

Online Appendix:

Managing bubbles in experimental asset markets with monetary policy

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APPENDIX A: ASSET PRICING MODEL WITH VARIABLE INTEREST RATE

A.1 Derivation of the market equilibrium price

The experiment is based on an asset pricing model with heterogeneous beliefs, as in Campbell, Lo, and MacKinlay (1997) and Brock and Hommes (1998). The asset market consists of I traders. At the beginning of each period, trader i can choose to invest in a risk-free asset or a risky asset. The risk-free asset (e.g. a savings account) pays a variable interest rate r_t over period t , which is known at the time of the investment decision.¹ The infinitely lived risky asset has a price p_t and pays an uncertain dividend y_t that is independently and identically distributed with mean \bar{y} . The number of shares z_{it} purchased by trader i in period t cost $(1 + r_t)p_t$ and yield a payoff $p_{t+1} + y_{t+1}$. The

¹This is the only extra assumption that is necessary to derive the market price when the interest rate is variable instead of fixed. It implies the interest rate can be taken out of the expectations operator. This is a standard assumption in macroeconomics (see, e.g., Bernanke, Gertler, and Gilchrist 1999). The payoff of the risk-free asset between period t and period $t + 1$ can be either defined as r_t or r_{t+1} , which is set by the central bank at the beginning of period t . These two equivalent forms of notation are both used in the literature.

realized wealth of the trader at the beginning of period $t + 1$ is thus given by

$$W_{i,t+1} = R_t W_{i,t} + (p_{t+1} + y_{t+1} - R_t p_t) z_{it}, \quad (\text{A1})$$

where $R_t = 1 + r_t$ is the gross rate of return of the risk-free asset in period t .

Traders differ in their beliefs about the conditional mean of the evolution of wealth, $E_{it}(W_{i,t+1})$. It is assumed that traders believe that the conditional variance of excess returns is constant and equal to σ^2 . Traders are myopic mean-variance optimizers, so the demand for shares z_{it} corresponds to the solution of

$$\max_{z_{it}} \left\{ E_{it}(W_{i,t+1}) - \frac{1}{2} a V_{it}(W_{i,t+1}) \right\} = \max_{z_{it}} \left\{ z_{it} E_{it}(p_{t+1} + y_{t+1} - R_t p_t) - \frac{1}{2} a \sigma^2 z_{it}^2 \right\},$$

where a measures the degree of risk aversion. Assume that the outside supply of shares z^s is zero. The market equilibrium condition then becomes

$$\sum_{i=1}^I z_{it} = \frac{1}{a \sigma^2} \sum_{i=1}^I E_{it}(p_{t+1} + y_{t+1} - R_t p_t) = z^s = 0. \quad (\text{A2})$$

Using that $E_{it}(y_{t+1}) = \bar{y}$ for all i and all t , the market equilibrium price is given by

$$p_t = \frac{1}{1 + r_t} \left[\frac{1}{I} \sum_{i=1}^I p_{i,t+1}^e + \bar{y} \right], \quad (\text{A3})$$

where $E_{it}(p_{t+1}) = p_{i,t+1}^e$ denotes the prediction by trader i in period t for the price in period $t + 1$.

A.2 Fundamental value of the risky asset

The fundamental value of the risky asset is the discounted sum of all future dividend payments. With a constant interest rate $r_t = r$, the fundamental is simply $p^f = \bar{y}/r$. This simplification can no longer be made when the interest rate is variable. To find the fundamental value, we iterate the market equilibrium price (equation (A3)) K steps

forward and apply the law of iterated expectations:

$$p_t = E_{it} \left[\prod_{k=0}^K \frac{1}{1+r_{t+k}} \left(\frac{1}{I} \sum_{i=1}^I E_{it}[p_{t+k+1}] \right) \right] + E_{it} \left[\sum_{j=0}^K \prod_{k=0}^j \frac{1}{1+r_{t+k}} \bar{y} \right]. \quad (\text{A4})$$

The transversality condition imposes that the first term in equation (A4) goes to zero, so that the fundamental price is given by

$$p_{it}^f = E_{it} \left[\sum_{j=0}^{\infty} \prod_{k=0}^j \frac{1}{1+r_{t+k}} \bar{y} \right]. \quad (\text{A5})$$

With a time-varying interest rate, the fundamental value is time-varying and depends on individual expectations of future interest rates.

A.3 Interest rate rule and zero lower bound

The interest rate is set according to a Taylor-type rule:

$$r_t = r^* + \phi \left(\frac{p_{t-1} - p^*}{p^*} \right), \quad (\text{A6})$$

where the target interest rate is $r^* = 0.05$ and the target price is $p^* = 60$, in line with the asset pricing model with a fixed interest rate.

With the interest rate rule in equation (A6), the interest rate becomes negative if $p_{t-1} < -(3/\phi) + 60$. This is only problematic for $\phi > 0.05$, since the condition is never satisfied for smaller values of ϕ . Hence, a zero lower bound (ZLB) on the interest rate must be implemented if $\phi > 0.05$. In our experiment, we use $\phi = 0.1$, so that means that the ZLB is reached when the price drops below 30. However, this does not have a large effect on the dynamics of the system. When the price is lower than 30, the interest rate without implementing the ZLB would be negative but relatively close to zero, so that the difference in the realized market price with or without implementing the ZLB is usually very small. In simulations with homogeneous expectations or a heuristics switching model, the difference in prices with or without a ZLB is barely visible and the dynamics are virtually the same. For our other parameter value, $\phi = 0.001$, the ZLB

does not play a role. Hence, to ease the derivations in this appendix, the interest rate rule in equation (A6) is taken without the ZLB.

