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Erotetic languages and the inquisitive hierarchy*

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Abstract

We start out postulating a notion of an erotetic language as a language that covers both informative and inquisitive semantic content. Next we postulate the concept of a classical erotetic language, where it is required that informative and inquisitive content are divided over two distinct syntactic categories: indicatives and interrogatives. From the general notion of an erotetic language we derive the fundamental logical-semantical concepts of inquisitive semantics, and sketch the contours of such a semantics for propositional erotetic languages. Then we first fill in the semantic details to arrive at a general inquisitive semantics for propositional erotetic languages. Next we restrict the syntax of the language in such a way that it becomes a classical erotetic language. The syntactic restrictions make this classical language semantically essentially poorer than the general one, though it is still richer than classical partition semantics, and can cope with conditional questions. The notion of the inquisitive hierarchy mentioned in the title of the paper plays a crucial role in explaining the difference. However, we go on to show that, while sticking to a classical erotetic language, the semantic restrictions can be lifted by generalizing interrogative formation from an operation on single sentences to one on sets of sentences. As a result, like non-classical general inquisitive semantics, classical inquisitive semantics can cope with alternative questions, and the general and the extended classical language turn out to be basically equivalent in expressiveness. They only differ in that in the classical case we sometimes need two sentences to express what in a general erotetic language can be expressed by a single sentence. For all systems under discussion, we show that they are conservative extensions of classical logic.

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†The paper is strongly based on joint work with Ivano Ciardelli and Floris Roelofsen.

1 Setting the stage

In Groenendijk and Stokhof (1997) a basic syntactic distinction between indicative and interrogative sentences is assumed and it is argued at length that for a proper interpretation of the interrogatives in a language, a specific *semantic domain* is required, distinct from the one that serves the interpretation of the indicatives in the language: whereas indicatives express propositions, interrogatives express questions.

What G&S argue against, might be called the Fregean position, which is that all sentences express propositions, and that the difference between a question and an assertion is not a difference in semantic content, but a difference in *force*. In slightly more modern terms, asking a question and making an assertion are different speech acts with the same propositional content. According to G&S, these different kinds of speech acts also have to come with a different kind of semantic content.[1]

The basic semantic picture argued for in Groenendijk and Stokhof (1984, 1997) is that, whereas an indicative expresses a proposition, viewed as a subset of the set of possible worlds, an interrogative expresses a question, viewed as a partition of the set of possible worlds.

Elegant as it is, in particular from a logical point of view, a partition semantics for interrogatives has its limitations. If we just consider a propositional language, there are already two basic interrogative constructions that cannot be dealt with in a straightforward way: conditional questions and alternative questions.[2]

Attempts to escape from the cells of partition semantics eventually have led to the development of the new paradigm of inquisitive semantics.[3] One of the main purposes of this paper is to provide a *conceptual argumentation* for inquisitive semantics. I want to deduce the conceptual framework as it is used in inquisitive semantics from some minimal assumptions by analytical means.

Although the conceptual analysis I will present is not inherently restricted to specific languages, in order to keep the abstract story a bit concrete, I will focus on the basic case of propositional languages. I take it to be part of the analytical methodology to be conservative, and stay as close as possible to classical propositional logic.

[1] A recent resurrection of the Fregean position can be found in the treatment of questions in dynamic epistemic logic. Cf. Van Benthem and Minică (2011); Minică (2011). Questions do not appear in the basic logical language of the system, which is purely assertoric, but they are only introduced in the dynamic part of the system as actions, i.e., as speech acts.

[2] See Mascarenhas (2009) for a critique of the partition semantics of questions on these and other grounds, from the perspective of inquisitive semantics.

[3] The two main references for inquisitive semantics, logic, and pragmatics, are Ciardelli and Roelofsen (2011); Groenendijk and Roelofsen (2009). See the web site `www.illc.uva.nl/inquisitive-semantics` for more publications and up to date information.

2 Inquisitive postulates

2.1 The erotetic postulate

The basic assumption from which we start is that a proper semantics for a language in which questions can be expressed has to distinguish two aspects of meaning, which we dub *inquisitiveness and informativeness*. We call a semantics in which these two semantic properties are distinguished an *inquisitive semantics*, and we call a language that comes with an inquisitive semantics an *erotetic language*.

Postulate 1 (Erotetic Languages)

A language $\mathcal{L}$ is an erotetic language *iff*

1. *The* semantics for $\mathcal{L}$ is inquisitive*: it distinguishes both informative and inquisitive content of the sentences in $\mathcal{L}$.*
2. *For some $\varphi \in \mathcal{L}$: φ is informative and for some $\varphi \in \mathcal{L}$: φ is inquisitive.*
3. *A sentence $\varphi \in \mathcal{L}$ is a* tautology *iff φ is neither informative nor inquisitive.*
4. *for some $\varphi \in \mathcal{L}$: φ is a tautology.*

The third clause is meant to guaranteee that no other aspects of semantic content than informativeness and inquisitiveness can play a role in $\mathcal{L}$. The last clause is meant to guarantee that there is a logic to be found for $\mathcal{L}$.[4] Together, the clauses (2)-(4) guarantee that for each of the two properties of being informative and being inquisitive, there are sentences that have, and sentences that lack that property.

2.2 Questions, assertions and hybrids

In introducing erotetic languages we started out from the notion of a "language in which questions can be expressed," but the semantic concept of a question has not entered the scene yet. It is introduced in the following stipulative definition:

Definition 1 (Assertions, questions, and hybrids)

Let $\mathcal{L}$ be an erotetic language, $\varphi \in \mathcal{L}$.

1. *φ is a* question *iff φ is not informative.*
2. *φ is an* assertion *iff φ is not inquisitive.*
3. *φ is* hybrid *iff φ is informative and inquisitive.*

[4] Later in the paper, we will identify the tautologies in $\mathcal{L}$ with the sentences in $\mathcal{L}$ which are logically valid.

Under this definition, being an assertion does not imply being informative, and being a question does not imply being inquisitive. But, given the definition of tautologies included in the definition of erotetic languages, φ is a non-tautological question iff φ is inquisitive and φ is a non-tautological assertion iff φ is informative. The definitions leave room for both tautological questions and tautological assertions. Since a tautology is a tautology, a tautological question counts as an assertion as well, and a tautological assertion is also a question. By definition, hybrid sentences are never tautological.

Note that being a question or an assertion is a *semantic property* a sentence may have. Contrary to what is at stake in Groenendijk and Stokhof (1997), questions as such are not characterized as *semantic objects* of a distinctive type, and we do not assume that questions and assertions are semantic objects of different types.

2.3 The classical erotetic postulate

We made no specific assumptions about the syntax of an erotetic language $\mathcal{L}$. First of all, nothing was required concerning the logical vocabulary of $\mathcal{L}$, such as the presence of an interrogative operator.

Perhaps more importantly, we did not assume, as is done in Groenendijk and Stokhof (1997), that there are distinct syntactic sentential categories in $\mathcal{L}$, nor the opposite. The definition only requires that the (non-)informative and the (non-)inquisitive sentences in $\mathcal{L}$ can be characterized as such semantically, not necessarily syntactically as well. We call a language a *classical erotetic language* if it does have such distinctive syntactic categories corresponding to assertions and questions.

Postulate 2 (Classical erotetic languages)

A language $\mathcal{L}$ is a classical erotetic language *iff*

1. *$\mathcal{L}$ is an erotetic language which has two syntactic sentential categories of* indicatives $\mathcal{L}_!$ and interrogatives $\mathcal{L}_?$*, where*
2. *$\mathcal{L} = \mathcal{L}_! \cup \mathcal{L}_?$ and $\mathcal{L}_! \subset \mathcal{L}$ and $\mathcal{L}_? \subset \mathcal{L}$ and $\mathcal{L}_! \cap \mathcal{L}_? = \emptyset$.*
3. *Every $\varphi \in \mathcal{L}_!$ is an assertion and every $\varphi \in \mathcal{L}_?$ is a question.*

Given the conditions in the last two clauses, there is no room in a classical erotetic language for hybrid single sentences. The general definition of an erotetic language does not require that it contains hybrid sentences, but it does not forbid it either. Actually, the definition in principle allows for erotetic languages where every non-tautological sentence is hybrid.

When we consider a natural language like Dutch or English, at most a fragment of it can form a classical erotetic language, since, e.g., imperatives are left out of consideration here. Presumably, it is a common assumption that the erotetic fragments of the languages of the world are always classical. Be that

as it may, we do not make this assumption in our general notion of what an erotetic language is.[5]

3 Inquisitive semantic concepts

3.1 Questions as classical tautologies

The classical format for defining a semantics for a language $\mathcal{L}$ is by means of a recursive truth-definition for the sentences in $\mathcal{L}$ relative to suitable models for $\mathcal{L}$, which we will call possible worlds. Let ω be the set of suitable worlds for $\mathcal{L}$, $v \in \omega$, and $\varphi \in \mathcal{L}$. We denote the classical notion of φ being true in v by $v \models_{cl} \varphi$.

