



UvA-DARE (Digital Academic Repository)

Time is suspect!

Heck, A.; Jordens, H.; Uylings, P.

DOI

[10.1088/1361-6552/acc297](https://doi.org/10.1088/1361-6552/acc297)

Publication date

2023

Document Version

Final published version

Published in

Physics Education

License

CC BY

[Link to publication](#)

Citation for published version (APA):

Heck, A., Jordens, H., & Uylings, P. (2023). Time is suspect! *Physics Education*, 58(3), Article 035022. <https://doi.org/10.1088/1361-6552/acc297>

General rights

It is not permitted to download or to forward/distribute the text or part of it without the consent of the author(s) and/or copyright holder(s), other than for strictly personal, individual use, unless the work is under an open content license (like Creative Commons).

Disclaimer/Complaints regulations

If you believe that digital publication of certain material infringes any of your rights or (privacy) interests, please let the Library know, stating your reasons. In case of a legitimate complaint, the Library will make the material inaccessible and/or remove it from the website. Please Ask the Library: <https://uba.uva.nl/en/contact>, or a letter to: Library of the University of Amsterdam, Secretariat, Singel 425, 1012 WP Amsterdam, The Netherlands. You will be contacted as soon as possible.

Time is suspect!

André Heck^{1,*} , Hans Jordens² and Peter Uylings³ 

¹ Korteweg-de Vries Institute for Mathematics, University of Amsterdam, Amsterdam, The Netherlands

² Physics Department, University of Groningen, Groningen, The Netherlands

³ Anton Pannekoek Institute for Astronomy, University of Amsterdam, Amsterdam, The Netherlands



E-mail: a.j.p.heck@uva.nl

Abstract

‘That it is possible to find the form of the Lorentz transformations without requiring the speed of light to be the same in every inertial system is rediscovered about every thirty years and, rightly, forgotten.’ This proposition alongside the doctoral dissertation of Hendrik Smid in 1986 provides food for thought: Is the light postulate of special relativity, which gives the speed of light c in vacuum its status of maximum and unchanging speed, indeed not necessary? What kind of physical world are you then left with? What could be the value of discussing this in class? In this paper we show how the Lorentz transformation, which describes the relationship between two coordinate systems moving relative to each other at a constant speed, can indeed be derived by replacing the light postulate by another assumption, namely the requirement of closure under composition of transformations that relate inertial frames of reference (i.e. two successive coordinate transformations of certain type can be written as one transformation of the same type). Although well known in specialised literature of physics research, the remarkable insight that c is somehow buried in nature is hardly mentioned in pre-university and bachelor’s physics courses and textbooks. Yet, the presented comprehensible approach may support the understanding and plausibility of the abstract subject of special relativity. Mathematics may go too far for an average pre-university student, but this subject would certainly not be out of place in an undergraduate physics course or a physics teacher course. Although many teachers do not know that the light postulate of special relativity can be replaced by another assumption or may not appreciate such replacement as part of their teaching

* Author to whom any correspondence should be addressed.



Original content from this work may be used under the terms of the [Creative Commons Attribution 4.0 licence](https://creativecommons.org/licenses/by/4.0/). Any further distribution of this work must maintain attribution to the author(s) and the title of the work, journal citation and DOI.

of the topic, it offers in our opinion an opportunity to inform students that a critical look at postulates and an investigation of alternatives is an essential part of the nature of science.

Keywords: special theory of relativity, Lorentz transformation, light postulate of special relativity

Supplementary material for this article is available [online](#)

1. Introduction

Many countries have recently introduced special relativity in secondary school physics in order to expose students to modern physics [1] and several teaching and learning sequences have been developed [2–7]. The postulates of special relativity and the Lorentz transformation, which describes the relationship between two coordinate systems moving relative to each other at a constant speed, play a role in almost all of them. The quotation of Smid [8] in the abstract suggests that the Lorentz transformations may equally be derived without using the light postulate. More accurately stated, one can replace the light postulate in the set of physical arguments used for the standard derivation of the Lorentz transformation (e.g. [9]) by the requirement that the coordinate transformations are closed under composition (i.e. two successive coordinate transformations of certain type can be written as one transformation of the same type). This remarkable result pointing towards the existence of a universal constant is well known in specialised literature of physics research [10–15]. The required mathematics knowledge and skills may go too far for an average pre-university student. But the topic would certainly not be out of place in a physics (teacher) course at undergraduate level, when students may be expected to have the mathematical competencies needed. We will show that a minimum of calculus, algebra and matrix algebra is sufficient to understand our teacher/student-friendly derivation. Apart from this, it is still worth conveying the fascinating result to younger students in a qualitative way so that they learn that having a critical look at foundational assumptions in theory formation and searching for alternatives are activities that scientists actually do in research practice.

