this section, we will assume that agents form expectations in a forward looking manner and hence stay relatively close to the assumption of rational expectations. However, we introduce some bounded rationality in expectation formation, in a way that is consistent with agent’s finite planning horizons. In particular, we assume that agents rationally use the model equations within their horizon to try to form model consistent expectations, but that they are not able to form in a sophisticated manner expectations for variables outside their horizon. Furthermore, we take an anticipated utility approach, by assuming that agents form expectations and make decisions based on their current relative valuation of future wealth. That is, when they make a consumption plan, agents do not consider how they might update their $\Lambda_i^t$ in the future.\(^\text{87}\)

Agents start with forming expectations about the final period of their horizon. To do this, they take the model equations of period $t + T$. However, in the IS curve and Phillips curve of that period (Equations (6.24) and (6.25) forwarded $T$ periods), finite sums with expectations about period $t + T + 1$ up to period $t + T + T$ appear. That is, in the model equation they are considering, expectations of variables outside their planning horizon show up. Agents therefore need to give these expectations a value, without being able to solve in a sophisticated manner what will happen in these periods (since they lie outside their planning horizon). Instead, they assume that in periods after their horizon, the model will have converged to steady state.\(^\text{88}\) With this assumption, they are able to solve for period $t + T$ variables in terms of the state variable $b_{t+T}$. They then move to the model equations of period $t + T - 1$. Here they plug in the solution of period $t + T$ variables as expectations, and again assume steady state levels for expectations of variables outside their horizon. They can then solve for period $t + T - 1$ variables in terms of state variable $b_{t+T-1}$. This process goes on until they have solved for all expectations within their horizon in terms of the observed state variable $b_t$. Expectations of variables for the periods within the horizon can then be

\(^{87}\)Assuming otherwise would make agents in a sense "hyper-rational", as argued by Branch and McGough (2016).

\(^{88}\)The steady state that the agents believe the model to converge to depends on their current relative valuation of future wealth ($\Lambda_i^t$). In Appendix 6.D we derive the steady state of the linearized model as a function of $\Lambda$. 

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obtained from these policy functions by plugging in the value of the current debt level.

When this expectation path implies that in period \( t + T \) the model will already be very close to the steady state that agents expect it to converge to, then the above algorithm results in a near perfect foresight path. If however, the model converges to the steady state too slowly relative to the forecasting horizon, then the short sightedness of the agents will lead to significantly biased expectations. Additional expectation errors arise because agents keep updating their valuation of future wealth (\( \Lambda_t \)) in a way that they were not able to foresee.

Finally, note that the fact that agents know the current value of the parameter \( \tilde{\tau}_{t-1}^{DT} \) in the tax rule, but that they do not know how this term \( \tilde{\tau}_{t-1}^{DT} \) is set, is an important assumption. As we show in Appendix 6.D, agents compute their perceived steady state using the model’s equations. If agents would know that \( \tilde{\tau}_{t-1}^{DT} \) is such that the model has a steady state with debt equal to its target, then they could use this to infer the optimal valuation of end of horizon wealth, \( \Lambda_t \), that is consistent with this steady state. This would allow them to behave as fully rational agents.

As we describe below, we consider the case where the government announces a change in its target for the debt ratio \( DT_{t-1} \) and in the parameter \( \tilde{\tau}_{t-1}^{DT} \) once and permanently. Therefore, once this happens, agents do not expect future changes in these parameters.

### 6.5.2 Parameterization

We now calibrate our model. We are interested in analyzing the effects of fiscal consolidations before, during and after implementation. We do not calibrate the model for a specific country. One period corresponds to one quarter. In our baseline calibration, we set the coefficient of relative risk aversion \( \sigma = 1 \), the inverse of the Frisch elasticity of labor supply \( \eta = 2 \), the elasticity of substitution \( \theta = 6 \), the Calvo parameter \( \omega = 0.75 \). The nonlinear steady state government spending as a share of GDP is set at \( G/Y = 0.21 \), the steady state debt target and debt ratio are set to 0.7, which requires a steady state tax rate of \( \tau \approx 0.26 \). In the benchmark calibration, we set \( \phi_1 = 1.3 \) and \( \phi_2 = 0 \) in the interest rate rule, while we set \( \gamma_0^{\tau} = 0.05 \) in the tax rule. This combination of active monetary policy and passive fiscal policy implies that the model is determinate, as can be seen in Figure 6.1. Finally, we set \( \gamma_g^{cons} = 0.3 \) and \( \gamma_{\tau}^{cons} = 0.36 \).\(^{89}\) In Section 6.5.5 we also consider what happens if fiscal and/or

\(^{89}\) In order to make tax-based and spending-based consolidations comparable we set \( \gamma_{\tau}^{cons} = \frac{1}{\omega} \gamma_g^{cons} \) so that the effect on tax *income* rather than the tax *rate* is the same as the effect on spending. Spending and tax-based consolidations then have an equal (direct) impact on the government budget constraint, as can be seen in Equation (6.50) in Appendix 6.B.3
monetary authorities respond more aggressively to debt and inflation.

6.5.3 Spending-based consolidations

In this section we proceed with the following experiment. We assume that initially the model is in the steady state consistent with a positive debt target \( \tilde{D}T_t = 0.05 \), which corresponds to a debt ratio of 75\% of GDP. Suddenly, the government decides that debt is too high, and lowers its debt target to \( \tilde{D}T_t = 0 \) (which corresponds to a debt ratio of 70\% of GDP) permanently. In the absence of consolidations, this would imply a very slow transition to a new steady state consistent with the lower debt target. Because the government wants to bring the debt ratio towards its target faster, the government additionally announces that it will start implementing consolidations in the next period, and will continue to do so for 12 quarters. The permanent drop of the publicly known debt target is therefore unanticipated. But once the drop has taken place, the imminent consolidation is anticipated. Moreover, we assume that the government credibly announces the nature of the consolidation (i.e. spending-based in this case) so that there is no uncertainty regarding the composition. In Appendix 6.E we consider what happens if the private sector initially does not know the composition of the upcoming consolidations.

In Figure 6.3 we plot the transition paths of the variables from the steady state with \( \tilde{D}T_t = 0.05 \) to the steady state with \( \tilde{D}T_t = 0 \) in case of spending-based consolidations, for different values of the planning horizon. The blue curves in Figure 6.3 depict the response to a drop in the debt target when \( T = 3 \), while the green curves depict the case of \( T = 8 \). The yellow curves show dynamics when \( T = 100 \), while the thin, black curve depicts the case of infinite planning horizons and fully rational expectations. In this case, the model can be written recursively, which is shown in Appendix 6.C. For all variables, the yellow and thin, black curves overlap and are indistinguishable from each other. This implies that when agents have planning horizons of 100 periods in our model, their actions are (almost) the same as if they had solved an infinite horizon optimization problem with rational expectations. The effects of consolidations in an economy where agents have long but finite planning horizons are therefore (almost) the same as in an economy with fully rational agents with an infinite planning horizon.

Let us first compare the initial with the new steady state values of debt, consumption, output, inflation and the nominal interest rate, and turn to the transition paths implied by the consolidations later. For \( T = 3 \) the model does not yet reach its new steady state within the 25 periods that are plotted (as will be discussed below). Therefore, we make a steady state comparison by comparing the initial and final values of
the other three curves in Figure 6.3. The lowering of the debt target implies a steady state with permanently lower debt, and therefore a lower interest rate burden for the government. In the new steady state the government can therefore afford lower taxes. These lower taxes imply that agents have more income and consume more, which also leads to a slight increase in output. Regarding inflation, the following occurs. The lower taxes increase households’ marginal return of labor and labor supply. At the same time, the increase in consumption increases labor demand. Therefore, wages and marginal costs do not change, and the new steady state inflation and interest rates are not affected by the drop in the debt target.

Let us now look at the transition paths towards the new steady state implied by
the spending-based consolidations, for different horizons. Regardless of the horizon, as soon as the debt target is lowered, agents realize that next period spending-based consolidations will start. They furthermore realize that the fall in government spending will lower aggregate demand and induce firms to lower both prices and labor demand. The latter leads to a decrease in wages, which will further lower inflation. Since firms expect low inflation in the periods to come, they already lower their prices in the period where consolidations are announced, hence the immediate drop in inflation.

Meanwhile, consumers expect a decrease in the nominal and real interest rate due to the periods of low inflation. However, how they react to this knowledge depends significantly on their planning horizon. When agents have a longer planning horizon, their consumption decision is for a large part driven by expectations about their disposable income in the future and about future real interest rates. If they expect lower future interest rates this leads them to consume more today. However, for shorter horizons, agents’ consumption depends less on their expectations about the future and relatively more on changes in their current wealth. Agents with shorter planning horizons realize that the consolidations will decrease their bond holdings in the coming periods. Since they are boundedly rational and do not immediately realize how this drop in debt should affect their valuation of future debt \((\hat{\Lambda}_t)\), they perceive the drop in their bond holdings as a more permanent decrease in their wealth. For this reason, they anticipate that they will consume less in the periods to come than agents with a longer planning horizon. Consumption smoothing then leads them to already consume relatively less in the anticipation period.

In Figure 6.3, it can be seen that consumption in case of a planning horizon of 100 and the case of an infinite planning horizon (the yellow/black curves) goes up considerably in the period of the announcement. For \(T = 3\) (blue) instead, we see that consumption goes up much less. For a planning horizon of three periods the effects of changes in next period’s expected wealth, as described above, thus dominate, leading to relatively low private consumption. For \(T = 8\) (green), agents also consume somewhat less than for \(T = 100\), but considerably more than for \(T = 3\).

