Fiscal policy and the business cycle: the impact of government expenditures, public debt, and sovereign risk on macroeconomic fluctuations

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Chapter 3

Expectations-Based Identification of Government Spending Shocks under Policy Foresight

Abstract

This chapter is concerned with the econometric problems of structural vector autoregressive analysis of the effects of government spending that are created by the presence of news or foresight about future fiscal policy. Using a combination of theory and stochastic simulations, the chapter investigates whether incorporating data that captures expectations on spending in VAR models is useful to address (i) the non-fundamentalness problem due to policy foresight and (ii) the problem of identifying structural spending shocks in the presence of policy foresight. In particular, the chapter demonstrates under which conditions and how expectations-based identifying restrictions can be applied to distinguish between anticipated and unanticipated shocks. In an application to U.S. data, the expectations-based approach that is proposed based on this analysis indicates a weaker impact of government spending on output, consumption, and investment than the most commonly applied fiscal VAR approach, which does not take into account the possibility of policy foresight.

3.1 Introduction

Discretionary changes in government spending can sometimes be anticipated in advance of their implementation, for example when spending programs are pre-announced in political speeches and news statements or when armed conflicts create expectations of military spending hikes. Such phenomena of policy foresight pose the following two challenges to attempts of quantifying the macroeconomic effects of government spending by empirical structural VAR (SVAR) methods.

First, if economic agents have access to information on fundamentals that affect future government spending but that are not yet incorporated in the actual data on government spending, then economic equilibria can have non-fundamental moving average representations. This means that the fundamental structural spending shocks cannot be recovered from present and past data on government spending by SVAR methods. As a consequence, ignoring the influence of policy foresight is likely to lead to biased estimates, as shown by Leeper, Walker, and Yang (2011), Ramey (2011b), and Yang (2005). Second, even if these non-fundamentalness issues could be circumvented, the structural shocks still need to be recovered from reduced-form VAR estimates by an appropriate identification strategy. This strategy not only needs to distinguish spending shocks from other economic shocks, but it also needs to distinguish between anticipated and unanticipated spending shocks. Good identifying restrictions are therefore especially hard to come by under policy foresight.

This chapter investigates whether incorporating expectations survey data in a vector autoregression can be useful to address these econometric difficulties. A number of recent studies indeed suggest to exploit information in forward-looking variables in fiscal VARs (e.g. Auerbach and Gorodnichenko, 2010; Ramey, 2011b; Sims, 2009) and some of those studies support this idea by simulation results. Laubach (2008) has also proposed to use direct measures of expectations on fiscal variables to address the econometric challenges posed by anticipation effects. To obtain a deeper understanding of such proposals and to make a precise link to the work of Leeper, Walker, and Yang (2011) and related work by Mertens and Ravn (2010), this chapter provides analytical results based on a standard growth model and tightly connected simulation evidence. In particular, the chapter studies the properties of identifying restrictions on government spending shocks that are based on model-consistent expectations. The candidate identification strategy is as follows: (i) an *unanticipated* or *surprise* spending increase is identified as an increase in the expectational error between today's spending and expectations thereof formed yesterday, while (ii) an *anticipated* spending increase is identified by an increase in expected spending tomorrow that is orthogonal to this expectational error.

The chapter first derives conditions under which incorporating expectations in a VAR model works to solve the non-fundamentalness problem due to policy foresight. For standard information flows, the requirement that is found to be sufficient is that expectations on future spending up to the anticipation horizon of economic agents should be included as an endogenous variable in the regression. Then, regarding the identification problem, the chapter shows that the success of the above identification approach depends on the type of endogenous reactions of government expenditures to the state of the economy, if one allows for the possibility that spending is not entirely exogenously determined. Surprise spending shocks are robustly identified if government spending reacts with some lag to other economic shocks, which is also assumed under the short-run restrictions of the standard recursive fiscal SVAR approach (see Blanchard and Perotti, 2002; Fatás and Mihov, 2001). However, the expectations-based approach may fail to correctly recover the effects of anticipated spending shocks even if government spending reacts to other shocks with a lag. The effects of anticipated shocks can only be correctly identified if all other relevant exogenous shocks are known, observed, and conditioned upon; that is, under fairly restrictive requirements. The expectations-based approach is thus found to be useful for the identification of surprise spending shocks when it is combined with standard short-run exogeneity restrictions.

Given these findings, the chapter focuses on the effects of surprise spending shocks in an application of the approach to the U.S., where data on expected federal government spending from the Survey of Professional Forecasters is used to measure expectations, presuming that this data does capture the expectations of economic agents (or market participants). This exercise reveals important differences in the effects of government spending according to the expectations-based approach in comparison to the standard fiscal SVAR approach. According to the expectations-based approach, an unexpected spending increase has positive short-run effects on output but negative effects at longer horizons, going along with pronounced declines in private consumption and investment. The standard SVAR approach, on the other hand, predicts an increase in consumption and investment in the short run and larger multipliers on GDP at longer horizons.

Following Ramey (2011b), the chapter also shows that the standard SVAR shocks are Granger-caused by the survey expectations (i.e. they are predictable) whereas the surprise shocks identified on the basis of expectational errors are indeed truly unpredictable. Taking into account policy foresight thus has important implications for the empirical effects of spending shocks.

The fact that anticipated shocks are not identified is of course a strong limitation if those shocks are the only relevant shocks to government spending or if they fall on different sub-items of total spending than unanticipated shocks. Surprise spending shocks are indeed more likely to fall on consumption expenditures under the standard definition of government spending, i.e. government consumption plus investment expenditures (see e.g. Blanchard and Perotti, 2002), because implementation lags are usually argued to be more relevant for government investment (see e.g. Leeper, Walker, and Yang, 2010). The empirical results in this chapter should be interpreted with these possible limitations in mind, despite the reassuring fact that some of the results reported below suggest that surprise spending shocks have been important driving forces of U.S. government spending in the past.[1]

Several other recent studies have addressed the econometric issues created by foresight on government spending. Ramey (2011b) applies a narrative approach which exploits military spending episodes, newspaper sources, and forecast errors based on survey data. However, although she also uses expectational errors to identify surprise spending shocks, Ramey does not add expectations on future spending in the VAR, which does *not* lead to a fundamental moving average representation. Fisher and Peters (2010) identify spending shocks by innovations to excess stock returns of military contractors, an approach which is applicable to defense-related expenditures only. Furthermore, Mertens and Ravn (2010) propose an SVAR estimator for permanent spending shocks based on Blaschke matrices, following Lippi and Reichlin (1994), which can be applied when the identifying assumptions pin down the Blaschke factor. Kriwoluzky (2010) directly estimates a vector moving average model, with model-based identifying restrictions on anticipated spending shocks. Forni and Gambetti (2010) es-

[1] See, in particular, the variation in the expectational errors in Figures 3.16, the tightness of the uncertainty bands in Figure 3.17, and the size of the identified shocks shown in Figure 3.20.

timate a large factor model which is arguably not affected by the non-fundamentalness problem (see Forni, Giannone, Lippi, and Reichlin, 2009), identifying a spending shock by various sign restrictions. However, the latter three approaches all rely on the correct specification of the identifying theoretical model. An expectations-based approach has the advantage of applying seemingly less restrictive identifying assumptions.

The remainder of the chapter is structured as follows. The next section explores the econometric problems created by foresight in a standard growth model and studies the usefulness of an expectations-based approach in addressing those problems. Section 3.3 provides connected simulation evidence. Section 3.4 analyzes the robustness of the approach to, not exclusively, alternative assumptions on the structure of the spending process, and proposes possible adjustments based on this analysis. Section 3.5 discusses the results of the empirical application. Section 3.6 concludes.

3.2 Policy foresight: problems and solutions

This section explores the problems induced by foresight on government spending in a simple analytical example, following Leeper, Walker, and Yang (2011) and Mertens and Ravn (2010). Leeper, Walker, and Yang (2011) analyze the econometric implications of foresight on future tax rates. Mertens and Ravn (2010) focus on government spending, in order to derive an SVAR estimator which is applicable in the face of permanent spending shocks. This section discusses potential solutions when economic agents' expectations can be observed. Throughout, the data is assumed to be generated by a version of the simple neoclassical growth model due to Hansen (1985). The main results would also hold in a larger DSGE model. The obvious advantage of using a simpler model is that analytical results can be derived more easily.[2]

3.2.1 Model description

The model economy is inhabited by a continuum of identical, infinitely lived households, whose instantaneous utility depends on consumption c_t and hours worked n_t. The

[2]This section focuses on a basic information structure with two periods anticipation and a single news shock, the latter following Mertens and Ravn (2010) and Ramey (2011b). The spending process could also be extended to longer anticipation horizons without affecting the main results.

households provide labor services and physical capital k_t to firms and they pay lump-sum taxes τ_t to the government. Time is indexed by $t = 0, 1, 2, \ldots, \infty$. All variables are denoted in real terms. The objective of a representative household in period t is to maximize expected discounted utility

$$E_t \sum_{s=0}^{\infty} \beta^s \left(\log c_{t+s} - A n_{t+s}\right), \qquad \beta \in (0,1), \qquad A > 0,$$

where E_t is the mathematical expectations operator conditional on the information available at time t. The household's optimization problem is subject to the period-by-period budget constraint

$$c_t + i_t + \tau_t = w_t n_t + r_t k_{t-1},$$

where w_t denotes the hourly wage and i_t denotes investment in physical capital at the rental rate r_t. Physical apital accumulates according to the law of motion

$$k_t = (1-\delta) k_{t-1} + i_t, \qquad \delta \in [0,1]. \tag{3.1}$$

There is also a continuum of identical, perfectly competitive firms that produce the final consumption good y_t. The technology of a representative firm is based on a Cobb-Douglas production function in capital and labor:

$$y_t = a_t k_{t-1}^{\alpha} n_t^{1-\alpha}, \qquad \alpha \in (0,1), \tag{3.2}$$

where a_t is total factor productivity (TFP) which is assumed to follow the law of motion

$$\log a_t = \rho_a \log a_{t-1} + \varepsilon_{a,t}, \qquad \rho_a \in [0,1), \tag{3.3}$$

with $\varepsilon_{a,t} \sim N(0, \sigma_a^2)$. Profit maximization yields the factor prices $w_t = (1-\alpha)\, y_t / n_t$ and $r_t = \alpha y_t / k_{t-1}$.

Government spending (i.e. purchases of the final good) g_t is assumed to be financed exclusively by lump-sum taxes, $g_t = \tau_t$, and it is modelled as an exogenous stochastic process:

$$\log(g_t/\bar{g}) = \rho_g \log(g_{t-1}/\bar{g}) + \varepsilon_{g,t}^{u} + \varepsilon_{g,t-2}^{a}, \qquad \rho_g \in [0,1), \tag{3.4}$$

where $\bar{g} = g$ is the non-stochastic steady state level of government spending, which is taken as given. This process allows for a surprise (unanticipated) shock to government spending $\varepsilon^u_{g,t} \sim N(0, \sigma^2_{g,u})$ and a news (anticipated) shock $\varepsilon^a_{g,t} \sim N(0, \sigma^2_{g,a})$. When a news shock occurs, the associated change in spending is known by economic agents two periods in advance of its implementation in terms of actual spending.

Combining the household's budget constraint with the government's budget constraint and the firm's first-order conditions, the feasibility constraint reads

$$c_t + i_t + g_t = y_t. \tag{3.5}$$

Substituting out the factor prices in the household's first-order conditions yields the labor/leisure trade-off and the consumption Euler equation:

$$\begin{aligned} Ac_t &= (1-\alpha)y_t/n_t, && (3.6) \\ c_t^{-1} &= \beta E_t R_{t+1}/c_{t+1}, && (3.7) \end{aligned}$$

where R_t denotes the real return on capital, that is

$$R_t = 1 - \delta + \alpha y_t/k_{t-1}. \tag{3.8}$$

The *rational expectations equilibrium* of this model is then the set of sequences $\{c_t, n_t, i_t, k_t, R_t, y_t, a_t, g_t\}_{t=0}^{\infty}$ satisfying (3.1) to (3.8) and the transversality condition for capital, for given initial values k_{-1}, a_{-1}, and g_{-1}, and given sequences of shocks $\{\varepsilon_{a,t}, \varepsilon^u_{g,t}, \varepsilon^a_{g,t}\}_{t=0}^{\infty}$.

To obtain an analytical solution to the model, the equilibrium system is log-linearized around the non-stochastic steady state and the log-linearized system is solved by the method of undetermined coefficients (see Uhlig, 1999), similarly as in Mertens and Ravn (2010). In particular, the log-linearized system can be reduced to the following two-dimensional, first-order stochastic difference equation in consumption and capital:

$$\begin{aligned} 0 &= \hat{c}_t - \phi_1 E_t \hat{c}_{t+1} + \phi_2 E_t \hat{a}_{t+1}, \\ 0 &= \phi_3 \hat{c}_t + \phi_4 \hat{k}_t - \phi_5 \hat{a}_t - \phi_6 \hat{k}_{t-1} + \phi_7 \hat{g}_t, \end{aligned}$$

Table 3.1: Benchmark calibration of the model

Parameter	Value	Description	Source/moment[a]
β	0.990	Subjective discount factor	KP (1982), Hansen (1985)
α	0.360	Capital share in production	KP (1982), Hansen (1985)
δ	0.025	Quarterly depreciation rate	KP (1982), Hansen (1985)
A	2.500	Disutility of labor supply	Time for market activities
a	1.000	Steady state TFP	Normalization
g/y	0.080	Government spending over GDP	Federal spending ratio
g	0.100	Steady state government spending	Federal spending ratio
ρ_a	0.950	AR(1) parameter TFP	Hansen (1985)
ρ_g	0.850	AR(1) parameter gov. spending	Galí et al. (2007)
σ_a	0.710	Std. dev. TFP shocks (%)	Hansen (1985)
$\sigma_{g,u}$	1.000	Std. dev. anticip. spend. shocks (%)	Benchmark
$\sigma_{g,a}$	1.000	Std. dev. unanticip. spend. shocks (%)	Benchmark

[a] KP (1982) refers to Kydland and Prescott (1982).

given the exogenous processes $\hat{g}_t = \rho_g \hat{g}_{t-1} + \varepsilon^u_{g,t} + \varepsilon^a_{g,t-2}$ and $\hat{a}_t = \rho_a \hat{a}_{t-1} + \varepsilon_{a,t}$, where $\hat{x}_t = \log(x_t/x)$ denotes the log deviation of variable x_t from its steady state value x. The parameters ϕ_i, $i = 1, 2, 3, \ldots, 7$ are functions of the model parameters and steady state values. The recursive laws of motion describing the solution are as follows:

$$\hat{k}_t = \eta_{kk}\hat{k}_{t-1} + \eta_{ka}\hat{a}_t + \eta_{kg}\hat{g}_t + \eta_{k\varepsilon,2}\varepsilon^a_{g,t-1} + \eta_{k\varepsilon,1}\varepsilon^a_{g,t}, \tag{3.9}$$

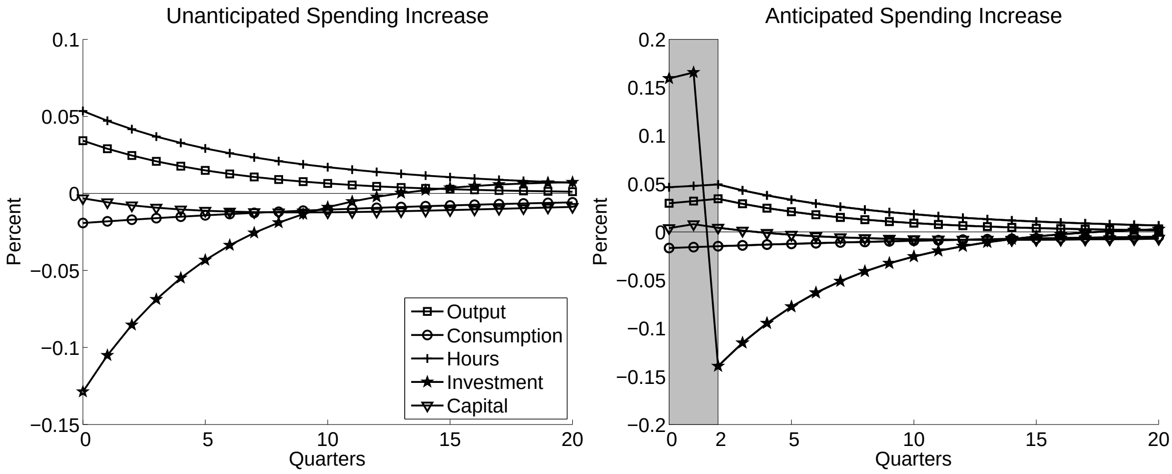
$$\hat{c}_t = \eta_{ck}\hat{k}_{t-1} + \eta_{ca}\hat{a}_t + \eta_{cg}\hat{g}_t + \eta_{c\varepsilon,2}\varepsilon^a_{g,t-1} + \eta_{c\varepsilon,1}\varepsilon^a_{g,t}, \tag{3.10}$$

where the coefficients η are non-linear functions of the model parameters and the parameters ϕ. A detailed derivation is provided in Appendix 3.A. Notice that, according to (3.9) and (3.10), the solution is characterized by the fact that the information set of economic agents in period t includes the shocks $\varepsilon^a_{g,t-1}$ and $\varepsilon^a_{g,t}$ as state variables.[3]

The model is calibrated in line with the real business cycle literature (see Hansen, 1985; Kydland and Prescott, 1982) and to match selected moments in U.S. quarterly

[3]The full information set includes $\{\varepsilon_{a,j}, \varepsilon^u_{g,j}, \varepsilon^a_{g,j}\}^t_{j=0}$. The shocks $\{\varepsilon_{a,j}\}^t_{j=0}$ are incorporated in $\hat{a}_t$ and (up to time $t-1$) in $\hat{k}_{t-1}$. The shocks $\{\varepsilon^u_{g,j}\}^t_{j=0}$ and $\{\varepsilon^a_{g,j}\}^{t-2}_{j=0}$ are also incorporated in $\hat{g}_t$ and $\hat{k}_{t-1}$ whereas $\varepsilon^a_{g,t-1}$ and $\varepsilon^a_{g,t}$ are announced but not yet implemented innovations to spending. Therefore, the latter appear as additional state variables in the recursive laws of motion.

Figure 3.1: Model impulse responses I – both types of spending shocks



Notes. Benchmark calibration; both panels show responses to one percent increases in government spending relative to its steady state value; left panel: surprise spending increase in quarter 0; right panel: news in quarter 0 that spending will increase in quarter 2; responses are measured as relative percentage deviations from steady state.

data over the period 1981Q4 to 2010Q1. The benchmark calibration is provided in Table 3.1. The subjective discount factor β is set to 0.99, which implies a steady state annual real interest rate of approximately four percent.[4] The capital share in production α is set to 0.36 and the quarterly depreciation rate δ is set to 0.025. The parameter A is set to 2.5, which implies that steady state hours worked is close to 1/3. The steady state ratio of government spending over GDP, g/y, is set to its empirical counterpart (the average federal government spending share) of eight percent. Finally, the standard deviation of TFP shocks is set to 0.71 percent and the AR(1) parameter of TFP is set to 0.95, following Hansen (1985). The standard deviations of the two spending innovations are set to one percent and the AR(1) parameter of government spending is set to 0.85 (see e.g. Galí et al., 2007).

Figure 3.1 shows impulse responses to one percent spending shocks of both types. As the model satisfies Ricardian equivalence, a surprise spending increase in quarter 0 (left panel) that is financed by lump-sum taxes has a negative wealth effect on the household's lifetime income. Consumption declines and, since leisure is a normal good,

[4]The average annual 3-month U.S. treasury bill secondary market rate was approximately equal to five percent over the period 1981Q4 to 2010Q1.

hours worked increase. Although the return on investment increases, the negative investment response is dictated by the feasibility constraint under the chosen calibration. On the other hand, if there is news in quarter 0 that spending will increase in quarter 2 (right panel) the investment response is positive during two quarters and then turns negative. There is an immediate negative wealth effect due to higher future taxes, so consumption declines immediately and hours worked and output increase immediately. The feasibility constraint allows investment to increase during the anticipation period since there is no government absorption of goods and services yet in that period.

