ACCRETION AND JETS
from stellar-mass to supermassive black holes
RILEY CONNORS
Accretion and jets from stellar-mass to supermassive black holes

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Riley Michael Thomas Connors

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Promotiecommissie:

Promotor: Prof. dr. S. B. Markoff Universiteit van Amsterdam
Copromotor: Prof. dr. R. A. M. J. Wijers Universiteit van Amsterdam

Overige leden: Prof. dr. J. Wilms University of Erlangen
Prof. dr. M. A. Nowak MIT Kavli Institute
dr. A. L. Watts Universiteit van Amsterdam
Prof. M. B. M. van der Klis Universiteit van Amsterdam
Prof. dr. A. de Koter Universiteit van Amsterdam
dr. P. Uttley Universiteit van Amsterdam
Prof. dr. A. R. King University of Leicester

Faculteit der Natuurwetenschappen, Wiskunde en Informatica

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Thanks to Chiara Ceccobello for designing the thesis cover.
A young Irishman sits in a small, lively pub in Kilrush, county Clare, Ireland, late 1960s, enjoying some (probably many) beers with his friends. The local guard walk into the pub and begin taking the names and addresses of every person present. Each responds in kind, until the guard approach the young Irishman in question:

Guard: "Name and address...."
Irishman: "Thirty-two Burton Street".
Guard: "And where’s that exactly?"
Irishman: "Well, next to thirty-f***in’-three!"

To my father, the Irishman.
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A wise man proportions his belief to the evidence.
— David Hume

1.1 On the history of black holes

The fascination we have, as a wondering humankind, with black holes, has been ever-present since the 18th century propositions of John Michell and Pierre-Simon Laplace that there may be "non-luminous bodies" in the universe (Montgomery et al. 2009). However our capability to understand the gravity of what we now term black holes was birthed by Albert Einstein and his theory of general relativity (GR) in 1915 (Einstein 1915a,b,c). It took just months for Karl Schwarzschild, in the trenches of World War I, to discover that one of the many potential solutions to Einstein’s field equations described a point mass (Schwarzschild 1916); thus the first real scientific description of the singularity. The discussion on the nature of the singularity was rife in the decades following the work of Einstein and Schwarzschild. One key identifier of a possible formation mechanism in nature came from Chandrasekhar (1931), who showed that if a non-rotating body of electron-degenerate matter exceeds the mass limit of $1.4 \, M_\odot$ (now known as the Chandrasekhar limit), it would be unstable to collapse. However it was not until the 1950s that the leap was made from recognising the Schwarzschild metric as describing a point mass, to its implications for the black hole spacetime and the event horizon. David Finkelstein made this breakthrough, showing that the Schwarzschild metric describes a surface within which there is only one direction of causality (Finkelstein 1958); what comes in, cannot escape, including objects travelling at the speed of light.
1 Introduction

It was the centres of galaxies outside our own which first directed the attention of most of the astronomical community toward black holes, due to indications that they harbour a bright non-stellar source of emission. This ultimately led to the realisation that accretion onto black holes must be powering the bright nuclei of galaxies, but the progression came in stages. The earliest observations of galaxies which indicated a component distinguishable from pure stellar emission came unwittingly from Fath (1909). Observations of spectroscopic emission lines in objects then perceived to be Galactic in origin were in fact propagating from the nuclei of NGC 1068 and Messier 81 (M81). Curtis (1918) later discovered a strange ray of emission on the sky that he identified as being connected to a nucleus via a straight line of matter; this was actually the jet propagating from Messier 87 (M87). When Carl Seyfert—the proponent of sources now classed as Seyfert galaxies—systematically identified broad emission lines in the nuclei of nearby galaxies (Seyfert 1943), the study of galaxies and their evolution began to take off. Significant developments came from radio astronomy, due to the overwhelming difference in luminosity between the nuclei and stellar components of galaxies at radio frequencies. The Galactic centre supermassive black hole (SMBH), Sagittarius A* (Sgr A*), was the source of the first extra-solar radio emission discovered by Karl Jansky, a pioneer of radio astronomy (Jansky 1933). Several of the canonical active galactic nuclei (AGN, the name given to the resolved bright nucleus of a galaxy) that are subject to intensive observational and theoretical investigation today were identified as radio emitters in the 1940s (M87, Centaurus A, Cygnus A; Bolton & Stanley 1948; Bolton et al. 1949; Stanley & Slee 1950), in the aftermath of world war II. Astronomers at this point understood that galaxies are more than a collection of stars and gas. Another iconic step was made when Schmidt (1963) discovered highly redshifted lines in the optical spectrum of high-redshift galaxy 3C 273, showing that it lies at a large distance, which would imply a luminosity in excess of $10^{59}$ erg s$^{-1}$. It was in the 1960s that accretion onto SMBHs was proposed as the mechanism responsible for the bright luminosities (Salpeter 1964; Zel’tovich 1964) observed at the centres of these galaxies, and later Donald Lynden-Bell suggested all galaxies may in fact harbour SMBHs, though most central engines of galaxies are 'dead' (Lynden-Bell 1969). These central engines are what we now broadly refer to as AGN.

The earliest attempts to understand how gas accretes onto massive bodies were made in the context of the interactions between stars and their gaseous neighbourhoods (the interstellar medium; ISM) (Hoyle & Lyttleton 1939; Bondi & Hoyle 1944; Bondi 1952). These early papers considered steady, spherically symmetric accretion of interstellar gas onto stars, yielding a simple expression for the mass accretion rate that depends on the mass of the star and the density and sound speed of the local gas. The spherical accretion of gas onto stars was initially extended to treat compact objects such as black holes by (Salpeter 1964; Zel’dovich 1964; Shapiro 1973; McCray 1973), focusing primarily on the mystery of supermassive black holes (SMBHs) at the
1.1 On the history of black holes

Table 1.1: A simple classification scheme for different types of AGN. The two broad properties used to define AGN type are radio loudness (radio-loud and radio-quiet) and optical emission line properties (narrow, broad, or unusual). QSO = quasi-stellar object, NLRG = narrow-line radio galaxy, BLRG = broad-line radio galaxy, SSRQ = steep-spectrum radio quasar, FSRQ = flat-spectrum radio quasar, FR = Faranoff-Riley. Both type 1 and type 2 AGN actually have detectable narrow lines, so the distinction is made by the presence of forbidden broad lines in type 1 AGN.

<table>
<thead>
<tr>
<th>Radio loudness</th>
<th>Optical emission line properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radio-quiet</td>
<td>Type 2 (narrow)</td>
</tr>
<tr>
<td></td>
<td>Seyfert 2</td>
</tr>
<tr>
<td></td>
<td>Type 1 (broad)</td>
</tr>
<tr>
<td></td>
<td>Seyfert</td>
</tr>
<tr>
<td></td>
<td>QSO</td>
</tr>
<tr>
<td>Radio-loud</td>
<td>Type 0 (unusual)</td>
</tr>
<tr>
<td>NLRG {</td>
<td>BLRG</td>
</tr>
<tr>
<td>FR I</td>
<td>SSRQ</td>
</tr>
<tr>
<td>FR II</td>
<td>FSRQ</td>
</tr>
<tr>
<td>Blazars {</td>
<td>BL Lac</td>
</tr>
<tr>
<td>FSRQ</td>
<td></td>
</tr>
</tbody>
</table>

centres of galaxies. Despite the simplicity of the spherical accretion assumption, one can use it to gain a sufficient first-principles understanding of the potential radiative output and efficiency of accreting black holes.

We have by now catalogued a large sample of AGN in the nearby and high-redshift universe with the use of telescopes and satellites covering the whole electromagnetic spectrum. Modern-day catalogues contain a plethora of detailed photometric and spectroscopic information on the AGN observed to date (e.g. the Veron-Cetty and Veron catalogue; Véron-Cetty & Véron 2010, the Fermi-LAC catalogue; Ackermann et al. 2011, SDSS; Pâris et al. 2017). The large amount of data available allows us to build a broad yet detailed picture of the behaviour of AGN. The picture is built by categorising the different AGN according to their observational properties across various wavebands. A key step in connecting the emission observed in AGN at different wavelengths was to recognise that several of the main classifiers of AGN types could be understood in terms of their orientation relative to the line of sight (see, e.g., Osterbrock et al. 1992, Antonucci 1993, Urry & Padovani 1995). An example of the basic distinctions between the different AGN classes is shown in Table 1.1.

If we observe a flat radio spectrum in a bright AGN, very long baseline interferometry (VLBI) typically shows a jet is resolved (e.g. the AGN sample of Kellermann et al. 1998). The radio waves emitted from these jets come in the form of synchrotron radiation (see, e.g., Ginzburg & Syrovatskii 1969; Begelman et al. 1984; Rybicki & Lightman 1986). The high brightness temperatures derived from their observed radio flux densities, and the low-frequency spectral turnovers coincident with such high brightness temperatures, shows that the radiating particles are at least partially non-thermal and the jets are self-absorbed (see, e.g., Williams 1963; Bridle 1967). We refer to this absorbed synchrotron emission in relativistic jets as synchrotron self-absorption (SSA).
1 Introduction

Figure 1.1: The $M_{\text{BH}}-\sigma_B$ relation, showing that there is a correlation between the mass of SMBHs at the centres of galaxies and the galaxy bulge mass (the velocity dispersion $\sigma_{\text{BH}}$ gives a measure of the total bulge mass). The plot is taken from Gültekin et al. (2009).

1.2 The big picture

Whilst the physics of accretion onto black holes and what drives outflows is interesting in itself, it fits within a much broader astrophysical and cosmological context which makes it an increasingly interdisciplinary area of research. For example, over the past two decades a tight correlation has been discovered, and verified, between the black hole mass of AGN and the masses of their bulges; the canonical $M_{\text{BH}}-\sigma_B$ relation, where $\sigma_B$ is the velocity dispersion of the bulge (Kormendy & Richstone 1995).
1.2 The big picture

Magorrian et al. (1998); Ferrarese & Merritt (2000); Gebhardt et al. (2000); Tremaine et al. (2002); Marconi & Hunt (2003); Gültekin et al. (2009); Kormendy & Ho (2013), shown in Figure 1.1. This correlation begs the question, what could be causally connecting the much larger kpc size-scales of the bulge with the Bondi capture radius within which accretion onto the SMBH occurs? Some explanations have been suggested, including major galaxy merger events, star-formation winds, and AGN-driven outflows (see e.g. Alexander & Hickox 2012 for a review). All three of these mechanisms may be contributing to the coincident evolution of the central engine and the surrounding environment (i.e. the bulge and broader intergalactic medium (IGM)), and may not be mutually exclusive. Though many authors have shown that AGN activity likely plays a significant role in the co-evolution of the spheroid bulges of galaxies (e.g., Silk & Rees 1998; Granato et al. 2004; Di Matteo et al. 2005), the $M_{\text{BH}} - \sigma_B$ relation may arise due to the natural progression of galaxies merging over time, i.e., the correlation requires no coupling between the AGN and the surrounding regions of the galaxy (Jahnke & Macciò 2011). Nonetheless, there is a consensus that AGN activity, star-formation in the host galaxy, and SMBH growth are all closely connected. Studies have found observational evidence that AGN activity quenches star-formation (e.g., Davies et al. 2007; González Delgado et al. 2001), and that the histories of SMBH accretion and star-formation are similar (e.g., Boyle & Terlevich 1998; Silverman et al. 2008; Aird et al. 2010; Merloni & Heinz 2013). Theoretical approaches to explain the $M_{\text{BH}} - \sigma_B$ correlation predict the contrary process in which the star formation rate reduces the mass accretion rate onto SMBHs (Power et al. 2011), implying a feedback loop, and others predict SMBH growth should coincide with star formation due to the progressive supply of cold gas to central regions of the host galaxy (e.g., Di Matteo et al. 2005; Somerville et al. 2008). Theoretical models also show that AGN activity can initiate further star formation due to the energy released into the surrounding environment (Nayakshin & Zubovas 2012; Zubovas et al. 2013; Nayakshin 2014). Although observations of high-luminosity AGN show they preferentially reside in star-forming hosts (Lutz et al. 2008; Bonfield et al. 2011), the same correlation is not seen to extend down to lower-luminosity AGN (Shao et al. 2010; Rosario et al. 2012; Harrison et al. 2012). However, Hickox et al. (2014) show that intrinsic AGN variability (i.e., variable accretion rates) may be the reason for this apparent weakening of the correlation.

