In memoriam Erik G.F. Thomas (1939-2011): "A good definition is half the work"


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“A good definition is half the work”

Erik G.F. Thomas, professor of mathematics at the University of Groningen, passed away on 13 September 2011 at age 72. His colleagues and former colleagues Boele Braaksma, Tom Koornwinder (coordinator), Jan Stegeman, Jacques Faraut, Gerrit van Dijk, Leo van Hemmen and Tony Dorlas look back on his life and work.

Erik Thomas was born in The Hague on 19 February 1939. He died in Groningen on 13 September 2011. He is survived by his wife Gerda and his daughters Karin and Christine.

Life and career (by Boele Braaksma)
Erik Thomas studied mathematics at the University of Paris, where in 1969 he obtained his PhD on the thesis L’intégration par rapport à une mesure de Radon vectorielle published in Annales de l’Institut Fourier [1]. His advisor was Laurent Schwartz, a Fields medalist, whose best known achievement is the foundation of the theory of distributions. After obtaining his PhD he stayed for a year as maître de conférence in Orsay before taking up the post of assistant professor of mathematics at Yale University. In 1973 he was appointed professor of mathematics at the University of Groningen.

Because of his background he brought a rich mathematical culture with him to Groningen. Erik was a passionate mathematician who conveyed his knowledge and enthusiasm to his many students and colleagues, and who often influenced them a great deal. His lectures and seminars were lucid and inspiring and showed many facets of the beauty of mathematics. He could explain very complicated pieces of mathematics in a transparent manner. His students were enthusiastic about his lectures and his inspiring personality. His door was always open to students and colleagues and he answered their questions with much care and without regard of his time.

Many problems posed to him came from other disciplines, in particular from Theoretical Physics and Applied Mathematics. Until recently he had a close collaboration with his colleague Joop Sparenberg from Technical Mechanics. Consequently he was advisor for several theses from these areas. Erik supervised his PhD students very closely and he had much influence on them. Their theses were valuable contributions to mathematics.

Erik had extremely high standards in his research. Although he had several unpublished works lining his shelves and despite the ‘publish or perish’ atmosphere during the
later part of his career, he submitted only his very best manuscripts for publication.

He also took his share in the administrative duties. From 1992 until 1994 he was chair-
man of the (nowadays Koninklijk) Wiskundig Genootschap. He remains in our memory as a man of the highest integrity and as both an inspired and inspiring mathematician of high calibre with a great desire to share his love and knowledge of mathematics with others.

Research (by Tom Koornwinder)

Erik Thomas has published 48 research pa-
pers. All of them are reviewed in Zentralblatt
MATH (but only 39 are found there with the
author identification thomas.erik-g-f, of
which two are wrong), while MathSciNet miss-
es two of these (but gives the other 46 with the
author identification Thomas, Erik G. F.).
Curiously, MathSciNet misses his 101 page
AMS Memoir from 1974. His papers can be
divided in six categories:

− Integration with respect to vector-valued
  This is the subject of his thesis [1] in Or-
say. See the contributions by J. Faraut and
J. Stegeman. The already mentioned AMS
Memoir [2] also belongs to this subject.

− General functional analysis and integra-
  A paper of 60 pages with Alain Belanger in
Canad. J. Math. [5] falls under this subject,
but it also relates to Analysis on Lie groups.

− Integral representation in convex cones (8
  See the contribution by J. Faraut.

− Analysis on Lie groups (9 papers, 1984–
  2005). See the contributions by J. Faraut
and G. van Dijk. His work on this subject
was strongly influenced by his expertise on
the previous subject.

− Applied mathematics (4 papers, 1981–
  2002; 2 reports, 1997).
  Under this heading fall collaborations with his
colleagues J.A. Sparenberg and J.C. Willems
(see also the contribution by B. Braaksma),
and with his former PhD student J.L. van Hem-
men, as described in the contribution by
Van Hemmen.

