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Market-Induced Rationalization and Welfare Enhancing Cartels

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Abstract

We show that incomplete cartels in quantity-setting oligopolies may increase welfare, without any efficiencies or synergies being internalized by cartel formation. The main intuition is that the cartel has an incentive to contract output and that the firms outside the cartel react to this by expanding output. If the outsiders are more efficient than the cartel firms average production costs go down. Even for relatively moderate differences in efficiency total welfare may increase due to this market-induced rationalization.

JEL classification: L10, L41, D43

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1 Introduction

It is generally accepted that collusion decreases social welfare. Price fixing agreements increase industry profits at the expense of consumer welfare and the resulting reduction in output creates deadweight losses. Some examples of 'efficient cartels' have been explored in the literature. Collusion may, for example, be the appropriate response to the non-existence of competitive equilibrium when there are large fixed costs or increasing returns to scale (Sjostrom, 1989). Moreover, it has been suggested that cartels may lead to increased output because: firms invest in capacity to increase their share in the cartel (Matsui, 1989); firms share information (Stennek, 2003); or the cartel acts as a Stackelberg leader in the presence of a competitive fringe (Daughety, 1990, Montero and Guzman, 2010). Nevertheless, competition authorities still treat collusion as unquestionably harmful to society.

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For mergers, on the other hand, efficiencies are considered to be a real possibility. Examples are the circumvention of fixed costs (Salant et al., 1983), or synergies leading to marginal costs for the merged entity that are lower than marginal costs for any of its constituent firms (Farrell and Shapiro, 1990, Cheung, 1992). A particular type of efficiency, requiring no technological advance, occurs when an efficient firm (that is, a firm producing against low marginal costs) merges with a less efficient firm. The merged entity can then shift production from the inefficient to the efficient plant, thereby reducing aggregate production costs. Farrell and Shapiro (1990) refer to this phenomenon as *rationalizing production.* However, for cartels without side payments it seems difficult to redistribute production – let alone to shut down plants – such that all firms benefit, while simultaneously increasing welfare. Bos and Pot (2012) explore this possibility.

In this paper we identify a different mechanism through which collusion (or a horizontal merger) may increase welfare, which we refer to – following Stennek (2003) – as *market-induced rationalization.* This occurs when firms that are less efficient than others collude and contract their output. The optimal response of the non-colluding, but more efficient, firms, is to expand their production, provided they compete in quantities. Aggregate output as well as average production costs will then decrease. If the reduction in costs is large enough, social welfare may increase. Note that the cartel does not internalize some particular efficiency, nor does it increase output; the welfare enhancement is solely due to the response of the market to collusion.

Using a stylized market structure with two products we consider a cartel of firms producing one of the products. We show that such an incomplete cartel may increase welfare through market-induced rationalization, a result that up till now has not been explicitly acknowledged in the literature.[1] Section 2 introduces the market structure, and conditions for the cartel to increase welfare are identified in Section 3. Section 4 concludes. The proofs can be found in the Appendix.

2 Market structure and incomplete cartels

Market demand originates from a representative consumer spending his income on commodities 1 and 2 and on a *composite* commodity 0, where the latter represents expenditures on all commodities other than the first two. Preferences of this consumer are represented by the utility function

$$U(Q_0, Q_1, Q_2) = Q_0 + a(Q_1 + Q_2) - \frac{1}{2}b\left[Q_1^2 + 2\vartheta Q_1 Q_2 + Q_2^2\right],$$

with Q_i consumption of commodity i, $i = 0, 1, 2$ and $a, b > 0$ preference parameters. The parameter $\vartheta \in (0, 1]$ measures the degree of substitutability between commodities 1 and 2, with $\vartheta = 1$ implying perfect substitutability. The consumer maximizes utility given his budget constraint $Q_0 + P_1Q_1 + P_2Q_2 \leq M$, where M is the available income, P_1 and P_2 are the prices of commodities 1 and 2 and the composite commodity's price