When the special theory of relativity was introduced after many years of experimentation

into the current upper secondary physics pre-university curriculum in the Netherlands and when it became one of four optional subdomains of which two have to be selected for the school examination of physics⁴, school physics teachers were not too worried. In contrast with the compulsory subdomain *Quantum World* of the country-wide examination [16], the explanation in the lecture halls of concepts such as inertial frames of reference, the Michelson–Morley experiment, and the premise that the speed of light in vacuum is the same in every inertial frame of reference were still fresh in many teachers’ minds. Nor were the mathematical derivations and algebraic manipulation too great an obstacle for the teachers.

The derivation of the formula for time dilation is an example of mathematical simplicity, which allows its discussion with upper secondary physics students. Imagine a thought experiment in which an observer in a frame of reference at rest measures that it takes a proper time interval Δt_r for a pulse of light to move vertically from the origin of the frame of reference to the ceiling. The height of the ceiling is then equal to $c \cdot \Delta t_r$. From the viewpoint of an observer with an inertial frame of reference moving parallel to the ceiling at velocity v to the right and passing by with coinciding origin in his/her frame of reference and the rest frame at the moment when the pulse of light is emitted, the light takes an oblique path; s/he measures a larger time interval Δt_m for the light signal to reach the ceiling and a longer path length of $c \cdot \Delta t_m$ that the pulse of light needs to reach the ceiling. In this time interval, the moving frame of reference is shifted $v \cdot \Delta t_m$ to the

⁴ Besides special theory of relativity, the three other optional subdomains in the school examination program are biophysics, geophysics and elementary particle physics. Two of these four subdomains must be elected for school assessment.

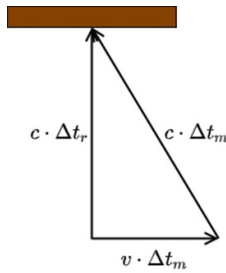


Figure 1. Rectangular triangle used for the derivation of the time dilation formula via the Pythagorean theorem.

right; see figure 1. The ratio of Δt_r and Δt_m can be determined by the Pythagorean theorem and basic algebraic manipulation leads to the well-known time dilation formula $\Delta t_m = \gamma \cdot \Delta t_r$, where $\gamma = 1/\sqrt{1 - v^2/c^2}$ is the so-called Lorentz factor.

At first sight, there seems nothing wrong with the above derivation of the formula for time dilation and many students will be completely satisfied with it. Yet intuition causes some friction here: a critical student might note that the difference between Δt_r and Δt_m arises from the fact that for the vertical side of the triangle and for the hypotenuse both the velocity of light has been used (and not, for instance, $\sqrt{c^2 + v^2}$). The light postulate of special relativity, which states that the speed of light in vacuum is the same for observers moving at constant speed relative to each other, is hidden in it. Telling that this status of c as maximum and unchanging speed of light is just the inevitable consequence of experimental results found by Michelson and Morley does not usually win the critical student over. It may help if the teacher notes that our intuition is simply formed in a world in which it seems as if the speed of light is infinite and that we therefore cannot escape from judging the results of the denial thereof, i.e. theory of relativity, as illogical and counterintuitive. If the speed of light, for example, were as small as the speed of a spacecraft, everyone would notice that time depends on the speed of the observer with the time measurement device and thus perceive time as unreliable. This is why we have chosen Einstein's expression 'time is suspect!' [17, p 48] as the title of this paper.

The suspect would even be greater when a teacher discusses with students the case that in

the frame at rest as well as in the moving frame at the moment that the origins of their frames of reference coincide a pulse of light is going vertically from the origin to ceiling of the same height: each observer now thinks that the pulse of light emitted in the frame of reference of the other observer takes longer time to reach the ceiling than in his/her own frame of reference.

Yet it would be nice to add that the light postulate does not appear out of the blue, but is embedded in the mathematical relationship between a frame of reference at rest and a moving inertial frame of reference. Three necessary and understandable conditions for the yet-to-be-determined coordinate transformation between two inertial frames of reference turn out to lead, even without the light postulate, to the Lorentz transformation containing a constant velocity C that, incidentally, can still be finite or infinite, or equal to the speed of light c .

But before we go into this derivation of the Lorentz transformation we first address a possible student's question: 'Why do we talk about the Lorentz transformation and not about the Einstein transformation?' The obvious, but incorrect answer is: 'Lorentz was the first person who wrote about it.' This would give the false image that theory formation in science is generally the result of work of a single person. It is true that Hendrik Lorentz did pioneering work in the context of electromagnetism: he came up with a transformation that made the electric and magnetic fields independent of the possible speed of an observer. He also formulated the notion of length contraction as explanation of the null result of the Michelson–Morley experiment. But other scientists contributed in the same period to the theory formation. Henri Poincaré was one of them and he introduced the name Lorentz transformation [18–21], probably in recognition of the leading role that Lorentz played. However, Lorentz did not believe in the physical reality of Δt_m : for him it only served as the auxiliary variable 'local' time. It was Einstein in 1905 who explained the fundamental physical value of the transformation, generalised it to uncharged particles, and derived the transformation from his relativity postulate and light postulate. By the way, the title of Einstein's work [22] refers indirectly to the work of Lorentz.

