Manipulation of ultracold Bose gases in a time-averaged orbiting potential
Cleary, P.W.

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Chapter 4

Measurements with long-lived condensates

4.1 Introduction

There are two hyperfine levels $F = 1$ and $F = 2$ of the electronic ground state in $^{87}\text{Rb}$ with three trappable magnetic substates $|F, m_F\rangle = |2, 1\rangle, |2, 2\rangle, |1, 1\rangle$. The previous successful experiments on our apparatus, creating and manipulating condensates, exclusively employed the $|2, 2\rangle$ state. Following the upgrades discussed in Chapters 3 and 4, it was possible to image BEC clouds with large atom numbers in the $|2, 2\rangle$ state, along and perpendicular to the cloud axis. This trappable state has the advantage that it can be imaged on a cycling transition but it has the strong disadvantage of a short lifetime due to collisions with $|2, 1\rangle$ atoms which are trapped spuriously. This limits the experimental possibilities to measurements that do not require a long sample lifetime. To enable the possibility of observing clouds over longer time periods, we developed a method of condensing atoms in the $|1, 1\rangle$ state, requiring modification of the apparatus. These modifications, the experiments on the $|1, 1\rangle$ samples and some of the underpinning theory will be discussed in this chapter. We begin in Section 4.2 by outlining the optical pumping, trapping, evaporative cooling and imaging of the atomic samples. In Section 4.3 we create a BEC of $F = 1$ atoms in the Ioffe-Pritchard trap and measure its properties including lifetime of the sample. We proceed to a BEC of $|1, 1\rangle$ atoms in a time-averaged orbiting potential (TOP) for both a single well and double well potential. Section 4.4 explains the origin of vorticity of the trapped condensate and outlines methods to create and detect vorticity in the sample. Finally, Section 4.5 describes our experiments in a rotating TOP and our efforts to induce and detect vortices.
Chapter 4. Measurements with Long-lived Condensates

4.2 Trapping and condensing the $|1, -1\rangle$ state

4.2.1 Optical pumping scheme

All sublevels are equally populated in the MOT and molasses stages, we modified both optical pumping and magnetic trapping to work with $|1, -1\rangle$. The concept of optical pumping has been outlined in Section 2.3 and we continue the nomenclature adopted there to describe levels, sublevels and the transitions of the D2 line. To collect atoms in the $|2, 2\rangle$ state it is necessary to drive the $\sigma^+$ transition $F = 2 \rightarrow F' = 2$, while also repumping the atoms from the $F = 1$ manifold back to $F = 2$ as illustrated in Fig. 4.1. When the goal is to collect the atoms in the $|1, -1\rangle$ state, the ideal scheme is to drive the $\sigma^-$ transition $F = 1 \rightarrow F' = 1$ and to clear the $F = 2$ manifold with $\pi$ transitions $F = 2 \rightarrow F' = 2$, with a decay channel to $F = 1$. To implement the $F = 1$ scheme we can utilize for the $F = 2$ clearing the same laser as used for the optical pumping in the $F = 2$ scheme. For the $\sigma^-$ optical pumping on the $F = 1 \rightarrow F' = 1$ transition an additional laser is required. Interestingly, we found that we can do without this additional laser. Optical pumping generally works imperfectly. In the case of optical pumping in the $F = 2$ scheme, typically 40% of
4.2. TRAPPING AND CONDENSING THE $|1, -1\rangle$ STATE

the atoms is transferred to the desired $|2, 2\rangle$ state, corresponding to some $10^9$ trapped atoms. In view of the five $F = 2$ sublevels this is a reasonable return. In the $F = 1$ optical pumping scheme we ended up with a similar efficiency. However, as there are only three $F = 1$ sublevels, the use of only the $F = 2$ clearing laser already resulted in the transfer of some 30% of the atoms. Hence, the gain by optical pumping was found to be too small to justify the overhead of the additional laser.

In the time-line of the experiment the MOT is driven as usual on the $F = 2 \rightarrow F' = 3$ cycling transition with 1 mW repumping light on the $F = 1 \rightarrow F' = 2$ transition along one of the MOT axes in the horizontal direction. After switching off the MOT coils we apply optical molasses with the same light. After 5 ms, we stop driving the cycling transition and switch to the $\pi$ transition $F = 2 \rightarrow F' = 2$ along a single horizontal MOT direction. Simultaneously, we switch off the $F = 1 \rightarrow F' = 2$ repumper and observe the decay to the $|1, -1\rangle$ state. In Fig. 4.2 we show the increase of the $|1, -1\rangle$ population as a function of the duration of the $\pi$ excitation. Also the associated loss of $F = 2$ atoms is shown. The actual measurement is made after recapturing the atoms for a short time in the magnetic trap. The imaging was done after a time of flight of 5 ms. With this method we typically collected $7 \times 10^8$ atoms in the $|1, -1\rangle$ state.

4.2.2 Magnetic transfer and evaporative cooling

In this section we discuss the sequence of steps under which a Bose-Einstein condensate is produced and emphasize the difference with the procedure used for $F = 2$ atoms.