[1] For example, although not explicitly mentioned, the numerical example of Cheung (1992, p.120), which deals with synergies from mergers, includes a welfare enhancing (but unprofitable) cartel (for the special case $\alpha = 1$).

equals 1. For M high enough, inverse demand functions for the first two commodities follow as

$$P_1(Q_1, Q_2) = a - bQ_1 - b\vartheta Q_2 \text{ and } P_2(Q_1, Q_2) = a - b\vartheta Q_1 - bQ_2.$$

Production takes place in industries 1 and 2, with n firms in industry 1 producing commodity 1 against constant marginal costs $c \geq 0$, and m firms in industry 2 producing commodity 2 against marginal costs $d \geq 0$. We assume $c \geq d$ and introduce

$$\Delta = \frac{c-d}{a-c}$$

as a measure of the production efficiency difference between producers of the two commodities (note that the denominator $a - c$ equals the maximum gains from trade from production of commodity 1).

Firms compete with quantities as strategic variable (Cournot competition) in both industries. Market equilibrium can be characterized as follows.

Lemma 1 *Let $0 \leq \Delta \leq \overline{\Delta}$, with*

$$\overline{\Delta} \equiv \frac{1 + m(1-\vartheta)}{m\vartheta}. \tag{1}$$

Then the Cournot-Nash equilibrium is given by all firms in industry 1 producing q_1^* and all firms in industry 2 producing q_2^*, where

$$q_1^* = \frac{m\vartheta(a-c)}{b((m+1)(n+1) - mn\vartheta^2)}\left(\overline{\Delta} - \Delta\right) \text{ and} \tag{2}$$
$$q_2^* = \frac{(n+1)(a-c)}{b((m+1)(n+1) - mn\vartheta^2)}\left(\Delta + \frac{1 + n(1-\vartheta)}{n+1}\right).$$

Total production is given by $Q_1^* = nq_1^*$ and $Q_2^* = mq_2^*$, equilibrium prices by $P_1^* = c + bq_1^*$ and $P_2^* = d + bq_2^*$ and firm profits by $\pi_1^* = (P_1^* - c)\, q_1^* = b\,(q_1^*)^2$ and $\pi_2^* = (P_2^* - c)\, q_2^* = b\,(q_2^*)^2$, respectively.

Note that for efficiency differences larger than $\overline{\Delta}$, firms in industry 1 are inactive in equilibrium.

We now analyze the effect of a cartel agreement between all firms in industry 1. The objective of the cartel is to set the total quantity of commodity 1 such that aggregate profits of firms from industry 1 are maximized. Firms from industry 2 still compete in quantities.

Proposition 2 *A cartel of the n firms from industry 1 increases consumer prices and output of commodity 2. It decreases output of commodity 1 and consumer welfare. The cartel is profitable for its members if and only if*

$$n > \left(\frac{m+1}{m+1-m\vartheta^2}\right)^2, \tag{3}$$

and is always beneficial to firms from industry 2.

The decrease in consumer welfare is consistent with the finding that mergers in Cournot competition with homogeneous goods increase consumer prices if no technological efficiencies materialize (Farrell and Shapiro, 1990).

Condition (3) also generalizes the well known *merger paradox* (Salant et al. 1983) which says that mergers (or incomplete cartels) in quantity competition are only profitable if enough firms join. In fact, for $\vartheta = 1$, our condition (3) reduces to $n > (m+1)^2$ which is equivalent to equation (3′) from Salant et al. (1983, p.191). It implies that the cartel in industry 1 is profitable only if industry 1 contains at least four times as many – but typically even much more – firms as industry 2. The reason is that the non-colluding firms respond to the contraction of output by the cartel by expanding output themselves. This decreases the cartel's profit and may outweigh the benefits of the agreement. The scope for profitable incomplete cartels widens for increasing marginal costs (Perry and Porter, 1985), price competition (Deneckere and Davidson, 1985), or nonlinear demand (Cheung, 1992). Limited substitutability also softens the merger paradox: the lower bound in (3) decreases with a decrease in ϑ. In particular, for $\vartheta < \sqrt{1 - \frac{1}{2}\sqrt{2}} \approx 0.54$ the cartel in industry 1 is profitable for all possible values of n and m.[2]

