Risk Sharing in Defined-Contribution Funded Pension Systems

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Abstract

This paper explores the introduction of collective risk-sharing elements in defined contribution pension contracts. We consider status-contingent, age-contingent and asset contingent risk-sharing arrangements. All arrangements raise aggregate welfare, as measured by equivalent variations. While working individuals hardly benefit or may even lose, retirees experience substantial welfare gains. An increase in the tax deductability of pension contributions can be beneficial for working cohorts, but comes at the cost of a reduction in aggregate welfare due to efficiency losses.

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1 Introduction

In many countries, pension systems are under reform (see OECD, 2011), because their sustainability is under threat. In particular, a number of countries have set up funded pension arrangements to complement or replace non-funded, pay-as-you-go arrangements, while other countries already featuring a sizable funded pillar are reconsidering its design. Newly funded systems, such as those in Israel and Norway, tend to be of the defined-contribution (DC) type. In the case of the United Kingdom defined-benefit (DB) funded pensions have on a large scale been replaced by individual funded DC arrangements,\(^1\) while in the Netherlands an increasing number of companies are putting their pension funds at arm’s length in order to prevent pension risks from spilling over on to the company’s balance sheet. This leads to the so-called funded collective defined contribution (CDC) arrangements. Many observers feel that this will be a first step towards a system of individual funded DC arrangements.

Experience from the U.K. and the U.S. has shown that the size of the DC pension benefit determined at retirement date is highly sensitive to the momentary levels of the interest rate and the stock markets. This is also confirmed by Burtless (2000), who shows that the annuity benefit of a male worker entering an individual retirement account plan at twenty-two, investing his contributions in financial markets and retiring at sixty-two in 1975 would have been only two-fifths of what he would have received in 1969 when retiring at the same age. In addition, Agnew (2003) shows that individuals fail to invest optimally in occupational DC accounts by investing all or nothing in equity, trading infrequently and holding a disproportionate share of their pension wealth in their own employer’s equity. Hence, pension fund participants run considerable risk in terms of the future benefits they may expect and this raises the question whether it is desirable to retain or reintroduce at least some collective elements in funded DC pension arrangements.

In this paper, we explore and quantify the potential benefits of incorporating such collective features into a DC pension arrangement. In particular, we allow for the sharing of financial market risks across the various generations participating in the fund. Financial market risks are mainly concentrated with retirees and older workers, because these groups hold the largest stocks of financial assets. However, precisely these groups have the shortest horizon to recover from potential negative shocks, while they have limited or no flexibility in terms of increasing their labour supply. Therefore, it is relevant to examine the potential welfare gains from shifting some of the financial markets risks from older individuals to younger individuals through the incorporation of some risk-sharing mechanism into a DC pension arrangement. We consider a variety of risk-sharing schedules. The "status-contingent" scheme sets the exposure of retirees to financial market risk to zero and shifts this risk in a uniform way to the various working generations. That is, the return that working generations receive on their asset holdings is blown up by a constant factor. The "age-contingent" scheme differs from the status-contingent scheme in that the vulnerability to financial market risks is made to fall with the age of the worker, the idea being that older workers have less time to restore from negative shocks and have less flexibility to make up for losses by increasing their labour supply. Moreover, older workers have larger financial asset holdings, which increases their vulnerability to bad shocks. We also consider an "asset-contingent" risk-sharing schedule, in which the increase in the sensitivity of individual compensation to financial market risk is larger for workers at lower ages and belonging to higher skill (income) groups.

Our risk-sharing schemes are all intra-temporally balanced. That is, the ex-post transfers that take place after the stochastic shocks have hit in a given period sum to zero. Moreover,

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\(^1\)See Workplace Retirement Income Commission (2011). The Commission has been installed out of dissatisfaction with the large share of the working population not covered or insufficiently covered by pension savings.
Our risk-sharing schemes do not involve ex-ante redistribution. However, they do lead to ex-post redistribution of wealth across skill classes. Therefore, we introduce a second "asset-contingent" scheme that avoids resource flows among skill groups. That is, after this risk-sharing scheme is applied the aggregate financial wealth of each skill group is unaffected.

Our results are obtained through stochastic simulation of a many-generation overlapping generations model with a pension system, intragenerational heterogeneity, endogenous labour supply and financial market and wage shocks. The pension system consists of a public pay-as-you-go first pillar and a funded second pillar.

We first analyse the special case in which skill differences are absent. The aggregate welfare effects as measured through equivalent variations are rather similar across the various risk-sharing schemes at the level of the entire alive population, as well as at the level of the groups of workers and retirees as a whole. Retirees as a group benefit under all schedules, while workers as a group may be better or worse off depending on the specific scheme in operation. However, the effects for the workers as a group are quantitatively small and are dominated by the benefits to the retired as a group, implying that aggregate welfare at the level of the entire initially alive population is positive. The almost negligible consequences for the group of workers are the result of two offsetting effects. The group loses from increased exposure to financial market risk during their working life, but this loss almost cancels against the benefit of reduced exposure when they are retired themselves.

The introduction of skill differences hardly affects the welfare consequences for the group of workers as a whole or the group of retirees as a whole. However, looking at the consequences for individual skill groups we see that the higher skilled is a retiree, the more he benefits from the introduction of a risk-sharing scheme. This is not surprising, because higher-skilled groups hold more assets during retirement and, hence, benefit more from a stabilisation of the return on those assets.

Finally, we explore the scope for making the risk-sharing scheme more attractive to workers by increasing the tax deductability of pension contributions. Such increase comes at the cost of higher consumption taxes, which are used to finance the tax deductions. An increase in the tax deduction of contributions can be beneficial for working cohorts, but comes at the cost of a reduction in aggregate welfare due to efficiency losses. Not surprisingly, retired generations always lose from enhanced tax deductability of contributions.

There already exists a substantial literature on intergenerational risk sharing in pension systems. For example, Wagener (2004) and Gottardi and Kubler (2011) study risk-sharing within PAYG systems, while Matsen and Thogersen (2004) investigate the optimal division between PAYG and funding from a risk-sharing perspective. Other works studying intergenerational risk-sharing within funded pension systems are Beetsma and Bovenberg (2009) and Cui et al. (2011). However, none of the articles mentioned so far introduces our type of risk-sharing schemes into individual DC schemes. Further, our framework incorporates features that are often absent in other articles. In particular, we allow for intragenerational heterogeneity and endogenous labour supply.

The remainder of this paper is structured as follows. Section 2 presents the model. Section 3 discusses the calibration. In Section 4 we present and discuss our results first for the case without skill differences and then for the case with skill differences. Section 5 investigates whether raising the tax deductability of pension contributions can better balance the benefits of risk-sharing between workers and retirees. Finally, Section 5 concludes the main body of the paper.

2Some papers do allow for endogenous labour supply, for example Bonenkamp and Westerhout (2010), Meekopf (2010) and Beetsma et al. (2011)
2 The model

2.1 General framework

Time is discrete and a period corresponds to one year. All the variables are expressed in real terms. We allow for three sources of shocks, namely to wage growth, stock returns and bond returns.