Figure 4.2: Increase of the $|1, -1\rangle$ population as a function of the $\pi$ excitation $F = 2 \rightarrow F' = 2$ (red circles). The associated loss of $F = 2$ atoms is also shown (black squares). At a later stage after removing stray $F = 1 \rightarrow F' = 2$ light, pumping could be completed within 0.2 ms with the number of $F = 2$ atoms reduced below the level of detectability.
Transfer to the magnetic trap

As mentioned in Section 2.6.1, the effective magnetic moment of the \( |1, -1\rangle \) state is a factor of 2 smaller than for the \( |2, 2\rangle \) state. This has consequences for the current settings of the magnetic trap. As the size of the MOT cloud is the same after \( F = 1 \) and \( F = 2 \) preparation, 40% more current is required to reproduce the optimal axial trap frequency \( \omega_z \) and radial trap frequency \( \omega_{\rho} \) for transfer to the magnetic trap. The transfer was done with the semi-adiabatic method \[59\]. Due to gravity, the position of the MOT does not match exactly that of the magnetic trap center in the vertical direction and so the cloud is allowed to fall for 1 ms before switching on the magnetic trap. This introduces an oscillation of the cloud in the trap and results after thermalization in a temperature too high to remain trapped. Therefore trap depth is increased before thermalization. Optimizing for the maximum number of atoms the best transfer was realized after a holding time of 50 ms in the shallow trap before compression. For the compression the current was increased to 400 A in 250 ms. In this procedure the cloud temperature increased to approximately 1 mK. The trap bottom field \( B_0 \) was then lowered over a period of 1.2 s, decreasing the share of the current through the compensation IGBT at the expense of the current through the pinch IGBT (see Section 2.6) until \( B_0 = 0.9 \) G is reached. This results in trapping frequencies of \( \omega_{\rho}/2\pi = 339 \) Hz and \( \omega_{z}/2\pi = 14.7 \) Hz for radial and axial directions, respectively. If \( B_0 \) is significantly lowered below this value the cloud suffers from Majorana losses, limiting the trapped cloud lifetime.

RF evaporation was initiated at a frequency of 100 MHz. The optimal evaporation curve was approximated by four linear frequency ramps. The ramp-down rate was optimized for maximum degeneracy parameter \( n_0^{\lambda^3} \). The cloud was then imaged after up to 27 ms time of flight.

4.2.3 Imaging

Converting from a system imaging \( F = 2 \) atoms to imaging for \( F = 1 \) atoms contains some subtle aspects. Imaging with \( \sigma^- \) light on the \( F = 1 \rightarrow F' = 2 \) transition sends the \( |1, -1\rangle \) atoms to the excited state sublevel \( |F', m_{F'}\rangle = |2, -2\rangle \). However, this is not a closed transition; there is a 40% chance of decay to the dark ground state sublevel \( |2, -2\rangle \) so that after absorption of just two photons most atoms are in this dark state. Scanning the detuning of the light and measuring the linewidth shows progressively larger effective linewidths for longer exposures as shown in the left graph of Fig. 4.3. Imaging on \( F = 1 \rightarrow F' = 0 \) solves this issue and produces a linewidth only slightly larger than the natural linewidth of the transition (6 MHz), neatly demonstrating the narrow laser bandwidth as shown on the right of Fig. 4.3. The latter method requires a dedicated laser for imaging because the ground states \( F = 1 \) and \( F = 2 \) are too far apart in frequency to be bridged by an AOM. An alternative way is to use the cooling laser for the imaging. In this scheme, we use excess (3 mW) repumping light.
4.3. $F = 1$ BEC

4.3.1 BEC characteristics and lifetime

Imaging the cloud perpendicular to the trap axis (radial imaging) we can observe the anisotropic expansion as the classic signature of reaching BEC (see Fig. 4.4). At temperatures just below the transition temperature, the cloud will be partially condensed and the expansion bimodal. By evaporating until the thermal cloud is no longer visible we ensure an essentially pure BEC of some $2 \times 10^5$ $F = 1$ atoms.

The cloud images can be used to measure characteristics of the trap for $F = 1$ atoms. The trap frequency is measured by observing parametric heating, sending a small modulation current (0.5 A) through one of the axial helper coils and observing atom loss from the BEC. With this method a radial trap frequency of $2\pi \times 395(3)$ Hz is found at $B_0 = 1.2$ G.

The lifetime of this cloud after an initial period of rapid loss is seen to be much longer than the sub-1 s lifetime of an $F = 2$ cloud but is still found to be less than the magnetic trap lifetime which is almost one minute. There are two main processes which can speed the decay of the Bose Einstein condensate, particularly at higher densities. The first is three-body recombination which occurs when three atoms of the condensate collide with one another, where two of the three atoms combine and...
the third carries away the binding energy as kinetic energy and so is lost from the trap [60]. Three body decay in condensate atom number can be written as

$$\frac{dN}{dt} = -L \int n^2(r) d^3r = -L \langle n^2 \rangle N$$  \hspace{1cm} (4.1)$$

where $\langle n \rangle = \int n(r) n(r) d^3r / N$ for any function $n$. This decay mechanism thus occurs proportional to the square of the condensate density $n^2$ with coefficient of decay $L$.