3.2.2 The non-fundamentalness problem

To characterize the non-fundamentalness problem induced by the news shock $\varepsilon^a_{g,t}$, notice that the coefficient η_{kk} is the stable root of the characteristic equation

$$0 = \phi_1\phi_4\eta_{kk}^2 - (\phi_1\phi_6 + \phi_4)\eta_{kk} + \phi_6.$$

In a unique saddle path solution, this equation has two real roots η^+_{kk} and η^-_{kk},

$$\eta^{\pm}_{kk} = (\phi_1^{-1} + \phi_6\phi_4^{-1})/2 \pm \sqrt{(\phi_1^{-1} + \phi_6\phi_4^{-1})^2/4 - \phi_6(\phi_1\phi_4)^{-1}}, \tag{3.11}$$

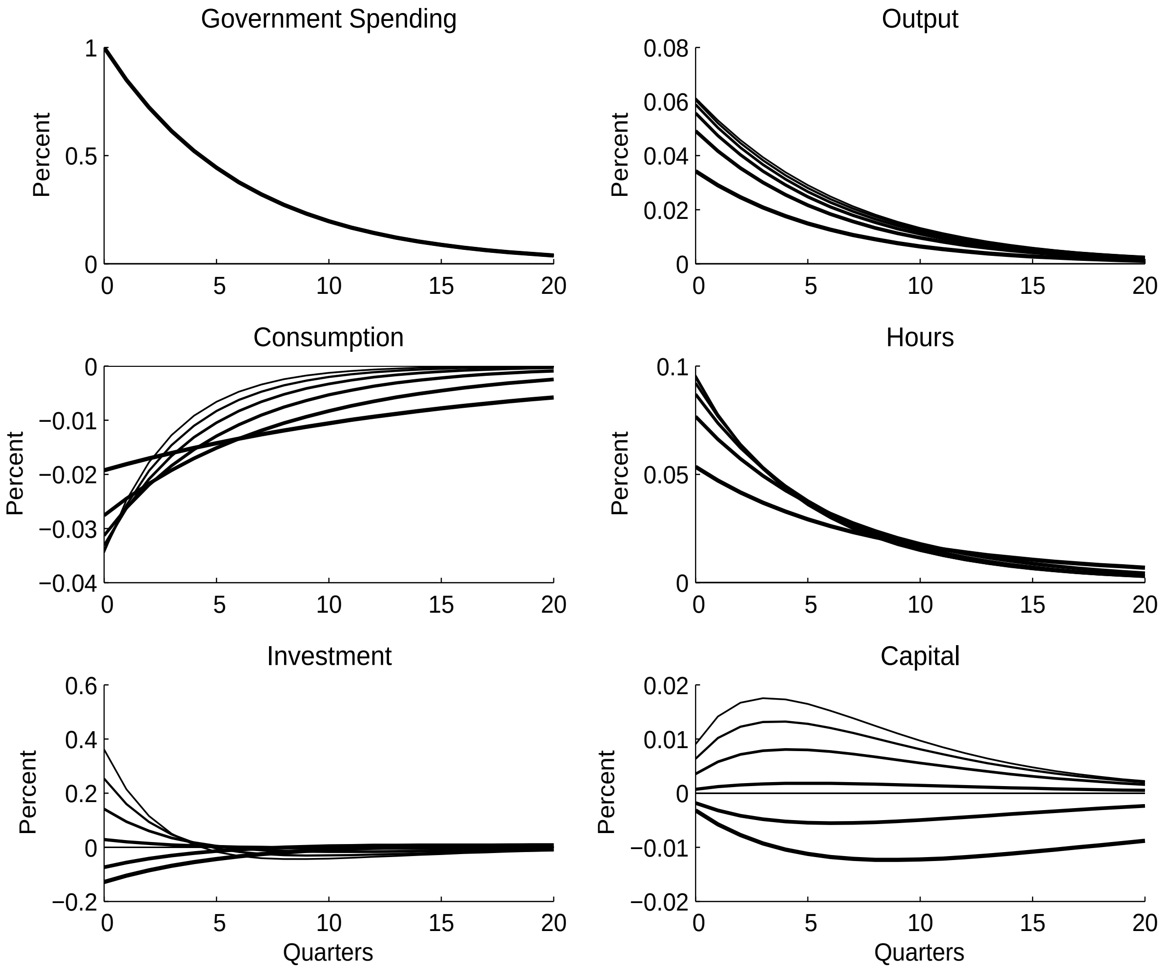
only one of which satisfies $|\eta_{kk}| < 1$. Furthermore, it is straightforward to show that the coefficients $\eta_{x\varepsilon,1}$ and $\eta_{x\varepsilon,2}$ $(x = k, c)$, are related with each other as follows:

$$\eta_{x\varepsilon,1} = \theta\eta_{x\varepsilon,2}, \qquad \theta = (\phi_1^{-1} + \phi_6\phi_4^{-1} - \eta_{kk})^{-1}.$$

Inserting the expression for θ in (3.11), it follows that $|\theta| < 1$.[5] This result implies that, when forming their decisions, economic agents discount more recent news on government spending $\varepsilon^a_{g,t}$ relative to more distant news $\varepsilon^a_{g,t-1}$ at a constant *anticipation rate* given by θ. The reason is that recent news affects spending later than distant news (see Leeper, Walker, and Yang, 2011; Mertens and Ravn, 2010). As noted by

[5] To see this, suppose that $\left|\eta^+_{kk}\right| = \left|\frac{1}{2}(\phi_1^{-1} + \phi_6\phi_4^{-1}) + [\frac{1}{4}(\phi_1^{-1} + \phi_6\phi_4^{-1})^2 - \phi_6(\phi_1\phi_4)^{-1}]^{1/2}\right| < 1$ such that $\left|\eta^-_{kk}\right| = \left|\frac{1}{2}(\phi_1^{-1} + \phi_6\phi_4^{-1}) - [\frac{1}{4}(\phi_1^{-1} + \phi_6\phi_4^{-1})^2 - \phi_6(\phi_1\phi_4)^{-1}]^{1/2}\right| > 1$. Then $\eta_{kk} = \eta^+_{kk}$ and, by direct calculation, $\theta = (\eta^-_{kk})^{-1}$, which implies that $|\theta| < 1$. Conversely, if $\left|\eta^+_{kk}\right| > 1$ and $\left|\eta^-_{kk}\right| < 1$ then $\eta_{kk} = \eta^-_{kk}$ and $\theta = (\eta^+_{kk})^{-1}$, which again implies that $|\theta| < 1$.

Figure 3.2: Model impulse responses II – unanticipated spending shock

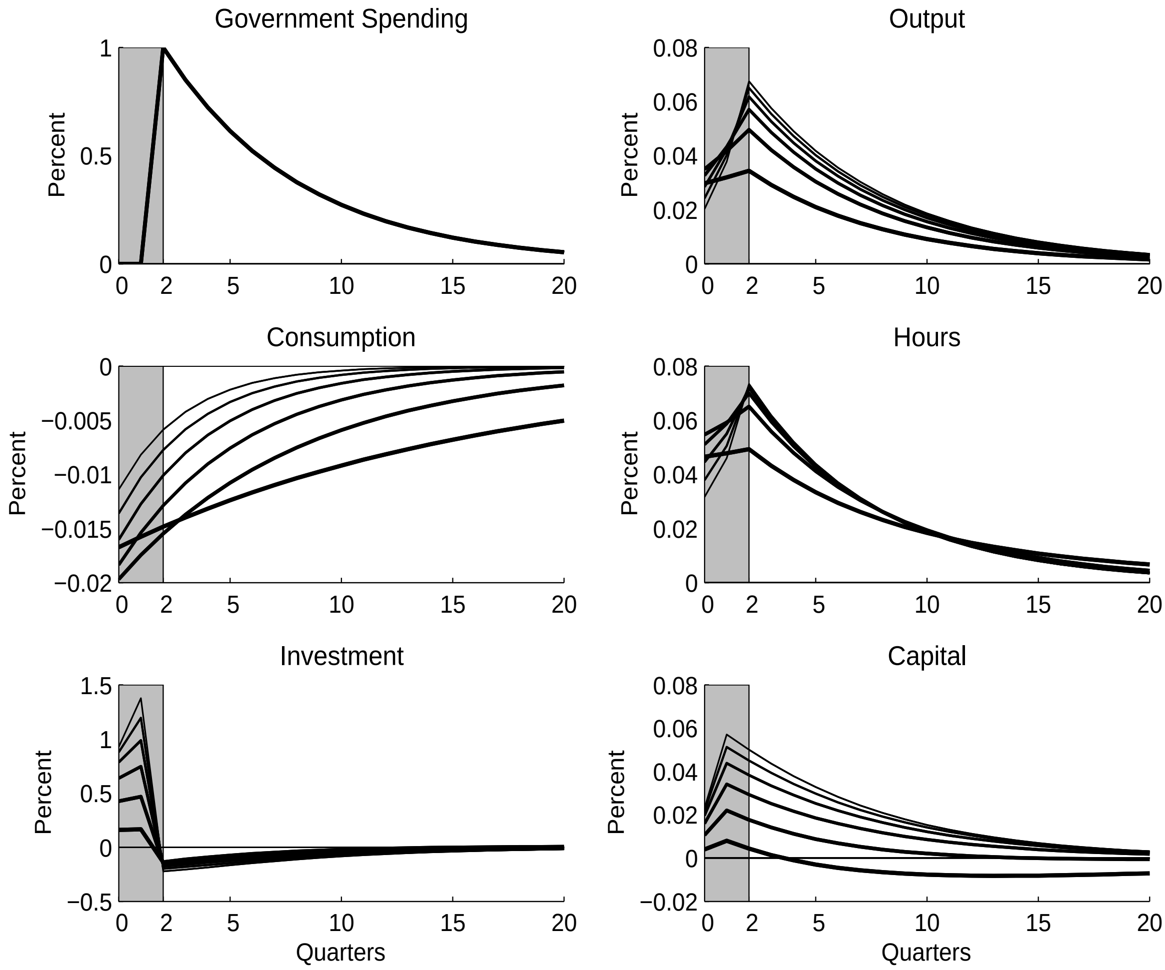


Notes. Changing anticipation rate θ; from thin to thick lines: θ is changed from 0.58 to 0.93 by changing the discount factor β from 0.8 to 0.99; thickest line: benchmark calibration; responses are measured as relative percentage deviations from steady state.

Mertens and Ravn (2010), the result of constant discounting generalizes to other settings (e.g. longer anticipation horizons, more control variables). Mertens and Ravn (2010) also show that the anticipation rate θ is, inter alia, monotonically increasing in the subjective discount factor β. This fact is exploited below.

Thus, Figures 3.2 and 3.3 show the impulse responses to the two types of spending shocks when β changes from 0.8 to 0.99 from thin to thick lines, implying values for θ from 0.58 to 0.93. The initial responses of consumption and hours to unanticipated shocks are uniformly stronger for lower discount factors (see Figure 3.2). The reason

Figure 3.3: Model impulse responses III – anticipated spending shock



Notes. News in quarter 0 that spending will increase in quarter 2; see Figure 3.2.

is that future utility is then discounted at a higher rate, so households have a lower preference for consumption smoothing. For the same reason, the anticipation rate falls when β decreases such that more recent news receives a heavier discount than more distant news. An anticipated increase in government spending thus affects several variables more strongly prior to its implementation in terms of actual spending for lower β's and θ's (see Figure 3.3). For those variables, the differences between the impulse responses to both types of shocks *after* spending has increased become larger for lower anticipation rates. Compare, for example, the investment responses which are uniformly negative from quarter 2 onwards after the anticipated spending increase but positive for some parameter values after the unanticipated spending increase. The implications of these outcomes are discussed in turn.

The phenomenon of constant discounting is indeed the root of the non-fundamentalness problem. To see this, following Leeper, Walker, and Yang (2011), suppose that an econometrician who is not aware of policy foresight estimates a VAR model in $\{\hat{g}_{t-j}, \hat{a}_{t-j}, \hat{k}_{t-j}\}_{j=0}^{\infty}$. According to the equilibrium representation implied by the underlying theoretical model, the econometrician's observables can be shown to follow the multivariate moving average process

$$\begin{bmatrix} \hat{g}_t \\ \hat{a}_t \\ \hat{k}_t \end{bmatrix} = \begin{bmatrix} \frac{1}{1-\rho_g L} & \frac{L^2}{1-\rho_g L} & 0 \\ 0 & 0 & \frac{1}{1-\rho_a L} \\ \frac{\eta_{kg}}{(1-\eta_{kk}L)(1-\rho_g L)} & \frac{\eta_{kg}L^2+\eta_{k\varepsilon,2}(1-\rho_g L)(\theta+L)}{(1-\eta_{kk}L)(1-\rho_g L)} & \frac{\eta_{ka}}{(1-\eta_{kk}L)(1-\rho_a L)} \end{bmatrix} \begin{bmatrix} \varepsilon^u_{g,t} \\ \varepsilon^a_{g,t} \\ \varepsilon_{a,t} \end{bmatrix},$$

or

$$\boldsymbol{y}_t = \boldsymbol{P}(L)\boldsymbol{\epsilon}_t, \tag{3.12}$$

where L denotes the lag operator, i.e. $L^s x_t = x_{t-s}$ for $s \geq 0$. If the process (3.12) is invertible in non-negative powers of L, then the econometrician can recover the structural shocks as a linear combination of present and past observables, i.e. $\boldsymbol{\epsilon}_t = \boldsymbol{P}^{-1}(L)\boldsymbol{y}_t$. A necessary and sufficient condition for $\boldsymbol{\epsilon}_t$ to be fundamental for $\boldsymbol{y}_t$ is that the zeroes of the determinant of $\boldsymbol{P}(z)$ do not lie inside the unit circle (see Hansen and Sargent, 1991). The determinant of $\boldsymbol{P}(z)$ is given by

$$\det \boldsymbol{P}(z) = \frac{-\eta_{k\varepsilon,2}\,(\theta+z)}{(1-\eta_{kk}z)\,(1-\rho_g z)\,(1-\rho_a z)},$$

which has a root inside the unit circle at $z = -\theta$. The structural shocks $\{\varepsilon^a_{g,j}, \varepsilon^u_{g,j}, \varepsilon_{a,j}\}_{j=0}^{t}$ thus cannot be recovered from the information set $\{\hat{g}_{t-j}, \hat{a}_{t-j}, \hat{k}_{t-j}\}_{j=0}^{\infty}$ since (3.12) is not a fundamental moving average (Wold) representation of the equilibrium time series process. Hence, if the econometrician only observes present and past spending along with present and past TFP and capital, his or her information set is misaligned with the information set of economic agents, who have knowledge of news shocks to future spending over their anticipation horizon already before those shocks have an impact on actual spending.

3.2.3 An expectations-based solution

A natural way to align the two information sets is to incorporate the agents' expectations on future spending in the econometrician's information set. Thus, suppose that instead of present and past TFP, the econometrician observes present and past expectations on spending two periods ahead conditional on time t information. The econometrician thus estimates a VAR in $\{\hat{g}_{t-j}, E_{t-j}\hat{g}_{t+2-j}, \hat{k}_{t-j}\}_{j=0}^{\infty}$, where

$$\begin{bmatrix} \hat{g}_t \\ E_t\hat{g}_{t+2} \\ \hat{k}_t \end{bmatrix} = \begin{bmatrix} \frac{1}{1-\rho_g L} & \frac{L^2}{1-\rho_g L} & 0 \\ \frac{\rho_g^2}{1-\rho_g L} & \frac{1}{1-\rho_g L} & 0 \\ \frac{\eta_{kg}}{(1-\eta_{kk}L)(1-\rho_g L)} & \frac{\eta_{kg}L^2+\eta_{k\varepsilon,2}(1-\rho_g L)(\theta+L)}{(1-\eta_{kk}L)(1-\rho_g L)} & \frac{\eta_{ka}}{(1-\eta_{kk}L)(1-\rho_a L)} \end{bmatrix} \begin{bmatrix} \varepsilon_{g,t}^{u} \\ \varepsilon_{g,t}^{a} \\ \varepsilon_{a,t} \end{bmatrix},$$

or

$$\boldsymbol{y}_t^* = \boldsymbol{P}^*(L)\boldsymbol{\epsilon}_t. \tag{3.13}$$

The determinant of $\boldsymbol{P}^*(z)$ is given by

$$\det \boldsymbol{P}^*(z) = \frac{\eta_{ka}(1+\rho_g z)}{(1-\rho_g z)(1-\eta_{kk}z)\,(1-\rho_a z)},$$

which has one root outside the unit circle at $z = -\rho_g^{-1}$ and three poles at $z = \rho_g^{-1}$, $z = \eta_{kk}^{-1}$, and $z = \rho_a^{-1}$. Hence, (3.13) is indeed an invertible moving average process such that the structural shocks in $\boldsymbol{\epsilon}_t$ can in principle be recovered from the information set $\{\hat{g}_{t-j}, E_{t-j}\hat{g}_{t+2-j}, \hat{k}_{t-j}\}_{t=0}^{\infty}$ by a linear combination of present and past observables. Through the inclusion of economic agents' expectations on government spending in the econometrician's information set, (3.13) is a fundamental Wold representation of the equilibrium time series process.

3.2.4 Confronting the identification problem

Fundamentalness of the structural shocks with respect to the econometrician's information set is necessary but not sufficient to be able to correctly estimate of their effects. In particular, after obtaining reduced-form estimates, the econometrician needs to recover the structural shocks through an appropriate identification strategy. The econometrician thus estimates an unrestricted VAR of the form

$$\boldsymbol{y}_t^* = \boldsymbol{B}_1\boldsymbol{y}_{t-1}^* + \boldsymbol{B}_2\boldsymbol{y}_{t-2}^* + \boldsymbol{B}_3\boldsymbol{y}_{t-3}^* + \cdots + \boldsymbol{u}_t = \boldsymbol{C}(L)\boldsymbol{u}_t, \quad \boldsymbol{u}_t \sim N(\boldsymbol{0}, \boldsymbol{\Sigma}), \tag{3.14}$$

where the $\boldsymbol{B}_i$, $i = 1, 2, 3, \ldots$, are matrices of coefficients and $\boldsymbol{C}(L)$ is the infinite order multivariate lag polynomial of the moving average representation in the innovations $\boldsymbol{u}_t$, satisfying $\boldsymbol{C}(0) = \boldsymbol{I}$. Assuming that there exists a linear mapping between the reduced-form innovations and the structural shocks, i.e. $\boldsymbol{u}_t = \boldsymbol{D}\boldsymbol{\epsilon}_t$, the moving average representation in the structural shocks is given by $\boldsymbol{y}_t^* = \boldsymbol{C}(L)\boldsymbol{D}\boldsymbol{\epsilon}_t$ with $\boldsymbol{\epsilon}_t = \boldsymbol{D}^{-1}\boldsymbol{u}_t$. Normalizing $\text{cov}(\boldsymbol{\epsilon}_t) = \boldsymbol{I}$, the impact matrix $\boldsymbol{D}$ must satisfy $\boldsymbol{D}\boldsymbol{D}' = \boldsymbol{\Sigma}$ or equivalently $\boldsymbol{D} = \boldsymbol{A}\boldsymbol{R}$, where $\boldsymbol{R}$ is an orthonormal matrix such that $\boldsymbol{R}\boldsymbol{R}' = \boldsymbol{I}$ and $\boldsymbol{A}$ is an arbitrary orthogonalization of the reduced-form covariance matrix $\boldsymbol{\Sigma}$. The latter could for example be achieved by a Cholesky decomposition of $\boldsymbol{\Sigma}$ such that $\boldsymbol{R} = \boldsymbol{I}$.

Suppose that the econometrician is only interested in the effects of government spending shocks. When observing $\{\hat{g}_{t-j}, E_{t-j}\hat{g}_{t+2-j}, \hat{k}_{t-j}\}_{j=0}^{\infty}$, the econometrician could identify the two types of spending shocks by taking the innovations to spending as the unanticipated shocks and by taking the innovations to expected spending that are orthogonal to the latter as the anticipated shocks. The shocks can thus be identified by a Cholesky decomposition of the reduced-form covariance matrix associated with (3.14) that has government spending ordered before its two-quarter ahead expectation and with capital ordered last. In fact, the impact matrix that is obtained by setting $L = 0$ in (3.13) is lower triangular:

$$\boldsymbol{P}^*(0) = \begin{bmatrix} 1 & 0 & 0 \\ \rho_g^2 & 1 & 0 \\ \eta_{kg} & \eta_{k\varepsilon,1} & \eta_{ka} \end{bmatrix},$$

which implies that the processes (3.13) and (3.14) have a Cholesky structure.