The meaning of φ, which classically coincides with its informative content, is then determined by $\mathsf{info}(\varphi) = \{v \in \omega \mid v \models_{cl} \varphi\}$, the set of worlds where φ is true. A sentence counts as informative, as not being a classical tautology iff $\mathsf{info}(\varphi) \neq \omega$.

This set-up will not work just like that for an inquisitive semantics for an erotetic language. There we meet questions, which, by definition, are not informative. So, if φ is a question then $\mathsf{info}(\varphi) = \omega$. Classically, when we only consider informative content, every question would be a tautology. But, of course, in terms of inquisitive content only some questions should be tautologies, those that are not inquisitive.

The old dictum has it that questions are neither true nor false. But given our analytical considerations, it would rather be the case that *when evaluated relative to a single world a question is always true.* The diagnosis is then, that stating the semantics for an erotetic language in terms of a truth definition, evaluating sentences relative to a single world, is not fine-grained enough to determine the meaning of inquisitive sentences.

We have arrived at a point where we have to make up our mind as to how to extend classical semantics in order to be able to deal with inquisitiveness. Our analytical methodology dictates that we have to be conservative and stay "as close as possible" to the classical setting. But it is not obvious that there cannot be different adequate extensions for which conservativity can be claimed, and of which, by lack of clear-cut criteria, it cannot be determined whether one is closer to classical logic than the other.[6]

So, for what is to follow, I do not want to claim that it is *the only way* to go, but only that it is *an easy way* in which to arrive at a conservative extension of classical logic that can deal with the semantics of erotetic languages.

[5] I was inspired by the persistent comments of Craige Roberts concerning the relation between natural language and inquisitive semantics, to investigate classical erotetic languages. To a large extent, the whole plan for this unplanned paper originates from that.

[6] Thanks to Johan van Benthem and Floris Roelofsen for pointing me in this direction.

3.2 From truth in a world to support in a state

Going by the observation that evaluation relative to a single world does not suffice, it is natural to lift the 'evaluation points' from single worlds to sets of worlds. Standardly, sets of worlds are used to model information states, where a state consists of the worlds that are still compatible with the information that the state contains. It is not unusual to denote the evaluation relation between sentences φ and states s by $s \models \varphi$ and to pronounce it as s *supports* φ.

The lift from truth in a world to support in a state immediately gives rise to a way to test whether we stay close to the classical setting by requiring that whenever we consider the case of evaluating a sentence relative to a singleton set consisting of one world, the result is classical, i.e., that $\{v\} \models \varphi$ iff $v \models_{cl} \varphi$, at least for the cases where φ is an assertion, the type of sentences which is under the jurisdiction of classical logic.

A state consisting of a single world corresponds to a state of maximal consistent information. This makes the link between $\{v\} \models \varphi$ and $v \models_{cl} \varphi$ for assertions a natural one. Also, that questions come out trivial when we only consider states of maximal consistent information is conceptually easy to digest.

Arguably, when we consider sets of worlds as evaluation points, we stay closer to the classical setting the smaller such sets can be. So, pairs of worlds would be the best, given that single worlds do not suffice. On the other hand, the conceptual link of sets of worlds with information states is then practically lost. What could be the intuitive idea behind evaluation relative to pairs of worlds?[7]

So, for conceptual reasons, let us stick to evaluation relative to arbitrary sets of worlds, which we can look upon as the familiar notion of information states. Also, if we go this way, we can always detect whether pairs would have been sufficient from a purely logical point of view. That is decidedly the case if it turns out to hold that: $s \models \varphi$ iff $\forall v, w \in s\colon \{v, w\} \models \varphi$, a property which we will call *pair-distributivy*.

[7] Well, inquisitive semantics originates from an update semantics where states were structured by an equivalence relation, a relation of indifference. Cf. Hulstijn (1997); Groenendijk (1998, 1999); ten Cate and Shan (2007). So, there we are dealing with an update version of partition semantics. In order to be able to deal with conditional questions (cf. Velissaratou (2000)) and alternative questions, states became structured by weaker indifference relations, lacking transitivity. By the way, the logical languages in those days were classical erotetic languages. Within such a framework, more or less by accident, inquisitive disjunction was 'discovered'. Cf. Mascarenhas (2009). By the way, precisely at this point classical erotetic languages were left behind. *The relation of indifference as structuring states has strong intuitive content.* But where there are relations, there are pairs. And, as was pointed out to me by Balder ten Cate, the whole update machinery with structured states can equivalently be formulated in logically much simpler and familiar terms: support relative to pairs of worlds. Cf. Groenendijk (2009). But here, increase of logical elegance led to loss of intuitive clarity, as is signalled by the question mark to which this note is attached. More on pairs and sets of worlds later.

3.3 Support semantics

Now that we have decided that we will employ a support semantics, let us be a bit more specific about the general features of such a framework, where we restrict ourselves to a support semantics for propositional languages.

Definition 2 (Worlds and states)

Let $\mathcal{L}$ be a propositional language with a set of proposition letters $\mathcal{P}$.

1. *The set of* suitable worlds *for $\mathcal{L}$, $\omega_{\mathcal{P}}$ is the set of subsets of $\mathcal{P}$.*
2. *The set of* suitable states for *$\mathcal{L}$, $S_{\mathcal{P}}$ is the set of subsets of $\omega_{\mathcal{P}}$.*

Whenever it is not strictly needed, we drop the subscript $\mathcal{P}$.

Let $p \in \mathcal{P}$ and $v \in \omega$, then we look upon $p \in v$ as: the fact that p holds in world v.

The set of suitable states for a language is partially ordered by the subset relation, we call it the relation of extension between states.

Definition 3 (Extension) *Let $s, t \in S$. s* is an extension of *t iff $s \subseteq t$.*

The unique maximal state under the extension ordering is ω, in which no world in the logical space is excluded by the information it contains. We call ω *the ignorant state*. The unique minimal element under the extension ordering is $\emptyset$, the state of inconsistent information. We call $\emptyset$ *the absurd state*. What we called 'states of maximal consistent information' above, are the minimal elements under the extension ordering if we ignore the absurd state.

We make the following general assumptions concerning a support semantics for a language relative to the set of suitable states for the language:

Assumption 1 (Support semantics) *Let S be the set of suitable states for $\mathcal{L}$.*

A support semantics *for $\mathcal{L}$ recursively defines the notion of when a state $s \in S$ supports a sentence $\varphi \in \mathcal{L}$, which we denote as $s \models \varphi$.*

The logical notions of validity, entailment *and* equivalence *are defined as:*

1. *$\models \varphi$ iff for all $s \in S$: $s \models \varphi$.*
2. *$\varphi \models \psi$ iff for all $s \in S$: if $s \models \varphi$, then $s \models \psi$.*
3. *$\varphi \equiv \psi$ iff $\varphi \models \psi$ and $\psi \models \varphi$.*

A sentence φ entails a sentence ψ if in every state where φ is supported, it cannot fail to be the case that ψ is supported as well. Two sentences are equivalent if they are supported in the same states. A sentence is valid if it is supported in every state.

3.4 Conservativity

There are two properties a sentence may or may not have under a support relation that are crucial for a support semantics for a langtuage $\mathcal{L}$. They can be named and characterized in different ways, but we call them *stability* and *additivity* here, and characterize them as follows.[8]

Definition 4 (Stability and additivity)

Let S be the set of suitable states for $\mathcal{L}$, $\varphi \in \mathcal{L}$.

1. *φ is* stable *iff for all $s \in S$: if $s \models \varphi$ and $t \subseteq s$, then $t \models \varphi$.*
2. *φ is* additive *iff for all $s, t \in S$: if $s \models \varphi$ and $t \models \varphi$, then $s \cup t \models \varphi$.*
3. *φ is* classical *iff φ is stable and additive.*

The reason why the properties of stability and additivity are crucial for a support semantics for a language $\mathcal{L}$ is that if every sentence in $\mathcal{L}$ has both properties, then the lift from worlds to states will not move us up or down.

Fact 1 (Stable plus additive is classical)

If φ is classical, then for all $s \in S$: $s \models \varphi$ iff $\forall v \in s$: $v \models_{cl} \varphi$.

We propose to use the following definition as a criterion to judge whether a support semantics for a propositional language can be considered to be a conservative extension of classical propositional logic.

Definition 5 (Conservative extension of classical logic.)

Let $\mathcal{L}$ be a propositional language with proposition letters $\mathcal{P}$, logical vocabulary λ, and interpreted by a support relation $\models$.