Making a quantitative link between AGN activity and the surrounding star-forming regions is complicated further by the uncertainty in determinations of the kinetic power output of the accreting SMBH in question. Progress has been made through detections of absorption lines in the outflows of AGN. For example, warm, absorbing outflows have been observed in Seyfert galaxies, indicating gas outflow (wind) velocities of $v_w \sim 1000$ km s$^{-1}$ (Reynolds 1997; Crenshaw et al. 2003). These winds were however shown to contain insufficient kinetic power to deliver enough kinetic energy into the surrounding medium (Blustin et al. 2005). Higher kinetic powers have been inferred in the ultra-fast outflows ($v_w \gg 1000$ km s$^{-1}$) of some quasars through ob-
servations of their broad UV absorption lines (Weymann et al. 1991; Ganguly et al. 2007), and similar wind velocities result from X-ray observations of AGN (Pounds et al. 2003; Reeves et al. 2009; Tombesi et al. 2010). However, once again, deriving the kinetic power of these winds is difficult, with uncertainties typically covering several orders of magnitude. Thus determining whether it is the winds or jets of AGN that dominate AGN feedback is a tall order (see, e.g., Fabian 2012).

The field of AGN feedback is evolving rapidly, and I refer the reader to a recent review by Harrison (2017) for further details on current observational constraints on the SMBH growth/star formation connection. What is clear from these extensive studies of AGN feedback is that the unanswered questions require a better understanding of what drives AGN activity, and the answers lie in determining the nature of accretion onto SMBHs and what drives changes in accretion rate.

It is challenging to build a complete picture of this feedback loop by observing AGN alone and relating the observational results back to the theoretical models of SMBH accretion and star formation. The difficulty arises for several reasons. Firstly, the timescale for changes in the accretion rates of AGN and responses in the system to those changes are well beyond any monitoring time we can hope to achieve in our human lifetime. We thus rely on piecing together observations of multiple AGN to build a picture of the duty cycle ($t_o/[t_o + t_q]$), where $t_o$ is the outburst time, and $t_q$ is the quiescence time) of any given object. Secondly, there are many components contributing to the broadband spectral energy distributions of AGN, due both to their complicated morphology and to bright star-forming regions outshining the AGN in some cases. Thirdly, due to the inverse relation between inner disc temperature and black hole mass ($T_{in} \propto M_{BH}^{-1/4}$; Shakura & Sunyaev 1973) the blackbody spectrum from the optically-thick accretion discs of higher-luminosity AGN (QSOs, Seyferts) peaks in the far-UV, even for the least massive SMBHs in the full range ($M_{BH} \sim 10^5$–$10^9 \, M_\odot$). UV-extinction in the host galaxy and our own Galaxy makes UV-observations of higher-luminosity AGN hard to achieve at sufficient sensitivity to characterise the accretion disc spectrum. For these reasons we need to use clever tricks to build a more complete picture of AGN inflows and outflows and their evolution.

Luckily, we have the answer right on our doorstep: black hole X-ray binaries (BHBs) are essentially scaled-down versions of AGN, all located just a few kpcs away in our own Galaxy (Merloni et al. 2003; Heinz & Sunyaev 2003; Markoff et al. 2003; Falcke et al. 2004; Done & Gierliński 2005; Jester 2005; Nipoti et al. 2005; Körding et al. 2006b; McHardy et al. 2006; Körding et al. 2007; Markoff et al. 2008; Gliozzi et al. 2010; Kelly et al. 2011; Plotkin et al. 2012; Markoff et al. 2015). Advances in observational detection of BHBs began with X-ray observations in 1960s, when we saw the discovery of extra-solar X-ray sources (Giacconi et al. 1962). The 1960s saw a significant advance in extra-solar X-ray astronomy, with the first X-ray observations occurring on board sounding rockets. It was not until 1972 however that one of the
discovered X-ray sources, Cygnus X-1, was speculated to be a binary system consisting of a star and black hole companion (Webster & Murdin 1972). The decades following saw the development of the study of BHBs, and the remarkable similarity between BHBs and AGN. The apparent similarities between BHBs and AGN lead to the terms ‘nanoquasar’ (Phinney 1982) to describe BHBs. However the concrete step in cementing this connection came from the discovery of superluminal motion in GRS 1915+105 with radio observations, and the discoverers Mirabel & Rodríguez (1994) gave the name ‘microquasars’ to describe the radio emission and superluminal motion in BHBs. This was a turning point which pushed the astronomical community to explore the BHB/AGN connection through broadband observations. The mass-invariance of accretion around black holes and its consequences is well-summarised in a review by Markoff (2010a). BHBs present us with data at much higher signal-to-noise than AGN due to their close proximity. Their accretion flows and jets can respond to changes in accretion rate on timescales of days (e.g., Mirabel et al. 1998), and the duty cycles of transient systems are on the order of decades (or as short as years in some systems), so we can observe multiple outburst cycles in the same system. In addition, BHB accretion discs peak in the soft X-ray band, so we can study their blackbody spectra in more detail that we can the discs in AGN which peak in the UV, and are typically not well-constrained.

BHBs are not only useful as probes for shining light on accretion physics in AGN, they may also have played a significant role in shaping our universe themselves. For instance, the X-ray emission from the earliest intermediate-and-supersmassive black holes and high-mass X-ray binaries (within globular clusters) have been proposed as significant contributors to the Hydrogen-ionising flux during the Epoch of reionization (e.g., Loeb & Barkana 2001; Ricotti & Ostriker 2004; Ricotti et al. 2005; Zaroubi et al. 2007; Power et al. 2009). If the ionising flux from BHBs within globular clusters comprised a portion comparable to that of the massive stars in those clusters, BHBs may be just as cosmologically relevant as the earliest AGN. What is clear from this is that the study of accretion and jet launching in BHBs and AGN, and our ability to determine what drives their radiative and kinetic output, has important consequences for our understanding of both the local and high-redshift universe.

As if the impact of black holes on our cosmos were not significant enough in the context of BHBs and AGN, black holes that could have formed at the inception of the universe (primordial black holes: PBHs) may also have played a part. The discussion regarding the possibility of dark matter (DM) being comprised primarily of PBHs is not a new one (Carr & Hawking 1974; Meszaros 1974; Carr 1975). However the question of the existence of these PBHs has come to the forefront following the first detection of a binary black hole merger (Abbott et al. 2016b) by the LIGO (Laser Interferometer Gravitational-Wave Observatory) experiment. The two black holes that merged were found to have masses of $36^{+5}_{-4} M_\odot$ and $29^{+8}_{-4} M_\odot$, and Bird et al. (2016) proposed that these could in fact be PBHs. LIGO has since made
Figure 1.2: A timeline of the history of the universe, highlighting the ‘Epoch of reionization’, the period during which the universe is optically-thick to the radiation from the earliest stars and galaxies, as well as accretion powered sources. These sources of radiation combined to reionize the universe such that it appears the way it does in the present day.
two more highly-significant detections of binary black hole mergers, one in which the component masses are similarly high \((31.2^{+8.4}_{-6.0} \, M_\odot, 19.4^{+5.3}_{-5.9} \, M_\odot)\); Abbott et al. (2017), and another with component masses closer to those found in BHBs, but still at the high-end of the distribution in terms of the population we have amassed up to now \((14.2^{+8.3}_{-3.9} \, M_\odot, 7.5^{+2.3}_{-2.3} \, M_\odot)\); Abbott et al. (2016a). The component masses lie within a range of possible PBH masses allowed by both microlensing and wide-binary disruption constraints (Carr et al. 2010; Monroy-Rodríguez & Allen 2014).

To summarise what I have discussed thus far, black holes are clearly important on many mass and size scales in the universe, the questions is to what extent, in both the present-day and distant past. In order to quantify the energetic contributions of accreting black holes across the mass scale (BHBs, AGN, and perhaps PBHs), we must first develop a more complete understanding of their evolution, and what drives changes in accretion rate and the accretion geometry. In the following sections of this introduction I discuss our observational and theoretical attempts to characterise accreting black holes across the mass scale. In Section 1.3 I present some of the basic formulae and physical arguments used to understand the principles of black hole accretion, as well as the radiative processes observed in the accretion flows black holes, in particular their jets. In Section 1.4 I give an overview of BHBs, both in terms of their observational characteristics and history, and the theoretical models adopted over the years to interpret the observations. In Section 1.5 I present the observed similarities between BHBs, during particular stages of their evolution, and their supermassive cousins, AGN. In Section 1.6 I briefly discuss the candidates for the DM in the universe and the current constraints on contributions of PBHs to the DM.

1.3 The basics of black hole accretion

The fundamentals of black hole accretion can be understood from a first-principles treatment of the spherical flow of gas onto the black hole, and an estimate of the conversion of the available gravitational potential energy into radiation. The first of these is addressed via the Bondi-Hoyle-Lyttleton formula in the context of accretion of interstellar gas onto an isolated black hole moving at constant velocity (Hoyle & Lyttleton 1939; Bondi & Hoyle 1944; Bondi 1952):

\[
\frac{dM}{dt} = \frac{4\pi G^2 M^2 \rho_\infty}{(v_{BH}^2 + c_s^2)^{3/2}},
\]

where \(G\) is the gravitational constant, \(M\) is the mass of the black hole, \(\rho_\infty\) is the gas density at infinity, \(v_{BH}\) is the velocity of the black hole, and \(c_s\) is the sound speed of the gas. The gas must be transonic in order to accrete onto the black hole, in that at \(r_\infty\) the gas is subsonic, whereas at the local environment of the black hole...
hole it must be supersonic. As such there is a location at which the gas becomes supersonic, known as the Bondi radius: \( R_B = \frac{GM}{(v_{BH}^2 + c_s^2)} \). This can be considered the location at which gas becomes gravitationally bound to the black hole. The radiative power (luminosity) produced by the accreted gas is given roughly by the rate at which energy is liberated at the event horizon of the black hole, \( \Delta L_{acc} = 2\eta GM_{BH}\dot{M}_{BH}/R_\star = \eta \dot{M}_{BH}c^2 \), where \( \eta \) is the efficiency of conversion of the rest mass energy of the particles into radiation, and \( R_\star = GM_{BH}/c^2 \) is the Schwarzschild radius of the black hole. The maximum luminosity in a steady, spherically symmetric, fully ionized flow is calculated by equating the gravitational force felt by the accreting ions (protons) with the radiation pressure force exerted on the leptons (electrons) by the outgoing radiation. The result is named the Eddington limit, after Arthur Eddington, the first to derive the relation:

\[
L_{\text{Edd}} = 4\pi GM_{BH}m_p c/\sigma_T \approx 1.3 \times 10^{38} \left( \frac{M}{M_\odot} \right) \text{ erg s}^{-1},
\]  

(1.2)

where \( m_p \) is the proton mass, \( c \) is the speed of light, and \( \sigma_T \) is the Thomson scattering cross-section. Equations 1.1 and 1.2 allow us to estimate of the size scales and luminosities typical of the accretion process. However, a particularly important ingredient for the relevance of accretion in BHBs is the surplus angular momentum associated with the accreted gas. In the following I will make use of arguments and formulae presented by Frank et al. (2002). One can derive the angular momentum in, for example, a binary system, using Kepler’s third law, \( 4\pi^2 a^3 = GM^2 P \), where \( a \) is the binary separation, \( M \) is the sum of the masses of the two bodies (\( M = M_1 + M_2 \)), and \( P \) is the orbital period of the binary. The assumption is that we can treat the orbits as circular, which is justified for binary systems since eccentric orbits tend to circularize due to tidal effects. Since angular momentum at any given location is the product of the moment of inertia and the angular velocity, \( J = mr^2 \omega \), and the angular velocity of the binary is \( \omega = \left( \frac{GM}{a^2} \right)^{1/3} \), we arrive at \( J = M_1 M_2 (Ga/M)^{1/2} \). The gas from the star will undergo Roche Lobe overflow and form a disc within the Roche Lobe of the primary due to the angular momentum it carries; typical tangential velocities will be \( v_\perp \sim 100 M_1^{1/3}(1+q)^{1/3} P_{\text{day}}^{-1/3} \) km s\(^{-1}\), where \( q = M_2/M_1 \) is the mass ratio of the two bodies, whereas radial velocities are on the order of the sound speed, \( v_\parallel \leq 10 \) km s\(^{-1}\). The gas in the disc must therefore lose a great deal of its angular momentum to accrete onto the black hole, so dissipative processes of some nature must convert the bulk kinetic energy of the flow into internal energy. It has been long understood that the leading candidate for such a processes is shear viscosity, wherein the interactions between concentric rings of gas in the disc generate a viscous torque.
1.3 The basics of black hole accretion

1.3.1 Multi-temperature disc blackbody spectrum

An optically-thick accretion flow, akin to the efficient thin accretion disc prescription of Shakura & Sunyaev (1973), can be treated as a blackbody radiator with a spectrum that depends on the temperature profile in the disc. One can show this by considering the disc to be comprised of optically-thick radial elements. The dissipation rate per unit face area is given by (Frank et al. 2002)

\[ D(R) = \frac{3GM\dot{M}}{8\pi R^3} \left[ 1 - \left( \frac{R_*}{R} \right)^{1/2} \right], \]  

(1.3)

where \( R \) is the radial distance from the black hole, and \( R_* = \frac{2GM}{c^2} \) is the radius of the black hole event horizon. Using Kirchhoff’s law for the emission coefficient \( (j_\nu = \alpha_\nu B_\nu(T)) \), where the emission mechanism is Bremsstrahlung) we can express the emission profile in terms of the temperature profile in the disc,

\[ T(R) = \left\{ \frac{3GM\dot{M}}{8\pi R^3\sigma} \left[ 1 - \left( \frac{R_*}{R} \right)^{1/2} \right] \right\}^{1/4}, \]  

(1.4)

where \( \sigma = 5.67 \times 10^{-5} \text{ erg cm}^{-2} \text{ s}^{-1} \text{ K}^{-4} \) is the Stefan-Boltzmann constant for a blackbody radiator. Thus an optically-thick accretion disc emits as a multi-temperature blackbody following a temperature profile \( T(R) \propto R^{-3/4} \). Examples of multi-temperature blackbody disc spectra as a function of inner disc temperature and radius are shown in Figure 1.3.