The fact that the cartel is less profitable for small values of n also has ramifications for cartel sustainability. As long as condition (3) holds the cartel is sustainable if the discount factor is high enough, but this critical discount factor is non-monotonic in n.[3] This is due to cartel profits being low – giving strong incentives to deviate and therefore requiring a high discount factor – either when n is small and condition (3) only just holds, or when n is large.

3 Welfare enhancing incomplete cartels

The impact of the cartel on total welfare, which we define as the sum of consumer utility and aggregate profits, is ambiguous: consumers are harmed, firms from industry 2 gain, and firms from industry 1 may gain or lose (see Proposition 2). Figure 1 illustrates, for the case of perfect substitutes ($\vartheta = 1$), how these different effects can be balanced against each other.

[2] Note that the efficiency difference Δ has no effect on profitability of the cartel, although it does determine market shares of individual firms. Therefore the minimal number of firms to join for profitability might represent an arbitrarily small market share. In particular, the market share of industry 1 firms, $Q_1^* / (Q_1^* + Q_2^*)$, equals $\frac{nm(\overline{\Delta} - \Delta)}{n+m+m\overline{\Delta}}$, for $\vartheta = 1$, which becomes arbitrary small for Δ close enough to $\overline{\Delta}$.

[3] It is straightforward to show that, assuming firms use grim trigger strategies, the critical discount factor for a sustainable cartel in the infinitely repeated game is given by

$$\delta^* = \frac{(n-1)\left((m+1)(n+1) - mn\vartheta^2\right)^2}{\left((m+1)(n-1) - mn\vartheta^2\right)\left((m+1)(n^2+6n+1) - (n+3)mn\vartheta^2\right)}.$$

This critical discount factor is increasing in m and ϑ, non-monotonic in n and strictly below 1 as long as condition (3) holds.

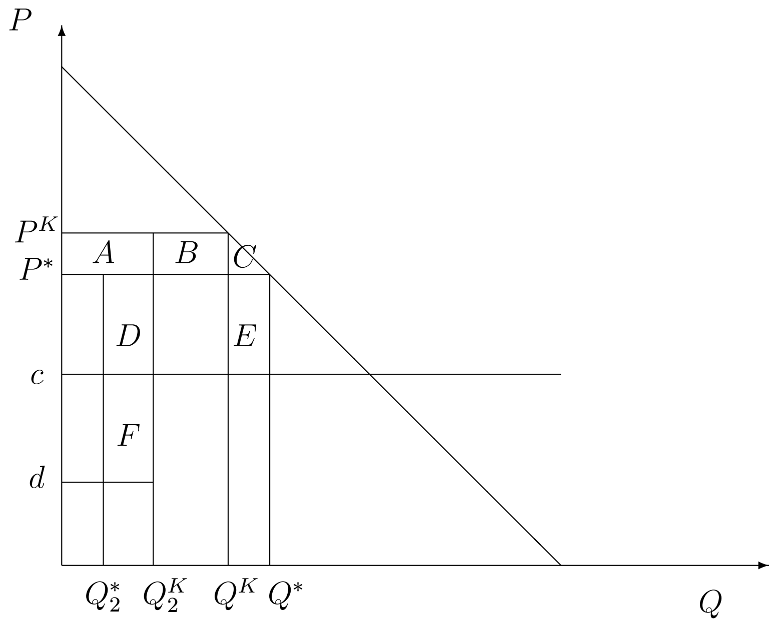

Figure 1: Illustration of welfare effects of incomplete cartel for perfect substitutes.