As consolidations are implemented and subsequently decrease in magnitude, under all horizons, consumption jumps on impact, and then slowly goes down. Note that since agents do not value future wealth correctly yet during this transition, agents with a planning horizon of \(T = 3\) and \(T = 8\) expect that in the new steady state, with lower bond holdings, they will consume less. For this reason, the green curve, and especially the blue curve, initially seem to converge to a lower consumption level than the yellow and the thin black ones. Interestingly, after 2 periods the spending cuts imply for \(T = 3\) that consumption is lower than its original steady state value for an
extended period of time. This is in line with the empirical evidence on a co-movement between private consumption and government spending (see Blanchard and Perotti (2002), Fatas and Mihov (2001) and Gali et al. (2007) among others).

Over time, agents start to update their valuation of future wealth, \( \hat{\Lambda}_t \), and all lines converge to the same steady state eventually. For \( T = 3 \) the consolidations have however been so unsuccessful that debt is still considerably above its steady state value after the 25 periods that are plotted. More consolidations would be needed to achieve the steady state within a reasonable number of periods.

Output dynamics are mainly dominated by government spending. However, for longer horizons output initially goes up considerably because of the increase in consumption in the anticipation period. Meanwhile, inflation dynamics also differ considerably for different horizons. Expectations about lower future bond holdings strongly affect inflation dynamics for shorter horizons. In particular, firms anticipate the economy to reach a state with lower consumption, lower wages, and hence lower inflation. This implies firms to already lower prices more today. Moreover, the lower consumption for shorter horizons lowers the inflation path even more. As agents start updating \( \hat{\Lambda}_t \), and eventually learn to value debt correctly in their new environment, inflation (just like consumption) starts converging to the level it would have taken in an economy with more rational agents with longer planning horizons.

Finally, we consider debt dynamics. As can be seen in Figure 6.3 debt is consolidated at the same rate when \( T = 8 \) as when \( T = 100 \) or when the horizon is infinite. For \( T = 8 \) we can therefore conclude that the lower tax income due to relatively lower output is offset by the drop in debt service costs that are implied by the relatively lower level of the real interest rate. For \( T = 3 \) however, the drop in output is so large, that the effect of the abrupt drop in the tax base dominates over the effect of the lower debt service costs. Consequently, the debt ratio falls at a lower pace when agents have very short horizons.

### 6.5.4 Tax-based consolidations

Now we turn to the case of tax-based consolidations. We conduct the same experiment as above with an unanticipated drop in the debt target followed by an anticipated consolidation after one quarter. Now, consolidations are implemented for 12 quarters through tax hikes only. The transition of the variables towards the new steady state is displayed in Figure 6.4.

In the case of tax-based consolidations, an increase in taxes leads to a lower marginal return of labor for the household, causing it to decrease its labor supply. Since agents
try to smooth consumption and since the tax increases do not lead to a very large loss in disposable income, agents do not lower their consumption by much. Aggregate demand and hence labor demand are therefore lowered by much less than labor supply. As a consequence, the tax increase causes a strong increase in real wages, which puts upward pressure on inflation. In the period before consolidations, consumers already anticipate slightly lower future consumption and decrease their consumption in anticipation of the tax increases. This result holds regardless of the planning horizon.

Comparing the paths for different planning horizons in Figure 6.4, consumption falls more, the shorter the horizon. This is because short sighted households weigh more changes in their current wealth, which is anticipated to go down during the coming
6.5. Fiscal Consolidations

periods of consolidation. The large fall in consumption for short horizons offsets the upward pressure of marginal cost on inflation, leading to a fall in the latter. Since short sighted firms expect to end up in a steady state where consumers have less wealth (lower bond holdings) and consume less, they also expect to end up in a steady state with low output and inflation. This leads them to lower prices already today which amplifies and moves forward the drop in inflation. When, in the next period, consolidations are implemented, marginal costs increase considerably, pushing up inflation somewhat. However, as taxes and wages start to go down again, the low expected future inflation and output dominate, and inflation falls.

For longer horizons though, households decrease their consumption to a lesser extent, since they base their consumption decision less on current wealth and more on future disposable income. In this case, the demand channel (lower output) is weaker than the supply channel (increase in marginal costs) causing a jump in inflation before and upon implementation. The different interaction between the demand and the supply channel of tax hikes also explains the differences in the monetary policy stance for different horizons. Monetary policy is expansionary for short horizons and contractionary for longer horizons.

As with spending-based consolidations, the yellow curves for $T = 100$ are almost indistinguishable from the responses depicted by the thin, black curves with infinite horizon rational expectations. Furthermore, as short sighted agents after some periods learn to value wealth correctly, all variables converge to the same steady state levels as when agents have an infinite planning horizon.

Finally, again as in the case of spending-based consolidations, for $T = 3$ output falls by so much that the resulting drop in the tax base causes consolidations to be less effective. For $T = 8$ however, the drop in the tax base is much smaller, and fully offset by lower real interest payments on existing debt. This causes debt to fall at the same rate as in the case of an infinite planning horizon.

### 6.5.5 Spending- versus tax-based consolidations

In this section, we compare the effects of the two types of consolidations under different monetary and fiscal policy stances. We will calculate present value multipliers of tax-based and spending-based consolidations and compare their impulse responses. We do this for the benchmark calibration used in Sections 6.5.3 and 6.5.4, but also for the cases of stronger consolidations and more aggressive monetary policy. Because we believe that the multipliers already tell most of the story, we limit the number of figures by only plotting impulse responses for the case of $T = 8$. As we saw in Sections 6.5.3
and 6.5.4, this case has representative features of both shorter and longer planning horizons.

Since we are interested in the effects of consolidations and not in the effects that a lowering of the debt target would have even in the absence of consolidations, we control for the latter when calculating multipliers and impulse responses. In particular, we normalize by subtracting in each period the value that a variable would have taken without consolidations from the value that it takes with the consolidation.

**Benchmark case**

Figure 6.5 compares the impulse responses of spending-based and tax-based consolidation for the case of $T = 8$ under the benchmark calibration that was used in Section 232.
In the figure, it can be seen that upon anticipation variables react differently under the two consolidation plans. Under spending-based consolidations output goes up, leading to a drop in the debt ratio, while under tax-based consolidations output contracts, leading to an increase in debt. However, the big recession that follows when spending cuts are implemented makes spending-based consolidations relatively less effective in the medium to long run.

This is also reflected in the present value multipliers of tax-based and spending-based consolidations. Following Mountford and Uhlig (2009) and Bi et al. (2013), we calculate present value multipliers as follows

\[
\Gamma_{t+k}^y = \sum_{j=0}^{k} \left( \prod_{i=0}^{j} r_{t+i}^{-1} \right) \left( Y_{t+j}^c - Y_{t+j}^{nc} \right) / \sum_{j=0}^{k} \left( \prod_{i=0}^{j} r_{t+i}^{-1} \right) \left( x_{t+j}^c - x_{t+j}^{nc} \right),
\]

where \( r_t \) is the real interest rate, and \( x \) denotes the type of fiscal consolidation: \( x = \tau_t \tilde{w} \tilde{H} \) for tax-based and \( x = -G_t \) for spending-based. \( Y_t^c \) and \( x_t^c \) indicate values taken when there are consolidations, and \( Y_t^{nc} \) and \( x_t^{nc} \) indicate values that would have occurred in the absence of consolidations.

<table>
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<tr>
<th>Panel a: ( \phi_1 = 1.3, \gamma_{\text{cons}}^y = 0.3 )</th>
<th>( \bar{\Delta Y}_{\Delta g} )</th>
<th>1qr</th>
<th>4qr</th>
<th>8qr</th>
<th>12qr</th>
<th>( \hat{\Delta Y}_{\Delta \tau} )</th>
<th>1qr</th>
<th>4qr</th>
<th>8qr</th>
<th>12qr</th>
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<td>( T = 3 )</td>
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<tr>
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<td>-0.58</td>
<td>-0.52</td>
<td>-0.46</td>
<td>( T = 8 )</td>
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<td>-0.45</td>
<td>( T = 100 )</td>
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<td>-0.21</td>
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<th>4qr</th>
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<th>12qr</th>
<th>( \hat{\Delta Y}_{\Delta \tau} )</th>
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<th>4qr</th>
<th>8qr</th>
<th>12qr</th>
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<td>0.16</td>
<td></td>
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<tr>
<td>( T = 100 )</td>
<td>-0.69</td>
<td>-0.66</td>
<td>-0.61</td>
<td>-0.56</td>
<td>( T = 100 )</td>
<td>-0.12</td>
<td>-0.07</td>
<td>0.03</td>
<td>0.17</td>
<td></td>
</tr>
</tbody>
</table>

<table>
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<tr>
<th>Panel c: ( \phi_1 = 1.8, \gamma_{\text{cons}}^y = 0.3 )</th>
<th>( \bar{\Delta Y}_{\Delta g} )</th>
<th>1qr</th>
<th>4qr</th>
<th>8qr</th>
<th>12qr</th>
<th>( \hat{\Delta Y}_{\Delta \tau} )</th>
<th>1qr</th>
<th>4qr</th>
<th>8qr</th>
<th>12qr</th>
</tr>
</thead>
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<tr>
<td>( T = 3 )</td>
<td>-0.65</td>
<td>-0.66</td>
<td>-0.62</td>
<td>-0.53</td>
<td>( T = 3 )</td>
<td>-0.47</td>
<td>-0.54</td>
<td>-0.59</td>
<td>-0.60</td>
<td></td>
</tr>
<tr>
<td>( T = 8 )</td>
<td>-0.50</td>
<td>-0.44</td>
<td>-0.35</td>
<td>-0.24</td>
<td>( T = 8 )</td>
<td>-0.32</td>
<td>-0.29</td>
<td>-0.22</td>
<td>-0.12</td>
<td></td>
</tr>
<tr>
<td>( T = 100 )</td>
<td>-0.52</td>
<td>-0.48</td>
<td>-0.52</td>
<td>-0.37</td>
<td>( T = 100 )</td>
<td>-0.33</td>
<td>-0.32</td>
<td>-0.30</td>
<td>-0.26</td>
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Table 6.1: Output Multipliers