− Path integrals (7 papers, 1996–2008).
  This was the subject in which he was much
interested during the last part of his career.
  See the contribution by T.C. Dorlas, with
whom he wrote his last published paper.
Of course, there is much interrelation be-
tween these categories. In particular they are
all fed by his impressive knowledge and mas-
tership of functional analysis and integration
theory. But he was the opposite of a narrow
specialist who only publishes ever increasing
technicalities in his own field. He enjoyed in-
spiration, interaction and collaboration with
people from other fields, both pure and ap-
plied mathematics and also physics. But he
avoided long series of papers with the same
co-authors. He had 12 different co-authors
with whom he wrote 11 papers.

A similar pattern can be seen from the sub-
jects of the PhD theses under his guid-
ance. According to the Mathematics Geneal-
ogy Project Thomas has had ten PhD stu-
dents. For three of them (Klamer, Pestman
and Capelle) he was the only advisor. All three
wrote a thesis in Analysis on Lie groups.
The other seven theses are with co-advisors,
often on applied topics, and sometimes defended
at another Dutch university.

Personally I got a closer acquaintance with
Erik when he started to come regularly to the
sessions of the Analysis on Lie groups semi-
nar during the eighties (see also the contribu-
tion by G. van Dijk). His own lectures there
were marvellous. But it gave also a great
added value to a session if Erik was in the
audience. By his frequent questions he really
wanted to understand what was said by the
speaker, and thus helped the speaker as well
to understand his own stuff better.

Erik brought joy and enthusiasm to the
annual sessions on Lie theory that Gerard
Helminck organized at Twente University for
a few days before Christmas. Even, in later
years, as Erik battled his disease, he kept at-
tending. His curiosity remained, and he could
inspire us as he always had done.

Orsay (door Jan Stegeman)

Graag wil ik iets vertellen over de bijzonde-
re relatie die ik meer dan veertig jaar met Erik
Thomas heb gehad. Onder de Nederland-
se wiskundigen neemt Erik een bijzondere
plaats in, omdat hij niet in ons land heeft ge-
studeerd. De middelbare school bezocht hij in
Engeland. Aanvankelijk was het de bedoeling
dat hij in Cambridge zou gaan studeren, maar
uiteindelijk kwam hij in Orsay terecht, 25 kli-
rometer ten zuiden van Parijs (tegenwoordig
onderdeel van de Université Paris-Sud). In zijn
eerste studiejaar raakte hij gefascineerd door
een analyse-college van Jacques Deny. Dit zou
bepalen zijn voor Erik latere carrière. Later
kwam hij in contact met Laurent Schwartz,
de vader van de distributietheorie. Deze gaf hem
enkele problemen die Erik wist op te lossen,
hetgeen uiteindelijk tot zijn promotie op 12
juni 1969 heeft geleid.

Doordat ik in 1969 een aanstelling in Orsay
had om aan mijn promotieonderzoek te wer-
ken, kwam ik met Erik in contact. Als land-
genoten trokken wij regelmatig met elkaar
op. Erik maakte mij wegwij in de Franse we-
reld, en onderzijds was hij geïnteresseerd in
het wiskundige leven in Nederland, want daar
wist hij heel weinig van. Ik heb toen voorge-
steld om samen het paascongres van het Wis-
kundig Genootschap bij te wonen, dat dat jaar
in Wageningen gehouden werd. Daar had ik
de gelegenheid hem bij diverse Nederlandse
wiskundigen te introduceren. Ik vlei mij soms
met het idee dat mede daardoor Erik enkele
jaren later er tot kwam om vanuit Amerika te
solliciteren naar een lectoraat in Groningen.

Erik en ik hebben altijd contact gehouden,
zowel wiskundig als op het persoonlijke vlak.

Bij mijn afscheid in Utrecht in 2000 was Erik
bereid iets over mijn werk te vertellen, terwijl
ik drie jaar later bij Erik afscheid een ver-
haal over Erik heb mogen houden ‘vanuit een
historisch perspectief’. Een samenvatting van
dit verhaal is opgenomen in het Groningse
Alumniews, no. 12, maart 2004. Men kan dit
nalezen op http://www.cs.rug.nl/jbi/History
/Thomas.