As an alternative, the econometrician could observe the one-period expectational error $\hat{g}_t - E_{t-1}\hat{g}_t$ instead of realized spending $\hat{g}_t$. Since $E_{t-1}\hat{g}_t = \rho_g\hat{g}_{t-1} + \varepsilon_{t-2}^a$, it follows that $\hat{g}_t - E_{t-1}\hat{g}_t = \varepsilon_t^u$. The VAR model in $\{\hat{g}_{t-j} - E_{t-1-j}\hat{g}_{t-j}, E_{t-j}\hat{g}_{t+2-j}, \hat{k}_{t-j}\}_{j=0}^{\infty}$ is then given by

$$\begin{bmatrix} \hat{g}_t - E_{t-1}\hat{g}_t \\ E_t\hat{g}_{t+2} \\ \hat{k}_t \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ \frac{\rho_g^2}{1-\rho_g L} & \frac{1}{1-\rho_g L} & 0 \\ \frac{\eta_{kg}}{(1-\eta_{kk}L)(1-\rho_g L)} & \frac{\eta_{kg}L^2+\eta_{k\varepsilon,2}(1-\rho_g L)(\theta+L)}{(1-\eta_{kk}L)(1-\rho_g L)} & \frac{\eta_{ka}}{(1-\eta_{kk}L)(1-\rho_a L)} \end{bmatrix} \begin{bmatrix} \varepsilon_{g,t}^u \\ \varepsilon_{g,t}^a \\ \varepsilon_{a,t} \end{bmatrix},$$

or

$$\boldsymbol{y}_t^{**} = \boldsymbol{P}^{**}(L)\boldsymbol{\epsilon}_t. \tag{3.15}$$

The determinant of $\boldsymbol{P}^{**}(z)$ is given by

$$\det \boldsymbol{P}^{**}(z) = \frac{\eta_{ka}}{(1-\rho_g z)(1-\eta_{kk}z)\,(1-\rho_a z)},$$

such that (3.15) is also an invertible moving average process. The econometrician could now identify the spending shocks by taking the expectational errors as the unanticipated shocks and by taking the innovations to expected spending that are orthogonal to the expectational errors as the anticipated shocks. Accordingly, the shocks can again be identified by a Cholesky decomposition as the impact matrix associated with (3.15) is also lower triangular; in fact, it satisfies $\boldsymbol{P}^{**}(0) = \boldsymbol{P}^{*}(0)$ as given above.

The two identification strategies just discussed are however not equivalent: the first variant relies on VAR forecasts to achieve identification while the second variant uses additional information from data on expectational errors. The following section thus compares the two variants using stochastic simulations.

3.3 Simulation evidence

This section tests the usefulness of the proposed identification strategies. In particular, model-based stochastic simulations are conducted to compare the expectations-based approach and the standard recursive SVAR approach, which does not take into account the possibility of policy foresight. Section 3.4 discusses modifications to the benchmark model to check the robustness of the expectations-based approach, for example to alternative assumptions on the government spending process.

3.3.1 Monte Carlo set-up

The approach implemented here is a Monte Carlo exercise, following e.g. Ramey (2011b). That is, M data samples of length T are generated from the calibrated model.[6] The different approaches are first evaluated in terms of their large-sample properties, setting $T = 10,000$ and $M = 100$. Small-sample results are discussed in Section 3.4. The estimated impulse responses for each of the M samples are ordered and the mean estimates are reported, with 90 percent two-sided error bands. The estimated responses are then compared to the impulse responses implied by the model.

3.3.2 Standard SVAR identification

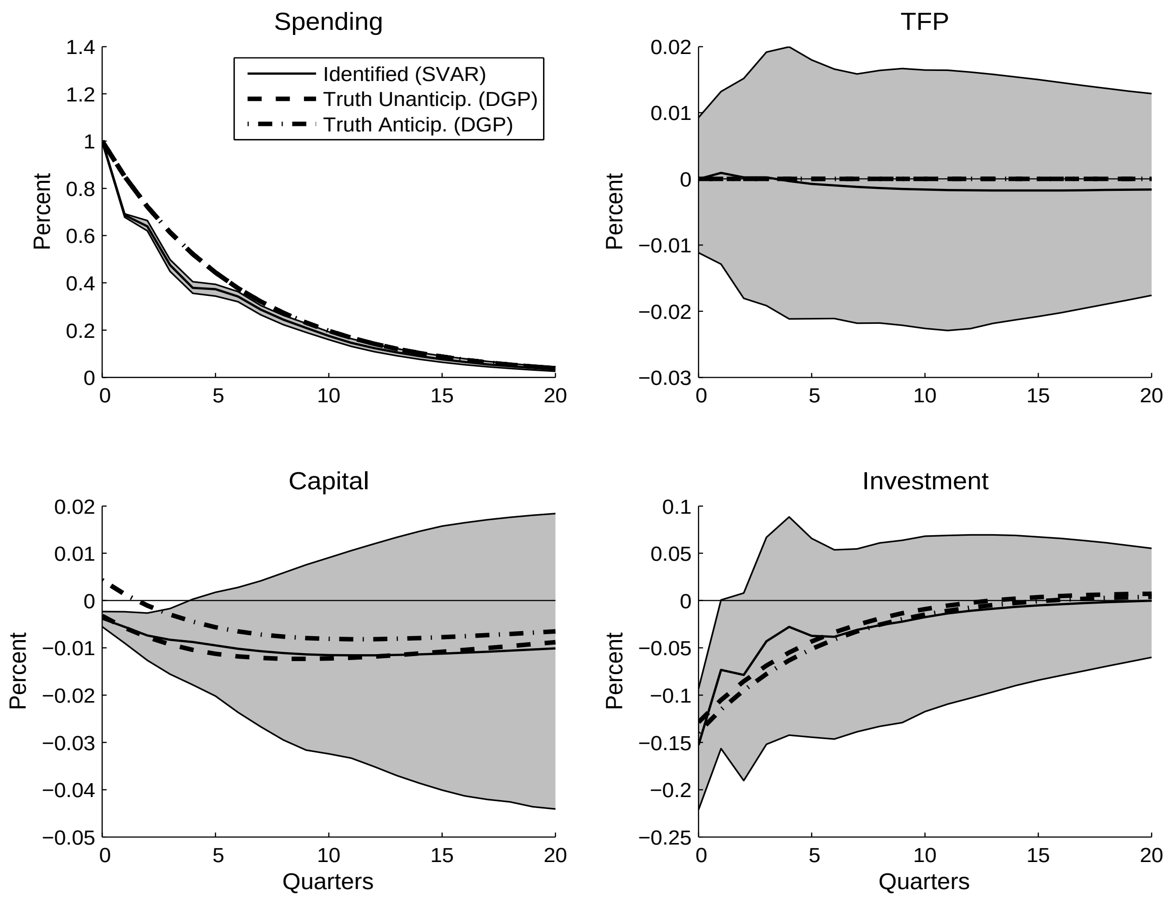
The properties of the standard SVAR identification approach are investigated first. Government spending, TFP, and capital are thus taken as observable and included in this order in the VAR model. As shown above, the equilibrium time series process associated with these observables has a non-fundamental equilibrium representation. Investment is added as a fourth variable to the regression equation. As there are only three shocks, to avoid stochastic singularity a measurement error on investment $\varepsilon_{i,t} \sim N(0, \sigma_i^2)$ with $\sigma_i = 0.0001$ is included in the data-generating process (DGP).

Figure 3.4 shows the estimated impulse responses for the benchmark calibration, where a government spending shock is identified as the innovation to government spending using a Cholesky decomposition of the reduced-form covariance matrix. The figure also shows the impulse responses to both shocks implied by the DGP from the quarter in which spending increases onwards. The impulse responses of course cannot pick up any variation in investment and capital due to anticipated shocks during the anticipation period. However, the results seem to suggest that the error with respect to the effects of surprise spending shock is not very large. The possibility of policy foresight may thus not matter much quantitatively even if it is ignored.

However, the latter is not true in general. Figure 3.5 reports the estimated impulse responses when the anticipation rate θ is reduced to 0.58 by reducing the subjective

[6] In this section, for convenience, the log-linearized model is solved numerically by the Gensys algorithm (see Sims, 2004).

Figure 3.4: Monte Carlo impulse responses – standard SVAR scheme I

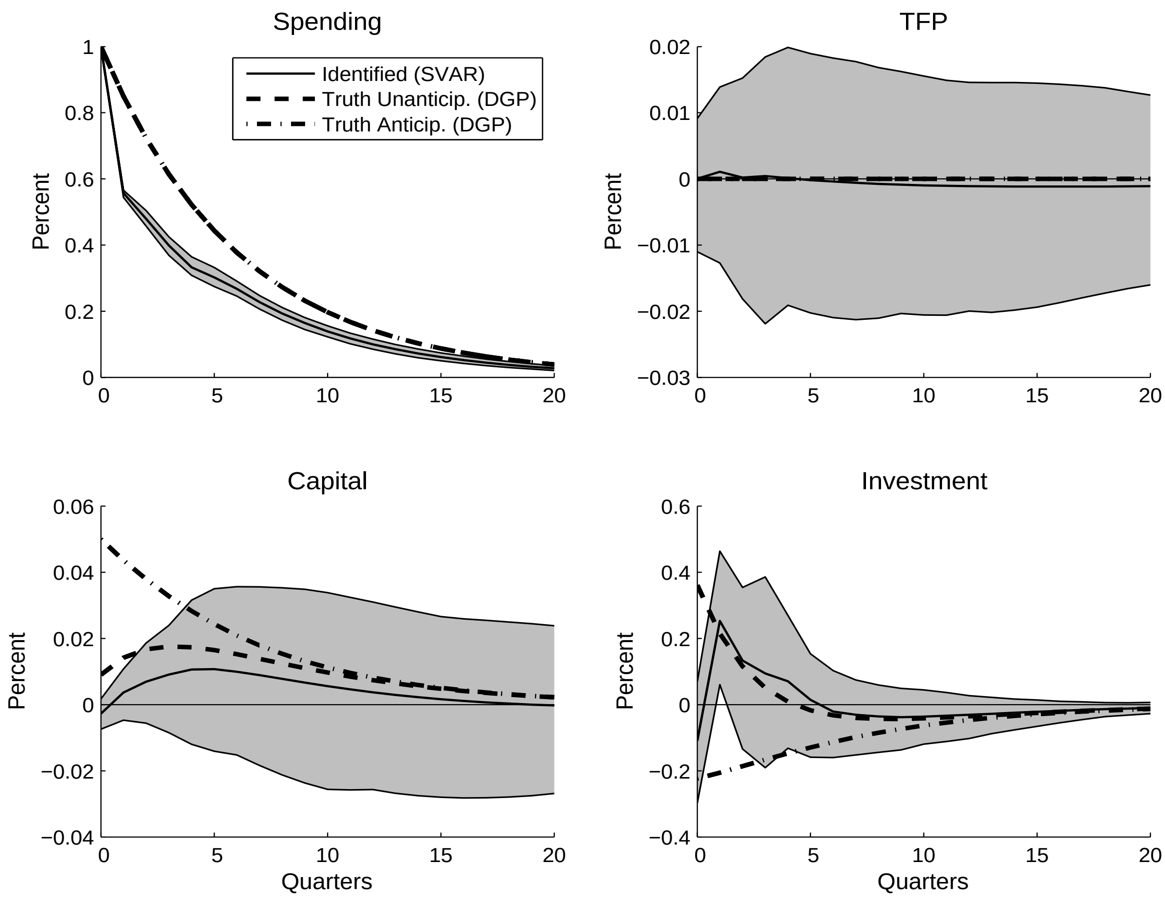


Notes. Benchmark calibration ($\theta = 0.93$); SVAR responses (means) and 90 percent error bands are based on 100 samples of 10,000 observations each; the spending shock is identified by ordering government spending first in a Cholesky decomposition; DGP responses to anticipated shock are plotted from spending increase onwards; responses are measured as relative percentage deviations from steady state.

discount factor β to 0.8.[7] The results show that the effects on investment and capital of neither of the two shocks are correctly estimated. The SVAR responses indicate an initial decline in investment followed by an increase, whereas the actual investment response to the unanticipated shock is positive for several quarters while the response to the anticipated shocks is uniformly negative (after the anticipation period). There is also a relatively strong downward bias in the estimated spending response, such that

[7]This is of course a relatively low value for β; alternatively, one could reduce the anticipation rate by increasing the intertemporal elasticity of substitution (which is equal to one with log utility) or lowering the capital share in production α (see Mertens and Ravn, 2010). A longer anticipation horizon would also create a stronger wedge between the effects of anticipated and unanticipated shocks.

Figure 3.5: Monte Carlo impulse responses – standard SVAR scheme II



Notes. Lower anticipation rate ($\theta = 0.58$, $\beta = 0.8$); see Figure 3.4.

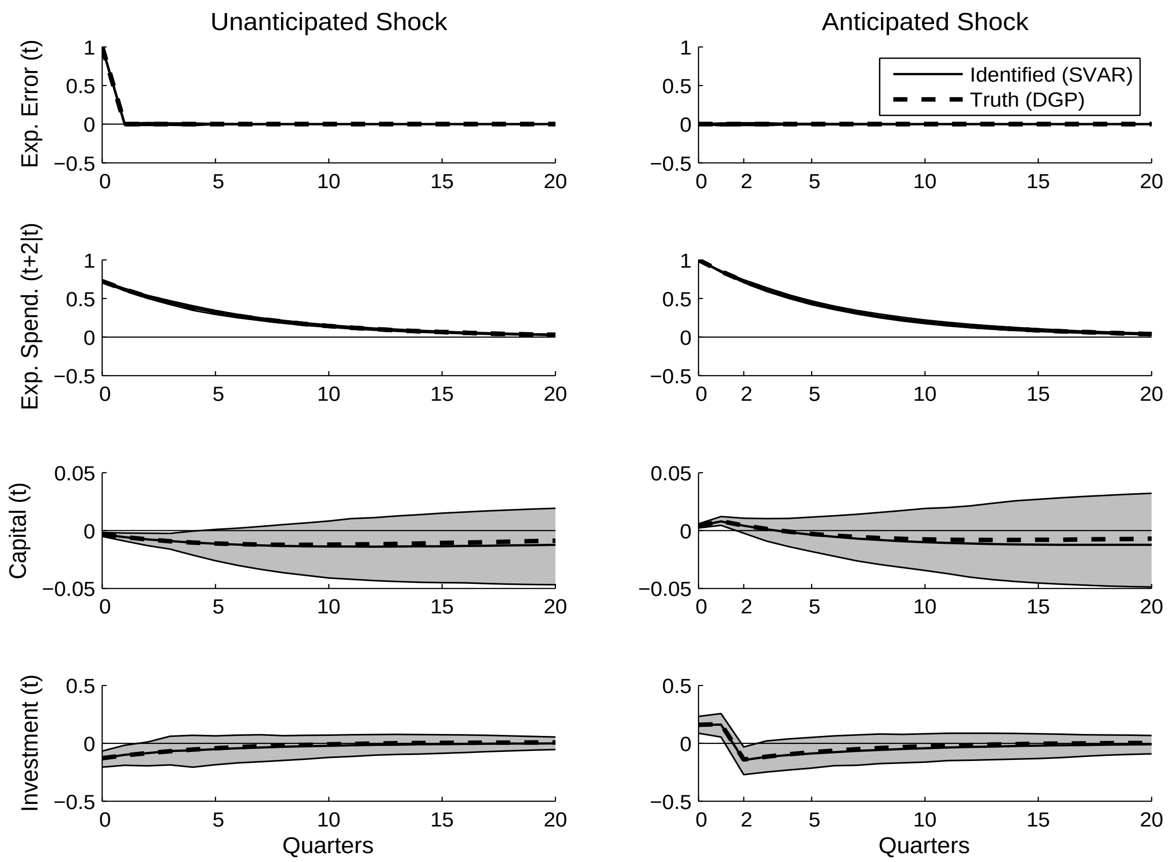
the econometrician would overstate the overall expansionary effect of government expenditures on investment. Hence, there are realistic cases where the standard recursive SVAR identification approach can lead to misleading conclusions.[8]

3.3.3 Expectations-based approach

The properties of the expectations-based identification approach are discussed next. The variant with expectational errors is analyzed first. The econometrician thus estimates a VAR in $\{\hat{g}_{t-j} - E_{t-1-j}\hat{g}_{t-j}, E_{t-j}\hat{g}_{t+2-j}, \hat{k}_{t-j}, \hat{i}^{obs}_{t-j}\}_{j=0}^{\infty}$. Adding investment with a measurement error to the regression goes without prejudice to the results discussed

[8]The bias in the estimated impulse responses turns even larger, and more in line with Ramey's (2011b) findings on the effects of anticipated shocks, if the standard deviation of anticipated shocks is increased relative to the standard deviation of unanticipated shocks (not reported).

Figure 3.6: Monte Carlo impulse responses – expectations-based scheme I



Notes. Benchmark calibration ($\theta = 0.93$); an unanticipated spending shock is identified by ordering the expectational error on spending first in a Cholesky decomposition, the shock is a one percent increase in the expectational error in quarter 0; an anticipated spending shock is identified by ordering the two-quarter ahead expectation of spending second, the shock being a one percent increase in the two-quarter ahead expectation in quarter 0; responses are measured as relative percentage deviations from steady state.