\(^{90}\)We multiply taxes by \( \tilde{w} \tilde{H} \) to get a change in tax income due to a changed tax rate, rather than the change in the tax rate itself. This facilitates compatibility with changes in government expenditure. Using the definitions of (log)-linearized variables, we can calculate the tax-based multipliers as

\[\Gamma_{t+k}^y = \sum_{j=0}^{k} \left( \prod_{i=0}^{j} r_{t+i}^{-1} \right) \left( Y_{t+j}^c - Y_{t+j}^{nc} \right) / \sum_{j=0}^{k} \left( \prod_{i=0}^{j} r_{t+i}^{-1} \right) \left( \tilde{w}(\tilde{\tau}_{t+j} - \tilde{\tau}_{t+j}^{nc}) \right)\]

and the spending-based multiplier as

\[\Gamma_{t+k}^y = \sum_{j=0}^{k} \left( \prod_{i=0}^{j} r_{t+i}^{-1} \right) \left( Y_{t+j}^c - Y_{t+j}^{nc} \right) / \sum_{j=0}^{k} \left( \prod_{i=0}^{j} r_{t+i}^{-1} \right) \left( \tilde{G}_{t+j} - \tilde{G}_{t+j}^{nc} \right)\]
In Panel a of Table 6.1 the impact multiplier and the cumulative multipliers after 4, 8 and 12 quarters under the benchmark calibration are displayed for both spending-based and tax-based consolidations and for different horizons. It can be seen that for $T = 8$ tax-based multipliers are considerably lower than spending-based ones, both on impact and after 4, 8 and 12 quarters. This reflects the deeper recession that arises under spending-based consolidations, observed in Figure 6.5. In this figure, it can furthermore be seen that output recovers quickly relative to government spending and taxes, and even turns positive under tax-based consolidations. This is also reflected in the present value multipliers of both type of consolidations in Table 6.1, which fall in absolute value as the number of periods over which they are calculated increase (from 1, to 4, to 8, to 12 quarters).

Turning to the case of $T = 100$, it can be seen in Panel a of Table 6.1 that the impact multipliers of both spending- and tax-based consolidations are slightly less negative than those obtained under $T = 8$. This is consistent with the smaller drop in output for $T = 100$ than for $T = 8$ that can be observed in Figures 6.3 and 6.4.

Finally, turning to the Multipliers of $T = 3$ in Panel a of Table 6.1, multipliers are much more negative than those of $T = 10$ and $T = 100$. Furthermore, unlike in those two cases, present value multipliers now become more negative rather than less negative as they are calculated over more quarters. We can conclude that when agents have very short planning horizons the consolidations are quite harmful to output in both the short and the medium run.

**Stronger consolidations**

Now, we consider the case where consolidations are stronger. In particular, we increase the consolidation coefficients in the spending and tax rule to $\gamma_g^{cons} = 0.6$ and $\gamma_{\tau}^{cons} = 1 - \frac{1}{\bar{w}} \gamma_g^{cons} = 0.72$. We plot impulse responses for the case of $T = 8$ in Figure 6.6. Comparing this figure with Figure 6.5, it can be seen that the cut in spending and the increase in taxes when consolidations start are more than twice as large now. For tax-based consolidations this also leads to a much faster reduction in debt. However, when consolidations are spending-based, the larger spending cuts also lead to a larger drop in output, which reduces the tax base and makes the consolidation less effective.

Turning to panel b of Table 6.1, which corresponds to this case of stronger consolidations, we see that for $T = 8$ spending-based multipliers are more negative than

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91 Note that the impact multiplier considers the period where consolidations are implemented for the first time. The period where consolidations are anticipated but not yet implemented, in which output goes up under anticipated spending-based consolidations but falls under anticipated tax-based consolidations, is therefore not included in the multipliers.
those of the benchmark calibration in panel a. This is in line with the large drop in output observed in Figure 6.5. However, for tax-based consolidations multipliers are less negative than in the benchmark case. This means that under stronger tax-based consolidations, taxes are increased by more, but that the extra output losses that these higher taxes imply are limited. This is indeed what is observed in Figure 6.6, and this also explains why debt falls more quickly in this figure.

Next, we turn to the multipliers for the case of $T = 3$ and $T = 100$ in panel b of Table 6.1. For $T = 100$ all multipliers are again slightly less negative than for $T = 8$. Moreover, the present value multiplier after 8 quarters now already is slightly positive. For $T = 3$ we see that spending multipliers do not change significantly compared to the benchmark case, but that tax-based multipliers are less negative.

We can conclude that when the government implements tax-based consolidations it
seems desirable to do this with large tax hikes. For all horizons, this leads to a faster reduction in debt without leading to significantly lower output. However, when the government implements spending cuts, doing so too strongly may be quite harmful to the economy, especially for longer horizons, and a more gradual approach might be preferable.

More aggressive monetary policy

Next, we turn to the case where the central bank responds more aggressively to inflation. Now $\phi_1 = 1.8$ in the Taylor rule. Figure 6.7 plots the corresponding impulse responses for the case of $T = 8$. Since spending-based consolidations imply deflation, a more aggressive response to inflation implies a lower real interest rate, which induces agents to consume more and hence limits the drop in output. This in turn implies a smaller fall in the tax base, making the consolidation more effective in reducing debt. In contrast, tax-based consolidations are slightly inflationary for $T = 8$, so that more aggressive monetary policy leads to a larger drop in output, making the consolidation less effective. Therefore, we see in Figure 6.7 that the drop in output under spending- and tax-based consolidations are of similar magnitude, and that the initial debt advantage that arises under spending consolidations because of the reduction in the anticipation period persists for much longer than in the benchmark case.

Turning to the multipliers in panel c of Table 6.1, we see that for all horizons spending-based multipliers are less negative than in the benchmark case, as expected. For tax-based consolidations it can be seen that for $T = 8$ and $T = 100$ multipliers are more negative under more aggressive monetary policy, but that they become less negative when $T = 3$. The reason for that, is that consolidations are deflationary rather than inflationary when agents are very short sighted and expect the consolidation to bring the economy to a new steady state with low output. More aggressive monetary policy hence decreases the real interest rate and leads agents to consume more rather than less for very short horizons.

6.5.6 The role of relative risk aversion

While the calibration of most of model parameters are not very important for our qualitative result, there is a parameter that crucially determines the relative performance of spending- and tax-based consolidations. This parameter is the relative risk aversion, $\sigma$. In this section, we show how dynamics change under both spending-based
6.5. Fiscal Consolidations

Figure 6.7: Impulse responses of spending-based (green) and tax-based (black) consolidations for $T = 8$, in case of more aggressive monetary policy. We now have $\phi_1 = 1.8$ instead of $\phi_1 = 1.3$

and tax-based consolidations when we change the relative risk aversion.\(^{92}\)

Figure 6.8 plots spending- and tax-based consolidations for $T = 8$ under the benchmark calibration with $\phi_1 = 1.3$ and $\gamma_{cons} = 0.3$, but now with a relative risk aversion of $\sigma = 0.157$ (as in e.g. Woodford (1999)) rather than $\sigma = 1$ (as in e.g. Clarida et al. (2000)). First focusing on spending-based consolidations (green), it can be seen that the recession caused by consolidations is much milder than in Figure 6.5. The reason for this is that agents with a lower relative risk aversion smooth consumption less, and therefore respond more strongly to temporary changes in their expected future disposable income and in expected future real interest rates. Hence, agents with a low relative

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\(^{92}\)Note that the changes in dynamics discussed in this section arise almost solely due to the change of the relative risk aversion parameter in the utility function $u(C^t_i, H^t_i))$. When we only change the relative risk aversion parameter in the value function $V^{t,x}(x)$ (given in Equation (6.3)) dynamics are hardly affected.
Figure 6.8: Impulse responses of spending-based (green) and tax-based (black) consolidations for \( T = 8 \), in case of lower relative risk aversion. We now have \( \sigma = 0.157 \) instead of \( \sigma = 1 \). Risk aversion increase their consumption much more in response to the anticipated low future real interest rates implied by spending cuts. Output then goes down by less, which causes the tax base to remain relatively high, so that debt drops faster.

Next we turn to tax-based consolidations (black). It can be seen that for tax-based consolidations output falls more and debt is reduced more slowly than in Figure 6.5. The intuition for this is similar to the case of spending-based consolidators. Agents with lower relative risk aversion care less about smoothing consumption and hence reduce their current consumption more when they anticipate tax increases to imply a lower labor income in the next few periods. This results in a bigger recession, less tax income for the government, and hence slower debt reduction.

Comparing spending- and tax-based consolidations, we see that the recession under spending-based consolidations is now considerably milder than under tax-based con-
6.6 Conclusions

In this chapter we analyze the effects of fiscal consolidations when agents are infinitely lived, but optimize over a finite number of periods. This is because agents do not have the cognitive ability to form expectations and make rational inferences over an infinite

Table 6.2: Output Multipliers for low relative risk aversion (σ = 0.157)

This is also reflected in the multipliers in Panel a of Table 6.2. In this table we reproduce the multipliers of Table 6.1, but with σ = 0.157 instead of σ = 1. It can be seen in Panel a for the case of T = 8 that the tax-based multipliers are much less negative than in Table 6.1 and that the present value multiplier after twelve quarters even turns positive. For tax-based consolidations on the other hand, multipliers are much more negative than in case of a σ = 1. It can further be seen by comparing the first panel of Table 6.2 and 6.1, that for T = 3 and T = 100 multipliers change in a similar way. Again multipliers of T = 100 are slightly less negative than those of T = 8 and multipliers of T = 3 are more negative than those of both other cases.

Finally, looking at panels b and c of Table 6.2 and comparing with Table 6.1, it can be seen that as we make consolidations stronger or monetary policy more aggressive, multipliers change in a similar way as for the case of higher relative risk aversion. These qualitative results therefore are robust to the calibration of σ.