Aggregate output and output of industry 2 in the Cournot–Nash equilibrium are given by Q^* and Q_2^*, respectively. The decrease in aggregate output to Q^K, due to the cartel contracting its output, is mitigated because firms from industry 2 respond by expanding their output from Q_2^* to Q_2^K. The price increases from P^* to P^K and consumers are worse off: $U^K - U^* = -(A + B + C) < 0$. Aggregate profits of firms in industry 2 go up by $m\left[\pi_2^K - \pi_2^*\right] = A + D + F > 0$ and the cartel's change in total profits equals $\pi_1^K - n\pi_1^* = B - D - E$.

The change in total welfare is $T^K - T^* = F - (C + E)$. The last part, $-(C + E)$, is the standard loss in consumer surplus and industry profits which arises when aggregate output falls below its competitive level. The other, non-standard, part, $F = (c - d)\left(Q_2^K - Q_2^*\right)$, gives cost savings that result from the redistribution of output to industry 2. If these savings are large enough total welfare may increase.

The efficiency difference Δ should be sufficiently high for the cost reduction to outweigh the deadweight loss from the decrease in production. Moreover, also the ability of the cartel to reduce aggregate output, and the strength of the output expansion response of industry 2 are important determinants of the scope for welfare increasing cartels.

The following result helps investigating the conditions under which welfare enhancing cartels exist.

Lemma 3 *Let n^* be the number of firms in industry 1 for which total welfare is maximized. Then there exist numbers Δ_L and Δ_H, with $\Delta_L < \Delta_H < \overline{\Delta}$, such that (i): for $\Delta_H \leq \Delta < \overline{\Delta}$ we have $n^* = 0$; (ii) if $\Delta_L > 0$, we have $n^* \geq n$ for $0 < \Delta \leq \Delta_L$.*

Lemma 3 implies that, for $\Delta_L < \Delta < \Delta_H$, welfare typically improves with a decrease in the number of firms in industry 1. This suggests that there exists a range of values of Δ (containing the interval $\left[\Delta_H, \overline{\Delta}\right]$) for which a cartel of industry 1 firms increases total welfare. Proposition 4, which is our main result, specifies this interval.

Proposition 4 *A cartel of the n firms in industry 1 increases total welfare if and only if*

$$\begin{aligned} \Delta &> H(n, m, \vartheta) \qquad (4) \\ &\equiv \overline{\Delta} - \frac{1}{m\vartheta} \frac{2\left(2(m+1) - m\vartheta^2\right)\left((m+1)(n+1) - mn\vartheta^2\right)}{(m+1)\left((n+3)m + 5n + 7\right) - m\left((n+1)m + 4n + 2\right)\vartheta^2}, \end{aligned}$$

This lower bound satisfies $\Delta_L < H(n, m, \vartheta) < \Delta_H$ and decreases in n and ϑ.

Note that the mechanism through which the cartel may improve welfare is closely related to the one responsible for the merger paradox: the more efficient non-colluding firms respond to output contraction of the cartel by expanding their output. This output expansion makes the cartel less profitable, and it decreases average production costs. The question therefore arises whether cartels exist that are simultaneously profitable and welfare enhancing. The shaded areas in Figure 2 provide examples of such market structures and suggest that they are a real possibility. The vertical lines correspond to condition (3) and the dashed curves correspond to condition (4). Departing from perfect substitutability (by decreasing ϑ below 1) increases the scope for profitable cartels substantially, but does not severely limit the possibility for an increase in total welfare.

Proposition 4 and Figure 2 show that the scope for welfare enhancing cartels increases with an increase in n or ϑ. When commodities are close substitutes an increase in m also makes welfare enhancing cartels more likely. The intuition for the effect of ϑ is straightforward: if commodities are closer substitutes industry 2 will expand output more in response to a cartel in industry 1. The effects of n and m are harder to interpret. An increase in n, for example, implies that the cartel will contract output more and consequently that industry 2 will expand output more. The net result of these two effects turns out to be always positive.