A.4 Rational expectations equilibrium

Substituting the interest rate rule (equation (A6)) into the market equilibrium price (equation (A3)), we obtain

$$p_t = \frac{60}{63 + \phi(p_{t-1} - 60)} \left[\frac{1}{I} \sum_{i=1}^I p_{i,t+1}^e + 3 \right]. \quad (\text{A7})$$

It is easy to verify that $p^* = 60$ and $r^* = 0.05$ form a steady state equilibrium, just as in the asset pricing model with a fixed interest rate. This is the only feasible steady state of the model.²

Any rational expectations (RE) solution must satisfy $p_t = p_{it}^e$, for all traders i and all periods t . Replacing the average price prediction in equation (A7) with p_{t+1} and rewriting the system in deviations from the steady state price, with $x_t = p_t - 60$, we obtain a first-order 2-D system:

$$\begin{aligned} x_{t+1} &= 1.05x_t + \frac{\phi}{60}x_t y_t + \phi y_t, \\ y_{t+1} &= x_t. \end{aligned} \quad (\text{A8})$$

This system describes all RE or perfect foresight solutions. The steady state $(x^*, y^*) = (0, 0)$ of this system is a saddle point for $0 \leq \phi < 2.05$ and an unstable node for $\phi > 2.05$. For the values of ϕ we consider in the experiment, $0 \leq \phi \leq 0.1$, the unique steady state is thus saddle-path stable.

Given two initial values $x_1, y_1 > 0$, the price in deviation from the steady state keeps growing. In the absence of monetary policy ($\phi = 0$), this “rational bubble” has a growth rate of $1 + r^* = 1.05$. When monetary policy is implemented ($\phi > 0$), the growth rate gets even larger and also increases over time, since the interest rate keeps increasing as

²Another steady state is $p = -(3/\phi)$ and $r = -\phi$, but this is not feasible since $\phi \geq 0$ and prices cannot be negative.

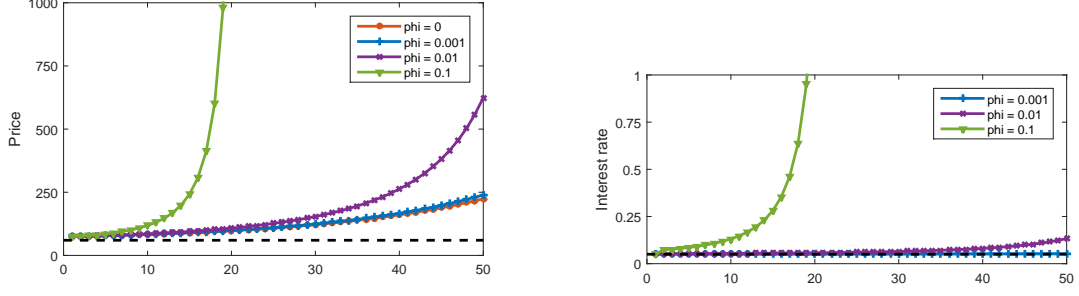


Figure A1: Simulations of rational bubbles

Notes: Initialization of the simulations: $x_1 = 15.75$ and $y_1 = 15$. The dashed lines indicate the steady state of $p^* = 60$ and $r^* = 0.05$.

well. The rational bubbles are illustrated in Figure A1.

A.5 Dynamics under homogenous expectations

To get an intuition for the dynamics of the asset pricing model, we investigate the stability of the system under homogeneous expectations, i.e. when $p_{i,t+1}^e = p_{t+1}^e$ for all i .

Adaptive expectations are given by

$$p_{t+1}^e = wp_{t-1} + (1-w)p_t^e = p_t^e + w(p_{t-1} - p_t^e), \quad (\text{A9})$$

with weight $w \in [0, 1]$. Naive expectations are a special case of this rule, obtained for $w = 1$. For $0 < w \leq 1$, the steady state is a stable node for $\phi < (w - 42)/(20w - 40)$. For $w = 0$, it is a saddle point for $0 \leq \phi < 1.05$. So for our parameter values, $0 \leq \phi \leq 0.1$, prices converge monotonically to the steady state (unless $w = 0$). This is illustrated in Figure A2a, which shows prices for adaptive expectations with $w = 0.65$.

Under trend-following expectations, we have

$$p_{t+1}^e = p_{t-1} + \gamma(p_{t-1} - p_{t-2}), \quad (\text{A10})$$

with extrapolation coefficient $\gamma > 0$. The dynamics of this system change for different combinations of ϕ and γ . For a weak trend-following rule with $\gamma = 0.4$ and $0 \leq \phi \leq 0.1$, the eigenvalues are real and inside the unit circle. The steady state is a stable node and there is monotonic convergence of the price, as illustrated in Figure A2b. For a