The second decay process is two body spin relaxation, which can occur due to the probability of a collision between two condensate atoms causing a spin flip out of the trapped state. This decay can be written

$$\frac{dN}{dt} = -G \int n^2(r) d^3r = -G \langle n \rangle N$$  \hspace{1cm} (4.2)$$

and thus occurs directly proportionally to the condensate density $n(r)$ with coefficient $G$.

It is easier to measure condensate numbers accurately than exact density distributions of the condensate. Following [61], we use the Thomas-Fermi approximation of a parabolic condensate density distribution to express the condensate density and its square in terms of $N$, the condensate atom number. Thus $\langle n \rangle = c_2 N^{2/5}$ and $\langle n^2 \rangle = c_3 N^{4/5}$ with $c_2 = (15^{3/5} / (14 \pi)) (\bar{\omega} / \hbar \sqrt{a})^{6/5}$ and $c_3 = (7/6) c_2^2$, where $a$ is the scattering length and $\bar{\omega}$ is the geometric average of the trap frequencies. The dynamic range of our data is insufficient to fit both processes so we adding a single-body
contribution to each equation to give

\[
\frac{1}{N} \frac{dN}{dt} = -LC_3N^{4/5} - \frac{1}{\tau} \tag{4.3}
\]

\[
\frac{1}{N} \frac{dN}{dt} = -GC_2N^{2/5} - \frac{1}{\tau} \tag{4.4}
\]

The value $\tau$ represents a lifetime due to other density independent decay processes such as scattering from thermal atoms or external heating of the cloud. Integrating these equations, we find the general form

\[
N = \frac{N_0}{((1 + k_3\tau)e^{\tau/m} - k_4\tau)^m} \tag{4.5}
\]

where $N_0$ is the initial atom number, and $k_3 = (N_0)^{9/5}LC_3$, $m = 5/4$ and $k_4 = (N_0)^{7/5}GC_2$, $m = 5/2$ for the three body and two body respectively. Fitting these equations to our data, we find for both cases that the influence of $\tau$ is actually very slight and to first order can be neglected (see Fig. 4.5). But for all values of $\tau$, the three body process is clearly the better fit to the data as the two body fit deviates strongly especially for longer times. Fitting for $\tau = 45$ s, the lifetime of a thermal cloud in our magnetic trap, we find $L = 4.9(5) \times 10^{-30}$ cm$^6$s$^{-1}$ for the three body decay.
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fit. Our value for the three body decay constant thus shows reasonable agreement with the results for \( F = 1 \) obtained by the JILA group of \( L = 5.8 \times 10^{-30} \text{ cm}^6\text{s}^{-1} \)\(^{[62]}\). This value is three times smaller than the rate for \( F = 2 \) atoms \(^{[61]}\) and five times smaller than the decay rate for a noncondensed cloud \(^{[62]}\) as predicted in \(^{[60]}\). All these aspects explain the long lived nature of the \( F = 1 \) condensate.

4.3.2 BEC of \(|1, -1\rangle\) atoms in TOP trap

The TOP trap is employed by applying a modulating current \( A_1 \cos \omega t \) to the horizontal (x-direction) pair of coils and a current \( A_2 \sin \omega t \) to the vertical (y-direction) pair of coils with \( \omega \) a frequency at least a decade higher than the radial trapping frequency \( \omega_\rho \) of the Ioffe Pritchard trap. Two Hewlett-Packard 3310 function generators were phase locked to produce these signals. The amplitudes \( A_1 \) and \( A_2 \) were controlled by analogue outputs of the signal conditioning rack (see Section 2.9). This results in a trap axis displaced over a distance \( \rho_m = B_m/\alpha \) (where \( B_m \) is the amplitude of the modulation field) and orbiting about the symmetry axis at frequency \( \omega \). As \( \omega \) is generally much faster than \( \omega_\rho \), the radial oscillation frequency of the atoms in the Ioffe Pritchard trap, this results in a time-averaged effective potential with TOP trap bottom \( \bar{B}_0 = \left( B_0^2 + B_m^2 \right)^{1/2} \), to be described in detail in Chapter 5.

The details of the evaporative cooling in the TOP trap differ from those in the Ioffe Pritchard trap, with the effective trap minimum \( B_0 \) taking the place of \( B_0 \). To investigate this we produced a BEC with and without the TOP trap. Evaporating without the TOP all atoms are removed at a final RF frequency of 0.65 MHz (equivalent to 0.91 G), and a BEC is formed at a frequency of 0.67 MHz (equivalent to 0.93 G). Obviously for a TOP, the RF frequencies which correspond to the removal of all atoms and the formation of a condensate depend on the strength of the modulation field \( B_m \). For \( B_m = 0.57 \) G, BEC of \( 8 \times 10^6 \) atoms is observed at an RF frequency of 0.84 MHz (equivalent to 1.17 G) slightly above \( B_0 = 1.07 \) G.