The required efficiency levels in Figure 2 range from rather low ($H(20, 3, 1) \approx 0.036$) to quite high values ($H(2, 1, 0.54) \approx 0.85$). The case of perfect substitutes combined with perfect competition in industry 1 gives $\lim_{n\to\infty} H(n, m, 1) = 1/\left[m(2m+5)\right]$ as a lower bound of the required efficiency difference. This lower bound become arbitrarily small for increasing m.

4 Concluding remarks

In this paper we introduced a mechanism by which incomplete cartels may (unintentionally) increase social welfare in quantity setting oligopolies. Non-colluding, but more efficient, firms expand output in response to the output contraction of the colluding firms. The resulting decrease in average production costs may outweigh the

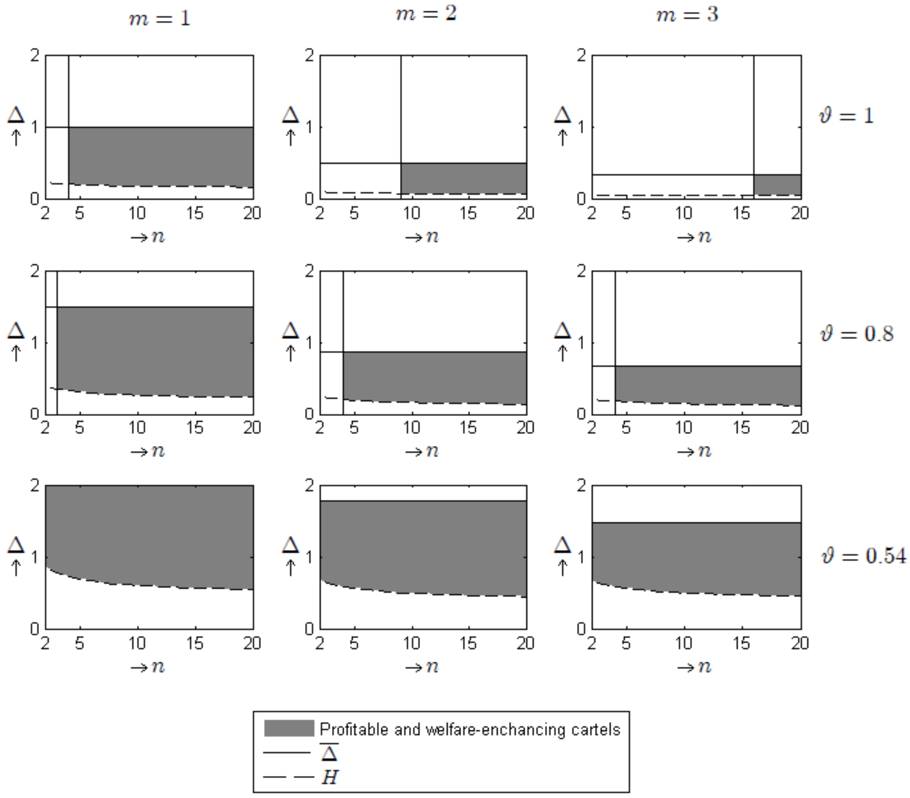

Figure 2: The scope for profitable and welfare enhancing incomplete cartels in the (n,Δ)-space for different values of m and ϑ.

deadweight loss from the decrease in aggregate production. These welfare enhancing cartels exist for reasonable parameter values in a stylized market with linear inverse demands and constant marginal costs, and do not require any synergies between the colluding firms or other special features. In a similar fashion an incomplete cartel between producers of low quality products may be welfare enhancing when it leads to an increase in the supply of high quality products.

The phenomenon identified here may be encountered in particular when a new technology or product is developed that is superior to the existing ones. This situation can typically be modeled by imperfect substitutability, like we do here. Incumbent firms then risk losing market share and profits, and may have a strong incentive to collude. Our analysis shows that this response might increase welfare and suggests that competition authorities should consider being lenient toward such cartels. Conversely, a cartel between efficient firms reduces welfare beyond the deadweight loss from decreased output, since it additionally leads to a shift of production to the inefficient firms.