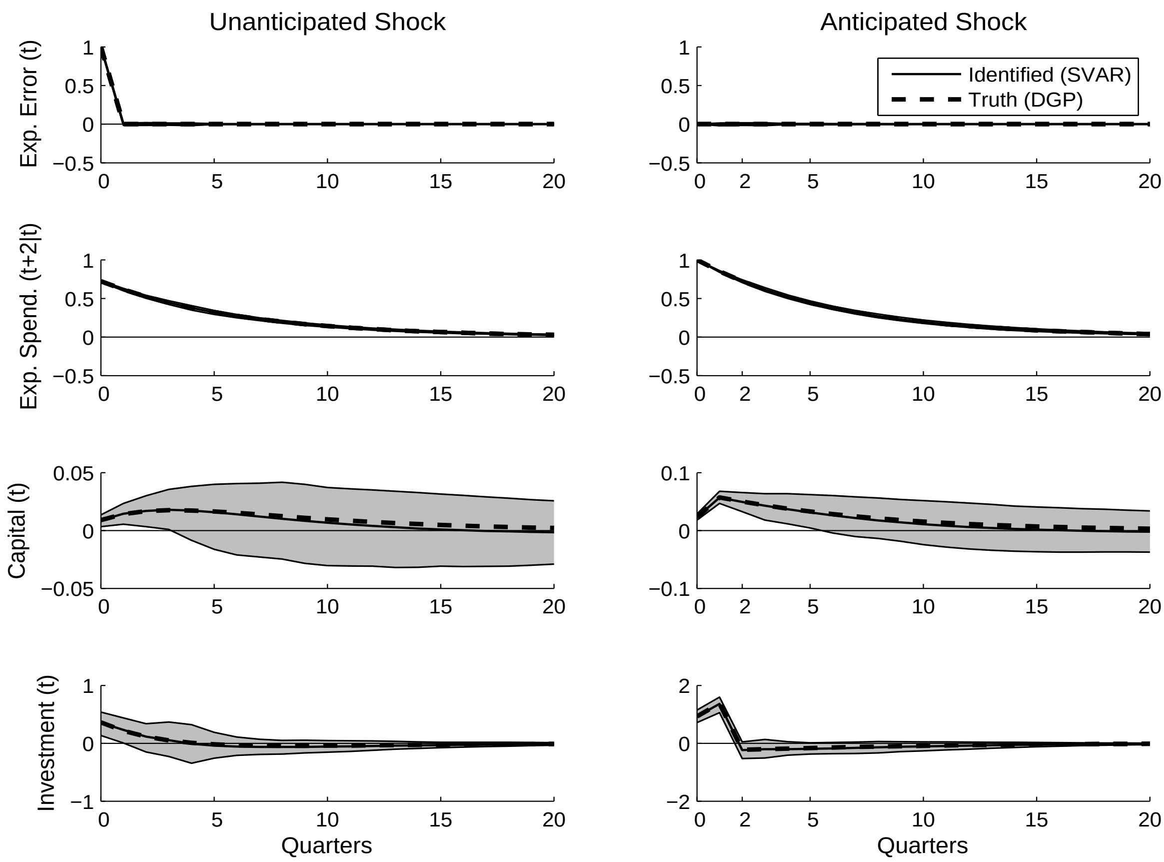
in the previous section. To see this, notice that the solution for investment is given by

$$\hat{i}_t = \eta_{ik}\hat{k}_{t-1} + \eta_{ia}\hat{a}_t + \eta_{ig}\hat{g}_t + \eta_{i\varepsilon,2}\varepsilon^a_{g,t-1} + \eta_{i\varepsilon,1}\varepsilon^a_{g,t}, \tag{3.16}$$

and observed investment is $\hat{i}^{obs}_t = \hat{i}_t + \varepsilon_{i,t}$. Hence, the econometrician's VAR reads

$$\begin{bmatrix} \hat{g}_t - E_{t-1}\hat{g}_t \\ E_t\hat{g}_{t+2} \\ \hat{k}_t \\ \hat{i}^{obs}_t \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \frac{\rho_g^2}{1-\rho_g L} & \frac{1}{1-\rho_g L} & 0 & 0 \\ \frac{\eta_{kg}}{(1-\eta_{kk}L)(1-\rho_g L)} & \frac{\eta_{kg}L^2+\eta_{k\varepsilon,2}(1-\rho_g L)(\theta+L)}{(1-\eta_{kk}L)(1-\rho_g L)} & \frac{\eta_{ka}}{(1-\eta_{kk}L)(1-\rho_a L)} & 0 \\ \Theta_1(L) & \Theta_2(L) & \Theta_3(L) & 1 \end{bmatrix} \begin{bmatrix} \varepsilon^u_{g,t} \\ \varepsilon^a_{g,t} \\ \varepsilon_{a,t} \\ \varepsilon_{i,t} \end{bmatrix},$$

Figure 3.7: Monte Carlo impulse responses – expectations-based scheme II



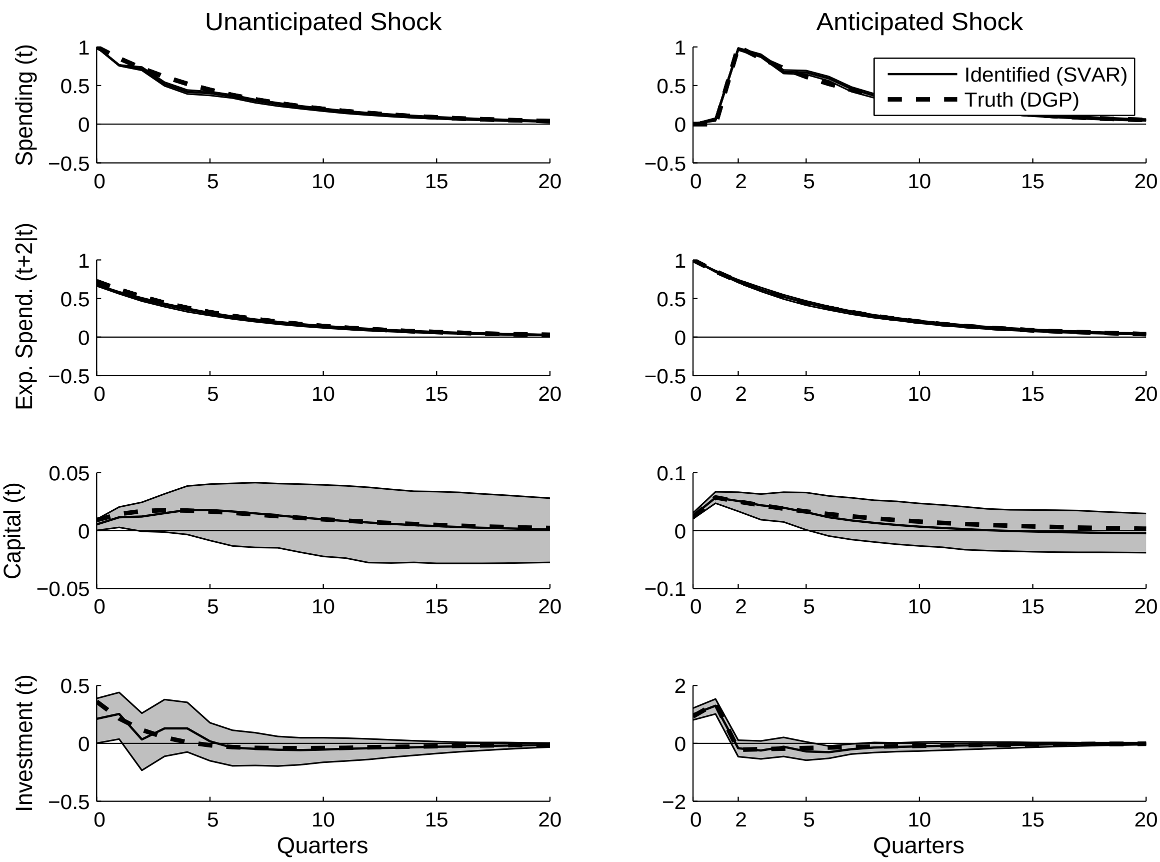
Notes. Lower anticipation rate ($\theta = 0.58$, $\beta = 0.8$); see Figure 3.6.

where the $\Theta_s(L)$, $s = 1, 2, 3$, follow from substituting out capital, TFP, and spending in (3.16). The determinant of the lag matrix is again equal to $\eta_{ka}/[(1-\rho_g z)(1-\eta_{kk}z)(1-\rho_a z)]$ as for (3.15), such that this modified process is also a fundamental one.

Figure 3.6 reports the expectations-based SVAR impulse respones to both types of spending shocks. The results show that the estimated effects closely match those of the DGP. Under the unanticipated shock, investment declines and the impact response of the two-quarter ahead expectation (expected spending in quarter $t + 2$ conditional on quarter t information) is close to the theoretical value of $\rho_g^2 = 0.72$. Under the anticipated shock, the two-quarter ahead expectation of spending increases by one percent in quarter 0. Importantly, investment increases during the anticipation period but it is negative from quarter 2 onwards, as implied by the DGP.

The previous exercise is now repeated when the anticipation rate θ is reduced

Figure 3.8: Monte Carlo impulse responses – alternative scheme



Notes. Lower anticipation rate ($\theta = 0.58$, $\beta = 0.8$); an unanticipated spending shock is identified by ordering spending first in a Cholesky decomposition, the shock is a one percent increase in spending in quarter 0; an anticipated spending shock is identified by ordering the two-quarter ahead expectation of spending second, the shock being a one percent increase in the two-quarter ahead expectation in quarter 0; responses are measured as relative percentage deviations from steady state.

to 0.58 by reducing β to 0.8, which was seen to increase the relevance of the non-fundamentalness problem of the standard recursive SVAR approach. However, Figure 3.7 shows that for the expectations-based approach the results are robust to changes in θ: the estimated effects of both shocks are similarly close to the DGP effects as under the benchmark calibration. The identification approach is also robust to changes in the relative volatility of the two types of spending shocks (not reported).

The variant of the expectations-based approach where actual spending is observed instead of the expectational errors is analyzed next. For brevity, only results for the lower anticipation rate are reported in Figure 3.8. The figure reveals some noticeable differences in the SVAR impulse responses and the DGP responses; in particular, the estimated investment response to the unanticipated shock is not as close to DGP response as under the variant with expectational errors. The additional information from data on expectational errors therefore seems useful to obtain precise estimates. The analysis thus proceeds with that variant, but Section 3.5 also checks the robustness of the empirical results when the variant with spending is used to achieve identification.

3.4 Robustness

This section discusses the results of three types of robustness exercises. First, the experiment of the previous section is repeated for a smaller sample size. Second, the spending process is modified by allowing feedbacks from other economic shocks (TFP shocks in the present model) on spending. Third, it is checked whether surprise spending shocks can also be correctly identified in a VAR model that does not include expectations on future spending but only expectational errors, as in Ramey (2011b) and Auerbach and Gorodnichenko (2010). The implications for the empirical application are discussed at the end of this section.

3.4.1 Small sample results

The results reported so far were based on large samples ($T = 10,000$). For Figure 3.9, the Monte Carlo exercise is repeated for an empirically realistic sample size of $T = 114$ and $M = 10,000$.[9] The reduction in the sample size implies that the data contains less information, so the error bands become wider. However, the point estimates remain close to the DGP responses. The bias of the standard SVAR approach is therefore still eliminated by the expectations-based approach.

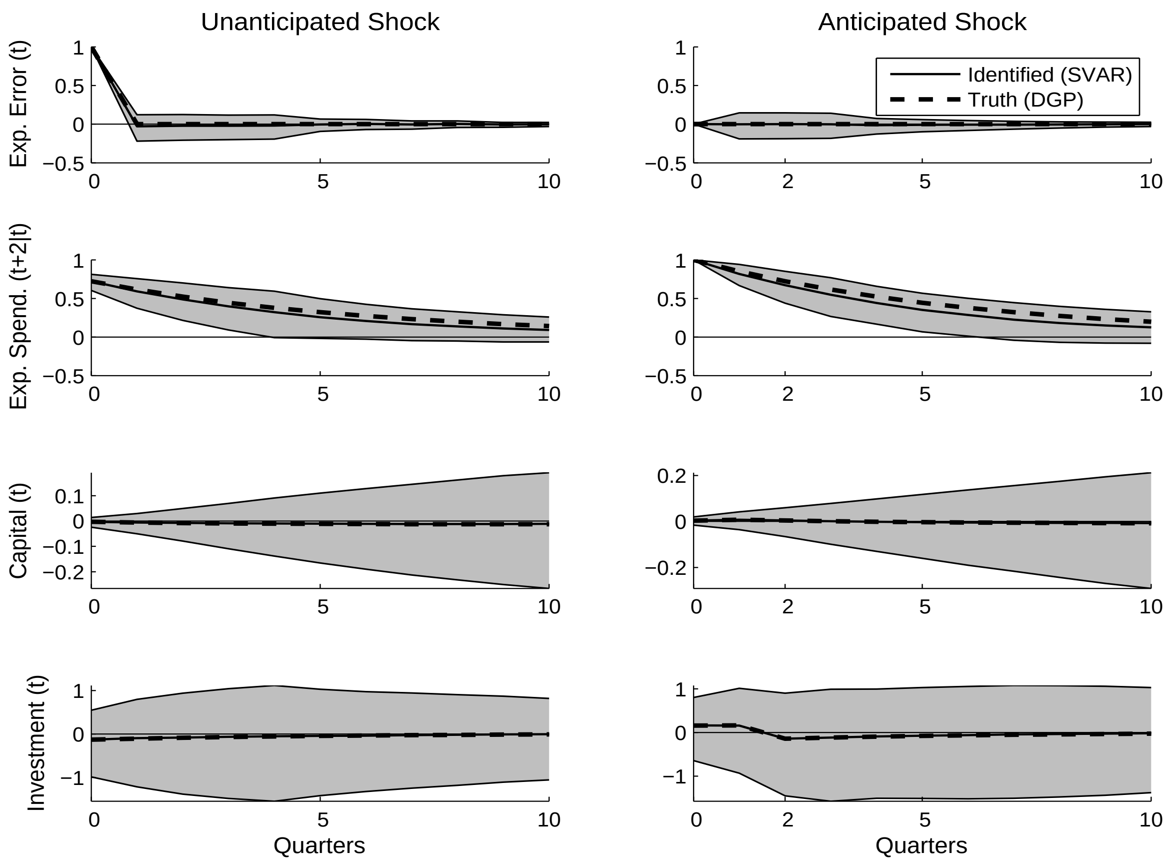
3.4.2 Spending reaction to lagged TFP

Consider now the following modification of the spending process (3.4):

$$\log(g_t/\bar{g}) = \rho_g \log(g_{t-1}/\bar{g}) + \rho_{ga} \log a_{t-1} + \varepsilon^u_{g,t} + \varepsilon^a_{g,t-2}, \qquad \rho_{ga} \in \mathbb{R}. \tag{3.17}$$

[9] The sample size is equal to the data sample below, i.e. 114 quarters from 1981Q4 to 2010Q1.

Figure 3.9: Robustness I – Monte Carlo impulse responses in small samples

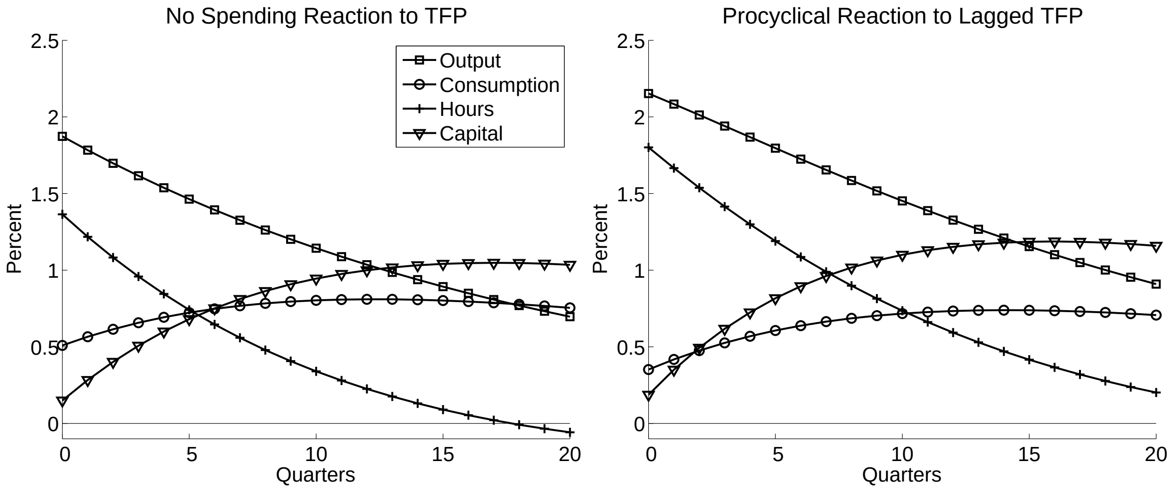


Notes. Sample size is $T = 114$; see Figure 3.6.

According to (3.17), government spending reacts with a one-period lag to fluctuations in productivity. This modification is a convenient shortcut for more complicated endogenous feedbacks on government spending (e.g. from movements in output or government revenues) that allows to obtain simple analytical expressions. Impulse responses to one percent productivity shocks in the model with (3.17) replacing (3.4) are shown in Figure 3.10. Absent any spending feedback ($\rho_{ga} = 0$, left panel), the shock increases consumption, hours, output, and investment. Hours increase because the substitution effect from higher productivity is larger than the positive wealth effect of higher productivity on lifetime income. When spending increases with TFP ($\rho_{ga} = 1$, right panel), there is a smaller wealth effect due to government absorption of goods and services, so the increase in hours and output (consumption) is stronger (weaker).

Under the modified spending process, the one-period expectational error is still

Figure 3.10: Robustness II – model impulse responses to a productivity shock when spending can react to lagged productivity



Notes. Both panels show responses to one percent surprise increases in TFP relative to its steady state value; left panel: no spending reaction to TFP ($\rho_{ga} = 0$); right panel: procyclical spending reaction to lagged TFP ($\rho_{ga} = 1$); responses are percentage deviations from steady state.

given by $E_{t-1}\hat{g}_t - \hat{g}_t = \varepsilon^u_{g,t}$, since spending only reacts with a lag to productivity shocks. However, two-quarter ahead expected spending becomes

$$E_t\hat{g}_{t+2} = \frac{\rho_g^2}{1-\rho_g L}\varepsilon^u_{g,t} + \frac{1}{1-\rho_g L}\varepsilon^a_{g,t} + \frac{\rho_{ga}\rho_a}{1-\rho_a L}\varepsilon_{a,t},$$

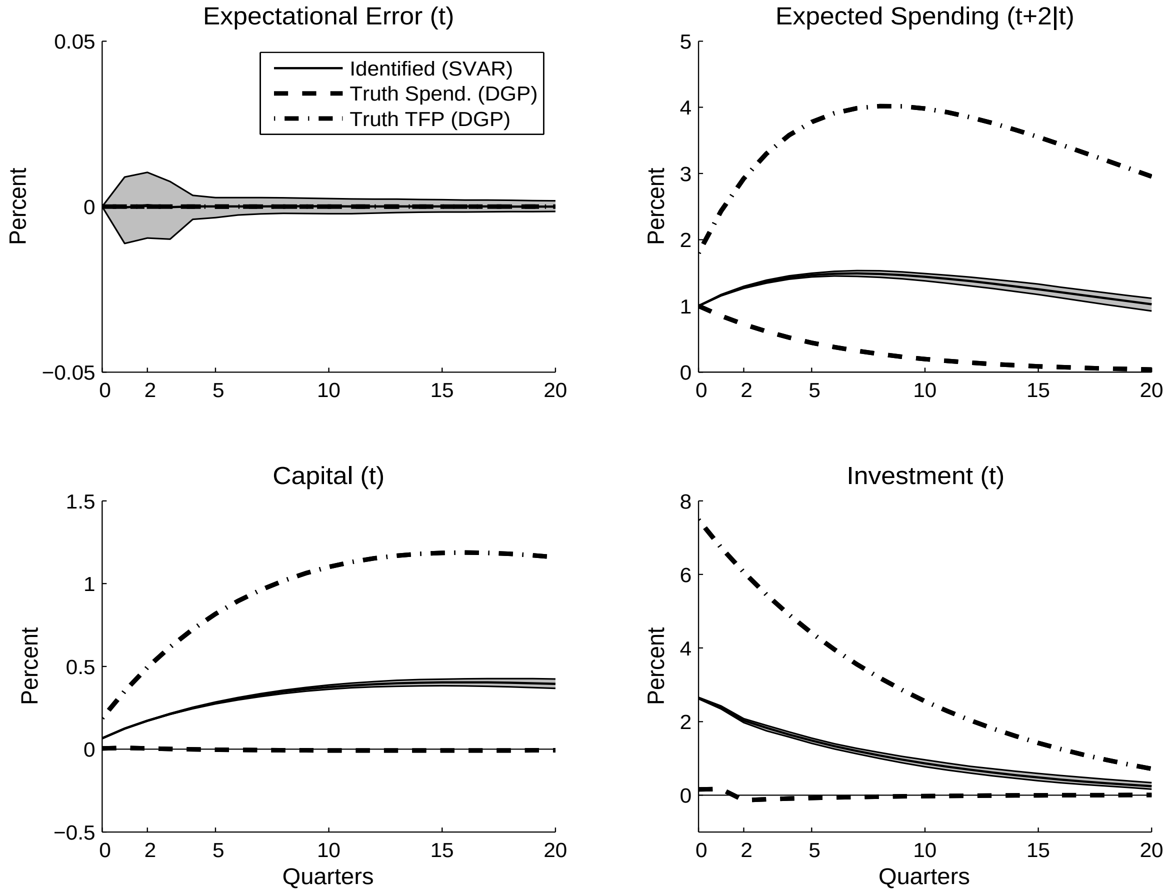
such that expected spending is affected by the current state of productivity. Suppose that the econometrician estimates a similar VAR as above:

$$\begin{bmatrix} \hat{g}_t - E_{t-1}\hat{g}_t \\ E_t\hat{g}_{t+2} \\ \hat{k}_t \\ \hat{i}_t^{obs} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \frac{\rho_g^2}{1-\rho_g L} & \frac{1}{1-\rho_g L} & \frac{\rho_{ga}\rho_a}{1-\rho_a L} & 0 \\ \frac{\eta_{kg}}{(1-\eta_{kk}L)(1-\rho_g L)} & \frac{\eta_{kg}L^2+\eta_{k\varepsilon,2}(1-\rho_g L)(\theta+L)}{(1-\eta_{kk}L)(1-\rho_g L)} & \frac{\eta^*_{ka}}{(1-\eta_{kk}L)(1-\rho_a L)} & 0 \\ \Theta_1(L) & \Theta_2(L) & \Theta_3^*(L) & 1 \end{bmatrix} \begin{bmatrix} \varepsilon^u_{g,t} \\ \varepsilon^a_{g,t} \\ \varepsilon_{a,t} \\ \varepsilon_{i,t} \end{bmatrix}.$$

The determinant of the lag matrix is equal to $\eta^*_{ka}/[(1-\rho_g z)(1-\eta_{kk}z)(1-\rho_a z)]$, such that the process is fundamental.[10] However, the presence of the term $\rho_{ga}\rho_a/(1-\rho_a L)$ makes the identification problem more difficult: the econometrician cannot distinguish changes in expected spending due to anticipated spending shocks and TFP shocks by

[10] Notice the presence of η^*_{ka}, which is equal to η_{ka} only for $\rho_{ga} = 0$. The coefficient η_{ca} also changes to η^*_{ca}, and similarly for $\Theta_3(L)$ which becomes $\Theta^*_3(L)$. Details are provided in Appendix 3.A.

