How birds weather the weather: avian migration in the mid-latitudes

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Appendix A: Explicit descriptions of flow-assistance equations

Each of the nine figures below (A.1 - A.9) contains a complete and explicit overview of a particular flow-assistance equation. Along the left column of each figure, from top to bottom, is the flow-assistance equation itself, a graphical representation of the equation’s components, a text summary describing the equation’s components and assumptions, and a circular plot indicating the resulting flow-assistance values for different combinations of flow speed (0 - 20 ms$^{-1}$) and direction (0 - 360°). In these circular plots, flow speed is indicated by the distance from the center of the circle with the edges corresponding to flow speeds of 20 ms$^{-1}$. In both the circular plots and the graphical representation of the equation’s components, a gray arrow indicates the preferred direction of movement (pdm). Note that in the graphic indicating the components of the model, the pdm is equivalent to 90°; whereas in the circular plot representing the resulting flow-assistance values, the pdm is equal to 225°. For the circular plots, the preferred air- or ground speed, whenever required by an equation, was set to 12 ms$^{-1}$, and, for Fig. A.9, the proportion of compensation (f) was set to 0.5. Along the right column of these figures are the partial derivatives (i.e. sensitivities) of the flow-assistance equation to each of its respective assumptions. For each of an equation’s assumptions, we provide the partial derivative equation and a circular plot indicating the sensitivity of the equation to that assumption for particular combinations of flow speed (0 - 20 ms$^{-1}$) and direction (0 - 360°). Some equations do not have assumptions that allow calculation of partial derivatives, so their sensitivity (i.e. the right column) contains only ‘NA’. As with the other circular plots, flow speed is indicated by the distance from the center of the circle with the
edges corresponding to flow speeds of 20 ms$^{-1}$. The contoured surface of these plots indicates the change in flow-assistance (ms$^{-1}$) that would result from a one-unit change in the specified assumption for the particular combination of flow speed and direction. In these plots, the pdm was set to 225°; any speed-related assumption was set to 12 ms$^{-1}$; and, for Fig. A.9, the proportion of compensation ($f$) was set to 0.5. We show the sensitivity of each equation to specific amounts of uncertainty in Figure 3.3 of the main text.

<table>
<thead>
<tr>
<th>Flow-assistance</th>
<th>Sensitivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>$FA = y$</td>
<td>$NA$</td>
</tr>
</tbody>
</table>

Where:

$y =$ flow speed

Explicit assumptions:

- Movement with the flow

![Fig. A.1 Equation FlowSpeed](image)
### Fig. A.2 Equation NegFlowSpeed

<table>
<thead>
<tr>
<th>Flow-assistance</th>
<th>Sensitivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>$FA = -1 \times y$</td>
<td>NA</td>
</tr>
</tbody>
</table>

Where:
- $y$ = flow speed

Explicit assumptions:
- Movement against the flow

![Diagram](image)
### Fig. A.3 Equation Binary

<table>
<thead>
<tr>
<th>Flow-assistance</th>
<th>Sensitivity</th>
</tr>
</thead>
</table>
| $FA = \begin{cases} 
0, & y \cos \theta \leq 0 \\
1, & y \cos \theta > 0 
\end{cases}$ | $NA$ |

Where:
- $y =$ flow speed
- $\theta =$ flow direction – pdm

Explicit assumptions:
- A preferred direction of movement (pdm)

Implicit assumptions:
- Full drift

---

![Diagram](image-url)
Fig. A.4 Equation Tailwind

Flow-assistance

\[ FA = y \cos \theta \]

<table>
<thead>
<tr>
<th>y cos \theta</th>
</tr>
</thead>
<tbody>
<tr>
<td>\theta</td>
</tr>
<tr>
<td>y</td>
</tr>
</tbody>
</table>

Where:
- \( y \) = flow speed
- \( \theta \) = flow direction – pdm

Explicit assumptions:
- A preferred direction of movement (pdm)

Implicit assumptions:
- Full drift

Sensitivity

\[ \frac{\partial FA}{\partial \theta} = -y \sin \theta \]

![Diagram](image-url)
### Fig. A.5 Equation Airspeed

<table>
<thead>
<tr>
<th>Flow-assistance</th>
<th>Sensitivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>( FA = y \cos \theta + \sqrt{z^2 - (y \sin \theta)^2} - z )</td>
<td>( \frac{\partial FA}{\partial \theta} = -y \sin \theta - \left( y^2 \sin \theta \cos \theta \cdot \left( z^2 - (y \sin \theta)^2 \right)^{\frac{1}{2}} \right) )</td>
</tr>
<tr>
<td>( \frac{\partial FA}{\partial z} = \left( z^2 - (y \sin \theta)^2 \right)^{\frac{1}{2}} - 1 )</td>
<td></td>
</tr>
</tbody>
</table>

Where:
- \( y \) = flow speed
- \( z \) = airspeed
- \( \theta \) = flow direction – pdm

**Explicit assumptions:**
- A preferred direction of movement (pdm)
- A constant airspeed
- Complete compensation by adjusting heading and groundspeed