For a TOP with \( B_0 = 0 \), the JILA group found that the lifetime of the BEC goes to zero as \( B_m \) goes to zero \(^{[63]}\). With a typical bias field of 0.65 G, the lifetime is 10 s. For our experiments in the TOP trap, the lifetime is determined by \( B_0 \), thus \( B_0 \) can be made significantly lower without compromising lifetime, once the cloud is smaller than the radius \( \rho_m \). Best results (highest condensate numbers) were achieved by evaporating in the magnetic trap with \( B_0 = 0.65 \) G and then quickly switching on the TOP with \( B_m = 0.68 \) G once condensation is reached. In this way condensate numbers of over a million atoms could be achieved with a cloud lifetime of about 10 s.

We also studied the effect on lifetime of the TOP frequency \( \omega \). Varying \( \omega \), clouds are shown to have progressively shorter lifetime at lower frequencies (see Fig. 4.6). A substantial BEC persists at \( \omega/2\pi = 2.3 \) kHz for \( \omega_\rho/2\pi \approx 0.3 \) kHz such that \( \omega < 8\omega_\rho \), but is unmeasurable at \( \omega/2\pi = 1 \) kHz where \( \omega \approx 3\omega_\rho \). In practice we use \( \omega/2\pi = 8 \) kHz for the remaining experiments in this chapter as this provided the
highest atom number from the frequencies we tried. Enscher found that for very high TOP frequencies the lifetime begins to drop again [63] but we did not investigate this regime.

**Optical Resolution**

Absorption imaging doesn’t work inside the magnetic trap. Therefore, to get information about the trapped condensate we tried time-of-flight imaging after negligible TOF (<50 μs). We image along the cloud axis, far off-resonance to avoid saturation and with 10× magnification to provide enough resolution. Under these conditions we measure a cloud diameter of some 10 pixels. Not surprisingly we found that proper setting of the imaging focal plane is very critical for measuring the size of the cloud. With 10× magnification a 10pixel cloud width corresponds to a real space gaussian diameter of 15 μm. As the calculated Thomas-Fermi diameter is much smaller (8 μm) we conclude that this method is not suited to gather information of *in-situ* clouds.

At longer time of flights (24 ms) our measured cloud sizes match well with expected values. We measure cloud widths of 96 μm and 93 μm with 4× and 10× imaging, which corresponds to $10^5$ and $8 \times 10^4$ atoms in the condensate respectively.

**The $F = 1$ double cloud**

Using the axial helper coils, $B_0$ can be also be decreased all the way to zero. When overcompensated we produce a double well potential (see Fig.2.13). In the double well Ioffe-Pritchard trap, the condensate is very short lived due to Majorana losses, so the use of the TOP field is imperative to study the double cloud. A long-lived double condensate was produced in a manner similar to that described in [44]. The cloud was evaporatively cooled in the TOP with a trap minimum $B_0 = 0.58$ G. The
current in the axial helper coils was ramped in steps of 0.1 A to final values of 1.6 A, 2 A, and 2.4 A respectively corresponding to −0.3 G (left), −0.4 G (middle) and −0.6 G. We then RF evaporate in each case to 0.94 MHz. The results are shown in Fig. 4.7. For a final axial helper coil current of 2.4 A \((B_0 = -0.6 \text{ G})\) this results in creation of a double BEC as evidenced by anisotropic expansion in TOF.

### 4.4 Vortex excitation and detection

#### 4.4.1 BEC and vortices

It is well known that a superfluid cannot rotate as a normal fluid. As a superfluid, the BEC has a superfluid velocity which can be written as

\[
\mathbf{v} = \frac{\hbar}{m} \nabla \phi
\]

where \(\phi\) is the phase of the condensate wave function. For a weakly interacting gas solutions of the Gross-Pitaevskii equation give the wavefunction of the condensate.

In an axially symmetric potential

\[
U_{\text{ext}}(\rho, z) = \frac{1}{2} m \omega_\rho^2 \rho^2 + \lambda^2 z^2,
\]

where \(\lambda\) is the trap anisotropy parameter and \(\rho, \phi, \text{ and } z\) are the cylindrical coordinates, the wave function is described as \(\psi_\phi(\mathbf{r}) = e^{i\ell \phi} |\psi_0(\rho)|\) with \(\ell\) the angular momentum quantum number and \(|\psi_0| = \sqrt{n}\), the condensate density [4].

It can be seen that the superfluid is irrotational since

\[
\nabla \times \mathbf{v} = \nabla \times \nabla \phi = 0,
\]

Integrating this velocity about a closed loop such as the circle described by the Thomas-Fermi radius of the cloud reveals a quantization known as the Onsager-Feynman [64, 65] condition.
4.4. VORTEX EXCITATION AND DETECTION

\[ \oint v \, dl = \frac{\hbar}{m} 2\pi l \]  \hspace{1cm} (4.9)

where \( l \) is an integer.