Terugblikkend op mijn verbluffend in Orsay be-
sef ik wat een geweldig instituut dat was,
waar Erik bijna tien jaar heeft kunnen stu-
deren en werken. Ter illustratie wil ik enkele
verdere namen noemen van wiskundigen (meest
analyticisti) die ik daar in 1969 heb mogen ont-
ommen: Jean-Pierre Kahane, Pierre Eymard,
Nicholas Varopoulos, Michel Demazure, Hen-
ri Cartan, Pierre Cartier, Paul Malliavin, Antoni
Zygmund, Lennart Carleson, Carl Herz, Adrien
Douady, Jacques Faraut, Yves Meyer. Men kan
zich voorstellen hoe Erik talenten, waarvan
het Groningse mathematisch instituut zoveel
profijt heeft gehad, zich in zo’n omgeving heb-
ken kunnen ontwikkelen.

Paris years and after (by Jacques Faraut)

I met Erik Thomas in Paris at the beginning of
the 60’s, when we were both students. Since
then we kept in touch, and we had still recent-
ly exchanges by mail and phone.

I recall with pleasure the time we were both
assistants at the University of Orsay, Paris-
Sud. Erik was showing much enthusiasm for
mathematics, communicating his enthusiasm
on every occasion. For instance, I remember
one evening in a restaurant in Paris when Erik
was explaining quantum mechanics to me.
Suddenly, all people at neighbouring tables
stopped talking and listened to Erik’s expla-
nations. At the University we organized to-
gether a workshop for the students, some-
thing rather unusual for assistants in these
days.
At that time Erik Thomas was writing his thesis under the supervision of Laurent Schwartz. The defense was a real event, which was attended by a large number of mathematicians.

After his doctorate, Erik Thomas worked for some years at American universities. While I was visiting him at Yale University, New Haven, Erik introduced me to the New York life.

Let me say a few words about some of his main mathematical achievements.

The thesis of Erik Thomas, defended in 1969, is devoted to the integration with respect to a vector-valued Radon measure. For a locally compact topological space $T$ and a real Banach space $E$, a Radon measure $\mu$ on $T$ with values in $E$ is a continuous map $K(T) \to E$, where $K(T)$ denotes the space of real-valued continuous functions with compact support. (More generally one considers a locally convex topological vector space $E$ which is quasi-complete.) One defines first the notion of extendable measure (mesure extensible). For such a measure every $w$-continuous bounded Borel function $f$ with compact support is integrable. Thomas established a domination theorem for unimodular Lie groups. In 

...
ing for which closed subgroups \(H\) in \(U(p, q)\) the pair \((H \ltimes H_\kappa, H)\) is a generalized Gelfand pair.

I had the chance to write with Thomas a paper [11] about the decomposition of unitary representations which are realized on Hilbert spaces of holomorphic functions, We gave a geometric criterion for multiplicity-free decomposition. (Our result has been reformulated in a much wider setting by T. Kobayashi, who introduced the concept of visible action.)

Thomas has been a very active mathematician until the last months of his life. An unpublished paper [15] written in June 2011 deals with multivariate completely monotonic functions.

**A true analyst** (by Gerrit van Dijk)

With Erik Thomas I shared a continuing interest in Gelfand pairs. It all started in 1979 with the doctoral dissertation of Erik’s student F.J.M. Klamer, entitled *Group representations in Hilbert subspaces of a locally convex space*, for which I was asked to serve in the examination committee.