Each individual will be identified by two indices, \( i = 1, \ldots, I \) and \( j = 1, \ldots, J \). The first index denotes the skill group, where a higher value of \( i \) corresponds to a higher skill level. Individuals born in a given skill group remain in the same skill group during their entire life. The second index denotes age, which is measured as the number of years since entry into the labour force. Each skill-age group consists of a continuum of individuals.

At the turn of each year the oldest generation dies and a new generation is born that is \( 1 + n \) times larger than the cohort born one period earlier. In each period, there are \( J \) overlapping generations. Under our assumption of a constant growth rate of the newborn cohort, the relative sizes of the various age groups remain constant over time. Each cohort consists of a continuum of individuals. Denoting by \( N_j, j = 1, \ldots, J \) the size of the cohort of age \( j \) it is easy to see that

\[
N_j = N_{j-1} \frac{\pi}{1 + n},
\]

where \( 0 < \pi < 1 \) is the constant probability that a person will survive at the end of each period.

Individuals start each period within given levels of personal and retirement savings. Then, workers choose their optimal consumption and labour supply. Retirees choose only their optimal consumption level, while they earn an income from government provided social security and from a private pension savings plan. Moreover, all individuals earn accidental bequests left by those who die. Personal and retirement savings are subject to the same market return. However, they differ in two major ways. First, retirement savings are less liquid than personal savings, which are immediately available for deposit and withdrawal at any moment. Retirement savings are instead available for withdrawal only at retirement and for deposit only while working. Second, investing in retirement savings is more rewarding, since contributions to the retirement pension scheme are partly tax deductible and matched by the employer.

2.2 The income process and retirement benefits

The pension system consists of two pillars. The first pillar, the social security system, is a pay-as-you-go (PAYG) arrangement. In each period the system receives contributions from workers and pays benefits to retirees. The second pillar is a funded DC arrangement.

Individuals work until the exogenous retirement age \( J + 1 \) and live for at most \( J \) years. Income prior to retirement \( (j \leq J) \) is described by

\[
y_{i,j,t} = \left(1 - \theta_t^{SS} - \theta_t^{DC}\right) w_{i,j,t} l_{i,j,t}, \quad j \leq J,
\]

where \( l_{i,j,t} \in [0, 1] \) is the amount of time spent working and \( w_{i,j,t} \) is the wage rate per unit of labour input, given by

\[
w_{i,j,t} = e_i s_j z_t,
\]

where \( e_i \ (i = 1, \ldots, I) \) is the efficiency index for skill group \( i \) and \( s_j \ (j = 1, \ldots, J) \) is a seniority index that for a given skill level causes income to vary with age. Moreover, \( \theta_t^{SS} \) and \( \theta_t^{DC} \) are the contribution rates to social security and the private retirement pension plan (in the sequel
referred to as "retirement plan"), respectively. Notice that all working individuals pay the same (mandatory) contribution rates, while, moreover, only the contribution rate to social security is time-dependent. Income depends on the exogenous process

$$z_t = (1 + g_t) z_{t-1},$$  

where $g_t = g + \epsilon_t^g$ is the growth rate of the process, which is the sum of a constant deterministic and a mean-zero stochastic component. We set $z_0 = 1$.

For convenience, we will define $x_{i,j,t}$ as the level of individual contributions to the retirement plan:

$$x_{i,j,t} = \theta_{DC} w_{i,j,t} l_{i,j,t}. \quad (3)$$

In line with U.S. arrangements and those in many other countries, retirement plan contributions are tax deductible up to a certain maximum $\nu_t = (1 + g_t) \nu_{t-1}$ that grows at the same rate as the wage rate. Hence, $\nu_t = z_t \frac{\nu_0}{z_0}$. The tax deduction $d_{i,j,t}$ is given by

$$d_{i,j,t} = \delta \min [x_{i,j,t}, \nu_t].$$

Further, the employer may match contributions to the retirement plan up to a certain maximum. The amount of matching by the employer $m_{i,j,t}$ is given by

$$m_{i,j,t} = \mu \min [x_{i,j,t}, \nu_t],$$

where $\mu$ is the match rate and $\nu_t$ is the match limit. Obviously, while the market return to personal and retirement savings is the same, investing in the retirement plan is actually more rewarding since contributions are tax deductible and matched by the employer.

Retirees ($j > J$) earn an income given by the benefits from social security ($b_{SS}^t$) and the retirement plan ($b_{DC}^t$):

$$y_{i,j,t} = b_{SS}^t + b_{DC}^t, \quad j > J.$$  

The social security benefit is a fixed fraction of the exogenous income process, identical for all retirees,

$$b_{SS}^t = \rho z_t, \quad 0 < \rho < 1.$$  

Hence, the social security system is progressive, in that less-skilled individuals pay lower contributions but receive the same benefits as richer individuals. The contribution rate $\theta_{SS}^t$ to the first pillar is set such that this pillar is balanced on a period-by-period basis, that is,

$$\sum_{j=1}^J N_j \sum_{i=1}^I \theta_{SS}^t x_{i,s_j} z_{i,j,t} = \sum_{j=J+1}^J N_j \sum_{i=1}^I \rho z_t.$$  

Notice that $z_t$ can be eliminated from (5) and, hence, the contribution rate depends on time only through the labour choice $l_{i,j,t}$.

In contrast to the social security benefit, the retirement plan benefit is a fraction of the accumulated plan savings, given by

$$b_{DC}^t = \frac{a_{i,j,t}}{\sum_{i=0}^\infty \left( \frac{\pi}{1 + r^f} \right)^i} = \left( 1 - \frac{\pi}{1 + r^f} \right) a_{i,j,t},$$  

where $r^f$ is the risk-free annual real rate of return. That is, the benefit from the retirement plan is computed in such a way that for given asset holdings $a_{i,j,t}$ the same constant level of benefits
benefit $b_{i,j,t}^{DC}$ can be provided up to death, when taking account of mortality risk and assuming risk-free market returns. However, from period to period this benefit level may fluctuate with the level of asset $a_{i,j,t}$. Hence, the benefit $b_{i,j,t}^{DC}$ differs from an annuity which pays out the same amount each period and individuals face uncertainty in retirement plan benefits, though not in the contribution rate to the plan.