Consequently upon rotation of the condensate about the axis, the superfluid velocity of the rotation of the fluid is given by

\[ v = \frac{\hbar}{m \rho} \mathbf{l}_\phi \]  \hspace{1cm} (4.10)

where \( \mathbf{l}_\phi \) is a unit vector in direction \( \phi \), which for \( l \neq 0 \) gives a vanishing wave function on the axis as \( \rho \to 0 \). Note that due to the quantization of circulation, after each closed loop, the phase of the fluid \( \phi \) must again equal to \( \phi \) or else differ by a multiple of \( 2\pi \). These accumulations of multiples of \( 2\pi \) are associated with vortices, which are singularities of the wave function and appear along the axis.

Far from the axis the density must again approach its normal value \( n \) and so \( \lvert \psi_0 \rvert \to \sqrt{n} \) suggesting the introduction of a dimensionless function \( f(\eta) \) such that \( \lvert \psi_0 \rvert = \sqrt{n} f(\eta) \) where \( \eta = \rho/\xi \) and \( \xi = \hbar/\sqrt{2mgn} \) is the healing length [4]. The function \( f(\eta) \) then satisfies

\[ \frac{1}{\eta} \frac{d}{d\eta} \left( \eta \frac{df}{d\eta} \right) + \left( 1 - \frac{\eta^2}{\xi^2} \right) f - f^3 = 0. \]  \hspace{1cm} (4.11)

Close to the axis \( \eta \to 0 \) and \( f \sim \eta^4 \), so the density \( n \) goes to zero on the axis. The signature for the presence of vortices in a condensate is this local drop in the density of the cloud. The distance over which the wavefunction increases from zero to the density of the rest of the condensate is of the order of the healing length \( \xi \).

4.4.2 Generation of vortices

The generation of vortices in a condensate was first shown in the JILA group for a two component condensate using microwave transitions between the components and a laser beam to engineer the phase profile of the second component [28, 67, 68].

Vortices can also be created analogously to the rotating bucket experiment with liquid Helium by rotating the confining potential of the condensate. The BEC can be created in a stationary trap and rotation induced by the addition of a rotating anisotropy, before removal of the anisotropy to allow vortices to nucleate. This was first done using a laser beam to stir the cloud in the ENS group [69, 70] and used to investigate the shape of a single vortex line in a condensate [71]. Using this method the presence of a vortex state can be determined by measuring the angular momentum of the cloud by inducing quadrupole shape oscillations [72] or by dislocations in the fringe pattern following coherent splitting and recombination of a BEC [73]. Similar methods, using an optical potential to spin up the condensate were implemented at MIT [74, 75] and used to show the formation and decay of vortex lattices [76].
The Oxford group created vortices by rotating the confining potential of the condensate by purely magnetic means [30]. This method was used to measure the angular momentum associated with the vorticity by precession of the scissors mode [77]. This method was also used to study vortex lattice dynamics [78]. The same method was used by the Arizona group to create a triangular vortex lattice [79].

Another method to induce vortices in a BEC was demonstrated by the MIT group, adiabatically manipulating a magnetic bias field to imprint a phase winding on the condensate [80, 81, 82]. The JILA approach described above was later followed by experiments which used a two photon transitions to produce doubly quantized vortices [83, 84] and later vortex-antivortex superposition states [85, 86].

An alternative to rotating a ready formed BEC is to begin with a rotating thermal cloud, then evaporatively cool to condensation to produce a rotating BEC as demonstrated by the JILA group [87]. Vortex lattice dynamics were studied in detail by this group with a string of interesting results [88, 89, 90, 91, 92, 93]. This approach allows large BEC rotation rates [94] close to the trap frequency where there is large atom loss. The critical rotation regime was also investigated at ENS in attempts to reach the lowest Landau level [95, 96, 97].

More recently the number of methods used to create and study vortex behaviour in BECs has proliferated greatly. One variation is to load the BEC into an optical lattice and rotate the lattice [98]. Studies on vortices in 2D clouds (BKT regime) have also been an area of interest [97, 99, 100, 101]. The method initiated at MIT has been successfully applied to produce vortices in fermionic systems as a proof of BCS superfluidity [102, 103, 104]. This method was also used elsewhere in a series of BEC experiments to investigate quadruply charged vortices [105, 106, 107]. A novel type of manipulation was demonstrated at NIST [108] where synthetic magnetic fields impart a Berry phase to produce vortices without rotation of the condensate.

Further methods which do not require rotation of the cloud have also been developed. Notable examples have been the use of condensate splitting and recombination to create "spontaneous vortices" [109, 33, 110] as well as the use of damping and quenching [111, 112] and helical imprinting [113]. Most recently the use and modelling of quantum turbulence to induce vorticity has been highly successful [114, 115]. Other recent developments have been the discovery of vortex dipoles [116] and an interesting technique taking multiples images from a single BEC to show the development of vorticity [117].