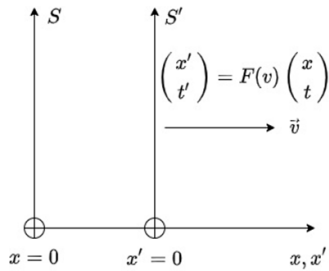


Figure 2. Two inertial frames moving with respect to each other along the x -axis with constant velocity v and the general transformation between the frames of reference.

2. Derivation of the Lorentz transformation

We consider two inertial frames of reference S and S' with position-time coordinates (x, t) and (x', t') , respectively. The frame of reference S' moves relative to S with constant velocity v along the x axis. One may think of a frame of reference S at rest and a moving frame of reference S' (see figure 2).

We derive in this section the coordinate transformation from the frame of reference S at rest to the moving frame of reference S' on the basis of the following three assumptions:

- (i) Exchangeability of frames of reference: frame S moves relative to S' with velocity $-v$ along the x axis.
- (ii) Space-time is homogeneous (i.e. the origin of the frames of reference can be located anywhere) and space is isotropic (i.e. it does not matter which direction is called positive).
- (iii) The composition of two coordinate transformations is again a coordinate transformation of the same kind.

These three assumptions turn out to be sufficient for the derivation of the Lorentz transformation without using the light postulate of a maximum speed of light in vacuum.

In the derivation we do not scale time to the dimension of length for three reasons: (i) in this way we get the form of the Lorentz transformation commonly found in physics textbooks; (ii) scaling is a technique that does not get much attention in education and therefore may be more confusing; and (iii) the argumentation and algebra are similar

for the derivation with and without time scaling. However, one could argue that it is from physical point of view more elegant to work in the coordinate transformation with scaled time coordinates pt and $p't'$ that have, just like the spatial coordinates x and x' the dimension of length. Here, p and p' are yet unknown scale factors with the dimension of velocity. By the way, teaching and learning sequences in special relativity at secondary school level that use spacetime Minkowski diagrams show that teachers expect that students can in fact deal with scaling (in this case with the speed of light c). In a supplementary document the authors present the derivation of the Lorentz transformation with time scaling included.

2.1. Derivation without time scaling

From the homogeneity of space-time follows that coordinate transformations between the frames of reference are linear mappings that depend continuously on the velocity v (cf [11, 13, 23]). The coordinates (x, t) of the frame of reference S at rest can be converted into the coordinates (x', t') of the moving frame of reference S via a velocity-dependent linear mapping, which we denote by the matrix $F(v)$. So:

$$\begin{pmatrix} x' \\ t' \end{pmatrix} = F(v) \cdot \begin{pmatrix} x \\ t \end{pmatrix} = \begin{pmatrix} f_{11}(v) & f_{12}(v) \\ f_{21}(v) & f_{22}(v) \end{pmatrix} \begin{pmatrix} x \\ t \end{pmatrix}.$$

The reverse transformation is also possible; the only thing that changes is the direction of the velocity

$$\begin{pmatrix} x \\ t \end{pmatrix} = F(-v) \cdot \begin{pmatrix} x' \\ t' \end{pmatrix} = \begin{pmatrix} f_{11}(-v) & f_{12}(-v) \\ f_{21}(-v) & f_{22}(-v) \end{pmatrix} \begin{pmatrix} x' \\ t' \end{pmatrix}.$$

The exchangeability of the frames of reference implies that $F(v)$ and $F(-v)$ are each other's inverse; in matrix form this can be written as:

$$F(v) \cdot F(-v) = I. \tag{1}$$

Assuming that space is isotropic so that it does not matter which direction we call positive, reverting the direction means that the frame of reference S' moves with velocity v in the opposite

direction with respect to the frame of reference S and that we get:

$$\begin{pmatrix} -x' \\ t' \end{pmatrix} = F(-v) \cdot \begin{pmatrix} -x \\ t \end{pmatrix}.$$

In matrix form we can write this as the following equation:

$$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \cdot F(v) = F(-v) \cdot \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}. \quad (2)$$

The third assumption about the coordinate transformation can be mathematically expressed as:

$$\text{For every } u \text{ and } v \text{ there exists a } w \text{ such that} \quad (3)$$

$$F(u) \cdot F(v) = F(w).$$

The matrix assumptions (1)–(3) for the coordinate transformation $F(v)$ determine its mathematical formula:

$$F(v) = \frac{1}{\sqrt{1 - \frac{v^2}{C^2}}} \cdot \begin{pmatrix} 1 & -v \\ -\frac{v}{C^2} & 1 \end{pmatrix}$$

$$= \gamma \cdot \begin{pmatrix} 1 & -v \\ -\frac{v}{C^2} & 1 \end{pmatrix}, \quad (4)$$

where C is a constant and $\gamma = 1/\sqrt{1 - v^2/C^2}$. We will prove this below.