1.3.2 Radiative processes in accreting black holes

Here I will give a brief description of some important details regarding the radiative processes associated with the accretion of gas onto black holes in order to give the reader an understanding of the more summarised descriptions and terms in the individual chapters. A particular influence is given to the radiative processes occurring within astrophysical, relativistic jets, since the bulk of the thesis focuses on the importance of jets in accreting black holes. The main assumptions and formulae presented are taken from Rybicki & Lightman (1986).

Bremsstrahlung

A key emission mechanism responsible for radiative dissipation in accretion flows is Bremsstrahlung, caused by accelerating charges through an electromagnetic field. In this case we can approximate the flow as an ionised gas comprised of ions and electrons, whereby the electrons are the key radiators, since the acceleration of a charge in an electromagnetic field is inversely proportional to the mass of the accelerating body \( (a \propto 1/m) \), and electrons are order of magnitude less massive than ions \( (m_e \sim \)
The radiation spectrum of a non-relativistic moving charge depends on the time-variation of the electric/magnetic fields and can be calculated from first principles using Maxwell’s equation of motion (Lorentz force) and an expression for the radiation field of a non-relativistic moving charge (Larmor’s formula):

\[
F = q \left( E + \frac{v}{c} \times B \right),
\]

\[
E_{rad} = \frac{q [n \times (n \times \dot{v})]}{Rc^2}; \quad B_{rad} = n \times E_{rad},
\]

where \( E \) is the electric field, \( B \) is the magnetic field, \( v \) is the particle velocity, \( q \) is the electric charge, \( F \) is the force experienced by the moving charge, \( R \) is the distance from the radiating particle, and \( n \) is the direction of the Poynting vector from the radiating particle; both \( E_{rad} \) and \( B_{rad} \) are orthogonal to \( n \).

Since the electromagnetic field of an isolated charge can be approximated as a dipole, we can express the equation of motion of an electron within this field in terms of \( m_i/1000 \). The following arguments can be found in detail in Rybicki & Lightman (1986), but I summarise the key points here in order to give context.

Figure 1.3: An example of Multi-temperature disc-blackbody spectra as a function of inner disc temperature and radius.
1.3 The basics of black hole accretion

Figure 1.4: The interaction of an electron in the coulomb field of an ion at distance $R$, moving at velocity $v$.

of its dipole moment,

$$\ddot{d} = -e\dot{v},$$

where $e$ is the electron charge, and $\mathbf{v}$ is the velocity vector of the electron. By transforming Equation 1.7 into Fourier space and interacting over the timescale of interaction (set by the distance-scale of the interaction between the ion and electron, and the velocity of the electron, as shown in Figure 1.4), and adopting a thermal distribution of particles in a plasma, we can derive a frequency-specific emissivity for the plasma as a whole,

$$\varepsilon_{ff}^\nu \equiv \frac{dW}{dV dt d\nu} = 6.8 \times 10^{-38} Z^2 n_e n_i T^{-1/2} \exp(-h\nu/kT) g_{ff}. \quad (1.8)$$

Here the dependence of the emissivity on temperature ($\varepsilon_{ff}^\nu \propto T^{-1/2}$) is due to the inverse proportionality of the spectrum on the velocity of the electron. The Gaunt factor, $g_{ff}$, corrects for the quantum effects that become important when the electron velocity becomes large (i.e., when the determination of the interaction time is governed by the uncertainty principle). Equation 1.8 describes a flat spectrum ($\log \varepsilon_{ff}^\nu$ vs. $\log \nu$) up to a cut-off at $h\nu \sim kT$, and applies to an optically-thin gas ($\tau < 1$). This has significance for determination of the radiation spectrum of an optically-thin accretion flow. One can see that $\varepsilon_{ff}^\nu \propto n^2$, and thus the radiative cooling rate of plasmas approaching low densities decreases significantly. The radiative efficiency of an accretion flow, considering just Bremsstrahlung as the emission mechanism for now, is a function of 1) the disc’s ability to dissipate gravitational potential into internal energy via viscosity (regardless of the driver of the viscosity) and 2) the optical thickness of the gas. The spherical accretion approach of McCray (1979) is thus an appropriate mathematical treatment of the gas dynamics for us to determine the efficiency of radiative dissipation in the limits where both these requirements are not being met in the accretion flow. If we assume the only heating mechanism of the gas is adiabatic compression in the spherical flow, such that the adiabatic index $\Gamma \sim 5/3$, we can
calculate the luminosity of the whole accretion flow using the frequency-integrated bremsstrahlung emissivity,\[ L = \int_{R_*}^{R_B} 4\pi \varepsilon ff(T)r^2 dr. \] (1.9)

This equation is appropriate for all spherically symmetric accretion flows. All one needs to do is derive the temperature profile for an adiabatic flow to calculate the luminosity. In a spherical flow we can determine the gas density profile using mass conservation, \( 4\pi r^2 n v = \text{constant} \), and adopting the free fall velocity in the gravitational potential of the black hole, \( v \sim (2GM/r)^{1/2} \). This gives \( n = n_0(R_B/r)^{3/2} \), and the temperature profile is then \( T = T_0(R_B/r) \), which follows if we assume the flow is adiabatic with \( \Gamma = 5/3 \) such that \( T \propto n^{\Gamma - 1} \).

**Synchrotron radiation**

Synchrotron radiation is emitted when relativistic charged particles (I consider just electrons in the following) interact with a magnetic field. In the non-relativistic limit, this process is termed cyclotron, and the frequency of the radiation is given by the gyro-frequency of the electrons around the magnetic field lines, \( \omega_g = eB/mv \). However, when the electrons are moving at relativistic velocities the frequency of the resultant radiation can extend far beyond \( \omega_g \). This is essentially due to the observer seeing time-dilated variations in the electric field variations. The relativistic beaming results in the observer seeing an apparent narrow set of light cones propagating from the direction of motion of the particle, which follows a helical path along a magnetic field line. This is shown in Figure 1.5. The emission cones have angular scale \( \Delta \theta \sim 1/\gamma_e \), and the angular extent of the particle motion about the direction of the field line is given by the pitch angle \( \alpha \). The power of the emitted radiation is found by referring back to the Lorentz force (Equation 1.5). One then finds a gyration frequency \( \omega_g = qB/\gamma_e mc \) and an angle-averaged synchrotron power
\[ P = \frac{4}{3} \sigma_T c^3 \beta \gamma_e^2 U_B, \] (1.10)
where \( \sigma_T = 6.65 \times 10^{-25} \text{ cm}^2 \) is the Thomson cross section for electron scattering, \( \beta = v/c \) is the dimensionless electron velocity, \( \gamma_e = E/mc^2 \) is the dimensionless electron energy and \( U_B = B^2/8\pi \) is the magnetic energy density. A single electron will emit a spectrum which depends on a critical frequency \( \omega_c = (3/2)\gamma_e^3 \omega_g \sin \alpha \) (this is calculated by considering the time difference between the arrival of successive pulses to the observer) given by
\[ P(\omega) = \frac{\sqrt{3} q^3 B \sin \alpha}{2\pi mc^2} F \left( \frac{\omega}{\omega_c} \right). \] (1.11)
1.3 The basics of black hole accretion

Figure 1.5: An adaptation of figure 6.2 from Rybicki & Lightman (1986) showing the synchrotron emission cones of a relativistic electron gyrating around a magnetic field line, as seen by an observer at infinity.

The function $F(\omega/\omega_c)$ in Equation 1.11 takes the form of a Bessel function, and I refer the reader to Rybicki & Lightman (1986) for full details on its treatment. The salient point here is that the radiation spectrum depends explicitly on the critical frequency, and thus we see that

$P(\omega) \propto F(\omega \gamma_e^4 m c \sin \alpha/q B)$,

and so the total radiative power in a physical plasma depends on the particle distribution and magnetic field. If one has an expression for the particle distribution, one can calculate the full radiation spectrum (in an optically-thin plasma).

If we assume the electron distribution is thermal, and we are in the relativistic regime, we can express the particle energies via the Maxwell-Jüttner distribution,

$f(\gamma_e) = \gamma_e^2 \beta_e \exp \left( -\gamma_e \Theta \right) K_2 \left( 1/\Theta \right)$, \hspace{1cm} (1.12)

where $\Theta \equiv kT/mc^2$ is the dimensionless average electron temperature and $K_2(1/\Theta)$ is a modified Bessel function of the second kind. The shape of the Maxwell-Jüttner distribution is shown in Figure 1.6 alongside the observed thermal synchrotron spectrum arising from the interaction of those electrons with a magnetic field, assuming that the electrons are within an optically-thin region of a mildly-relativistically outflowing jet.

If we consider a power-law distribution of electrons instead, such that $f(\gamma_e) d\gamma_e = C \gamma_e^{-p} d\gamma_e$, and substitute Equation 1.11 in for the synchrotron power of a single electron, the total synchrotron power as a function of photon frequency is given by

$P_{tot}(\omega) = C \int_{\gamma_{e,1}}^{\gamma_{e,2}} P(\omega) \gamma_e^{-p} d\gamma_e \propto \int_{\gamma_{e,1}}^{\gamma_{e,2}} F \left( \frac{\omega}{\omega_c} \right) \gamma_e^{-p} d\gamma_e$.

(1.13)

If we substitute $x \equiv \omega/\omega_c$ into the Bessel function, we find $P_{tot}(\omega) \propto \omega^{-(p-1)/2}$. This means an optically-thin medium comprised of a power-law distribution of electrons
interacting with a magnetic field will radiate as a power-law with index \( \alpha = -(p-1)/2 \).