4.4.3 Excitation by rotation in a magnetic trap

In our experiments we use a rotating magnetic potential to stir the condensate. This is done by breaking the cylindrical symmetry of the trapping potential $U_{ext}(r)$ as defined in Eq. (4.7) such that the radial frequencies become $\omega_\perp = \sqrt{T - \epsilon} \omega_\perp$, $\omega_\parallel = \sqrt{T + \epsilon} \omega_\parallel$ where $\omega_\perp = \omega_\rho = \sqrt{\omega_\rho - \omega_\perp}$ and $\epsilon$ is the trap anisotropy.

Applying rotation of the major axis of the potential with frequency $\Omega$, angular
momentum can be imparted to the system. For \( \Omega \approx \omega_\perp \), a center of mass instability occurs. The cloud behaves analogously to a classical particle in a rotating elliptical potential and is ejected from the trap due to the coupling of the oscillation frequencies. At lower rotation frequencies the angular momentum introduced can be sufficient to induce vortices. The equation for the wave function of a condensate with a vortex can also be used to calculate the energy associated with the vortex state [4]. For a cylindrical cloud of length \( L \) and radius \( R \) this is shown to be

\[ E_v = L \pi \frac{\hbar^2}{m} \ln \left( \frac{1.46}{\xi R} \right) \]

resulting in a critical angular velocity

\[ \Omega_c = \frac{E_v}{N\hbar} - \frac{\hbar}{mR^2} \ln \left( \frac{1.46}{\xi R} \right) \]

For a cigar-shaped cloud, it is calculated that vorticity can appear for \( \Omega \geq 0.3\omega_\perp \) [118] and that below this value vorticity cannot be excited.

Experimentally, for low anisotropy \( \epsilon \) vorticity can be induced around \( \Omega = 0.7\omega_\perp \) [77, 69]. This is attributed to the quadrupole mode \( (l = 2) \) excited by the rotation of the elliptical condensate. For a small anisotropy this mode is excited at \( \Omega = \omega_\perp / \sqrt{2} \) and induces large amplitude oscillations. These oscillations result in the dynamical instability necessary for vortex creation [119]. For larger eccentricities, the dynamical instabilities occur in a range of frequencies around \( \Omega = \omega_\perp / \sqrt{2} \).

There are two methods to induce this instability via rotation of the magnetic trapping potential of the cloud. The first is to fix the anisotropy \( \epsilon \) and slowly ramp the rotation frequency \( \Omega \). The second is to begin rotating the condensate in the normal state, cool below the condensate transition temperature and then adjust the anisotropy to form a vortex state. Both have been shown to create vortices following a settling time of some seconds [77, 33].

### 4.4.4 Vortex detection

For a condensate in an axially symmetric trapping potential, the vortex core can take the form of a line along the axis of the cloud. In general the size of the vortex \( \xi_0 \) is much smaller than the size of the condensate \( \rho_0 \) and so below the optical resolution of most imaging systems. The size of the cloud is expanded by time of flight imaging and in this process also the size of the vortex-generated hole in the cloud increases.

The free expansion from a trap for a cloud during time-of-flight is affected by the presence of a vortex state [66]. Defining the aspect ratio of the cloud during expansion as \( A = r_p / \sqrt{2}r_a \), where \( r_p \) and \( r_a \) are the radial and axial sizes of the cloud, the ratio \( A_p/A_a \) of the aspect ratios of rotating to a non-rotating condensate can be calculated. This calculation depends on anisotropy parameter \( \lambda \) used in Eq. (4.7) and
the parameter \( N/a_0 \), where \( N \) is the condensate atom number, \( a \) is the scattering length and \( a_0 = \sqrt{\hbar/m\bar{\omega}} \) is the harmonic oscillator length scale with \( \bar{\omega} \) the geometric average of the trap frequencies \( \omega_x, \omega_y \) and \( \omega_z \).

For an isotropic trap (\( \lambda = 1 \)), the deviations from a ratio of \( A_f/A_n = 1 \) are shown to be very slight for all but the smallest cloud atom numbers. For a cigar shaped trap such as ours where \( \lambda \approx 0.05 \), and \( N/a_0 = 100 \) (corresponding to \( 5 \times 10^3 \) atoms in the condensate), the aspect ratio can be increased by up to 25%.

The development of the size of the vortex core is also shown to be dependent on the parameter \( N/a_0 \). For \( N/a_0 = 20 \) the vortex core size \( \rho_i \) grows from \( 0.2\rho \) up to \( 0.25\rho \) during 25 ms TOF expansion. For our numbers, the vortex core corresponds to about \( 0.06\rho \) or about \( 15 \mu m \) after the expansion associated with a 25 ms time-of-flight, and so should be just observable with the imaging system outlined in Chapter 3.