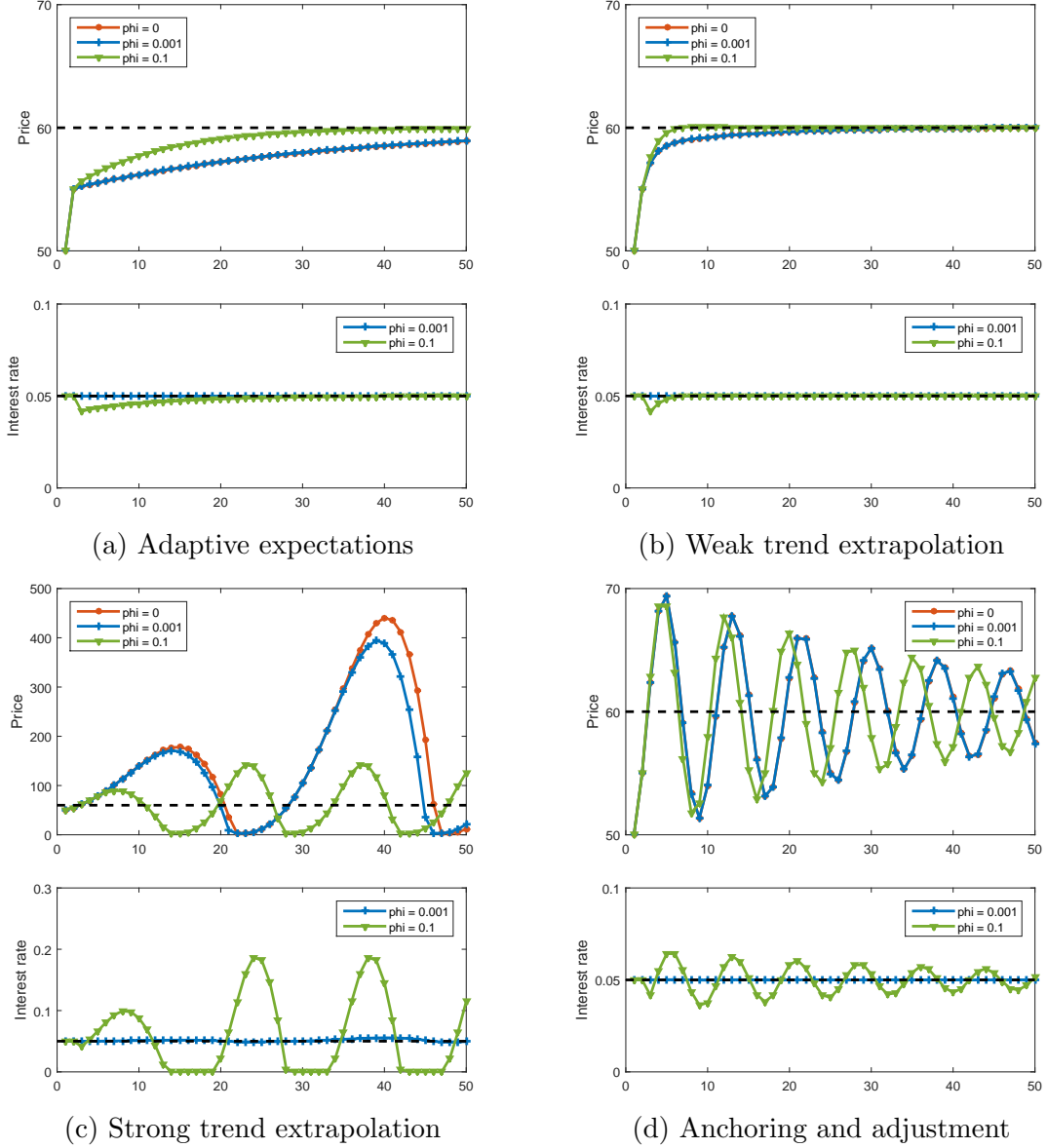


Figure A2: Simulations with homogeneous expectations

Notes: Initialization of the simulations: $p_1 = 50$, $p_2 = 55$ and $p_3^e = 55$. The simulations implement the ZLB. The dashed lines indicate the steady state of $p^* = 60$ and $r^* = 0.05$. Note that the scale of the vertical axis differs in the four figures.

strong trend-following rule with $\gamma = 1.3$ and $0 \leq \phi \leq 0.1$, the eigenvalues are complex and outside the unit circle, so the steady state is an unstable focus. The simulations (including ZLB) in Figure A2c shows that prices and interest rates oscillate and converge to cycles.

Under anchoring and adjustment, expectations are given by

$$p_{t+1}^e = 0.5(p^* + p_{t-1}) + (p_{t-1} - p_{t-2}). \quad (\text{A11})$$

Again, the dynamics depend on the value of ϕ . For $0 \leq \phi \leq 0.1$, the eigenvalues are complex and inside the unit circle, so the steady state is a stable focus. Convergence is oscillatory, as can be seen from the simulations in Figure A2d.

References

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- Brock, William, and Cars Hommes (1998). “Heterogeneous beliefs and routes to chaos in a simple asset pricing model”. *Journal of Economic Dynamics and Control*, 22(4), pp. 1235–1274.
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APPENDIX B: INSTRUCTIONS EXPERIMENT

The instructions below are used for the Weak Rule, Strong Rule and Sample Average treatments.

For the No Information treatment, “a known interest rate” is replaced by “an interest rate that is known to the pension fund” and the interest rate is not mentioned in the information about the forecasting task of the financial advisor. Furthermore, the second sentence of the information about the investment strategies of the pension funds is changed into “The bank account of the risk-free investment pays a target interest rate of 5% each time period” and the following three sentences about the interest rate are removed.

For the Communication treatment, a sentence is added after the fourth sentence of the information about the investment strategies of the pension funds: “The policy of the central bank is to raise the interest rate above 5% when it considers the stock price to be too high, and to cut the interest rate below 5% when it considers the stock price to be too low.”

INSTRUCTIONS

General information

You are a **financial advisor** to a pension fund that wants to optimally invest a large amount of money. The pension fund has two investment options: a risk-free investment and a risky investment. The risk-free investment is putting all money on a bank account paying a known interest rate. The alternative risky investment is an investment in the stock market with uncertain return. In each time period the pension fund has to decide which fraction of its money to put on the bank account and which fraction of its money to spend on buying stocks. In order to make an optimal investment decision the pension fund needs an accurate prediction of the price of the stock. As their financial advisor, you have to predict the stock market price during 51 subsequent time periods. Your

earnings during the experiment depend upon your forecasting accuracy. The smaller your forecasting errors in each period, the higher your total earnings.

Forecasting task of the financial advisor

The only task of the financial advisors in this experiment is to forecast the stock market index in each time period as accurate as possible. The stock price has to be predicted two time periods ahead. At the beginning of the experiment, you have to predict the stock price in the first two periods, given the risk-free interest rate. It is very likely that the stock price will be between 0 and 100 in the first two periods. After all participants have given their predictions for the first two periods, the stock market price for the first period will be revealed and, based upon your forecasting error, your earnings for period 1 will be given. After that you have to give your prediction for the stock market index in the third period, given the risk-free interest rate. After all participants have given their predictions for period 3, the stock market index in the second period will be revealed and, based upon your forecasting error, your earnings for period 2 will be given. This process continues for 51 time periods.

The available information in period t for forecasting the stock price for period $t + 1$ consists of

- the current interest rate for period t and all past interest rates,
- all past prices up to period $t - 1$, and
- all past predictions up to period t , and
- total earnings up to period $t - 1$.

Information about the stock market

The stock market price is determined by equilibrium between demand and supply of stocks. The stock market price in period t will be that price for which aggregate demand equals supply. The supply of stocks is fixed during the experiment. The demand for

stocks is determined by the aggregate demand of a number of large pension funds active. Each pension fund is advised by a participant of the experiment.

