Fitting model parameters in conformal geometric algebra to Euclidean observation data
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Background on the Hypergeometric Function

In this section we provide some brief background discussion about concepts needed to express the results in section 5.4.

Given any function, one can approximate its value at the point $x = 0$ by its power series expansion

$$f(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \ldots$$  \hspace{1cm} (C.1)

Conversely, given a sequence of coefficients \{$a_0, a_1, a_2, \ldots$\}, one can attempt to find a function that generates this sequence as coefficients of the respective power in its power series expansion. This function is called a generating function of the sequence. For example, the sequence \{1, 1, 1, \ldots\} is generated by the function $f(x) = \frac{1}{1-x}$.

A sequence which fulfills the property that the ratio of consecutive terms can be written as a rational function of polynomials in the index, is called a hypergeometric sequence, i.e.

$$\frac{c_{k+1}}{c_k} = \frac{P(k)}{Q(k)} = \frac{(k + a_1)(k + a_2)\ldots(k + a_p)}{(k + b_1)(k + b_2)\ldots(k + b_q)(k + 1)}$$  \hspace{1cm} (C.2)

where the factor $(k + 1)$ in the denominator is present for historical reasons of notation. A hypergeometric function is a function that has a hypergeometric series (i.e. one whose coefficients
form a hypergeometric sequence) as its power series expansion, i.e.

\[ pF_q(x) = pF_q(a_1, \ldots, a_p, b_1, \ldots, b_q, x) = \sum_k c_k x^k = \sum_k \frac{(a_1)_k(a_2)_k \ldots (a_p)_k x^k}{(b_1)_k(b_2)_k \ldots (b_q)_k k!}, \tag{C.3} \]

where consecutive coefficients \( c_k \) and \( c_{k+1} \) fulfill (C.2) and \((a)_k\) is the Pochhammer symbol or rising factorial \((a)_k = \frac{\Gamma(a+k)}{\Gamma(a)} = a(a+1) \ldots (a+k-1)\).

Some hypergeometric functions have specific names. For example, \( _0F_1 \) is called the

confluent hypergeometric limit function

and often arises in statistical physics as well as in section 5.4. For more details about hypergeometric functions see for example [LBC10].

Note that some well-known functions can be expressed as hypergeometric functions, e.g.

\[ _0F_1 \left( \frac{1}{2}, \frac{x^2}{4} \right) = \cosh(x), \]
\[ _0F_1 \left( 1, \frac{x^2}{4} \right) = I_0(x), \]
\[ _0F_1 \left( \frac{3}{2}, \frac{x^2}{4} \right) = \frac{\sinh(x)}{x}, \]

where \( I_0(x) \) denotes the modified Bessel function of the first kind.

Among many other representations, the hypergeometric limit function \( _0F_1 \) has the integral representation

\[ _0F_1(b, x) = \frac{\Gamma(b)}{\sqrt{\pi} \Gamma\left(b - \frac{1}{2}\right)} \int_0^\pi \exp\left(-2\sqrt{x} \cos t\right) (\sin t)^{2b-2} \, dt, \tag{C.4} \]

if the real part of the argument \( b, \text{Re}(b) \), is larger than 1/2. If we substitute \( b = n/2 \) with \( n > 1 \), we get

\[ _0F_1 \left( \frac{n}{2}, x \right) = \frac{\Gamma\left(\frac{n}{2}\right)}{\sqrt{\pi} \Gamma\left(\frac{n-1}{2}\right)} \int_0^\pi \exp\left(-2\sqrt{x} \cos t\right) (\sin t)^{n-2} \, dt. \tag{C.5} \]

We invoked this result (C.5) in section 5.4.