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Financial fairness and conditional indexation

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ABSTRACT

Collective pension contracts can generate advantages for their participants by implementing forms of risk sharing. To ensure the continuity of a collective scheme, it has to be monitored whether the contracts offered to participants are financially fair in terms of their market value. When risk sharing is implemented by means of optionalities such as conditional indexation, the analysis of financial fairness is not straightforward. In this paper, we use a stylised overlapping generations model to study financial fairness for a conditional indexation scheme. We find that financial fairness for all participants at all times is not feasible within a scheme of this type, unless the nature of indexation is such that the scheme is reduced to DC. However, financial fairness for incoming generations at the moment of entry can be realised. We show how to compute the fair contribution rate as a function of the current nominal asset/liability ratio for a given level of nominal entitlements. At low levels of the ratio, the fair contribution for incoming generations is also relatively low; nevertheless, the joining of a new generation still has a positive effect on the asset/liability ratio.

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1. Introduction

Improved mortality and increasing uncertainty about investment returns have led to the closure of many traditional defined benefit (DB) pension schemes. As an alternative, defined ambition schemes (DA) have been suggested, for example in The Netherlands. While in traditional DB schemes certain annuity levels were guaranteed, subject to the pension plan or the employer remaining solvent, the annuity levels of DA schemes represent a target or an ambition, rather than a guarantee. For a more detailed discussion of the DA scheme in The Netherlands, see for example [Bovenberg et al. \(2015\)](#).

A characteristic of DA schemes is the presence of more or less complex rules for adaptation of benefit payments and accrual of pension rights to changing circumstances, arising for instance from fluctuations in investment returns or from revisions of mortality forecasts. In this way, DA schemes implement forms of intergenerational risk sharing. Discussions of various schemes in the literature include [Gollier \(2008\)](#), [Dai & Schumacher \(2009\)](#), [Cui et al. \(2011\)](#), [Kleinow \(2011\)](#), and [Bovenberg & Mehlkopf \(2014\)](#).

The sustainability of collective pension schemes hinges on the willingness of all involved stakeholders to continue their participation. As pointed out by [Bovenberg et al. \(2007\)](#), it may not be attractive for new generations to enter an existing fund when the fund is running a deficit. Of course,

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whether or not incoming generations do have a motive to refrain from participation depends on the rules of the pension scheme. In this paper, we analyse the participation constraint in the context of a particular type of scheme known as conditional indexation (CI) scheme.

Contrary to [Beetsma et al. \(2012\)](#), our definition of the participation constraint is in terms of market value rather than in terms of utility. In other words, we consider the open market as an alternative to participation in the collective. A generation that enters a collective scheme with specified rules effectively acquires a contingent claim. The only contingencies that can affect the payoffs relate to events that occur between the time at which the generation enters the system and the time of its death; indeed, events that occur earlier are not contingencies, and events that occur later cannot be taken into account. Financial contracts that depend on events that take place during the generation's lifetime are also offered in the open market. If a generation does not get a 'fair deal' on a market value basis in the collective scheme (that is, if the market-consistent value of the contingent claim that the generation receives from the scheme is less than the market-consistent value of the contributions that are charged), then, under sufficient market completeness, it could achieve higher utility by making use of suitable financial instruments that are traded in the open market. The importance of financial fairness for the sustainability of collective pension schemes has been stressed by [Kocken \(2012\)](#).

In debates on pension reform, different opinions can be heard on whether the market completeness referred to above is indeed present. Market frictions reduce the threat of deviations from market consistency to the continuity of schemes that are well designed in terms of utility improvement. Deviations that are too large, however, would endanger sustainability. A certain 'tolerance band' might be identified, but in this paper we will stick to strict financial fairness in market value terms. We will work under the tacit assumption that, from a utility point of view, participation in the scheme that we consider, is sufficiently attractive to successive generations. This assumption allows us to avoid explicit modelling of preferences.

In the design of risk sharing systems over time, the constraints imposed by incoming participants are not the only ones to be taken into account. There is a rich literature on the conditions for continued risk sharing in overlapping generations models applied to financial intermediation, starting from [Diamond & Dybvig \(1983\)](#). Here we consider the interests of three parties: incoming generations, incumbent generations, and an infinitely lived sponsor. We derive the consequences of financial fairness for each of these stakeholders.

In order to analyse financial fairness, one must specify the rules of the scheme that is considered. In this paper we focus attention on CI schemes. In such schemes, pension benefits are subject to an indexation function which refers to an exogenous index such as inflation or a salary index, but which also depends on the solvency of the fund: if the asset/liability ratio is low, there will be no or little indexation, or, in some cases, promised pensions might be reduced.

In the CI scheme that we assume, pension benefits are regularly adjusted by a factor consisting of an exogenous index multiplied by a CI function which depends on the nominal asset/liability ratio of the pension fund. The nominal asset/liability ratio, or the nominal funding ratio as we shall call it, is defined as the ratio of assets to *nominal* liabilities, that is, liabilities without taking uncertain future indexation into account. This contrasts with what might be called the *market-consistent* funding ratio, which is the ratio of assets to the market value of liabilities, given the rules of the indexation scheme. For the nominal funding ratio, only the accrued benefits for existing members are relevant, while potential increases of those benefits in the future are ignored.

Using an overlapping generations model, we find that the value of benefits promised to a new generation depends on the contribution that this generation makes to the assets of the fund. However, it is also necessary to consider the unknown contributions that future generations will make. Therefore, the valuation of promised benefits in a CI pension scheme turns out to be a non-standard option pricing problem with two major complications: the premium itself has an impact on the payoff, and the premium paid by future generations affects the payoff as well. In this respect, our approach is different from the approach used by [de Jong \(2008\)](#), who does not consider the feedback effect between the level of new contributions and the indexation of benefits. Instead, his research

focuses on optimal portfolio choice. In contrast, we assume that the asset allocation is fixed. By this assumption, which we believe is a good approximation of common practice, we can study the impact of current and future contributions on the value of current pension promises more effectively.

The paper makes two main contributions to the existing literature. Firstly, we consider the financial position of the three stakeholders in the proposed CI pension scheme, namely: existing members, new members and the sponsor, for example, the employer. We then show that it is not possible to treat all three stakeholders fairly at all points in time, unless the indexation ladder is of such a nature that effectively a defined contribution (DC) policy is implemented.

Secondly, we consider a scheme in which only the young generation (new members) will be treated fairly at the time when they join the scheme, and in which any generation is obliged to allow any future generation to join the scheme subject to the future generation paying a fair premium. We calculate the fair value of the pension contract at the time a generation joins the scheme. We find that the fair level of contribution to be paid by the incoming generation is driven mainly by the current nominal funding ratio; when the funding ratio is low, the young generation should pay less than when the funding ratio is high. Our calculations indicate that, even when the contribution of an incoming generation is reduced when the funding ratio is low, the older generations still benefit from the joining of the new generation.

By treating the young generation fairly, we ensure that they are willing to join the pension scheme rather than being forced to join a pension scheme that they might perceive as unfair, given the information they have at the time of joining. In such a scheme each generation is driven only by self-interest. However, we show that even when decisions are driven by self-interest only, risk sharing between generations takes place.

Those conclusions are drawn in a very stylised setting. The applied overlapping generations model only considers three generations at each point in time, and each generation only makes one contribution at the start of their working life and only receives a lump sum benefit at retirement. While this is a rather simple setting, it allows us to obtain a clear derivation of the main conclusions mentioned above. We discuss some extensions to our model in Section 4.