Finally, the government finances the tax deductions of the contributions to the retirement plan through a value-added tax on consumption. Consumption is taxed at a rate $\tau_t$. Hence, to enjoy $c_{i,j,t}$ units of consumption, an individual pays $(1 + \tau_t) c_{i,j,t}$. We require the government’s budget to be balanced on a yearly basis:

$$\sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{t=1}^{T} N_j \tau_t c_{i,j,t} = \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{t=1}^{T} N_j d_{i,j,t}. \quad (7)$$

### 2.3 Accidental bequests

The only role of accidental bequests in the model is to ensure that resources do not "disappear" because people die. At the start of period $t$ the government collects the personal assets of the individuals who die at the end of $t-1$. The sum of the personal assets collected by the government is

$$S_t = \sum_{j=2}^{J} (1 - \pi) \frac{N_j}{T} \sum_{i=1}^{I} s_{i,j,t},$$

where $s_{i,j,t}$ are the personal assets at the end of period $t-1$ of an individual from cohort $j$ and skill class $i$. The government redistributes $S_t$ equally over all the alive individuals. Hence, each individual receives a transfer $h_t$ that is credited to his personal savings account:

$$h_t = \frac{S_t}{\sum_{j=1}^{J} N_j}.$$  

Similarly, the pension fund collects the retirement savings of those who die:

$$A_t = \sum_{j=2}^{J} (1 - \pi) \frac{N_j}{T} \sum_{i=1}^{I} a_{i,j,t}.$$  

The fund redistributes $A_t$ equally over all the alive individuals. Hence, each individual receives a transfer $q_t$, that is credited to his retirement savings account:

$$q_t = \frac{A_t}{\sum_{j=1}^{J} N_j}.$$  

### 2.4 Risk sharing

Elderly are heavily exposed to financial market shocks, because they hold a relatively large fraction of assets and they have a relatively short remaining lifespan to recover from losses. In this section we describe schemes intended to shift financial market risk from the elderly to younger cohorts.

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3Because each skill-age group consists of a continuum of individuals, the fractions of the group that are alive in future periods are known with certainty. Hence, individual longevity risks are shared and the pension fund that is responsible for executing the retirement plans is always able to meet its obligations.
Personal savings \( s_{i,j,t} \) evolve as
\[
s_{i,j,t} = \begin{cases} 
(1 + r_t) (s_{i,j-1,t-1} + h_{i,t-1} + y_{i,j-1,t-1} - (1 + \tau) c_{i,j-1,t-1} + d_{i,j-1,t-1} ) & j \leq J \\
(1 + r_t) (s_{i,j-1,t-1} + h_{i,t-1} + y_{i,j-1,t-1} - (1 + \tau) c_{i,j-1,t-1} ) & j > J \end{cases}, \tag{8}
\]
while pension savings evolve as
\[
a^b_{i,j,t} = \begin{cases} 
(1 + r_t) (a_{i,j-1,t-1} + q_{t-1} + x_{i,j-1,t-1} + m_{i,j-1,t-1} - b^{DC}_{i,j-1,t-1} ) & j \leq J \\
(1 + r_t) (a_{i,j-1,t-1} + q_{t-1} - b^{DC}_{i,j-1,t-1} ) & j > J \end{cases}, \tag{9}
\]
where \( a^b_{i,j,t} \) are pension asset holdings before (after) our risk-sharing scheme has been applied, as will be explained shortly. Further, \( r_t \) is the return on the individual’s personal and pension investment portfolios. It is given by
\[
r_t = \xi r^a_t + (1 - \xi) r^b_t,
\]
where \( \xi \) is the (exogenous and, by assumption, identical) share of both portfolios that is invested in stocks. Here, \( r^a_t \) and \( r^b_t \) are the returns on stocks, respectively bonds, which are given by
\[
\begin{align*}
r^a_t &= r^a + \epsilon^a_t, \\
r^b_t &= r^b + \epsilon^b_t,
\end{align*}
\]
where \( r^a \) and \( r^b \) are the corresponding averages and \( \epsilon^a_t \) and \( \epsilon^b_t \) are the corresponding mean-zero shocks to the asset returns. Overall, an individual faces individual shocks to mortality as well as aggregate shocks to wages and stock and bond returns. The aggregate shocks jointly follow a multivariate mean-zero normal distribution with variance-covariance matrix \( \Sigma \),
\[
\begin{pmatrix} \epsilon^a_t \\ \epsilon^b_t \end{pmatrix} \sim N (0, \Sigma). \tag{10}
\]
The DC pension fund redistributes over the population shocks to returns in retirement savings, subject to the balanced budget condition
\[
\sum_{j=2}^{J} N_j \sum_{i=1}^{I} a_{i,j,t} = \sum_{j=2}^{J} N_j \sum_{i=1}^{I} a^b_{i,j,t}, \tag{11}
\]
This expression says that the total amount of retirement savings after risk-sharing schedule has been applied (the left-hand side) be equal to the total amount of retirement savings before the risk-sharing schedule is applied (the right-hand side). Hence, the risk-sharing mechanisms that we present below are budgetarily neutral. Retirement savings before risk sharing evolve as in equation (9), while retirement savings after risk sharing are given by:
\[
a_{i,j,t} = \begin{cases} 
(1 + r_t) (a_{i,j-1,t-1} + q_{t-1} + x_{i,j-1,t-1} + m_{i,j-1,t-1} + t_{i,j,t} ) & j \leq J \\
(1 + r_t) (a_{i,j-1,t-1} + q_{t-1} + b^{DC}_{i,j-1,t-1} + t_{i,j,t} ) & j > J \end{cases}, \tag{12}
\]
where \( t_{i,j,t} \) is the transfer resulting from the risk-sharing mechanism.

Risk sharing takes place as follows. Each individual will get her expected retirement savings, which is the amount accumulated at the end of the previous year, \( a_{i,j-1,t-1} + q_{t-1} + x_{i,j-1,t-1} + m_{i,j-1,t-1} \), grossed up by the expected market return \( r_t \), rather than the actual market return \( r_t \), plus a transfer \( t_{i,j,t} \). We define the transfer as a function of the difference between the actual
and the expected net return on retirement savings, multiplied by the function \( f(i,j) \), which is a function of the skill level and the age.

\[
t_{i,j,t} = \begin{cases} 
   f(i,j) (r_t - r) (a_{i,j-1,t-1} + q_{t-1} + x_{i,j-1,t-1} + m_{i,j-1,t-1}), & \text{if } j \leq J \\
   f(i,j) (r_t - r) (a_{i,j-1,t-1} + q_{t-1} - b_{i,j-1,t-1}^{DC}), & \text{if } j > J.
\end{cases}
\]

Hence, after the risk-sharing scheme has been applied an individual of working age will have retirement savings equal to

\[
a_{i,j,t} = \begin{cases} 
   (1 + r + (r_t - r) f(i,j)) (a_{i,j-1,t-1} + q_{t-1} + x_{i,j-1,t-1} + m_{i,j-1,t-1}), & \text{if } j \leq J \\
   (1 + r + (r_t - r) f(i,j)) (a_{i,j-1,t-1} + q_{t-1} - b_{i,j-1,t-1}^{DC}), & \text{if } j > J.
\end{cases}
\] (13)

The function \( f(i,j) \) governs the transfers after the shocks have materialised. It will be chosen so as to protect the elderly (high \( j \)) and (in case) the less-skilled individuals (low \( i \)) relatively more against unexpected shocks. The rationale for these choices is that (1) the elderly have relatively little flexibility to respond to shocks and, hence, they may benefit from less uncertainty about the benefits they receive, and (2) policymakers want to protect the less-skilled, and hence poorer, individuals from too large fluctuations in their income. As a result, for the elderly and less-skilled individuals, \( f(i,j) \) will be smaller in absolute magnitude.

