Collective Memory, Consensus, and Learning explained by Network Connectivity

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Collective Memory, Consensus, and Learning Explained by Network Connectivity

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Abstract

Humans cluster in social groups where they discuss their shared past, problems, and the potential solutions; they learn collectively when they repeat activities; they synchronize when they sing or dance together; and they bond through social cohesion. By representing a group network with a Laplacian matrix, the outcomes of these activities, as well as the group's cohesion, can be predicted by its second smallest eigenvalue, called the algebraic connectivity. It predicts when processes converge toward a consensus or focal activity, but it cannot predict divergence, such as division of labor or polarization.

Humans live in social groups, which yield advantages and disadvantages. Advantages include access to information and collective goods, but they come at a cost, including the maintenance of social ties, and there are risks of exploitation and misinformation. Psychology has a long tradition of studying interdependent individuals through the lens of network analysis [4, 19, 32]. In this paper, I focus on a common feature of social networks underlying a number of advantages and disadvantages of group life, which have been examined separately in different fields. The crucial characteristic turns out to be the network's *algebraic connectivity* [17], which interconnects and provides new insights into previously unrelated findings about group learning [2, 52], collective memory [13, 33], agreement [26, 9, 41], synchronization [24, 30], and social cohesion [51]. To see how, we revisit a number of experiments.

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Results

Experimental studies about communicating individuals have focused on different kinds of shared information, such as memories [13, 33], network node coloring [26], conventions [9], opinions [36, 20], and visual cues of body movement [28]. In all these experiments, pairwise differences between individuals' memories, opinions, ideas, and movements decreased, which is also at the heart of the contact hypothesis [40].

Convergence in a group can be modeled as a grand total of decreasing pairwise differences. Formally, when individuals i and j discuss their positions on a given issue, y, their difference is $y_j - y_i$. By using matrix notation, the process for determining group consensus in a network, based on these pairwise differences, can be written as

$$\frac{d\mathbf{y}}{dt} = -\mathbf{L}\mathbf{y}, \tag{1}$$

where $\mathbf{L}$ is the network Laplacian (see Methods) and $\mathbf{y}$ is a vector of individuals' positions [38]. This approach is an efficient notation for well-known and well-replicated social influence models [48, 18]. The added value is in the eigenvalue spectrum of $\mathbf{L}$. For initial opinions $\mathbf{y_0}$ at t_0, the speed of convergence decreases with initial opinion diversity, but increases with the second-smallest eigenvalue of the Laplacian [38], denoted λ_2 and called *algebraic connectivity* [17]. This also holds true for the synchronization of connected oscillators [24], for example singing or dancing individuals [30], even though models of synchronization are different [29]. Algebraic connectivity is maximal for fully connected networks, zero for networks with disconnected parts, and close to zero if the network is very sparse between dense parts [14], such as social groups (for example, Fig. 1b).

The two networks in Fig. 1 were used in an experiment [13] that drew considerable attention [47]. The participants conversed about their past (i.e., a story made up by the researchers that subjects first had to memorize) for a certain amount of time. The authors concluded that in the network with more mnemonic alignment, in Fig. 1a, clustering is lower and the average distance (hops through the net) is smaller than that in Fig. 1b. Their conclusions are correct, but the Laplacian model (Eq. 1; Fig. 1c) contributes to the explanation of group dynamics, namely how group outcomes emerge from interacting individuals. The model also makes it possible to relate specific findings, such as this one, to numerous others. The first that comes to mind is a variation of the memory experiment, where different ties were used over time [34]. In this variant, there were four clusters with four subjects in each, with connections between the clusters (SM). If intercluster communication