The remainder of the paper is organised as follows. The model that we use is developed in Section 2 below. In Section 3, we ask whether it would be possible, within the CI schemes that we consider, to guarantee financial fairness for all involved parties at all times, and we arrive at the conclusion that this is not feasible unless an indexation policy is chosen which essentially reduces the scheme to DC. We briefly discuss in Section 4 how this result extends to models with multiple overlapping generations. We then continue under the assumption that entry implies commitment to future participation. In Section 5 we analyse the effect of incoming generations on older generations under this assumption, and we obtain a formula for the fair level of contributions for generations entering an infinitely lived scheme. The formula justifies an approximation that is based on a finite horizon; we use this in Section 6 to obtain quantitative results. The intergenerational risk sharing effects are described in Section 7, and conclusions follow in Section 8.

2. Model and notation

We consider an overlapping generations model in which each generation works for two periods and then retires at the end of the second period. Such a model was first introduced by Samuelson (1958) and has been applied widely in the economic literature; see for example Weil (2008). In our model, members of each generation pay into a pension fund and receive a lump sum payment at retirement. We will call the generation that enters at time $t = 1, 2, \dots$ generation t . We will assume that each generation t makes a single contribution, C_t , at time t (start of working life). In return, generation t is promised a nominal benefit to be paid as a lump sum upon retirement at time $t + 2$. In addition, the initial nominal benefits are adjusted at the end of each period, i.e. at times $t + 1$ and $t + 2$. The nominal benefits promised at time s to generation t are denoted by N_s^t , so that N_t^t is the initial promised benefit, N_{t+1}^t is the nominal promise at time $t + 1$ after adjustments at the end of the first

period, and N_{t+2}^t is the actual pension payment received by generation t when that generation retires at time $t + 2$. We assume that the sizes of generations are equal, so that the quantity N_{t+2}^t can also be viewed as the pension benefit per member of generation t .

Let A_t denote the value of the pension fund's assets at time t after the payment N_{t+2}^{t-2} has been made to the generation that retires at t , but before new contributions are received from generation t . It follows that the asset value after new contributions have been received is $A_t + C_t$, and the value of those assets at $t + 1$ before pension payments N_{t+1}^{t-1} is $(A_t + C_t) \exp(R_{t+1})$ where R_{t+1} denotes the return during the period $[t, t + 1)$. After the payment of N_{t+1}^{t-1} to generation $t - 1$ has been made at time $t + 1$, the value of the assets reduces to

$$A_{t+1} = (A_t + C_t) \exp(R_{t+1}) - N_{t+1}^{t-1} \quad (1)$$

and the cycle starts again from that new asset value.

The adjustment of nominal benefits is called indexation in this paper. Our study focuses on the calculation of fair contributions C_t to be made by generation t for a given initial nominal benefit N_t^t , and conditionally on an agreed indexation procedure. Pension schemes differ in that respect. In a pure DC scheme there are no guarantees, and therefore no promises are made at time t . The payment N_{t+2}^t at retirement will be decided at that time, depending on the assets available. At the other extreme end of the scale we have DB schemes, where an initial promise is made, and the promise is adjusted throughout working life independently of the available assets until the promised payment is made at retirement. We ignore here adjustments during retirement since we assume a lump sum payment, N_{t+2}^t at the end of the second period. Although we study the risk for the pension fund sponsor in this paper, we also assume that the pension fund cannot default on its obligations because the sponsor will always fund outstanding pension benefits.

DC and DB schemes have been widely discussed in the literature, and the risk sharing between pension fund sponsors (employer, state) and pension fund members (employees, citizens) is well understood; see for example [Exley et al. \(1997\)](#), [Josa-Fombellida & Rincón-Zapatero \(2004\)](#) and [Vigna & Haberman \(2001\)](#).

In contrast to this literature, the focus of the present paper is on pension schemes where indexation is conditional on the value of the assets available to the pension fund. The basic argument is that, if asset values are low, then indexation is low, while high asset values lead to higher indexation. To be precise, we assume for the nominal promises made to generations $t - 1$ and t that

$$N_{t+1}^{t-1} = I_{t+1} N_t^{t-1}, \quad N_{t+1}^t = I_{t+1} N_t^t \text{ with } I_{t+1} = g\left(\frac{A_t + C_t}{N_{t+1}^{t-1} + N_t^t} \exp(R_{t+1})\right) \quad (2)$$

where I_{t+1} is the indexation factor and g is the indexation function which depends on the ratio of available assets $(A_t + C_t) \exp(R_{t+1})$ at $t + 1$ before payments and before new contributions, to the total liabilities $N_{t+1}^{t-1} + N_t^t$ of the scheme, which consist of the promises made earlier to the two generations $t - 1$ and t . The function g is often called the indexation ladder or the policy ladder. In more realistic situations, the indexation factor I_t would be influenced by other factors as well, such as realised inflation; for simplicity we ignore such additional factors here. Note that the same indexation factor I_{t+1} in (2) is applied to the promised nominal benefits of both generations t and $t - 1$ which are in the scheme at the time of indexation.

The effect of CI is such that a given increase in value of the assets does not lead to a proportional increase of the market-consistent funding ratio, because the market value of liabilities is increased as well, as a result of better perspectives for indexation. If the policy ladder g in (2) is too steep, then it may even happen that an increase of asset value causes a decline of the market-consistent funding ratio. For this reason we include an upper bound on the steepness of the ladder function in

the standing assumption below. We also include as standard requirements that the ladder function is not constant and takes only positive values.

(A1) The ladder function g is a continuous and piecewise differentiable function satisfying the following properties:

- (i) $0 \leq g'(x) \leq 1$ for all x ;
- (ii) $g'(x) > 0$ for some $x > 0$.

We specify an additional requirement, which reflects the typical situation in practice in which there is an upper bound to indexation. This assumption will be a standing assumption starting from Section 5 in this paper.

(A2) There is a constant $g_{\max}$ such that $g(x) \leq g_{\max}$ for all x .

For simplicity of notation, it will be assumed as well that all amounts are expressed in terms of a suitable numéraire, so that interest rates can be taken to be zero. We assume furthermore that a common valuation operator is applied by all stakeholders to assign a value at time t to uncertain payoffs that will be realised at time $s > t$. Under our assumption that amounts are stated in terms of a numéraire, the valuation operator takes the form $E_t[X_s]$ where X_s is an uncertain payoff realised at time s , expectation is taken with respect to the pricing measure that corresponds to the chosen numéraire, and the subscript t indicates conditioning with respect to information that is available at time t . In the context of valuation, the term ‘expectation’ when used below will always refer to expectation under the pricing measure. It will be assumed furthermore that successive asset returns R_t are exogenous random variables which are independent under the pricing measure, and which satisfy the no-arbitrage condition $E_t[\exp(R_s)] = 1$ for $s > t$. It will moreover be assumed that the support of the distribution of asset returns is all of the real line.

In addition to the generations that pay contributions and receive benefits, we include a sponsor among the stakeholders. The sponsor could either be the state, an employer, or a regulator. It could also be an institution like the Pension Protection Fund (PPF) in the UK, or the Pension Benefit Guaranty Corporation (PBGC) in the US, who would cover deficits when there is an insolvency of the employer. Naturally, the sponsor has an interest in preventing deficits. We consider two different settings below, and in both of these, deficits are, in principle, excluded. In the first setting (‘financial fairness for everyone’), this condition is stated explicitly. In the second setting (‘financial fairness for new members only’), we impose the condition that incoming generations pay a fair contribution for the contingent benefit promise that they receive. This means that, at least theoretically, it would be possible to hedge the benefit payoff in the financial markets, so that no risk for the sponsor remains. In practice, deficits may still arise, due to ineffective (or absent) hedging and other causes.