4.5 Experiments rotating the TOP

Previous work [35] suggested a method to produce vortices with the aid of a rotating linear TAP (time averaged potential). In combination with a Ioffe-Pritchard trap (\( B_0 > 0 \)) this produces a rotating elliptical potential. This idea can be generalized for arbitrary \( B_0 \) by adding a rotating elliptical TOP (time-averaged orbiting potential) to the IP trap. For \( B_0 > 0 \) this is very similar to the rotating linear TAP and for \( B_0 < 0 \) it gives rise to a pair of rotating elliptical TOP traps. The equation for the TOP field can be written as:

\[
B_X : A \cos \omega t \cos \Omega t - B \sin \omega t \sin \Omega t \\
B_Y : A \cos \omega t \sin \Omega t + B \sin \omega t \cos \Omega t
\]

where \( \omega/2\pi = 8 \) kHz is the frequency of the TOP, \( \Omega \) is the frequency of rotation of the elliptical potential and the amplitudes \( A \) and \( B \) can be set in the range \( 0 - 1.2 \) G. The resulting radial anisotropy of the total potential (IP trap plus TOP trap) is defined as

\[
\varepsilon = \frac{\omega_+^2 - \omega_-^2}{\omega_+^2 + \omega_-^2},
\]

where \( \omega_+ \) and \( \omega_- \) are the major and minor frequencies of our total potential, respectively. For \( A = B \) the TOP field is circular; for \( A > B \) it is elliptical with the major axis in the direction \( \theta = \Omega t \). For \( B = 0 \), the TOP field corresponds to the TAP field mentioned above. In this manner we can produce an anisotropy of up to \( \varepsilon = 0.25 \). The signals \( A \cos \omega t \) and \( B \sin \omega t \) were produced by the Hewlett-Packard 3310 function generators also used to produce the circular TOP, while the factors \( \sin \theta \) and \( \cos \theta \) were generated from a Krohn-Hite function generator. The outputs of these function generators were then multiplied with a pair of AD835 multipliers to produce the final inputs to the current supplies. Technical details are found in...
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Section 2.7. A possibility to trim the offset of the multiplier output was included to avoid wobbling of the trap axis and the associated heating.

4.5.1 Results with radial imaging

Imaging from the radial direction is used to establish the properties of the BEC. This imaging is set up for a large field of view to enable convenient TOF imaging. We can follow the evolution of the cloud shape over a maximum of 27 ms. First we produced a BEC in a circular TOP with $A = B = 0.68$ G rotating at an angular frequency $\Omega$. For this we used the same method described for the ordinary circular TOP ($\Omega = 0$). Ramping the amplitude $B$ to 0.45 G in 15 ms did not result in significant loss of the condensate. The latter condition corresponds to an anisotropy $\varepsilon \approx 0.14$, $\omega_{\perp}/2\pi = 294$ Hz with $\omega_{\perp}/2\pi = 316$ Hz, $\omega_{\perp}/2\pi = 272$ Hz. The measured trap frequency was $\omega_{\perp}/2\pi = 300(10)$ Hz. This frequency was measured by varying the spinning frequency $\Omega$ and observing the atom loss. We also measured substantial losses when spinning a circular TOP, which is an indication of residual anisotropy. In this way we measured $\omega_{\perp}/2\pi = 290(10)$ Hz for $A = B = 0.68$ G and $\omega_{\perp}/2\pi = 310(10)$ Hz for $A = B = 0.45$ G. In Fig. 4.8 we show our results for the trap losses in the rotating TOP. The spinning frequency $\Omega$ is normalized to the measured radial trap frequency. We not only observe a loss dip at $\Omega = \omega_{\perp}$, but also at $\Omega = 2\omega_{\perp}$ and $\Omega = \frac{3}{2}\omega_{\perp}$. These results suggest that we were not successful in completely removing the presence of anisotropy possibly due to the presence of gravitational sag.

As explained in Section 4.4.2, the optimal frequency range to produce a vortex under these conditions is around the quadrupole excitation frequency $\Omega \approx \omega_{\perp}/\sqrt{2} \approx 2\pi \times 220$ Hz, so we concentrated our search at TOP frequencies in the range 170 – 230 Hz. For this purpose amplitude $A$ was again set to 0.68 G, while $B$ was first set at 0.68 G to produce a condensate and then ramped down to 0.45 G. Measurements were done for spinning times $t_{\text{spin}} = 30$ ms, 300 ms, and 1300 ms. After each spinning period the BEC was still found to be intact. This was also the case for the linear TAP ($\varepsilon \approx 0.25$). The estimated BEC lifetime while stirring at $\Omega/2\pi = 230$ Hz was 3.7 s, long enough for vortices to form according to the literature [77, 33].

With this procedure we observed some indications of vorticity by looking at the aspect ratio of the cloud after a fixed TOP (see Table 4.1). We find that when no radial anisotropy is introduced to cloud ($A = B$) the aspect ratio of the cloud is independent of $\Omega$. Then at $\Omega/2\pi = 230$ Hz, for $B = 0.45$ G we see an increase by some 16% in the aspect ratio as we spin for longer times, corresponding well with the predictions of Lundh et al [66].