**Synchrotron self-absorption**

Any plasma entangled in a magnetic field which is emitting synchrotron photons will see those photons absorbed in similar fashion. This occurs via the process of spontaneous absorption, prescribed by the Einstein coefficients of emission and absorption, and described as SSA in astrophysical systems. Once again following the logic of Rybicki & Lightman (1986), the absorption coefficient, \( \alpha_\nu \), is defined as

\[
\alpha_\nu = \frac{h\nu}{4\pi} \sum_{E_1} \sum_{E_2} \left[ n(E_1)B_{12} - n(E_2)B_{21} \right] \delta_{21}(\nu),
\]

where \( E_1 \) and \( E_2 \) are the lower and upper particle energy states respectively, \( B_{21} \) and \( B_{12} \) are the stimulated emission and absorption coefficients respectively, and \( \delta_{21} \) is a delta function restricting photon energies to those matching the transition energy difference \( h\nu = E_2 - E_1 \). Treating the continuous energy of a charged particle via these discrete states at the microscopic level allows us to calculate the macroscopic absorption coefficient intrinsic to Kirchhoff’s law, the relation between the emissivity \( (j_\nu) \), absorption \( (\alpha_\nu) \), and source term \( (S_\nu) : j_\nu = \alpha_\nu S_\nu \). In an optically-thick medium \( (\tau > 1) \) the microphysical treatment provided by these Einstein coefficients leads to Kirchhoff’s law on the macroscopic scale, and we can calculate the observed spectrum from an astrophysical source in which electrons are emitting synchrotron radiation and subsequently absorbing it. This is particularly relevant in studies of astrophysical relativistic jets in both AGN and BHBs. Using the Einstein relation between spontaneous emission, \( A_{21} \), and stimulated emission, \( A_{21} = (2h\nu^3/c^2)B_{21} \),
one arrives at an expression for the absorption coefficient given in Equation 1.14 in terms of the synchrotron power, $P$,

$$ \alpha_\nu = \frac{c^2}{8\pi h \nu^3} \sum_{E_2} [n(E_2 - h\nu) - n(E_2)] P(\nu, E_2), $$

(1.15)

where I have replaced $E_1$ with $E_2 - h\nu$. In order to arrive at an expression for $\alpha_\nu$ for a given particle distribution, we need to treat the energy states of the particles in terms of the particle momenta. One can then express the summed energy states into an integral of the distribution of momenta:

$$ \sum_{E_2} [n(E_2 - h\nu) - n(E_2)] P(\nu, E_2) = \int d^2 p_2 [f(p_2^\ast) - f(p_2)], $$

where $p_2^\ast$ represents the momentum state corresponding to the level $E_2 - h\nu$. Now we can calculate the absorption coefficient and the resultant spectrum from a thermal and non-thermal distribution of particles emitting synchrotron radiation in an optically-thick medium. If we assume a thermal distribution of electrons with temperature $T$ in a magnetic field for example, we have $f(p) \propto \exp[-E(p)/kT]$, and by inserting this expression into the integral over momentum states we arrive at

$$ \alpha_{\nu, th} = \frac{c^2}{8\pi h \nu^3} \left( e^{h\nu/kT} - 1 \right) \int d^3 p_2 f(p_2) P(\nu, E_2), $$

(1.16)

where $(c^2/8\pi h \nu^3)(e^{h\nu/kT} - 1) \equiv 1/B_\nu(T)$ is the source term and the term in the integral is the emission coefficient, $4\pi j_\nu$. Hence Equation 1.16 is the full expression of Kirchhoff’s law for the absorption coefficient of an optically-thick, thermal medium. In the Rayleigh-Jeans limit ($h\nu \ll kT$) the integral in Equation 1.16 evaluates to give $\alpha_{\nu, th} = j_\nu c^2/2\nu^2 kT$, such that $S_\nu = j_\nu/\alpha_\nu \propto \nu^2$. Take instead a non-thermal distribution of electrons, $f(p) \propto E^{-\gamma}$, and following a similar calculation to that shown in section 1.3.2 one finds a source term given by $S_\nu \propto \nu^{5/2}$.

**Inverse Compton scattering**

IC scattering concerns the interaction of photons with high-energy charged particles (again I will consider these particles to be electrons). The IC spectrum observed in accreting black holes, whether in the form of an inflowing (e.g., Lightman & Eardley 1974; Eardley et al. 1975; Shapiro et al. 1976; Haardt & Maraschi 1993; Narayan & Yi 1994, 1995a; Esin et al. 1997), or outflowing (e.g., Beloborodov 1999; Markoff et al. 2005) plasma, is governed by the fractional photon energy gain per scattering, and the mean number of scatterings, which is characterised by the Compton-$y$ parameter. If we assume a thermal distribution of electrons and an isotropic distribution of seed photons, the average energy shift imparted to the photons will depend on whether we assume the electrons are relativistic or non-relativistic. In the Thomson limit ($\gamma_e E_0 \ll mc^2$, where $E_0$ is the initial photon energy), and assuming the electrons are more energetic than the incident photons, the energy transfer for IC scattering by
Introduction

The energy gain of electrons is given by \( \Delta E = \frac{E_0}{mc^2}(4kT) \) in the non-relativistic case, and \( \Delta E = \frac{4}{3}\gamma^2_e E_0 \) in the relativistic case. This energy gain is referred to as the amplification factor, \( A \). The measure of how much a photon will change in energy in traversing the medium of electrons is given by the Compton-\( y \) parameter, defined as the average energy gain per scattering multiplied by the average number of scatterings within the medium. The latter is characterised by the optical depth of the medium, \( \tau \), and will scale linearly with optical depth if \( \tau \ll 1 \), or go as \( \tau^2 \) if \( \tau \gg 1 \). Thus in the non-relativistic and relativistic cases respectively,

\[
y = \frac{4kT}{mc^2}\text{Max}(\tau, \tau^2),
\]

and,

\[
y = 16 \left( \frac{kT}{mc^2} \right)^2 \text{Max}(\tau, \tau^2).
\]

One can use Equation 1.18 to calculate the emergent spectrum from iterative photon scatterings off relativistic electrons and show that the spectrum will resemble a power law with index \( \alpha \equiv -\ln \tau / \ln A \).

In the case \( \gamma_e E_0 \ll mc^2 \), photons will experience a boost in energy of \( \gamma^2_e \) in the observer frame. In the limit \( \gamma_e E_0 \rightarrow mc^2 \), photons can only be scattered to energies \( \sim \gamma_e mc^2 \).

If we considers an isotropic distribution of photons scattering off an isotropic distribution of high-energy electrons, we arrive at an expression for the radiative power of IC scattering,

\[
P_{IC} = \frac{4}{3} \sigma_T c \gamma^2_e \beta^2_e U_{ph},
\]

where \( U_{ph} \) is the photon energy density. Note that this is a simplified expression in the Thomson limit \( (\gamma_e E_0 \ll mc^2) \). By comparison with Equation 1.10 one arrives at the interesting result that in a synchrotron-radiating plasma, the ratio of radiative losses due to synchrotron and IC emission is given by the ratio of magnetic-to-photon energy density,

\[
\frac{P_{syn}}{P_{IC}} = \frac{U_B}{U_{ph}}.
\]

The inherent similarity between the formalism of synchrotron and IC power reflects the fact that both processes are physical occurrences of the interaction between electromagnetic fields and charges in motion.

If the electrons doing the scattering are responsible for the seed photon field through synchrotron emission, the process is referred to as synchrotron self-Compton...
1.4 Black hole X-ray binaries

Black hole X-ray binaries (BHBs) are binary systems consisting of a black hole and a star in orbit, termed X-ray binaries due to their initial detection and extensive study with X-ray telescopes. BHBs appear bright in X-rays due to the process of accretion, a phenomenon of great importance and hence deeply studied in astrophysics (see e.g. Frank et al. 2002 for a comprehensive review, though note the field has surpassed the knowledge summarised there).

Figure 1.7: Observed spectra resulting from IC scattering of jet-synchrotron photons (SSC) and disc blackbody photons as a function of the jet power ($N_j$) and mean temperature of the electron energy distribution ($\Theta$).

(SSC). One can see from Equation 1.10 and the dependence of the number of scattering on the optical depth, that in an optically-thin $\tau \leq 1$ plasma the SSC spectrum will scale with the density $n_e$ of electrons in the plasma. Figure 1.7 shows SSC spectra within the base of a mildly-relativistic jet as a function of jet power and electron temperature, assuming a thermal distribution of relativistic electrons and equipartition between the magnetic field and electron energy densities.

1.4 Black hole X-ray binaries
1.4.1 X-ray spectral states

The first indication that BHBs actually evolve through definitive spectra states came from Tananbaum et al. (1972) with the discovery that Cygnus X-1 had shown a drop in soft X-ray flux by several factors, and a coincident constant hard X-ray flux. In addition to changes in the X-ray spectrum, a radio counterpart was observed in Cygnus X-1, leading to the suggestion that these two events were connected and likely indicative of a structural change in the system. Further work that identified an evolution of states within individual sources came from Miyamoto et al. (1988) and Miyamoto et al. (1993). Cui et al. (1997) later identified a consistent shift in hard-to-soft dominated X-ray spectra in Cyg X-1, and alongside the proposal of Esin et al. (1997), the discussion regarding state changes in BHBs gained momentum. There are now multiple identifiable BHB spectral states based on the hardness of the emitted X-ray spectrum, changes in the strength of X-ray variability across a wide frequency range, and radio flux variations (see, e.g., reviews by van der Klis 1995; Tanaka & Lewin 1995; Nowak 1995; Remillard & McClintock 2006b). The X-ray spectra of BHBs are composite, being composed of a thermal blackbody and a non-thermal, power law component. The thermal component dominates in the soft states and is well-explained by a multi-temperature blackbody, originating in the optically-thick, geometrically-thin accretion disc. The power law component dominates in the hard states.

A physical mechanism for the transient nature of BHBs was first introduced by Osaki (1974) to explain the evolution of dwarf novae (outbursts from cataclismic variable stars), known as the disc instability model (see, e.g., Lasota 2001 for a detailed review of the disc instability). The core principle of the disc instability model is that high, unstable periods of increased mass accretion rate are triggered by the rapid ionisation of Hydrogen in the disc, characterised by the interplay between radiative cooling and viscous heating in the disc (prescribed by the $\alpha$-viscosity parameter). Hydrogen ionises in the disc when its surface density exceeds a critical limit. During these stages of high accretion rate, the thin accretion disc extends close to the black hole (within a few gravitational radii), and is optically thick.

The evolution of a transient BHB through states is typically presented via the hardness-intensity diagram (HID), a plot of X-ray intensity against X-ray hardness in a soft and hard energy band (e.g. Homan et al. 2001; Belloni 2004). A conceptual view of the HID is shown in Figure 1.8. BHBs rise out of quiescence where the thin accretion disc is either absent or heavily truncated, and hence the blackbody component is not observed. As the source rises and progresses through the hard state, the X-ray spectrum is dominated by power law emission, and the disc inner edge of the accretion disc moves inwards until the disc blackbody component begins to dominate and the spectrum softens, bringing the BHB into an intermediate, transient state between the longer-lived hard and soft states. The location on the HID at which
the disc ceases to be truncated is a matter of intense debate among the community in recent years, with early models proposing a unified accretion disc model in which truncation occurs in the hard state (e.g. Esin et al. 1997), and more recent challenges to this paradigm claiming the disc remains close to the innermost stable circular orbit (ISCO) during the hard state (e.g. Miller et al. 2006). The debate over disc truncation is just one among many within the study of BHBs. A full description of the nature of the power-law emitting region is still not pinned down. Initial models focused on the X-ray emission alone and postulated an optically-thin inner accretion flow, or corona, associated with hot electrons with cooling times exceeding the accretion time of the gas (Lightman & Eardley 1974; Eardley et al. 1975; Shapiro et al. 1976). In
1.4.2 Radio/X-ray correlations and broadband spectra

Our understanding of the jets propagating from the inner regions of BHB accretion flows has advanced due to the discovery of radio/X-ray correlations in the hard state (Hannikainen et al. 1998; Corbel et al. 2000, 2003; Gallo et al. 2003; Merloni et al. 2003; Corbel et al. 2008, 2013; Miller-Jones et al. 2011; Gallo et al. 2014). Even though the jets in BHBs are not always spatially resolved, we typically attribute the observed radio emission to the jets due to the superluminal motions observed in some systems (e.g., Mirabel & Rodríguez 1994; Harmon et al. 1995; Hjellming et al. 1998), their high brightness temperatures, and high degree of polarisation (Fender 2006). Figure 1.9 shows the most recent compilation (Corbel et al. 2013) of radio and X-ray observations of BHBs, in which some sources (GX 339–4, XTE J1118+480 and V404 Cygni) track the slope of the correlation during multiple outbursts. The correlation is a power law given by $L_R = L_X^m$, where $L_R$ is the radio luminosity of the jet radio core in the 5 GHz band, and $L_X$ is typically measured across the 2–10 keV band. The index of the correlation is constrained to $m \sim 0.6–0.7$. This scaling can be interpreted at the very least as an energetic connection between the radiatively inefficient X-ray emitting regions where $L_X \propto \dot{m}^2$, and the outer radio-emitting regions of the jet (e.g., Markoff et al. 2003; Falcke et al. 2004). Such a tight correlation implies a physical connection between the inner regions of the accretion flow and the jet. However we still do not fully understand this connection, due largely to the degeneracy in models of the X-ray spectra of BHBs, and the difficulties involved in modelling broadband spectral observations. Multiwavelength observing campaigns of transient BHBs across radio/IR/optical/X-ray bands (e.g. GX 339–4; Hannikainen et al. 1998; Corbel et al. 2000, 2003; Russell et al. 2006; Gandhi et al. 2008, 2010, 2014; XTE J1118+480; Markoff et al. 2001a) have shown that the optically-thick synchrotron spectrum of the jet extends from radio down to IR frequencies, with a turnover before the optical bands. In addition to this, the contribution of optically-thin synchrotron emission in the jets of transient BHBs may contribute significantly to the optical emission in the hard state. However, the extent of the jet’s contribution in the X-ray is still uncertain, and difficult to distinguish from IC emission from a static corona or inner accretion flow. Markoff et al. (2001a), Markoff et al. (2005) and Maitra et al. (2009) present broadband semi-analytic jet modelling of simultaneous multiwavelength BHB observations and show that not only can the spectrum be fully

the corona the hot electrons inverse Compton (IC) scatter photons from the accretion disc (Haardt & Maraschi 1993; Stern et al. 1993; Dove et al. 1997). The electrons are hot, typically at temperatures of $T_e \sim 100$ keV (e.g., Ichimaru 1977; Sunyaev & Titarchuk 1980; Dove et al. 1997; Ibragimov et al. 2005), producing an approximate power law spectrum with a cut-off, where the photon index ($\Gamma \sim 1.6–2$) depends on the optical depth ($\tau$) and $T_e$. 
1.4 Black hole X-ray binaries

Figure 1.9: Radio vs. X-ray luminosity of hard state BHBs, adapted from Corbel et al. (2013). Most hard state BHBs follow the inefficient track given by $L_R \propto L_X^m$, where $m \sim 0.6–0.7$. Some sources deviate from this inefficient track and appear to follow a track similar to the more efficient X-ray emitting soft states.

explained by jet emission alone, but the radio/X-ray scaling relations are naturally predicted by such models.