The theory developed by Klamer appeared to have immediate implications for my own work on Gelfand pairs: pairs of groups \((G, K)\) with \(K\) compact, with the property that every irreducible unitary representation of \(G\), when restricted to \(K\), contains the trivial representation of \(K\) at most once. Klamer’s dissertation gave rise to an extension to pairs \((G, H)\) with \(H\) a closed, not necessarily compact subgroup of \(G\). This was a breakthrough which excited Erik and me. A lot of new questions arose. Would it be possible to generalize in some form the well-known criterion of Gelfand for showing that \((G, K)\) is a Gelfand pair, to pairs \((G, H)\)? On a beautiful day in July 1980 I received at my home address a letter from South Africa. Upon opening, the letter appeared to contain the solution. I was very thrilled by the elegance of the result, but I also wondered why this letter was posted in South Africa. Later this became clear to me: Erik was visiting South Africa to make his acquaintance with the family of his future wife Gerda.

After returning to the Netherlands, Erik reported extensively on his new mathematical results in the seminar ‘Analysis on Lie Groups’, chaired by Tom Koornwinder and myself. He proved, in passing, also an important result [4] for classical Gelfand pairs: if \(G\) is connected, then the pair \((G, K)\) is a Gelfand pair if the algebra of \(G\)-invariant differential operators on \(G/K\) is commutative. A little later it dawned upon us that Helgason had proven the same result, almost simultaneously.

Erik always impressed us with an excellent presentation of his lectures. His enthusiasm infected us, his independent thinking roused admiration. The importance of a good presentation he also successfully emphasized to his students. I have been a witness of this several times because some of his master students later wrote their doctoral dissertation under my guidance. With the passing of Erik we have lost a pure analyst and a true colleague.

**Clarity of exposition** (by Leo van Hemmen)

Erik Thomas was striving for mathematical clarity all his life, both while teaching and while discussing open problems. The way in which he practiced this clarity was fascinating and at the same time totally convincing. I was effectively Erik’s first graduate student. In fact, I had two doctoral thesis advisors, Nico Hugenholtz in theoretical physics and Erik Thomas in mathematics. My topic was ‘ergodic theory’ for a dynamical system with *a priori* infinitely many particles; in my case, the infinite harmonic crystal in thermodynamic equilibrium, a problem that I knew from my solid-state physics days.

What I learned from Erik while working on my doctoral dissertation *Dynamics and ergodicity of the infinite harmonic crystal* (University of Groningen, 1976; *Phys. Rep.* 65, 1980, 43–149) was focusing on total mathematical clarity. To quote him: “A good definition is half the work.” How true, but easily forgotten. As for buying books: “Only the very best is good enough.” This wise advice has saved me from the nuisance of having seductively cheap but in reality boring books looking down upon me. We always kept contact and two decades later we embarked on another project, that suited him even better.

Suppose we have a differential equation

\[
\frac{dx}{dt} = f(x) + \varphi
\]

in some Banach space \(E\); for example, \(\mathbb{R}^n\). The system \(\frac{dx}{dt} = f(x)\) is autonomous. It is supposed to have an equilibrium state; without restriction we can take it to be \(x = 0\), i.e., \(f(0) = 0\). Quite often the full \(f\) is neither known nor accessible to experiment, except for some of its ‘components’ where also the time-dependent input \(\varphi\) lives, and which can be sampled experimentally.

What we are hunting for is the solution operator that generates the time evolution induced by (1). Since Volterra (*Theory of functional and of integral and integrodifferential equations*, Blackie, London, 1930; Dover, 1959) one has often represented the solution to (1) as a series, which one now calls the Volterra series, with respect to increasing powers of \(\varphi\). A canonical representation of the Volterra series expansion for the scalar case \(\mathbb{R}^n\) with \(n = 1\) reads

\[
x(t) = \kappa_0 + \sum_{n \geq 1} \int_{-\infty}^t ds_1 \cdots \int_{-\infty}^t ds_n \kappa_n(t - s_1, \ldots, t - s_n) \cdot \varphi(s_1) \cdots \varphi(s_n).
\]

At reception after G. van Dijk’s farewell lecture, Leiden, 2004
Since \( x = 0 \) is our equilibrium point, we substitute \( \varphi = 0 \) and find \( k_0 = 0 \). In real life \( x(t) \), or part of it, is given experimentally and we would like to determine the kernels \( k_n \). In Banach space Erik Thomas, my former graduate student Werner Kistler and I could solve this problem fully [12].