$\mathcal{L}$ is a conservative extension of classical logic *iff there is a fragment $\mathcal{L}_{cl}$ of $\mathcal{L}$ with proposition letters $\mathcal{P}$ and logical vocabulary $\lambda_{cl} \subseteq \lambda$ such that:*

1. *$\mathcal{L}_{cl}$ is a functionally complete language of classical propositional logic;*
2. *for all $\varphi \in \mathcal{L}_{cl}$: φ is classical.*
3. *for all $\varphi \in \mathcal{L}_{cl}$: $\models \varphi$ iff $\models_{cl} \varphi$.*

The three clauses in the definition concern the syntax, the semantics, and the logic of the language, respectively. The requirement that the logic of the fragment is classical follows from the requirement that its semantics is classical.

[8] Support semantics is a simplistic version of data semantics, as introduced in Veltman (1985). The notion of an information state appears there, but I am not sure whether it originates from data-semantics. The support–terminology is not yet used there, but it is in Veltman's later work on update semantics. Cf. Veltman (1996). Veltman (1985) uses the more long-winding dual notion of 'truth/falsity relative to the evidence available at s'. Stability is a crucial notion there. It comes in two versions: T-stable and F-stable. The epistemic modality *may*-φ is not T-stable, and the epistemic modality *must*-φ is not F-stable. In our simplified version, we would only be able to deal with the first of these two modalities, and it would count as non-stable. But, in fact, inquisitive semantics proposes a radically different way of dealing with *may*-φ, or rather *might*-φ, using the terminology of Veltman (1996), where *might*-φ concerns neither informative nor inquisitive semantic content, but attentive content. Cf. Ciardelli *et al.* (2009a).

3.5 Informative and inquisitive support

Our main task is now to determine what support in a state amounts to for the sentences in an erotetic language, and which role informativeness, and in particular inquisitiveness, play in that. In order to do so, we make three assumptions.

The first assumption is that under a support semantics, the semantic notion of φ being a tautology in an erotetic language coincides with the logical notion of φ being valid, i.e., φ being supported in every state.

Assumption 2 (Tautologies and validity)

Let $\mathcal{L}$ be an erotetic language, $\varphi \in \mathcal{L}$.

$\models \varphi$ *iff* φ *is a tautology.*

The second assumption is that the semantic notion of a sentence φ being a tautology, of φ being insignificant, can be relativized to states.

Assumption 3 (Tautological in a state)

Let $\mathcal{L}$ be an erotetic language, $\varphi \in \mathcal{L}$, and s a suitable state for $\mathcal{L}$.

φ *is tautological in s iff φ is neither informative nor inquisitive in s.*

If we put these two assumptions together, then we arrive at a notion of a sentence φ being 'valid in a state s', suitably denoted as $s \models \varphi$, i.e., as the support relation. So, taking these two assumptions for granted, we arrive at the following notion of support of a sentence in a state:

Definition 6 (Support)

Let $\mathcal{L}$ be an erotetic language, $\varphi \in \mathcal{L}$, and s a suitable state for $\mathcal{L}$.

$s \models \varphi$ *iff* φ *is neither informative nor inquisitive in s.*

It remains to be explained what it means for a sentence to be informative or inquisitive in a state.

As for informativeness, we will be conservative. The third and last assumption we make is that the notion of informativeness remains classical.

Assumption 4 (Classical informativeness)

Let $\mathcal{L}$ be an erotetic language, $\varphi \in \mathcal{L}$, and ω the set of suitable worlds for $\mathcal{L}$.

The informative content of φ, $\mathsf{info}(\varphi) = \{v \in \omega \mid \{v\} \models \varphi\}$.

φ *is* informative *iff* $\mathsf{info}(\varphi) \neq \omega$.

Non-informativeness of a sentence φ in a state s can then be taken to mean that the update of s with the informative content of φ has no effect.

Definition 7 (Informativeness in a state)

Let $\mathcal{L}$ be an erotetic language, $\varphi \in \mathcal{L}$, and s a suitable state for $\mathcal{L}$.

φ *is* informative *in s iff* $s \cap \mathsf{info}(\varphi) \neq s$.

Note that the assumed absolute notion of informativeness of a sentence φ coincides with φ being informative in the state of ignorance ω.

Then, finally, our task it to figure out when a sentence is inquisitive in a state. In principle, although this does not hold for questions nor for assertions, a sentence may be hybrid and hence be both informative and inquisitive in a state. So, in general, if a state does not support a sentence this may be due to informativeness and to inquisitiveness. But since we have already decided what informativeness in a state means, we can neutralize that aspect. Just add the informative content of a sentence to the information that is already contained in a state, and if the state then still does not support the sentence, then it must be because it is inquisitive in that state.

Definition 8 (Inquisitiveness in a state)

Let $\mathcal{L}$ be an erotetic language, $\varphi \in \mathcal{L}$, and s a suitable state for $\mathcal{L}$.

φ is inquisitive in s iff $s \cap \mathsf{info}(\varphi) \not\models \varphi$.

At this point, we have more or less achieved our main purpose. Starting from postulating what an erotetic language is (the notion of a classical erotetic language did not influence our deliberations so far), we arrived in a largely analytical fashion at the basic conceptual apparatus as it is used in inquisitive semantics.

The only non-analytical step in our proceedings was the choice for the format of a support semantics. However, what is clearly a fortunate aspect of that choice, is that it made it easily possible to formulate clear-cut criteria by which we can determine whether a support semantics for an erotetic language is a conservative extension of classical logic.

4 General inquisitive semantics

4.1 Propositional erotetic languages: the basics

So far our consideration concerning erotetic languages and inquisitive semantics were fully abstract and analytical. Now we will get more concrete and will actually present propositional erotetic languages and provide an inquisitive support semantics, to which the concepts that we derived above apply.

We start from a standard language $\mathcal{L}$ for classical propositional logic, where the non-logical vocabulary of the language consists of a set of proposition letters $\mathcal{P}$, and the logical vocabulary consists of the connectives: $\bot, \neg, \wedge, \vee, \rightarrow$. Some connectives may be basic and others definable in terms of the basic ones.

When we move to erotetic propositional languages it is natural to allow for an extra sentential interrogative operator ?. The interrogative operator is, of course, intended to create questions. For reasons of symmetry, we also make room for a declarative operator !, in case we need or want to syntactically force a sentence to be an assertion.

If we only allow for an interrogative operator, we implicitly assume that there is a basic declarative language to which interrogatives are *added*. But in

case we consider a language with assertions and questions, it makes at least as much sense to start from a basic 'neutral' language, where in order to obtain assertions and questions both of them can or have to be marked explicitly as such.

The full set of connectives we now allow for is: $\bot, \neg, \wedge, \vee, \rightarrow, ?, !$. Also relative to the full set it might be the case that there is a basic subset in terms of which the rest can be defined. And it might just as well be that ? or ! are definable in terms of the standard connectives, once we add inquisitive content to a propositional language.

We have bound ourselves to a support semantics relative to information states, and we have already determined in Definition 2, what the suitable states for a propositional language are.

4.2 Inquisitive support semantics

We start out by presenting the system InqB of general inquisitive semantics for a propositional language, following Ciardelli (2009); Ciardelli and Roelofsen (2011); Groenendijk and Roelofsen (2009). The B in InqB stands for 'basic', and it has become customary to refer by this acronym to the version of inquisitive semantics discussed here.

In my presentation of InqB, I will concentrate on those logical and semantical aspects that are immediately related to the specific concerns of the present paper. So, I do not intend to give a proper general introduction to inquisitive semantics here, but refer the reader to the papers I just mentioned. I will also draw heavily from the logical results in the first two papers I mentioned, without explicitly mentioning this case by case.

An InqB-language $\mathcal{L}$ is a standard language of propositional logic with as basic connectives $\bot, \wedge, \vee, \rightarrow$. The one-place connectives $\neg, ?, !$ are introduced by definition. Let $\mathcal{P}$ be the set of propositional variables in $\mathcal{L}$, and S the set of suitable states for $\mathcal{L}$. We recursively define when a sentence $\varphi \in \mathcal{L}$ is supported in a state $s \in S$.

Definition 9 (General inquisitive semantics)

1. $s \models p$ *iff* $\forall v \in s : p \in v$ *for all* $p \in \mathcal{P}$
2. $s \models \bot$ *iff* $s = \emptyset$
3. $s \models \varphi \wedge \psi$ *iff* $s \models \varphi$ *and* $s \models \psi$
4. $s \models \varphi \vee \psi$ *iff* $s \models \varphi$ *or* $s \models \psi$
5. $s \models \varphi \rightarrow \psi$ *iff* $\forall t \subseteq s :$ *if* $t \models \varphi$ *then* $t \models \psi$

The operations of negation, non-inquisitive closure, and non-informative closure, are introduced by definition.