1.4.3 Accretion models

The dynamics of optically-thick, geometrically thin accretion discs observed in the hard state of BHBs was characterised in depth in the early 1970s by several works (Pringle & Rees 1972; Shakura & Sunyaev 1973; Novikov & Thorne 1973), who all attempted to describe their structure and resultant spectrum, multi-temperature black-body radiation. Such a spectrum emerges due to the optically-thick nature of the disc, and the scaling of gas temperature with disc radius, $T(R) \propto R^{-3/4}$. This temperature scaling is derived intuitively from the gravitational potential of the black hole, and the condition that gas can rid itself of angular momentum. Shakura & Sunyaev (1973) first characterised the mechanism for momentum transport in the disc via the $\alpha$-viscosity parameter, such that viscosity is given by $\nu = \alpha c_s H$ where $H$ is the scale height of the disc, and $c_s$ is the sound speed of the gas. The obvious question that
should spring to the reader’s mind is, what is the nature of this viscosity, and how
does it arise? The proposed mechanism which withstands the most scrutiny is that
of magnetohydrodynamic (MHD) turbulence (though kinetic turbulence is likely a
non-negligible contributor). MHD turbulence was postulated as being an important
driver of dissipation in rotational fluids immersed in a poloidal field by Balbus &
Hawley [1991], inspired by the earlier works of Velikhov [1959] and Chandrasekhar
[1960, 1961]. The weak field instability described by Balbus & Hawley [1991] is
known as the magnetorotational instability (MRI), which can grow in differentially
rotating discs in the presence of a weak poloidal magnetic field. The strength of the
turbulence does not explicitly depend on the field strength, only the condition that
thermal energy density dominates over magnetic energy density.

The story of accretion disc theory branched out into different avenues in the
decades following the breakthrough of Shakura & Sunyaev [1973]. It was first no-
ticed by Lightman & Eardley [1974] and Shapiro et al. [1976] that the standard
thin disc solutions are actually unstable, the former proposing a thick inner region
of low density within $\sim 100 r_g$ of the black hole, and the latter complementing the
former by prescribing such a solution to the fluid equations. In an attempt to explain
the observed low radiative efficiencies of accreting black holes (the main proponent
being Sgr A*, the Galactic centre SMBH) Narayan & Yi [1994, 1995a,b] showed
that the solutions presented by Shapiro et al. [1976] are thermally unstable, and sta-
ble solutions were produced in which not only are the ions and electrons decoupled
($T_{\text{ion}} > T_e$), but the gas can become unbound, meaning such accretion flows may pro-
duce outflows. The key aspect of this solution which differentiates it from the previous
unstable solutions of Shapiro [1973] is the advective nature of the gas: the flow has
a dominant radial component close to the black hole, such that accretion timescales
become much shorter than cooling timescales. These solutions were thus dubbed ad-
vection dominated accretion flows (ADAF), and provided a natural explanation for
the inefficient X-ray emission in the hard state of BHBs. Further models for accre-
tion flows with similar properties have been presented to explain inefficient accretion
flows. Blandford & Begelman [1999] found accretion flow solutions in which the bulk
of the accreting gas is removed from the system via outflows (advected-dominated
inflow/outflow solutions; ADIOS), and Quataert & Gruzinov [2000] explored solutions
whereby convection drives angular momentum inwards as opposed to outwards
(convection-dominated accretion flow; CDAF). This class of models, which I shall now
refer to as radiatively-inefficient accretion flows (RIAFs) have become instrumental to
our understanding of compact objects accreting at low rates (low $\dot{M}$). Another key
point one should remember about RIAFs is their similarity to the spherical accretion
flows first investigated in the 1940s (Hoyle & Lyttleton [1939], Bondi & Hoyle [1944];
Bondi [1952]), in that the flow is mostly radial and radiatively inefficient. The en-
lightening developments then are to show that 1) such solutions may naturally occur
in accreting systems with inherently high initial specific angular momentum, i.e. in
1.4 Black hole X-ray binaries

Several mechanisms have been proposed to explain how jets are launched and why their energetics are so closely tied to the X-ray emitting regions. The two earliest classes of models came from Blandford & Znajek (1977) and Blandford & Payne (1982), the former proposing the black hole spin as the reservoir of angular momentum, and latter positing that this reservoir originated in the accretion flow itself, both requiring magnetic fields to channel this angular momentum into an outflow. Extensions to these models do of course exist. Meier (2001) and Livio et al. (2003) proposed models in which the jet power depends on the poloidal field anchored into the accretion flow, showing that geometrically thick discs favour the channelling of magnetic energy into outflows. This scenario agrees with models for the inner accretion flow of BHBs in the hard state (Esin et al. 1997). Tagger & Pellat (1999) and Varnièrè & Tagger (2002) proposed a slightly different model in which instabilities in the magnetic-dominated accretion disc channels angular momentum vertically into the jet.

1.4.4 X-ray reflection spectroscopy and black hole spin

Until now I kept the discussion of the observations of BHBs to direct emission from the accretion disc or inner regions of the accretion flow (either within a RIAF, corona, or jet). In fact the high-energy, non-thermal, power law X-rays emanating from the inner regions irradiate the thin accretion disc. Due to the ionisation levels in the accretion disc, the emergent spectrum contains fluorescent emission-line features, and a characteristic Compton hump due to the down-scattering of the high-energy X-rays by disc free electrons. A prominent Iron-$K_\alpha$ emission line appears in the X-ray spectrum between 6–7 keV, and if the accretion disc lies close to the ISCO where the gravitational effects of the black hole are significant, the line will show up in the spectrum with a relativistically-broadened red wing. Calculations of disc reflection in accreting systems were first introduced by Lightman & Rybicki (1980) and Lightman et al. (1984), and have since evolved to highly sophisticated reflection models that allow self-consistent calculation of the photoionisation in the disc, Compton scattering, and the full relativistic broadening effects on both the incoming and outgoing radiation (Ross et al. 1978; Ross 1979; Ross & Fabian 1993, 2005; García & Kallman 2010; García et al. 2011, 2013, 2015b; Dauser et al. 2010, 2013, 2014), with angular dependence included.

Reflection studies are powerful because the emergent spectrum depends heavily on the spin of the black hole. Reflection models are used to derive the spin of black holes, and the location of the inner edge of the accretion disc, in both AGN (e.g., Risaliti et al. 2013; Marinucci et al. 2014; Walton et al. 2014) and BHBs (e.g., Steiner et al. 2011; García et al. 2015b; Walton et al. 2017) by modelling their X-ray spectra,
and advanced high-resolution telescopes such as *NuSTAR* (Harrison et al. 2013) are improving the constraints on key reflection model parameters. Nonetheless, reflection spectroscopy has similar drawbacks to models of coronae and jets, in that we cannot be sure we are capturing the system geometry accurately. Reflection models must assume an emissivity profile of irradiation as a function of accretion disc radius, and this is closely tied to the geometry of the irradiator. Assumptions regarding the structure of the X-ray irradiating source, i.e., compact in the form of a ‘lamppost’ (Martocchia & Matt 1996), or extending vertically along the jet axis, produce key differences in the observed iron-line profile as a function of black hole spin (see, e.g., Dauser et al. 2013). In addition, the high-energy irradiating X-ray flux drives strong photoionisation effects in the low-energy portion of reflection spectra (Ross & Fabian 2005; García & Kallman 2010; García et al. 2015a). For this reason, in any observation of BHBs or AGN in which reflection features are observed, one must treat the reflection and irradiating spectra self-consistently in order to constrain the high-energy cut-off in the power law spectrum. This reinforces the need to treat all radiation components within the inner regions and jets of hard state BHBs, and LLAGN. We should look to determine the fractional contribution from X-ray-emitting components in order to get better constraints on reflection-model parameters. These improvements could lead to better measurements of black hole spin. I present our progress in attempting to develop more self-consistent models of disc reflection of irradiating jets in Chapter 5.

### 1.5 The fundamental plane of black hole activity

The radio/X-ray correlations observed in BHBs were shown to extend to low-luminosity classes of AGN (LLAGN) by Merloni et al. (2003) and Falcke et al. (2004) using large samples of AGN and BHBs (Merloni et al. 2003 adopt a sample of ∼100 AGN comprising Liners, Seyfert 1s, 2s, Narrow-Line Seyfert 1s, quasars, and BL Lacs, Falcke et al. 2004 instead include Liners, FR Is and BL Lacs), and the resultant relation between radio luminosity, $L_R$, X-ray luminosity, $L_X$, and black hole mass, $M_{BH}$, is known as the Fundamental Plane of Black Hole Activity (from here on, FP). Heinz & Sunyaev (2003) presented a model showing that jets emitting only synchrotron can naturally produce the observed scaling law. Heinz & Sunyaev (2003) remove all the model-dependence from the expression for the flux $F_\nu$ by using a mathematical description for any dynamically relevant quantity of the jet, such as magnetic field and electron density. The formalism leads to a model-independent scaling of the flux with mass, $F_\nu \propto M^{\xi_M}$, where $\xi_M \sim 17/12 - \alpha/3$ (for both ADAF and standard thin disc accretion modes), and $\alpha = (p - 1)/2$ where $p$ is the power law index of the electron distribution. Observationally the FP implies that the X-ray and radio emitting regions are energetically linked consistently across black holes accreting at low luminosities,
1.5 The fundamental plane of black hole activity

**Figure 1.10:** The FP, showing the radio and X-ray luminosities of BHBs and LLAGN as a 2D plane in 3D space where the masses of the accreting black holes are the 3rd variable (Plotkin et al. 2012). The red triangles in the bottom left of the relation are BHBs in their hard states, in which their X-ray spectra are power law-dominated. The AGN on the relation toward the top right include LLAGN, FR I galaxies, and BL Lacs, all coincident with radio jets, showing a characteristic flat-to-inverted self-absorption spectrum in the radio bands.

independent of their masses. This relation then adds weight to a canonical connection between the accretion disc and jet of an accreting black hole. Since for scale invariant models the mathematical formalism adopted by Heinz & Sunyaev (2003) to achieve the coefficients of this relation is model independent (though they still depend upon the accretion model, since the normalisation is still an undetermined variable), any scale invariant jet model applied to these sources must obey this relation. This proves quite powerful, since any simple scale invariant model can in principle provide a lot of information in regard to the connection between different sources.

Merloni et al. (2003) note that despite there being a clear FP relation, the scatter is very large and likely due to a number of factors, for example, the variation in the radio spectral index $\alpha_R$, the intercept of the plane (model used), and the calculation of relativistic beaming in the sources, particularly because BL Lac objects are heavily beamed due to their low inclination. They also find, in contrast to Falcke et al. (2004), that the X-ray emission in this scaling relation is best explained by radiatively inefficient accretion flow (RIAF) models, rather than optically-thin synchrotron emission, explaining that the spectral slope of synchrotron would have to be too high ($p \geq 2.5$). Körding et al. (2006a) further refine the FP by attempting to improve the parameter estimation and estimating the scatter around the plane. By estimating the scatter whilst alternating the inclusion of objects in the low/high luminosity states, Körding et al. (2006a) show that hard state objects fit on the plane with low levels of scatter, consistent with the errors, and can be explained by an uncooled synchrotron model,
1 Introduction

whereas the inclusion of higher luminosity sources requires a disc component as well as the jet spectrum. Such a conflict in findings argues strongly for multiwavelength spectral fitting campaigns that can reveal how spectral components are reconciled with one another.