Figure 3.11: Robustness III – Monte Carlo impulse responses to an anticipated spending shock when spending reacts to lagged productivity



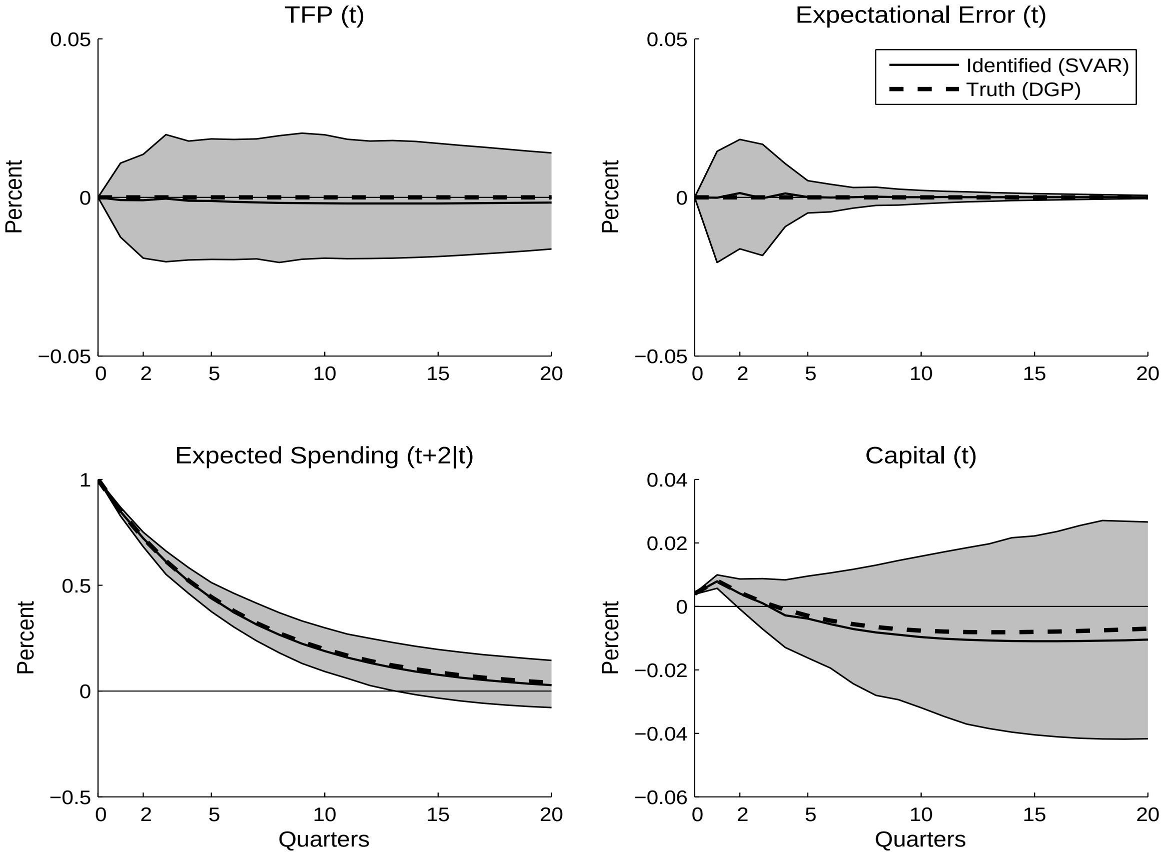
Notes. Spending reacts procyclically to lagged TFP ($\rho_{ga} = 1$); see Figure 3.6.

conditioning on $\varepsilon^u_{g,t}$, because the modified process does not have a Cholesky structure.

Figure 3.11 demonstrates the implications of the missing Cholesky structure, if the econometrician nevertheless attempts to estimate the effects of anticipated spending shocks by identifying the latter as increases in expected spending that are orthogonal to expectational errors. The estimated effects are seen to be located in between the responses to anticipated spending shocks and TFP shocks implied by the DGP. Of course, the bias becomes smaller with a smaller reaction of spending to the state of productivity. However, the identification scheme produces a bias even for relatively small feedbacks ρ_{ga}; for negative ρ_{ga}, the bias turns negative.

One way to address those issues is to condition on TFP in the VAR model. That

Figure 3.12: Robustness IV – Monte Carlo impulse responses to an anticipated spending shock when spending reacts to lagged productivity (observed)



Notes. Spending reacts procyclically to lagged TFP ($\rho_{ga} = 1$); an anticipated spending shock is identified by ordering the two-quarter ahead expectation of spending third in a Cholesky decomposition, the shock being a one percent increase in the two-quarter ahead expectation in quarter 0; the expectational error on spending is ordered second and TFP is ordered first; responses are measured as relative percentage deviations from steady state.

is, suppose that the econometrician includes $\hat{a}_t$ as the first variable in the VAR:

$$\begin{bmatrix} \hat{a}_t \\ \hat{g}_t - E_{t-1}\hat{g}_t \\ E_t\hat{g}_{t+2} \\ \hat{k}_t^{obs} \end{bmatrix} = \begin{bmatrix} \frac{1}{1-\rho_a L} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \frac{\rho_{ga}\rho_a}{1-\rho_a L} & \frac{\rho_g^2}{1-\rho_g L} & \frac{1}{1-\rho_g L} & 0 \\ \frac{\eta_{ka}^*}{(1-\eta_{kk}L)(1-\rho_a L)} & \frac{\eta_{kg}}{(1-\eta_{kk}L)(1-\rho_g L)} & \frac{\eta_{kg}L^2+\eta_{k\varepsilon,2}(1-\rho_g L)(\theta+L)}{(1-\eta_{kk}L)(1-\rho_g L)} & 1 \end{bmatrix} \begin{bmatrix} \varepsilon_{a,t} \\ \varepsilon_{g,t}^u \\ \varepsilon_{g,t}^a \\ \varepsilon_{k,t} \end{bmatrix}.$$

Notice that investment and its measurement error have been dropped and instead there is a measurement error on capital, $\varepsilon_{k,t} \sim N(0, \sigma_k^2)$ with $\sigma_k = 0.0001$. Furthermore, the surprise spending shock is now ordered second and the news shock is ordered third.

This is again a fundamental process, the determinant of the lag matrix being equal to $[(1-\rho_g L)(1-\rho_a L)]^{-1}$, and the impact matrix has a Cholesky structure.

Figure 3.12 shows that, by conditioning on TFP, the anticipated spending shock is again well identified. However, the requirements on the econometrician's information set have become more stringent under this modified identification scheme, since TFP needs to be available as an observable variable for the scheme to work.

3.4.3 Spending reaction to current TFP

If there is a contemporaneous feedback from TFP on spending, the expectational error on spending is a weighted average of unanticipated spending shocks and TFP shocks, where the weight on TFP shocks is given by the strength of the feedback. That is, if

$$\log(g_t/\bar{g}) = \rho_g \log(g_{t-1}/\bar{g}) + \rho_{ga} \log a_t + \varepsilon^u_{g,t} + \varepsilon^a_{g,t-2}, \qquad \rho_{ga} \in \mathbb{R}, \tag{3.18}$$

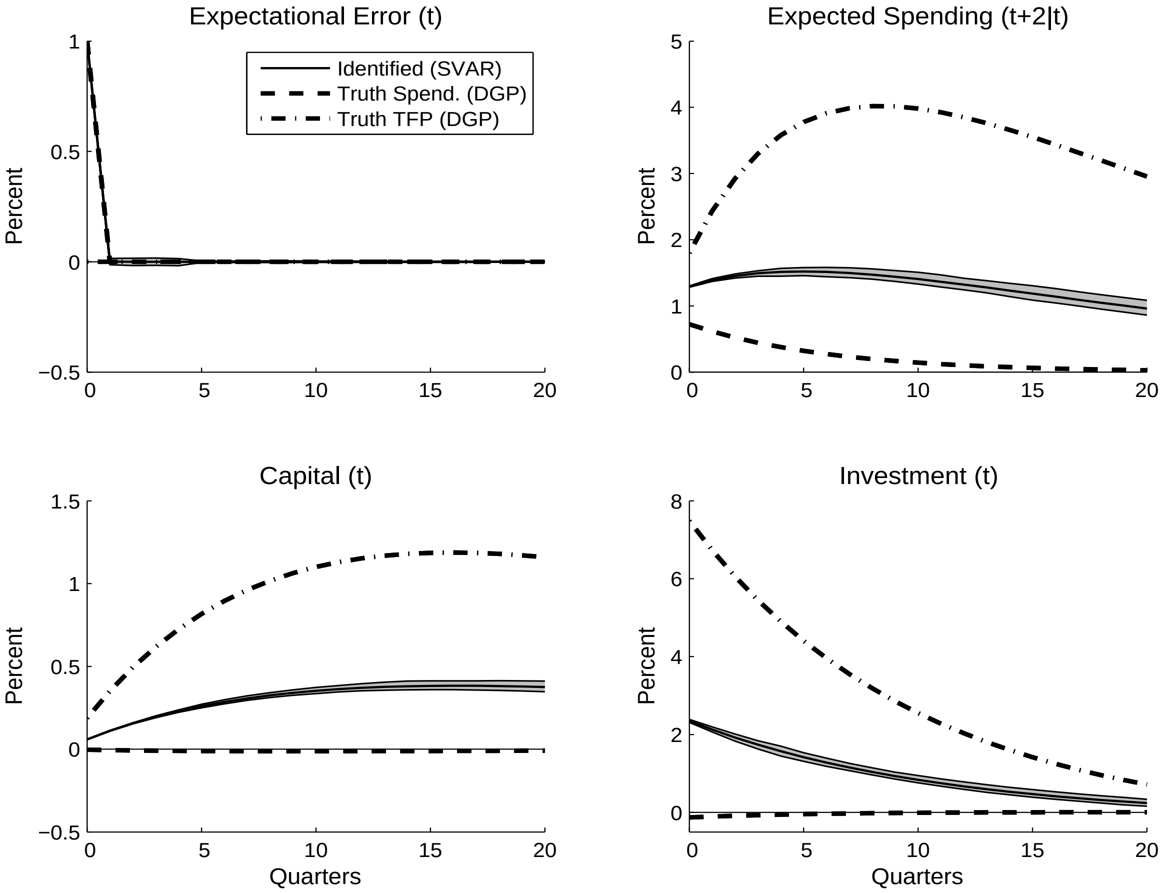
the expectational error on spending is a mix of TFP shocks and surprise spending shocks: $\hat{g}_t - E_{t-1}\hat{g}_t = \rho_{ga}\varepsilon_{a,t} + \varepsilon^u_{g,t}$. If TFP remains unobserved, the identification of surprise spending shocks based on expectational errors would therefore fail. This is demonstrated in Figure 3.13, which shows the estimated responses to an unanticipated spending shock that is identified by the associated expectations-based scheme. Similarly as above, the estimated responses are biased as they are located in between the responses to spending shocks and TFP shocks implied by the DGP.

However, if TFP can be observed, the econometrician could estimate the model

$$\begin{bmatrix} \hat{a}_t \\ \hat{g}_t - E_{t-1}\hat{g}_t \\ E_t\hat{g}_{t+2} \\ \hat{k}^{obs}_t \end{bmatrix} = \begin{bmatrix} \frac{1}{1-\rho_a L} & 0 & 0 & 0 \\ \rho_{ga} & 1 & 0 & 0 \\ \frac{\rho_{ga}\rho_a^2}{1-\rho_a L} & \frac{\rho_g^2}{1-\rho_g L} & \frac{1}{1-\rho_g L} & 0 \\ \frac{\eta^{**}_{ka}}{(1-\eta_{kk}L)(1-\rho_a L)} & \frac{\eta_{kg}}{(1-\eta_{kk}L)(1-\rho_g L)} & \frac{\eta_{kg}L^2+\eta_{k\varepsilon,2}(1-\rho_g L)(\theta+L)}{(1-\eta_{kk}L)(1-\rho_g L)} & 1 \end{bmatrix} \begin{bmatrix} \varepsilon_{a,t} \\ \varepsilon^u_{g,t} \\ \varepsilon^a_{g,t} \\ \varepsilon_{k,t} \end{bmatrix},$$

and apply the expectations-based identification scheme, conditioning on TFP when estimating the effects of spending shocks. The estimated impulse responses to a surprise spending shock are shown in Figure 3.14. The results show that the effects of the shock are well identified under the adjusted scheme.

Figure 3.13: Robustness V – Monte Carlo impulse responses to an unanticipated spending shock when spending reacts to current productivity



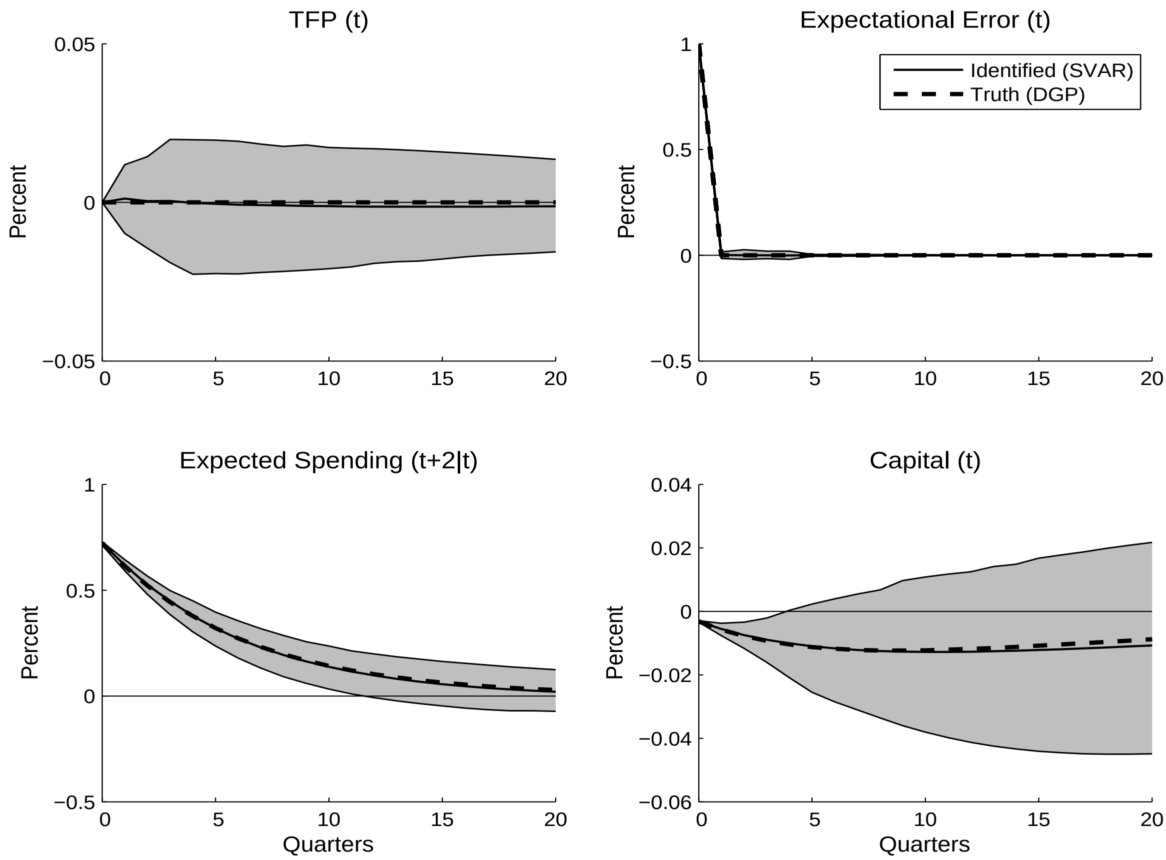
Notes. Spending reacts procyclically to current TFP ($\rho_{ga} = 1$); see Figure 3.6.

3.4.4 Surprise shocks under non-fundamentalness

Ramey (2011b) and Auerbach and Gorodnichenko (2010) suggest to extract the unanticipated component of exogenous movements in government spending through expectational errors on spending. In both of these studies an innovation to the forecast error is interpreted as an unanticipated spending shock. However, the studies do not include expectations on future spending in the regression. It can be shown, similarly as above, that a VAR that is specified in this way is not fundamental. Would an econometrician who applies this type of "non-fundamental" identification strategy still correctly estimate the effects of unanticipated spending shocks?

Figure 3.15 compares point estimates for ten simulated data sets from the bench-

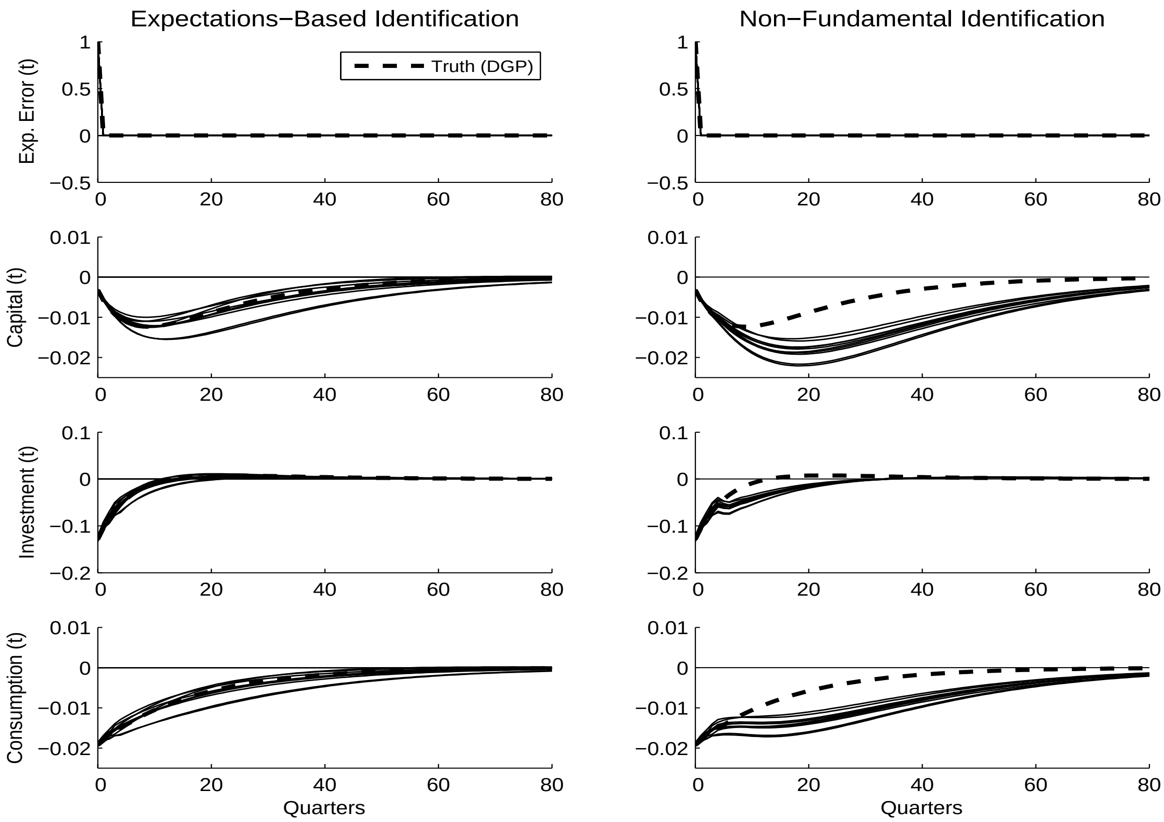
Figure 3.14: Robustness VI – Monte Carlo impulse responses to an unanticipated spending shock when spending reacts to current productivity (observed)



Notes. Spending reacts procyclically to current TFP ($\rho_{ga} = 1$); an unanticipated spending shock is identified by ordering the expectational error on spending second in a Cholesky decomposition, the shock being a one percent increase in the expectational error in quarter 0; the two-quarter ahead expectation of spending is ordered third and TFP is ordered first; responses are measured as relative percentage deviations from steady state.

mark specification of the model that was used for Figure 3.6. The left-hand panels show the estimates under the expectations-based scheme, where the two-quarter ahead expectation of spending is included in the VAR. The right-hand panels show the estimates under the non-fundamental scheme, where expected spending is not included in the regression. The estimated regressions include otherwise identical simulated data. In both cases, consumption is added as an additional observed variable with a small measurement error of 0.001 on observed consumption in the model. Unlike the fully expectations-based identification, the non-fundamental identification produces a downward bias in the estimated responses of investment, capital, and consumption,

Figure 3.15: Robustness VII – Monte Carlo impulse responses under non-fundamental scheme



Notes. Benchmark calibration; an unanticipated spending shock is identified by ordering the expectational error on spending first in a Cholesky decomposition, the shock being a one percent increase in the expectational error in quarter 0; left panels: two-quarter ahead expectation of spending is included as the second variable; right panels: two-quarter ahead expectation is not included; responses are measured as relative percentage deviations from steady state.

especially at longer horizons. The results of this excercise therefore suggest that an SVAR identification strategy based on expectational errors only is not appropriate to estimate the effects of surprise spending shocks.