Definition 10 (Abbreviations)

1. $\neg\varphi := \varphi \to \bot$
2. $!\varphi := \neg\neg\varphi$ (non-inquisitive closure)
3. $?\varphi := \varphi \vee \neg\varphi$ (non-informative closure).

General inquisitive semantics has the property of persistence, whenever a state supports a sentence φ, so do all of its substates, all of its extensions: if $s \models \varphi$, then for all $t \subseteq s$: $t \models \varphi$. As information grows, support of φ cannot get lost. In Definition 4, we called sentence φ *stable* if it has this property.

Fact 2 (Support is persistent) *For all $\varphi \in \mathcal{L}$: φ is stable.*

Above we defined notions of informativeness and inquisitiveness relative to states (Definitions 7 and 8). We already noted there that the absolute notion of informativeness amounts to informativeness in the ignorant state. Given persistence, the same holds for the absolute notion of inquisitiveness.

Fact 3 (Informativeness and inquisitiveness)

φ is informative iff $\mathsf{info}(\varphi) \neq \omega$.

φ is inquisitive iff $\mathsf{info}(\varphi) \not\models \varphi$.

Persistence implies that if $s \models \varphi$, then for all $v \in s$: $\{v\} \models \varphi$. Support of a sentence φ in a state of maximal consistent information $\{v\}$ amounts to truth of φ in v in classical propositional logic.

Fact 4 (Singleton states behave classically)

For all $\varphi \in \mathcal{L}$ and for all $v \in \omega$: $\{v\} \models \varphi$ iff $v \models_{cl} \varphi$.

This implies that for all $\varphi \in \mathcal{L}$: $\mathsf{info}(\varphi)$, the informative content of φ, is equal to the meaning of φ in classical propositional logic. But $\mathsf{info}(\varphi)$ does not in general exhaust the meaning of a sentence φ in inquisitive semantics.

Given that singleton states behave classically, if φ is an inquisitive sentence and $\mathsf{info}(\varphi) \not\models \varphi$, then there must be states s and t such that $s \models \varphi$ and $t \models \varphi$, but $s \cup t \not\models \varphi$. Inquisitive sentences are non-additive according to Definition 4.

Fact 5 (Inquisitive is non-additive) *φ is inquisitive iff φ is not additive.*

We saw in Section 3.4 that a support semantics is not called for if all sentences in the language are both stable (persistency of support) and additive. But if InqB meets the erotetic language postulate there must be inquisitive sentences in the language, and the use of a support semantics is then not spurious.

4.3 Hybrids, assertions, and questions

Let us now make sure that InqB is in accordance with the erotetic language postulate. This mainly means that we have to make sure that there are informative and inquisitive sentences in $\mathcal{L}$. We can show this to be the case with a single sentence, provided that there are at least two proposition letters $p, q \in \mathcal{P}$.

Fact 6 (Hybrid disjunction) *$p \vee q$ is informative and inquisitive.*

The disjunction $p \vee q$ is informative: $\mathsf{info}(p \vee q) \neq \omega$, and $p \vee q$ is also inquisitive: $\mathsf{info}(p \vee q) \not\models p \vee q$. That $\mathsf{info}(p \vee q) \not\models p \vee q$ results from the interpretation of disjunction in the inquisitive support semantics, which says that a state supports a disjunction if it supports one of its disjuncts. After we have added the information that $p \vee q$ to the ignorant state ω, it does not contain sufficient information to support either p or q. Since $p \vee q$ is both informative and inquisitive, it is, by definition, a hybrid sentence.

It is also easy to see that $p \vee q$ lacks the property of additivity, and hence is not a classical sentence under Definition 4. Whereas $\mathsf{info}(p) \models p \vee q$ and $\mathsf{info}(q) \models p \vee q$, $\mathsf{info}(p) \cup \mathsf{info}(q) = \mathsf{info}(p \vee q) \not\models p \vee q$.

We can turn $p \vee q$ into a non-inquisitive sentence with the same informative content by applying non-inquisitive closure $!(p \vee q)$, where non-inquisitive closure is defined as double negation, and negation as $\varphi \to \bot$. Negation behaves classically in InqB.

Fact 7 (Negation is classical) *$s \models \neg\varphi$ iff $\forall v \in s\colon \{v\} \not\models \varphi$.*

Hence, double negation, i.e., non-inquisitive closure, behaves classically as well.

Fact 8 ($!\varphi$ is an assertion) *For all $\varphi \in \mathcal{L}$: $!\varphi$ is not inquisitive.*

So, the operator ! behaves as was expected of it when we allowed it to be present as an operator in an erotetic language: when applied to a sentence it forces it to be an assertion in the language. We can actually use it to characterize assertions.

Fact 9 (Assertions) *For all $\varphi \in \mathcal{L}$: φ is an assertion iff $\varphi \equiv !\varphi$.*

Since non-inquisitive closure behaves classically and assertions and only assertions are equivalent with their non-inquisitive closure, the assertions in the language coincide with the classical sentences.

Fact 10 (Assertions are classical)

For all $\varphi \in \mathcal{L}$: φ is an assertion iff φ is a classical sentence.

We obtain similar facts for the non-informative closure $?\varphi$ of a sentence φ, as we did for non-inquisitive closure. Since $?\varphi$ is defined as $\varphi \vee \neg\varphi$, it holds that:

Fact 11 ($?\varphi$ is a question) *For all $\varphi \in \mathcal{L}$: $?\varphi$ is not informative.*

In this case we can also characterize questions by means of non-informative closure.[9]

Fact 12 (Questions) *For all $\varphi \in \mathcal{L}$: φ is a question iff $\varphi \equiv ?\varphi$.*

That φ is a question does not mean that φ is inquisitive, e.g., $?\bot$ is a tautology. Every state supports $\neg\bot$. So, every state supports $?\bot$. Hence $\models ?\bot$, it is a validity in InqB, i.e., a tautology, a sentence which is neither inquisitive nor informative. The presence of such sentences was also required by the erotetic languages postulate.

Fact 13 (Inquisitive questions)

$?\varphi$ is an inquisitive question iff $\varphi \not\equiv \bot$ and $\neg\varphi \not\equiv \bot$.

The moment φ is informative and not contradictory, $?\varphi$ will be inquisitive.

I think we can safely take it that we have shown that:

Fact 14 (InqB meets the erotetic postulate)

Let $\mathcal{L}$ be an InqB-language with proposition letters $\mathcal{P}$.

$\mathcal{L}$ is an erotetic language iff $\mathcal{P} \neq \emptyset$.

Even in case there is only a single proposition letter $p \in \mathcal{P}$ in $\mathcal{L}$, the erotetic postulate can be met with ease. Then p, $\neg p$ and $p \wedge \neg p$ are three non-equivalent assertions in $\mathcal{L}$; $?p$ is an inquisitive question in $\mathcal{L}$; and $!?p$ is a tautology in $\mathcal{L}$. Only, there are no hybrids in such a language. But that is not required by the erotetic postulate. If $\mathcal{P} = \emptyset$, then every sentence in the language is either a contradiction, an informative assertion, or a tautology, a non-informative assertion. So, such an almost empty language does not meet the erotetic postulate.

Since this is relevant for what follows, we end this subsection with two observations concerning the coverage of types of questions by InqB. First, conditional questions can be expressed, such as $p \to ?q$, to give the simplest example. A state supports $p \to ?q$ if it supports $p \to q$ or if it supports $p \to \neg q$. These two assertions are indeed intuitively the two answers to this conditional question.

Secondly, InqB allows us to express alternative questions. The simplest example is $?(p \vee q)$, which is supported by a state if it supports p or supports q or supports $\neg p \wedge \neg q$. The first two of these three assertions correspond to proto-typical answers, when we read $?(p \vee q)$ as an alternative question. And if $?(p \vee q)$ is to count as a *question*, and hence has to be non-informative, the third assertion should also be present as a possible response.[10]

[9] Note that these convenient closure facts hold because of the way in which questions and assertions where characterized in Definition 1, not requiring questions to be inquisitive nor assertions to be informative.

[10] This is certainly not the whole story about alternative questions. If only because there are whole chapters in that story that involve aspects of meaning that are out of the reach of InqB, such as implicatures, presuppositions, non-at issue content, etc. But the simple analysis InqB offers for alternative questions seems at least a lot better than what partition semantics

4.4 Conservativity

The following fact gives sufficient syntactic conditions for a sentence to be an assertion or a question in InqB.

Fact 15 (Sufficient conditions for assertionhood and questionhood)

1. *p is an informative assertion, for all $p \in \mathcal{P}$*
2. *$\bot$ is an informative assertion*
3. *If φ and ψ are assertions, then $\varphi \wedge \psi$ is an assertion*

   If φ and ψ are questions, then $\varphi \wedge \psi$ is a question
4. *If ψ is an assertion, then $\varphi \to \psi$ is an assertion*

   If ψ is a question, then $\varphi \to \psi$ is a question
5. *If either φ or ψ is a question, then $\varphi \vee \psi$ is a question*

It follows immediately from this fact that disjunction is the only source of inquisitiveness in the language. All sentences that can be constructed without using disjunction are assertions.