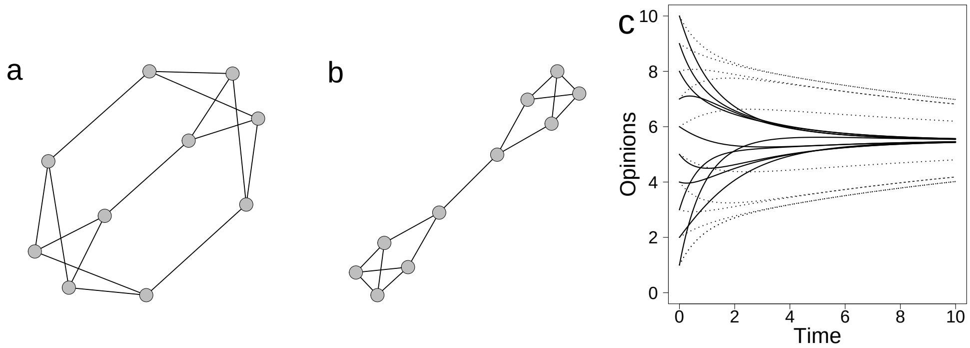

Figure 1: Two networks of the same size and density were used in a mnemonic convergence experiment (adapted from Coman et al. 2016). (**a**) Network with $\lambda_2 = 0.333$. (**b**) Network with $\lambda_2 = 0.074$. (**c**) Memory, or opinion, convergence in network a (continuous lines) and network b (dotted lines) according to the Laplacian model (Eq. 1).

precedes within-cluster communication, there is more convergence of memories than if the sequence is reversed. Although algebraic connectivity cannot distinguish between the two treatments, because the same network is used in both, the Laplacian model, solved numerically over subsequent rounds, predicts the experimental outcome (SM). Again, the model contributes precision and explanatory power to the authors' positing that communication through bridging ties early on facilitates mnemonic convergence. Furthermore, the explanatory power of the Laplacian model, with or without its algebraic connectivity, turns out to be robust. The predictions generated by the model are impaired neither by asynchronous (versus simultaneous) interactions nor by categorical (versus continuous) issues, where differences $y_j - y_i$ are, strictly speaking, undefined.

Another experiment with categories involved the formation of a convention among multiple alternatives proposed by the participants [9]. Subjects interacted over 25 rounds, communicating with one alter in each round. Three networks were compared, each with $n = 24$: a circular lattice where everyone had four ties ($\lambda_2 = 0.084$); a random network where everyone had four ties on average ($\lambda_2 = 0.256$); and a network where subjects were connected to a next alter in every subsequent round, homogeneously spread across the network. In the latter network, this procedure resulted in a clique (fully connected network) at round 23 ($\lambda_2 = 1.043$), when everyone had interacted with everyone else. In the two networks with low connectivity (lattice and random), a group convention was never achieved in time, in contrast to the clique, where it was always achieved.

Along with predicting experimental outcomes, algebraic connectivity can also be used to measure social cohesion, that is, the network structure that bonds a group together and facilitates information transmission [35]. Because information transmitted over longer distances tends to deteriorate [36, 16] or not arrives at all [15], it is important that distances to sources are short for group members, and there is network redundancy such that a noisy or biased message through one path (concatenation of ties) can be corrected by messages through other, nonoverlapping, paths [51], as in Fig. 1a. Algebraic connectivity is larger in networks with shorter distances and multiple nonoverlapping paths (Methods). It is also larger when chords cross-connect these paths, an aspect of redundancy that has been largely overlooked in the literature, whereas they turned out to be crucial for reliable transmission in an experiment with long paths [16]. Network redundancy and proximity increase the robustness of a network, which, together with rapid consensus on important issues and interaction rituals [12], such as synchronous singing and dancing [42], contributes to the bonding of a group and facilitates collective action [7]. We note that in general, algebraic connectivity is only loosely related to density because sparse connections between (sub)groups strongly weaken the effect of increasing density within (sub)groups, even though adding ties to a network cannot decrease algebraic connectivity [5]. Therefore, assessing the algebraic connectivity of (sub)groups is in most cases more informative than assessing the algebraic connectivity of a large network, which tends to be very small or zero.