**Implicit assumptions:**
- If airspeed is insufficient to maintain pdm, equation returns no real solution.
### APPENDIX A

#### Fig. A.6 Equation Groundspeed

<table>
<thead>
<tr>
<th>Flow-assistance</th>
<th>Sensitivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>$FA = x - \sqrt{x^2 + y^2 - 2xy \cos \theta}$</td>
<td>$\frac{\partial FA}{\partial \theta} = -xy \sin \theta \times \left(\frac{x^2 + y^2 - 2xy \cos \theta}{\sqrt{x^2 + y^2 - 2xy \cos \theta}}\right)^{\frac{1}{2}}$</td>
</tr>
</tbody>
</table>

Where:
- $y =$ flow speed
- $x =$ groundspeed
- $\theta =$ flow direction – pdm

Explicit assumptions:
- A preferred direction of movement (pdm)
- A constant groundspeed

Implicit assumptions:
- Complete compensation by adjusting airspeed and heading
- Tailwinds faster than the specified groundspeed are suboptimal

$$\frac{\partial FA}{\partial x} = 1 - (x - y \cos \theta) \sqrt{x^2 + y^2 - 2xy \cos \theta}^{\frac{1}{2}}$$
**Fig. A.7 Equation C. Groundspeed**

<table>
<thead>
<tr>
<th>Flow-assistance</th>
<th>Sensitivity</th>
</tr>
</thead>
</table>
| $FA = \begin{cases} 
  x - \sqrt{x^2 + y^2 - 2xy\cos\theta}, & y\cos\theta \leq x \\
  y\cos\theta - y\sin\theta, & y\cos\theta > x 
\end{cases}$ | $\frac{\partial FA}{\partial \theta} = \begin{cases} 
  -xy\sin\theta + \left(x^2 + y^2 - 2xy\cos\theta\right)^{\frac{1}{2}}, & y\cos\theta \leq x \\
  -y\sin\theta - y\cos\theta, & y\cos\theta > 0 \\
  \text{Undefined}, & \sin\theta = 0 \\
  -y\sin\theta + y\cos\theta, & y\sin\theta < 0 
\end{cases}$ |

$\sqrt{x^2 + y^2 - 2xy\cos\theta}$

(cosine law)

Where:
- $y$ = flow speed
- $x$ = groundspeed
- $\theta$ = flow direction – pdm

Explicit assumptions:
- A preferred direction of movement (pdm)
- A constant groundspeed

Implicit assumptions:
- Complete compensation
  - Unless supportive axial flow is faster than specified groundspeed (in which case, no specific behavior is assumed)
- Supportive axial flow faster than the specified groundspeed is increasingly beneficial.
**Fig. A.8 Equation M. Groundspeed**

<table>
<thead>
<tr>
<th>Flow-assistance</th>
<th>Sensitivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x = z + y \cos \theta$</td>
<td>$\frac{\partial WP}{\partial \theta} = 0.5 y \sin \theta \cdot \left(-2 y \sin \theta - 2 y^2 \sin \theta \cos \theta + 2 y x \sin \theta\right) \cdot \left{\left(x y \cos \theta\right)^2 + y^2 - 2 y x \cos \theta\right}^{-\frac{3}{2}}$</td>
</tr>
<tr>
<td>$FA = x - \sqrt{x^2 + y^2 - 2 y x \cos \theta}$</td>
<td></td>
</tr>
</tbody>
</table>

Where:
- $y =$ flow speed
- $z =$ groundspeed in still conditions
- $x =$ groundspeed
- $\theta =$ flow direction – pdm

**Explicit assumptions:**
- A preferred direction of movement (pdm)
- Groundspeed equal to groundspeed in still conditions plus flow component along the pdm.

**Implicit assumptions:**
- Complete compensation by adjusting heading and airspeed

[Diagram of vector components and flow direction]
The figure illustrates the equation for partial speed, denoted as $FA$, which is given by:

$FA = y \cos \theta + \sqrt{z^2 - (f \times y \sin \theta)^2} - z$

Where:
- $y$ = flow speed
- $z$ = airspeed
- $\theta$ = flow direction – pdm
- $f$ = proportion of compensation

Explicit assumptions:
- A preferred direction of movement (pdm)
- A constant airspeed
- Partial compensation by adjusting heading and groundspeed
- The animal will compensate for a proportion of the flow lateral to the pdm as described by $f$

Implicit assumptions:
- If airspeed is insufficient to compensate the specified amount, equation returns no real solution.
- $f = 1$ is complete compensation
- $f = 0$ is full drift

The sensitivity of $FA$ with respect to $\theta$ and $z$ is given by:

$\frac{\partial FA}{\partial \theta} = -y \sin \theta - f^2 \times y^2 \sin \theta \cos \theta \sqrt{z^2 - (f \times y \sin \theta)^2}$

$\frac{\partial FA}{\partial z} = \frac{1}{2} \frac{z^2 - (f \times y \sin \theta)^2}{\sqrt{z^2 - (f \times y \sin \theta)^2}}$