Fact 16 (Disjunction is the source of inquisitiveness)

In the disjunction-free fragment of $\mathcal{L}$ all sentences are assertions.

From this it follows that InqB is a conservative extension of classical logic.

Fact 17 (Conservativity) *InqB is a conservative extension of classical logic.*

Let $\mathcal{L}'$ be the disjunction free fragment of $\mathcal{L}$. $\mathcal{L}'$ is a functionally complete language of classical propositional logic. Furthermore, all sentences $\varphi \in \mathcal{L}'$ are assertions, and hence classical sentences (Fact 10). Hence, by the criteria specified in Definition 5, InqB is a conservative extension of classical logic. It is a proper extension in the sense that there are sentences in $\mathcal{L}$ such that $\models_{cl} \varphi$ and $\not\models \varphi$, as soon as $\mathcal{P} \neq \emptyset$. If $\mathcal{P} = \{p\}$, then $p \vee \neg p$ is such a sentence.

4.5 The Inquisitive Hierarchy

We noted in Section 3.2, where we decided to choose arbitrary sets of worlds as evaluation points, that the smaller such sets could be, the closer we might remain to the classical setting. As has been shown in Ciardelli (2009); Ciardelli and Roelofsen (2011), in the context of InqB, it is not possible to set a general limit to the size of states and correctly capture the notion of inquisitiveness. In stating this fact we will use the following auxiliary notion:

can achieve, which cannot come up with anything better than $?p \wedge ?q$ as a representation of an alternative question. Cf. Groenendijk and Stokhof (1982). More fine-grained analyses of alternative questions in inquisitive semantics and pragmatics can be found in Groenendijk and Roelofsen (2009); Roelofsen and van Gool (2010); Pruitt and Roelofsen (2011).

Definition 11 (n-inquisitiveness)

Let S^n denote the set of suitable states s for $\mathcal{L}$ such that $|s| \leq n$, and let $\varphi \in \mathcal{L}$.

φ *is* n-inquisitive *iff for some* $X \subseteq S^n$: *for all* $s \in X$: $s \models \varphi$ *and* $\bigcup X \not\models \varphi$.

First we note that if for all $s \in X$: $s \models \varphi$, whereas $\bigcup X \not\models \varphi$, this can only be caused by the fact that φ is inquisitive in $\bigcup X$. If φ is not informative in states s and in t, then φ cannot be informative in $s \cup t$ either. That φ is not informative in s implies that $s \subseteq \mathsf{info}(\varphi)$, if that holds for s and t it holds for $s \cup t$ as well.

Roughly speaking, a sentence φ is n-inquisitive if its inquisitiveness only becomes apparent when we evaluate φ relative to states with at least cardinality n.

The following fact expresses that in order to detect inquisitiveness, sets consisting of a single world never suffice.

Fact 18 *For any sentence $\varphi \in \mathcal{L}$: φ is not 1-inquisitive.*

S^1 states consist of at most a single world. Only the absurd state and states of maximal consistent information count as such. Clearly, in such states, no issues remain.

Finally, the following theorem expresses that if we were to decide to restrict ourselves to states up to a certain size n, then if the language is sufficiently rich, there will always be sentences that are inquisitive, but of which their inquisitiveness escapes us because it only shows up in bigger states than the ones we decided to limit ourselves to.

Theorem 1 (Inquisitive Hierarchy) *Let $\mathcal{L}$ be an InqB-language with a countably infinite set of proposition letters $\mathcal{P}$.*

For any number $n > 1$ there is a sentence $\varphi \in \mathcal{L}$ such that φ is n-inquisitive and φ is not k-inquisitive for all $k < n$.

The inquisitive hierarchy theorem played a decisive role in the development of inquisitive semantics. For quite a while inquisitive semantics was stated in a way that is, or amounts to, a pair-semantics. See, e.g., Groenendijk (2009); Mascarenhas (2009). The theorem shows that such a framework is inherently too limited, and has to be replaced by the general inquisitive framework presented above.

The short-comings of the pair-semantics also show up in another way. It predicts that the sentence $(p \vee q \vee r)$ is not only, as is to be expected, supported in states s where in every $v \in s$: $p \in v$, or in every $v \in s$: $q \in v$, or in every $v \in s$: $r \in v$, but unexpectedly also, e.g., in a state consisting of the four worlds: $\{\{p, q, r\}, \{p, q\}, \{p, r\}, \{q, r\}\}$. Note that in any two of these four worlds either p or q or r holds in both of them. It is evaluating just relative to pairs of worlds that causes this erroneous result.[11]

[11] At Christopher Potts web site computational tools can be found by which you can calculate the interpretation of formulas, both in a pair-semantics, and in general inquisitive semantics. `http://christopherpotts.net/ling/implementations/is/`. By using these tools you can convince yourself of the different interpretations the two semantics assign to disjunctions with three disjuncts.

5 Classical inquisitive semantics

5.1 Two classical erotetic languages

The general inquisitive system InqB involves a language which does not meet the classical erotetic postulate: it does not distinguish sentential categories of indicatives and interrogatives in the syntax, such that in the semantics all the indicatives have the property of being an assertion, and all interrogatives the property of being a question. And, contrary to what is allowed in a classical erotetic language, we met hybrid sentences in InqB.

In this section we will subsequently investigate two classical erotetic systems. First we obtain a system InqC by starting from InqB and restricting the syntax in a way that it meets the requirements for a classical erotetic language. We will see that this leads to an essentially poorer language that escapes the inquisitive hierarchy and can be captured by a pair-semantics.

Essentially, InqC covers partition semantics extended with conditional questions, which do not correspond to partitions. But it does not cover alternative questions, generally. Adding them to InqC we arrive at the classical erotetic system InqA, of which we will show in Section 6 that it basically has the same expressive power as the non-classical system InqB.

5.2 Indicatives and interrogatives

We will now consider the changes we have to make in the syntax and the semantics of InqB, in order to obtain a classical erotetic system InqC.

Since syntactic assumptions played no role in the way in which we arrived at the conceptual semantic framework that we used for InqB, we take it that we can use the same semantic framework for InqC. Of course, on the syntactic side we will meet restrictions which may have semantic consequences as well, but we may expect that these will show at most that not the 'full force' of the semantics is put to use.

This being so, in deciding on the syntax of InqC we can let ourselves be guided by Fact 15 concerning sufficient conditions for assertionhood and questionhood in InqB. Given that in a classical erotetic language indicatives have to be assertions, and interrogatives have to be questions, and every sentence has to be one of the two, the sufficient conditions for assertionhood and questionhood in InqB provide constraints that the syntax of InqC necessarily has to meet. You can read them from Fact 15 by substituting *indicative* for *assertion* and *interrogative* for *question*.

This helps us to decide that atomic sentences and $\bot$ are to be categorized as indicatives, and it helps us to determine the syntactic clause(s) for conjunction, if we realize that implicitly the fact tells us that if the conjuncts are not of the same category, then there is no certainty concerning the category of the conjunction, it could be a hybrid, which are not allowed to occur in InqC.

As for implication, the fact tells us that an implication inherits the category of its consequent, where the category of the antecedent, indicative or inter-

rogative, does not matter. Note that when we stick to $\neg\varphi$ being defined as $\varphi \to \bot$, negation could apply freely to indicatives and interrogatives, invariably resulting in an indicative, which is bound to be a contradiction if we negate an interrogative.

Here, from a logical-semantical point of view, we make the reasonably arbitrary decision that the antecedent of an implication has to be an indicative. There are some more subtle reasons for it as well, but let us just provide the (debatable) linguistic motivation that in natural languages with classical erotetic features, like Dutch and English, except for some rare constructions, interrogatives do not appear as antecedents of conditional sentences, nor can they be directly negated.

Finally, disjunction, where the sufficient conditions for assertionhood and questionhood in InqB tell us that there is no guarantee that a disjunction delivers an assertion, not even when both disjuncts are assertions. This much we knew already, $p \vee q$ is a hybrid sentence in InqB, but it shouldn't be in InqC.

We could replace the semantic clause for disjunction by a classical one in the semantics for InqC, but then we can just as well define $\varphi \vee \psi$ in terms of $\neg(\neg\varphi \wedge \neg\psi)$, and delete the clause for disjunction from the semantics of InqB. Given that, by decisions we made already, only indicatives can appear under negation, only indicatives can then appear as disjuncts of a disjunction. And that a disjunction as a whole is an assertion, and hence must be an indicative, is assured by the outermost negation in $\neg(\neg\varphi \wedge \neg\psi)$.

