Accreting neutron stars: strong gravity and type I bursts
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Chapter 2

Discovery of an accreting millisecond pulsar in the eclipsing binary system Swift J1749.4–2807


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Abstract: We report on the discovery and the timing analysis of the first eclipsing accretion-powered millisecond X-ray pulsar (AMXP): SWIFT J1749.4–2807. The neutron star rotates at a frequency of $\sim 517.9$ Hz and is in a binary system with an orbital period of 8.8 hrs and a projected semi-major axis of $\sim 1.90$ lt-s. Assuming a neutron star between 0.8 and 2.2 $M_\odot$ and using the mass function of the system and the eclipse half-angle, we constrain the mass of the companion and the inclination of the system to be in the $\sim 0.46-0.81$ $M_\odot$ and $\sim 74.4^\circ - 77.3^\circ$ range, respectively. To date, this is the tightest constraint on the orbital inclination of any AMXP. As in other AMXPs, the pulse profile shows harmonic content up to the 3rd overtone. However, this is the first AMXP to show a 1st overtone with rms amplitudes between $\sim 6\%$ and $\sim 23\%$, which is the strongest ever seen, and which can be more than two times stronger than the fundamental. The fact that SWIFT J1749.4–2807 is an eclipsing system which shows uncommonly strong harmonic content suggests that it might be the best source to date to set constraints on neutron star properties including compactness and geometry.
2.1 Introduction

The first accreting millisecond X-ray pulsar (hereafter AMXP) was discovered in 1998 (SAX J1808.4–3658, see Wijnands & van der Klis, 1998) and since then, a total of 13 AMXPs have been found and studied in detail (Patruno, 2010). Most AMXPs show near sinusoidal profiles during most of their outbursts. This is consistent with a picture in which only one of the hotspots (at the magnetic poles) is visible (see ref. below). Deviations from a sinusoidal profile (i.e., an increase in harmonic content) are generally interpreted as being caused by the antipodal spot becoming visible, perhaps as accretion rate falls and the disk retreats (see, e.g., Poutanen & Gierliński, 2003; Ibragimov & Poutanen, 2009, and references therein).

Although the amplitude of the 1st overtone may reach that of the fundamental late in the outburst (see, e.g., Hartman et al., 2008, 2009), no AMXP so far has shown pulse profiles where the 1st overtone is generally stronger than the fundamental throughout the outburst.

The stability of the pulse profiles in some of the AMXPs means that pulse profile modeling can be used to set bounds on the compactness of the neutron star and hence the dense matter equation of state (see, e.g., Poutanen & Gierliński, 2003; Poutanen et al., 2009, and references therein). Unfortunately, there is often a large degeneracy between the parameters due to the number of free parameters needed to construct the model profile. One of these parameters is the inclination of the system, which to date has not been well-constrained for any AMXP.

In this Letter we report on the discovery and timing of the accretion-powered millisecond X-ray pulsar SWIFT J1749.4–2807. Thanks to the observed eclipses (Markwardt et al., 2010), we set the tightest constraint on system inclination for any AMXP. This, coupled with the fact that the amplitude of the first overtone is higher/comparable to that of the fundamental for much of the outburst and that the amplitude of the first overtone is unusually high, allows to put tight constraints on pulse profile models. We show that SWIFT J1749.4–2807 has the potential to be one of the best sources for this approach to constraining the neutron star mass-radius relation and hence the EoS of dense matter.

2.2 SWIFT J1749.4–2807

SWIFT J1749.4–2807 was discovered in June 2, 2006 (Schady et al., 2006), when a bright burst was detected by the Swift burst alert telescope (BAT).
Figure 2.1: Top panel: 2-10 keV flux as measured from RXTE/PCA and Swift/XRT observations. The flux of the last PCA observation (MJD 55307.5) is not shown; the spectrum of this observation was used as an estimate background emission (see text). Upper limits are quoted at 95% confidence level. We calculated the flux during the last PCA and last Swift/XRT detection using WebPIMMS (assuming a power law spectrum with index 1.8). Middle panels: Fractional rms amplitude and 95% confidence level upper limits of the fundamental and three overtones as a function of time. Detections (> 3σ single trial) and upper limits are from ~ 500 and ~ 3000 sec datasets, respectively. Bottom panel: Ratio between the fractional rms amplitude of the 1st overtone and fundamental. Blue triangles represent points in which both harmonics are significantly detected in ~ 500 second datasets. Black circles represent the ratio between the fractional rms amplitude of the 1st overtone and the 95% confidence level upper limit to the amplitude of the fundamental. Grey circles represent the same ratio but when the fundamental is significantly detected and not the 1st overtone. This means that black circles represent lower limits while grey circles are upper limits. These ratios are independent of the background.
Wijnands et al. (2009) presented a detailed analysis of the Swift/BAT and Swift/XRT data and showed that the spectrum of the 2006 burst was consistent with that of a thermonuclear Type I X-ray burst (Palmer et al., 2006; Beardmore et al., 2006, see, also) from a source at a distance of 6.7 ± 1.3 kpc.