From equations (1) and (2), and from multiplication properties of the determinant of a matrix follows that

$$\det(F(v))^2 = 1. \quad (5)$$

So $\det(F(v)) = \pm 1$. Because $\det(F(0)) = \det(I) = 1$ and the coordinate transformation $F(v)$ depends continuously on velocity v , it must be true that

$$\det(F(v)) = 1, \text{ for all } v. \quad (6)$$

This formula simplifies the computation of the formula of the inverse matrix of $F(v)$, which according to the matrix condition (1) equals $F(-v)$. Elementary linear algebra leads to:

$$F(-v) = \begin{pmatrix} f_{22}(v) & -f_{12}(v) \\ -f_{21}(v) & f_{11}(v) \end{pmatrix}. \quad (7)$$

We can rewrite the matrix equation (2) as

$$F(-v) = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \cdot F(v) \cdot \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \quad (8)$$

and obtain from this

$$F(-v) = \begin{pmatrix} f_{11}(v) & -f_{12}(v) \\ -f_{21}(v) & f_{22}(v) \end{pmatrix}. \quad (9)$$

Formulas (8) and (9) imply that $f_{11}(v) = f_{22}(v)$ and that we can write

$$F(v) = \begin{pmatrix} f_{11}(v) & f_{12}(v) \\ f_{21}(v) & f_{11}(v) \end{pmatrix}. \quad (10)$$

Now, let x' coincide with the origin of the moving frame of reference S' . Then $x' = 0$ and thus $0 = f_{11}(v)x + f_{12}(v)t$, which can be rewritten as

$$-\frac{f_{12}(v)}{f_{11}(v)} = \frac{x}{t} = v. \quad (11)$$

This means that we can rewrite formula (10) of $F(v)$ as

$$F(v) = \begin{pmatrix} f_{11}(v) & -vf_{11}(v) \\ f_{21}(v) & f_{11}(v) \end{pmatrix}. \quad (12)$$

We introduce in the crucial matrix element $f_{21}(v)$ a new, in principle velocity-dependent quantity, $C(v)^2$ defined as

$$C(v)^2 = -\frac{vf_{11}(v)}{f_{21}(v)}. \quad (13)$$

The coordinate transformation $F(v)$ can now be written as

$$F(v) = f_{11}(v) \cdot \begin{pmatrix} 1 & -v \\ -\frac{v}{C(v)^2} & 1 \end{pmatrix}. \quad (14)$$

Because $\det(F(v)) = 1$ for all v (formula (6)) it follows from expression (14) for $F(v)$ that

$$f_{11}(v)^2 \cdot \left(1 - \frac{v^2}{C(v)^2}\right) = 1. \quad (15)$$

So we can rewrite the coordinate transformation $F(v)$ as

$$F(v) = \gamma(v) \cdot \begin{pmatrix} 1 & -v \\ -\frac{v}{C(v)^2} & 1 \end{pmatrix}, \quad (16)$$

where $\gamma(v) = 1/\sqrt{1 - v^2/C(v)^2}$.

Let us now compute the composition of two coordinate transformation with velocities u and v , respectively. Matrix multiplication gives:

$$\begin{aligned} F(u)F(v) &= \gamma(u)\gamma(v) \begin{pmatrix} 1 + \frac{uv}{C(v)^2} & -u - v \\ -\frac{u}{C(u)^2} - \frac{v}{C(v)^2} & 1 + \frac{uv}{C(u)^2} \end{pmatrix}. \end{aligned} \quad (17)$$

Assumption (iii) states that the composition must be a coordinate transformation of the same kind and in particular the diagonal elements must be equal to each other. This implies in formula (17) that in fact $C(u)^2 = C(v)^2$ for all velocities u and v . In other words, $C(v)$ is independent of velocity v and thus equal to some constant C .

Without experimental data we do not know whether this constant C is a finite number or infinity. Taking the limit for $C \rightarrow \infty$ yields the Galilean transformation:

$$F_{\text{Galilean}}(v) = \lim_{c \rightarrow \infty} F(v) = \begin{pmatrix} 1 & -v \\ 0 & 1 \end{pmatrix}, \quad (18)$$

that is, $x' = x - vt$ and $t' = t$, and the classical mechanics addition formula of velocities can be applied. However, when C is finite, time is transformed and therefore dependent on the relative velocity v of the inertial frames: ‘time is suspect!’ The addition formula is now more complicated and follows from rewriting formula (17) as

$$F(u) \cdot F(v) = F\left(\frac{u+v}{1 + \frac{uv}{C^2}}\right). \quad (19)$$

Thus velocities u and v add up to a velocity w given by

$$w = \frac{u+v}{1 + \frac{uv}{C^2}}. \quad (20)$$

In conclusion, we have found the Lorentz transformation $F(v)$ without reference to the light postulate of a maximum speed of light, constant