Information about the investment strategies of the pension funds

The precise investment strategy of the pension fund that you are advising and the investment strategies of the other pension funds are unknown. The bank account of the risk-free investment pays a known interest rate each time period. The interest rate is initially set at 5% per period, but it is variable. This means that it is possible, but not certain, that the interest rate will change in later periods. The current interest rate will be given in each period. The holder of the stock receives a dividend payment in each time period. These dividend payments are uncertain however and vary over time. Economic experts of the pension funds have computed that the average dividend payments are 3 euro per time period. The return of the stock market per time period is uncertain and depends upon (unknown) dividend payments as well as upon price changes of the stock. As the financial advisor of a pension fund you are **not** asked to forecast dividends, but you are only asked to forecast the price of the stock in each time period. Based upon your stock market price forecast, your pension fund will make an optimal investment decision. The higher your price forecast is, the larger will be the fraction of money invested by your pension fund in the stock market, so the larger will be their demand for stocks.

Earnings

Your earnings depend only on the accuracy of your predictions. The earnings shown on the computer screen will be in points. If your prediction is p_t^e and the price turns out to be p_t in period t , your earnings are determined by the following equation:

$$earnings_t = \max\left\{1300 - \frac{1300}{49}(p_t^e - p_t)^2, 0\right\}.$$

The maximum possible points you can earn for each period (if you make no prediction error) is 1300, and the larger your prediction error is, the fewer points you earn. You will

earn 0 points if your prediction error is larger than 7. The earnings table below shows the number of points you earn for different prediction errors. At the end of the experiment, your total earnings in points will be converted into euros, at an exchange rate of **0.5 euro for 1300 points**.

Earnings table									
1300 points equal 0.5 euro									
error	points	error	points	error	points	error	points	error	points
0.1	1300	1.5	1240	2.9	1077	4.3	809	5.7	438
0.15	1299	1.55	1236	2.95	1069	4.35	798	5.75	423
0.2	1299	1.6	1232	3	1061	4.4	786	5.8	408
0.25	1298	1.65	1228	3.05	1053	4.45	775	5.85	392
0.3	1298	1.7	1223	3.1	1045	4.5	763	5.9	376
0.35	1297	1.75	1219	3.15	1037	4.55	751	5.95	361
0.4	1296	1.8	1214	3.2	1028	4.6	739	6	345
0.45	1295	1.85	1209	3.25	1020	4.65	726	6.05	329
0.5	1293	1.9	1204	3.3	1011	4.7	714	6.1	313
0.55	1292	1.95	1199	3.35	1002	4.75	701	6.15	297
0.6	1290	2	1194	3.4	993	4.8	689	6.2	280
0.65	1289	2.05	1189	3.45	984	4.85	676	6.25	264
0.7	1287	2.1	1183	3.5	975	4.9	663	6.3	247
0.75	1285	2.15	1177	3.55	966	4.95	650	6.35	230
0.8	1283	2.2	1172	3.6	956	5	637	6.4	213
0.85	1281	2.25	1166	3.65	947	5.05	623	6.45	196
0.9	1279	2.3	1160	3.7	937	5.1	610	6.5	179
0.95	1276	2.35	1153	3.75	927	5.15	596	6.55	162
1	1273	2.4	1147	3.8	917	5.2	583	6.6	144
1.05	1271	2.45	1141	3.85	907	5.25	569	6.65	127
1.1	1268	2.5	1134	3.9	896	5.3	555	6.7	109
1.15	1265	2.55	1127	3.95	886	5.35	541	6.75	91
1.2	1262	2.6	1121	4	876	5.4	526	6.8	73
1.25	1259	2.65	1114	4.05	865	5.45	512	6.85	55
1.3	1255	2.7	1107	4.1	854	5.5	497	6.9	37
1.35	1252	2.75	1099	4.15	843	5.55	483	6.95	19
1.4	1248	2.8	1092	4.2	832	5.6	468	error ≥ 7	0
1.45	1244	2.85	1085	4.25	821	5.65	453		

CONTROL QUESTIONS

- Suppose in one period, your prediction for the market price is 45.5, and the market price turns out to be 45.75. How many points do you earn for the forecasting task in this period (round it to the nearest integer)? (*Answer: 1298*)
- Suppose a financial advisor predicts that the stock price goes up in period 10, and goes down in period 20, and the pension fund acts according to this prediction. In which period does the pension fund increase its demand for stocks, period 9 or period 19? (*Answer: period 9*)

- In which of the following cases will the stock price go up?
 - A. When advisors think the price will go down and the pension funds buy very little.
 - B. When advisors think the price will go up and the pension funds buy a lot.

(Answer: B)
- *NOT for treatment No Information:*

Which of the following statements is true?

 - A. The current interest rate is known, so the bank account is always a risk-free investment.
 - B. The interest rate is variable, so the bank account and the stock are both risky investments.

(Answer: A)
- *ONLY for treatment Communication:*

Suppose the current interest rate is 10%. Does this mean that the central bank considers the stock price to be too high or too low? *(Answer: Too high)*
- Suppose by the end of the experiment you have earned 26,000 points, how much is this worth in euros? *(Answer: 10 euro)*