A solution operator is said to be nonanticipative, or causal, if for each \( t \) the solution \( \mathcal{A}(\varphi)(t) \) depends on the restriction of \( \varphi \) to \( (-\infty, t] \) only, i.e., on the past of the input. What we have actually done is obtaining the unique nonanticipative solution operator \( \varphi \to \mathcal{A}(\varphi) \) for (1) so that \( x = \mathcal{A}(\varphi) \) solves (1). The Volterra expansion (2) amounts to expanding the solution operator \( \mathcal{A}(\varphi) \) into a Taylor series around \( \varphi = 0 \),

\[
\mathcal{A}(\varphi) = \sum_{n=1}^{\infty} \mathcal{A}_n(\varphi),
\]

\[
\mathcal{A}_n(\varphi) = \frac{1}{n!} \left. \frac{D^n \mathcal{A}(0)}{D^n \varphi} \right|_{\varphi = 0} (\varphi, \ldots, \varphi),
\]

where \( D^n \mathcal{A}(0)(\varphi_1, \ldots, \varphi_n) \) denotes the \( n \)-th order directional derivative at 0 in the directions \( \varphi_1, \ldots, \varphi_n \), the dependence upon \( t \) being understood; a glance at (2) may be helpful.

What Wiener (Nonlinear problems in random theory, MIT Press, 1958) did was substituting white noise (wn) for \( \varphi \), averaging (arithmetically) over \( t \) runs (while taking advantage of the strong law of large numbers), and exploiting a key property of white noise in that its mean gives a Dirac delta measure:

\[
\langle \varphi_{wn}(t) \varphi_{wn}(t+s) \rangle = \delta_s \text{ with, as usual, } \langle \cdots \rangle \text{ denoting the stochastic mean.}
\]

To see how this works, we take a simple example, viz., the linear case in (2) with \( x(t) = \int ds k_1(t-s) \varphi(s) \), multiply this by \( \varphi_{wn}(t') \), average over finitely many runs and obtain as approximation for \( t \) finite but large

\[
\langle x(t) \varphi_{wn}(t') \rangle = \int_0^\infty ds k_1(t-s) \delta(s-t') = k_1(t-t').
\]

Generating white noise is nontrivial, so why not use a delta measure as input? White noise has been used extensively in e.g. auditory experiments but why not use a click? A click sounds like, and is, an approximate delta measure (see W. A. Yost, Fundamentals of hearing, Academic Press, 1994), but the approximation is easy to produce and quite good. However, for using Dirac delta measures as input \( \varphi \) the kernels \( k_n \) must be continuous. Together with Erik Thomas [12, Section 3] we could prove that they are even real analytic, under the fairly general condition of \( f \) in (1) being an analytic function satisfying a Lipschitz condition, which suffices for most purposes.

Now one could complain that white noise may well be hard to generate but in experiment a delta function is not perfect either. True. That is why we have also proven a continuity theorem [12, Section 6] showing that for \( E = \mathbb{R}^N \) the approximate kernels approach the exact ones as the sampling, approximating, click becomes an exact Dirac delta measure. In passing we note that, though quite a bit clumsier, white-noise averaging is completely justified too once the kernels \( k_n \) such as those in (2) are continuous.