3. Financial fairness for everyone

Our aim in this section is to investigate whether it is possible to construct a CI scheme that treats all involved parties fairly at every point in time.

We consider a scheme at time t , after indexation has been applied and payments to generation $t - 2$ have been made, but before generation t has joined. The pension fund has assets with a value of A_t and a nominal liability of $N_t^{t-1} = N_{t-1}^{t-1} I_t$ since only generation $t - 1$ is still a member of the fund.

There are three stakeholders who should be treated financially fairly when new members make their contribution and are given nominal promises. These are: the new members, the existing members and the pension fund sponsor (employer/state for example). These three parties now need to negotiate how much the new generation should contribute in return for a nominal promise of N_t^t which will be adjusted according to a given indexation function g . In a CI pension scheme where all three stakeholders are treated fairly at any point in time, the new generation might decide not to join if new contributions are perceived as too high, or the new generation might not be allowed to join by

the sponsor or the old generation if they disagree on the new contribution C_t to be paid by the new generation.

The existing members of the fund, generation $t - 1$, will argue that the value at time t of the random payment that they will receive at $t + 1$ should not be reduced by the fact that generation t joins. Specifically, in the context of a CI scheme, the payment that generation $t - 1$ will receive when they retire at time $t + 1$ is

$$N_{t+1}^{t-1} = N_t^{t-1} g \left(\frac{A_t + C_t}{N_t^{t-1} + N_t^t} \exp(R_{t+1}) \right) \quad (3)$$

which depends on the contribution C_t and the nominal benefits N_t^t . We will use the notation V_s^t to indicate the market value at time s of the pension promise held by generation t . The value of the pension promise to generation $t - 1$ at time t depends on the contribution C_t made by the incoming generation at time t as well as on the nominal promise N_t^t that this generation receives; therefore we write $V_t^{t-1} = V_t^{t-1}(C_t, N_t^t)$. The older generation will only be treated fairly if any new contributions and promises do not reduce the fair value of its pension deal, that is:

$$V_t^{t-1}(C_t, N_t^t) \geq V_t^{t-1}(0, 0) \quad (4)$$

where the right-hand side in (4) is the market value of benefits if no generation joins at time t .

As mentioned in Section 2, the sponsor, for example the employer, will have to deal with all shortfalls in the pension scheme. The recent closure of many DB schemes has shown that the sponsor will not always agree to new members joining a pension scheme. The sponsor will consider the surplus (or deficit), denoted by S_t , of the pension fund using the same valuation operator as the other two stakeholders. The surplus in the pension fund after new members joined the fund at time t depends again on the contribution that those new members make:

$$S_t(C_t, N_t^t) = A_t + C_t - \left[V_t^{t-1}(C_t, N_t^t) + V_t^t(C_t, N_t^t) \right] \quad (5)$$

where $A_t + C_t$ is the value of the available assets at time t after generation t has joined, and $V_t^{t-1}(C_t, N_t^t) + V_t^t(C_t, N_t^t)$ is the market value (at time t) of the combined pension promises made to generations $t - 1$ and t .

The sponsor will compare this surplus to the surplus of the fund without generation t joining, that is,

$$S_t(0, 0) = A_t - V_t^{t-1}(0, 0). \quad (6)$$

Again, we argue that financial fairness for the sponsor means that

$$S_t(C_t, N_t^t) \geq S_t(0, 0) \quad (7)$$

and that the sponsor will close the fund for new members if that condition is not fulfilled. Combining Equations (5), (6) and (7), and solving for $V_t^{t-1}(0, 0)$, we obtain

$$V_t^{t-1}(C_t, N_t^t) + V_t^t(C_t, N_t^t) - C_t \leq V_t^{t-1}(0, 0). \quad (8)$$

Finally, new members will also demand to be treated fairly or decide not to join the pension scheme. They require that their contributions do not exceed the value of the nominal promises they receive. The restriction imposed by the incoming generation is

$$C_t \leq V_t^t(C_t, N_t^t). \quad (9)$$

The constraints imposed by the three stakeholders are (4), (8), and (9). The last two inequalities imply

$$V_t^{t-1}(C_t, N_t^t) \leq V_t^{t-1}(0, 0). \quad (10)$$

Combining this with (4) leads to

$$V_t^{t-1}(C_t, N_t^t) = V_t^{t-1}(0, 0). \quad (11)$$

This equation in combination with (8) leads to the inequality

$$C_t \geq V_t^t(C_t, N_t^t) \quad (12)$$

which together with (9) implies that

$$C_t = V_t^t(C_t, N_t^t). \quad (13)$$

We find that the three inequalities (4), (8), and (9) can only be satisfied if all inequalities are in fact equalities. This happens when the two conditions (11) and (13) are both satisfied. On the one hand, the condition in Equation (13) says that new members will pay a contribution which is equal to the value of the promises they receive. On the other hand, the condition in Equation (11) says that any new contributions and promises should not change the value of promises made to existing members.

Using the representation of the valuation operator in terms of an expectation under the pricing measure, we can write the market value at time t of the pension promise to generation $t - 1$ as

$$\begin{aligned} V_t^{t-1}(C_t, N_t^t) &= E_t \left[N_{t+1}^{t-1} \mid N_t^{t-1}, A_t \right] \\ &= E_t \left[N_t^{t-1} g \left(\frac{A_t + C_t}{N_t^{t-1} + N_t^t} \exp(R_{t+1}) \right) \mid N_t^{t-1}, A_t \right]. \end{aligned} \quad (14)$$

Furthermore, we assume that the indexation function g is non-decreasing and not constant, see Assumption (A1) in Section 2. We then obtain that (11) is only fulfilled if

$$\frac{A_t + C_t}{N_t^{t-1} + N_t^t} = \frac{A_t}{N_t^{t-1}}. \quad (15)$$

Therefore, to ensure financial fairness for all three stakeholders involved, the nominal funding ratio should not change when generation t joins and makes new contributions. Solving (15) for C_t , we find that this holds if and only if

$$C_t = \frac{A_t}{N_t^{t-1}} N_t^t. \quad (16)$$

The question arises under what conditions on the policy ladder g the conditions (11) and (13) can be simultaneously satisfied. To answer this, we consider the evolution of pension promises under the assumption that incoming generations pay contributions according to (11). We introduce a new variable Z_t by

$$Z_t = \frac{A_t}{N_t^{t-1}} = \frac{A_t + C_t}{N_t^{t-1} + N_t^t}. \quad (17)$$

Since $A_{t+1} = (A_t + C_t) \exp(R_{t+1}) - N_{t+1}^{t-1} = Z_t(N_t^{t-1} + N_t^t) \exp(R_{t+1}) - I_{t+1} N_t^{t-1}$, and $I_{t+1} = g(Z_t \exp(R_{t+1}))$, we can write

$$Z_{t+1} = \frac{A_{t+1}}{N_{t+1}^t} = \frac{Z_t(N_t^{t-1} + N_t^t) \exp(R_{t+1}) - g(Z_t \exp(R_{t+1})) N_t^{t-1}}{g(Z_t \exp(R_{t+1})) N_t^t}. \quad (18)$$

This expression for Z_{t+1} contains the variable N_t^{t-1} , for which we can write an update rule as follows:

$$N_{t+1}^t = g(Z_t \exp(R_{t+1}))N_t^t. \quad (19)$$

We assume that the nominal promises to successive generations N_t^t are fixed in advance as a deterministic sequence. Then the two Equations (17) and (19) show that the variables Z_t and N_t^{t-1} can be viewed as state variables, which at time t determine the distribution of all future pension payments. In particular, we can look at the value at time t of the nominal promise received by generation t as a function of Z_t , the nominal funding ratio, and N_t^{t-1} , the nominal pension promise to the older generation. The financial fairness condition (13) states that this value must be equal to the actual contribution made by the generation entering at time t . To ensure that relation (11) is satisfied, this contribution must be given by (16), or, using Z_t :

$$C_t = Z_t N_t^t. \quad (20)$$

That is to say, the contribution contains a premium (or discount), relative to the nominal promise, which is equal to the nominal funding ratio at time t .