APPENDIX C: EXPERIMENTAL RESULTS PER MARKET

Table C1: Summary statistics

	Prices				RAD	RD	Interest rates (%)			
	mean	st.dev.	min	max			mean	st.dev.	min	max
Weak Rule										
Average	184.11	173.32	22.09	607.74	2.29	2.07	5.20	0.29	4.94	5.91
Group 1	181.78	205.97	7.27	926.88	2.36	2.03	5.20	0.34	4.91	6.44
Group 2	176.87	222.75	19.58	898.36	2.11	1.95	5.19	0.37	4.93	6.40
Group 3	61.45	22.55	12.85	94.80	0.31	0.02	5.00	0.04	4.92	5.06
Group 4	337.74	288.66	35.86	930.62	4.69	4.63	5.45	0.48	4.96	6.45
Group 5	264.05	317.40	12.23	942.35	3.79	3.40	5.33	0.53	4.92	6.47
Group 6	340.72	310.28	26.19	934.73	4.78	4.68	5.46	0.52	4.94	6.46
Group 7	53.43	2.53	46.83	57.30	0.11	-0.11	4.99	0.00	4.98	5.00
Group 8	56.87	16.39	15.93	76.91	0.20	-0.05	4.99	0.03	4.93	5.03
Strong Rule										
Average	65.07	29.13	23.04	140.64	0.37	0.08	5.99	4.61	0.55	18.44
Group 1	69.82	38.19	22.30	164.8	0.47	0.16	6.72	6.18	0	22.47
Group 2	64.79	25.87	26.92	107.39	0.38	0.08	5.81	4.23	0	12.90
Group 3	61.12	24.89	17.63	112.52	0.34	0.02	5.31	3.90	0	13.75
Group 4	62.83	27.81	6.70	141.48	0.28	0.05	5.74	4.12	0	18.58
Group 5	81.12	42.82	25.17	169.08	0.64	0.35	8.49	7.04	0	23.18
Group 6	57.64	21.16	17.38	98.87	0.28	-0.04	4.76	3.24	0	11.48
Group 7	62.98	50.44	11.66	262.92	0.57	0.05	6.02	7.86	0	38.82
Group 8	60.22	1.86	56.57	68.02	0.02	0.00	5.04	0.31	4.43	6.34
No Information										
Average	87.62	42.50	28.12	201.07	0.62	0.46	9.55	7.00	0.33	28.51
Group 1	62.58	21.76	40.45	166.42	0.21	0.04	5.42	3.59	1.74	22.74
Group 2	63.21	12.33	32.06	85.85	0.16	0.05	5.52	2.04	0.34	9.31
Group 3	61.95	13.94	29.26	94.03	0.16	0.03	5.32	2.30	0	10.67
Group 4	95.09	36.92	29.43	196.75	0.65	0.58	10.74	6.14	0	27.79
Group 5	73.84	54.68	24.08	268.94	0.55	0.23	7.33	8.97	0	39.82
Group 6	85.35	36.21	23.98	184.41	0.54	0.42	9.16	5.97	0	25.73
Group 7	176.77	122.88	33.47	440.50	2.04	1.95	24.08	20.46	0.58	68.42
Group 8	82.19	41.31	12.23	171.69	0.60	0.37	8.82	6.55	0	23.61
Communication										
Average	62.44	16.59	29.47	103.07	0.23	0.04	5.48	2.63	0.75	12.18
Group 1	60.99	6.76	36.19	70.50	0.08	0.02	5.16	1.12	1.03	6.75
Group 2	66.28	29.14	24.20	193.99	0.36	0.10	6.09	4.72	0	27.33
Group 3	62.25	14.31	34.66	89.59	0.18	0.04	5.37	2.36	0.78	9.93
Group 4	59.21	15.28	20.79	85.34	0.21	-0.01	4.90	2.45	0	9.22
Group 5	59.43	2.22	46.35	60.77	0.01	-0.01	4.91	0.37	2.72	5.13
Group 6	52.19	28.63	9.16	103.69	0.42	-0.13	4.29	4.01	0	12.28
Group 7	81.79	29.63	25.66	146.98	0.47	0.36	8.57	4.89	0	19.50
Group 8	57.41	6.79	38.72	73.73	0.09	-0.04	4.58	1.12	1.45	7.29
Sample Average										
Average	81.67	60.91	18.04	249.68	0.76	0.36	7.98	6.58	0.80	26.60
Group 1	110.4	93.04	8.56	383.03	1.17	0.84	9.51	8.85	0	33.46
Group 2	93.89	107.43	7.10	442.09	1.00	0.56	10.63	12.93	0	49.19
Group 3	158.2	173.94	6.00	583.00	2.07	1.64	11.85	13.06	0	50.70
Group 4	51.29	2.64	43.29	56.26	0.15	-0.15	5.17	0.47	3.74	5.97
Group 5	72.96	38.64	18.59	154.97	0.55	0.22	7.34	5.27	0	18.06
Group 6	76.25	45.75	11.27	236.81	0.58	0.27	7.43	6.26	0	31.63
Group 7	43.96	19.94	15.16	85.97	0.37	-0.27	6.61	4.64	0	16.65
Group 8	46.37	5.87	34.32	55.32	0.23	-0.23	5.29	1.19	2.63	7.16