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Appendix: Proofs

Proof of Lemma 1. Firm i in industry 1 sets its quantity q_{1i} in order to maximize $\left[P\left(\sum_{k\neq i} q_{1k} + q_{1i}, \sum_k q_{2k}\right) - c\right] q_{1i}$ and firm j in industry 2 sets its quantity q_{2j} to maximize $\left[P_2\left(\sum_{k\neq j} q_{2k} + q_{2j}, \sum_k q_{1k}\right) - d\right] q_{2j}$. The $n+m$ first order conditions are given by

$$a - b\left(\sum_{k\neq i} q_{1k} + 2q_{1i}\right) - b\vartheta \sum_k q_{2k} - c = 0, \qquad i = 1, \ldots, n,$$

$$a - b\left(\sum_{k\neq j} q_{2k} + 2q_{2j}\right) - b\vartheta \sum_k q_{1k} - d = 0, \qquad j = 1, \ldots, m.$$

The solution of which is given by $q_{1i} = q_1^*$ for $i = 1, \ldots, n$ and $q_{2j} = q_2^*$ for $j = 1, \ldots, m$. Prices and profits follow straightforwardly from this. ■

Proof of Proposition 2. Although n is an integer, for mathematical convenience we may treat it as a continuous variable. Differentiating q_1^* and q_2^* with respect to n we find that both are decreasing in the number of firms in industry 1:

$$\frac{\partial q_1^*}{\partial n} = -\frac{m\left(1-\vartheta^2\right)+1}{(m+1)(n+1) - mn\vartheta^2} q_1^* < 0 \text{ and } \frac{\partial q_2^*}{\partial n} = -\frac{\vartheta}{(m+1)(n+1) - mn\vartheta^2} q_1^* < 0 \tag{5}$$

The change in aggregate output then is:

$$\frac{\partial Q_1^*}{\partial n} = q_1^* + n\frac{\partial q_1^*}{\partial n} = \frac{m+1}{(m+1)(n+1) - mn\vartheta^2} q_1^* > 0 \text{ and } \frac{\partial Q_2^*}{\partial n} = m\frac{\partial q_2^*}{\partial n} < 0.$$

Moreover, both prices decrease:

$$\frac{\partial P_1}{\partial n} = b\frac{\partial q_1^*}{\partial n} < 0 \text{ and } \frac{\partial P_2}{\partial n} = b\frac{\partial q_2^*}{\partial n} < 0.$$

Since the consumer faces lower prices and consumes positive amounts of both commodities he will be strictly better off with an increase in n.

Consequently, formation of a cartel in industry 1, which is formally equivalent with a decrease of the number of firms in that industry from n to 1 will increase both prices, decreases production of commodity 1 and increases production of commodity 2. Moreover, the consumer will be strictly worse off, and firms from industry 2, who now sell more and at a higher price, will be better off. The cartel is profitable if $\pi_1(1,m) \geq n\pi_1(n,m)$, or when $q_1(1,m) > \sqrt{n} q_1(n,m)$ – that is, when the cartel does not restrain production by too much. This is the case for the values of n given by condition (3). ■

Proof of Lemma 3. Denote by $q_1(n,\Delta) = q_1^*$ and $q_2(n,\Delta) = q_2^*$ the Cournot–Nash equilibrium quantities when there are n firms independently producing commodity 1 (where, for notational convenience, we suppress the dependence of q_1^* and q_2^* on m, ϑ and $(a-c)/b$). Consumer utility is then given by

$$V(n,\Delta) = M + \frac{1}{2}b\left[n^2[q_1(n,\Delta)]^2 + 2\vartheta nmq_1(n,\Delta)q_2(n,\Delta) + m^2[q_2(n,\Delta)]^2\right].$$