In qualitative terms we would expect this to be the outcome under a utilitarian planner who decides about the allocation of resources over the population. Since the total amount of retirement savings must be unaffected by the ex-post transfers, some groups (the younger and more-skilled individuals) will actually face retirement savings that are more volatile than in the absence of the risk-sharing scheme. Notice that the risk-sharing mechanism amounts to pure risk sharing. There is no ex-ante redistribution, because \( E_{t-1} [a_{i,j,t}] = E_{t-1} [a_{b,i,j,t}] \) for all \( i, j \) and \( t \).

We consider five different schedules for the rescaling function \( f(i,j) \).

\begin{itemize}
  \item \textbf{a. Uniform risk-sharing scheme (benchmark)}
  
  This schedule serves as the benchmark. Under this schedule,

  \[ f(i,j) = 1, \quad \forall i,j, \]

  and, hence, \( a_{b,i,j,t} = a_{i,j,t} \) for all \( i \) and \( j \). Hence, there is no risk-sharing among individuals.\(^4\) We refer to this case as the pure DC scheme.

  \item \textbf{b. Status-contingent risk-sharing scheme} 
  
  Retirees face no financial market uncertainty and always receive the expected return on their retirement savings. In contrast, all workers face proportionally more uncertainty than under the uniform scheme:

  \[ f(i,j) = \begin{cases} 
     a_1, & \text{if } j \leq J \\
     0, & \text{if } j > J.
\end{cases} \]

  The idea behind this scheme is that, unlike retirees, workers can react to shocks in their asset returns by changing their labour input, while, moreover, they have a relatively large amount of time left to recover from adverse shocks in their asset returns.

  \item \textbf{c. Age-contingent risk-sharing scheme} 
  
  \(^4\)Longevity risks are always shared (Footnote 3). Hence, when referring to "no risk sharing", we refer to the sharing of the other, aggregate risks.
Under this scheme the uncertainty about the returns to retirement savings falls linearly with working age to zero at the retirement age \( J + 1 \) and remains zero in all the retirement years:

\[
f(i, j) = \begin{cases} 
\alpha_2 (1 + J - j), & j \leq J \\
0, & j > J
\end{cases}
\]

This scheme is a refinement of the previous one and its motivation is analogous. Younger workers have more room to vary their labour input and to restore from adverse developments in the financial markets.

d. Asset-contingent risk-sharing scheme-I

Uncertainty is zero for retirees and for individuals with the largest amount of retirement savings in their skill group (i.e., those exactly at retirement age \( J \)). It is instead different from zero for all the other workers. In particular, it is larger for workers at lower ages and belonging to higher skill groups. Function \( f(i, j) \) then decreases with age progressively more quickly for more highly-skilled groups. Hence, for a given working age \( j \), function \( f(i, j) \) takes on larger values for more highly-skilled groups, the idea being that higher-income individuals can manage more risk in their asset income. To compute the rescaling function we take for retirement savings the levels based on a history of average market returns, which we denote by \( a_{i,j,0} \):

\[
f(i, j) = \begin{cases} 
\alpha_3 (\max_j \{a_{i,j,0} - a_{i,j,0}\}), & j \leq J \\
0, & j > J
\end{cases}
\]

e. Asset-contingent risk sharing scheme-II

Asset-contingent scheme-I gives rise to a rescaling function that differs by age and skill group. It leads to ex-post redistribution along two dimensions, age and skill. Asset-contingent scheme-II closely resembles asset-contingent scheme-I, except that it is specifically designed to avoid ex-post redistribution across skill groups. In the case when all individuals have identical skills asset-contingent scheme-II coincides with asset-contingent scheme-I. The rescaling function for asset-contingent scheme-II is

\[
f(i, j) = \begin{cases} 
\alpha_{4i} (\max_j \{a_{i,j,0} - a_{i,j,0}\}), & j \leq J \\
0, & j > J
\end{cases}
\]

where the parameter \( \alpha_{4i} \) is skill-specific.

The parameters \( \alpha_1, \alpha_2, \alpha_3 \) are larger than zero and chosen such that equation (11) is satisfied. The parameters \( \alpha_{4i}, i = 1, \ldots, I \), are also larger than zero, but each one of them is chosen to satisfy the budget balance equation for its corresponding skill level \( i \),

\[
\sum_{j=2}^{J} N_j a_{i,j,t} = \sum_{j=2}^{J} N_j a_{i,j,t}^b,
\]

implying that there are no net ex-post transfers across skill groups, but that all the ex-post redistribution takes place within the same skill group. Choosing all the \( I \) parameters this way implies that equation (11) is respected.

We take the time \( t = 0 \) distribution of retirement savings to avoid the circularity problem of having retirement savings that depend on the rescaling function that in turn depends on retirement savings. The \( t = 0 \) distribution is generated through a preliminary run of the model before the actual simulation starts on which our calculations are based. More details are given below.
2.5 The individual decision problem

The individual’s value function is:

\[
V_{i,j,t}(s_{i,j,t}, a_{i,j,t}) = \max_{c_{i,j,t}, l_{i,j,t}} \{ u(c_{i,j,t}, l_{i,j,t}) + \beta \pi E_t [V_{i,j+1,t+1}(s_{i,j+1,t+1}, a_{i,j+1,t+1})] \},
\]

(15)

where period utility \( u \) is the following function of consumption \( c_{i,j,t} \geq 0 \) and the labour supply \( l_{i,j,t} \in [0, 1] \),

\[
u_{i,j,t} = \frac{1}{1 - \gamma} \left( e_{i,j,t}^{1-\phi} (1 - l_{i,j,t})^\phi \right)^{1-\gamma}.
\]

Equation (15) is subject to the dynamics of (8) and (13). Maximization of the value function (15) yields the following set of first-order conditions:

\[
\begin{align*}
\frac{\partial u(c_{i,j,t}, l_{i,j,t})}{\partial c_{i,j,t}} &= \beta \pi E_t \left[ (1 + r_{t+1}) \frac{\partial u(c_{i,j+1,t+1}, l_{i,j+1,t+1})}{\partial c_{i,j+1,t+1}} \right] \\
&\quad + \frac{\partial u(c_{i,j+1,t+1})}{\partial c_{i,j,t}} c_{i,j,t} \left( 1 - \theta_{i,t} - \theta_{DC} \left( 1 - \delta 1_{\{x_{i,j,t} < \nu_{i,t}\}} \right) \right) \\
&\quad - \beta \pi E_t \left[ (1 + r_{t+1}) \frac{\partial u(c_{i,j+1,t+1}, l_{i,j+1,t+1})}{\partial c_{i,j+1,t+1}} \right] \\
&\quad + \frac{\partial u(c_{i,j+1,t+1})}{\partial c_{i,j,t}} c_{i,j,t} \left( 1 - \theta_{i,t} - \theta_{DC} \left( 1 - \delta 1_{\{x_{i,j,t} < \nu_{i,t}\}} \right) \right) \\
&= \beta \pi E_t \left[ (1 + r_{t+1}) \frac{\partial u(c_{i,j+1,t+1}, l_{i,j+1,t+1})}{\partial c_{i,j+1,t+1}} \right] \\
&\quad + \frac{\partial u(c_{i,j+1,t+1})}{\partial c_{i,j,t}} c_{i,j,t} \left( 1 - \theta_{i,t} - \theta_{DC} \left( 1 - \delta 1_{\{x_{i,j,t} < \nu_{i,t}\}} \right) \right) \\
\end{align*}
\]