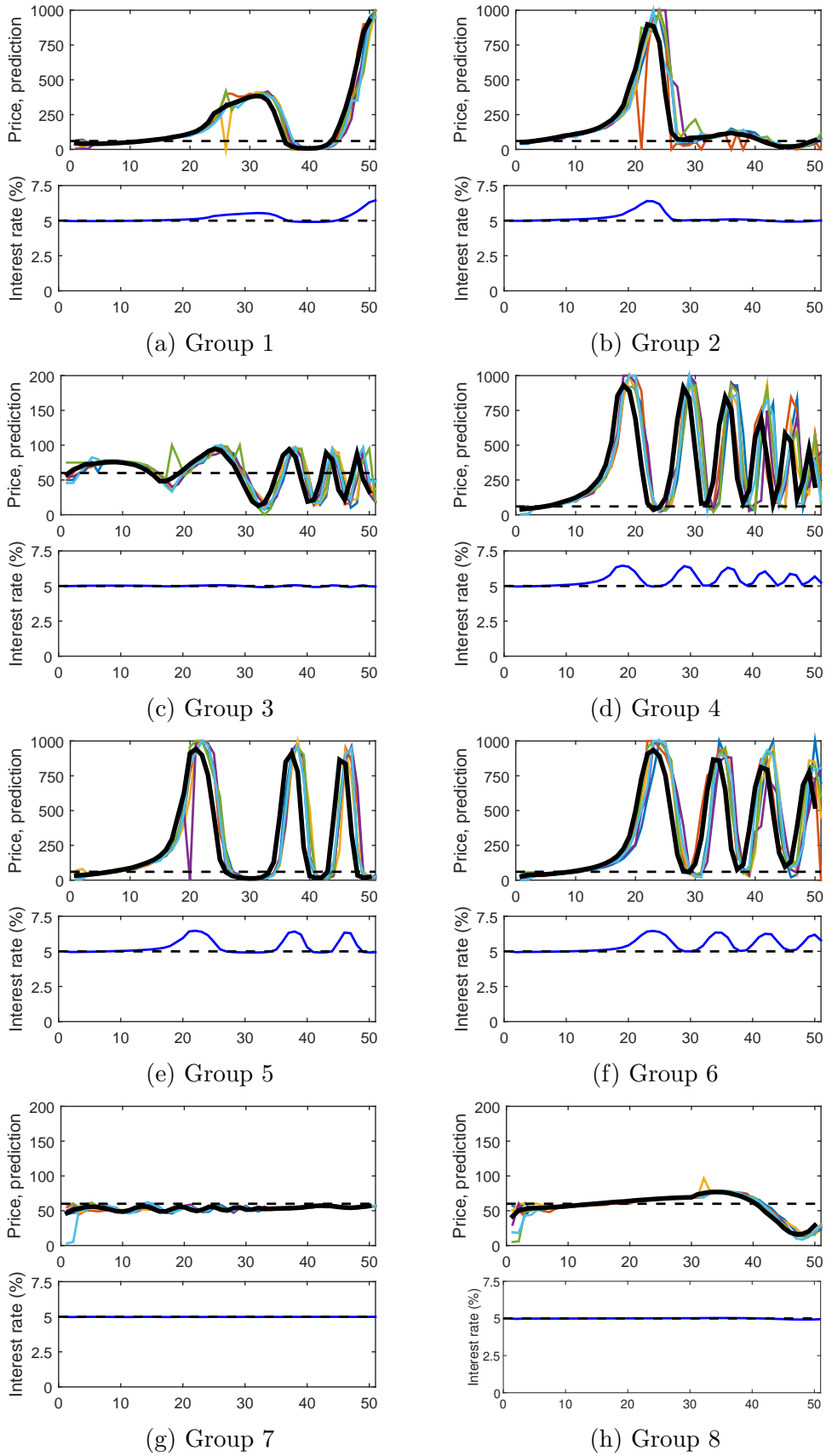


Figure C1: Market prices, predictions and interest rates in treatment Weak Rule
Notes: The dashed lines indicate the steady state price of 60 and the steady state interest rate of 5%. Note that the scale of the vertical axis may differ per group.

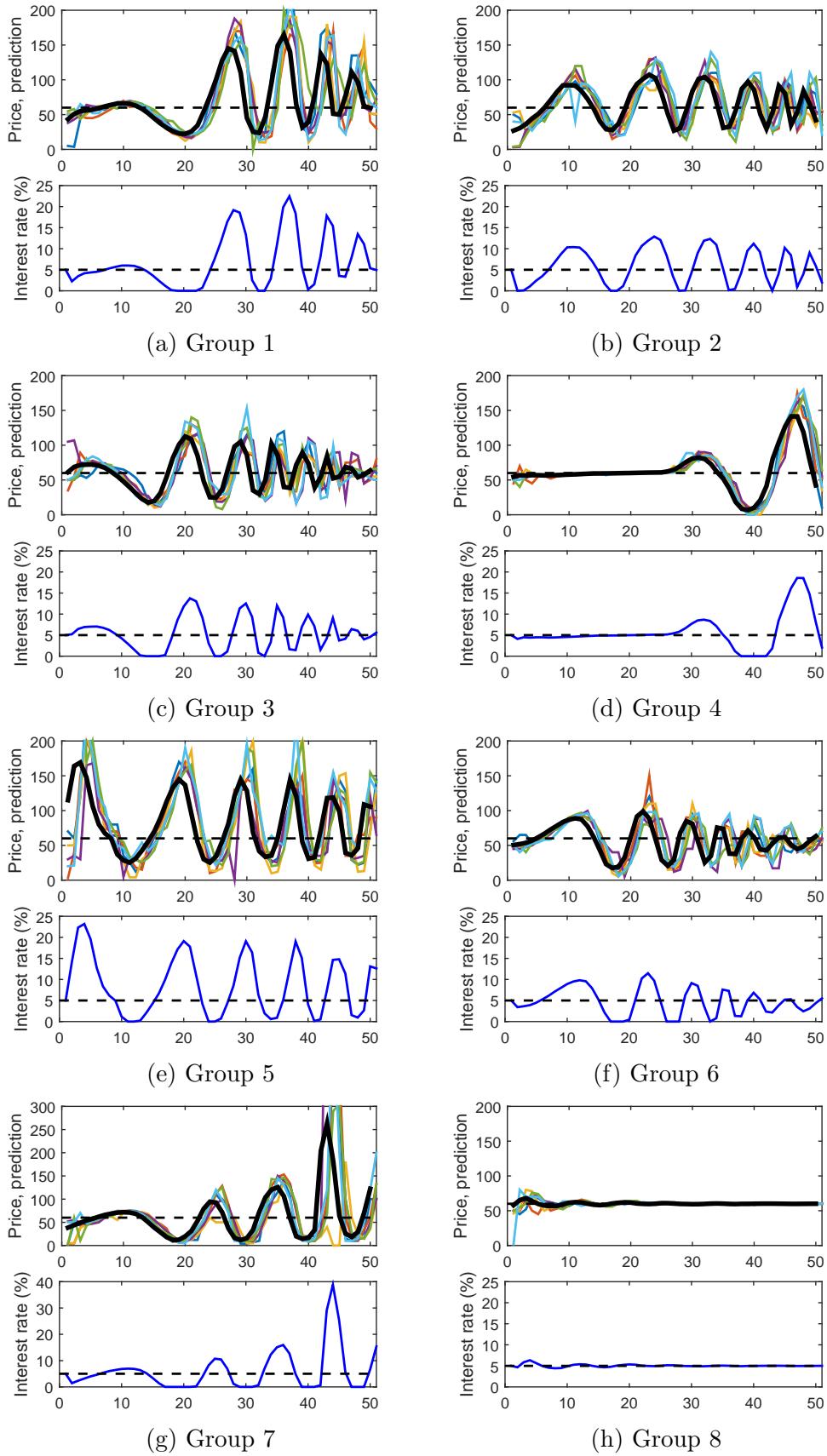


Figure C2: Market prices, predictions and interest rates in treatment Strong Rule
Notes: The dashed lines indicate the steady state price of 60 and the steady state interest rate of 5%. Note that the scale of the vertical axis may differ per group.