This robs us from disjunctions of which at least one disjunct is an interrogative, which, according to the fact concerning sufficient conditions for assertionhood and questionhood, would be harmless as long as we categorize such constructions as interrogatives, because they are bound to be questions.

More or less arbitrarily, from a logical-semantical point of view, we decide not to allow for such interrogative constructions. We would need a separate disjunctive operator next to indicative disjunction, and, as in the case of implications with interrogative antecedents, we point at the (disputable) fact that in natural languages with classical erotetic features, like Dutch and English, disjunctions of interrogatives are rare animals, and where they may seem to occur, as in (1), they do not have the meaning InqB assigns to $?p \vee ?q$, which is rather what corresponds to (2).

(1) Will Alf go to the party or will Bea go?

(2) Answer one of the following two questions:
   a. Will Alf go to the party?
   b. Will Bea go to the party?

Conclusion, we omit disjunction as a basic operation in InqC, and introduce it by definition in terms of negation and conjunction.

However, since disjunction is the only source of inquisitiveness in InqB, we have to introduce an interrogative operator ? as a basic operator in InqC, with, of course, the same semantic effect that non-informative closure $?\varphi$ has in InqB, i.e., that $s \models ?\varphi$ iff $s \models \varphi$ or $s \models \neg\varphi$. But this can easily be done.

5.3 Classical erotetic syntax and support semantics

An InqC-language is a propositional language $\mathcal{L}$ with two syntactic sentential categories: $\mathcal{L}_!$, the indicatives in $\mathcal{L}$, and $\mathcal{L}_?$, the interrogatives in $\mathcal{L}$. The basic connectives in $\mathcal{L}$ are $\bot, \wedge, \rightarrow, ?$. The connectives $\neg$ and $\vee$ are introduced by definition. Given that $\mathcal{P}$ is the set of proposition letters in $\mathcal{L}$, the deliberations in the previous subsection motivate the following syntax for $\mathcal{L}$.

Definition 12 (Classical erotetic syntax)

1. *$\varphi \in \mathcal{L}_!$, for all $\varphi \in \mathcal{P}$*
2. *$\bot \in \mathcal{L}_!$*
3. *If $\varphi \in \mathcal{L}_!$, then $?\varphi \in \mathcal{L}_?$*
4. *If $\varphi \in \mathcal{L}_!$ and $\psi \in \mathcal{L}_{c \in \{!,?\}}$, then $(\varphi \rightarrow \psi) \in \mathcal{L}_c$*
5. *If $\varphi, \psi \in \mathcal{L}_{c \in \{!,?\}}$, then $(\varphi \wedge \psi) \in \mathcal{L}_c$*
6. *If Φ is a finite subset of $\mathcal{L}_! \cup \mathcal{L}_?$, then $\Phi \in \mathcal{L}$*

We added an extra and unusual clause to the syntax which collects sentences of both categories in a set. The corresponding clause in the semantics will require that a state supports such a set of sentences if it supports all sentences in the set. So, the interpretation follows the pattern of conjunction. Whereas single sentences in an InqC-language cannot be hybrid, sets of them can have that semantic property. We could try and tell an interesting story about this additional feature of InqC, but we mainly introduce it to facilitate comparison with InqB.

Next we turn to the semantics. Let $\mathcal{P}$ be the set of propositional variables in $\mathcal{L}$, and S the set of suitable states for $\mathcal{L}$. We recursively define when a sentence $\varphi \in \mathcal{L}$ is supported in a state $s \in S$.

Definition 13 (Classical inquisitive semantics)

1. *$s \models p$ iff $\forall v \in s : p \in v$ for all $p \in \mathcal{P}$*
2. *$s \models \bot$ iff $s = \emptyset$*
3. *$s \models ?\varphi$ iff $s \models \varphi$, or for all $t \subseteq s$: if $t \models \varphi$, then $t = \emptyset$*
4. *$s \models \varphi \wedge \psi$ iff $s \models \varphi$ and $s \models \psi$*
5. *$s \models \varphi \rightarrow \psi$ iff $\forall t \subseteq s$: if $t \models \varphi$ then $t \models \psi$*
6. *$s \models \Phi$ iff for all $\varphi \in \Phi$: $s \models \varphi$*

Despite the fact that the syntax of InqC is more involved than in InqB, the only difference in the semantics, apart from the additional clause that deals with sets of sentences, is that the clause for disjunction has been replaced by a clause for basic interrogatives. The remaining clauses read the same as in InqB. The semantics makes no reference to the syntactic categories. We add the following two classical abbreviations:

Definition 14 (Abbreviations)

1. $\neg\varphi := (\varphi \to \bot)$
2. $(\varphi \vee \psi) := \neg(\neg\varphi \wedge \neg\psi)$

The reader can check, that InqC is indeed a classical erotetic language according to the specifications we postulated in the beginning of the paper.

Fact 19 (InqC meets the classical erotetic postulate)
Let $\mathcal{L}$ be an InqC-language with proposition letters $\mathcal{P}$.

$\mathcal{L}$ is a classical erotetic language iff $\mathcal{P} \neq \emptyset$.

Also, it can easily be seen that InqC is a conservative extension of classical logic. The indicatives in $\mathcal{L}_!$ form a fragment of an InqC-language $\mathcal{L}$ that is a functionally complete language of classical logic. Since all indicatives are assertions, for the same reasons as we gave for InqB, InqC also meets the semantical and logical criteria given in Definition 5 for InqC to be a conservative extension of classical logic.

Fact 20 (Conservativity) *InqC is a conservative extension of classical logic.*

We note one more fact and make one general observation before we directly move on to the most crucial feature of InqC.

Fact 21 (Basic questions) *$s \models ?\varphi$ iff $s \models \varphi$ or $s \models \neg\varphi$.*

Obviously, any basic question $?\varphi$ corresponds to a bi-partition of the logical space ω. Any conjunction of such basic questions will also correspond to a partition, potentially with more blocks. There is one more construction in which basic questions can enter, they can appear as consequent of an implication. Typically in that case, we leave the domain of partitions. For example, the conditional question $p \to ?q$ does not deliver a partition, but a cover of ω with two overlapping elements.[12]

Actually, if we would further syntactically restrict implication as to also require that its consequent is an indicative, the result is a good-old partition semantics for a propositional language. That is the most elementary case of a classical erotetic propositional language.

[12]But see Isaacs and Rawlins (2008) for a rather involved partition semantic approach to conditional questions, arguing against Velissaratou (2000) for not being a partition-approach, where the latter is much in line with the syntactic and semantic analysis of conditional questions in InqC.

5.4 Escape from the inquisitive hierarchy

Although the semantics used for InqC is largely like the general one, the syntax at which we arrived, taking care to meet the classical erotetic postulate, for better or worse heavily restricts the expressive power of InqC-languages. This is best illustrated by the following fact, which implies that the inquisitive hierarchy theorem, which is highly characteristic for general inquisitive semantics, does not apply to InqC.

Fact 22 (Pair distributivity) *Let $\mathcal{L}$ be an InqC-language.*

For all $\varphi \in \mathcal{L}$: $\forall s \in S$: $s \models \varphi$ iff for all $v, w \in s$: $\{v, w\} \models \varphi$.

That pair-distributivity holds for the partition semantic fragment of InqC is more or less obvious from the fact that partitions correspond to equivalence relations. Adding conditional questions, which is basically what InqC does, steers us away from partitions, but not so far away that it cannot be captured by a pair-semantics anymore, it is still relational.[13]

The pair-distributivity fact can be proved by induction on the complexity of φ. It is trivial whenever only indicatives are involved, and straightforward in case of basic questions and conjunction. The only case of interest are implications with an interrogative consequent. Given that the antecedent of an implication is an indicative in InqC, and that the support relation is persistent, as it is generally also in InqB (Fact 2), the following fact holds in InqC:

Fact 23 (Implication in InqC) $s \models \varphi \to \psi$ *iff* $s \cap \mathit{info}(\varphi) \models \psi$.

Under the assumption that ψ is pair-distributive, it is easy to see from this fact that $\varphi \to \psi$ is then pair-distributive as well.

5.5 Adding classical alternative questions

In InqC no single sentence is a hybrid, but by using a mixed set of indicatives and interrogatives we can express hybrid meanings. It is possible, e.g., to express the meaning of the hybrid disjunction $p \vee q$ in InqB with the pair of InqC-sentences in (3).

(3) $\{p \vee q, \neg(p \wedge q) \to ?p\}$.

Given the fact that the semantics for InqC is pair-distributive, it is not possible to adequately represent hybrid disjunctions in InqB with more than two disjuncts, such as $(p \vee q \vee r)$, in a similar fashion.