where \( 1_{\{x_{i,j,t} < \nu_{i,t}\}} \) is a dummy variable equal to 1 if \( x_{i,j,t} < \nu_{i,t} \), and 0, otherwise. The second first-order condition is only relevant for \( f \) and \( s \) yields the following set of \( f \) first-order conditions:

\[
\begin{align*}
\frac{\partial u(c_{i,j,t}, l_{i,j,t})}{\partial l_{i,j,t}} &= \beta \pi E_t \left[ (1 + r_{t+1}) \frac{\partial u(c_{i,j+1,t+1}, l_{i,j+1,t+1})}{\partial l_{i,j+1,t+1}} \right] \\
&\quad + \frac{\partial u(c_{i,j+1,t+1})}{\partial l_{i,j,t}} l_{i,j,t} \left( 1 - \theta_{i,t} - \theta_{SS} \left( 1 - \delta 1_{\{x_{i,j,t} < \nu_{i,t}\}} \right) \right) \\
&\quad - \beta \pi E_t \left[ (1 + r_{t+1}) \frac{\partial u(c_{i,j+1,t+1}, l_{i,j+1,t+1})}{\partial l_{i,j+1,t+1}} \right] \\
&\quad + \frac{\partial u(c_{i,j+1,t+1})}{\partial l_{i,j,t}} l_{i,j,t} \left( 1 - \theta_{i,t} - \theta_{SS} \left( 1 - \delta 1_{\{x_{i,j,t} < \nu_{i,t}\}} \right) \right) \\
&= \beta \pi E_t \left[ (1 + r_{t+1}) \frac{\partial u(c_{i,j+1,t+1}, l_{i,j+1,t+1})}{\partial l_{i,j+1,t+1}} \right] \\
&\quad + \frac{\partial u(c_{i,j+1,t+1})}{\partial l_{i,j,t}} l_{i,j,t} \left( 1 - \theta_{i,t} - \theta_{SS} \left( 1 - \delta 1_{\{x_{i,j,t} < \nu_{i,t}\}} \right) \right) \\
\end{align*}
\]

2.6 Welfare comparisons between policies

We define the equivalent variation \( EV^{A,B}_{i,j,1} \) for skill group \( i \) of cohort \( j \) alive at \( t = 1 \) as the one-time transfer of resources that this particular group should receive extra in scenario \( A \) to obtain the same utility as in scenario \( B \):

\[
V^{A}_{i,j,1}(\bar{s}_{i,j,1} + EV^{A,B}_{i,j,1}, \bar{\pi}_{i,j,1}) = V^{B}_{i,j,1},
\]

where \( (\bar{s}_{i,j,1} + EV^{A,B}_{i,j,1}, \bar{\pi}_{i,j,1}) \) are the arguments of the value function, that is the sum \( \bar{s}_{i,j,1} + EV^{A,B}_{i,j,1} \) of the initial level of personal assets and the equivalent variation and the initial level of retirement assets \( \bar{\pi}_{i,j,1} \). As an example, suppose that \( EV^{A,B}_{i,j,1} = 0.050 \). If the initial wage rate is unity, this implies an equivalent variation of 5% of the initial wage rate. Newly-born individuals start with zero pension and private savings, hence \( \bar{s}_{i,1,t} = \bar{\pi}_{i,1,t} = 0 \).

The equivalent variations for the various groups can be added up to produce an aggregate welfare comparison:

\[
EV^{A,B} = \sum_{j=1}^{J} N_j \sum_{i=1}^{I} EV^{A,B}_{i,j,1}.
\]

(16)

In addition, we consider the percentage of individuals in favour of the alternative policy:

\[
PER^{A,B} = \sum_{j=1}^{J} N_j \sum_{i=1}^{I} 1 \{ V^{B}_{i,j,1} > V^{A}_{i,j,1} \},
\]

where \( 1 \{ \} \) is an indicator equal to 1, if the condition within the curly brackets holds, and 0, otherwise. We report also these aggregate measures for the groups of alive workers and retirees separately.
To understand the extent to which different skill levels affect our conclusions, we first present results for the case in which skill differences are absent. In so doing, we construct the relevant aggregate equivalent variation as

$$\text{EV}^{A,B}_{*} = \sum_{j=1}^{J} N_j \text{EV}^{A,B}_{*_j,1},$$

where $\text{EV}^{A,B}_{*_j,1}$ is the equivalent variation for cohort $j$ at time $t = 1$, belonging to the only skill group in the model, and we construct the percentage of individuals in favour of the alternative policy as

$$\text{PER}^{A,B}_{*} = \sum_{j=1}^{J} N_j \mathbb{I}\{V^{B}_{*_j,1} > V^{A}_{*_j,1}\},$$

where $V^{B}_{*_j,1}$ ($V^{A}_{*_j,1}$) is the value function under scenario B for cohort $j$ at time $t = 1$.

We focus the analysis on generations that are born at $t = 1$ or earlier. We do not consider future-born generations as this would provide no further insights. In fact, in this setting, shocks are i.i.d. and there is no memory of the past. Hence, the ex-ante situation of an agent who is born at $t = 1$ and experiences a risk-sharing scheme for all his life coincides with that of an agent born later, because both agents face identical conditions at birth.

3 Calibration and simulation set-up

3.1 Calibration

The model includes exogenous and endogenous parameters. The social security contribution rate $\theta^SS_t$, the consumption tax rate $\tau$ and the parameters $\alpha_1, \alpha_2, \alpha_3$ and $\alpha_4$ of the various rescaling functions are endogenous. The social security contribution rate is derived from equation (5). At the start of each simulation run it equals 13.209%. The parameters $\alpha_1, \alpha_2,$ and $\alpha_3$ are determined from equation (11) and are equal to 1.92, 0.16, and 0.44, respectively. In contrast, the parameters $\alpha_4i$ are derived from equation (14) and they range from 3.48 (skill group 1) to 0.25 (skill group 10). These parameters induce the rescaling functions shown in Figure 1 below. Rescaling differs by skill group only under the two asset-contingent risk-sharing schemes.

![Rescaling functions](image1.png)

**Figure 1.** Rescaling functions
Table 1 lists the exogenous parameters of the model and their calibration under the benchmark analysis. The parameters are chosen such that they reproduce main features of the U.S. economy. The economically active life of an individual starts at real-life age 25, which in the model we reset to $j = 1$. He then works for $J = 37$ years, which corresponds to a real-life age of 62. At that moment retirement starts. This retirement age is in line with median figures in Munnell et al. (2008) and calibrations in Laibson et al. (2007) and Bucciol (2011). Individuals live for at most $J = 75$ years after entry into the labour force, which corresponds to real-life age 100. We calibrate the conditional annual survival probability $\pi$ such that life expectancy is 53 more years at the moment of entry into the labour force. Hence, life expectancy is set at age 78, which is consistent with current figures from various sources (for instance, the Social Security Administration, see Bell and Miller, 2005). Using (1), the constant population growth rate $n$ is determined to produce a dependency ratio (the ratio of retirees over working-age individuals) of 33%.