The realised pension payment that generation t receives upon its retirement at time $t+2$ is given by $I_{t+2}I_{t+1}N_t^t$. Combining (13) and (20), the condition of financial fairness can therefore be written as $Z_t N_t^t = E_t[I_{t+2}I_{t+1}]N_t^t$, or

$$Z_t = E[I_{t+2}I_{t+1} | Z_t, N_t^{t-1}]. \quad (21)$$

This condition states that the premium/discount factor applied to the contribution of the incoming generation should be equal to the expected value, under the pricing measure, of cumulative indexation until retirement.

There is a policy ladder that satisfies this condition, namely the one given by $g(x) = x$ for all x . Indeed, in this case it follows from (17) that $Z_{t+1} = 1$ irrespective of the values of Z_t and N_t^{t-1} , and hence $I_{t+2} = g(Z_{t+1} \exp(R_{t+2})) = \exp(R_{t+2})$. Consequently, we obtain

$$E[I_{t+2}I_{t+1} | Z_t, N_t^{t-1}] = E[\exp(R_{t+2})Z_t \exp(R_{t+1})] = Z_t$$

since asset returns are independent and $E[\exp(R_t)] = 1$ for all t , as discussed in Section 2. The policy $g(x) = x$ may be referred to as the policy of *immediate restoration*. After each period, the funding ratio is restored to 100% by adaptation (either upwards or downwards) of all liabilities. The factor Z_t that applies to the contribution of the incoming generation according to the rule (20) equals the expected value under the pricing measure of the indexation that will take place at time $t+1$. Since at that point the funding ratio is restored to 100%, the expected value under the pricing measure of the subsequent indexation at time $t+2$ is 1. Overall, the expected indexation under the pricing measure is Z_t . In this way, the funding ratio preserving contribution (20) is consistent with financial fairness.

However, a pension fund that follows the immediate restoration policy is in fact a pure DC fund, since the value of benefits directly follows asset returns. CI has been originally conceived as a modification of DB systems, aimed at introducing a certain amount of flexibility to help stabilise the funding ratio. Therefore, policy ladders that are used in practice are typically not of the form $g(x) = x$. Frequently, there is a cap on the upside, referred to as ‘full indexation’. On the downside, protection is provided, which keeps the indexation from sinking as low as the funding ratio.

If contributions are determined by the funding ratio preserving rule (20), then the use of a policy ladder that is not as steep as immediate restoration will in general lead to deviations from financial fairness. To illustrate this, we consider the impact of existing liabilities. The partial derivative with

respect to N_t^{t-1} of the value at time t of the pension promise to the incoming generation is given by

$$\begin{aligned} & \frac{\partial E[I_{t+2}I_{t+1}N_t^t \mid N_t^{t-1}, Z_t]}{\partial N_t^{t-1}} \\ &= E[g'(Z_{t+1} \exp(R_{t+1}))[Z_t \exp(R_{t+1}) - g(Z_t \exp(R_{t+1}))] \exp(R_{t+2}) \mid Z_t, N_t^{t-1}]. \end{aligned} \quad (22)$$

Under the financial fairness condition (21), this partial derivative should be equal to zero for any given value of Z_t and N_t^{t-1} . However, when the policy ladder is such that $g(x) < x$ for large x and $g(x) > x$ for small x , then, under typical assumptions on the asset return R_{t+1} , the right-hand side is positive for large Z_t and negative for small Z_t .¹ Consequently, financial fairness does not hold.

Henceforth we shall assume in this paper that there is a cap on indexation, i.e. Assumption (A2) in Section 2 holds in addition to Assumption (A1). Under this condition, the two requirements (13) and (16) cannot hold simultaneously. Indeed, the relation (21) cannot hold for all values of Z_t , since the right-hand side is bounded above by $g_{\max}^2$. The situation in which the requirement of fairness to the old generation is maintained has already been discussed above. Below, we study the reverse situation in which the requirement (13) is dropped, and instead financial fairness to the incoming generations is maintained. Before going into this, we briefly discuss a possible extension to an overlapping generations model with more than three generations.

4. Extension to multiple generations

Our model can be extended to an overlapping generations model in which a new generation joins the scheme at the start of each period and then works for several periods making contributions at the start of each period. Our conclusions in Section 3 that financial fairness for all generations is impossible also extend to this setting, as long as all contributions that generation t will make during their working life are agreed at time t when they join the scheme. In that case C_t represents the present value of all contributions made by generation t . We argue that any generation will then be indifferent between making several contributions throughout their working life with present value C_t or a single contribution C_t at the time they join the pension scheme.

In such an extended model, there would be several old generations in the pension scheme at time t . The quantity V_t^{t-1} in Section 3 would then represent the combined market value of all promises made to generations that joined prior to time t . We have seen that those generations collectively cannot be treated fairly, without reverting to DC, if the sponsor and the new members are treated fairly. However, it would clearly be possible that a mechanism is introduced by which some of the existing generations accept a reduction in the market value of their promised benefits, while other generations' benefits remain unaffected by new members. For example, it could be agreed that it is always generation $t - 1$ that accepts potential losses caused by generation t joining. In this case, each generation would be treated fairly at the time they join; in return, they would be required to accept potential losses when the next generation joins, but would then be unaffected by any further generations joining. In that way generation $t - 1$ would compensate all previous generations for potential losses caused by generation t joining.

Our conclusions in Section 3 can therefore be extended in so far as it is not possible outside DC to treat the new generation, the sponsor and the collective of all earlier generations that are still in the scheme fairly. In such an extended model, however, the rules of the pension scheme could be changed such that contributions are negotiated at the time the contributions are made rather than at the time a generations joins the scheme. The analysis of such a scheme is beyond the scope of this paper.

¹It may seem counterintuitive that the value of the promise to the incoming generation can be impacted positively by the value of the promise to the older generation. However, recall that this impact is measured at a fixed value of the funding ratio. If the funding ratio is high even while the value of the promise to the older generation is also high, this means that the fund is in excellent shape and that the perspectives for indexation are good.

5. Financial fairness for new members only

We will now consider a pension scheme in which new members are always allowed to join at time t as long as this generation t pays its fair contribution C_t , which is derived as a solution of $C_t = V_t^t(C_t, N_t^t)$ for a given promised benefit N_t^t and an agreed valuation operator V_t^s . We argue that such an arrangement is reasonable for both generations since the old generation $t - 1$ enjoyed the same privilege when they were young and joined the scheme. Their continued participation in the scheme has been priced into the contribution that they paid.

Before we turn to the calculation of C_t under this assumption, we discuss the effect of the new generation on the other two stakeholders (existing members and sponsor). Under the assumption of market completeness, both the sponsor and the old generation are able to hedge completely against the effect of the entry of the new generation, since all incoming generations pay a market-consistent price for the rights they receive. The implementation of a complete or partial hedging strategy may be more feasible for the sponsor than for the participants, and we focus below in particular on the consequences of the joining of the new generation for the old generation.