6.6 Conclusions
horizon. Moreover, agents are uncertain about the correct valuation of their wealth at the end of their planning horizon and learn about the correct valuation using a constant gain learning mechanism. The government uses either spending or taxes periodically in order to consolidate debt when the latter is too far above its target. Agents are aware of the upcoming consolidations once this happens and adjust their expectations accordingly. Hence, our model can also capture anticipation effects.

Following a consolidation, we show that the magnitude of the responses of inflation, output and private consumption is sensitive to the agents’ planning horizon. More importantly, we show that for short horizons the effects on output, inflation and consumption change qualitatively as well. In particular, for short planning horizons private consumption falls following spending-based consolidations. This result is in line with empirical evidence on the positive co-movement between private consumption and government spending. On the contrary, as we increase the planning horizon of households, the response of consumption changes sign, resulting in a negative co-movement with government spending, in line with one of the weaknesses of standard neoclassical models with agents optimizing over an infinite horizon.

The virtue of our framework is that it captures the finite horizon aspect of life and the time inconsistency of consumption plans. Consequently, it does not generate the strong wealth effects inherent in standard neoclassical models where agents optimize and form expectations for the infinite future. In those models, such wealth effects imply correlations between macro variables that are not consistent with the actual data and as such they may lead to the wrong conclusions regarding the effects of consolidations.

We find that in our benchmark calibration tax-based consolidations perform better than spending-based consolidations, both in terms of their effectiveness in reducing debt and in terms of output losses. However, we show that the performance of spending-based consolidations can improve considerably when we increase the aggressiveness of monetary policy in its reaction to inflation fluctuations. Moreover, increasing the strength of consolidations seems to make tax-based consolidations more effective in reducing debt without leading to larger output losses, while the opposite holds for spending-based consolidations. Interestingly, we show that for low degrees of relative risk aversion spending-based consolidations outperform tax-based resulting in lower costs in terms of output losses and a faster reduction of debt. In fact, this is in line with empirical evidence as regards the output costs of fiscal consolidations. This result regarding the relative risk aversion was not found in Chapter 5, and crucially depends on the fact that all our agents are now forming expectations and making decisions over a larger number of periods. All other results are in line with the findings of Chapter 5.
Appendix 6.A  Steady state non-linear model

In this section we derive the steady state of the non-linear model, where gross inflation equals 1.

Evaluating (6.16) at the zero inflation steady state gives

\[ n\bar{c} = \frac{\theta - 1}{\theta}. \] (6.36)

From the first order conditions of the households it follows that in this steady state we must have

\[ 1 + \bar{i} = \frac{1}{\beta}. \] (6.37)

Furthermore, normalizing technology \( A \), to 1, it follows from (6.10) that

\[ \bar{H} = \bar{Y}. \] (6.38)

Next, we solve the steady state aggregate resource constraint, (6.20), for consumption, and write

\[ \bar{C} = \bar{Y}(1 - \bar{g}). \] (6.39)

Plugging in these steady state labor and consumption levels in the steady state version of (6.5) gives

\[ \bar{w} = \frac{\bar{Y}^\eta(\bar{Y}(1 - \bar{g}))^\sigma}{1 - \bar{\tau}} = \frac{\bar{Y}^{\eta+\sigma}(1 - \bar{g})^\sigma}{1 - \bar{\tau}} = \frac{\theta - 1}{\theta}. \] (6.40)

Where the last equality follows from \( n\bar{c} = \bar{w} \) and (6.36). We can thus write

\[ \bar{Y} = \left( \frac{\theta - 1}{\theta} \right) \frac{1 - \bar{\tau}}{(1 - \bar{g})^{\sigma}}. \] (6.41)

Then we turn to the government budget constraint. In steady state, (6.19) reduces to

\[ \bar{b} = \frac{\bar{\tau}\bar{w} - \bar{g}}{1 - \beta} = \frac{(\frac{\bar{\tau}}{\bar{g}} - 1 - \bar{g})}{1 - \beta}, \] (6.42)

where we used (6.37) to substitute for the interest rate.
Steady state government spending and taxes are given by

\[ \bar{g} = g_1 - \zeta \gamma_{\text{cons}} \mathbf{1}_{\text{cons}} (\bar{b} - \bar{DT}), \]

and

\[ \bar{\tau} = \bar{\tau}^{DT} + (\gamma_\tau^0 + (1 - \zeta) \gamma_{\text{cons}} \mathbf{1}_{\text{cons}}) (\bar{b} - \bar{DT}), \]

where we assume that \( \bar{\tau}^{DT} \) is chosen such that steady state debt equals the steady state debt target. It follows from 6.42 that this requires

\[ \bar{\tau}^{DT} = \frac{\theta}{\theta - 1} \left( (1 - \beta) \bar{DT} + \bar{g} \right). \]

We then have \( \bar{g} = g_1 \) and \( \bar{\tau} = \bar{\tau}^{DT} \), as well as \( \bar{b} = \bar{D}T \).

Finally we, assume that households value real bond holdings relative to consumption such that, in steady state, they make optimal decisions. It then follows from (6.9) and (6.42) that

\[ \bar{\Lambda} = \left( \frac{\bar{b}}{1 - \bar{g}} \right)^\sigma = \left( \frac{(\bar{\tau}^{\tau - 1} - \bar{g})}{(1 - \bar{g})(1 - \beta)} \right)^\sigma. \]

Appendix 6.B  Log-linear model

6.B.1  Optimal consumption decision

The log-linearized optimality conditions (including budget constraints) are given by

\[ \dot{C}_s = \dot{C}_{s+1} - \frac{1}{\sigma} (E_{t+1}^i s - E_{t+1}^i \pi_{s+1}), \quad s = t, t + 1, \ldots t + T - 1, \]

\[ \dot{b}_{t+T+1} = \dot{b} \dot{C}_{t+T} + \frac{\bar{b}}{\sigma} E_{t+T}^i \pi_t + \frac{\bar{b}}{\sigma} \dot{\Lambda}_t, \]

\[ \eta \dot{H}_s = -\sigma \dot{C}_s - \frac{E_{t+1}^i \bar{e}_s}{1 - \bar{s}} + E_{t+1}^i \dot{w}_s, \quad s = t, t + 1, \ldots t + T, \]
Iterating the log-linearized budget constraints from period \( t + T \) backward and multiplying both sides by \( \beta^{T+1} \) gives

\[
\beta^{T+1} b_{t+T+1} = \frac{\bar{w}}{\beta} (1 - \bar{g}) \sum_{s=0}^{T} \beta^s (\tilde{C}_{t+s}^i) + \frac{\bar{w}}{Y} \sum_{s=0}^{T} \beta^s (E_t^i \tilde{w}_t + \bar{H}_t^i - E_{t+s}^i \tilde{\tau}_t + 1) - \bar{g} \tilde{C}_t^i, \tag{6.51}
\]

where we used that \( \bar{H} = \bar{Y} \) and \( \frac{\bar{C}}{\bar{Y}} = 1 - \bar{g} \).

We can plug in \( \tilde{b}_{t+T+1} \) from (6.48) and labor from (6.49) to get

\[
\beta^{T+1} b \tilde{C}_{t+T+1}^i + \frac{\bar{b}}{\sigma} E_{t+T}^i - \beta^{T+1} b \Lambda_t^i = \frac{\bar{b}}{\sigma} \tilde{b}_t^i - (1 - \bar{g}) \sum_{s=0}^{T} \beta^s (\tilde{C}_{t+s}^i) + \frac{\bar{w}}{Y} \sum_{s=0}^{T} \beta^s (E_t^i \tilde{w}_t + \bar{H}_t^i - E_{t+s}^i \tilde{\tau}_t + 1) - \bar{g} \tilde{C}_t^i. \tag{6.52}
\]

Next we use the Euler equation to substitute for future consumption. Iterating the Euler equation gives

\[
\tilde{C}_{t+s}^i = \tilde{C}_t^i + \sum_{j=0}^{s-1} \frac{1}{\sigma} (E_{t+j}^i - E_{t+j+1}^i), \quad T - s \geq 1. \tag{6.52}
\]
CHAPTER 6. CONSOLIDATIONS AND FINITE PLANNING HORIZONS

Rearranging (6.51) and substituting for future consumption gives

\[
\beta^{T+1} \bar{b}_i \left( \hat{C}_i^t + \sum_{j=0}^{T-1} \frac{1}{\sigma} (E^i_{t+j+1} - E^i_{t+j}) \right) + \beta^{T+1} \bar{b}_i E^i_{t+T} + \beta^{T+1} \bar{b}_i \hat{\Lambda}_i^t
\]

\[
= \hat{b}_i^t - \left( \frac{\sigma}{\eta} \bar{w}(1 - \bar{\tau}) + (1 - \bar{g}) \right) \sum_{s=0}^{T} \beta^s \left( \hat{C}_i^t + \sum_{j=0}^{s-1} \frac{1}{\sigma} (E^i_{t+j} - E^i_{t+j+1}) \right)
\]

\[
+ \bar{\Xi} \sum_{s=0}^{T} \beta^s (E^i_{t+s}) + \bar{b} \sum_{s=0}^{T} \beta^s (\beta E^i_{t+s} - E^i_{t+s})
\]

\[
+ \bar{w} \sum_{s=0}^{T} \beta^s ((1 + 1/\eta) (1 - \bar{\tau}) E^i_{t+s}) - E^i_{t+s}).
\]

Taking contemporaneous consumption to one side of the equation gives the current decision of consumer \(i\)

\[
(\beta^{T+1} \bar{b} + \left( \frac{\sigma}{\eta} \bar{w}(1 - \bar{\tau}) + (1 - \bar{g}) \right) \frac{1 - \beta^{T+1}}{1 - \beta} ) \hat{C}_i^t =
\]

\[
- \beta^{T+1} \bar{b} \bar{\Lambda}_i^t + \hat{b}_i^t + \bar{w} \sum_{s=0}^{T} \beta^s ((1 + 1/\eta) (1 - \bar{\tau}) E^i_{t+s} - E^i_{t+s})
\]

\[
- \left( \frac{\sigma}{\eta} \bar{w}(1 - \bar{\tau}) + (1 - \bar{g}) \right) \sum_{s=1}^{T} \beta^s \sum_{j=0}^{s-1} \frac{1}{\sigma} (E^i_{t+j} - E^i_{t+j+1})
\]

\[
+ \bar{b} \sum_{s=0}^{T} \beta^s (\beta E^i_{t+s} - E^i_{t+s}) - \beta^{T+1} \bar{b} \sum_{j=0}^{T-1} \frac{1}{\sigma} (E^i_{t+j} - E^i_{t+j+1}) - \beta^{T+1} \bar{b} \sigma E^i_{t+T}.
\]

Aggregating this equation over all households yields an expression for aggregate

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consumption as a function of aggregate expectations about aggregate variables, only.