Preference parameters

We set the discount factor at $\beta = 0.96$, a rather common choice in the macroeconomic literature (e.g., see Imrohoroglu, 1989, or Krebs, 2007), and the coefficient of relative risk aversion at $\gamma = 3$, which accords quite well with the assumed risk aversion in much of the macroeconomic literature (see, e.g., Imrohoroglu et al., 2003) as well as estimates at the individual level (for example, Gertner, 1993, and Beetsma and Schotman, 2001). The parameter $\phi$, which determines the substitution between consumption and leisure, is chosen to produce an average leisure choice of $l = 0.5$ (half of the time is devoted to work and half to leisure).\footnote{We assume that a full day is 16 hours. Hence, we exclude 8 hours of sleep, while $l = 0.5$ corresponds to the conventional working day of 8 hours.}

Income

The efficiency index $\{e_i\}_{i=1}^I$ is based on the income deciles for the U.S. for the year 2000 reported by the World Income Inequality Database (WIID, version 2.0c, May 2008). We normalise the index such that it has an average value of unity. The seniority index $\{s_j\}_{j=1}^J$ uses the average of Hansen’s (1993) estimation of median wage rates by age group. We take the averages between males and females and interpolate the data using the spline method.

Portfolio investment and financial markets

We assume that $\xi = 0.5$, implying that always half of the personal and pension portfolios consist of equity. The average risk-free rate is set to $r^f = 1.533\%$, which is the historical yield of three-month U.S. Treasury bills over the sample period 1986-2005, covered at monthly frequency. This yield is corrected for inflation, measured as the growth rate in the CPI for all urban consumers.

Shocks

The averages, variances and covariances of wage growth, stock returns and bond returns are estimated at monthly frequency from U.S. historical data covering the period 1986-2005. This period is selected to end well before the start of the recent financial and economic crisis in which economies have behaved in an unusual way. Specifically, we take series on personal income net of government transfers, rents and revenues from financial assets (source: U.S. Bureau of Economic Analysis), the MSCI U.S. stock index (source: Datastream) and the Merrill Lynch U.S. corporate and government master index (source: Datastream). Wage growth and returns are then converted into real terms by subtracting the CPI index for all urban consumers.

We estimate the average real wage growth rate, stock return and bond return at, respectively, $g = 2.665\%$, $r^s = 7.934\%$, and $r^b = 5.567\%$. The standard deviations associated with the corre-
sponding shocks are, respectively, 1.964%, 15.584%, and 5.961%. The three variables show little correlation, comprised between 3.29% for the correlation between wage growth and the stock return and 8.51% for the correlation between the stock and bond returns. With these figures we fill the variance-covariance matrix of the shocks Σ.

Social security and retirement assets

Social security in the U.S. provides a replacement rate, i.e. the ratio between the first pension benefit and the final salary payment of around 40%. We set the parameter ρ = 0.2, obtained as 0.4 times the average labour choice of 0.5 over working life. We set the contribution rate to the retirement plan at θ_{DC} = 0.10, implying that the benefit from the retirement plan is roughly equal to that from social security. We then set the three parameters describing the features of the retirement plan to match the features of a typical 401(k) plan. Following the calibration in Love (2006), we assume a match rate by the employer of μ = 0.5 and an initial match limit ν₀ = 0.06. That is, the employer matches 50 cents for each dollar contributed by the employee, with the match capped at the level that corresponds to employee contributions equal to 6% of compensation. We also assume a deduction rate of δ = 0.2, in line with the effective federal tax rate (see Congressional Budget Office, 2007). These assumptions give rise to an effective return on retirement assets above the return on personal assets, as frequently documented in the literature (see Brennan and Subrahmanyam, 1996, and Pàstor and Stambaugh, 2003). Finally, the consumption tax rate is obtained from equation (7) and is τ = 0.012 in the benchmark case.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Calibration</th>
</tr>
</thead>
<tbody>
<tr>
<td>J</td>
<td>Length of working life</td>
<td>37</td>
</tr>
<tr>
<td>J</td>
<td>Maximum death age after entry into labour force</td>
<td>75</td>
</tr>
<tr>
<td>n</td>
<td>Annual survival probability</td>
<td>0.991</td>
</tr>
<tr>
<td>β</td>
<td>Discount factor</td>
<td>0.96</td>
</tr>
<tr>
<td>γ</td>
<td>Relative risk-aversion parameter</td>
<td>3</td>
</tr>
<tr>
<td>φ</td>
<td>Consumption-leisure parameter</td>
<td>0.25</td>
</tr>
<tr>
<td>ei</td>
<td>Efficiency index (10 groups)</td>
<td>WIID (2008)</td>
</tr>
<tr>
<td>si</td>
<td>Seniority index</td>
<td>Hansen (1993)</td>
</tr>
<tr>
<td>ξ</td>
<td>Portfolio share in stocks</td>
<td>0.5</td>
</tr>
<tr>
<td>r_f</td>
<td>Risk-free real return</td>
<td>1.533%</td>
</tr>
<tr>
<td>g</td>
<td>Average real wage growth rate</td>
<td>2.665%</td>
</tr>
<tr>
<td>r_s</td>
<td>Average real stock return</td>
<td>7.934%</td>
</tr>
<tr>
<td>r_b</td>
<td>Average real bond return</td>
<td>5.567%</td>
</tr>
<tr>
<td>θ</td>
<td>Benefit scale factor</td>
<td>0.2</td>
</tr>
<tr>
<td>θ_{DC}</td>
<td>Retirement asset contribution rate</td>
<td>0.10</td>
</tr>
<tr>
<td>ν₀</td>
<td>Match limit</td>
<td>0.06</td>
</tr>
<tr>
<td>μ</td>
<td>Match rate</td>
<td>0.5</td>
</tr>
<tr>
<td>δ</td>
<td>Tax deduction parameter</td>
<td>0.2</td>
</tr>
</tbody>
</table>

5Standard deviations are calculated on returns in excess of the return to the risk free asset, to remove the part of volatility related to the asset that we consider risk free.
6With our calibration contributions to the retirement pension scheme are on average equal to x_{i, j, t} = \theta_{DC} e_{i,s} z_{i, j, t} = 0.05, thus very close to ν₀ = 0.06. This means that the employer usually matches most contributions.
7Below, we will explore how our results are affected if we vary the tax deductability parameter δ of pension contributions.
3.2 Simulation set-up

We run 1,000 simulations of the model for \( Y = 2J - 1 = 149 \) years in total. The simulations are based on random draws of the nominal wage growth rate and market returns from the shock process (10). The initial \( J - 1 \) years of the simulation are treated as "shadow" years. These observations are not used in the calculation of the statistics. During the shadow years there is no uncertainty. This part of the simulation merely serves to produce a set of initial asset holdings for the cohorts of different ages. We use the remaining simulation years, when individuals do face shocks in the returns to their private and pension asset portfolios and, hence, engage in risk sharing, for the calculation of our statistics. We reset time, so that the first period of this part of the simulation becomes \( t = 1 \). In this year a contingent risk-sharing scheme may be started. We assume that all individuals in \( t = 1 \) start with the levels of personal and retirement assets that are obtained in the latest "shadow" year of the simulation.