5.1. Effect on sponsor and old generation

We consider a scheme in which new generations pay a contribution that is equal to the market value of the pension promise they receive, and in which they are always allowed to join regardless of the effect on the nominal funding ratio which determines indexation. In such a scheme, the joining of any generation t will almost surely have an effect on the financial positions of generation $t - 1$ and the sponsor.

5.1.1. Increased nominal funding ratio

We first discuss the situation in which the nominal funding ratio increases due to contributions made by generation t , that is,

$$\frac{A_t + C_t}{N_t^{t-1} + N_t^t} > \frac{A_t}{N_t^{t-1}}.$$

In this situation, the value $V_t^{t-1}(C_t, N_t^t)$ of the pension promised to generation $t - 1$ will be larger than the value $V_t^{t-1}(0, 0)$ of those promises without generation t . The reason is that g is a non-decreasing function. Therefore, for any realised return R_{t+1} , the pension payment N_{t+1}^{t-1} will be larger than without the new generation joining.

On the other hand, the surplus in the pension fund will be reduced: $S_t(C_t, N_t^t) < S_t(0, 0)$. This follows from the inequality $V_t^{t-1}(C_t, N_t) > V_t^{t-1}(0, 0)$ and the equality $C_t = V_t^t(C_t, N_t)$, since

$$\begin{aligned} S_t(C_t, N_t) &= A_t + C_t - (V_t^{t-1}(C_t, N_t) + V_t^t(C_t, N_t)) \\ &= A_t - V_t^{t-1}(C_t, N_t) < A_t - V_t^{t-1}(0, 0) = S_t(0, 0). \end{aligned}$$

This seems to be counterintuitive, since it might be argued that any increase in the funding ratio should improve the solvency of the fund. However, in the pension scheme considered, an increased funding ratio also increases the liabilities of the fund, leading to a reduction of the surplus. This reduction is to the detriment of the sponsor, who is the owner of the surplus.

Instead of the difference $S_t(C_t, N_t^t) - S_t(0, 0) < 0$, which measures the absolute change in the surplus caused by generation t , we may also consider a funding ratio based on the market (risk-neutral) value $V_t^{t-1}(C_t, N_t^t) + V_t^t(C_t, N_t^t)$ of the total liabilities. In case the current surplus is positive ($A_t > V_t(0, 0)$), we find that

$$\frac{A_t + C_t}{V_t^{t-1}(C_t, N_t^t) + V_t^t(C_t, N_t)} = \frac{A_t + C_t}{V_t^{t-1}(C_t, N_t^t) + C_t} < \frac{A_t + C_t}{V_t^{t-1}(0, 0) + C_t} < \frac{A_t}{V_t^{t-1}(0, 0)} \quad (23)$$

which means that the market-consistent funding ratio will be reduced.

However, if the current surplus is negative ($A_t < V_t(0, 0)$), the market-consistent funding ratio is improved by the incoming new generation if the condition

$$V_t^{t-1}(C_t, N_t^t) - V_t^{t-1}(0, 0) < \left(\frac{V_t^{t-1}(0, 0)}{A_t} - 1 \right) C_t \quad (24)$$

is satisfied, that is, when the increase in market value of the rights of the existing generation is relatively small compared to the contribution made by the incoming generation. In this scenario, the deficit (negative surplus) would still increase, while the market-consistent funding ratio would improve. The new funding ratio (left hand side in (23)) will still be less than 100%, but it will be closer to 100% than the funding ratio without the new generation (right-hand side in (23)).

5.1.2. Decreased nominal funding ratio

Similar arguments hold if the entry of the new generation lowers the nominal asset/liability ratio. In that case, the surplus of the fund would increase, and the value of the promises made to generation $t - 1$ would decrease. The funding ratio based on market values would increase if the fund runs a deficit, but it might decrease if the fund runs a surplus depending on a condition similar to (24).

5.2. Valuation

Having investigated the effect of new generations on the financial positions of the sponsor and the old generation, we are now interested in quantifying the fair contribution required by any new generation for a fixed initial nominal promise. To this end we will first study in more detail how the value $V_t^{t-1}(C_t, N_t^t)$ of the pension contract of existing members is affected by the contribution C_t of new members and the nominal promises N_t^t made to them. In a second step we will consider the conditional expectation of $V_t^{t-1}(C_t, N_t^t)$ at time $t - 1$ to find the fair contribution required from generation $t - 1$.

Using risk-neutral valuation we obtain for the value at time t of actual pension payments N_{t+1}^{t-1} made at time $t + 1$:

$$V_t^{t-1}(C_t, N_t^t) = E_t[N_{t+1}^{t-1}] = E_t \left[N_t^{t-1} g \left(\frac{A_t + C_t}{N_t^{t-1} + N_t^t} \exp(R_{t+1}) \right) \right]. \quad (25)$$

We know that

$$V_t^{t-1}(C_t, N_t^t) = V_t^{t-1}(0, 0) \iff C_t = \frac{A_t}{N_t^{t-1}} N_t^t.$$

In words, the funding ratio is not changed by new contributions and new rights.

We now introduce the following notation, which we will then use to formulate our result about the fair contribution C_{t-1} that solves (13) for generation $t - 1$:

$h_t^{t-1}(n)$ is the sensitivity of the valuation function V_t^{t-1} in (25) with respect to the contribution C_t at time t at the point $C_t = \frac{A_t}{N_t^{t-1}} n$, that is, near the contribution that does not change the asset liability ratio:

$$h_t^{t-1}(n) = \frac{\partial}{\partial c} V_t^{t-1}(c, n) \Big|_{c = \frac{A_t}{N_t^{t-1}} n}. \quad (26)$$

$\tilde{C}_t$ is the fair contribution of the generation entering at time t assuming that the next generation will not change the nominal funding ratio at time $t + 1$, i.e.

$$\tilde{C}_t = E_t \left[V_{t+1}^t \left(\frac{A_{t+1}}{N_{t+1}^t} N_{t+1}^{t+1}, N_{t+1}^{t+1} \right) \right] = E_t [V_{t+1}^t(0, 0)].$$

$\hat{C}_t$ is the contribution of the generation entering at time t such that the nominal funding ratio is not changed at time t , i.e.

$$\hat{C}_t = \frac{A_t}{N_t^{t-1}} N_t^t.$$

We can now formulate the following proposition about the fair contribution at time $t - 1$. The proof can be found in Appendix 1.

Proposition 1: *The fair contribution to be paid by generation $t - 1$, given all future nominal promises, is*

$$C_{t-1} = \tilde{C}_{t-1} + \sum_{k=0}^{\infty} E_{t-1} \left[\left\{ \tilde{C}_{t+k} - \hat{C}_{t+k} \right\} \prod_{n=0}^k h_{t+n}^{t+n-1} (N_{t+n}^{t+n}) \right]. \quad (27)$$

Our conclusion is, therefore, that the fair contribution at time $t - 1$, given all future nominal promises, consists of two parts. The first part is the fair contribution at $t - 1$ assuming that the next generation will not change the nominal funding ratio. To that we add the expected value of all changes to the nominal funding ratio made by future generations. However, instead of the actual change in the funding ratio resulting from $C_{t+k} - \hat{C}_{t+k}$, which we cannot calculate without knowing future contributions, we only require $\tilde{C}_{t+k} - \hat{C}_{t+k}$ in (27) for all future generations.