(6.55)

\[
\left( \beta^{T+1} \bar{b} + \left( \frac{\sigma}{\eta} \bar{w}(1 - \bar{\tau}) + (1 - \bar{g}) \right) \frac{1 - \beta^{T+1}}{1 - \beta} \right) \hat{C}_t =
\]

\[
- \beta^{T+1} \frac{\bar{b}}{\sigma} \hat{\Lambda}_t + \hat{b}_t + \bar{w} \sum_{s=0}^{T} \beta^s \left( (1 + \frac{1}{\eta}) (1 - \bar{\tau}) \bar{E}_t \hat{w}_{t+s} - \bar{E}_t \hat{\pi}_{t+s} \right) + \bar{w} \sum_{s=0}^{T} \beta^s (\bar{E}_t \hat{\xi}_{t+s})
\]

\[
- \left( \frac{\sigma}{\eta} \bar{w}(1 - \bar{\tau}) + (1 - \bar{g}) \right) \sum_{s=1}^{T} \beta^s \sum_{j=0}^{s-1} \frac{1}{\sigma} (\bar{E}_t \hat{\pi}_{t+j} - \bar{E}_t \hat{\pi}_{t+j+1})
\]

\[
+ \bar{b} \sum_{s=0}^{T} \beta^s (\bar{E}_t \hat{\pi}_{t+s} - \bar{E}_t \hat{\pi}_{t+s}) - \beta^{T+1} \frac{\bar{b}}{\sigma} \sum_{j=0}^{T-1} \frac{1}{\sigma} (\bar{E}_t \hat{\pi}_{t+j} - \bar{E}_t \hat{\pi}_{t+j+1}) - \beta^{T+1} \frac{\bar{b}}{\sigma} \bar{E}_t \hat{\pi}_{t+T}.
\]

6.B.2 Optimal pricing decision

Log linearizing (6.16) gives

\[
\hat{p}_t^* (j) - \hat{p}_t = \frac{1 - \omega \beta}{1 - \omega^{T+1} \beta^{T+1}} \left[ \hat{m}_c_t + \bar{E}_t \sum_{s=1}^{T} \omega^s \beta^s \left( \hat{m}_c_{t+s} + \hat{\pi}_{t+s} - \hat{\pi}_t \right) \right].
\]

Next, (6.17) can be log-linearized to

\[
\hat{p}_t = \omega \hat{p}_{t-1} + (1 - \omega) \int_0^1 \hat{p}_t^* (j) dj,
\]

from which it follows that

(6.57)

\[
\hat{\pi}_t = \frac{1 - \omega}{\omega} \left( \int_0^1 \hat{p}_t^* (j) dj - \hat{p}_t \right).
\]

Aggregating (6.56) and plugging this in in the above expression gives

(6.58)

\[
\hat{\pi}_t = \frac{(1 - \omega)(1 - \omega \beta)}{\omega (1 - \omega^{T+1} \beta^{T+1})} \left( \hat{m}_c_t + \sum_{s=1}^{T} \omega^s \beta^s \bar{E}_t \hat{m}_c_{t+s} + \sum_{s=1}^{T} \omega^s \beta^s \sum_{\tau=1}^{s} \bar{E}_t \hat{\pi}_{t+\tau} \right).
\]
6.B.3 Final model

To complete the model we first log-linearize the government budget constraint, (6.19), to

\[ \tilde{b}_{t+1} = \frac{1}{\beta} \tilde{g}_t - \frac{\bar{w}}{\beta} (\tilde{r}(\tilde{w}_t + \tilde{H}_t) + \tilde{\tau}_t) + \frac{1}{\beta} \tilde{b}_t + \tilde{b}(\tilde{\tau}_t - \frac{1}{\beta} \tilde{\pi}_t). \]  

Next, we log-linearize the market clearing condition, (6.20)

\[ \hat{Y}_t = (1 - \bar{g}) \hat{C}_t + \tilde{g}_t, \]

and write wages and marginal costs as

\[ \hat{mc}_t = \hat{w}_t = \eta \hat{H}_t + \sigma \hat{C}_t + \frac{\tilde{\tau}_t}{1 - \tilde{r}} = (\eta + \frac{\sigma}{1 - \bar{g}}) \hat{Y}_t - \sigma \frac{\tilde{g}}{1 - \bar{g}} + \frac{\tilde{\tau}_t}{1 - \tilde{r}}. \]

Finally, log-linearizing profits of firm \( j \)

\[ \hat{\Xi}_t(j) = \frac{1}{1 - \hat{mc}} (\hat{p}_t(j) - \hat{p}_t) + \hat{Y}_t(j) - \frac{\hat{mc}}{1 - \hat{mc}} \hat{mc}_t. \]

Aggregate profits can be written as

\[ \hat{\Xi}_t = \hat{Y}_t - (\theta - 1) \hat{mc}_t, \]

where we used that \( \hat{mc} = \frac{\theta - 1}{\theta} \).

Using (6.60) in (6.55) results an expression for aggregate output.

\[ \hat{Y}_t = \frac{1}{\rho} \hat{b}_t + \hat{g}_t + \delta \sum_{s=0}^{T} \beta^s ((1 - \bar{r}) \bar{E}_t \tilde{w}_{t+s} - \bar{E}_t \tilde{\tau}_{t+s}) + \frac{\tilde{w}}{\bar{Y}_\rho} \sum_{s=0}^{T} \beta^s (\bar{E}_t \tilde{\Xi}_{t+s}) \]

\[ - \mu \sum_{s=1}^{T} \beta^s (\bar{E}_t \tilde{\pi}_{t+j+1}) + \frac{\tilde{b}}{\rho} \sum_{s=0}^{T} \beta^s (\beta \bar{E}_t \tilde{\pi}_{t+s} - \bar{E}_t \tilde{\pi}_{t+s}) \]

\[ - \beta^{T+1} \frac{\tilde{b}}{\sigma \rho} \sum_{j=0}^{T-1} (\bar{E}_t \tilde{\pi}_{t+j} - \bar{E}_t \tilde{\pi}_{t+j+1}) - \beta^{T+1} \frac{\tilde{b}}{\sigma \rho} \bar{E}_t \tilde{\pi}_{t+T} - \beta^{T+1} \frac{\tilde{b}}{\sigma \rho} \tilde{\lambda}_t, \]

\[ \delta = \frac{\tilde{w} \eta + 1}{\rho \eta}, \]
6.B. Log-linear model

\[ \mu = \frac{1}{\rho} \left( \frac{\bar{w}}{\eta} (1 - \bar{\tau}) + \frac{1 - \bar{g}}{\sigma} \right), \]

\[ \rho = \frac{1}{1 - \bar{g}} \left[ \beta^{T+1} \bar{b} + \left( \frac{\sigma}{\eta} \bar{w} (1 - \bar{\tau}) + (1 - \bar{g}) \right) \frac{1 - \beta^{T+1}}{1 - \beta} \right]. \]

We now assume that agents know, or have learned about the above relations between aggregate variables (which hold in every period). Therefore, expectations about wages and profits can be substituted for, using (6.61) and (6.63). This gives the following system of 3 equations that, together with a specification of monetary and fiscal policy, completely describe our model

\[ (1 - \nu_y) \hat{Y}_t = \frac{1}{\rho} \hat{b}_t + g_t + \nu \sum_{s=0}^{T} \beta^s (\tilde{E}_t \tilde{\tau}_{t+s}) + \nu_g \sum_{s=0}^{T} \beta^s (\tilde{E}_t \tilde{g}_{t+s}) + \nu_g \sum_{s=1}^{T} \beta^s (\tilde{E}_t \tilde{y}_{t+s}) \]

\[ - \mu \sum_{s=1}^{T} \beta^s \sum_{j=1}^{s} (\tilde{E}_t \tilde{r}_{t+j-1} - \tilde{E}_t \tilde{\tau}_{t+j}) + \frac{\bar{b}}{\rho} \sum_{s=0}^{T} \beta^s (\beta \tilde{E}_t \tilde{\tau}_{t+s} - \tilde{E}_t \hat{\tau}_{t+s}) \]

\[ - \beta^{T+1} \frac{\bar{b}}{\sigma \rho} \sum_{j=0}^{T-1} (\tilde{E}_t \tilde{r}_{t+j} - \tilde{E}_t \tilde{\tau}_{t+j+1}) - \beta^{T+1} \frac{\bar{b}}{\sigma \rho} \tilde{E}_t \tilde{r}_{t+T} - \beta^{T+1} \frac{\bar{b}}{\sigma \rho} \hat{\Lambda}_t, \]

\[ \pi_t = \tilde{\kappa} (\eta + \frac{\sigma}{1 - \bar{g}}) \sum_{s=0}^{T} \omega^s \beta^s \tilde{E}_t \tilde{y}_{t+s} + \frac{\tilde{\kappa} \sigma}{1 - \bar{g}} \sum_{s=0}^{T} \omega^s \beta^s \tilde{E}_t \tilde{y}_{t+s} \]