A second, and perhaps more important limitation of the expressiveness of InqC concerns alternative questions. As we saw above, in InqB, the sentence $?(p \vee q)$ can be taken to correspond to an alternative question. Superficially the

[13] This means, by the way, that the semantics for conditional questions as given in Velissaratou (2000), which is stated as a pair-semantics, but for the rest is much like InqC, is in fact equivalent with InqC.

same sentence in InqC corresponds to a *yes/no*-question, i.e., to $?!(p \vee q)$ in InqB. However, by means of the conjunction of questions in (4) we obtain a sentence in InqC that has the same meaning as the alternative question $?(p \vee q)$ in InqB.

(4) $\quad ?(p \vee q) \wedge (\neg(p \wedge q) \rightarrow ?p)$

But, again, given that InqC is pair-distributive, it is not possible to adequately express alternative questions in InqC with more than two alternatives, such as $?(p \vee q \vee r)$.

In order to remedy this, we extend the syntax and the semantics of InqC by replacing the clause for interrogative formation that operates on a single indicative sentence, by one that operates on sets of indicatives.[14] We refer to the resulting system as InqA.

Definition 15 (Adding classical alternative questions)

- *If Φ is a finite subset of $\mathcal{L}_!$, then $?\Phi \in \mathcal{L}_?$.*
- *$s \models ?\Phi$ iff $\exists \varphi \in \Phi \colon s \models \varphi$, or $\forall \varphi \in \Phi \colon s \models \neg\varphi$.*

The semantic clause is chosen in a way that the interpretation of classical alternative questions meets the one in general inquisitive semantics. In principle, there seem to be different options here as well that may be worth considering.

Note that nothing changes with respect to the indicatives in the language, which means that InqA is as much a conservative extension of classical logic as InqC. However, unlike for InqC, once we add classical alternative questions, pair-distributivity does not hold anymore, the inquisitive hierarchy theorem applies just as much to InqA as to InqB. The comparison between the two in the next section will make this clear.

6 General and classical inquisitive semantics

6.1 Global comparison

We can suitably compare a pair of languages $\mathcal{L}_{\mathsf{InqA}}$ and $\mathcal{L}_{\mathsf{InqB}}$ if they are based on the same set of proposition letters $\mathcal{P}$, which we denote by $\mathcal{L}_{\mathsf{InqA}} \simeq_{\mathcal{P}} \mathcal{L}_{\mathsf{InqB}}$. If this is the case, then $\mathcal{L}_{\mathsf{InqA}}$ and $\mathcal{L}_{\mathsf{InqB}}$ have the same set of suitable states S, and we can define a notion of equivalence across the two languages.

Definition 16 (Equivalence across $\mathcal{L}_{\mathsf{InqA}}$ and $\mathcal{L}_{\mathsf{InqB}}$)

Let $\mathcal{L}_{InqA} \simeq_{\mathcal{P}} \mathcal{L}_{InqB}$, and let $\varphi \in \mathcal{L}_{InqA}$ and $\psi \in \mathcal{L}_{InqB}$.

φ is equivalent with ψ, $\varphi \Leftrightarrow \psi$ *iff for all* $s \in S \colon s \models \varphi$ *iff* $s \models \psi$.

[14] In erotetic logic it is quite common to introduce interrogatives in the logical language in terms of sets of indicative sentences. See, e.g., Wiśniewski (1996, 2001).

We will compare $\mathcal{L}_{\mathsf{InqB}}$ and $\mathcal{L}_{\mathsf{InqA}}$ by way of meaning preserving translation procedures. We can translate any single sentence and any set of sentences in $\mathcal{L}_{\mathsf{InqA}}$ into a single sentence in $\mathcal{L}_{\mathsf{InqB}}$ by a recursive procedure. We take it that in the definition we consider the 'official' language $\mathcal{L}_{\mathsf{InqA}}$, i.e., without the non-basic connectives that were introduced by definition.

Definition 17 (Translation from $\mathcal{L}_{\mathsf{InqA}}$ to $\mathcal{L}_{\mathsf{InqB}}$) *Let $\mathcal{L}_{InqA} \simeq_{\mathcal{P}} \mathcal{L}_{InqB}$.*

1. $(\varphi)^\dagger = \varphi$ *for all* $\varphi \in \mathcal{P}$
2. $(\bot)^\dagger = \bot$
3. $(?\{\varphi_1, \ldots, \varphi_n\})^\dagger = ?(\varphi_1 \vee \ldots \vee \varphi_n)$
4. $(\varphi \to \psi)^\dagger = (\varphi \to (\psi)^\dagger)$
5. $(\varphi \wedge \psi)^\dagger = ((\varphi)^\dagger \wedge (\psi)^\dagger)$
6. $(\{\varphi_1, \ldots, \varphi_n\})^\dagger = ((\varphi)^\dagger \wedge \ldots \wedge (\varphi_n)^\dagger)$

The translation is meaning preserving.

Fact 24 ($(\varphi)^\dagger$ is meaning preserving) *For all $\varphi \in \mathcal{L}_{InqA}$: $\varphi \Leftrightarrow (\varphi)^\dagger$.*

As is to be expected, a translation in the other direction is less straightforward. For a start, given that in $\mathcal{L}_{\mathsf{InqA}}$ no hybrid single sentences occur, it will not be possible to translate every sentence in $\mathcal{L}_{\mathsf{InqB}}$ into a *single sentence* in $\mathcal{L}_{\mathsf{InqA}}$ in a meaning preserving way.

It is possible, however, to translate each sentence in $\mathcal{L}_{\mathsf{InqB}}$ into a *pair of sentences* in $\mathcal{L}_{\mathsf{InqA}}$. What plays a crucial role is that in $\mathcal{L}_{\mathsf{InqB}}$ every sentence can be 'divided' into a purely non-inquisitive and a purely non-informative 'part'.

Fact 25 (Division) *For all $\varphi \in \mathcal{L}_{InqB}$: $\varphi \equiv !\varphi \wedge ?\varphi$*

Since $!\varphi$ is an assertion in $\mathcal{L}_{\mathsf{InqB}}$, it is in principle possible to translate it into an indicative sentence in $\mathcal{L}_{\mathsf{InqA}}$, and since $?\varphi$ is a question in $\mathcal{L}_{\mathsf{InqB}}$, it is in principle possible to translate it into an interrogative sentence in $\mathcal{L}_{\mathsf{InqA}}$.

Given that the interpretation of a set of sentences in $\mathcal{L}_{\mathsf{InqA}}$ amounts to (hybrid) conjunction, the fact of division makes it in principle possible to translate any sentence φ in $\mathcal{L}_{\mathsf{InqB}}$ into a pair of sentences in $\mathcal{L}_{\mathsf{InqA}}$: an indicative which covers $!\varphi$, and an interrogative which covers $?\varphi$. Of course, if φ is a question or an assertion, and not hybrid, a single sentence can suffice.

First we consider the non-inquisitive part $!\varphi$ of the division of φ. Let $\mathcal{L}^{\vee}_{\mathsf{InqA}}$, be the official language $\mathcal{L}_{\mathsf{InqA}}$ plus disjunction, as it was introduced in the language by definition. Then any sentence φ in the official language $\mathcal{L}_{\mathsf{InqB}}$, is also an indicative sentence in $\mathcal{L}^{\vee}_{\mathsf{InqA}}$. Of course, in general, the meaning of φ in these two languages is not the same. But that *is* the case if φ is an assertion in $\mathcal{L}_{\mathsf{InqB}}$.

Fact 26 (One half of division) *Let $\varphi \in \mathcal{L}_{InqB}$. Then $\varphi \in \mathcal{L}^{\vee}_{InqA}$ and $\varphi \Leftrightarrow !\varphi$.*

This means that we can simply use φ as such as a suitable translation in $\mathcal{L}^{\vee}_{\mathsf{InqA}}$ for the non-inquisitive closure $!\varphi$ of φ in $\mathcal{L}_{\mathsf{InqB}}$.

Next, we consider disjunction, which is the source of inquisitiveness in InqB. The following fact says that if we consider a disjunction φ in the official language $\mathcal{L}_{\mathsf{InqB}}$, then if all disjuncts of φ are assertions, we can construct an interrogative sentence $?\Phi$ in $\mathcal{L}^{\vee}_{\mathsf{InqA}}$ which is equivalent with the non-informative closure $?\varphi$ of φ.[15]

Fact 27 (The other half of division)

Let $\varphi \in \mathcal{L}_{InqB}$, where $\varphi = (\psi_1 \vee \ldots \vee \psi_n)$ and $\psi_i \equiv !\psi_i$ for $i \leq n$. Then

$$?\Phi = ?\{\psi_1, \ldots, \psi_n\} \in \mathcal{L}^{\vee}_{InqA} \text{ and } ?\Phi \Leftrightarrow ?\varphi.$$

Note that if φ is a disjunction of assertions, then given that $?\varphi = \varphi \vee \neg\varphi$, and that $\neg\varphi$ is an assertion, it is also the case that $?\varphi$ is a disjunction of assertions.