for all inertial frames of reference. Only some constant C with the dimension of velocity pops up in the derivation. There is at this stage yet no reason to identify this constant C with the light of speed $-c$. What we know about C is that if an object moves in one inertial frame of reference with velocity C , then it moves in any other inertial frame of reference with the same velocity. Applying the linear mapping defined in formula (4) to the case of $x = Ct$ leads to $x' = Ct'$. We will come back to this in the final example of section 3. In other words, C is an invariant speed, i.e. it is the same in all inertial frames of reference. Moreover, C is a maximum as any existing larger velocity would violate causality. Maxwell’s theory of electromagnetism stipulates that the speed at which electromagnetic waves travel in vacuum is constant in any inertial frame of reference and light is an electromagnetic wave. No experiments have contradicted this. As a result, we can identify C with the speed of light c . Alternatively, application of the relativity postulate to Maxwell’s equations yields the Lorentz transformation with the identification $C = c$.

3. Discussion

In the previous section we have shown that knowledge and skills in elementary calculus and linear algebra are sufficient to derive the Lorentz transformation without the light postulate of special relativity. But physics theory cannot do without experimental validation: one cannot escape from experimental determination of the speed of light. This is probably why the proposition alongside the doctoral dissertation of Smid [8], quoted in the abstract, contains the word ‘rightly’. Yet, our approach shows how fundamental the Lorentz transformation actually is! An elegant aspect of the primarily mathematical approach is that one can find on the basis of rather simple principles about space and time transformations, depending on the choice of an unknown speed C , either classical or relativistic transformations. Maybe Einstein already realised in 1916 that the Lorentz transformation could be derived without the light postulate: his derivation and sigh ‘Das hätte ich einfacher sagen können’ (I could have said that more simply) [24] do point at this.

The simplicity and the ease of acceptance of the assumption that a composition of

transformations of certain type is again a transformation of the same kind does not automatically mean that it replaces the light postulate in education. Actually this is not what we advocate: certainly at secondary school level, the derivation of the light postulate from the new assumption is, as we wrote upfront, probably too difficult for the students. We are just of opinion that inclusion of the discussion of postulates and alternatives adds to the scientific literacy of students, as described by Hodson [25, 26]. They learn more about the way scientists work, not in solitude but within a research community, and gain a better understanding of the history and philosophy of science. Improving students' scientific literacy, presenting concepts of modern physics in authentic contexts, and increasing their motivation and engagement with modern physics were in many countries also driving forces in the curriculum reforms in the last two decades [e.g. 27–29].

Change of an underpinning assumption in the special relativity theory also does not provide an easy solution for many of the students' difficulties that are reported in the research literature. This is clear when one reads about student's alternative conceptions in the review and analysis of the body of research on special relativity theory education at secondary and undergraduate level by Alstein *et al* [1] and references herein. But one must keep in mind that this review study is restricted to articles that address conceptual understanding of special relativity and Galilean relativity 'without presupposing an understanding of the formalism of Lorentz transformation or spacetime geometry.' [p 2] So, these authors take the stand that at secondary and lower undergraduate level a conceptual approach to special relativity theory, in which abstract concepts are expressed through the use of thought experiments [2], rather than mathematical or geometrical formalism, which can be derived as a complement afterwards. They developed a teaching and learning approach in which the light postulate, which is notoriously difficult for students to grasp [3, 30, 31], is chosen as a central concept and in which students perform relativistic thought experiments using event diagrams as reasoning tool, try different interpretations of the light postulate, and experience how the new concepts meet the requirements for a good theory [2]. This interesting instructional sequence was developed as part of a design research-based

doctoral study [32], but as the authors [2, p 96] admit: 'the current version of our design cannot be directly implemented in the classroom as it relies heavily on a teacher working closely with students individually to guide their reasoning.' Yet such detailed studies have great value in exploring challenges and opportunities with regard to teaching special relativity at secondary and lower undergraduate level.

Inspired by the analysis of college physics textbooks by Dimitiadi and Halkia [3], Alstein *et al* [1] organised their overview of learning difficulties of secondary and lower undergraduate students into three main categories: (i) frames of reference; (ii) postulates of special relativity theory; and (iii) relativistic effects such as relativity of simultaneity, time dilation and length contraction. Any description of relative motion is inextricably linked to the concept of frames of reference, in which observers perform measurements to determine spatial and temporal coordinates of events by means of measuring rods and synchronised clocks, and to an operational definition of the time of a distant event. Understanding of the transformation of measurement outcomes found in several frames of reference is therefore of key importance in any approach to the special relativity theory. Actually similar difficulties exist in Galilean relativity education. Below we describe research findings about students' alternative of frames of reference.