4.5.2 Results with axial imaging

The drawback to the large field of view of our radial imaging set-up is that resolution is not good enough to see details such as vortex lines. Therefore to search for vortices
we used axial imaging. In this way the imaging is along the vortex axis and the appropriate magnification can be selected with microscope objectives. To check that the elliptical stirring is really taking place, we cool a cloud in a TOP with a stable and large anisotropy $A = 0.1$ G, $B = 0.5$ G, corresponding to $\omega_+ / 2\pi = 380$ Hz, $\omega_- / 2\pi = 293$ Hz and an anisotropy $\varepsilon = 0.25$. We then set the major axis at an angle $\theta = 0, 45^\circ$ and $90^\circ$ and evaporate to a temperature just above $T_c$. Imaging with a TOP $\leq 50$ μs, we find elliptically shaped thermal clouds with the major axis reproducing the three angle settings of $\theta$.

Using axial imaging we tried to excite vortices with an elliptical TOP spinning at $\Omega / 2\pi = 230$ Hz for $t_{\text{spin}} = 30$ ms, 300 ms, and 1300 ms. In all cases the imaging was done by ramping back to circular conditions in 15 ms and using a 24 ms TOF. However, no vortices were observed in these measurements. We then varied the modulation frequency $\Omega$, set the spinning time $t_{\text{spin}} = 1300$ ms and returned for times $t_{\text{hold}} = 200$ ms, 500 ms, 700 ms to the circular TOP. We then imaged with $4\times$ and $10\times$ magnification. The number of photons per pixel is inversely proportional

Table 4.1: Ratio of aspect ratio for a nonrotating to a rotating condensate.

<table>
<thead>
<tr>
<th>$\Omega / 2\pi$</th>
<th>$B$</th>
<th>$t_{\text{spin}}$</th>
<th>$A_+/A_-$</th>
</tr>
</thead>
<tbody>
<tr>
<td>100 Hz</td>
<td>0.68 G</td>
<td>300 ms</td>
<td>1</td>
</tr>
<tr>
<td>230 Hz</td>
<td>0.68 G</td>
<td>300 ms</td>
<td>1.0</td>
</tr>
<tr>
<td>230 Hz</td>
<td>0.45 G</td>
<td>300 ms</td>
<td>1.12</td>
</tr>
<tr>
<td>230 Hz</td>
<td>0.45 G</td>
<td>1500 ms</td>
<td>1.16</td>
</tr>
</tbody>
</table>
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Figure 4.9: Above: Images taken after 20 ms TOF and imaged with 4× magnification. Below: corresponding profiles through the cloud showing optical density. At $0.34\omega_\rho = 2\pi \times 100$ Hz and above $0.68\omega_\rho = 2\pi \times 200$ Hz absorption is high but no evidence of vortices is present. At $0.58\omega_\rho$, $0.61\omega_\rho$, and $0.65\omega_\rho$ absorption is lower due to atom loss close to half the trap frequency but dips in absorption several pixels wide point to possible onset of vortices.

to the magnification and so was quite low with 10× magnification. In the trade-off between signal to noise and resolution 4× magnification was found to be optimal. Between $\Omega/2\pi = 200$ Hz and $\Omega/2\pi = 230$ Hz the cloud could be imaged with both 4× and 10× magnification but no vortices were observed. Below 200 Hz identification of vortices was difficult due to the loss dip around $\Omega = 0.5\omega_\perp$. The cloud was lost entirely by spinning at $\Omega/2\pi = 160$ Hz, which corresponds to $0.5\omega_\perp$. Some indications for vorticity were present in certain images at $\Omega/2\pi = 170$ Hz, 180 Hz and 190 Hz. These images show drops in absorption over areas of several pixels in diameter corresponding to about 5% of a cloud diameter. The frequencies mentioned correspond to $0.6\omega_\perp - 0.65\omega_\perp$, where the onset of vorticity could be expected. However in this frequency range losses already limit the contrast and makes it difficult to be conclusive. Possible vortex depth is less than 20% and reproducibility of vortex number for a given frequency is poor. Looking at the radial size of the cloud in these images suggests also some evidence for vorticity but is inconclusive.

4.5.3 Conclusions

We can speculate that curving of the vortex along the imaging axis might obscure the visibility of the contrast. However, our aspect ratio is comparable to those used in experiments on vortex shape at ENS, where despite evident bending of the vortex lines, vortices could still be imaged head on [69]. The lack of observation of a vortex above $\Omega/2\pi = 200$ Hz corresponding to $\Omega = 0.68\omega_\rho$ (where losses are minimal) leads to a conclusion that no vortices could be reliably nucleated. A possible reason for the lack of vortices is the radial anisotropy of the TOP which is circular to no better than 1 part in 80. Several authors suggest significantly better homogeneity is
necessary in order to nucleate vortices, as high as 1 in 1000 is used in nucleation experiments at JILA [120]. It is also unclear what role the uncontrolled switch-on of the TOP field had on the reproducibility of these experiments. We changed our apparatus to produce a more reliable TOP field for the experiments described in Chapter 5 but did not repeat the search for vortices.