3.4.5 Implications for applied research

What are the implications of the above findings for applied research on the macroeconomic effects of government spending?

First, the results show that an expectations-based identification approach can help to solve the non-fundamentalness problem that distorts econometric inference under the standard recursive SVAR identification approach. For standard information flows, the sufficient condition is that expectations on future spending up the anticipation horizon of economic agents should be included in the VAR.

Second, with respect to the identification problem of distinguishing anticipated spending shocks from unanticipated spending shocks and other economic shocks, the conclusions are mixed. If spending is affected by other shocks, these shocks need to be known, observed, and conditioned upon. However, there is significant uncertainty on which shocks do affect government spending. In addition, most structural shocks are unobserved state variables, so they cannot be included in the econometrician's information set.[11] The expectations-based identification of both types of spending shocks is therefore prone to significant problems.[12]

The good news is that, by exploiting variation in expectational errors, surprise spending shocks can be robustly identified if expected future spending is included in the VAR and if spending only reacts with some lag to other economic shocks. Since government spending is usually defined as government final consumption plus government investment and thus net of transfer and interest payments, the assumption that spending does not react within a quarter to other shocks seems justified. Government spending is then arguably acyclical, such that there is no automatic reaction of spending to movements in the business cycle, and a discretionary fiscal response to economic shocks is unlikely to occur within a quarter due to implementation lags in the policy process. Overall, the expectations-based approach is thus found to be useful for the identification of surprise spending shocks when it includes standard short-run exogeneity restrictions.

[11]Few exceptions such as multifactor productivity estimates are available from official sources but usually only on an annual basis (e.g. U.S. productivity estimates from the Bureau of Labor Statistics).

[12]Of course, things are even worse for the econometrician if there is not only news on future fiscal variables but also on future economic shocks (e.g. productivity news, see Beaudry and Portier, 2006) to which spending might react in the future. The econometrician would then need to condition on *expectations* of future unobserved state variables!

3.5 Empirical application

This section discusses the results of an empirical application to the U.S. that uses survey data on federal government spending from the Survey of Professional Forecasters to measure economic agents' (or market participants') expectations. Given the findings of the previous sections, the empirical application focuses on the effects of surprise spending shocks. The discussion further concentrates on a comparison of estimates from the expectations-based approach and the standard recursive SVAR identification approach (see e.g. Blanchard and Perotti, 2002; Fatás and Mihov, 2001; Perotti, 2005), which does not take into account the possibility of policy foresight.

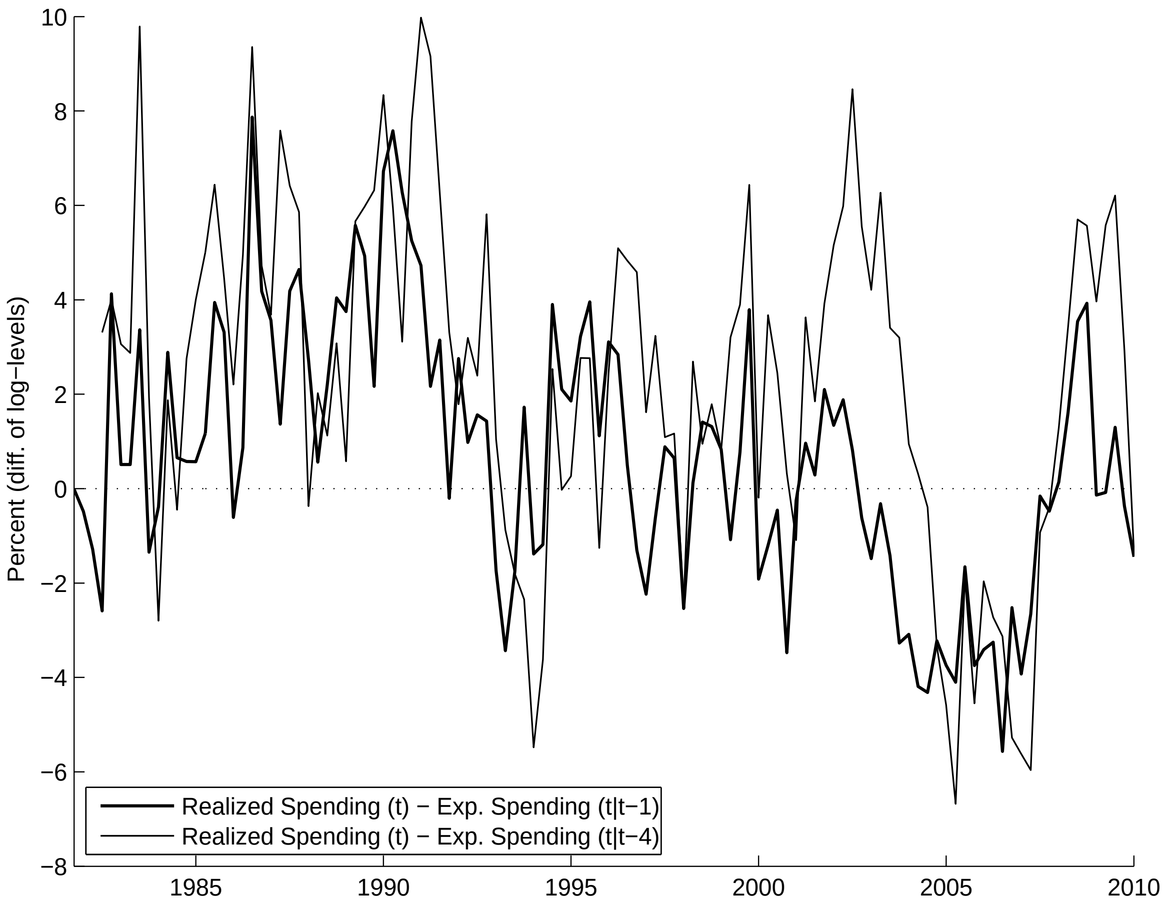
3.5.1 Data description

Figure 3.16 shows the deviations of real federal government spending from the predictions of the respondents to the Philadelphia Fed's Survey of Professional Forecasters over the period 1981Q4 to 2010Q1. Government spending is defined as the sum of government consumption and gross investment. Details on data definitions are provided in Appendix 3.B. The expectational errors are computed on the basis of the average predictions across all panelists made one and four quarters earlier. The forecasts submitted in quarter t are also taken conditional on quarter t information, although the official documentation of the survey takes forecasts made in t conditional on $t-1$ information.[13] The reason for the latter is that the questionnaires are sent out right after the advance report of the Bureau of Economic Analysis (BEA) is released, which contains the first estimates of GDP and its components for the previous quarter. However, the forecasters form their expectations conditional on all information which they have available in period t, and which is not necessarily restricted to the BEA report. Conditioning on the information set at the time when the forecast is made thus seems reasonable.[14]

[13] The official documentation is available from `http://www.philadelphiafed.org`.

[14] The forecasts for levels were originally scaled to the national accounts base year that had been in effect at the time the survey questionnaire was sent to the forecasters. Over time, as benchmark revisions to the data occur, the scale changes. As there have been a number of base year changes in U.S. national accounts since the survey began, the forecasts were therefore scaled to the current base year, 2005, through backcasting by the actual growth rates and imposing the average growth rate over the sample at the break points.

Figure 3.16: Expectational errors on U.S. federal government spending

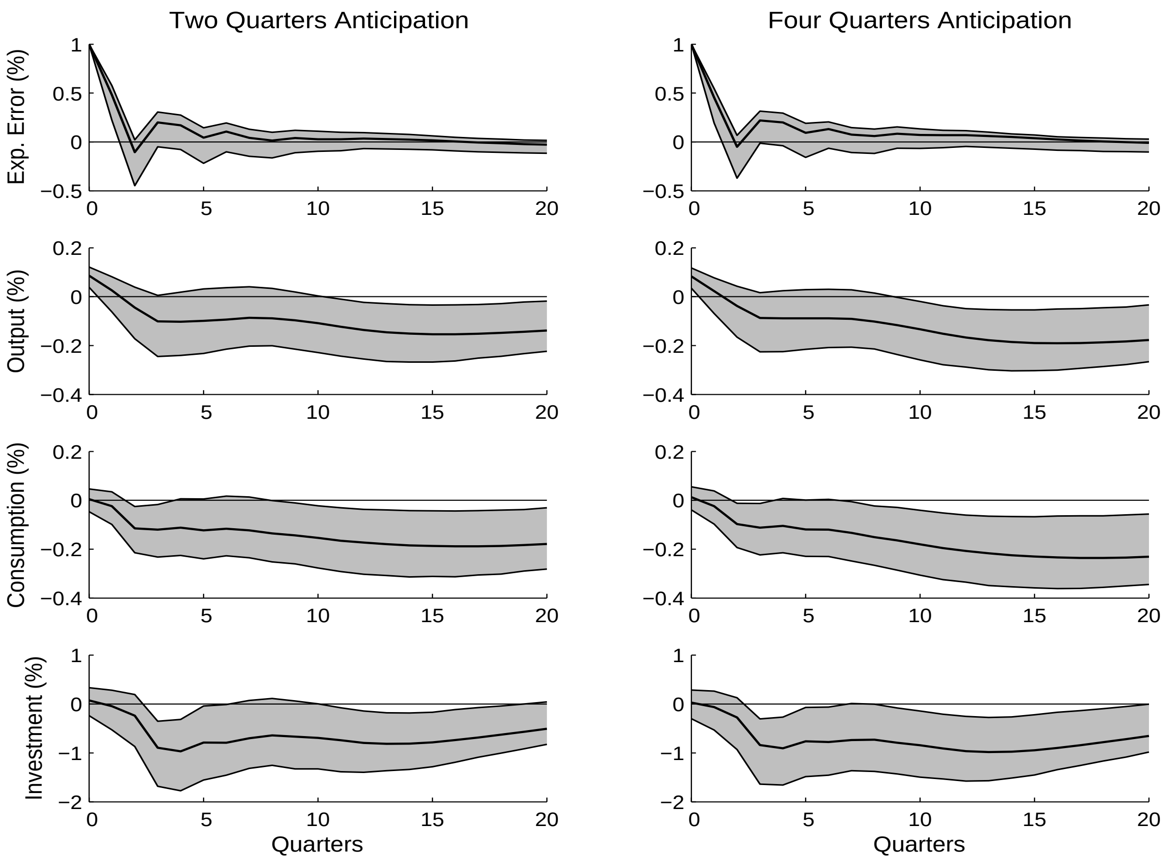


Notes. Quarterly data, period 1981Q4-2010Q1; data on expectations is taken from Philadelphia Fed's Survey of Professional Forecasters; expectational errors are computed as log differences (in percent) of real spending in quarter t and the prediction of real spending made in quarters $t-1$ (thick line) and $t-4$ (thin line).

3.5.2 Expectations-based identification

The expectational errors shown in Figure 3.16 indicate the presence of a pronounced unanticipated component in federal government spending. A natural next step is to exploit this variation to estimate the effects of surprise spending shocks by an application of the identication strategy analyzed above. To achieve fundamentalness, expectations on future government spending are included in the VARs in log-levels. As the precise anticipation horizon of economic agents is uncertain, two reduced-form VARs are estimated by OLS which include, respectively, the two- and four-quarter ahead expectations of spending. Real GDP, private consumption, and private investment are added

Figure 3.17: Empirical impulse responses I – expectations-based scheme



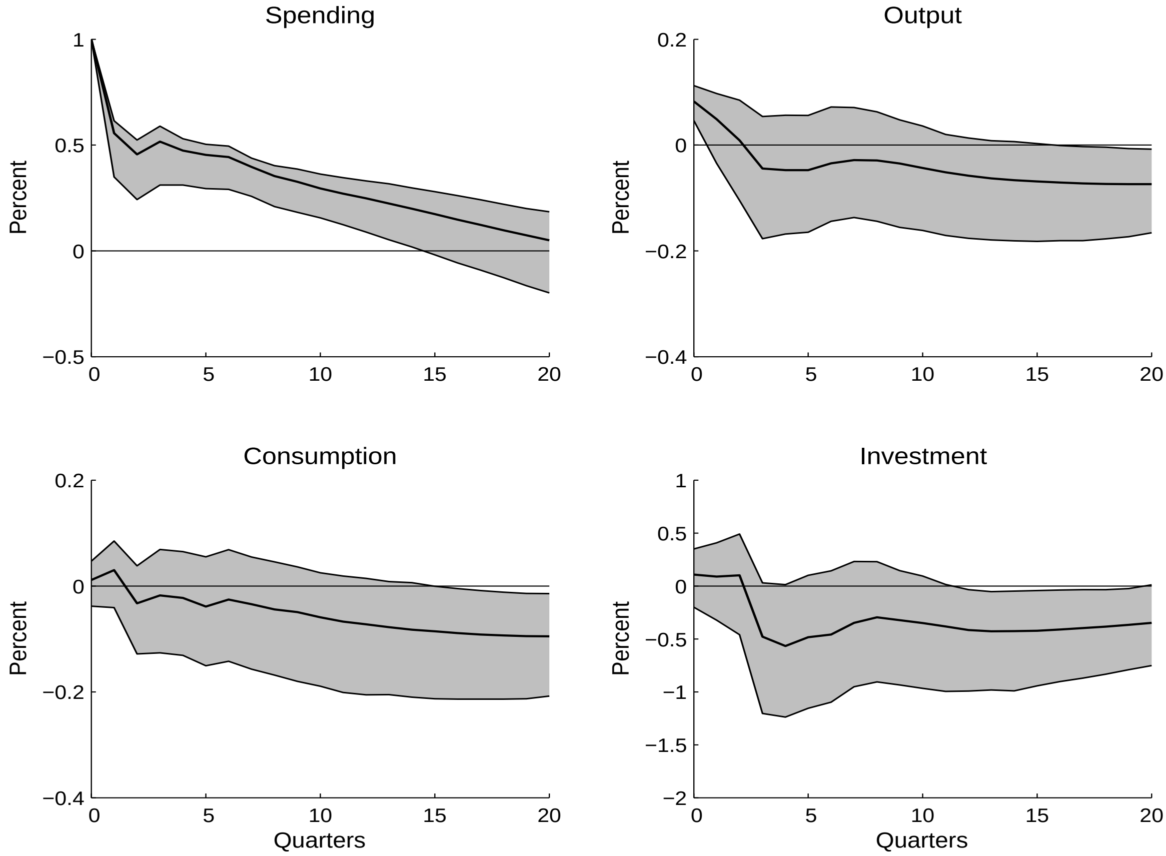
Notes. Normalized one percent increase in expectational error on federal government spending in quarter 0; the VAR models include the one-quarter expectational errors as the first variable and the two-quarter (left panels) and four-quarter (right panels) ahead expectations of spending as the second variable (responses not shown); 90 percent two-sided confidence bands around mean impulse responses are reported, calculated by 1,000 bootstrap replications; data definitions are provided in Appendix 3.B.

as additional variables in log-levels. Both VARs include four lags of the endogenous variables, a constant, and a quadratic time trend.[15]

Surprise spending shocks are then identified as the innovations to expectational errors that have a contemporaneous impact on all other variables in a system with expected spending, output, consumption, and investment, by a Cholesky decomposition of the estimated reduced-form covariance matrix. Figure 3.17 shows the estimated mean impulse responses of the expectational errors, output, consumption, and investment to one percent shocks of this type. The figure also shows 90 percent two-sided

[15]The results are robust to the use of a linear trend and three or five lags (not reported).

Figure 3.18: Empirical impulse responses II – standard SVAR scheme



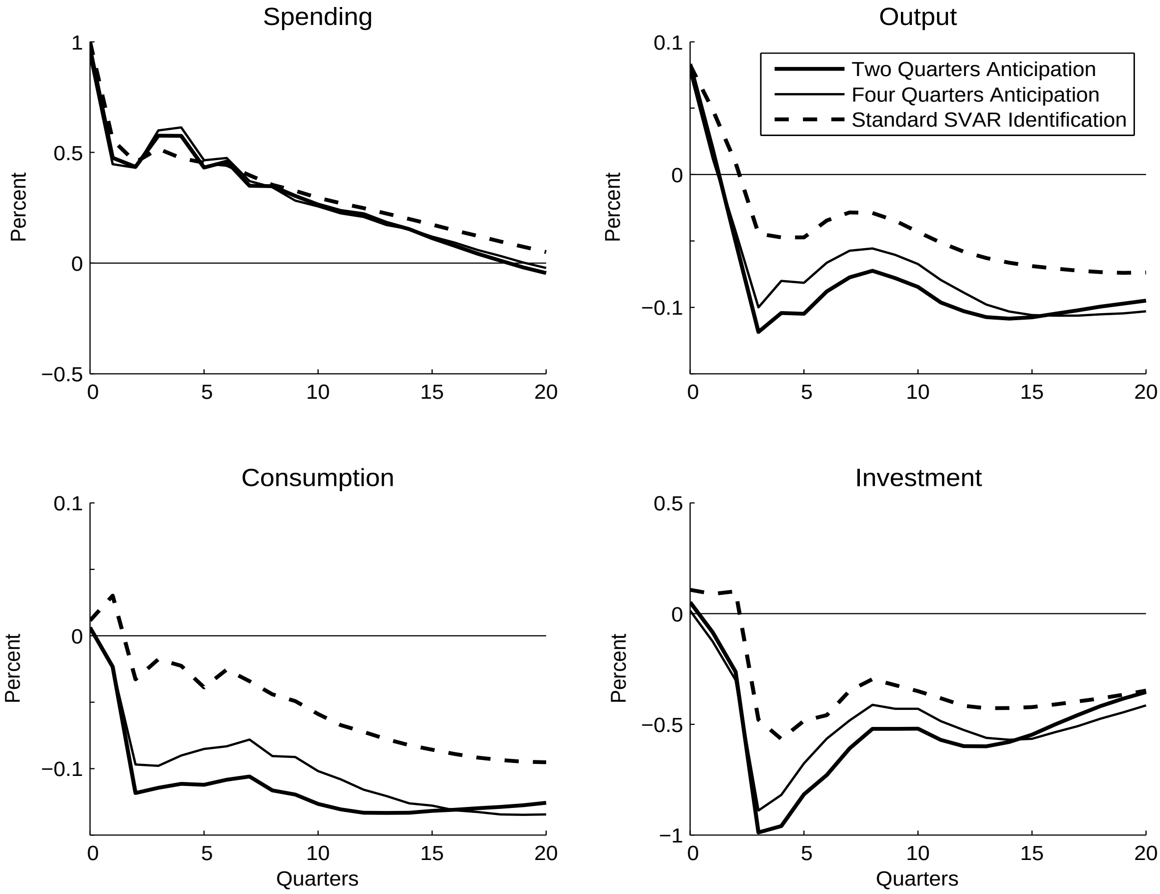
Notes. Normalized one percent increase in federal government spending in quarter 0; the VAR model does not include expectations data; the spending shock is identified by ordering spending before the remaining variables in a Cholesky decomposition; 90 percent two-sided confidence bands around mean impulse responses are reported, calculated by 1,000 bootstrap replications; data definitions are provided in Appendix 3.B.

bootstrap confidence bands for the estimated impulse responses. According to both VARs, a surprise spending shock leads, on average, to an initial increase in output. Consumption and investment hardly react on impact but start to decline shortly after the shock. After a few quarters, both consumption and investment turn significantly negative, which is associated with a reversal of the output effect.