\[ + \frac{\tilde{\kappa}}{1 - T} \sum_{s=0}^{T} \omega^s \beta^s \tilde{E}_t \tilde{\tau}_{t+s} + \tilde{\kappa} \sum_{s=1}^{T} \omega^s \beta^s \tilde{E}_t \tilde{\tau}_{t+s} \]

\[ \tilde{b}_{t+1} = \frac{1}{\beta} \tilde{b}_t - \frac{\bar{w}}{\beta} \left[ \tilde{\tau} \left( (1 + \eta + \frac{\sigma}{1 - \bar{g}}) \hat{Y}_t - \sigma \frac{\tilde{y}_t}{1 - \bar{g}} + \frac{\tilde{\tau}_t}{1 - \bar{\tau}} \right) + \frac{\hat{r}_t}{1 - \beta} + \tilde{b}_t \right] + \frac{1}{\beta} \tilde{b}_t + \bar{b} (i_t - \frac{1}{\beta} \tilde{\pi}_t), \]

with

\[ \nu_y = \frac{1}{\theta \rho} + \left( \frac{\delta (1 - \tau) - \theta - 1}{\theta \rho} \right) (\eta + \frac{\sigma}{1 - \bar{g}}), \]

\[ \nu_g = \left( \frac{\theta - 1}{\theta \rho} - \delta (1 - \tau) \right) \frac{\sigma}{1 - \bar{g}}. \]
Appendix 6.C  Model under infinitely forward looking rational expectations

When we let the planning horizon, \(T\), go to infinity (6.64) reduces to

\[
\hat{Y}_t = \frac{1}{\rho} \hat{b}_t + \hat{g}_t + \delta \sum_{s=0}^{\infty} \beta^s ((1 - \bar{\tau}) \hat{E}_t \hat{\pi}_{t+s} - \hat{E}_t \hat{\pi}_{t+s}) + \frac{\tilde{\kappa}}{\bar{\nu} \rho} \sum_{s=0}^{\infty} \beta^s (\hat{E}_t \hat{\xi}_{t+s})
\]

\[
- \mu \sum_{s=1}^{\infty} \beta^s \sum_{j=0}^{s-1} (\hat{E}_t \hat{i}_{t+j} - \hat{E}_t \hat{\pi}_{t+j+1}) + \frac{\hat{b}}{\rho} \sum_{s=0}^{\infty} \beta^s (\beta \hat{E}_t \hat{\pi}_{t+s} - \hat{E}_t \hat{\pi}_{t+s}),
\]

which can be written as

\[
\hat{Y}_t = \frac{1}{\rho} \hat{b}_t + \hat{g}_t + \delta \sum_{s=0}^{\infty} \beta^s ((1 - \bar{\tau}) \hat{E}_t \hat{w}_{t+s} - \hat{E}_t \hat{w}_{t+s}) + \frac{1}{\bar{\nu} \rho} \sum_{s=0}^{\infty} \beta^s (\hat{E}_t \hat{w}_{t+s})
\]

\[
- \mu \sum_{s=1}^{\infty} \beta^s \sum_{j=0}^{s-1} (\hat{E}_t \hat{i}_{t+j} - \hat{E}_t \hat{\pi}_{t+j+1}) + \frac{\hat{b}}{\rho} \sum_{s=0}^{\infty} \beta^s (\beta \hat{E}_t \hat{\pi}_{t+s} - \hat{E}_t \hat{\pi}_{t+s}).
\]

Leading this equation 1 period and taking expectations gives

\[
\hat{E}_t \hat{Y}_{t+1} = \frac{1}{\rho} \hat{E}_t \hat{b}_{t+1} + \hat{E}_t \hat{g}_{t+1} + \delta \sum_{s=1}^{\infty} \beta^{s-1} ((1 - \bar{\tau}) \hat{E}_t \hat{w}_{t+s} - \hat{E}_t \hat{w}_{t+s})
\]

\[
+ \frac{1}{\theta \rho} \sum_{s=1}^{\infty} \beta^{s-1} (\hat{E}_t \hat{\xi}_{t+s}) - \frac{\mu \beta}{1 - \beta} \sum_{j=1}^{\infty} \beta^{j-1} (\hat{E}_t \hat{i}_{t+j} - \hat{E}_t \hat{\pi}_{t+j+1})
\]

\[
+ \frac{\hat{b}}{\rho} \sum_{s=1}^{\infty} \beta^{s-1} (\beta \hat{E}_t \hat{\pi}_{t+s} - \hat{E}_t \hat{\pi}_{t+s}).
\]

We can therefore rewrite output recursively as

\[
\hat{Y}_t = \beta \hat{E}_t \hat{Y}_{t+1} + \frac{1}{\rho} (\hat{b}_t - \beta \hat{E}_t \hat{b}_{t+1}) + (\hat{g}_t - \beta \hat{E}_t \hat{g}_{t+1}) + \delta (1 - \bar{\tau}) \hat{w}_t - \delta \hat{\tau}_t + \frac{\hat{b}}{\rho} \hat{\xi}_t
\]

\[
+ \left( \beta \frac{\hat{b}}{\rho} - \frac{\mu \beta}{1 - \beta} \right) \hat{i}_t - \frac{\hat{b}}{\rho} \hat{\pi}_t + \frac{\mu \beta}{1 - \beta} \hat{E}_t \hat{\pi}_{t+1}.\]
Plugging in (6.61) and (6.63) this reduces to

\[
(1 - \nu_y)\hat{Y}_t = \beta \hat{E}_t \hat{Y}_{t+1} + \frac{1}{\rho} (\ddot{b}_t - \beta \hat{E}_t \ddot{b}_{t+1}) + (1 + \nu_g) \ddot{g}_t - \beta \ddot{g}_{t+1} + \nu \ddot{\tau}_t
\]

\[
+ \left( \beta \frac{\ddot{b}}{\rho} - \frac{\mu \beta}{1 - \beta} \right) \dot{\pi}_t - \frac{\ddot{b}}{\rho} \ddot{\pi}_t + \frac{\mu \beta}{1 - \beta} \hat{E}_t \ddot{\pi}_{t+1}.
\]

The same kind of derivation on the production side results in the standard New Keynesian Phillips curve. For infinite horizons we get

\[
\hat{\pi}_t = (1 - \omega)(1 - \omega \beta) \left( \sum_{s=0}^{\infty} \omega^s \beta^s \hat{E}_t \hat{mc}_{t+s} + \frac{\omega \beta}{1 - \omega \beta} \sum_{s=1}^{\infty} \omega^s \beta^s \hat{E}_t \hat{\pi}_{t+j+1} \right).
\]

Leading 1 period and taking expectations gives

\[
\hat{E}_t \hat{\pi}_{t+1} = (1 - \omega)(1 - \omega \beta) \left( \sum_{s=1}^{\infty} (\omega \beta)^{s-1} \hat{E}_t \hat{mc}_{t+s} + \frac{\omega \beta}{1 - \omega \beta} \sum_{s=1}^{\infty} (\omega \beta)^{s-1} \hat{E}_t \hat{\pi}_{t+j+1} \right).
\]

So that inflation writes recursively as

\[
\hat{\pi}_t = \omega \beta \hat{E}_t \hat{\pi}_{t+1} + (1 - \omega) \beta \hat{E}_t \hat{\pi}_{t+1} + \hat{\kappa} \hat{mc}_t = \beta \hat{E}_t \hat{\pi}_{t+1} + \hat{\kappa} \hat{mc}_t,
\]

\[
\hat{\kappa} = (1 - \omega)(1 - \omega \beta) \frac{\omega}{1 - \omega \beta}.
\]

Using (6.61) then gives

\[
\hat{\pi}_t = \beta \hat{E}_t \hat{\pi}_{t+1} + \hat{\kappa} (\eta + \frac{\sigma}{1 - \hat{g}}) \hat{Y}_t - \hat{\kappa} \sigma \frac{\ddot{g}_t}{1 - \hat{g}} + \hat{\kappa} \frac{\ddot{\tau}_t}{1 - \hat{\tau}}.
\]

**Appendix 6.D  Steady states log-linear model for given Λ**

Below we derive steady state relations that hold for a given value of the valuation of end of horizon wealth: \( \Lambda_t = \Lambda \). This is the steady state that agents expect the model to have reached after their horizon.\(^{93}\)

\(^{93}\)The steady state that the model will reach once \( \Lambda_t \) has converged can then be obtained by plugging the value of \( \Lambda \) that solves \( \Lambda = (1 - \gamma) \Lambda + \gamma \left( \frac{z - \sigma}{y - \bar{g}} Y + \frac{\sigma}{y - \bar{g}} \bar{g} - \bar{i} \right) \).