Given the expression in Proposition 1, the question arises whether the contributions C_{t-1} are finite.

Proposition 2: *If the initial nominal promises made to future generations are constant, that is, $N_{t+k}^{t+k} = N$ for all $k > 0$ where $N > 0$ is a constant, then $C_{t-1} < \infty$ for all t , where C_{t-1} is given in Proposition 1.*

The proof of Proposition 2 can be found in Appendix 2.

In the proof of Proposition 2 we find in particular that the product $\prod_{n=0}^k h_{t+n}^{t+n-1} (N_{t+n}^{t+n})$ in (27) is a decreasing function of the number of factors, which means that generations which are distant in the future have less influence on the fair contribution today. This result justifies calculating fair contributions under the assumption that there is only a finite number of future generations, since the impact of generations $t + k$ for large k can be ignored.

6. Fair contributions with a finite number of future generations

The results in the previous section justify an approximation in which only a finite number of future generations are considered as long as Assumption (A1) is satisfied and initial promises are constant. In this case we do not need to use the approximation in (27), but we can calculate the fair contributions numerically by using a recursive scheme starting with the last generation. To be specific, we assume throughout that $N_t^t = 1$; in other words, we work with a fixed initial promise and generation size.

Our purpose in this section is therefore to compute the fair value of contributions in the conditional indexation scheme described by (1–2), with a fixed initial pension promise, under the assumption that the scheme will be terminated at a given future time T . In essence, this is an option pricing problem: the contribution is considered as the premium to be paid by the incoming generation at time t for the contingent payoff that they will receive at time $t + 2$. There are two major complications compared to standard option pricing problems: first, the premium itself has an impact on the payoff, and secondly, the premium paid by the next generation impacts the payoff as well. In order to deal with these complications, we use a computational scheme that employs two auxiliary functions:

- the expected value (under the pricing measure) of the payoff at time $t + 2$, as a function of existing liabilities at time $t + 1$ and the asset value at time $t + 1$ *before* the contribution by the generation that comes in at time $t + 1$ has been paid, under the assumption that this contribution will occur at fair value (one-step-ahead expectation);

- the expected value (under the pricing measure) of the payoff at time $t + 2$, as a function of existing liabilities at time t and the asset value at time t after the contribution that comes in at time t has been paid (two-step-ahead expectation).

With an abuse of notation, the first function will be denoted by $E[N_{t+2}^t | A_{t+1}, N_{t+1}^t]$ as in (14), and the second by $E[N_{t+2}^t | A_t + C_t, N_t^{t-1}]$. As in (13), the fair contribution at time t is determined as a function of asset values at time t , namely A_t , and existing liabilities at time t , N_t^{t-1} , by the equation

$$C_t = E[N_{t+2}^t | A_t + C_t, N_t^{t-1}]. \quad (28)$$

The two auxiliary functions mentioned above can be used to evaluate the right-hand side, since

$$E[N_{t+2}^t | A_t + C_t, N_t^{t-1}] = E[E[N_{t+2}^t | A_{t+1}, N_{t+1}^t] | A_t + C_t, N_t^{t-1}]$$

where A_{t+1} and N_{t+1}^t are expressed in terms of A_t , C_t , and N_t^{t-1} according to the equations of the CI scheme.

Since the scheme terminates at time T , there is no new generation coming in at time $T - 1$. Within the equations above, this can be expressed by $N_{T-1}^{T-1} = 0$ and $C_{T-1} = 0$. The above equations then allow computation of the fair contribution C_{T-2} as a function of A_{T-2} and N_{T-2}^{T-3} . Using this, the function C_{T-3} can be computed, and so on.

Figure 1 shows the results of computations in a particular case. We assume a quite simple policy ladder that applies no indexation if the nominal funding ratio falls below 105%, and full indexation when the funding ratio is above 140%; linear interpolation is applied between these two bounds. Thinking of generations as following each other by 25 years, we set full indexation to 35%, which corresponds to real wage growth on a 25-year horizon if annual real wage growth equals 1.2%. Our indexation function is then given by:

$$g(x) = \begin{cases} 1 & \text{for } x < 1.05 \\ x - 0.05 & \text{for } 1.05 \leq x \leq 1.4 \\ 1.35 & \text{for } 1.4 < x \end{cases}$$

Asset returns are assumed to be lognormally distributed with 25% volatility; on a 25-year horizon, this corresponds to a quite conservative investment policy (5% annual volatility).

The fair contribution is determined largely by the nominal funding ratio, but the structure of liabilities (pension rights held by different generations) also plays a role. Figure 1 shows two curves, corresponding to the older generation having obtained no indexation at the previous time step ('low liability') or full indexation ('high liability'). The two curves are seen to cross, which can be explained by the same reasoning as in footnote 1. Also drawn in Figure 1 is the level of contribution that would keep the nominal funding ratio constant. It is seen that at levels up to about 160%, the fair contribution helps to improve the funding ratio; at higher levels, the effect of the fair contribution is to reduce it.

The graph shows the situation five steps before termination, because this appears already enough to represent a stationary situation. The fair contribution is computed, for a fixed value of assets A_t and a fixed level of the existing liabilities N_t^{t-1} , by the iterative procedure that is naturally suggested by Equation (28). The iteration converges quickly; at a tolerance level of 10^{-5} (compared to the level of the contribution between 1 and 2), convergence typically takes place within three or four iteration steps. Computations were carried out on a grid of 150 steps in the direction of the asset value and 20 steps in the direction of the pension promise to the older generation. Total computation time, using an implementation in MATLAB on a standard PC, was in tens of seconds.

The stability of the scheme that is obtained in this way can be investigated in time series simulation. Figure 2 shows a scatter plot of funding ratios after 20 periods vs. funding ratios after 10 periods in

Figure 1. The plot shows the fair contribution of the generation entering at time t as a function of the funding ratio at time t , in two cases. 'Low liability' means that the promise made to the older generation is at level 1 (no indexation in previous time step); 'high liability' means that the promise to the older generation is at level 1.35 (full indexation in previous time step). The dotted line represents contribution levels that leave the funding ratio unchanged.

1000 simulated scenarios. The initial funding ratio in this scenario set is taken to be 100%, and it is assumed that no indexation was granted to the generation that was mid-career at the initiation of the scheme. Furthermore, given uncertainties that exist over the long period lengths that we work with, and given also that we assume a quite conservative investment policy, the excess expected return parameter is set equal to zero. Even under this rather cautious assumption, high funding ratios appear in many scenarios. The spread of funding ratios is wider after 20 periods than it is after 10 periods, which is a sign of instability. The figure shows that underfunding after 10 periods may well be followed by substantial recovery, suggesting that the 'sinking giant' phenomenon is not strongly present. Given the highly stylised form of our model, these observations are very preliminary in nature. In a more realistic model with many generations, the size of the contribution made by the incoming generation would be much smaller relative to the size of the liabilities. On the other hand, in reality contributions are not paid as a lump sum at the start of the career, but rather in instalments. If contributions during the career are adjusted on the basis of current fair value, as suggested in Section 4, they may still support recovery to an extent that is comparable to what we find in the simplified model.