SWIFT J1749.4–2807 was detected again between April 10th and 13th, 2010 using INTEGRAL and Swift observations (Pavan et al., 2010; Chen-evez et al., 2010). We promptly triggered approved RXTE observations on this source to study X-ray bursts and to search for millisecond pulsations (Proposal ID:93085-09, PI: Wijnands). The first RXTE observation was performed on April 14th and lasted for about 1.6 ksec. We found strong coherent pulsations at ≈ 517.9 Hz and at its first overtone ≈ 1035.8 Hz, showing that SWIFT J1749.4–2807 is an accreting millisecond X-ray pulsar (Altamirano et al., 2010). RXTE followed up the decay of the outburst on a daily basis. Preliminary results on the rms amplitude of the pulsations, orbital solution, discovery of eclipses, evolution of the outburst and upper limits on the quiescent luminosity were reported in Astronomer’s Telegrams (Altamirano et al., 2010; Bozzo et al., 2010; Belloni et al., 2010; Strohmayer & Markwardt, 2010; Markwardt et al., 2010; Yang et al., 2010; Chakrabarty et al., 2010). No optical counterpart has been identified as yet, with a 3σ lower limit in the i-band of 19.6 (Yang et al., 2010).

2.3 Observations, spectral analysis and background estimation

We used data from the Rossi X-ray Timing Explorer (RXTE) Proportional Counter Array (PCA, for instrument information see Jahoda et al., 2006). Between April 14th and April 21st there were 15 pointed observations of SWIFT J1749.4–2807, each covering 1 or 2 consecutive 90-min satellite orbits.

We also analyzed data from Swift’s X-ray telescope (XRT; Burrows et al., 2005). There were a total of 10 observations (target ID 31686), all obtained in the Photon Counting (PC) mode.

We used standard tools and procedures to extract energy spectra from PCA Standard 2 data. We calculated response matrices and ancillary files for each observation using the FTOOLS routine PCARSP V10.1. Background spectra were estimated using the faint-model in PCABACKEST (version 6.0). For the XRT, we used standard procedures to process and analyze the PC mode data.

\[1\text{http://www.swift.ac.uk/XRT.shtml}\]
2.3 Observations, spectral analysis and background estimation

When necessary, an annular extraction region was used to correct for pile-up effects. We generated exposure maps with the task XRTEXPOMAP and ancillary response files were created with XRTMKARF. The latest response matrix files (v. 11) were obtained from the CALDB database.

We used an absorbed power-law to fit all PCA/XRT observations. We first fitted all XRT spectra and found an average interstellar absorption of $3.5 \times 10^{22} \text{ cm}^{-2}$; $N_H$ was fixed to this value when fitting all (standard) background-subtracted PCA spectra. When comparing the fluxes estimated by PCA and XRT we found that the PCA fluxes were systematically higher. Only on one occasion RXTE and Swift observations were performed simultaneously (MJD 55306.69, i.e. at the end of the outburst) and in this case the flux difference was $\approx 1.4 \times 10^{-10} \text{ erg cm}^{-2} \text{ s}^{-1}$. This is consistent with that seen a day later. Since (i) the count rates during the last RXTE observation (when the source was below the PCA detection limit but detected by Swift/XRT) are consistent with those we measure during the eclipses (see Section 2.6) and (ii) these count rates are consistent with the offset we find between PCA and XRT, we conclude that there is an additional source of background flux in our PCA observations. To correct for this, we also use the background-corrected spectrum of the last PCA observation (OBSID: 95085-09-02-08, MJD 55307.5) as an estimate of the additional source of background flux. This approach is optimal in crowded fields near the Galactic plane, where the contribution from the Galactic ridge emission and other X-ray sources in the $1^\circ$ PCA FoV becomes important (see, e.g., Linares et al., 2007, 2008).

Background estimates and the fractional rms amplitudes

Given the low flux during our observations, it is very important to accurately estimate the background emission before calculating the pulse fractional rms amplitudes. The fact that the extra source of background photons is unknown complicates the estimation of total background flux as a function of time. For example, the background flux could be intrinsically varying; even in the case of a constant distribution of background flux in the sky, it would be possible to measure flux variations if the collimator (i.e. PCA) orientation on the sky changes between observations. Given the background uncertainties, we arbitrarily adopt as total 3–16 keV background per observation the modeled background plus a constant offset of $\approx 17.5 \pm 2 \text{ ct/s/PCU}$. This takes into account the $\approx 19.5 \text{ ct/s/PCU}$ as estimated by the eclipses, the last PCA observation and the PCA–XRT offset (which is equivalent to $\approx 18 – 19 \text{ ct/s/PCU}$ in the
Table 2.1: Timing parameters for the AMXP SWIFT J1749.4–2807

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Orbital period, $P_{\text{orb}}$ (days)</td>
<td>0.3673696(2)</td>
</tr>
<tr>
<td>Projected semi major axis, $a_x \sin i$ (lt-s)</td>
<td>1.89953(2)</td>
</tr>
<tr>
<td>Time of ascending node, $T_{\text{asc}}$ (MJD)</td>
<td>55300.6522536(7)</td>
</tr>
<tr>
<td>Eccentricity, $e$ (95% c.l.)</td>
<td>&lt; $5.2 \times 10^{-5}$</td>
</tr>
<tr>
<td>Spin frequency $\nu_0$ (Hz)</td>
<td>517.92001395(1)</td>
</tr>
<tr>
<td>Pulsar mass function, $f_x$ ($M_\odot$)</td>
<td>0.0545278(13)</td>
</tr>
<tr>
<td>Minimum companion mass$^a$, $M_c$ ($M_\odot$)</td>
<td>0.5898</td>
</tr>
</tbody>
</table>

Note: All errors are at $\Delta \chi^2 = 1$.