3.5.3 Comparison with standard SVAR identification

Figure 3.18 reports the results of an application of the standard recursive SVAR identification scheme, according to which government spending shocks are identified by a

Figure 3.19: Empirical impulse responses III – both schemes



Notes. The expectations-based VAR models (solid lines) include the one-quarter expectational errors (ordered first), realized spending (ordered second), and the two-/four-quarter ahead expectations of spending (ordered third); surprise spending shocks are identified as in Figure 3.17 and the standard SVAR scheme (dashed lines) goes as in Figure 3.18; mean estimated impulse responses are reported.

Cholesky decomposition as the innovations to spending that have a contemporaneous impact on output, consumption, and investment. In contrast to the previous results, a shock that is identified in this way leads to increases in consumption and investment during two quarters, which are however not significant at the 90 percent level. In addition, both impulse responses and also the output response are more persistent than under the expectations-based approach; in particular, they do not turn significantly negative until towards the end of the horizon considered.

The expectations-based VAR models estimated for Figure 3.17 do not include the level of government spending as an endogenous variable but the standard VAR model

Table 3.2: Government spending multipliers[a]

	Impact	4 qrts.	8 qrts.	12 qrts.	20 qrts.
Two quarters anticipation[b]					
GDP	1.10	-1.57	-1.02	-1.28	-1.29
Spending	1.00	0.59	0.36	0.24	-0.02
Four quarters anticipation[b]					
GDP	1.07	-1.37	-0.79	-1.09	-1.44
Spending	1.00	0.64	0.40	0.24	0.00
Standard SVAR identification					
GDP	1.06	-0.57	-0.37	-0.66	-0.95
Spending	1.00	0.52	0.40	0.27	0.07

[a] The multipliers on GDP are computed based on mean impulse responses according to the following formula: multiplier in quarter t = GDP response in quarter t/(spending response in quarter 0 times average share of spending over GDP over the sample).

[b] For the expectations-based identification, the multipliers are computed from two different VAR models which include the two-quarter and four-quarter ahead expectations of government spending, respectively.

does. Thus, to make the results comparable, two additional regressions are estimated where the level of spending is added (ordered second) next to the expectational errors and the expectations of future spending. The point estimates of the impulse responses from those models are compared to the point estimates from the standard VAR model in Figure 3.19. The results show that, despite similar spending responses in terms of size and persistence, the effects on output, consumption, and investment are, on average, substantially smaller under the expectations-based identification scheme.

To compare the magnitudes of the estimated fiscal multipliers, following Blanchard and Perotti (2002), Table 3.2 compares the dollar change in GDP due to the initial dollar change in government spending at different horizons for the VAR models from Figure 3.19. The entries in the rows for GDP can also be interpreted as multipliers on output due to a fiscal shock leading to an initial increase in the level of government spending of size 1% of GDP. The results show that both the expectations-based approach and the standard recursive SVAR approach yield multipliers on GDP of approximately 1.1 on impact. However, the expectations-based approach yields multipliers smaller than minus one at longer horizons, whereas the multipliers implied by the standard recursive

SVAR approach are uniformly larger than that.

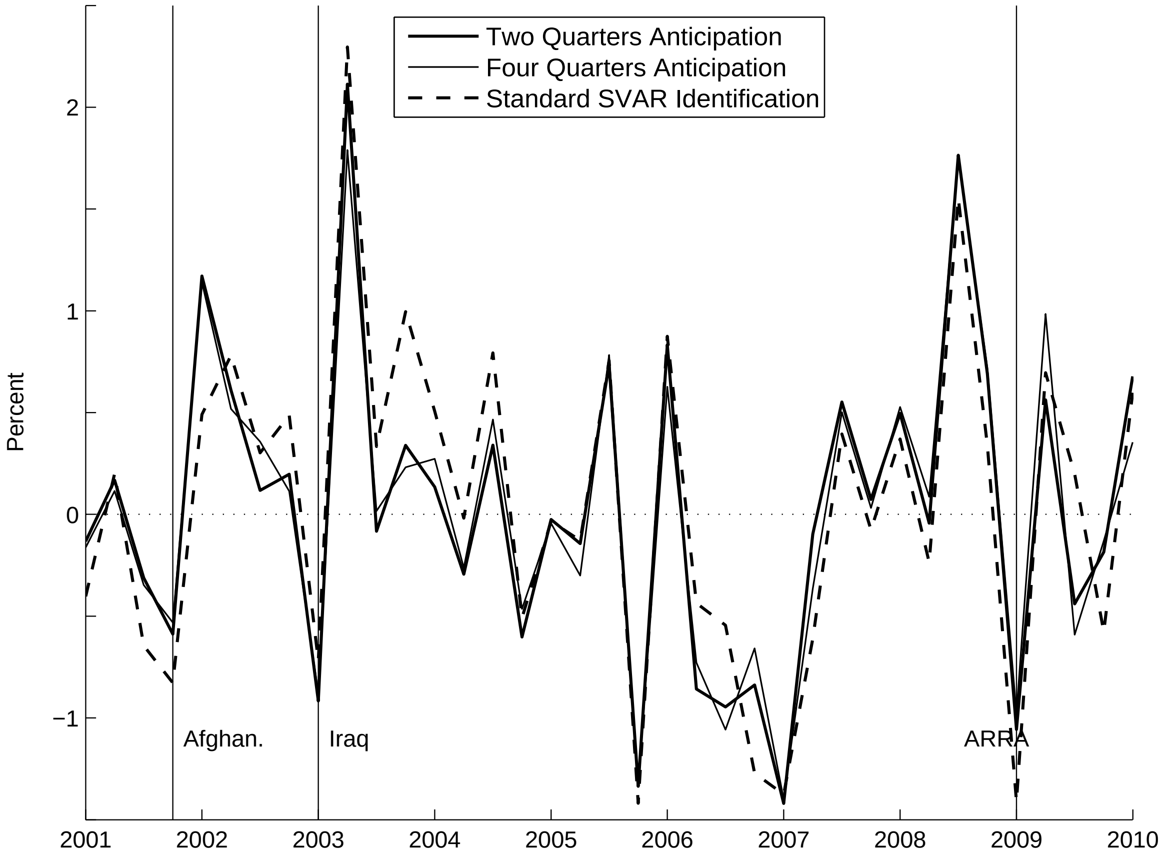
3.5.4 Predictability of shocks

A possible explanation for the differences between the results from the two alternative approaches is that the impulse responses to the standard SVAR shocks may incorporate some of the effects of anticipated spending shocks. The standard recursive SVAR approach may then pick up the upward-sloping paths of the responses of consumption and investment to shocks that were anticipated some quarters in advance, whereas the econometrician treats the spending increase as if it was unanticipated. In fact, Ramey (2011b) shows that standard SVAR shocks for federal government spending are Granger-caused by professional forecasts made one to four quarters earlier; that is, the SVAR shocks are predictable. A likely implication of this finding is that by ignoring policy foresight the econometrician would not capture the true economic impact of discretionary changes in government spending, even if surprise spending shocks are the only object of interest.

Figure 3.20 compares the identified shocks from the two approaches for the period 2001Q1 to 2010Q1. During this period, three easily recognizable events are likely to have affected U.S. federal government spending. The first two are the wars in Afghanistan and Iraq which began, respectively, on October 7, 2001 and March 20, 2003. The third event is the American Recovery and Reinvestment Act (ARRA) which was signed into law by President Obama on February 17, 2009. These events are marked by vertical lines in Figure 3.20. The figure shows that spending shocks are identified by all approaches immediately after the three events. However, the standard SVAR approach identifies sizeable positive shocks during about two years after the beginning of the war in Iraq, whereas the expectations-based approach does not identify any large surprise spending shocks during this period. Based on those results, one might suspect that some of the shocks identified by the standard SVAR approach were indeed anticipated by economic agents.

To investigate whether this is the case, following Ramey (2011b), it is analyzed whether the professional forecasts Granger-cause the identified shocks from the two approaches. In particular, a series of F-tests are performed where the unrestricted test

Figure 3.20: Identified spending shocks



Notes. Period 2001Q1-2010Q1; the expectations-based VAR model includes two-quarter ahead expectations of spending; see Figures 3.17 and 3.18.

equation takes the form

$$x_t = a_0 + a_1 x_{t-1} + a_2 x_{t-2} + \cdots + a_p x_{t-p} + b_1 f_{t|t-1} + b_2 f_{t|t-2} + \cdots + b_h f_{t|t-h},$$

and where the restricted test equation is given by

$$x_t = a_0 + a_1 x_{t-1} + a_2 x_{t-2} + \cdots + a_p x_{t-p},$$

where x_t denotes the identified shocks in quarter t and $f_{t|t-1}, \ldots, f_{t|t-h}$ the log first differences of forecasts on real federal government spending made up to h quarters earlier. The null hypothesis is thus that the forecasts $f_{t|t-1}, \ldots, f_{t|t-h}$ do not Granger-cause the shocks; that is, the null hypothesis states that the shocks are not actually

Table 3.3: Granger causality tests on identified shocks

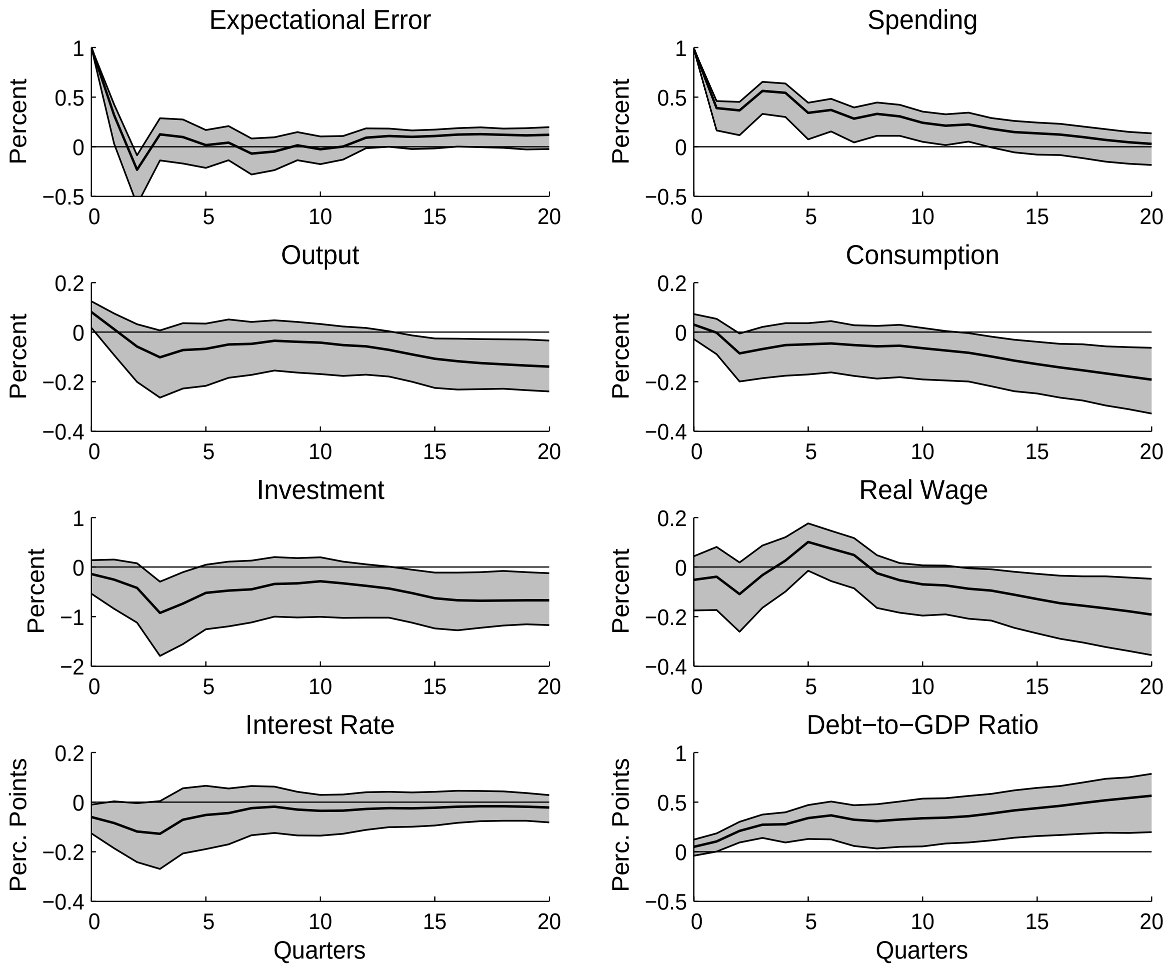
Independent variable[a,b]	F-statistic		5% critical value		p-value	
	$p = 1$	$p = 4$	$p = 1$	$p = 4$	$p = 1$	$p = 4$
Standard SVAR identification						
One-quarter ahead forecasts	1.87	3.36*	3.93	3.93	0.17	0.07
Two-quarter ahead forecasts	2.02	2.21	3.93	3.93	0.16	0.14
Three-quarter ahead forecasts	5.52**	5.77**	3.93	3.93	0.02	0.02
Four-quarter ahead forecasts	3.50*	3.07*	3.93	3.93	0.06	0.08
All forecasts simultaneously	4.60**	4.39**	3.93	3.93	0.03	0.04
Two quarters anticipation[c]						
Two-quarter ahead forecasts	0.00	0.10	3.93	3.93	0.95	0.75
All forecasts simultaneously	0.04	0.35	3.93	3.94	0.85	0.56
Four quarters anticipation[c]						
Four-quarter ahead forecasts	0.96	1.19	3.93	3.93	0.33	0.28
All forecasts simultaneously	0.80	1.24	3.93	3.94	0.38	0.27

[a] The dependent variables are the identified shocks in quarter t, which are regressed on a constant, p own lags, and the log difference of forecasted spending for quarter t made one to four quarters earlier.
[b] The null hypothesis is that the forecasts do not Granger-cause the shocks. ** indicates rejection of the null hypothesis at the five percent significance level, * at the ten percent significance level.
[c] For the expectations-based identification, the identified shocks from the VAR models which include the two-quarter and four-quarter ahead expectations of government spending are taken as dependent variables, respectively.

forecastable by the professional forecasters' predictions.

Table 3.3 reports the test results when the forecasts are included individually and jointly as independent variables in the unrestricted regression, for both $p = 1$ and $p = 4$ lags of the dependent variables. The results show that, indeed, the standard SVAR shocks are on average forecastable by the professional forecasters' predictions. When the forecasts are included jointly in the unrestricted regression, the null hypothesis that the forecasts do not Granger-cause the shocks is clearly rejected at the five percent significance level. On the other hand, the null hypothesis cannot be rejected for the shocks identified on the basis of expectational errors. These results suggest that the expectations-based approach is successful in extracting the unpredictable part of exogenous spending changes. Any bias of the standard SVAR approach should therefore be eliminated by the expectations-based identification scheme.

Figure 3.21: Empirical impulse responses IV – extended specification



Notes. The VAR model includes two-quarter ahead expectations of spending; see Figure 3.17.

3.5.5 Extended regression specification

As a next step, the impact of surprise spending shocks on a broader set of indicators is investigated. That is, the real wage, the 3-month treasury bill rate, and the federal government debt-to-GDP ratio are added as additional endogenous variables in the regression. The real wage is added since it is an important variable in the controversy on the effects of government spending shocks.[16] The T-bill rate is added to assess the impact of spending shocks on interest rates. The debt-to-GDP ratio is included to capture financing aspects and also to address the problem of omitted state variables of

[16]See, for instance, Linnemann and Schabert (2003), Perotti (2008), or Ramey (2011b).

SVAR analysis (see Chari, Kehoe, and McGrattan, 2005).

The results are reported in Figure 3.21. The estimated VAR model includes the two-quarter ahead expectation of spending and the level of spending.[17] Also according to the extended specification, the spending increase has small effects on output and leads to a decline in consumption and investment. The response of the real wage is insignificant in the short run and negative at longer horizons. The interest rate declines immediately. In terms of financing, the spending shock is associated with a sustained increase in the debt-to-GDP ratio. Hence, also in the extended specification a surprise spending shock is estimated to have contractionary effects over the medium term, in contrast to most of the previous SVAR literature but in line with the findings of, for instance, Mountford and Uhlig (2009) and Ramey (2011b).

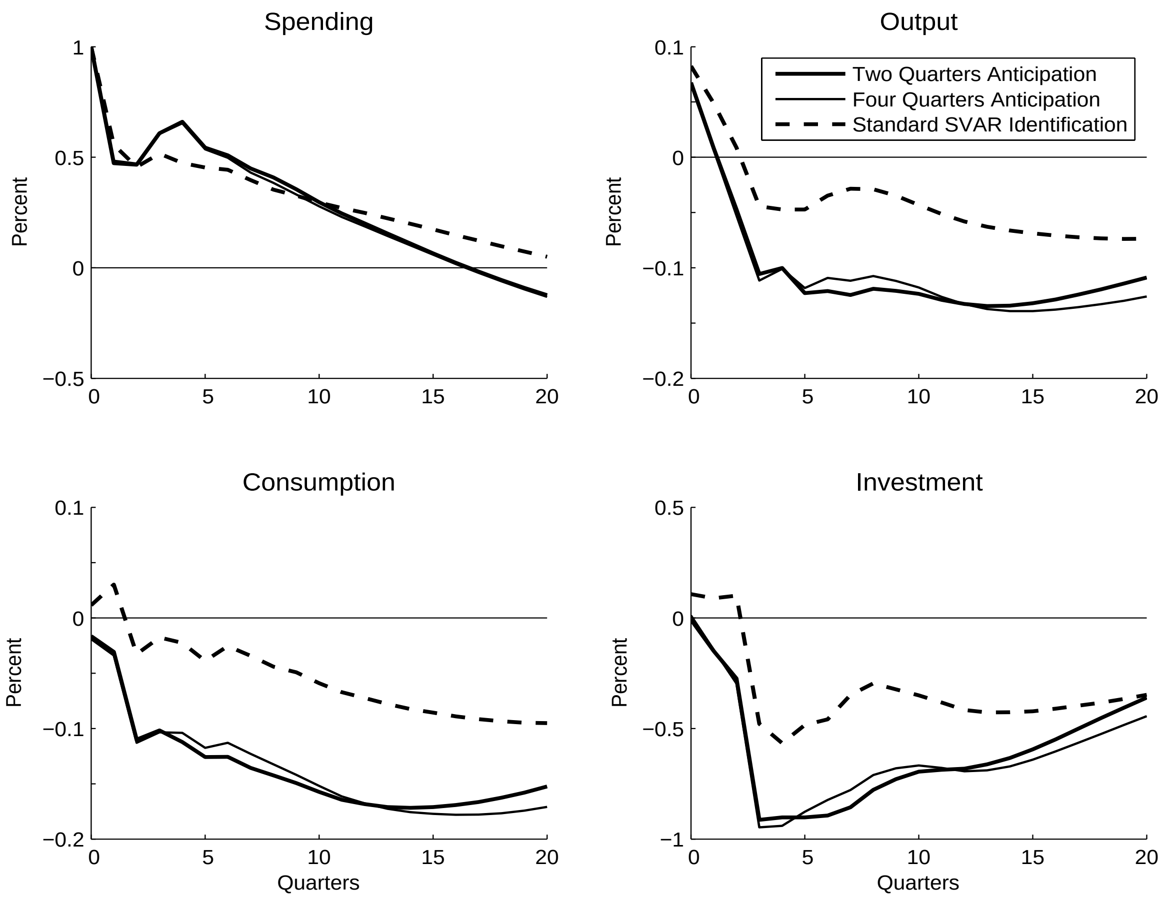
3.5.6 Alternative identification scheme

As a final step of the analysis, the variant of the expectations-based scheme that relies on VAR forecasts in the identification of surprise spending shocks is applied. As discussed in Section 3.2.4, this scheme has government spending ordered first before expected spending as the second variable in the VAR model, and takes the innovations to spending that have a contemporaneous impact on all variables (by a Cholesky decomposition) as the unanticipated spending shocks.