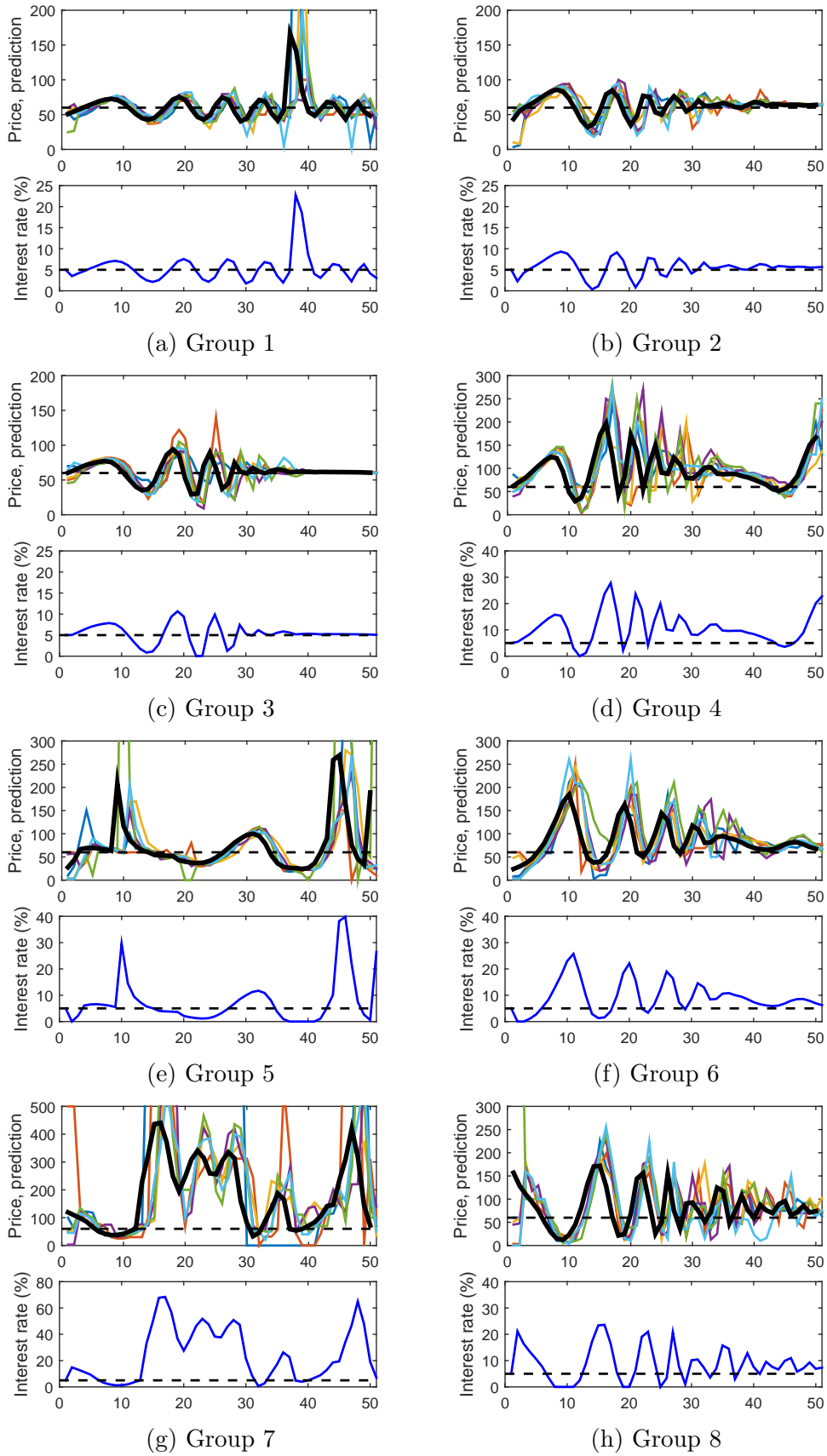


Figure C3: Market prices, predictions and interest rates in treatment No Information
Notes: The dashed lines indicate the steady state price of 60 and the steady state interest rate of 5%. Note that the scale of the vertical axis may differ per group.