$^a$: The companion mass is estimated assuming a neutron star of 1.4 $M_\odot$.

3–16 keV band as estimated with WebPIMMS\(^2\) and the best fit model to the XRT data), and the $\approx 15.5$ offset we would obtain if the additional source of background photons could change by $\sim 20\%$. This conservative adopted possible background range results in conservative errors on the pulsed fractions we report, i.e. the errors are probably overestimated.

### 2.4 Outburst Evolution

In Figure 2.1 we show the 2.0–10.0 keV unabsorbed flux of SWIFT J1749.4–2807 as measured from all available RXTE/PCA and Swift/XRT observations. Our dataset samples the last 7 days of the outburst, during which the flux decayed exponentially. We find that between MJD 55306.5 and 55307.5 SWIFT J1749.4–2807 underwent a sudden drop in flux of more than an order of magnitude, less abrupt than the 3 orders of magnitude drop in flux observed in the previous outburst of SWIFT J1749.4–2807 (Wijnands et al., 2009). Similar drops in flux have been seen for other AMXPs (see, e.g. Wijnands et al., 2003; Patruno et al., 2009a). If we take into account the fact that SWIFT J1749.4–2807 was first detected on MJD 55296 (Pavan et al., 2010), we estimate an outburst duration of about 12 days.

### 2.5 Pulsations

Adopting a source position $\alpha = 17^h49^m31^s.94$, $\delta = -28^\circ08'05'' .8$ (from XMM-Newton images, see Wijnands et al., 2009), we converted the photon arrival times to the Solar System barycenter (Barycentric Dynamical Time) with the FTOOL faxbary, which uses the JPL DE-405 ephemeris along with the

\(^2\)http://heasarc.gsfc.nasa.gov/Tools/w3pimms.html
spacecraft ephemeris and fine clock corrections to provide an absolute timing accuracy of $3.4\,\mu s$ (Jahoda et al., 2006).

We created power spectra of segments of 512 sec of data and found strong signals at frequencies of $\approx 517.92\,\text{Hz}$ and $\approx 1035.84\,\text{Hz}$ (Altamirano et al., 2010); these signals were not always detected simultaneously with a significance greater than $3\sigma$.

To proceed further, we used the preliminary orbital solution reported by Strohmayer & Markwardt (2010) and folded our dataset into 87 pulse profiles of $\approx 500\,\text{sec}$ each. We then fitted the profiles with a constant plus 4 sinusoids representing the pulse frequency and its overtones. We then phase-connected the pulse phases by fitting a constant pulse frequency plus a circular Keplerian orbital model. The procedure is described in detail in Patruno et al. (2010). In Table 2.1 we report the best fit solution and in Figure 2.2 we show one example of the pulse profile.

It is known that the timing residuals represent a significant contribution to the X-ray timing noise, which if not properly taken into account can affect the determination of the pulse frequency and the orbital solution (see, e.g. Hartman et al., 2008; Patruno et al., 2010). There is a hint of a correlation

Figure 2.2: Pulse profile obtained by folding $\approx 3400\,\text{sec}$ of data (ObsId 95085-09-01-02, 2-16 keV range).
Figure 2.3: Eclipse observed on observation 95085-09-02-02. The dataset starts on MJD 55302.9531, during the eclipse and shows that the egress occurs at orbital phase \( \approx 0.282 \) (where orbital phase zero is the time of passage through the ascending node). We estimate the duration of the eclipse by assuming that eclipse is symmetric around orbital phase 0.25. The light curve was only corrected by the standard modeled background (see text). Count rates are in the 2-16 keV band.

between the X-ray timing noise and the X-ray flux, especially between MJD 55302 and 55303, where a slight increase of the X-ray flux is accompanied by a jump in the pulse phases, similarly to what was reported for 6 other AMXPs by Patruno et al. (2009b). A complete discussion of timing noise in this source is beyond the scope of this paper and will be presented elsewhere.

In the middle panels of Figure 2.1 we show the fractional rms amplitude of the fundamental, and of the first, second and third overtone when the signal was \( > 3\sigma \) significant in \( \sim 500 \) sec datasets. The 95% confidence level upper limits are are estimated using \( \sim 3000 \) sec datasets (excluding detections) and plotted separately for clarity. When detected significantly, the rms amplitudes of the fundamental and 1st overtone are in the \( \sim 6 - 29\% \) and \( \sim 6 - 23\% \) ranges, respectively; the highest values are reached at the end of the outburst, where the uncertainties in our measurements also increase. Amplitudes for the fundamental as high as 15-20\% rms have been seen before for at least one source (although for a brief interval, see Patruno et al., 2010), however, no other AMXP shows a 1st overtone as strong as we detect it in SWIFT J1749.4–2807. In order to compare the strength of both signals, in Figure 2.1 (lower panel) we show the ratio between the fractional rms amplitude of the 1st
overtone and that of the fundamental. As can be seen, there are periods in which the ratio is approximately one, but also periods where the ratio is 2 or more. We note that these ratios are independent of the uncertainties on the background.