7. Intergenerational risk sharing and sustainability of the pension scheme

We have seen in Section 5.1 how any new generation affects the financial positions of existing members and the sponsor of the CI pension scheme. We have considered the two scenarios of decreased or increased nominal funding ratios. Figure 1 shows that, for low nominal funding ratios, the fair contribution made by generation t exceeds the contribution $C_t = \frac{A_t}{N_t^{t-1}} N_t^t$, which does not change the nominal funding ratio. This is the case in both the low liability and the high liability scenario in Figure 1. It follows that low nominal funding ratios are increased by the new contributions made by generation t , with the consequences discussed in Section 5.1. In particular, the new contributions improve the financial position of existing members. It follows that existing members benefit after a period of low returns from the contributions made by generation t .

On the other hand, we find in Figure 1 that the fair contribution of generation t will be less than $\frac{A_t}{N_t^{t-1}} N_t^t$ if the nominal funding ratio at time t is rather high, that is, after the existing members and previous generations have enjoyed high returns R_t .

In that way generation t participates in the high or low returns experienced in previous periods. This risk sharing mechanism is achieved despite each generation only paying the financially fair contribution conditional on the information available at the time when they join the scheme. This contrasts with risk sharing mechanisms which rely on a social planner, and on forcing each new

Figure 2. Funding ratio after 20 periods vs. funding ratio after 10 periods in 1000 simulated scenarios.

generation to join. We argue that in the CI pension scheme new generations will always be happy to join since they are only required to pay a fair contribution for the benefits they receive.

One could imagine a pension scheme in which new generations are always forced to join and to make a contribution which will restore the funding ratio to a specified level. Although such a pension scheme would have a more stable funding ratio, new generations would not be treated fairly and might not join given the choice. On the other hand, the CI pension scheme combines financial fairness with the risk sharing mechanisms described above.

8. Conclusions and recommendations

Our purpose in this paper has been to investigate the consequences of imposing financial fairness on a CI scheme. We have only considered a very simple form of such a collective pension scheme. Nevertheless, we believe that some conclusions and policy recommendations can be drawn from our findings.

We have argued that financial fairness at all times for all cohorts cannot be achieved without essentially breaking up the scheme. On the other hand, it is possible to realise financial fairness for incoming generations. This means that the collective scheme requires a commitment from participating generations, which guarantees that they will continue their participation, and that they will allow new generations to enter. Under some circumstances, older generations must accept that the value of their pension rights will be reduced. However, in a scheme that is constructed on the basis of financial fairness for incoming generations, the fact that such circumstances may arise has already been taken into account in the contribution to be paid by these generations. In that sense the conditional reduction of rights is still fair. Obviously this point needs to be clearly communicated to plan members.

We recommend that, in the design of collective schemes which allow for adjustment of accrued rights (either by CI or by other rules), calculations be made analogous the ones that have been presented here, in order to ensure at least approximate financial fairness for incoming generations. The sustainability of the system would be at stake if this condition is not fulfilled. From a technical point of view, the computations are not straightforward. In Section 6 we have outlined a method that can be used for the particular option pricing problem that arises here. It is a recursive method, which makes use of suitable auxiliary functions. Scaling up the computational procedure to determine fair contributions in a more real-life situation would, of course, still require a substantial amount of work.

It is encouraging to observe in the example that the fair contribution level is determined largely by the current nominal funding ratio. It depends only to a minor extent on the structure of existing liabilities, which, in a multi-generation framework, can be a complicated object.

Our conclusion is that, in CI schemes which aim for financial fairness at entry, the contribution to be paid by incoming generations should be reduced when the funding ratio is low. This is fair, since a low funding ratio means that indexation is less likely. We have kept the nominal rights constant for simplicity, but clearly an alternative way to achieve financial fairness would be to keep contributions constant, and to grant higher nominal rights to incoming generations when the funding ratio is low than when it is high.

The recommendation to reduce contributions for incoming generations in case of low funding ratios, and to increase contributions in case of high funding ratios, may raise the question whether the funding ratio would not thereby be destabilised. Calculations that we have made in a specific case, however (see Figure 1), show that such an effect need not occur. At low funding ratios, the reduction of the contribution is limited to such an extent that the joining of the new generation still helps to increase the funding ratio. Conversely, when the funding ratio is sufficiently high, it will be reduced by the arrival of the new generation. Of course, the result depends on the specific assumptions we made in the example, and calculations need to be done anew in other cases. Further information about the variability of the funding ratio can be obtained from more extensive ALM studies.

In our analysis, we have focused on the effects of financial fairness in the context of a given conditional indexation scheme. Clearly, these effects may depend on the CI rule that is used, as well as on other parameters of the collective scheme, in particular the investment policy. An assumption that the indexation function is not too steep has been used above, as part of mathematical support for the intuitive notion that generations in the very distant future should be less important for setting the current level of contributions; see Assumption (A1). It may be surmised that sufficient flatness of the indexation function is also important in relation to the stability of the funding ratio. If this is confirmed in further studies, we obtain in this way a policy recommendation regarding indexation functions. The consequences of imposing financial fairness are influenced as well by the investment policy. Consequently, policy recommendations regarding investments may be derived from an investigation of this dependence.

In this paper we have assumed the investment mix to be fixed in time, both for simplicity and because it is common practice. However, one might envisage that the investment policy could be adapted, for instance in response to the funding ratio. A situation in which management has complete freedom to change the portfolio at any time has been discussed by Kleinow (2011) for a related pension contract. For an analysis along the lines of the present paper, it would be necessary to assume that the composition of the investment portfolio is determined according to a specified rule, rather than being at the discretion of the trustees. Financial fairness can then in principle again be determined by solving an option pricing problem, although the complexity of this problem would be increased.

The analysis of this paper, while applied in a highly idealised situation, indicates that collective schemes that implement risk sharing through adjustment of accrued rights can be designed in such a way as to ensure financial fairness for incoming generations. Our findings suggest that such schemes offer a route to effective risk sharing between generations driven by self-interest only, rather than by enforcement.

A potential route for further research would be to consider more general overlapping generations models with several generations of active members continuously contributing to the scheme, and also several generations of retired members who are receiving an annuity rather than a lump sum payment. Relevant research questions would then be the following. Can the scheme be designed to treat all members fairly in the sense of Section 3? And if not, is it possible to treat at least all generations fairly that contribute to the scheme at any given time? And, would all or some generations still share risks with each other?

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Appendix 1. Proof of Proposition 1

We use a Taylor expansion of $V_t^{t-1}(c, n)$ (defined in (25)) in c for a given value of $n = N_t^t$:

$$\begin{aligned} V_t^{t-1}(C_t, N_t^t) &= V_t^{t-1} \left(\frac{A_t}{N_t^{t-1}} N_t^t, N_t^t \right) + \left[C_t - \frac{A_t}{N_t^{t-1}} N_t^t \right] \frac{\partial}{\partial c} V_t^{t-1}(c, N_t^t) \Big|_{c=\frac{A_t}{N_t^{t-1}} N_t^t} \\ &\quad + O \left(\left[C_t - \frac{A_t}{N_t^{t-1}} N_t^t \right]^2 \right). \end{aligned}$$

The value at time $t - 1$ of the pension contract is then, in first-order approximation,

$$\begin{aligned} V_{t-1}^{t-1} &= E_{t-1} \left[V_t^{t-1}(C_t, N_t^t) \right] \\ &= E_{t-1} \left[V_t^{t-1} \left(\frac{A_t}{N_{t-1}^t} N_t^t, N_t^t \right) \right] - E_{t-1} \left[\frac{A_t}{N_{t-1}^t} N_t^t h_t^{t-1}(N_t^t) \right] + E_{t-1} \left[C_t h_t^{t-1}(N_t^t) \right] \end{aligned} \quad (A1)$$

where h_t^{t-1} was defined in (26).