Looking backwards, I realize that working with Erik Thomas was a fascinating experience where we all greatly enjoyed his deep insight, his clear explanations, and his great enthusiasm for clarifying why mathematical structures give new insight. You ‘only’ need to see them, as Erik did.
Path integrals (by Tony Dorlas)
For me as a student at Groningen University, Erik Thomas was one of my favourite lecturers. I really intended to study theoretical physics but his inspiring lectures persuaded me to complete a degree in mathematics as well. He used to give his lectures entirely without notes and would often wear a round-necked jumper which he took off during the lectures. The lectures were always a model of clarity and organisation, showing a real mastery of the subject. Later, as a lecturer myself, it was always his lecturing style that I tried to imitate. Having just returned from Yale, and having studied in France, Erik sometimes used uncommon words during the lectures. Thus, I first learnt the word digression (Dutch for digression) from him. Despite his encouragement, I nevertheless decided to do my PhD in theoretical physics, but kept in close contact with Erik through his weekly seminar, which touched on many interesting subjects, but especially harmonic analysis, which was his main interest at the time. It consisted of a small group of students, giving a number of lectures in turn, often studying a particular book or article. I have learnt a lot from those seminars, not just mathematics but also how to present a talk. Around Christmas time, Erik would often invite us home for dinner with his wife Gerda. Here his French habits were also apparent. We would have a cognac and a plate of lettuce before the main meal. The conversation was often quite philosophical in nature, in keeping with Erik’s interests.

Although not trained in physics, Erik did have an interest in it and in the 90’s he started working on the mathematical definition of the Feynman path integral. This is the Lagrangian formulation of quantum mechanics introduced by Feynman after a tentative suggestion by Dirac. Originally, Feynman formulated his path integral as an alternative way of expressing the solution of the Schrödinger equation in non-relativistic quantum mechanics, but then he generalised it to the relativistic case and quantum field theory. It proved to be a particularly useful tool in perturbation theory giving rise to his introduction of Feynman diagrams. This led to much shorter calculations of relevant quantities than the traditional Hamiltonian approach.

However, the concept is still poorly understood in a mathematical sense. Many alternative formulations have already been suggested to give a mathematical meaning to this concept, but none is particularly satisfactory. The most fruitful to date is the Euclidean approach. Here one makes an ‘analytic continuation’ of the time variable to imaginary time. This turns the ill-defined oscillatory Feynman path integral into a well-defined Wiener integral. This was done by Kac, and is known as the Feynman–Kac path integral. It is such a powerful tool that many theoretical physicists today think in terms of Euclidean space rather than Minkowski space quite routinely. As Erik remarked, however, this does not really answer the question what mathematical entity corresponds to the Feynman path integral itself. It is known that it cannot be a (complex-valued) measure. Cecile De Witt-Morette suggested that it should be a distribution of some kind, but she did not give a more detailed construction.

A proper mathematical definition was given by Albeverio and Hoegh-Krohn after a suggestion by Ito, in terms of the Fourier transform of a bounded measure. Although this is indeed a proper mathematical formulation, Erik was not happy with it. He argued that the space of Fourier transforms of bounded measures is an unwieldy space. He initiated a new approach, exploring various simplified scenarios. One of those was to discretise space, another to discretise time.

At the time I was a lecturer at the University of Swansea, where professor Truman had also worked on the Feynman path integral. As I was interested myself, I invited Erik to Swansea, where we started a collaboration. At that time he had already worked out a discrete-time formulation [7], and we considered a possible continuous-time limit. Unfortunately, the result was negative. This discouraged me more than him, and I turned my attention to other projects. More than ten years later, on a visit to Groningen I discussed the matter again with him. It turned out that he had only published his work on a finite space version but thought that it could be generalised to infinite discrete space. I offered to try and work this out, and sent him a draft version of a paper some time later. Although Erik was already unwell at that time, he nevertheless sent some comments and we communicated about its publication. We decided to send it to the Journal of Mathematical Physics, where it was accepted [14].

Just this year, I had a new post-doc (Matieu Beau) with whom I decided to work on the path integral again. So far, we have been able to extend Erik’s work on the discrete-time integral, simplifying his approach somewhat and considering more general boundary conditions. Sadly, we will have to do without his comments and encouragement.

Selected papers by Erik Thomas
15. Completely monotonic functions and elementary symmetric polynomials, unpublished manuscript, 2011.