### 2.6 Eclipses and the inclination of the system

We searched the RXTE data for the occurrence of X-ray bursts and found none. Following Markwardt et al. (2010), we also searched for possible signatures of eclipses and found two clear cases in the RXTE data (ObsIDs: 95085-09-02-02 and 95085-09-02-04, beginning at MJD 55302.97 and 55305.87, respectively). PCA data on MJD 55306.97 (OBSID: 95089-09-02-11) samples an ingress, however, the count rate is too low to extract useful information (but see Markwardt & Strohmayer, 2010; Ferrigno et al., 2011).

The first and clearest case of an eclipse is shown in Figure 2.3. The average 3–16 keV count rate at the beginning of the observation is about \( \approx 18.5 - 19.5 \) cts/sec/PCU2 (only the standard modeled background has been subtracted) for the first \( \approx 600 \) sec. Then the countrate increases within a few seconds to an average of \( \approx 36 \) cts/sec/PCU2 and remains approximately constant for the rest of the dataset. The other dataset also shows a similar low-to-high count rate transition although at lower intensities: the observation samples less than 275 sec of the eclipse (at a rate of \( \approx 18 - 19 \) cts/sec/PCU2); the count rate after the egress is about \( \approx 22 \) cts/sec/PCU2, i.e. much lower than in the previous case.

Within the uncertainties on the unmodeled background, both egresses occur between orbital phases of \( \approx 0.2823 - 0.2825 \). During the eclipse the count rates in these two observations are consistent with the expected background of \( \approx 18 - 19 \) cts/sec/PCU2, implying that that SWIFT J1749.4–2807 most probably shows total eclipses; however, given the uncertainties in the background (see Section 2.3) and the sensitivity of the PCA, this should be tested and better quantified with observations from instruments like XMM-Newton, Suzaku or Chandra (see also Pavan et al., 2010; Ferrigno et al., 2011).

Using the best solution reported in Table 2.1, we also searched for pulsations in the 600 second period during which the companion star is eclipsing the neutron star (see above). We found none. Upper limits are unconstrained.

With our improved orbital solution and the measured times of the two egresses we determine the phase of egress to be no larger than 0.2825. Assuming the eclipses are centered around neutron star superior conjunction, the eclipse half-angle is \( \approx 11.7^\circ \), corresponding to an eclipse duration of \( \approx 2065 \)
Figure 2.4: Inclination of the binary system vs. the neutron star mass. For each point we also mark the mass of the companion star $M_c$ (in units of $M_\odot$) and the mass ratio $q = M_c/M_{NS}$.

We do not detect any evidence of absorption in the form of dips in the light curves, probably due to the fact that our dataset only samples $\approx 1.5$ orbital periods. These dips are common in other eclipsing LMXBs and thought to be due to the interaction of the photons from the central X-ray emitting region with structure on the disk rim or by what is left of the stream of incoming matter (from the companion) above and below the accretion disk. These dips are known to be highly energy dependent; both eclipses and egresses in our data are energy independent.

Assuming that the companion star is a sphere with a radius $R$ equal to the mean Roche lobe radius, then the radius of the companion star can be approximated as

$$R_L = a \cdot \frac{0.49 \cdot q^{2/3}}{0.6 \cdot q^{2/3} + \ln (1 + q^{1/3})},$$

where $a$ is the semi-major axis of the system and $q = M_c/M_{NS}$ is the...
2.7 Constraining neutron star properties via pulse-profile modeling

Knowing the inclination to a high degree of precision is useful for pulse profile modeling to constrain neutron star properties including compactness and geometry. To explore what could be done, we tried fitting simple model lightcurves to the pulse amplitude observations (along the lines explored by Pechenick et al. 1983, Nath et al. 2002 and Cadeau et al. 2007). The code we use has been tested against, and is in good agreement with, the results of Lamb et al. (2009).

We assume isotropic blackbody emission from one or two antipodal circular hot spots, and no emission from the rest of the star or the disc. At this stage we ignore both Comptonization (which might be important Gierliński & Poutanen 2005) and disc obscuration.

We consider as free parameters stellar mass and radius, and the colatitude \( \alpha \) and angular half-size \( \delta \) of the hotspot(s). Using only points where both fundamental and 1st overtone are detected with at least 3\( \sigma \) significance, we search for models that fit all observations (amplitude of fundamental and ratio of first overtone to fundamental) and which have the same mass and radius. Hotspot size and position are permitted to vary between observations since accretion flow is expected to be variable.
Although it is possible to obtain a high degree of harmonic content, due to GR effects, from a single visible hotspot (see also Lamb et al. 2009), we find that the strength of the harmonic is such that two antipodal hotspots must be visible in order to fit the data. We are also able to constrain system geometry. The $1\sigma$ confidence contours restrict us to models with $\alpha \simeq 50^\circ$ and $\delta = (45 - 50)^\circ$; the $2\sigma$ contours permit a wider range of parameters but still require models where $\alpha = (40 - 50)^\circ$ and $\delta = (30 - 50)^\circ$ (hotspots must be smaller if they are located closer to the pole). These results, within the frame of our simple model, suggest a substantial offset between rotational and magnetic pole in this source.