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In steady state, Equations (6.21) through (6.29) reduce to

\begin{align}
(6.81) & \quad i = \phi_1 \pi + \phi_2 Y, \\
(6.82) & \quad g = -1_{cons} \alpha \gamma_{g}^{cons} (b - DT), \\
(6.83) & \quad \tau = \tau^{DT} + (\gamma_0^{\tau} + 1_{cons}(1 - \alpha)\gamma_{\tau}^{cons})(b - DT).
\end{align}

Plugging in these steady states values for monetary and fiscal variables in the steady state versions of Equations (6.24) through (6.26), we can rearrange these three equations for output, inflation and debt to

\begin{align}
(6.84) & \quad Y = \frac{1}{\bar{\rho}} b + \left( \phi_1 \frac{\psi_i}{\epsilon} - \frac{\psi_{\pi}}{\epsilon} \right) \pi - \gamma_{DT} (b - DT) - \gamma_{\tau} \tau^{DT} - \beta^{T+1} \frac{\bar{b}}{\sigma \bar{\rho}} \Lambda, \\
(6.85) & \quad \pi = \chi \left( \eta + \frac{\sigma}{1 - g} \right) Y + \chi \sigma_1 (b - DT) + \chi \frac{\tau^{DT}}{1 - \bar{\tau}}, \\
(6.86) & \quad \beta b = b + \left( \phi_2 \beta \bar{b} - \bar{w} \bar{\pi} (1 + \eta + \frac{\sigma}{1 - g}) \right) Y \\
& \quad \quad + (\beta \phi_1 - 1) \bar{b} \pi - \delta_{DT} (b - DT) - \delta_{\tau} \tau^{DT},
\end{align}

With

\begin{align}
(6.87) & \quad \epsilon = 1 - \frac{1}{1 - \beta} \nu_g - \phi_2 \psi_i, \\
(6.88) & \quad \bar{\rho} = \rho \epsilon, \\
(6.89) & \quad \psi_i = \left( -\beta^{T+1} \frac{\bar{b}}{\sigma \bar{\rho}} (T + 1) + \frac{\beta b (1 - \beta^{T+1})}{\rho} \frac{1}{1 - \beta} - \frac{\mu (T - T - 1) \beta^{T+1} + \beta}{(1 - \beta)^2} \right), \\
(6.90) & \quad \psi_{\pi} = \left( -\beta^{T+1} \frac{\bar{b}}{\sigma \bar{\rho}} T + \frac{\bar{b} (1 - \beta^{T+1})}{\rho} \frac{1}{1 - \beta} - \frac{\mu (T - T - 1) \beta^{T+1} + \beta}{(1 - \beta)^2} \right),
\end{align}
6.D. Steady states log-linear model for given $\Lambda$

\begin{align}
\gamma_{DT} &= \frac{1}{\epsilon} \left[ I_{\text{cons}} \alpha \gamma_{\text{cons}}^g + \frac{1 - \beta^{T+1}}{1 - \beta} \left( -\nu_s (\gamma_{\tau}^0 + I_{\text{cons}} (1 - \alpha) \gamma_{\tau}^\text{cons}) + \nu_{g} I_{\text{cons}} \alpha \gamma_{\text{cons}}^g \right) \right], \\
\gamma_{\tau} &= -\frac{1}{\epsilon} \nu_s \frac{1 - \beta^{T+1}}{1 - \beta}, \\
\chi &= \frac{\kappa \left( \frac{1 - \omega^{T+1} \beta^{T+1}}{1 - \omega^{T}} \right)}{\kappa (\omega^{T+1} \beta^{T+1} + \omega^{T} 1)} , \\
\sigma_1 &= \sigma \frac{I_{\text{cons}} \alpha \gamma_{\text{cons}}^g}{1 - \bar{g}} + \frac{(\gamma_{\tau}^0 + I_{\text{cons}} (1 - \alpha) \gamma_{\tau}^\text{cons})}{1 - \bar{\tau}}, \\
\delta_{DT} &= \left[ I_{\text{cons}} \alpha \gamma_{\text{cons}}^g + \bar{w} \left( (\gamma_{\tau}^0 + I_{\text{cons}} (1 - \alpha) \gamma_{\tau}^\text{cons}) + \bar{\tau} \sigma_1 \right) \right], \\
\delta_{\tau} &= \bar{w} \left( 1 + \frac{\bar{\tau}}{1 - \bar{\tau}} \right).
\end{align}

Now, plugging in (6.85) in (6.84) gives

\begin{align}
Y &= \frac{1}{\rho \nu} b - \beta^{T+1} \frac{\bar{b}}{\sigma \rho \nu} \Lambda + \frac{1}{\nu} \left[ \chi \sigma_1 (\phi \frac{\psi_i}{\epsilon} - \frac{\psi_\pi}{\epsilon}) - \gamma_{DT} \right] (b - DT) \\
&\quad + \frac{1}{\nu} \left[ \chi \frac{1}{1 - \bar{\tau}} (\phi \frac{\psi_i}{\epsilon} - \frac{\psi_\pi}{\epsilon}) - \gamma_{\tau} \right] \tau_{DT}, \\
\nu &= 1 - (\phi \frac{\psi_i}{\epsilon} - \frac{\psi_\pi}{\epsilon}) \lambda \left( \eta + \frac{\sigma}{1 - \bar{g}} \right).
\end{align}

Plugging in (6.85) in (6.86) gives

\begin{align}
\beta b &= b + \sigma_2 Y + \left[ - \delta_{DT} + (\beta \phi_1 - 1) \bar{b} \chi \sigma_1 \right] (b - DT) \\
&\quad + \left[ - \delta_{\tau} + (\beta \phi_1 - 1) \bar{b} \chi \frac{1}{1 - \bar{\tau}} \right] \tau_{DT}.
\end{align}
CHAPTER 6. CONSOLIDATIONS AND FINITE PLANNING HORIZONS

\[ \sigma_2 = \left( \phi_2 \beta \tilde{b} - \tilde{w} \tau (1 + \eta + \frac{\sigma}{1 - g}) + (\beta \phi_1 - 1) \tilde{b} \chi (\eta + \frac{\sigma}{1 - g}) \right). \]

Finally plugging in (6.97) in (6.99) and rearranging gives

\[ b = \frac{1}{\sigma_3} \left( \omega_{\text{DT}} DT - \omega_{\tau} \tau DT - \beta^T \frac{\tilde{b}}{\sigma \rho v} \Lambda \sigma_2 \right), \]

with

\[ \sigma_3 = 1 - \frac{1}{\beta} \left[ 1 + \frac{1}{\rho v} \sigma_2 \right] + \omega_{\text{DT}}, \]

\[ \omega_{\text{DT}} = \frac{\delta_{\text{DT}}}{\beta} - \frac{1}{\beta} (\beta \phi_1 - 1) \tilde{b} \chi \sigma_1 - \frac{1}{\beta} \sigma_2 \frac{1}{v} \left[ \chi \sigma_1 \left( \frac{\psi_i}{\epsilon} - \frac{\psi_x}{\epsilon} \right) - \gamma_{\text{DT}} \right], \]

\[ \omega_\tau = \frac{\delta_\tau}{\beta} - \frac{1}{\beta} (\beta \phi_1 - 1) \tilde{b} \chi \frac{1}{1 - \tau} - \frac{1}{\beta} \sigma_2 \frac{1}{v} \left[ \chi \frac{1}{1 - \tau} \left( \frac{\psi_i}{\epsilon} - \frac{\psi_x}{\epsilon} \right) - \gamma_{\tau} \right]. \]

Steady state output and inflation now can by plugging in (6.101) in (6.97) and plugging in the resulting steady state output, as well as (6.101), in (6.85).

Appendix 6.E Uncertainty about type of consolidation

In the chapter, we have assumed that when the debt target drops, agents not only know that consolidations will be implemented next period, but also whether these consolidations will be spending or tax-based. In this appendix, we assume that agents initially think consolidations are a mix of 50% spending-based and 50% tax-based. This reflects the case where the government does not credibly announce which type of consolidations it will implement. When consolidations are implemented for the first time, agents observe the type of consolidation, and update their beliefs accordingly.

This case of initial uncertainty about the type of consolidations is plotted in Figure 6.9, where we assume \( T = 8, \phi_1 = 1.3 \) and \( \gamma_{\text{cons}} = 0.3 \), as in Figure 6.5. We see that now consumption and hence output do not react much in the period where the drop in the debt target is announced (period 2). This is because agents do not expect a big change in their future consumption, since spending-based and tax-based consolidations have opposite effects on that variable. As a result, debt does not change much either.
in the period where the debt target is dropped. Comparing this with Figure 6.5, under spending-based consolidations (green) output is now lower, while debt drops with a slower pace. The opposite holds for tax-based consolidations (black).

In the first period where consolidations are implemented (period 3), agents still do not know what the type will be. Hence, output is again lower under spending-based consolidations than in Figure 6.5, while it is higher in case of a tax-based consolidation. This results in a relative advantage of tax-based consolidations in reducing debt, already in the short run. In the following periods agents have updated their beliefs about the type of consolidations, and dynamics are very similar to those in Figure 6.5. Slight differences arise because of slightly different debt levels, and hence different values of taxes and government spending.

The other figures of Section 6.5 change in a similar fashion under uncertainty about
the type of consolidation. That is, the initial two periods favor tax-based consolidations and hurt spending-based consolidations compared to the case of fully anticipated consolidations, while medium to long run dynamics are hardly affected.
Chapter 7

Summary

The recent economic developments that followed the 2007 financial crisis have called for new modeling approaches that can better explain observed macroeconomic phenomena and that can help in designing more robust economic policies. In particular, we would like to understand why recovery of economic growth and inflation have been low, with interest rates at their zero lower bound, and what policy measures could be successful in restoring the economy to a more healthy state. Additionally, it is important to understand how the rising European debt levels can best be managed, and what the consequences of consolidations on economic activity can be.

This thesis addresses these issues from the perspective of bounded rationality in five related, but independent chapters. By deviating from the traditional assumption of homogeneous rational expectations and by allowing for less sophisticated and heterogeneous expectation formation, new insights are obtained with important policy implications.

Chapter 2 explicitly models the credibility of the central bank, and allows this credibility to evolve endogenously with the central bank’s past ability to achieve its targets. It is shown that the stability region of policy parameters in the central bank’s interest rate rule is strictly larger under heterogeneous expectations than under rational expectations. Furthermore, with optimal monetary policy, global stability of the fundamental steady state can be achieved, implying that the system always converges to the targets of the central bank. This result however no longer holds when the zero lower bound on the nominal interest rate is accounted for. Self-fulfilling deflationary spirals can then occur, even under optimal policy. The occurrence of these liquidity traps crucially depends on, and is inversely related to, the credibility of the central bank. Deflationary spirals can be prevented with a high inflation target, aggressive monetary easing, or a more aggressive response to inflation.
Chapter 3 studies monetary policy when there is an evolving distribution of heterogeneous expectations. Agents may choose from a continuum of forecasting rules and adjust their expectations based on relative past performance. The extent to which expectations are anchored to the fundamentals of the economy turns out to be crucial in determining whether the central bank can stabilize the economy. When expectations are strongly anchored, little is required of the central bank for local stability. Only when expectations are unanchored, the Taylor principle (which needs to be satisfied under rational expectations) becomes a necessary condition. More aggressive policy may however be required to prevent coordination on almost self-fulfilling optimism or pessimism and the existence of multiple equilibria. When the zero lower bound on the nominal interest rate is accounted for, the inflation target must furthermore be high enough, in order to prevent coordination on self-fulfilling liquidity traps and deflationary spirals.

