On the cutting edge of semiconductor sensors: towards intelligent X-ray detectors

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Charge spread

In addition to blur inferred by the non-negligble range of the photoelectron as well as that of Compton scattered photons and possibly generated fluorescence photons or Auger electrons, image quality may degrade due to charge spread caused by diffusion and electrostatic repulsion.

C.1 Diffusion

As described in Section 2.1.4, the generated charge carriers diffuse due to a concentration gradient. In absence of recombination, charge can be considered a conserved quantity. As a result, charge diffusion follows the continuity equation:

$$\frac{\partial n(\vec{r},t)}{\partial t} + \vec{\nabla} \cdot \vec{J}(\vec{r},t) = 0\,. \tag{C.1}$$

It states that a temporal change in carrier concentration leads to a change in current density. Using Fick's law, the continuity equation for electrons becomes:

$$\frac{\partial n(\vec{r},t)}{\partial t} = \vec{\nabla} \cdot (D\vec{\nabla} n(\vec{r},t)) = D\Delta n(\vec{r},t)\,. \tag{C.2}$$

The solution of this differential equation is a Gaussian function, which describes the charge-carrier distribution relative to the centre of mass of the charge cloud as a function of the the drift time t.

$$n(\vec{r},t) = n_0 \left(\frac{1}{\sqrt{(4\pi Dt)}}\right)^3 e^{-\dfrac{r^2}{4Dt}}\,, \tag{C.3}$$

with a standard deviation σ of

$$\sigma = \sqrt{2Dt}\,. \tag{C.4}$$

For a constant drift field, this results in the following expression for the lateral spread caused by diffusion:

$$\sigma = \sqrt{2Dt_{\mathrm{drift}}} = \sqrt{2\frac{kT\mu}{q}\frac{d}{v_{\mathrm{drift}}}} = \sqrt{2\frac{kT\mu}{q}\frac{d}{\mu\vec{E}}} = \sqrt{2\,\frac{kTd^2}{qU}}\,, \tag{C.5}$$

where the Einstein relation (see equation 2.21 on page 28) is used and it is assumed that t is the drift time over the full thickness of the sensor.
An alternative expression for the lateral extent of a distribution is the full-width-half-maximum (FWHM). The diffusion spread can therefore be defined as:

$$\mathrm{FWHM} = 2\sqrt{2\ \ln 2}\ \sigma = 4\sqrt{\ln 2 \frac{kTd^2}{qU}}\,. \tag{C.6}$$

Note that the spread is only dependent on the temperature, the drift distance and the applied bias voltage. It is independent of the mobility, material used and the energy of the incoming X-ray photon.

C.2 Electrostatic repulsion

Another effect that causes charge spread is the electrostatic repulsion of same-sign charge carriers in the generated electron-hole charge cloud. In [123], the radius of the charge cloud is obtained from the part of the continuity equation that describes the charge-cloud expansion due to repulsion only.

$$\frac{\partial Q(r,t)}{\partial r}\mu E = \frac{\partial Q(r,t)}{\partial r}\mu\frac{1}{4\pi\epsilon_0 r^2} = -\frac{\partial Q(r,t)}{\partial t}\,, \tag{C.7}$$

where $Q(r,t)$ is the total amount of charge inside a spherical charge cloud and μ is the mobility. By using the method of separation of variables and satisfying the condition that the net force on the charge carriers in a sphere of radius r only depends on the number of charge carriers inside the sphere, r can be calculated by

$$r(t) = \sqrt[3]{3\frac{q}{4\pi\epsilon_r\epsilon_0}\mu N t} = \sqrt[3]{3\frac{q}{4\pi\epsilon_r\epsilon_0}N\frac{d^2}{U}}\,, \tag{C.8}$$

where ϵ_r the dielectric constant of the material and N the number of created electron-hole pairs.