Many students consider a frame of reference as local instead of infinite: it never extends beyond the object where the observer is [33, 34] and an observer can only know what happens at his/her location [35]. Many students link every observer or object to a private frame of reference; so they think that there can be no more than one observer in a frame of reference. In other words, many students fail to consider a frame of reference as a system of observers that determines the same position and time for any event, and they interpret statements of the frame-dependence of the time of an event to mean that signals are received from events at different times by observers at different locations [35]. Conversely, Scherr *et al* [36] showed that students tend to treat observers at the same position as being in the same frame of reference, regardless of their relative motion. Many students (initially) see motion and velocity as an intrinsic property of a moving object and

find it therefore difficult to refer to velocities and trajectories with respect to a frame of [37, 38]. Some students think that events may be simultaneous for one observer and not simultaneous for another person, even when both observers are located in the same frame of reference, and many students cannot give an answer only depending on the location of the observer when his or her velocity is mentioned as if the movement contaminated the event [39]. Many students consider length as a property of an object that cannot change and therefore they look at length contraction as an optical illusion and they think that, when several observers do not get the same measurements of length, some of them must have made a mistake [3]. Students frequently do not spontaneously or consciously consider alternative frames of reference besides the default frame of reference, which is generally the earth. Many of them tend to believe that in special relativity theory there is a 'privileged observer' position and that time dilation and length contraction happen only for the moving observer [30]. Scherr *et al* [36] pointed out that many of the student difficulties may arise from too vague and/or unclear definitions of basic concepts such as frame of reference, event, observer, 'intelligent observer' (taking signal travel time into account), simultaneity, and spacetime leading to the construction of a conceptual framework in which the ideas of absolute and relative simultaneity harmoniously coexist (see also [40]). Entire conferences [41] have been devoted to students difficulties with frames of reference and ways to overcome obstacles. Much attention has gone and still goes into development of ICT-based learning activities that enrich students' experiences, allow them to engage with more abstract and sometimes counterintuitive physics concepts through simulations, and help them deepen their understanding of modern physics. In conclusion, Scherr *et al* [42] concluded that many students who study special relativity at the undergraduate to graduate levels fail to develop a functional understanding, partly because of a lack of understanding of more basic, underlying concepts. These authors found [p 1247] that 'after standard instruction many students lack operational definitions for fundamental ideas such as time of an event, simultaneity, and reference frame—concepts that should be familiar to them from Galilean relativity.'

Recently, Hughes and Kersting [43] pointed out how the language of special relativity can mislead and confuse students, or even worse is a likely source of misconceptions. In particular, the words 'observer' and 'observation' may blur the distinction between seeing and measuring. For example, the formula for length contraction does not describe what one sees, but what one measures. What is measured corresponds to a specific measuring procedure in which simultaneously separate coordinates on a moving object are located. In contrast, what is seen depends on the differing time light takes to reach the eyes of the observer for different parts of a moving object. Similar confusion happens with time dilation. Research in quantum physics education also suggests that secondary school students tend to conceptualise 'observation' as seeing rather than measuring.

In light of the above discussion of student difficulties linked to frames of reference, we believe that students may only better understand that formulas in special relativity mainly deal with measurement when they are at early stages of education already deeply exposed to frames of reference and apply them in many exercises in classical mechanics, Galilean relativity, and finally in special relativity. Then students may learn to understand how measurement is inextricably linked to a choice of a frame of reference. Scaling of an axis is actually nothing else than choosing another unit of the quantity linked to that axis. Students better learn at an early stage how coordinate transformations link differing frames of reference: rotation changes the orientation, and translation allows for a differing choice of the origin. After such a learning path full of discussion of frames of reference and coordinate transformations, students can understand that the Lorentz transformation is just another transformation of a two-dimensional space, only a little bit more complicated than they have seen before. They may better understand the Lorentz transformation and not believe anymore that the transformation of time constitutes a correction for signal travel time, a misconception reported by Scherr *et al* [36].

Use of the Lorentz transformation does not necessarily mean that the student performs calculations. Use can also be in the form of qualitative algebraic reasoning. In this way, for example the loss of simultaneity among equivalent

observers and other relativistic effects such as time dilation and length contraction can be analysed by students. As concrete example we discuss the spacecraft question designed by Scherr *et al* [36]. It involved two volcanoes. Mt. Rainier and Mt. Hood, that erupt simultaneously in the frame of reference S of an observer at rest on the ground, midway between the volcanoes. A spacecraft moving at a given constant relativistic velocity v from Mt. Rainier to Mt. Hood is directly over Mt. Rainier when it erupts. Event 1 and Event 2 are defined to be ‘Mt. Rainier erupts’ and ‘Mt. Hood erupts,’ respectively. The question is what the order of eruptions is in the moving frame of reference S' of the spacecraft, i.e. does event 1 occur before, after, or at the same time as event 2 in the frame of reference of the spacecraft observer. A correct answer can be obtained by qualitative reasoning, by use of spacetime diagrams, and by application of the Lorentz transformation. We discuss the latter approach. The spatial separation between the two events in the frame of reference S is defined as $\delta x_{12} = x_2 - x_1$, where x_1 and x_2 are the locations of event 1 and event 2, respectively. Taking the positive direction to be directed from Mt. Rainier to Mt. Hood we have $\delta x_{12} > 0$ and $v > 0$. Similarly the temporal separation between the two events is defined as $\delta t_{12} = t_2 - t_1$, where t_1 and t_2 are the times at which event 1 and event 2 occur, respectively. In this question $\delta t_{12} = 0$ because the volcanoes erupt simultaneously. The Lorentz transformation relates measurements in the ground frame S and the spacecraft frame S' . The general Lorentz transformation of time is $\delta t'_{12} = \gamma \cdot (\delta t_{12} - v \cdot \delta x_{12}/c)$ and specialises in this case to $\delta t'_{12} = -\gamma \cdot v \cdot \delta x_{12}/c < 0$ because $\delta t_{12} = 0$, $\delta x_{12} > 0$ and $v > 0$. A negative value of $\delta t'_{12}$ means that in the spacecraft reference frame Mt. Hood erupts before Mt. Rainier. We believe that the above reasoning is less error-prone than many other approaches. The final result is of course still counterintuitive.