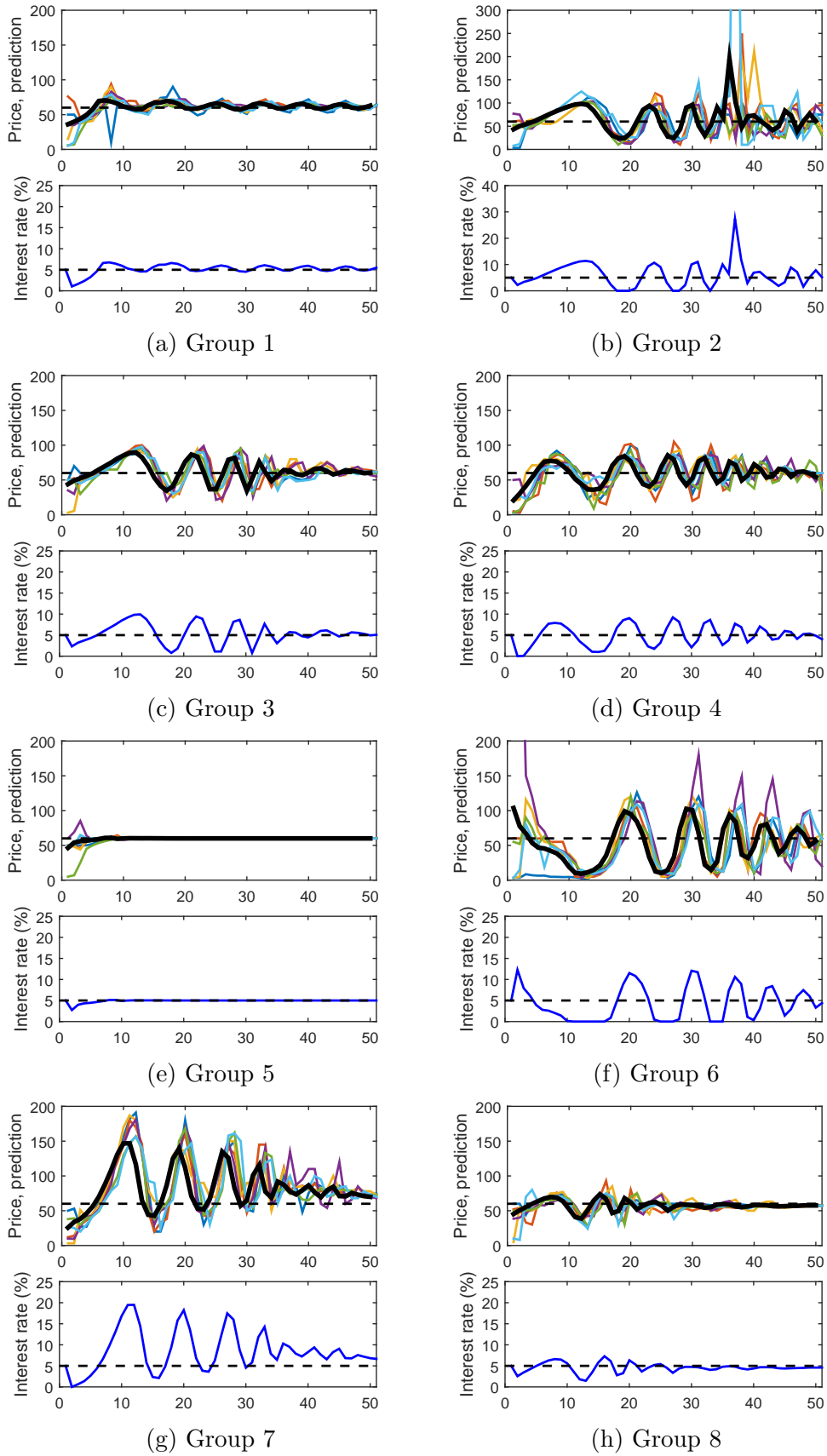


Figure C4: Market prices, predictions and interest rates in treatment Communication
Notes: The dashed lines indicate the steady state price of 60 and the steady state interest rate of 5%. Note that the scale of the vertical axis may differ per group.

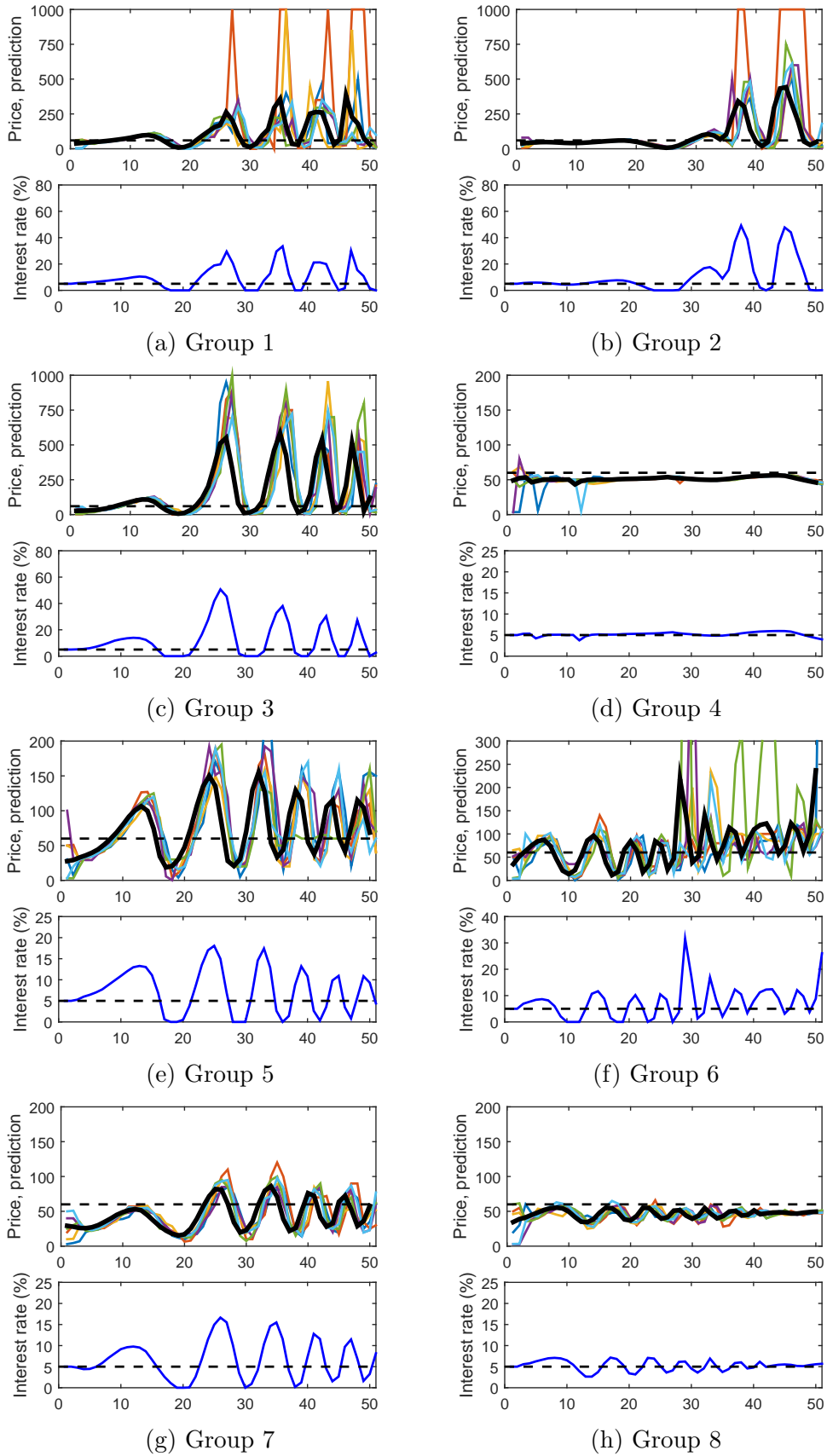


Figure C5: Market prices, predictions and interest rates in treatment Sample Average

Notes: The dashed lines indicate the steady state price of 60 and the steady state interest rate of 5%. Note that the scale of the vertical axis may differ per group.