Advances in the robustness of the FP have been made by adopting only dynamically-measured masses of the SMBHs of LLAGN in the samples (Gültekin et al. 2009). The most recent work on FP statistics is presented by Plotkin et al. (2012), outlining the Bayesian technique (Kelly et al. 2011) they use to regress the FP in order to better constrain the coefficients of the relation and to determine which mechanisms could best explain the X-ray emission. Optically-thin synchrotron radiation from a jet can explain the relation found, with X-ray from a RIAF excluded at 3σ significance. On top of this Plotkin et al. (2012) discuss the need to consider synchrotron cooling of jet emission from the highest mass black holes to avoid biasing the FP relation. The empirical evidence available to us therefore points to a clear connection between radio/X-ray emission across the black hole mass scale, and this is something we can explore directly with models capable of capturing their broadband spectra.

Markoff et al. (2015) conducted a test of the FP by making a direct comparison between a BHB in the low/hard state and a LLAGN, V 404 Cygni ($M_{\text{BH}} = 12 M_\odot$; Shahbaz et al. 1994) and M 81* ($M_{\text{BH}} = 7 \times 10^7 M_\odot$), both sources that fall on the FP relation. They show that the principle of scale-invariance of the jet physics in both sources holds, and the fundamental properties of the jets that drive their radiative outputs can scale surprisingly well with mass. These properties include the equipartition of energy between radiating particles and magnetic field, the size scales of the jet (in units of $r_g$), and the power law index of accelerated particles. Building on such comparisons may lead to a better understanding of precisely which properties of accretion and jets in both BHBs and LLAGN are actually scale-invariant, and this will allow us to start building a more quantitative picture of what is driving the evolution of AGN over long timescales.

1.6 Dark matter and primordial black holes

DM is shown to comprise $\sim 85\%$ of matter in the universe (see, e.g., Bertone 2010 and references therein), but its nature has been elusive to astronomers and particle physicists up to now (see, e.g., Jungman et al. 1996; Bertone et al. 2005; Bertone 2010). Multiple candidates have been proposed for what constitutes DM, and some of the leading candidates include those of a particle nature such as weakly-interacting massive particles (Jungman et al. 1996, WIMPs), Axions (Visinelli & Gondolo 2009, Sikivie 2010), and sterile neutrinos (Shaposhnikov 2010; Boyarsky et al. 2009), and larger scale candidates, i.e. massive compact halo objects (MACHOs), such as PBHs (Carr & Hawking 1974; Meszaros 1973; Carr 1975). Despite the preference within
1.6 Dark matter and primordial black holes

those searching for DM to focus on WIMP-like candidates, the idea that PBHs could in fact constitute a significant fraction of the DM is still actively pursued (Ivanov et al. 1994; Khlopov 2010; Carr et al. 2010; Blais et al. 2002; Afshordi et al. 2003; Frampton et al. 2010). In the context of the discovery of binary black hole mergers comprised of preferentially high-mass black holes by LIGO (Abbott et al. 2016b,a, 2017), it has been proposed that LIGO is probing the much-sought-after mass window for the existence of PBHs (Bird et al. 2016). The arguments made by Bird et al. (2016) are based on a lack of exclusionary constraints on PBHs between 10–100 $M_\odot$. Current constraints on the fraction of DM in the form of PBHs come from microlensing (Alcock et al. 2001), halo wide binaries (Quinn et al. 2009; Monroy-Rodríguez & Allen 2014), the survival of central star clusters in faint dwarf galaxies (Brandt 2016; Li et al. 2016), and signatures in the cosmic microwave background (CMB) (Ricotti et al. 2008; Clesse & García-Bellido 2016; Chen et al. 2016).

Attempts to associate the imprint on CMB anisotropies with accretion onto PBHs (Ricotti et al. 2008) are compromised by our lack of understanding of the dynamics of accretion flows in early-universe conditions. As such it may be more informative to look for present-day signals of the emission from accreting PBHs. We present a method for attempting to detect PBHs in our own Galaxy in Chapter 6.

1.6.1 Jet modelling

Chapters 2, 3, 4, and 5 of this thesis centre on the implementation and use of agnjet (Markoff et al. 2005; Maitra et al. 2009), a semi-analytical, multi-zonal, model of a mildly-relativistic jet, in modelling the broadband spectra of BHBs and LLAGN. The respective chapters give sufficient details on the assumptions, dynamics, radiative processes, and model parameters associated with agnjet, however it is useful at this point to show how the radiative processes discussed in Section 1.3.2 contribute to the broadband emission observed in BHB and LLAGN jets. Figure 1.12 shows the observed broadband spectrum of agnjet for an arbitrary selection of model parameters. This model has been shown to explain simultaneous broadband spectra of BHBs and LLAGN to remarkable accuracy despite the huge differences in black hole mass (Markoff et al. 2005, 2008; Maitra et al. 2009; Prieto et al. 2016). As I show in this thesis, we have made further steps in improving the finer details of agnjet and modelling techniques to obtain further constraints on the X-ray-emitting regions in BHBs and LLAGN.
Figure 1.11: The various constraints placed on the fraction of DM comprised of PBHs, taken from (Clesse & García-Bellido 2015). Constraints come from femtolensing of gamma-ray bursts (Jedamzik & Niemeyer 1999), the extragalactic gamma-ray background (Carr et al. 2010), microlensing with the EROS and MACHO projects (Alcock et al. 1998; Tisserand et al. 2007), and neutron-star capture rates (Capela et al. 2013). One can see a mass window
1.6 Dark matter and primordial black holes

Figure 1.12: The observed broadband spectrum produced by the agnjet model, used in modelling simultaneous observations of hard state BHBs and LLAGN. The black long-dashed line shows a multitemperature disc blackbody spectrum from a disc with a parameterised inner-disc temperature and radius, and an outer-disc radius, following an $R^{-3/4}$ temperature profile. The blue long-short-dashed line shows an optically-thin thermal synchrotron emission from a Maxwell-Jüttner distribution of relativistic electrons. Both the disc and thermal synchrotron photons are seeds for multiple IC scattering in the jet, shown with the green short-dashed line (though we label the component as SSC as the predominant seed photons are the synchrotron photons in its own rest frame). The red short-short-long dashed line shows post-particle-acceleration synchrotron emission, with an optically-thin power law portion, progressing into a self-absorbed flat-to-inverted radio spectrum. The thin black line shows the total spectrum of the jet + disc.
Mass-scaling as a method to constrain outflows and particle acceleration from low-luminosity accreting black holes

Never be a spectator of unfairness or stupidity. Seek out argument and disputation for their own sake: the grave will supply plenty of time for silence.
—Christopher Hitchens


Abstract

The ‘fundamental plane of black hole accretion’ (FP), a relation between the radio luminosities \(L_R\), X-ray luminosities \(L_X\), and masses \(M_{BH}\) of hard/quiescent state black hole binaries and low-luminosity active galactic nuclei, suggests some aspects of black hole accretion may be scale invariant. However, key questions still exist concerning the relationship between the inflow/outflow behaviour in the ‘classic’ hard state and quiescence, which may impact this scaling. We show that the broadband spectra of A0620−00 and Sgr A* (the least luminous stellar mass/supermassive black holes on the FP) can be modelled simultaneously with a physically-motivated outflow-dominated model where the jet power and all distances are scaled by the black hole mass. We find we can explain the data of both A0620−00 and Sgr A* (in its non-thermal flaring state) in the context of two outflow-model scenarios: (1) a
Using mass-scaling to study accretion

synchrotron-self-Compton dominated state in which the jet plasma reaches highly sub-equipartition conditions (for the magnetic field with respect to that of the radiating particles), and (2) a synchrotron dominated state in the fast-cooling regime in which particle acceleration occurs within the inner few gravitational radii of the black hole and plasma is close to equipartition. We show that it may be possible to further discriminate between models (1) and (2) through future monitoring of its submm/IR/X-ray emission, in particular via time lags between the variable emission in these bands.
2.1 Introduction

Accreting black holes span an enormous range of masses, from black holes in stellar X-ray binaries (BHBs) of a few $M_\odot$ to active galactic nuclei (AGN) harbouring supermassive black holes (SMBH) ranging from $\sim 10^6$–$10^{10} M_\odot$. The accretion physics of BHBs has been extensively studied, and their accretion evolution is well characterised by a disc instability model (see Lasota 2001 for a review). Observationally, BHBs (high mass and low mass alike) are classified via particular ‘states’ based on their spectral and timing properties, of which there are many (e.g. Nowak 1995, Gierliński & Done 2003, Remillard & McClintock 2006b, Belloni 2010), but the two longest-lived and thus ‘canonical’ states are the so-called ‘hard’ and ‘soft’. A basic underlying definition can be given to the hard and soft states of BHBs, wherein hard refers to an X-ray spectrum dominated by higher energies ($> 10$ keV) and a non-thermal power-law spectrum, and soft is dominated by lower X-ray energies (2–10 keV) and a thermal blackbody spectrum. An idea currently under exploration concerns observational comparisons between AGN and BHBs that point to an identification of some AGN classifications with BHB states (e.g. Körding et al. 2006b).

Whilst the classic thermal-blackbody thin accretion disc component provides a good model representation of the soft state (Shakura & Sunyaev 1973), there is not yet an agreed upon paradigm for the hard state. This latter situation is well demonstrated in the case of Cyg X-1 (a well-studied BHB with a high mass companion star), wherein multiple models with different inflow/outflow geometries (thermal/non-thermal coronae, jets) are all capable of explaining the observed hard-state X-ray spectrum (Nowak et al. 2011): spectral modelling of BHBs in the hard state is degenerate. Furthering our understanding of the accretion, ejection and radiative processes of BHBs thus calls for novel methods to strip out this spectral fitting degeneracy and determine how gravitational energy is re-distributed in the hard state.

Observations of BHBs at both radio and X-ray wavelengths during the hard state, which we define now as sources having an Eddington scaled X-ray luminosity ($L_X$) in the range $L_X/L_{\text{Edd}} \equiv l_X \sim 10^{-6}$–$10^{-2}$, reveal a correlation between the respective luminosities, $L_X \propto L_R^{0.6}$–$0.7$ (Corbel et al. 2000, 2003; Gallo et al. 2003; Corbel et al. 2008, 2013; see also Miller-Jones et al. 2011 and Gallo et al. 2014). This scaling relation indicates a coupling between the radio/X-ray emission mechanisms during the hard state, pointing to a connection between the accretion flow and the jet—since radio emission in BHBs is identified with a steady compact jet, as directly imaged in BHBs GX 339-4, Cyg X-1, and GRS 1915+105 (Fender 2001; Stirling et al. 2001; Miller-Jones et al. 2005). This scaling relation also presents the possibility of breaking model degeneracies (discerning the dominant spectral components in hard

$$L_{\text{Edd}} = 4\pi GMm_p c / \sigma_T = 1.25 \times 10^{38} (M/M_\odot) \text{ erg/s},$$
where $G$ is the gravitational constant, $m_p$ is the proton mass, $c$ is the speed of light, $\sigma_T$ is the Thomson cross-section, $M$ is the black hole mass, and $M_\odot$ is the mass of the Sun.
Using mass-scaling to study accretion

This concept of scaling has broader implications when we compare these hard state BHBs with AGN showing similar compact jets, since a common scaling would imply the discovered correlation (and thus inflow/outflow coupling in these particular states) is independent of black hole mass. It has been shown that when one includes low-luminosity AGN with jet cores (low-luminosity AGN (LLAGN): including LINERS, FR1, and BL Lacs), the correlation extends to the so-called Fundamental Plane of Black Hole Accretion (FP), relating the X-ray luminosities, \( L_X \), radio luminosities, \( L_R \), and masses, \( M_{BH} \) of the selected LLAGN and hard-state BHBs (Merloni et al. 2003; Falcke et al. 2004; Körding et al. 2006a; Plotkin et al. 2012). Efforts to derive these scaling laws begin by expressing all luminosities in terms of their dependence on mass and mass-accretion rate (expressed in mass-scaling Eddington units \( \dot{m} = \dot{M} / \dot{M}_{\text{Edd}} \), where \( \dot{M}_{\text{Edd}} = L_{\text{Edd}} / \left(0.1c^2\right) \)). For example, it can be shown through full calculation of the scaling relations that in order to satisfy the observed correlation, weakly accreting black holes must be radiating inefficiently (\( L \propto \dot{m}^q \), where \( q \approx 2 \)) (Markoff et al. 2003; Heinz & Sunyaev 2003; Plotkin et al. 2012); which includes synchrotron, inverse Compton and bremsstrahlung processes.