Our models also put limits on stellar compactness. The $1\sigma$ confidence contours exclude models with $M/R > 0.17M_\odot/km$, while the $2\sigma$ contours exclude models with $M/R > 0.18M_\odot/km$. Although this does not rule out any common equation of state (Lattimer & Prakash, 2007), it does exclude some viable regions of dense matter parameter space.

Our simple calculations, while certainly not conclusive, illustrate the potential of this source. With better models, and phase-resolved spectroscopy using high spectral resolution observations, this system is an extremely promising candidate for obtaining tight constraints from pulse profile fitting.

Acknowledgements: We thank J. Poutanen for useful discussions. AP and ML acknowledge support from the Netherlands Organization for Scientific Research (NWO) Veni and Rubicon Fellowship, respectively.

2.8 Appendix: Raytracing code details$^3$

Overview

In order to measure mass and radius, what we do is to generate lightcurves and then fit them to the data. Then, to calculate lightcurves we have to know how many photons per frequency bin reach the observer at a given time. This depends on how photons are emitted at the surface (assuming no sink or source along the ray paths), the direction of the outgoing ray and the energy as measured at the surface of the NS. We therefore need to know the blueshifted (with respect to the observer at infinity) emitted frequency of the photons and the path the rays follow. This information is encoded in the four-velocity

$^3$This Appendix was not part of the published Letter, due to page limits. We have included it in this thesis to give full details of the raytracing code that was used to obtain constraints on stellar mass and radius, since this was the main contribution of the candidate.
Figure 2.5: The scheme for the lightcurve calculation. The star and observer parameters and the resolutions required are shared by the three programs: the equatorial geodesics calculator, the general geodesics calculator, which transforms the equatorial geodesics to those for the observer inclination, and the lightcurve calculator. Moreover, the first program uses some subroutine for integration and the lightcurve generator may use additional modules for the emission patterns and spectra.

tangent to the light null-geodesics connecting the NS surface to the observer at infinity.

Basically, the code works as follows (Figure 2.5). First, the null-geodesics are calculated. These depend on the NS mass $M$ and radius $R$ and on the inclination $i$ of the rotational axis with respect to the observer’s line of sight.
Geodesics provide information about the directions of single rays from the NS surface and about the redshift the photons experience. Second, we specify emission pattern and spectrum. Only then can we calculate the actual lightcurve.

In more details, the actual geodesic calculation itself is further divided into two substeps. For rotating NSs, the rotational axis breaks the spherical symmetry of the system and the colatitude of the observer (the inclination \(i\)) changes the set of geodesics that can connect the different patches to the detector. A non rotating NS metric, on the other hand, has full spherical symmetry and can be described by the outer Schwarzschild solution. In this case, it can be assumed with no loss of generality that the observer and all the geodesics lie on the equatorial plane, since a plane can be always found passing through the centre of the star, the observer and the centre of each patch. If we define this as the equatorial plane (which can always be done, thanks to the spherical symmetry) it can be proven that the geodesic connecting the patch and the observer will never leave such plane (de Felice & Clarke, 1992).

Direct testing of full, axisymmetric, calculations with approximate, spherically symmetric, ones showed that except in cases with very high spin rate, approximating the outer metric as the Schwarzschild solution does not introduce significant errors (Cadeau et al., 2007, who point out that the greatest source of error is the possible rotation–induced oblateness of the NS). We therefore first generate the geodesics on the equatorial plane of a given mass and radius NS and only subsequently transform these to the full set for different inclinations. The rationale behind splitting the code in this way is that the same geodesics can be used for different emission patterns. Transforming the equatorial plane geodesics to the different geometries of observer inclination further reduces the computational time when exploring the phase space of \(M\), \(R\) and \(i\).

The surface of the NS is divided into a grid of equal area patches. The number of photons emitted along a ray is proportional to this area and the infinitesimal solid angle subtended by each ray. The proportionality is given by the intensity (see below), which depends on the direction of the ray and the frequency of the photons. The frequency is set by blueshifting the frequency detected at infinity. Blueshift and direction (with respect to the normal to the surface) are calculated according to the local metric of an observer sitting on the star and corotating with it. In this way the Doppler shift and the aberration due to rotation and the redshift due to gravity are automatically taken into account.

Finally we calculate the time dependence. All rays leave the surface at
the same time, but due to differences in the flight times, they are split across different time bins (see Cadeau et al., 2007). After all the contributions to the same time bin are summed, the result is multiplied by the detector response curve and the lightcurve is produced.