Aggregate profits in industry 1 and industry 2 are given by $\Pi_1(n,\Delta) \equiv n\pi_1^* = bn[q_1(n,\Delta)]^2$ and $\Pi_2(n,\Delta) = m\pi_2^* = bm[q_2(n,\Delta)]^2$, respectively. Total welfare, as a function of n and $\triangle$, is

$$T(n,\Delta) = V(n,\Delta) + \Pi_1(n,\Delta) + \Pi_2(n,\Delta) \tag{6}$$
$$= M + \frac{1}{2}b\left(n(n+2)[q_1(n,\Delta)]^2 + 2\vartheta nmq_1(n,\Delta)q_2(n,\Delta) + m(m+2)[q_2(n,\Delta)]^2\right).$$

Since $T(n,\Delta)$ is continuous in n we can take the derivative with respect to n. Using (5) from the proof of Proposition 2 we find

$$\frac{2}{b}\frac{\partial T(n,\Delta)}{\partial n} = \frac{2q_1}{(m+1)(n+1) - mn\vartheta^2}\left((m+1)q_1(n,\Delta) - m\vartheta q_2(n,\Delta)\right).$$

Moreover, $(m+1)q_1(n,\triangle) - m\vartheta q_2(n,\triangle)$ is decreasing in n. This implies that $T(n,\Delta)$ is unimodal in n and has a unique global maximum which is given by the solution to $\frac{\partial T(n,\Delta)}{\partial n} = 0$. Now if $(m+1)q_1(n,\triangle) \geq m\vartheta q_2(n,\triangle)$ for the *actual* value of n, then in the social optimum at least n firms are active in the production of commodity 1. This gives

$$\Delta_L = \frac{(m+1)^2 - m(m+n(1-\vartheta)+2)\vartheta}{m(m+n+2)\vartheta}.$$

On the other hand, if $(m+1)q_1(0,\Delta) \leq m\vartheta q_2(0,\Delta)$ then in the social optimum no firm should be active in the production of commodity 1. This gives

$$\Delta \geq \Delta_H = \frac{(m+1)^2 - m(m+2)\vartheta}{m(m+2)\vartheta}.$$

It is easily checked that $\Delta_L < \Delta_H < \overline{\Delta}$ for $\vartheta > 0$. ■

Proof of Proposition 4. Substituting the expressions for $q_1(n,\Delta)$ and $q_2(n,\Delta)$ from (2) into (6) and solving $T(1,\Delta) > T(n,\Delta)$ for Δ we obtain, after some straightforward but tedious calculations, $\triangle > H(n,m,\vartheta)$.

The fact that $T(n,\Delta)$ is unimodal in n (see the proof of Lemma 3) implies that $\Delta_L < H(n,m,\vartheta) < \Delta_H$ for all admissible values of n, m and ϑ. Differentiating $H(n,m,\vartheta)$ with respect to n shows that $\frac{\partial H}{\partial n}$ has the same sign as

$$-\left(\left(2-\vartheta^2\right)m + 2\right)\left((m+1)^2 - (m+2)m\vartheta^2\right),$$

which is always negative. The derivative with respect to ϑ is much more complicated but turns out to be negative for any admissible value of n, m and ϑ. ■

Corollary 5 *Consider the case of perfect substitutes, $\vartheta = 1$. A cartel of primary industry firms is profitable for the colluding firms when $n > (m+1)^2$ and welfare enhancing when*

$$\Delta > H(n, m, 1) = \frac{2m + n + 3}{m(2m^2 + (2n + 8)m + 5n + 7)}. \tag{7}$$

Furthermore, $H(n, m, 1)$ is decreasing in n and m.

Proof of Corollary 5. The profitability condition and condition (7) follow immediately from Propositions 2 and 4. Furthermore, the derivatives with respect to n and m are given by

$$\frac{\partial H(n, m, 1)}{\partial n} = -\frac{2(m+2)^2}{m(2m^2 + (2n+8)m + 5n + 7)^2} < 0 \text{ and}$$
$$\frac{\partial H(n, m, 1)}{\partial m} = -\frac{2m(m(4m + 5n + 17) + 2(n+4)(n+3)) + (n+3)(5n+7)}{m^2(2m^2 + (2n+8)m + 5n + 7)^2} < 0,$$

respectively. ■