4 The results

4.1 All individuals have identical skills

Table 2 reports the aggregate welfare effects of a switch from no risk sharing to risk sharing. We omit the case of asset-contingent-II indexation scheme, as it coincides with an asset-contingent-I indexation scheme in the absence of skill differentiation. We report both the percentages of individuals in favour of the shift as well as aggregate equivalent variations. We do this for current workers, current retirees and all currently alive. While a vast majority of the currently retired are in favour of some form of risk sharing, the shares of workers in favour vary substantially across the various schemes. In terms of equivalent variation, the retired as a group gain substantially from risk sharing. This is not surprising, because they shed a substantial amount of risk by limiting the consequences of fluctuations in asset returns for the returns on their portfolio of pension savings. This benefit is strengthened by the fact that the elderly have on average little time left to recover from bad shocks. Current workers lose under the status-contingent scheme and gain under the other schemes. However, the welfare effects for this group as a whole are substantially smaller than for the retired.

<table>
<thead>
<tr>
<th>Table 2. Aggregate welfare effects - no skill differences</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Cohorts</strong></td>
</tr>
<tr>
<td>% in favour of contingent risk sharing</td>
</tr>
<tr>
<td>Status-contingent</td>
</tr>
<tr>
<td>Age-contingent</td>
</tr>
<tr>
<td>Asset-contingent-I</td>
</tr>
<tr>
<td>Equivalent variation</td>
</tr>
<tr>
<td>Status-contingent</td>
</tr>
<tr>
<td>Age-contingent</td>
</tr>
<tr>
<td>Asset-contingent-I</td>
</tr>
</tbody>
</table>

The reason for the much-smaller welfare effect for this group is that the loss they experience from larger fluctuations in the value of their pension asset portfolio during their working life, which result in a higher variance of asset holdings on retirement date under the alternatives to the pure DC scheme (see Table 3), is compensated by smaller (zero) fluctuations in the value of their
pension asset portfolio once they are retired. The welfare consequences are suppressed further by the intertemporal consumption and leisure smoothing, which reduces the welfare consequences of any shock. This is, in particular, the case for the very youngest, who have the longest period over which to smooth the effects of shocks on consumption and labour supply. Also, the very youngest have relatively little assets and, hence, the impact of shocks on consumption is small.

Figure 2. Welfare comparison (all schemes) - no skill differences

Table 3. Assets at retirement date: summary statistics - no skill differences

<table>
<thead>
<tr>
<th>Indexation</th>
<th>median</th>
<th>mean</th>
<th>std. dev.</th>
<th>coeff. of variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uniform</td>
<td>11.536</td>
<td>12.136</td>
<td>4.285</td>
<td>0.353</td>
</tr>
<tr>
<td>Status-contingent</td>
<td>9.384</td>
<td>11.194</td>
<td>7.849</td>
<td>0.701</td>
</tr>
<tr>
<td>Age-contingent</td>
<td>8.535</td>
<td>12.049</td>
<td>10.658</td>
<td>0.885</td>
</tr>
<tr>
<td>Asset-contingent I</td>
<td>9.479</td>
<td>12.234</td>
<td>7.483</td>
<td>0.612</td>
</tr>
</tbody>
</table>

4.2 Differences in skills

Now we assume the presence of ten different and equally-sized skill groups and repeat the analysis of the previous subsection. Table 4 aggregates the welfare effects over all skill groups for workers as a group and retirees as a group. Comparing the figures in Table 4 with those in Table 2, we see that at the aggregate level the introduction of skill differences has hardly any impact.
Table 4. Aggregate welfare effects - skill differences

<table>
<thead>
<tr>
<th>Cohorts</th>
<th>workers</th>
<th>retired</th>
<th>all</th>
</tr>
</thead>
<tbody>
<tr>
<td>% in favour of risk sharing</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Status-contingent</td>
<td>5.689</td>
<td>97.321</td>
<td>28.425</td>
</tr>
<tr>
<td>Age-contingent</td>
<td>45.533</td>
<td>97.321</td>
<td>58.383</td>
</tr>
<tr>
<td>Asset-contingent-I</td>
<td>68.080</td>
<td>97.321</td>
<td>75.336</td>
</tr>
<tr>
<td>Asset-contingent-II</td>
<td>19.648</td>
<td>97.491</td>
<td>38.963</td>
</tr>
</tbody>
</table>

Equivalent variation

| Status-contingent        | -0.087  | 1.773   | 1.249 |
| Age-contingent           | 0.115   | 1.773   | 1.306 |
| Asset-contingent-I       | 0.008   | 1.773   | 1.276 |
| Asset-contingent-II      | 0.106   | 1.773   | 1.303 |

Figure 3. Welfare comparison

All retirees are in favour of the switch to a risk-sharing scheme. However, the size of the benefit differs substantially across the skill groups – see Figure 3. The higher the skill group of a retiree, the more she or he benefits from the switch. This is not surprising, because higher-skilled groups hold more assets during retirement and, hence, benefit more from a stabilisation of the return on those assets.

Table 5 shows the effects of the switch to workers for a selected set of individual skill groups. For status-contingent transfers large majorities of workers in all skill groups are opposed to the
switch. In contrast, for the asset-contingent-I scheme it is only the highest skill groups that are in majority against the switch, because they absorb a large share of the volatility in the asset returns from the other groups. Asset-contingent-II scheme is on average unpopular to workers from all skill groups as it protects the workers with the highest asset levels, i.e. those close to retirement, from financial market volatility at the cost of younger workers.

Comparing workers of different ages, we find that older workers tend to be in favour of risk sharing. Specifically, when we consider cohorts (having aggregated over the skill groups) we see that under status-contingent indexation the oldest seven cohorts of workers are in favour of a switch to risk sharing. Under the other scenarios, many more workers prefer the switch: the oldest fifteen cohorts under age-contingent indexation and the oldest twenty-three cohorts under asset-contingent-I indexation prefer the switch.