In more cohesive groups, not only does convergence to consensus proceed faster but also the collective production of goods, such as building a house. Collective production follows the learning curve [52]: The time it takes for a group to finish an activity, t, decreases at a decreasing rate with the number of times, x, the activity has been executed in the past. The functional form often used is a power law [2], $t = ax^b$, with $b < 0$. Due to repetition, a group network becomes more efficient through individuals finding shortcuts in their social "problem space" [37] with increasing probability [46], connecting to knowledgeable colleagues at closer network distances. It has been proven mathematically that this probabilistic tie relaying (keeping density fixed) results in a learning curve [46, 23]. To determine how algebraic connectivity matters, we revisited an experiment wherein every subject was randomly assigned an initial color, and was instructed to synchronize their color with their network neighbors [26]. (The number of colors to choose from was the chromatic number.) The time it required to achieve one overall color (i.e., consensus), t seconds, was measured in six networks, all with the same density and 36 subjects (SM). The first had six densely connected clusters with one tie connecting each cluster to the next; the remaining five networks were obtained by randomly rewiring the ties of the first network with increasing probability, $q \in \{0, 0.1, 0.2, 0.4, 0.6, 1\}$, while keeping the, progressively sparser, clusters connected to each other. In contrast to teams and business companies [2], where individuals find shortcuts in their network by themselves, the researchers implemented the shortcuts in the coloring experiment. Apart from this difference, the sequence of treatments was networkwise identical to the sequence of snapshots of a learning process. Bearing this in mind, writing λ_2 instead of x to capture the effect of network shortcuts, a learning curve was fitted to the data from the coloring experiment (Fig. 2; $b = -0.434; P = 0.0004; n = 6$). This parameter is close to the average in industry ($b \approx -1/3$; *ibid*). The learning curve predicts the data much better than does the authors' model with a concave, rather than convex, curve (SM).

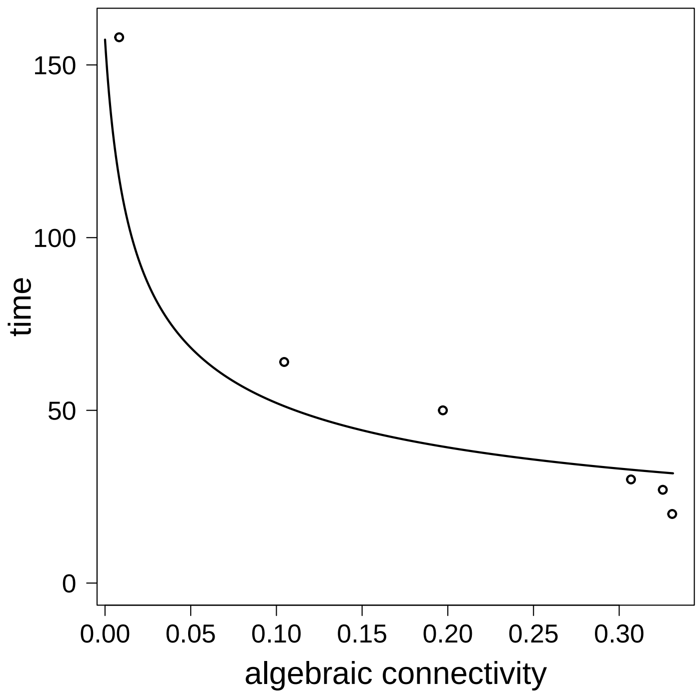

Figure 2: Time to reach consensus as a function of the algebraic connectivity of the network (data from Kearns et al. 2010). Note that the data were obtained for given chances of random tie relaying, but here, pertinent levels of algebraic connectivity are plotted.

Discussion

An important lesson from the coloring experiment is that the incremental effect size of algebraic connectivity is large at low levels, whereas further increasing connectivity to higher levels has small effects. In other words, to gain most of the advantages of learning, a modest level of connectivity suffices. This aligns with the findings of other studies, in which the effect of messages through multiple ties was examined. In one of these studies, the chance to win subjects over to accept a health improving behavior increased with the number of ties through which they received health messages, but at a decreasing rate, and more than three ties had no significant effect on or above three ties [8]. The only exception we encountered was the presence of a time limit, when a group result had to be achieved in a short period of time, in the convention experiment [9].