This means that if *every* sentence in $\mathcal{L}_{\mathsf{InqB}}$ can be transformed into a disjunction of assertions, then the non-informative closure $?\varphi$ of any sentence $\varphi \in \mathcal{L}_{\mathsf{InqB}}$ can be translated in a meaning preserving way in $\mathcal{L}^{\vee}_{\mathsf{InqA}}$. This is indeed the case.

Fact 28 (Disjunctive normal form)

There is a recursive procedure that transforms any sentence φ in $\mathcal{L}_{InqB}$ into a sentence $DNF(\varphi)$ in $\mathcal{L}_{InqB}$, such that:

$$DNF(\varphi) \text{ is a disjunction of assertions and } DNF(\varphi) \equiv \varphi.$$

For reasons of space I will not give the recursive definition of the disjunctive normal form for InqB here, but refer the reader to Ciardelli *et al.* (2009b).

Given the existence of the disjunctive normal form for $\mathcal{L}_{\mathsf{InqB}}$, and the division facts given above, we can define a general translation procedure from the sentences in $\mathcal{L}_{\mathsf{InqB}}$ into pairs of sentences in $\mathcal{L}^{\vee}_{\mathsf{InqA}}$.

Definition 18 (Translation from $\mathcal{L}_{\mathsf{InqB}}$ to $\mathcal{L}^{\vee}_{\mathsf{InqA}}$) *Let $\mathcal{L}_{InqA} \simeq_{\mathcal{P}} \mathcal{L}_{InqB}$.*

$$(\varphi)^{\sharp} = \{\varphi, ?\{\psi \mid \psi \text{ is a disjunct of } DNF(\varphi)\}\}.$$

The translation is meaning preserving.

Fact 29 ($(\varphi)^{\sharp}$ is meaning preserving) *For all $\varphi \in \mathcal{L}_{InqB}$: $(\varphi)^{\sharp} \Leftrightarrow \varphi$.*

Despite the considerable differences in their syntax, general and classical inquisitive semantics are globally speaking equivalent erotetic logical systems.

[15] In the fact below I present disjunctions in $\mathcal{L}_{\mathsf{InqB}}$ as if they were n-ary, whereas in fact disjunction is a binary operation. It may not be completely trivial to make this formally correct, but I take it that it can be done.

6.2 Comparison in detail

Globally $\mathcal{L}_{\mathsf{InqB}}$ and $\mathcal{L}_{\mathsf{InqA}}$ are equivalent in expressiveness, but not in detail. Just one example to show the nature of the differences between the general and the classical language. Consider the question in (5) and the two different answers in (6).

(5) If Alf goes to Paris or London, will he fly KLM?

(6) a. If Alf goes to Paris or London, he will fly KLM.
b. If Alf goes to Paris he will fly KLM, but not if he goes to London.

The questions in (7) and (8) below are equivalent in $\mathcal{L}_{\mathsf{InqB}}$. Under the 'literal' translation of (5) into (7), both answers in (6) are predicted to be direct answers to (5) by the semantics.

(7) $(p \vee q) \rightarrow ?r$

(8) $(p \rightarrow ?r) \wedge (q \rightarrow ?r)$

In $\mathcal{L}_{\mathsf{InqA}}$ (7) and (8) are not equivalent, (8) properly entails (7). Under the 'literal' translation of (5) into (7) in $\mathcal{L}_{\mathsf{InqA}}$, only the first answer in (6) is predicted to be a direct answer to (5). To account for the second answer in (6) as a direct answer to it, (5) is to be ambiguous and (8), or the set consisting of the two conjoined questions, has to count as an alternative translation in $\mathcal{L}_{\mathsf{InqA}}$. The ambiguity can be accounted for in $\mathcal{L}_{\mathsf{InqB}}$ by a second translation of (5) into (9).

(9) $!(p \vee q) \rightarrow ?r$

6.3 Concluding remark

Inquisitive semantics is first and foremost a general conceptual system in which the notions of inquisitiveness and informativeness are analyzed in an integrated way. It is not as such a specific semantic theory of questions, nor of anything else. It is a well-studied and well-behaved logical system in which quite different such theories can be formulated and formally compared, as our case studies of InqC and InqA in relation to InqB were intended to show.

7 Finally, getting the picture

We started this paper with a brief sketch of the non-Fregean position of G&S, according to which indicatives and interrogatives form distinct syntactic categories with the distinct corresponding semantic domains of propositions and questions, which are parts and partitions of logical space, respectively. This, literally, gives nice pictures of the meanings of such sentences.

With as basic motivation an extension of the semantics to cover types of questions that do not straightforwardly correspond to partitions, we took a new start. We reasoned from the existence of two *semantic properties* of inquisitiveness and informativeness, rather than assuming separate *semantic domains*,

and arrived at an inquisitive support semantics. But what about the *meanings* of sentences? How to picture the meanings inquisitive semantics gives rise to? Sofar, in this paper at least, we have not said much about that.

First, we note that in partition semantics, the picture of a question is not immediately given. The intension of an interrogative $?\varphi$ (here G&S are Fregean) is of semantic type $\langle s, \langle s, t\rangle\rangle$, a relation on the set of worlds. It is an equivalence relation, because it corresponds to: the true and complete answer to $?\varphi$ is the same in v and w. Hence, a question corresponds to a partition of logical space. You can construct the blocks in the partition by taking maximal sets of worlds such that all of them are related to each other, i.e., such that in all of them the true and complete answer to $?\varphi$ is the same.

In general inquisitive semantics we get pictures of meaning in a structurally similar way. Given the support semantics the meaning of a sentence φ is determined to be $[\varphi] = \{s \in S \mid s \models \varphi\}$, the set of states that support φ. We call the maximal elements of $[\varphi]$ under the extension relation, the *possibilities for* φ.[16] Since the support semantics is persistent the possibilities for a sentence φ suffice to fully characterize its meaning, and we are entitled to refer to the set of possibilities for φ as the *proposition expressed by* φ. We could also call it *the picture of the meaning of* φ, where the possibilities for φ are what we depict.

Consider the fragment of InqC where the consequent of an implication must be an indicative. This blocks conditional questions. The result is partition semantics. Both the picture of the meaning of a question and the picture of the meaning of an assertion is the same as in partition semantics. In case of an assertion φ, you get $\mathsf{info}(\varphi)$ as the only possibility for φ to be depicted.

So, remarkably enough, we get at the same pictures of meaning, while in the case of G&S partition semantics the meanings of the two categories of sentences are of different semantic types, whereas in the case of inquisitive semantics they are of the same type, the single type of a proposition as a set of possibiities.

When we take full InqC, the picture of the meaning of a question is a cover of logical space, and not always a partition. It has the specific property that as soon as we remove one of the depicted possibilities from the picture, we do not cover the whole of logical space anymore. Although, unlike in the case of a partition, possibilities may overlap, part of every possibility is not contained in any of the other possibilities.

This specific feature of the possibilities for a sentence in InqC is lost as a general property in InqA and InqB. This is related to the fact that InqC escapes from the inquisitive hierarchy, and can be captured by a pair-semantics.

So, basically, the pictures of meaning in inquisitive semantics are generalizations of the pictures in partition semantics. But we arrive at them with a single unified notion of what the meaning of a sentence is, irrespective of its category.

There still is a fundamental difference between classical InqA and InqC on the one hand and, general InqC on the other. In the classical case every sentence either has a single possibility, or its possibilities form a cover of logical space. In

[16] In the propositional case such maximal supporting states are bound to exist under reasonable assumptions. In the first order case this is problematic. Cf. Ciardelli (2010).

case of a tautology the two options coincide. Here, you can reasonably say that the semantic domains for indicatives and interrogatives are different subtypes within the same type.

In the general semantic system InqB, the picture of meaning is that the possibilities for a sentence φ form a cover of its informative content $\mathsf{info}(\varphi)$, which only if φ is a question is the whole of logical space. And if φ is an assertion, it is just the single possibility $\mathsf{info}(\varphi)$ that covers itself, so to say.

So, now it is no longer questions on the one side and assertions on the other, and nothing in between. Questions and assertions are now the borderline cases of a general notion of meaning that covers informative and inquisitive semantic content. Questions are not informative, assertions are not inquisitive, but in between there are sentences which are a bit of both.

There is no logical reason for these two semantic properties to exclude each other, and for keeping them apart as belonging to distinct syntactic categories. From an empirical perspective, but that was not our point of view in this exercise in conceptual semantics, there is every reason to believe that this feature of general inquisitive semantics has its applications in natural language semantics and pragmatics, both in and beyond an analysis of questions and answers.

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