We end this paper with the following thought experiment that shows how the Lorentz transformation can be used to illustrate the light postulate. The mathematics does not go beyond upper secondary level. Consider an observer in the origin of a three-dimensional frame of reference S at rest. When s/he creates a flash of light at time $t = 0$, then the light will be after time t on a sphere

with radius $r = ct$, where c is the speed of light in vacuum. A second observer in a spacecraft with constant velocity passes at time $t' = t = 0$ the origin of S , thus exactly at the time of emission of the flash of light. What shape has the light in his/her moving frame of reference S' . To simplify the algebra, we rewrite the Lorentz transformation and its inverse in the following more symmetric format:

$$\begin{aligned}x' &= \gamma \cdot (x - \beta \cdot ct) & x &= \gamma \cdot (x' + \beta \cdot ct') \\y' &= y & y &= y' \\z' &= z & z &= z' \\ct' &= \gamma \cdot (ct - \beta x) & ct &= \gamma \cdot (ct' + \beta \cdot x')\end{aligned}$$

where

$$\beta = \frac{v}{c} \text{ and } \gamma = \frac{1}{\sqrt{1 - \beta^2}}.$$

These transformations can be used to explain that the second observer also concludes that the light will be after some time on a sphere in his reference frame. First, square the equation $r = ct$ and rewrite it as $x^2 + y^2 + z^2 = (ct)^2$. Substitute the inverse Lorentz transformation:

$$\gamma^2 \cdot (x' + \beta \cdot ct')^2 + y'^2 + z'^2 = \gamma^2 \cdot (ct' + \beta \cdot x')^2.$$

Expand the brackets in the previous equation and use the relation $\gamma^2(1 - \beta) = 1$ to rewrite it as $x'^2 + y'^2 + z'^2 = (ct')^2$. Thus, in the frame of reference of the second observer, the light will be after time t' on a sphere with radius $r' = ct'$. This mathematical approach, well within reach of upper secondary students, does not obscure physical aspects and does not hinder the process of acquiring physical intuition of special relativity. On the contrary, it shows that the constancy of the speed of light in vacuum in any frame of reference does not come out of the blue.

Data availability statement

No new data were created or analysed in this study.

ORCID iDs

André Heck  <https://orcid.org/0000-0002-7923-6360>

Peter Uylings  <https://orcid.org/0000-0001-6876-2472>

Received 11 November 2022, in final form 7 February 2023
Accepted for publication 8 March 2023
<https://doi.org/10.1088/1361-6552/acc297>