The results of the application of this scheme in comparison to the standard recursive scheme are shown in Figure 3.22. The figure shows that the previous conclusions remain intact from a qualitative point of view: the estimated effects on output, consumption, and investment are, on average, substantially smaller under the expectations-based scheme than under the standard recursive scheme. Quantitatively, the effects are even weaker than under the benchmark expectations-based scheme especially at longer horizons (cf. Figure 3.19). The latter strengthens the conclusion that unanticipated discretionary changes in federal government spending were not very effective in raising U.S. economic activity over the period considered.

[17]The results are robust to using the four-quarter ahead expectation of spending instead.

Figure 3.22: Empirical impulse responses V – alternative scheme



Notes. The expectations-based VAR models (solid lines) include realized spending (ordered first), and the two-/four-quarter ahead expectations of spending (ordered second); the alternative scheme identifies surprise spending shocks as one percent increases in spending by a Cholesky decomposition in this model; see Figure 3.18 for the standard SVAR scheme (dashed lines); mean estimated impulse responses are reported.

3.6 Conclusion

This chapter has demonstrated how the econometric problems created by foresight on government spending can be addressed when economic agents' expectations on future spending can be incorporated, e.g. through survey data, in a VAR model. By a combination of theory and stochastic simulations, the chapter has shown that incorporating expectations not only solves the non-fundamentalness problem created by foresight but also makes it possible to identify structural shocks through an appropriate expectations-based scheme. In particular, when expectations-based identifying assumptions are combined with standard short-run exogeneity restrictions, the expectations-based approach is found to be useful for the identification of surprise spending shocks.

The application of the approach to U.S. data supports concerns raised by Leeper, Walker, and Yang (2011) and Ramey (2011b) on the validity of the results of previous empirical studies due to the influence of policy foresight. The expectations-based approach indicates positive short-run output effects of federal government expenditures, but negative medium-term effects due to falling consumption and investment. The standard SVAR identification scheme, on the other hand, predicts stronger and more persistent effects on consumption, investment, and output. However, Granger causality tests suggest that, unlike the surprise shocks identified by the expectations-based scheme, the shocks identified by the standard scheme are forecastable.

In addition to policy foresight, several alternative explanations for the differences to previous studies are conceivable. In particular, the post-1980 period is often argued to have smaller average fiscal multipliers than the pre-1980 period.[18] In addition, the structure of government spending is likely to matter, given that more than 70 percent of U.S. federal spending falls on defense-related expenditures. An investigation of the effects of other types of expenditures such as state and local government spending, for which expectations data is also available from the Survey of Professional Forecasters, would thus be a useful extension of the analysis conducted in this chapter.

3.A Analytical solution

This appendix provides a detailed derivation of the analytical solution of the model. The steps are as follows. First, the non-stochastic steady state solution is derived from the non-linear equilibrium conditions. The equilibrium system is then log-linearized around the non-stochastic steady state and reduced to a two-dimensional first-order linear difference equation in capital and consumption, given the stochastic processes for TFP and government spending. Finally, the parameters in the recursive laws of motion for capital and consumption are derived by the method of undetermined coefficients (see Uhlig, 1999).

[18]See, for instance, Bilbiie et al. (2008) and Blanchard and Perotti (2002).

Non-linear equilibrium. The non-linear equilibrium conditions are as follows:

$$\begin{aligned}
\text{Labor/leisure} &: \quad Ac_t = (1-\alpha)y_t/n_t, \\
\text{Euler equation} &: \quad c_t^{-1} = \beta E_t R_{t+1}/c_{t+1}, \\
\text{Real return} &: \quad R_t = 1-\delta+\alpha y_t/k_{t-1}, \\
\text{Production} &: \quad y_t = a_t k_{t-1}^{\alpha} n_t^{1-\alpha}, \\
\text{Feasibility} &: \quad y_t = c_t + k_t - (1-\delta)k_{t-1} + g_t, \\
\text{TFP} &: \quad \log a_t = \rho_a \log a_{t-1} + \varepsilon_{a,t}, \\
\text{Gov. expenditures} &: \quad \log\left(g_t/\bar{g}\right) = \rho_g \log\left(g_{t-1}/\bar{g}\right) + \varepsilon_{g,t}^{u} + \varepsilon_{g,t-2}^{a}.
\end{aligned}$$

where investment has been eliminated in the feasibility constraint using the capital accumulation equation.

Steady state. Let variables without time subscript denote non-stochastic steady state values. The TFP process implies $a = 1$ since $\rho_a \in (0,1]$. The Euler equation yields $R = \beta^{-1}$. The real return equation can be solved for the output-to-capital ratio:

$$\frac{y}{k} = \frac{R-1+\delta}{\alpha} = \frac{\beta^{-1}-1+\delta}{\alpha}.$$

The production function implies that

$$\frac{y}{n} = \frac{k^{\alpha}n^{1-\alpha}}{n} = \left(\frac{k}{n}\right)^{\alpha}, \qquad \frac{y}{k} = \frac{k^{\alpha}n^{1-\alpha}}{k} = \left(\frac{k}{n}\right)^{\alpha-1}.$$

From the second equation, we have $k/n = (y/k)^{1/(\alpha-1)} = (y/k)^{-1/(1-\alpha)}$. Substituting this expression into the first equation yields an expression for the output-to-labor ratio:

$$\frac{y}{n} = \left(\frac{y}{k}\right)^{-\frac{\alpha}{1-\alpha}} = \left(\frac{\beta^{-1}-1+\delta}{\alpha}\right)^{-\frac{\alpha}{1-\alpha}}.$$

The labor/leisure tradeoff then yields $c = A^{-1}(1-\alpha)y/n$. Dividing the feasibility constraint by y and re-writing yields

$$n = \frac{c(y/n)^{-1}}{1-\delta k/y - g/y}.$$

Taking $s_g = g/y$ as given, the government spending equation implies $g = \bar{g}$, since $\rho_g \in [0, 1)$. The remaining steady state solutions follow as

$$y = (y/n)n, \qquad k = (k/y)y, \qquad g = \bar{g} = s_g y.$$

Log-linearized system. The log-linearized system is given by

$$\begin{aligned}
\text{Labor/leisure} &: \hat{n}_t = \hat{y}_t - \hat{c}_t, && (3.19)\\
\text{Euler equation} &: 0 = E_t[\hat{c}_t - \hat{c}_{t+1} + \widehat{R}_{t+1}], && (3.20)\\
\text{Production} &: \hat{y}_t = \hat{a}_t + \alpha \hat{k}_{t-1} + (1-\alpha)\hat{n}_t, && (3.21)\\
\text{Feasibility} &: c\hat{c}_t = y\hat{y}_t - k\hat{k}_t + (1-\delta)k\hat{k}_{t-1} - g\hat{g}, && (3.22)\\
\text{Real return} &: \widehat{R}_t = \frac{\alpha}{R}\frac{y}{k}(\hat{y}_t - \hat{k}_{t-1}), && (3.23)\\
\text{TFP} &: \hat{a}_t = \rho_a \hat{a}_{t-1} + \varepsilon_{a,t}, && (3.24)\\
\text{Gov. expenditures} &: \hat{g}_t = \rho_g \hat{g}_{t-1} + \varepsilon^u_{g,t} + \varepsilon^a_{g,t-2}, && (3.25)
\end{aligned}$$

where $\hat{x}_t = \log(x_t/x)$ such that $x_t = x\exp(\hat{x}_t) \approx x(1 + \hat{x}_t)$ for $\hat{x}_t \approx 0$.

Difference equations. The log-linearized system is now reduced to the two first-order linear difference equations reported in the main text. Substituting (3.19) into (3.21), leading the result by one period, and re-arranging terms yields

$$\hat{y}_{t+1} = \frac{\hat{a}_{t+1} + \alpha\hat{k}_t - (1-\alpha)\hat{c}_{t+1}}{\alpha}. \tag{3.26}$$

Substituting (3.23) into (3.20) yields

$$0 = E_t\left[\hat{c}_t - \hat{c}_{t+1} + \frac{\alpha}{R}\frac{y}{k}(\hat{y}_{t+1} - \hat{k}_t)\right].$$

Combining the latter two expressions gives the first difference equation:

$$\begin{aligned}
0 &= E_t\left[\hat{c}_t - \hat{c}_{t+1} + \frac{\alpha}{R}\frac{y}{k}\left(\frac{1}{\alpha}\hat{a}_{t+1} + \hat{k}_t - \frac{1-\alpha}{\alpha}\hat{c}_{t+1} - \hat{k}_t\right)\right],\\
&= E_t\left[\hat{c}_t - \left(1 + \frac{y}{k}\frac{1-\alpha}{R}\right)\hat{c}_{t+1} + \frac{y}{k}\frac{1}{R}\hat{a}_{t+1}\right].
\end{aligned}$$

Using (3.21) in (3.22) yields

$$c\hat{c}_t = y\hat{a}_t + [\alpha y + (1-\delta)k]\,\hat{k}_{t-1} + (1-\alpha)y\hat{n}_t - k\hat{k}_t - g\hat{g}_t.$$

Substituting out (3.19) in the last expression, using (3.26) lagged by one period, rearranging terms, and taking expectations yields the second difference equation:

$$0 = E_t\left[\left(c + y\frac{1-\alpha}{\alpha}\right)\hat{c}_t + k\hat{k}_t - \frac{y}{\alpha}\hat{a}_t - (y + (1-\delta)k)\,\hat{k}_{t-1} + g\hat{g}_t\right].$$

The reduced system is thus given by

$$\begin{aligned} 0 &= E_t[\hat{c}_t - \phi_1\hat{c}_{t+1} + \phi_2\hat{a}_{t+1}], && (3.27)\\ 0 &= E_t[\phi_3\hat{c}_t + \phi_4\hat{k}_t - \phi_5\hat{a}_t - \phi_6\hat{k}_{t-1} + \phi_7\hat{g}_t], && (3.28)\end{aligned}$$

where $\phi_1 = 1 + R^{-1}(1-\alpha)y/k$, $\phi_2 = R^{-1}y/k$, $\phi_3 = c + y(1-\alpha)/\alpha$, $\phi_4 = k$, $\phi_5 = y/\alpha$, $\phi_6 = y + (1-\delta)k$ and $\phi_7 = g$.

Recursive laws of motion. Next, guess the recursive laws of motion

$$\begin{aligned} \hat{k}_t &= \eta_{kk}\hat{k}_{t-1} + \eta_{ka}\hat{a}_t + \eta_{kg}\hat{g}_t + \eta_{k\varepsilon,1}\varepsilon^a_{g,t} + \eta_{k\varepsilon,2}\varepsilon^a_{g,t-1},\\ \hat{c}_t &= \eta_{ck}\hat{k}_{t-1} + \eta_{ca}\hat{a}_t + \eta_{cg}\hat{g}_t + \eta_{c\varepsilon,1}\varepsilon^a_{g,t} + \eta_{c\varepsilon,2}\varepsilon^a_{g,t-1}.\end{aligned}$$

Repeatedly substituting the latter into (3.27) and (3.28) and using $E_t\hat{g}_{t+1} = \rho_g\hat{g}_t + \varepsilon^a_{g,t-1}$ and $E_t\hat{a}_{t+1} = \rho_a\hat{a}_t$ yields, after some tedious but straightforward algebra:

$$\begin{aligned} 0 &= [(1-\phi_1\eta_{kk})\,\eta_{ck}]\,\hat{k}_{t-1}\\ &\quad + [\eta_{ca}(1-\phi_1\rho_a) + \phi_2\rho_a - \phi_1\eta_{ck}\eta_{ka}]\,\hat{a}_t + [\eta_{cg}(1-\phi_1\rho_g) - \phi_1\eta_{ck}\eta_{kg}]\,\hat{g}_t\\ &\quad + [\eta_{c\varepsilon,1} - \phi_1(\eta_{c\varepsilon,2} + \eta_{ck}\eta_{k\varepsilon,1})]\,\varepsilon^a_{g,t} + [\eta_{c\varepsilon,2} - \phi_1(\eta_{cg} + \eta_{ck}\eta_{k\varepsilon,2})]\,\varepsilon^a_{g,t-1}, \quad (3.29)\\ 0 &= [\phi_3\eta_{ck} + \phi_4\eta_{kk} - \phi_6]\,\hat{k}_{t-1}\\ &\quad + [\phi_3\eta_{ca} + \phi_4\eta_{ka} - \phi_5]\,\hat{a}_t + [\phi_3\eta_{cg} + \phi_4\eta_{kg} + \phi_7]\,\hat{g}_t\\ &\quad + [\phi_3\eta_{c\varepsilon,1} + \phi_4\eta_{k\varepsilon,1}]\,\varepsilon^a_{g,t} + [\phi_3\eta_{c\varepsilon,2} + \phi_4\eta_{k\varepsilon,2}]\,\varepsilon^a_{g,t-1}.\end{aligned}$$

Solving for the dynamics. Finally, one can solve for the coefficients η in the recursive laws of motion. Both of the above equations must hold with equality for all values of the state variables. First, set $\hat{a}_t = \hat{g}_t = \varepsilon^a_{g,t} = \varepsilon^a_{g,t-1} = 0$:

$$\begin{aligned} 0 &= \left[\left(1 - \phi_1 \eta_{kk}\right) \eta_{ck}\right] \hat{k}_{t-1}, \\ 0 &= \left[\phi_3 \eta_{ck} + \phi_4 \eta_{kk} - \phi_6\right] \hat{k}_{t-1}. \end{aligned}$$

Since both equations also need to hold for any value of $\hat{k}_{t-1}$, it must be that

$$\begin{aligned} 0 &= \left(1 - \phi_1 \eta_{kk}\right) \eta_{ck}, \\ 0 &= \phi_3 \eta_{ck} + \phi_4 \eta_{kk} - \phi_6. \end{aligned}$$

The second equation implies

$$\eta_{ck} = \frac{\phi_6}{\phi_3} - \frac{\phi_4}{\phi_3}\eta_{kk},$$

and the first equation implies

$$0 = \phi_1 \phi_4 \eta_{kk}^2 - \left(\phi_1 \phi_6 + \phi_4\right) \eta_{kk} + \phi_6.$$

with solutions

$$\eta_{kk}^{\pm} = \frac{\phi_1^{-1} + \phi_6/\phi_4}{2} \pm \sqrt{\left(\frac{\phi_1^{-1} + \phi_6/\phi_4}{2}\right)^2 - \frac{\phi_6}{\phi_1 \phi_4}}.$$

Similarly, comparing coefficients on $\hat{a}_t$ gives

$$\eta_{ka} = \frac{\phi_5}{\phi_4} - \frac{\phi_3}{\phi_4}\eta_{ca}, \qquad \eta_{ca} = \frac{\phi_1(\phi_5/\phi_4)\eta_{ck} - \phi_2 \rho_a}{1 - \phi_1 \rho_a + \phi_1(\phi_3/\phi_4)\eta_{ck}}.$$

Comparing coefficients on $\hat{g}_t$ yields

$$\eta_{kg} = -\left(\frac{\phi_7}{\phi_4} + \frac{\phi_3}{\phi_4}\eta_{cg}\right), \qquad \eta_{cg} = \frac{-\phi_1(\phi_7/\phi_4)\eta_{ck}}{1 + \phi_1\left[(\phi_3/\phi_4)\eta_{ck} - \rho_g\right]}.$$

Further, comparing coefficients on $\varepsilon^a_{g,t-1}$ gives

$$\eta_{c\varepsilon,2} = -\frac{\phi_4}{\phi_3}\eta_{k\varepsilon,2}, \qquad \eta_{k\varepsilon,2} = \frac{-\eta_{cg}}{\eta_{ck} + \phi_4/(\phi_1\phi_3)}.$$

Finally, comparing coefficients on $\varepsilon^a_{g,t}$ yields

$$\eta_{c\varepsilon,1} = -\frac{\phi_4}{\phi_3}\eta_{k\varepsilon,1}, \qquad \eta_{k\varepsilon,1} = \frac{-\eta_{c\varepsilon,2}}{\eta_{ck} + \phi_4/(\phi_1\phi_3)}.$$

Modifications. When spending reacts to lagged TFP, the coefficient on $\hat{a}_t$ in (3.29) changes to $\eta_{ca}(1-\phi_1\rho_a) + \phi_2\rho_a - \phi_1(\eta_{ck}\eta_{ka} + \eta_{cg}\rho_{ga})$. Thus, the only coefficients in the recursive laws of motion that are affected are η_{ca} and η_{ka}. For $\rho_{ga} \in \mathbb{R}$, they are

$$\eta^*_{ka} = \frac{\phi_5}{\phi_4} - \frac{\phi_3}{\phi_4}\eta^*_{ca}, \qquad \eta^*_{ca} = \frac{\phi_1(\phi_5/\phi_4)\eta_{ck} - \phi_2\rho_a + \rho_{ga}\phi_1\eta_{cg}}{1 - \phi_1\rho_a + \phi_1(\phi_3/\phi_4)\eta_{ck}}.$$

If $\rho_{ga} = 0$, it follows that $\eta^*_{ka} = \eta_{ka}$ and $\eta^*_{ca} = \eta_{ca}$.

When spending reacts to current TFP, the coefficient on $\hat{a}_t$ in (3.29) changes to $\eta_{ca}(1-\phi_1\rho_a) + \phi_2\rho_a - \phi_1(\eta_{ck}\eta_{ka} + \eta_{cg}\rho_{ga}\rho_a)$. In this case,

$$\eta^{**}_{ka} = \frac{\phi_5}{\phi_4} - \frac{\phi_3}{\phi_4}\eta^{**}_{ca}, \qquad \eta^{**}_{ca} = \frac{\phi_1(\phi_5/\phi_4)\eta_{ck} - \phi_2\rho_a + \rho_{ga}\rho_a\phi_1\eta_{cg}}{1 - \phi_1\rho_a + \phi_1(\phi_3/\phi_4)\eta_{ck}}.$$

If $\rho_{ga} = 0$, it follows that $\eta^{**}_{ka} = \eta^*_{ka} = \eta_{ka}$ and $\eta^{**}_{ca} = \eta^*_{ca} = \eta_{ca}$.

3.B Detailed data description

This appendix provides details on data sources and data definitions. Throughout, NIPA refers to the National Income and Product Accounts of the Bureau of Economic Analysis, BLS to the Bureau of Labor Statistics, ALFRED to the Archival Federal Reserve Economic Data of the Federal Reserve Bank of St. Louis, and SPF to the Survey of Professional Forecasters of the Federal Reserve Bank of Philadelphia. All time series are provided in seasonally adjusted terms from the original sources, except the data on federal debt and the T-bill rate which are not seasonally adjusted.

- *Government spending, realization:* Real federal government consumption plus gross investment; the nominal series is taken from NIPA Table 1.1.5. Line 22; the series is then scaled to constant 2005 prices by its deflator (NIPA Table 1.1.4. Line 22) and converted into natural logarithms.

- *Government spending, forecasts:* One to three-quarter ahead forecasts of real federal government consumption plus gross investment; the level series is the mean prediction of SPF variable RFEDGOV; given breaks in levels due to NIPA base year changes, the forecasts are scaled to constant 2005 prices by backcasting the actual growth rates and imposing the average growth rate over the sample at the break points.

- *Government spending, expectational error:* First difference of natural logarithms of realized spending and the prediction thereof made one quarter earlier.

- *Output:* Real gross domestic product; the nominal series is taken from NIPA Table 1.1.5. Line 1; the series is then scaled to constant 2005 prices by its deflator (NIPA Table 1.1.4. Line 1) and converted into natural logarithms.

- *Consumption:* Real personal consumption expenditure; the nominal series is taken from NIPA Table 1.1.5. Line 2; the series is then scaled to constant 2005 prices by its deflator (NIPA Table 1.1.4. Line 2) and converted into natural logarithms.

- *Investment:* Real gross private investment; the nominal series is taken from NIPA Table 1.1.5. Line 7; the series is then scaled to constant 2005 prices by its deflator (NIPA Table 1.1.4. Line 7) and converted into natural logarithms.

- *Real wage:* Real hourly compensation in the business sector; BLS series ID: PRS84006153; the original series is converted into natural logarithms.

- *Interest rate:* 3-month treasury bill rate, secondary market rate; series TB3MS in ALFRED database; the interest rate is expressed in annual terms.

- *Debt-to-GDP ratio:* total end-of-period federal government debt divided by nominal GDP; public debt data: series GFDEBTN in ALFRED database.