There are prevalent difficulties with attempts to distinguish between these allowed radiative processes, and thus accretion models, capable of reproducing this \( L \propto \dot{m}^2 \) dependence, due primarily to the degeneracy in spectral modelling. For instance various radiatively inefficient accretion flow (RIAF) models (Narayan & Yi 1994; Yuan et al. 2003) and outflow models (Yuan et al. 2002, 2003) have the inefficient (\( q \approx 2 \)) scaling predicted by the FP. In addition to broadband spectral modelling, degeneracies can be disentangled by introducing further observational data, such as X-ray variability studies (van der Klis 1995; Remillard & McClintock 2006b), broadband variability studies (Casella et al. 2010; Cantrell et al. 2010; Kalamkar et al. 2016), variability comparisons of AGN and BHBs (Uttley & McHardy 2003; McHardy et al. 2006), and polarisation measurements (Shahbaz et al. 2008; Russell & Fender 2008). Sgr A*, the SMBH at the Galactic centre, is a prime example of how combining these individual diagnostics leads to a better physical interpretation of the emission mechanisms (Bower et al. 2003; Genzel et al. 2003; Ghez et al. 2004; Eckart et al. 2006; Marrone et al. 2006; Witzel et al. 2012; Neilsen et al. 2015; Li et al. 2015; Dibi et al. 2016). However, even with all such techniques, degeneracies still exist in the physical interpretation of the hard X-ray emission mechanisms of hard state BHBs and LLAGN. Markoff et al. (2015) attempt a new approach in breaking the degeneracy between the SSC and synchrotron-dominated scenarios, testing the extent to which the scale invariance implied by the FP holds. They jointly model the broadband spectral energy distributions (SEDs) of two black holes on opposite ends of the mass scale (the LLAGN M81*, and the BHB V404 Cyg in a low-luminosity hard state), accreting at similar Eddington rates (\( l_X \sim 10^{-6} \)). The same model with half of the fitted parameters at the same value (in mass-scaled units) provides a good fit to both
sources, and a model in which synchrotron emission dominates the high-energy spectra provides the most reasonable fit. Now that a proof-of-concept has shown the method of joint spectral modelling provides physical insight into black holes across the mass scale, we want to extend the study to quiescence \((l_X < 10^{-6})\). Is quiescence a direct continuation of the hard state, or does the physics change below some accretion rate, as indicated by the increase in the X-ray power-law spectral index (Kong et al. 2002b; Tomsick et al. 2003, 2004; Corbel et al. 2006, 2008; Plotkin et al. 2013)? Plotkin et al. (2013) model the broadband spectrum of BHB XTE J1118+480 in its quiescent state and compare to previous modelling of its hard state emission (Maitra et al. 2009), showing that the transition from the hard to quiescent state of XTE J1118+480 may be characterised by a decrease in particle acceleration efficiency (see e.g. Markoff 2010b)—it is also interesting to note that the radio/X-ray correlation slope of XTE J1118+480 is consistent with those of other sources on the trend over 5 dex in \(l_X\). Here we adopt the method presented in Markoff et al. (2013), fitting an outflow-dominated model (the details of which can be found in Markoff et al. 2005 and Maitra et al. 2009, from here on MNW05 and M09 respectively) to two black holes deep in quiescence yet on opposite ends of the mass scale; quiescent \((l_X \sim 10^{-8.5})\) BHB A0620−00 and SMBH Sgr A* \((l_X \sim 10^{-9})\) during bright, non-thermal flares.

In Sections 2.2 and 2.3 we give an overview of the theoretical and observational history (and therefore the source properties determined to date) of Sgr A* and A0620−00 respectively and a brief description of the data used in our modelling. In Section 4.3 we describe the model (including the updates made to the model in Sections 2.4.1 and 2.4.2) we apply to both sources. In Section 4.4 we describe the methodology behind fitting the broadband spectra. In sections 2.6 and 2.7 we present results of individual fits to both sources, as well as the new joint fits. In section 2.8 we discuss which of our model scenarios are most plausible when applied to both sources, and posit possible future observations and work.

### 2.2 Sgr A*

Our own galaxy harbours an extremely weakly accreting SMBH, Sgr A*, that during intermittent non-thermal flaring seems to fit the criteria of a source in the universally regulated state associated with the FP (see Melia & Falcke 2001, Markoff 2005, Genzel et al. 2010, Markoff 2010b and Yuan & Narayan 2014 for full reviews on the features of Sgr A* and the Galactic centre). Sgr A* has a mass of \(4.1 \times 10^6 M_\odot\) (Genzel et al. 2003; Ghez et al. 2008; Gillessen et al. 2009) and lies at a distance of 8 kpc (Reid 1993; Ghez et al. 2008; Reid et al. 2009). The unabsorbed X-ray (2–10 keV) luminosity of Sgr A* during quiescence is a few times \(10^{33} \text{ erg s}^{-1}\) (Baganoff et al. 2003) or \(l_X \sim 10^{-11}\), making it the most weakly accreting black hole observed
to date. Wang et al. (2013) present the results of 3 Msec of Chandra X-ray Observatory imaging of the Galactic centre as part of an X-ray Visionary Project (see http://www.sgra-star.com), resolving the accreting gas around Sgr A* during quiescence. The results confirm that the steady quiescent spectrum of Sgr A* can be fit with a thermal bremsstrahlung model from a hot plasma near the Bondi radius, consistent with earlier predictions by e.g. Narayan et al. (1995); Quataert (2002).

Frequent X-ray monitoring of Sgr A* also resulted in the discovery of flares (Baganoff et al. 2001) lasting as long as 10 ks with a peak luminosity $\sim 50\times$ brighter than the quiescent emission, with the flare emission best fit by a power law with a significantly harder spectrum than that detected in quiescence ($dI_\nu/d\nu = \nu^{-\alpha}$, $\alpha \sim 0.3$, with $\alpha \sim 1.2$ in quiescence). Subsequent observations of the flare emission have found peak luminosities reaching $130\times$ (Nowak et al. 2012) and $400\times$ (Haggard et al., in prep.) higher than the quiescent level, originating from much smaller radii than the quiescent emission. Attempts to model the variable (flare) emission of Sgr A* now include jet models capable of producing a synchrotron + inverse Compton (in particular SSC) (Falcke & Markoff 2000; Markoff et al. 2001b), and hybrid models including RIAF components, both thermal (Yuan et al. 2002) and with a non-thermal population of particles (Yuan et al. 2003). The 3 Msec of additional observing time with Chandra allowed the first detailed observations of the flaring emission, doubling the population of known flares within a year (Neilsen et al. 2013). These flares range in duration from a few 100 seconds to 8 ks, and in luminosity from $\sim 10^{34}$ erg s$^{-1}$ to $2 \times 10^{35}$ erg s$^{-1}$, bringing Sgr A* to fluxes consistent with the FP relation (in quiescence Sgr A* lies $\sim 2$ orders of magnitude in $L_X$ below the FP relation). The timescales of the flares indicate an emission region of 5–400 $r_g$, though large excursions from quiescence likely originate from within $\approx 5 \times r_g$ (Barrière et al. 2014).

We are interested in modelling Sgr A* during bright X-ray flares (when Sgr A* approaches the FP relation), and thus we select the 3 brightest X-ray flares, with peak count rates 0.15–0.25 cts/s, whereby this grouping of flares contains sufficient cumulative counts for us to perform $\chi^2$ statistics—see Figure 2.1. The details of the Chandra observations and data reduction can be found in Neilsen et al. (2013), and in section 2.5.1 we detail how the spectra are binned/grouped and subsequently modelled.

X-ray flares observed from Sgr A* are coincident with an IR counterpart, but the opposite is not always true (Eckart et al. 2006; Hornstein et al. 2007). This characteristic hints at the nature of the physical connection between the IR/X-ray emission, however lack of coverage combined with uncertainties regarding the IR flux distribution make simultaneous modelling a difficult task (Dodds-Eden et al. 2011; Trap et al. 2011; Witzel et al. 2012). We thus select the median IR H and ks-band fluxes found by Bremer et al. (2011), $3.61 \pm 1.62$ and $6.03 \pm 1.85$ mJy respectively, and the mid-IR 3$\sigma$ upper limit of 58 mJy found by Haubois et al. (2012). Using the median NIR fluxes allows us to somewhat represent the flux uncertainties during the brightest X-ray flares, whilst the mid-IR upper limit allows us to put prior constraints on our
2.3 A0620$-$00

Figure 2.1: The splitting of X-ray flares into 3 categories of peak count rate. The plot show how the 39 X-ray flares (Neilsen et al. 2013) are divided into 3 sections, based on CF levels $\leq 1200$ (red lines), $1200 \leq CF \leq 2200$ (green squares), and $CF > 2200$ (blue circles).

model parameters (since the thermal synchrotron spectrum cannot exceed this upper limit).

We require a quasi-simultaneous broadband spectrum to perform time-independent modelling. Sgr A* only becomes significantly variable (up to $\sim 40\%$) at submm wavelengths, when the emitting region is optically thin (Lu et al. 2011; Bower et al. 2015), though there has been a rise in flux of $\sim 20\%$ in the 5–20 GHz range over the past decade (An et al. 2005; Bower et al. 2015), as shown in Figure 2.2. We compile an average radio-to-submm spectrum that encompasses this short and long term variability, with appropriate coverage across the 330 MHz - 850 GHz range. The resulting data table can be found in Table 2.5 in the Appendix.

2.3 A0620$-$00

First discovered at X-ray wavelengths when it went into outburst in 1975 (Elvis et al. 1975), A0620$-$00 (hereafter A0620) settled into quiescence 15 months later. It has been in a persistent quiescent state since then, and we now know that the system consists of a K-type donor star transferring mass to a black hole via an accretion disc (McClintock & Remillard 1986). The mass, distance, and orbital inclination ($i$) are found by Cantrell et al. (2010) to be $6.6 \pm 0.25 \, M_\odot$, $1.06 \pm 0.12 \, \text{kpc}$ and $51.0^\circ \pm 0.9$ respectively. Garcia et al. (2001) and Kong et al. (2002b) find a quiescent X-ray lumi-
Figure 2.2: The radio to submm spectrum of Sgr A* as observed over the past 20 years. The key shows the observing windows of the following works in order from top to bottom; Zhao et al. (2001); Serabyn et al. (1997); Zylic et al. (1995); Nord et al. (2004); Falcke et al. (1998); Roy & Pramesh Rao (2004); An et al. (2005); Lu et al. (2011); Brinkerink et al. (2015); Bower et al. (2015). The top panel shows the mean flux density as a function of frequency ranging from 330 MHz to 850 GHz. The bottom panel shows the fractional uncertainty (uncertainty/flux) of each flux measurement. The dotted line shows the boundary below which measurement uncertainty due to the scattering screen of electrons along the line of sight starts to dominate the intrinsic variability of Sgr A*.

A0620 has an 8.5 GHz radio flux density of \(51 \pm 7 \, \mu Jy\) (Gallo et al. 2006), interpreted as self-absorbed synchrotron emission from a jet/outflow. Comparison of the radio/X-ray flux confirms A0620 as the lowest-luminosity source on the FP. Mid-IR detections suggest the self-absorbed synchrotron emission extends up to the mid-IR given the flat spectral index between radio-mid-IR, though a circumbinary disc component cannot be ruled out (Muno & Mauerhan 2006; Gallo et al. 2007).

Froning et al. (2011) present a broadband spectral energy distribution (SED) including X-ray, UV, optical, NIR, and radio observations of A0620, adding to the already existing broadband coverage (Narayan et al. 1996; McClintock & Remillard 1998).
2.4 The model

Through modelling of the broadband spectrum, Froning et al. (2011) show that 90% of the disc mass is lost between the outer and inner accretion flow, indicative of either an outflow prior to capture by the black hole (an ADIOS-like solution, e.g. Blandford & Begelman 1999, 2004), or mass loss to the black hole (an ADAF-like solution, e.g. Narayan & Yi 1994). Wang et al. (2013) find a similar accretion disc density profile explains the low accretion rates onto Sgr A*, providing a further analogy between black holes of varying mass accreting at very sub-Eddington rates. Since A0620 extends the radio/X-ray correlation down to the most quiescent luminosities, and we know that brighter hard-state sources show a radio jet, it seems reasonable to assume the presence of a jet at the lowest luminosities.