The geodesics

In the Schwarzschild metric

\[
\text{ds}^2 = (1 - r_S/r') c^2 \text{d}t'^2 - (1 - r_S/r')^{-1} \text{d}r'^2 - r'^2 (\text{d}\theta'^2 + \sin^2 \theta' \text{d}\varphi'^2),
\]

(2.3)

where \(\text{ds}\) is the path length, \(t'\) the coordinate time, \(r', \theta'\) and \(\varphi'\) the spherical spatial coordinates and \(r_S = 2GM/c^2\) is the Schwarzschild radius. The planar, equatorial (\(\theta' = \pi/2\)) geodesics are calculated from the four velocity \(v^i\) (Pechenick et al., 1983)

\[
v'^i = E_\infty \begin{pmatrix} (1 - r_S/r')^{-1} , \sqrt{1 - (1 - r_S/r') b'^2/r'^2} , 0 , b'/r'^2 \end{pmatrix}^T
\]

\[
\frac{\text{d}t'}{\text{d}r'} = \frac{1}{c (1 - r_S/r') \sqrt{1 - (1 - r_S/r') b'^2/r'^2}}
\]

(2.4)

\[
\frac{\text{d}\varphi'}{\text{d}r'} = \frac{b'}{r'^2 \sqrt{1 - (1 - r_S/r') b'^2/r'^2}}
\]

(2.5)

where \(b'\) is the impact parameter of the ray, i.e. the ratio of its conserved angular momentum (times c) and its conserved energy, \(E_\infty\), as measured by a static observer at infinity (see below).

For the actual implementation Eqs. (2.4) and (2.5) are changed into

\[
\frac{\text{d}\Delta t'}{\text{d}r'} = \frac{1 - \sqrt{1 - (1 - r_S/r') b'^2/r'^2}}{c (1 - r_S/r') \sqrt{1 - (1 - r_S/r') b'^2/r'^2}}
\]

(2.6)

\[
\frac{\text{d}\Delta \varphi'}{\text{d}y} = \sqrt{\frac{1 - r_S}{R} \frac{1 - \sin^2 y/b'}{1 - \sin^2 y}}^{-1}
\]

(2.7)

where \(R\) is the star radius, \(\hat{b}' = b'/b_{\text{max}}\), \(b_{\text{max}} = R/\sqrt{1 - r_S/R}\) and \(y \in [0; \sin^{-1} \hat{b}']\) (Nath et al., 2002). Eq. (2.6) actually expresses the lag with respect to the flight time of the shortest-path ray and Eq. (2.7) is the maximum change in the angular coordinate from one end of the ray to the other. If the observer is at \(\varphi' = 0\), then \(\Delta \varphi'\) is the initial \(\varphi'\) of the ray. We use \(\hat{b}' \in [0, 1]\), which corresponds to rays coming only from the negative \(\varphi'\) half of the equatorial plane.
Eqs. (2.6) and (2.7) are integrated by a Runge-Kutta integrator with adaptive step as described in Press et al. (1992). The extremes of the integration for the time lag are \( R \) and \( r_\infty \), which is set to an arbitrarily large number. It is clear that the geodesic values produced at this stage depend only on the compactness of the star, encoded in \( b_{\text{max}} \) via \( r_S/R \). The number of geodesics calculated, i.e. the different values for \( \hat{b}' \), are set by a parameter (which should be high enough to ensure good resolution for interpolation, see below).

These results are then transformed to their form in the system with the vertical axis aligned with the rotational axis, where the observer is at colatitude \( i \), according to the spatial change of coordinate (a rotation):

\[
\begin{pmatrix}
\sin \theta \cos \varphi \\
\sin \theta \sin \varphi \\
\cos \theta 
\end{pmatrix}
= r
\begin{pmatrix}
\sin i & \cos i & -\sin \psi \cos i \\
0 & \sin \psi & \cos \psi \\
\cos i & -\cos \psi \sin i & \sin \psi \sin i
\end{pmatrix}
\begin{pmatrix}
\sin \theta' \cos \varphi' \\
\sin \theta' \sin \varphi' \\
\cos \theta'
\end{pmatrix}
\tag{2.8}
\]

It is the combination of a rotation along the y axis that brings the line of sight, originally lying on the equator on the x axis, to an inclination \( i \) and a rotation of \( \psi \) around the new observer line of sight. Therefore, since \( \theta' = \pi/2 \),

\[
t = t' \tag{2.9}
\]
\[
r = r' \tag{2.10}
\]
\[
\theta = \cos^{-1} \left( \cos i \cos \varphi' - \sin i \cos \psi \sin \varphi' \right) \tag{2.11}
\]
\[
\varphi = \tan^{-1} \left( \frac{\sin \psi \sin \varphi'}{\sin i \cos \varphi' + \cos i \cos \psi \sin \varphi'} \right) \tag{2.12}
\]

\( \varphi' \) is given by \( \Delta \varphi' \) from Eq. (2.7) and \( \psi \) spans the range \([0,2\pi]\).