**Table 5. Aggregate welfare effects for workers by skill group**

<table>
<thead>
<tr>
<th>Skill group</th>
<th>1</th>
<th>3</th>
<th>5</th>
<th>6</th>
<th>8</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>% in favour of risk sharing</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Age-contingent</td>
<td>55.474</td>
<td>52.980</td>
<td>48.379</td>
<td>44.985</td>
<td>43.465</td>
<td>22.442</td>
</tr>
<tr>
<td>Asset-contingent-I</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>91.896</td>
<td>24.521</td>
<td>12.859</td>
</tr>
</tbody>
</table>

**Equivalent variation**

<table>
<thead>
<tr>
<th>Skill group</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Status-contingent</td>
<td>0.001</td>
<td>0.000</td>
<td>-0.014</td>
<td>-0.027</td>
<td>-0.085</td>
<td>-0.528</td>
</tr>
<tr>
<td>Age-contingent</td>
<td>0.002</td>
<td>0.026</td>
<td>0.064</td>
<td>0.087</td>
<td>0.158</td>
<td>0.411</td>
</tr>
<tr>
<td>Asset-contingent-I</td>
<td>0.002</td>
<td>0.032</td>
<td>0.081</td>
<td>0.110</td>
<td>0.173</td>
<td>-0.699</td>
</tr>
<tr>
<td>Asset-contingent-II</td>
<td>0.002</td>
<td>0.025</td>
<td>0.062</td>
<td>0.084</td>
<td>0.150</td>
<td>0.357</td>
</tr>
</tbody>
</table>

Table 6 reports for an agent born at $t=1$ under asset-contingent scheme-I the standard deviations of the gross asset returns after risk-sharing has taken place, i.e. $(1 + r + (r_t - r) f(i,j))$, at ages $1$ and $J-1$. It also reports the average of those standard deviations across all ages. Clearly, the gross asset returns are most volatile for the youngest workers and least volatile for the oldest workers. In addition, for given age they are more volatile for higher-skilled workers than for lower-skilled workers. The first six skill groups face lower uncertainty over their asset returns than under the benchmark of pure DC where the life-cycle standard deviation is 8.318%, while the other skill groups face higher uncertainty. Also from this table it is clear that the high-skilled groups absorb a large part of the burden of the switch to this risk-sharing scheme. This effect is strengthened by the facts that it is these groups that hold the largest amounts of wealth at retirement and that the wealth distribution is highly skewed with average wealth at retirement date of skill group 10 being more than twice average wealth at retirement of skill group 8.
Table 6. Standard deviation of asset returns after risk sharing, asset-contingent-I scheme

<table>
<thead>
<tr>
<th>Average</th>
<th>Age 1</th>
<th>Age J – 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average</td>
<td>9.145</td>
<td>27.389</td>
</tr>
<tr>
<td>skill 1</td>
<td>1.719</td>
<td>5.148</td>
</tr>
<tr>
<td>skill 3</td>
<td>4.550</td>
<td>13.620</td>
</tr>
<tr>
<td>skill 5</td>
<td>6.952</td>
<td>20.814</td>
</tr>
<tr>
<td>skill 6</td>
<td>8.235</td>
<td>24.652</td>
</tr>
<tr>
<td>skill 8</td>
<td>11.825</td>
<td>35.399</td>
</tr>
<tr>
<td>skill 10</td>
<td>24.035</td>
<td>72.066</td>
</tr>
</tbody>
</table>

4.3 Varying the tax deductibility of pension contributions

The preceding analysis showed that the gains from introducing a risk-sharing schedule are highly unevenly distributed with the elderly as a group enjoying a substantial welfare gain and the young as a group enjoying only a small welfare gain or even losing from the arrangement. The question is whether the welfare gains of such an arrangement can be better distributed across the two groups. In this regard, the obvious policy option would be to increase the tax deductability of the second pillar contributions. For the case in which skill differences are absent, Table 7 shows that variations in tax deductability do have some effect although this effect is quantitatively rather small.\textsuperscript{10} The table also reports the implied average tax rate over time.\textsuperscript{11} On the one hand, due to efficiency losses caused by an increased distortion in the consumption-leisure trade-off there is always a negative effect of a rise in tax deductibility on aggregate welfare. On the other hand, for the working cohorts there is an offsetting positive effect arising from the benefit of increased tax deductability of their pension contributions that is partly paid for by the retired generations. In the case of the age-contingent and the asset-contingent-I scheme working generations benefit on net, while in the case of status-contingent risk-sharing the negative efficiency effect prevails. The group of retirees always loses from an increase in the deductability of pension contributions.

\textsuperscript{10}The percentages in favour of the switch to contingent risk sharing remain the same as in Table 2 for all values of $\delta$ that we consider, hence these percentages are not reported again.

\textsuperscript{11}The tax rate hardly varies over time.
Table 7. Equivalent variation with changing tax deductability

<table>
<thead>
<tr>
<th>Consumption tax</th>
<th>Status-contingent</th>
<th>Age-contingent</th>
<th>Asset-contingent-I</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta = 0$ ($\tau = 0$)</td>
<td>-0.041</td>
<td>1.846</td>
<td>1.314</td>
</tr>
<tr>
<td>$\delta = 0.1$ ($\tau = 0.006$)</td>
<td>0.110</td>
<td>1.846</td>
<td>1.357</td>
</tr>
<tr>
<td>$\delta = 0.3$ ($\tau = 0.018$)</td>
<td>0.126</td>
<td>1.846</td>
<td>1.364</td>
</tr>
<tr>
<td>$\delta = 0.4$ ($\tau = 0.024$)</td>
<td>-0.043</td>
<td>1.836</td>
<td>1.306</td>
</tr>
<tr>
<td>$\delta = 0.4$ ($\tau = 0.024$)</td>
<td>0.137</td>
<td>1.836</td>
<td>1.357</td>
</tr>
<tr>
<td>$\delta = 0.4$ ($\tau = 0.024$)</td>
<td>-0.046</td>
<td>1.816</td>
<td>1.291</td>
</tr>
<tr>
<td>$\delta = 0.4$ ($\tau = 0.024$)</td>
<td>0.140</td>
<td>1.816</td>
<td>1.336</td>
</tr>
<tr>
<td>$\delta = 0.4$ ($\tau = 0.024$)</td>
<td>0.048</td>
<td>1.806</td>
<td>1.283</td>
</tr>
<tr>
<td>$\delta = 0.4$ ($\tau = 0.024$)</td>
<td>0.142</td>
<td>1.806</td>
<td>1.337</td>
</tr>
</tbody>
</table>

5 Conclusion

In this paper, we have explored the introduction of collective risk-sharing elements in individual DC pension arrangements. The motivation for this research is that participants of individual DC arrangements run substantial risk during retirement because they cannot share their risks with other groups. We measure the consequences of the introduction of risk-sharing arrangements in terms of equivalent variations and find that the various schedules we consider work out in rather similar ways at the level of the current population and at the level of the groups of workers and retirees. Retirees as a group benefit under all schedules from the reduction in their exposure to financial market risk, while workers as a group may be better or worse off depending on the specific scheme in operation. However, the effects for the group of workers as a whole are quantitatively small, implying that the aggregate welfare effects for the initially-alive generations are always positive.

On the one hand, workers suffer from increased exposure during their working life, while on the other hand they benefit from reduced exposure once they are retired. An increase in the tax deductability of their pension contributions is beneficial for workers under some schemes, although quantitatively the effect is small, but it needs to be traded off against efficiency losses that always reduce aggregate welfare as measured by our equivalent variations.

References