Along with diminishing advantages, there are increasing disadvantages. If people too easily converge to the majority, the result is not necessarily wisdom of the crowd but oftentimes a form of groupthink: a commitment to an inferior solution to a given problem [25], a lack of innovation due to cultural homogeneity [39], a memory of events that never happened [13], the adoption of a conspiracy theory [22], or the punishment of critics with valuable feedback [21]. For information from multiple sources to be advantageous, two conditions must be fulfilled. First, knowledgeable people should not let themselves be influenced by sources with inferior information [6, 10] (Eq.4 in Methods). Second, the least knowledgeable individuals should either be aware that they can learn from their average social environment [10] or specifically know which sources are best, and relay their ties accordingly [6]. Both conditions are easier to meet at modest levels of algebraic connectivity and density when the network is not too efficient and social pressure is not too strong.

In sum, most of the benefits of algebraic connectivity can be had at relatively low levels, whereas further increasing connectivity by means of extra ties entails costly tie maintenance, increasing information overload, and higher risks of groupthink while rarely yielding additional benefits. A challenge for future research is to determine the sweet spot.

In this paper, we have shown that the Laplacian model with its algebraic connectivity predicts the outcomes of various processes that converge. (The mathematical conditions for convergence are described by Proskurnikov [41].) However, it cannot predict divergence, for example choosing a color differing from one's neighbors [27] or the division of labor in general. Nor can it predict when a discussion about a controversial issue leads to polarization instead of convergence [3, 43, 1] or how conflicts evolve in general. Furthermore, there are challenges to social groups beyond convergence/divergence, for example, finding solutions to complex problems [45], for which algebraic connectivity has limited use. Nonetheless, a Laplacian network representation of social groups interconnects numerous findings from multiple fields and improves their explanations. It is also practical; if your meetings last too long, increase their algebraic connectivity!

Methods

Laplacian matrix

In a set of n individuals, indexed i and j, their social ties are denoted $a_{ij} \geq 0$. If i pays attention to j, tie $a_{ij} > 0$, and the social influence of all others on i's position on issue y are written as the sum total of pairwise differences [18], $\sum_{j=1}^{n} a_{ij}(y_j - y_i)$. To write this for everyone concisely (Eq. 1), a Laplacian matrix is used

$$\mathbf{L} = \mathbf{D} - \mathbf{A}, \tag{2}$$

with $\mathbf{A}$ the adjacency matrix of the network. $\mathbf{D}$ is a diagonal matrix with $d_{ii} = \sum_j a_{ij}$ and $d_{ij} = 0$ if $i \neq j$ [11]. The definition of $\mathbf{L}$ implies that its smallest eigenvalue is $\lambda_1 = 0$. For the remainder I assume that the network is a strong component (from every node, there is a path to every other node).

Results are most straightforward if $\mathbf{A}$ is binary and symmetric ($a_{ij} = 1$ or $a_{ij} = 0$, and $a_{ij} = a_{ji}$); however, this yields the illusions that larger groups have stronger cohesion and reach consensus more quickly, which are both false. To sidestep these incorrect ideas, the row-normalized adjacency matrix $\mathbf{W}$ is used instead of $\mathbf{A}$, with $w_{ij} = a_{ij}/\sum_{j=1}^{n} a_{ij}$ such that $\sum_j w_{ij} = 1$, in line with social influence models in general [20]. Then, $d_{ii} = 1$ for all individuals who pay attention to at least one social contact. Accordingly, $\mathbf{L} = \mathbf{D} - \mathbf{W}$ was used for all the calculations. If $w_{ij} \neq w_{ji}$, this poses no problem in general; however, completely nonreciprocal ties ($w_{ij} > 0$ and $w_{ji} = 0$) entail complex eigenvalues without meaningful interpretation in social life, and hardly contribute to social cohesion. Therefore it is best to remove these ties from the data.