According to the same transformation, the metric is

\[
ds^2 = (1 - r_S/r) c^2 dt^2 - (1 - r_S/r)^{-1} dr^2 - r^2 (d\theta^2 + \sin^2 \theta \, d\varphi^2), \tag{2.13}
\]

and the initial components of a light ray four velocity are

\[
v^i = E_\infty \begin{pmatrix}
c / (1 - r_S/r) \\
\sqrt{1 - \hat{b}'^2} \\
(\partial \theta / \partial \varphi') \hat{b}' / R^2 \\
(\partial \varphi / \partial \varphi') \hat{b}' / R^2
\end{pmatrix} \tag{2.14}
\]

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where

\[
\frac{\partial \theta}{\partial \varphi'} = \frac{\cos i \sin \varphi' + \sin i \cos \psi \cos \varphi'}{\sin \theta} \tag{2.15}
\]

\[
\frac{\partial \varphi}{\partial \varphi'} = \frac{\sin \psi \sin i}{\sin^2 \theta} \tag{2.16}
\]

For later convenience, the last component of \(v^i\) can be written as \(b/(R^2 \sin^2 \theta)\) and that defines \(b\) as \(b = b' \sin \psi \sin i\).

It is only at this second stage that the inclination \(i\) of the observer is introduced. The transformations are actually done in reverse order: the angular resolutions in \(\cos \theta\) and \(\varphi\) are set, again as parameters, and for each couple \([\theta, \varphi]\) the corresponding \([\psi, \varphi']\) are derived inverting Eqs. (2.11) and (2.12). The value of \(b'\) is uniquely defined by \(\varphi'\) by interpolating linearly the results of the previous step. This is why it is necessary to calculate a dense sample of geodesics. Finally, \(\psi\) and \(b'\) are used to calculate \(b\).

**Angles and area element**

As we said in the beginning, after having calculated the geodesics, we need to know the area and angle measurements at the surface, in order to know the emitted number of photons. In the following we follow Cadeau et al. (2007).

Every measure on the surface is done in the system of the observer corotating with the star. Their four velocity is:

\[
u^i = \frac{1}{\sqrt{1 - r_S/R} \sqrt{1 - u^2/c^2}} \begin{pmatrix} 1 \\ 0 \\ 0 \\ \Omega/c \end{pmatrix} \tag{2.17}
\]

\[u = R \sin \theta \Omega / \sqrt{1 - r_S/R} \text{ and } \Omega \text{ is the spin of the NS as seen at infinity.}\]

The lengths measured at the star are given by the orthogonal projector of the observer (a metric for the components of vectors on the plane perpendicular to the time axis of the observer, i.e. perpendicular to his four velocity, see de Felice & Clarke, 1992):

\[
h_{ab} = g_{ab} + u_a u_b = \begin{bmatrix}
(1 - r_S/R) u^2 / c^2 & 0 & 0 & -R^2 \sin^2 \theta \Omega / c \\
0 & (1 - r_S/R)^{-1} & 0 & 0 \\
0 & 0 & R^2 & 0 \\
-R^2 \sin^2 \theta \Omega / c & 0 & 0 & R^2 \sin^2 \theta / (1 - u^2 / c^2)
\end{bmatrix} \tag{2.18}
\]
Angles between any four vectors \( l^a \) and \( s^b \) are measured as follows

\[
\cos \xi = \frac{h_{ab} l^a s^b}{\sqrt{h_{ab} l^a l^b \sqrt{h_{ab} s^a s^b}}} \tag{2.19}
\]

So that the angle between a light ray and the normal to the spherical surface, \( n = \left( 0, \sqrt{1 - r_s/R}, 0, 0 \right) \), is

\[
\cos \xi = \frac{\sqrt{1 - u^2/c^2}}{1 - \hat{b} \Omega/c} \sqrt{1 - \hat{b}^2} \tag{2.20}
\]

To find the area of each patch we need the spatial, projected, two dimensional metric of the surface (de Felice & Clarke, 1992).

The generators for this surface are \( \hat{\theta} \) and \( \hat{\varphi} \). The metric element \( h_{ab} (a, b = \theta, \varphi) \) is given by

\[
(2) \quad h_{ab} = \left( \hat{x}_a \cdot \hat{x}_b \right) = h_{ij} \hat{x}_a^i \hat{x}_b^j \tag{2.21}
\]

The infinitesimal area element is then given by the square root of the determinant of such metric multiplied by \( d\theta d\varphi \). So:

\[
(2) \quad h_{ab} = \begin{bmatrix} R^2 & 0 \\ 0 & \frac{R^2 \sin^2 \theta}{1 - u^2/c^2} \end{bmatrix} \tag{2.22}
\]

\[
(2) \quad h = R^4 \sin^2 \theta / \left( 1 - u^2/c^2 \right) \tag{2.23}
\]

And finally the finite area of any patch is:

\[
\Delta A_e = \frac{R^2}{\sqrt{1 - u^2/c^2}} \Delta \cos \theta \Delta \varphi \tag{2.24}
\]

In the code, the term \( \Delta \cos \theta \Delta \varphi \) is constant by construction for all the patches.