Chapter 4 presents a learning-to-forecast experiment to study whether central banks can manage market expectations by means of forward guidance. Forward guidance takes the form of one-period ahead inflation projections that are published by the central bank in each period. Subjects in the experiment observe these projections along with the historic development of the economy and subsequently submit their own one-period ahead inflation forecasts. In this context, we find that the central bank can significantly manage market expectations through forward guidance and that this management strongly supports monetary policy in stabilizing the economy and reducing forecast errors. Moreover, strategically optimistic forward guidance drastically reduces the probability of a deflationary spiral after strong negative shocks to the economy. However, pessimistic forward guidance announcements after negative shocks can instead initiate coordination on a deflationary spiral. Although the credibility of the central banks forecasts depends on both the central banks forecasts errors and on credibility in earlier periods, we do not find evidence that a central bank with a better forecasting track record is better able to mitigate recessions than a central bank with less credible past forecasts.

Chapter 5 analyzes fiscal consolidations using a model where agents have heterogeneous expectations and are uncertain about the composition of consolidations. We look at spending-based and tax-based consolidations and analyze their effects separately. We find that the effects of consolidations and the output multipliers are sensitive to heterogeneity in expectations before and after implementation of a specific fiscal plan. Depending on the beliefs about the type of consolidation prior to implementation, we show that heterogeneity in expectations may lead to optimism in the economy, improving thus the performance of a specific fiscal plan, or can work towards
the opposite direction leading to pessimism, amplifying the contractionary effects of
the consolidation. Interestingly, we find that wrong beliefs about the composition of
fiscal consolidation may improve or harm the effectiveness of consolidations, depending
on the degree of heterogeneity.

Chapter 6 analyzes fiscal consolidations in a model where agents are homogeneous,
but have a finite planning horizon. Both consumers and firms are infinitely lived,
but only plan and form expectations up to a finite number of periods into the future.
The length of agents’ planning horizons plays an important role in determining how
spending cuts or tax increases affect output and inflation. We find that for low degrees
of relative risk aversion spending-based consolidations are less costly in terms of output
losses, in line with empirical evidence. A stronger response of monetary policy to
inflation makes spending-based consolidations more favorable as well. Interestingly,
for short planning horizons, our model is able to capture the positive comovement
between private consumption and government spending.
Bibliography


Samenvatting (Summary in Dutch)

Door de financiële crisis in 2007, en de economische ontwikkelingen die daar op volgden, is er vraag gekomen naar nieuwe economische modellen die de waargenomen macro-economische ontwikkelingen beter kunnen verklaren en die kunnen helpen bij het opstellen van meer robuust economisch beleid. In het bijzonder willen we graag weten waarom de economische groei en inflatie laag waren, met rentes dicht bij 0, en welke beleidsmaatregelen de economie weer in een gezonde toestand kunnen brengen. Ook is het belangrijk om te begrijpen hoe we het beste kunnen omgaan met de groeiende Europese staatsschulden en wat de gevolgen voor budgettaire consolidaties zijn voor economische activiteit.

Dit proefschrift behandelt deze kwesties vanuit het perspectief van begrensde rationaliteit in vijf gerelateerde, maar onafhankelijke hoofdstukken. Door uit te gaan van heterogene en minder geavanceerde verwachtingsvorming door deelnemers aan de economie (agenten) en hiermee af te wijken van de traditionele aanname van homogene en rationele verwachtingen, kunnen nieuwe inzichten worden verkregen met belangrijke beleidsimplicaties.

Hoofdstuk 2 modelleert de geloofwaardigheid van de centrale bank expliciet. Deze geloofwaardigheid ontwikkelt zich bovendien endogeen met het vermogen van de centrale bank om zijn doel te bereiken in het verleden. Het hoofdstuk laat zien dat de stabilitieitsregio van beleidsparameters in de renteregel van de centrale bank strikt groter is onder heterogene verwachtingen dan onder rationele verwachtingen. Bovendien kan met optimaal monetair beleid globale stabiliteit van de fundamentele rusttoestand worden bereikt, wat impliceert dat het systeem altijd naar de doelwaardes van de centrale bank convergeert. Dit resultaat houdt echter geen stand als er rekening wordt gehouden met de nul-ondergrens op de nominale rente. Dan kunnen namelijk zelfvervullende deflationaire spiralen ontstaan, zelfs onder optimaal beleid. Of dit gebeurt hangt cruciaal (met een negatief verband) af van de geloofwaardigheid van de centrale bank. Deflationaire spiralen kunnen worden voorkomen door te streven naar hogere inflatie, door de rente agressief te verlagen als deze dicht bij 0 komt, of door de rente sterker te laten
HOOFDSTUK 3 onderzoekt monetair beleid in het geval dat er een statistische verdeling van heterogene agenten is die zich endogeen ontwikkelt. Agenten kiezen uit een continue verdeling van voorspellingsregels, gebaseerd op relatieve prestaties van die regels in het verleden. De mate waarin verwachtingen verankerd zijn aan de fundamentele waarden van de economie blijkt cruciaal te zijn bij het bepalen of de centrale bank de economie kan stabiliseren. Als verwachtingen sterk verankerd zijn, hoeft de centrale bank weinig te doen om lokale stabiliteit te bereiken. Alleen als verwachtingen niet verankerd zijn, is het zogenoemde "Taylore principe" (waaraan voldaan moet worden onder rationele verwachtingen) een noodzakelijke voorwaarde. Er is echter agressiever beleid nodig om coördinatie op bijna zelfvervullend optimisme of pessimisme te voorkomen en om het bestaan van niet-fundamentele evenwichten uit te sluiten. Als er rekening wordt gehouden met de nul-ondergrens op de nominale rente moet er bovendien gestreefd worden naar hogere inflatie om coördinatie op zelfvervullende liquiditeitsvallen en deflationaire spiralen te voorkomen.

HOOFDSTUK 4 presenteert een 'learning-to-forecast' experiment waarmee onderzocht wordt of de centrale bank marktverwachtingen kan beïnvloeden met zo genaamde 'forward guidance'. Forward guidance bestaat hier uit inflatievoorspellingen voor een periode in de toekomst die elke periode gepubliceerd worden door de centrale bank. Deelnemers aan het experiment zien deze voorspellingen samen met de historische ontwikkelingen van de economie waar ze aan deelnemen en moeten dan hun eigen inflatievoorspelling opgeven. We vinden in dit experiment dat de centrale bank met forward guidance een significante invloed kan uitoefenen op marktverwachtingen en dat deze invloed monetair beleid sterk ondersteunt bij het stabiliseren van de economie. Ook leidt het tot kleinere voorspellingsfouten in de economie. Verder vinden we dat strategisch optimistische forward guidance de kans sterk vermindert dat de economie in een deflationaire spiraal geraakt na grote negatieve schokken. Daarentegen kan pessimistische forward guidance na negatieve schokken juist leiden tot coördinatie op een deflationaire spiraal. Hoewel de geloofwaardigheid van de voorspellingen van de centrale bank afhanger van zowel de voorspellingsfouten van de centrale bank als van de geloofwaardigheid in eerdere perioden, vinden we geen bewijs dat een centrale bank met betere voorspellingen in het verleden beter in staat is een recessie te verhelpen dan een centrale bank met minder geloofwaardige voorspellingen in het verleden.

HOOFDSTUK 5 analyseert budgettaire consolidaties met een model waar agenten heterogene verwachtingen hebben en onzeker zijn over de vorm van de consolidatie. Consolidaties kunnen uitgevoerd worden in de vorm van bezuinigingen en van belastingverhogingen en we analyseren de effecten van deze maatregelen apart. We vinden dat
effecten van de consolidaties en de bijbehorende multipliers afhangen van de heterogeniteit in verwachtingen voor en na de implementatie van het specifieke fiscale plan. We laten zien dat, afhankelijk van de verwachtingen over de vorm van consolidatie voordat deze wordt geïmplementeerd, heterogeniteit in verwachtingen kan leiden tot optimisme in de economie, waarmee de consolidatie tot betere resultaten leidt, of juist tot pessimisme, waardoor de recessioneire effecten van de consolidatie worden versterkt. Een interessant resultaat is dat verkeerde verwachtingen over de vorm van de consolidatie de effecten van de consolidatie zowel kunnen verbeteren als verslechteren, afhankelijk van de mate van heterogeniteit.

Hoofdstuk 6 analyseert budgettaire consolidaties in een model waar agenten homogeen zijn, maar waar ze een eindige planningshorizon hebben. Zowel consumenten als bedrijven leven oneindig lang, maar plannen slechts een eindig aantal perioden vooruit en vormen ook geen verwachtingen over wat daarna gebeurt. De lengte van de planningshorizon van agenten speelt een belangrijke rol bij het bepalen hoe bezuinigingen en belastingverhogingen economische groei en inflatie beïnvloeden. We vinden dat in het geval dat consumenten een lage relatieve risico-afkerigheid hebben, bezuinigingen minder schadelijk zijn voor economische groei dan belastingverhogingen, wat overeenstemt met empirische resultaten. Een sterkere reactie van monetair beleid op inflatie maakt bezuinigingen ook relatief beter. Voor korte planningshorizons is ons model in staat de positieve correlatie tussen private consumptie en overheidsuitgaven - die in data wordt gevonden - te reproduceren.
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