When $d\mathbf{y}/dt = -\mathbf{L}\mathbf{y}$ (Eq. 1) is integrated, the solution, $\mathbf{y}_t = e^{-\mathbf{L}t}\mathbf{y}_0$, is unpacked in terms of the eigenvectors, $\mathbf{v}_i$, and eigenvalues of $\mathbf{L}$ [44]. Because $\mathbf{y}_0 = b_1\mathbf{v}_1 + b_2\mathbf{v}_2 + ... + b_n\mathbf{v}_n$,

$$\mathbf{y}_t = b_1 e^{-\lambda_1 t}\mathbf{v}_1 + b_2 e^{-\lambda_2 t}\mathbf{v}_2 + ... + b_n e^{-\lambda_n t}\mathbf{v}_n. \tag{3}$$

The outcome is dominated by the smallest eigenvalue larger than zero, λ_2.

When there is variation across individuals' susceptibilities to social influence ($s_{ii} \neq s_{jj}$), Eq.1 can be expanded to

$$\frac{d\mathbf{y}}{dt} = -\mathbf{SLy}, \tag{4}$$

with diagonal matrix $\mathbf{S} = diag(s_{ii})$ and for all $i \neq j$, $s_{ij} = 0$. The larger the variation in susceptibility is, the less accurate algebraic connectivity is at predicting the speed of convergence, which does not impede numerically solving Eq. 4. The indicates that stubborn individuals (with low s_{ii}) have greater influence on $\mathbf{y}_t$. This is also true for leaders to whom others direct their strongest ties.

Algebraic connectivity and social cohesion

The majority of mathematical results [49] are based on symmetric binary matrices, instead of row-normalized matrices, including the results in this section. The presentation follows the order of the main text.

First, the expected path length, $E(\mathcal{D})$, imposes an upper bound on algebraic connectivity

$$\lambda_2 \geq \frac{2}{(n-1)E(\mathcal{D}) - 0.5(n-2)}, \tag{5}$$

and so does $\mathcal{D}_{max}$, the diameter, $\lambda_2 \geq 4/(n\mathcal{D}_{max})$. The proofs are given in [49].

Second, for all networks that are no cliques (i.e., are at least one tie short), $\lambda_2 \leq \mathcal{K}$, where $\mathcal{K}$ is the minimum number of nodes that have to be removed to make the network fall apart [17]. $\mathcal{K}$ also equals the minimum number of non-overlapping paths connecting arbitrary pairs of nodes [31, 51], for which λ_2 thus imposes a lower bound.

Third, to show that chords between long paths increase λ_2 while keeping the density constant, we take the smallest chordless cycle in the network (usually a triad), we remove one tie from it, and we set it aside. Now, we take the longest chordless cycle and cross-connect it midway with the tie just removed. By means of this tie relaying, the average distance decreases, similar to random relays do in the famous small-world model [50]. The procedure may be repeated. To finish the proof, we apply the expected path length theorem (Eq. 5).

Furthermore, $\lambda_2 \leq min(k)$, is the smallest degree in the network [49].

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Code and data availability

The R code, including all data, is available at `https://osf.io/8wpxt/`

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Supplementary Material

Memory convergence

In the memory experiment with bridges versus clusters [34], there were two treatments, each with four rounds (Fig. 3). In each round, every subject communicated with one other subject. In the first round of treatment 1, subjects communicated between the clusters, not within. In the remaining rounds, they communicated within the clusters, in each round with someone else. In treatment 2, the inter-cluster communication took place in the last round instead of the first; for details, see Fig. 2 in [34].

For the vector of initial memories, I randomly assigned 0 or 1 to each network node with equal probability. I took the output of the previous round as input vector for the next round. The numerical experiment was repeated 1000 times, and then averaged. Based on the result of the lab experiment, the hypothesis was: $sd(treatment\ 2) - sd(treatment\ 1) > 0$. The numerical outcome was $sd(treatment\ 2) - sd(treatment\ 1) = 0.0297$. In social life, the effect size will be larger because there tends to be more diversity of information across groups than within groups [7].

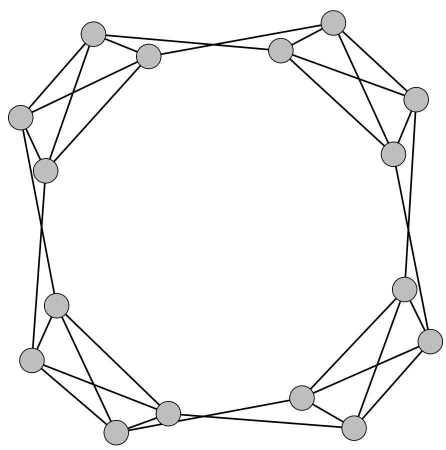

Figure 3: Network with four clusters, used in a mnemonic convergence experiment (adapted from Momennejad et al. 2019).