**Lightcurves**

First of all, we need to know the relation between the frequency of the photons at the surface and at infinity. The frequency is related to the energy by \( h\nu = E \), and \( E \) is the time component of the light four velocity \( v^i \), i.e. it is the component parallel to the observer four velocity. On the NS this is given by

\[
h\nu_e = -u_i v^i = E_\infty \frac{1 - \hat{b} \Omega/c}{\sqrt{1 - r_s/R} \sqrt{1 - u^2/c^2}} \tag{2.25}
\]
The frequency measured by the static observer at infinity, four velocity $(1, 0, 0, 0)$, is simply:

$$h\nu_\infty = E_\infty$$  \(2.26\)

so that the redshift between the two systems is

$$1 + z = \frac{\nu_e}{\nu_\infty} = \frac{1 - b\Omega/c}{\sqrt{1 - r_s/R}\sqrt{1 - u^2/c^2}}$$  \(2.27\)

To construct the specific flux at infinity we exploit the fact that the number density of photons is a conserved quantity, so that

$$\frac{I(\nu_e)}{\nu_e^3} = \frac{I_\infty(\nu_\infty)}{\nu_\infty^3}$$  \(2.28\)

Where $I(\nu_e)$ is the intensity emitted at the surface and $I(\nu_\infty)$ that observed at infinity. Moreover, we use (Pechenick et al., 1983)

$$\Delta A_\infty \Delta \Omega_\infty = \cos \xi \Delta A_e \Delta \Omega_e,$$  \(2.29\)

where the $\Delta \Omega$s are solid angles and we consider both $\Delta \Omega_e$ and $\Delta A_\infty$ as constants.

Combining Eqs. (2.20), (2.24) and (2.27) we include the correction factor $(1 - b\Omega/c)^{-1}$, which accounts for the effects of the rotation of the star during the finite extent of the time bins used to sample the light curve, as explained by Cadeau et al. (2007). The specific flux $F(\nu_\infty)$ as measured at infinity is

$$F(\nu_\infty) \propto \frac{I(\nu_e)}{(1 + z)^3} \frac{R^2 \cos \xi}{\sqrt{1 - u^2/c^2} \sqrt{1 - r_s/R} (1 - b\Omega/c)}.$$  \(2.30\)

The bolometric flux $F$ is derived integrating Eq. (2.30) by $d\nu_\infty = d\nu_e/(1 + z)$:

$$F \propto \frac{I_e}{(1 + z)^4} \frac{R^2 \cos \xi}{\sqrt{1 - u^2/c^2} \sqrt{1 - r_s/R} (1 - b\Omega/c)},$$  \(2.31\)

where $I_e$ is the integrated intensity.

At this point all the ingredients are ready to calculate the lightcurve. The rotation period is divided into time steps (equal in number to the chosen $\varphi$ resolution). For each time step the surface emission pattern is calculated. For example, in the case of a single spot, whose colatitude $\alpha$ and angular half-size $\delta$ are defined as parameters, the $[\theta, \varphi]$ position of the spot with respect to the line of sight changes in the $\varphi$ direction by $\Omega \Delta t_e$ at each time step. The contribution to Eqs. (2.30) and (2.31) of each patch is then calculated
according to the chosen function for the intensity $I(\nu_e)$. For example, for a blackbody:

$$I(\nu_e) = \frac{\nu_e^3}{e^{\frac{\nu_e}{kT_e}} - 1}$$  \hspace{1cm} (2.32)$$

$$I_e = T_e^4$$  \hspace{1cm} (2.33)$$

Finally, these contributions are distributed among the receiver’s time bins. For the shortest–path ray, each rotation time step is mapped to a full receiver time bin, but for all the other rays each rotation time step contribution has to be subdivided among different time bins according to the lag in Eq. (2.6).

As last features, detector response curves can be added if desired, they have to be specified as functions of $\nu_\infty$, and it is easy to incorporate to the code additional emission spectra and surface patterns thanks to its modular structure.

**Code testing**

Finally, in Section 2.7 we mentioned we tested our results against the work of Lamb et al. (2009). We did not have access to the raw data, but compared as accurately as we could against the plots in fig. 1 of their paper. Figure 2.6 shows our code values for the parameters of their figs. 1 a,b,d,e. We plot contours of the amplitudes of the fourier components into which the theoretical pulse profiles are decomposed, projected on the plane ($\alpha$,i) of the colatitude of the spot and the inclination of the observer. There are two cases: either one spot, where $A_1$ is the amplitude of the fundamental and $A_2$ is the first overtone, or two antipodal spots, where $A_1$ is the first overtone that would be detected and actually corresponds to the frequency of the star, while $A_1/2$ is the detected fundamental at half the spin frequency. For the amplitudes we follow the definition given in Lamb et al. (2009):

$$f(t) = A_0 + \sum_{l=1}^{\infty} A_l \cos [2\pi l \Omega t - \varphi_l]$$  \hspace{1cm} (2.34)$$

where $f(t)$ is the pulse profile as a function of time, $A_l$ the amplitude of the $l$th overtone, $\varphi_l$ its phase and $\Omega$ is the spin frequency of the star. Our curves are not smooth since we did not sample the parameter phase space as finely as Lamb et al. Keeping in mind also the difference in scales between the two sets of figures, careful analysis of the two sets shows very good agreement of the two codes.
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Figure 2.6: The contours of the powers of the fundamental and the first harmonic of the predicted pulse profiles in the case of 1 spot (Top) and 2 spots (bottom) with angular radius $\delta = 25^{\circ}$, for a star of $M = 1.4 \, M_{\odot}$ and $R = 5 \, M$, spinning at 400 Hz. The horizontal axis is the angular position of the spot $\alpha$ (the second one, if present, is antipodal), while the vertical one is the inclination of the observer $i$. 
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