References

- [1] Alstein P, Krijtenburg-Lewerissa K and van Joolingen W R 2021 *Phys. Rev. Phys. Educ. Res.* **17** 023101
- [2] Kamphorst F, Vollebregt M J, Savelsbergh E R and van Joolingen W R 2023 *Sci. Educ.* **32** 57–100
- [3] Dimitiradi K and Halkia K 2012 *Int. J. Sci. Educ.* **34** 2565–82
- [4] Arriasecq I and Greca I M 2012 *Sci. Educ.* **21** 827–51
- [5] Kersting M and Blair D 2021 *Teaching Einsteinian Physics in Schools* (London: Routledge)
- [6] Kersting M, Hernriksen E K, Bøe M V and Angell C 2018 *Phys. Rev. Phys. Educ. Res.* **14** 010130
- [7] Leonardi A M, Mobilio S and Fazio V 2022 *Phys. Educ.* **57** 045002
- [8] Smid H 1986 Studies of configuration interaction calculations with complete series Doctoral dissertation University of Amsterdam (available at: <https://pid.uba.uva.nl/ark:/88238/b1990000892980205131>)
- [9] Szego L 1989 *Eur. J. Phys.* **10** 158–9
- [10] von Ignatowsky W 1911 *Arch. Math. Phys.* **17** 1–24 (available at: [https://de.wikisource.org/wiki/Das_Relativitätsprinzip_\(Ignatowski\)](https://de.wikisource.org/wiki/Das_Relativitätsprinzip_(Ignatowski)))
- [11] Berzi V and Gorini V 1969 *J. Math. Phys.* **10** 1518–24
- [12] Lee A R and Kalotas T M 1975 *Am. J. Phys.* **43** 434–7
- [13] Lévy-Leblond J M 1976 *Am. J. Phys.* **44** 271–7
- [14] Sardelis D A 1982 *Eur. J. Phys.* **3** 96–99
- [15] Pelissetto A and Testa M 2015 *Am. J. Phys.* **83** 338–40
- [16] Stadermann K 2022 Connecting secondary school quantum physics and nature of science Doctoral dissertation University of Groningen (<https://doi.org/10.33612/diss.195942915>)
- [17] Shankland R S 1963 *Am. J. Phys.* **31** 47–57
- [18] Poincaré H 1906 *Rend. Circ. Mat. Palermo* **21** 129–76
- [19] Schwartz H M 1971 *Am. J. Phys.* **39** 1287–94
- [20] Schwartz H M 1971 *Am. J. Phys.* **40** 862–72
- [21] Schwartz H M 1971 *Am. J. Phys.* **40** 1282–7
- [22] Einstein A 1905 *Ann. der Phys.* **34** 825–54
- [23] Eisenberg L J 1967 *Am. J. Phys.* **35** 649
- [24] Chao S 2017 *Eur. J. Phys.* **38** 1–11
- [25] Hodson D 2008 *Towards Scientific Literacy* (Rotterdam: Sense Publishers)
- [26] Hodson D 2009 *Teaching and Learning about Science* (Rotterdam: Sense Publishers)
- [27] Van Weert C G, Pieters M L M and Eijkelhof H M C 2006 *Natuurkunde leeft [Physics is alive]—visie op het vak natuurkunde in havo en vwo Commissie Vernieuwing Natuurkundeonderwijs havo/vwo*. Nederlandse Natuurkundige Vereniging (available at: www.leraar24.nl/app/uploads/Visiedocument_natuurkunde1-1.pdf)
- [28] Millar R 2006 *Int. J. Sci. Educ.* **28** 1499–521
- [29] Mikelskis S and Duit R 2007 *MNU* **60** 265–73
- [30] Villani A and Pacca J L 1987 *Int. J. Sci. Educ.* **9** 1499–521
- [31] Kamphorst F, Vollebregt M J, Savelsbergh E R and van Joolingen W R 2019 *Phys. Rev. Phys. Educ. Res.* **15** 02012301
- [32] Kamphorst F 2021 Introducing special relativity in secondary education Doctoral dissertation Utrecht University (<https://doi.org/10.33540/1071>)
- [33] Panse S, Ramadas J and Kumar A 1994 *Int. J. Sci. Educ.* **16** 63–82
- [34] Tanel Z 2013 *J. Balt. Sci. Educ.* **13** 573–82
- [35] Scherr R E 2001 An investigation of student understanding of basic concepts in special relativity Doctoral Thesis University of Washington (available at: www.physics.umd.edu/perg/papers/scherr/dissertation/)
- [36] Scherr R E, Shaffer P S and Vokos S 2001 *Am. J. Phys.* **69** S24–S35
- [37] Saltiel E and Malgrange J L 1980 *Eur. J. Phys.* **1** 73–80
- [38] Ramadas J, Barve S and Kumar A 1996 *Int. J. Sci. Educ.* **18** 615–29
- [39] De Hosson C, Kermen I and Parizot E 2010 *Eur. J. Phys.* **31** 1527–38
- [40] Scherr R E 2007 *Am. J. Phys.* **75** 272–80
- [41] Kühnelt H, Berndt M, Staszek M and Turlo J (eds) 1992 *GIREP 1991 Conf. Proc. Teaching about Reference Frames: From Copernicus to Einstein* (Nicholas Copernicus University Press)
- [42] Scherr R E, Shaffer P S and Vokos S 2002 *Am. J. Phys.* **70** 1238–48
- [43] Hughes T and Kertsing M 2021 *Phys. Educ.* **56** 025011



André Heck earned MSc degrees in mathematics and chemistry, and a doctoral degree in mathematics and science education. He is senior lecturer at the Faculty of Science of the University of Amsterdam. His research area is the application of ICT in mathematics and science education.



Hans Jordens earned a doctoral degree in theoretical physics, and in physics and mathematics education. He worked ten years as teacher at a secondary school and thirty years as lecturer at the University of Groningen, Netherlands. During 35 years he was involved in many physics and science competitions for secondary education amongst which the International Physics Olympiad, of which he was president for 10 years.



Peter Uylings graduated in physics and mathematics, and he obtained his PhD in theoretical physics. He worked part-time as teacher at a secondary school and as physics teacher educator. After retirement he went back to his first love in physics research and restarted work in the field of atomic physics.