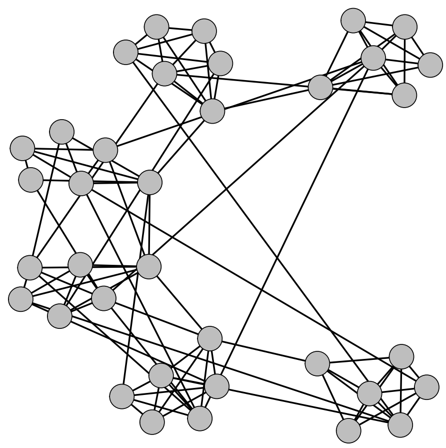

Figure 4: Network with six clusters and a random tie-rewiring chance of $q = 0.1$, used in a node coloring experiment (adapted from Kearns et al. 2010).

Network node coloring

Fig. 4 depicts the network with six clusters from the node coloring experiment [26]. Table 1 contains the data. Because the original data were unavailable, I enlarged Fig. 4 from [26] and used a measuring rod to assess the number of seconds required for everyone to have the same color, t. The inter-rater correlations with two colleagues were 0.9997 and 0.9996; the reported times in the table are averages of our three measurements.

I calculated the algebraic connectivity at each probability of rewiring, $q \in \{0, 0.1, 0.2, 0.4, 0.6, 1\}$, by averaging over 1000 simulated networks, starting out with the maximally clustered network ($q = 0$), and rewiring randomly for given q. The full range of values from 1000 simulations is depicted in Fig. 5. Because the variation shown here is an artifact of the inaccessibility of the original data (which have one λ_2 at each q), I fitted a learning curve to the mean values. For this curve, I used the `nlstools` package in R; I do not report R^2 because for the response variable, $\sum(y_i - \bar{y})^2 \neq \sum(\hat{y}_i - \bar{y})^2 + \sum(y_i - \hat{y}_i)^2$. The predictions from the original myopic heuristic model [26] are shown in red triangles. Despite the uncertainty of the original data, the improvement over the original model is clear.

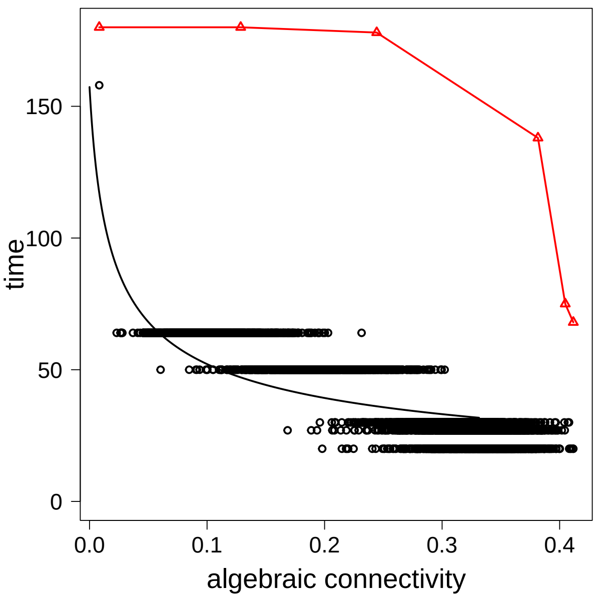

Figure 5: Variation of algebraic connectivity across simulated networks. The predictions from the original myopic heuristic model are depicted in red triangles.

treatments	q	0	0.1	0.2	0.4	0.6	1
outcomes	t	158	63	49	29.5	27	20.5
	λ_2	0.0083	0.1050	0.1974	0.3038	0.3267	0.3301
	myop	180	180	178	138	75	68

Table 1: Data and predictions from the network node coloring experiment.