The first two terms in (A1) can be calculated since they are independent of contributions made by future generations. The third term contains C_t and can therefore not be computed immediately. However, treating new members fairly at any time means that $V_{t-1}^{t-1} = C_{t-1}$. We therefore establish an approximate relationship between contributions at successive times:

$$C_{t-1} = E_{t-1} \left[V_t^{t-1} \left(\frac{A_t}{N_{t-1}^t} N_t^t, N_t^t \right) \right] - E_{t-1} \left[\frac{A_t}{N_{t-1}^t} N_t^t h_t^{t-1}(N_t^t) \right] + E_{t-1} \left[C_t h_t^{t-1}(N_t^t) \right]. \quad (A2)$$

Using this formula recursively we obtain

$$\begin{aligned} C_{t-1} &= E_{t-1} \left[V_t^{t-1} \left(\frac{A_t}{N_{t-1}^t} N_t^t, N_t^t \right) \right] - E_{t-1} \left[\frac{A_t}{N_{t-1}^t} N_t^t h_t^{t-1}(N_t^t) \right] \\ &\quad + E_{t-1} \left[E_t \left[V_{t+1}^t \left(\frac{A_{t+1}}{N_{t+1}^t} N_{t+1}^{t+1}, N_{t+1}^{t+1} \right) h_t^{t-1}(N_t^t) \right] \right] \\ &\quad - E_{t-1} \left[E_t \left[\frac{A_{t+1}}{N_{t+1}^t} N_{t+1}^{t+1} h_{t+1}^t(N_{t+1}^t) \right] h_t^{t-1}(N_t^t) \right] \\ &\quad + E_{t-1} \left[E_t \left[C_{t+1} h_{t+1}^t(N_{t+1}^{t+1}) \right] h_t^{t-1}(N_t^t) \right] \\ &\quad \vdots \\ &= E_{t-1} \left[V_t^{t-1} \left(\frac{A_t}{N_{t-1}^t} N_t^t, N_t^t \right) \right] \\ &\quad + \sum_{k=0}^{\infty} E_{t-1} \left[\left\{ V_{t+k+1}^{t+k} \left(\frac{A_{t+k+1}}{N_{t+k+1}^{t+k}} N_{t+k+1}^{t+k+1}, N_{t+k+1}^{t+k+1} \right) - \frac{A_{t+k}}{N_{t+k}^{t+k-1}} N_{t+k}^{t+k} \right\} \prod_{n=0}^k h_{t+n}^{t+n-1}(N_{t+n}^{t+n}) \right] \\ &= E_{t-1} \left[V_t^{t-1} \left(\frac{A_t}{N_{t-1}^t} N_t^t, N_t^t \right) \right] \\ &\quad + \sum_{k=0}^{\infty} E_{t-1} \left[\left\{ E_{t+k} \left[V_{t+k+1}^{t+k} \left(\frac{A_{t+k+1}}{N_{t+k+1}^{t+k}} N_{t+k+1}^{t+k+1}, N_{t+k+1}^{t+k+1} \right) \right] - \frac{A_{t+k}}{N_{t+k}^{t+k-1}} N_{t+k}^{t+k} \right\} \prod_{n=0}^k h_{t+n}^{t+n-1}(N_{t+n}^{t+n}) \right] \\ &= E_{t-1} \left[V_t^{t-1} \left(\frac{A_t}{N_{t-1}^t} N_t^t, N_t^t \right) \right] \\ &\quad + \sum_{k=0}^{\infty} E_{t-1} \left[\left\{ E_{t+k} \left[V_{t+k+1}^{t+k} (0, 0) \right] - \frac{A_{t+k}}{N_{t+k}^{t+k-1}} N_{t+k}^{t+k} \right\} \prod_{n=0}^k h_{t+n}^{t+n-1}(N_{t+n}^{t+n}) \right] \end{aligned} \quad (A3)$$

where we used the tower rule for conditional expectations and

$$V_{t+1}^t \left(\frac{A_{t+1}}{N_{t+1}^t} N_{t+1}^{t+1}, N_{t+1}^{t+1} \right) = V_{t+1}^t(0, 0) \quad \forall t.$$

The result in Proposition 1 then follows from (A3) and the definition of $\tilde{C}$ and $\hat{C}$ in section 5.2.

Appendix 2. Proof of Proposition 2

The differences $\tilde{C}_{t+k} - \hat{C}_{t+k}$ in (A3), are multiplied by $\prod_{n=0}^k h_{t+n}^{t+n-1} (N_{t+n}^{t+n})$ in (27). Using (25) and the definition of h in (26), we obtain for those derivatives

$$\begin{aligned}
 \frac{\partial}{\partial c} V_t^{t-1}(c, N_t^t) &= \frac{\partial}{\partial c} E_t \left[N_t^{t-1} g \left(\frac{A_t + c}{N_t^{t-1} + N_t^t} \exp(R_{t+1}) \right) \right] \\
 &= E_t \left[N_t^{t-1} \frac{\partial}{\partial c} g \left(\frac{A_t + c}{N_t^{t-1} + N_t^t} \exp(R_{t+1}) \right) \right] \\
 &= E_t \left[N_t^{t-1} g' \left(\frac{A_t + c}{N_t^{t-1} + N_t^t} \exp(R_{t+1}) \right) \frac{1}{N_t^{t-1} + N_t^t} \exp(R_{t+1}) \right] \\
 &= \frac{N_t^{t-1}}{N_t^{t-1} + N_t^t} E_t \left[g' \left(\frac{A_t + c}{N_t^{t-1} + N_t^t} \exp(R_{t+1}) \right) \exp(R_{t+1}) \right] \\
 &\leq \frac{N_t^{t-1}}{N_t^{t-1} + N_t^t} \max_x g'(x) E_t [\exp(R_{t+1})] \leq \frac{g_{\max}}{1 + g_{\max}} =: N_2 < 1.
 \end{aligned} \tag{B1}$$

The inequality in the last line follows from the standing Assumption (A1), together with the assumption that $N_t^t = N$ for all t . Also note that we are taking expectations under a pricing measure, and prices are taken relative to a numéraire, so that $E_t[\exp(R_{t+1})] = 1$.

From (B1) we obtain

$$\begin{aligned}
 \sum_{k=0}^{\infty} E_{t-1} \left[\tilde{C}_{t+k} \prod_{n=0}^k h_{t+n}^{t+n-1} (N_{t+n}^{t+n}) \right] &\leq \sum_{k=0}^{\infty} E_{t-1} [\tilde{C}_{t+k}] N_2^{k+1} < \infty, \\
 \sum_{k=0}^{\infty} E_{t-1} \left[\hat{C}_{t+k} \prod_{n=0}^k h_{t+n}^{t+n-1} (N_{t+n}^{t+n}) \right] &\leq \sum_{k=0}^{\infty} E_{t-1} [\hat{C}_{t+k}] N_2^{k+1} < \infty
 \end{aligned}$$

since, by the assumption that $N_{t+k}^{t+k} = N$, we have the following bounds which do not depend on the index k :

$$E_{t-1} [\tilde{C}_{t+k}] \leq g_{\max}^2 N$$

and

$$E_{t-1} [\hat{C}_{t+k}] = E_{t-1} \left[\frac{A_{t+k}}{N_{t+k}^{t+k-1}} N_{t+k}^{t+k} \right] \leq E_{t-1} [A_{t+k}] \frac{N_{t+k}^{t+k}}{N_{t+k}^{t+k-1}} \leq A_{t-1}.$$

The finiteness result now follows from the expression (27).