We model the full radio-to-X-ray quasi-simultaneous spectrum of A0620. This includes the simultaneous radio/IR/optical/X-ray observations taken in August 2005 presented by Gallo et al. (2006), the IR observations taken 5 months prior in March 2005 by (Gallo et al. 2007), and full IR/optical/UV observations taken by Froning et al. (2011) in March 2010. We refer the reader to the relevant observational papers for a full description of the data reduction and analysis. Combining these datasets gives us good coverage from radio to X-ray frequencies whilst accounting for the optical/UV variability of A0620 during its “active” state (Cantrell et al. 2008). Although this results in added $\chi^2$ residuals in our fits, it is more informative to include this variability and have a representative time-averaged spectrum. We also note that the constraints that come from fitting across 8 orders of magnitude in spectral energy outweigh the residuals accrued by modelling data over these two epochs (see e.g. Markoff et al. 2008). We deredden the IR-FUV fluxes with $E(B-V)$ = 0.39 in agreement with Gallo et al. (2007). The full radio-FUV dereddened flux values are shown in table 2.4 in the Appendix. The X-ray spectrum is identical to that modelled in Gallo et al. (2007).

2.4 The model

We explore statistical fits of the agnjet model (MNW05, M09) to multiwavelength spectra of both Sgr A* and A0620 separately, and then perform joint fitting of both sources, tying parameters that represent the scale invariance (this is discussed in detail in Section 2.7).

MNW05 and M09 (and references therein) give a full description of agnjet including its assumptions and parameters, and subsequent work explores model fits to both BHBs and LLAGN (Plotkin et al. 2015; Markoff et al. 2015; Prieto et al. 2016). Here we give a basic outline and a description of the agnjet parameters. In agnjet, a relativistic plasma of adiabatic index $\Gamma = 4/3$ is injected in a nozzle at the base of the jet (or rather both axially symmetric jets) following assumptions for the hydrodynamics as laid out in Falcke & Biermann 1995; Falcke 1996. At the jet base,
the internal energy density, $U_J = U_B + U_e + U_{tu}$ (where $U_B$ is the magnetic energy density, $U_e$ is the relativistic electron energy density, and $U_{tu}$ is the turbulent plasma energy density), is assumed to be equal to the rest-mass energy density. Zdziarski (2016) points out that in fact the internal energy density could be arbitrarily large. However, the small Lorentz factors found in BHB jets ($\gamma_j \sim \text{a few}$), as well as implied by the variability in Sgr A* (Falcke et al. 2009; Brinkerink et al. 2015), require that the jet’s internal energy density not exceed the rest-mass energy density by a factor of more than a few at the base of the jet. We therefore suggest that such a scenario applies for all low-luminosity sources (lying on the FP) such as Sgr A* and A0620. Limiting the internal energy density of the jet in this way means we are not describing what would be classed as a Poynting-flux dominated jet (Blandford & Znajek 1977) in any of our modelling; we cannot have $U_B > n m_p c^2$, since the dynamics are not correctly calculated in such a scenario. In that sense our jet is consistent with being matter-dominated (Blandford & Payne 1982). The plasma is assumed to expand freely with an initial sound speed $\beta_{s,0} = \sqrt{\Gamma (\Gamma - 1)} / (\Gamma + 1) \sim 0.43$ in the lateral direction, and longitudinal pressure gradients accelerate the jet to supersonic speeds along its axis.

The main parameters of interest are displayed in Table 4.3. The most important fitted parameter here is $N_j$, the normalised jet power, since it acts as the model normalisation, and the entire spectrum is very sensitive to its value. $N_j$ is thus the total power fed into the base of the jet. Since the jet base represents a steady state, the inflow rate given by the power and the initial sound speed ($\beta_{s,0}$) and nozzle radius ($r_0$) sets the energy density. The partition factor $k$ parameterises the division of the jet energy density between the magnetic field and electrons ($U_B/U_e$, where $k \sim 1$ is referred to as equipartition. Since there is no radial structuring in agnjet, once $r_0$ is known the radial jet profile $r(z)$ is calculated, defining the jet opening angle by evaluating the velocity profile along the jet, a solution to the relativistic Euler equation for a roughly isothermal jet (Falcke 1996). The radius of the jet base $r_0$ is a very influential parameter, since the initial energy density comprising the magnetic field and particles depends inversely on the square of the radius ($U \propto r_0^{-2}$). Thus decreasing the jet-base radius increases the radiative output significantly, and it also affects the synchrotron/SSC radiative outputs differently. The height $h_0$ has an effect, though somewhat less than the radius. Increasing the height of the nozzle will cause an increase in the thermal synchrotron flux, as well as provide more particles to inverse Compton upscatter the synchrotron photons. We explore this parameter during fitting by both fixing it and allowing it to vary freely to explore SSC-dominated fits.

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2 We are currently exploring extending the agnjet model to cases in which the internal energy density of the plasma is not equal to the rest-mass energy density at the jet nozzle. We will address this topic in detail in a forthcoming research note (Crumley et al., in prep.), since it resides outside the scope of this paper.

3 This is essentially the inverse of the plasma beta parameter ($k = 1/\beta$).
Table 2.1: A list of the main input parameters of the \texttt{agnjet} model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_j$</td>
<td>the normalised jet power, in units of $L_{\text{Edd}}$.</td>
</tr>
<tr>
<td>$r_0$ and $h_0$ ($r_g$)</td>
<td>the radius and height (length) of the jet nozzle.</td>
</tr>
<tr>
<td>$T_e$ (K)</td>
<td>the electron temperature of the input distribution.</td>
</tr>
<tr>
<td>$k$</td>
<td>the ratio of magnetic to electron energy density, $U_B/U_e$, otherwise known as the partition factor.</td>
</tr>
<tr>
<td>$p$</td>
<td>the power-law index of the accelerated electron distribution.</td>
</tr>
<tr>
<td>$z_{acc}$ ($r_g$)</td>
<td>the distance from the black hole along the jet axis where particle acceleration begins.</td>
</tr>
<tr>
<td>$n_{nth}$</td>
<td>the fraction of particles accelerated at a distance $z_{acc}$ from the black hole along the axis of the jet.</td>
</tr>
<tr>
<td>$f_{sc}$</td>
<td>$\beta_{sh}^{-2}/(\lambda/R_{gyro})$ where $\beta_{sh}$ is the shock speed relative to the plasma, $\lambda$ is the scattering mean free path in the plasma at the shock region, and $R_{gyro}$ is the gyroradius of the particles in the magnetic field. In reality we do not require a shock so this parameterisation can generally be seen as a measure of the acceleration efficiency.</td>
</tr>
<tr>
<td>$\epsilon_{nth}$</td>
<td>the fraction of energy density in non-thermal electrons injected at the jet base.</td>
</tr>
<tr>
<td>$\Delta_{fac}$</td>
<td>the multiplication factor giving the maximum energy of the injected non-thermal electrons, $\gamma_{\text{max, nth}} = \Delta_{fac}\gamma_{\text{min, nth}}$.</td>
</tr>
</tbody>
</table>

**Notes.** Parameters $\epsilon_{nth}$ and $\Delta_{fac}$ only apply in model cases (c) and (d), when a mixed distribution of thermal and non-thermal particles is injected at the base of the jet.

The particles entering the base of the jet are assumed to be advected from the accretion flow with a thermal distribution at temperature $T_e$ (we note that this temperature may be different to the equilibrium temperature of the accretion flow, due to any heating processes at work), and are subsequently accelerated into a power law energy distribution at a distance $z_{acc}$ from the black hole along the axis of the jet (also a free parameter). An additional parameter $n_{nth}$ is included to specify the fraction of these particles accelerated into the power law (this is a separate parameterisation from $\epsilon_{nth}$, the fraction of energy density in non-thermal electrons injected at the jet base, discussed in Section 2.4.2), which we initially fix at 0.6, reflective of typical values used in previous applications of \texttt{agnjet} (MNW05, M09). We represent the
acceleration rate efficiency with the parameter $f_{ac}$, which incorporates uncertainty in
the acceleration mechanism, and whose derivation is presented in Jokipii (1987). We
stress that we are not asserting that the mechanism must be diffusive shock acceleration,
but rather these are convenient parameterisations of acceleration in general.

Additional parameters include the inner temperature and radius of the accretion
flow, $T_{in}$ and $r_{in}$, used to describe the thermal accretion disc blackbody emission,
and these photons are included in the photon field which undergoes inverse Compton
scattering at low optical depth (a maximum of one scattering per photon) with the
jet electrons. These are not included in Table 4.3 because for both Sgr A* and A0620
there is no discernible thin disc component to model; in the case of Sgr A* we adopt
a bremsstrahlung model to fit the quiescent thermal spectrum (Wang et al. 2013),
assumed to be produced by a RIAF as discussed in Section 2.2. Fixed parameters
include the mass of the black hole, $M_{BH}$, the inclination of the jet axis to the line of
sight, $\theta_i$, and the distance to the source, $D$.

The model agnjet produces 3 main components of emission—thermal synchrotron
(at $z < z_{acc}$), non-thermal synchrotron (produced at $z > z_{acc}$), and SSC (with a con-
tribution from inverse Compton scattering of disc photons, a minimal component in
fits to Sgr A* and A0620)—and as previously discussed in Section 4.1, there is sig-
nificant degeneracy between these components when fitting the spectra of hard-state
BHBs and LLAGN, particularly the X-ray spectra. The jet becomes self-absorbed
once it becomes optically thick at a given frequency, and hence exhibits a spectral
break (also often referred to as the spectral turnover or self-absorption frequency) at
$\nu_{SSA}$ below which we see the flat/inverted radio spectrum.

2.4.1 Synchrotron cooling

The accelerated distribution of electrons had in previous versions of agnjet been
assumed to be maintained by dissipative processes along the post-acceleration regions
of the jet, with no prescription for the cooling of electrons due to synchrotron radiation
(Falcke & Markoff 2000, MNW05, M09, Plotkin et al. 2013, Markoff et al. 2015). We
have updated the code to include a prescription for synchrotron cooling that is based
on solutions to the electron kinetic equation obtained by Kardashev (1962). If we
consider an electron distribution in which fresh power-law electrons are continually
injected and allowed to evolve with time in our adiabatically expanding jet, there will
be a break in the spectrum due to the balance between supply and radiative cooling,
found at

$$ E_{br} = \frac{4}{AB^2t}, $$

where $A = \sigma_T / (6\pi m_e^2 c^3)$, $B$ is the magnetic field, $m_e$ is the electron mass. Equation
2.1 tells us how the break energy of the electron distribution will evolve with time, but
since our model is time-independent we instead quantify the break energy analytically
using characteristic timescales. We do this by setting \( t = t_{dyn} \) where \( t_{dyn} = \Delta z/\beta_j c \) is the dynamical time during which electrons travel through a jet segment of height \( \Delta z \) at a bulk-flow velocity \( \beta_j \). Synchrotron cooling in balance with a continuous particle injection rate yields a broken powerlaw electron distribution given by

\[
\frac{dN}{dE} = \begin{cases} 
CE^{-p}dE, & E \leq E_{br} \\
CE^{-(p+1)}dE, & E > E_{br}
\end{cases}
\] (2.2)

where, after substituting \( t = t_{dyn} \) into (2.1), \( E_{br} \) is given by

\[
E_{br} = \frac{4\beta_j c}{AB^2\Delta z} = \frac{24\pi\beta_j m_e^2c^4}{\sigma_T B^2\Delta z}.
\] (2.3)

If the spectrum without cooling is given by \( I_\nu \propto \nu^{-\alpha} \), then cooling produces a steepening in the spectrum from \( \alpha \) to \( \alpha + 0.5 \) at the corresponding critical break frequency, \( \nu_{br} = (eB/2\pi m_e c)^2 \), where \( \gamma_{br} = E_{br}/m_e c^2 \). We can understand how the cooling break will evolve along the jet by considering its dependence on the variable quantities, \( E_{br} \propto \beta_j B^{-2}\Delta z^{-1} \). Since the jet is accelerating, and the overall energy budget is inversely proportional to the Mach number (this is the dominant cooling term), we know that the particle number density and magnetic field strength decrease with jet height. Thus it is clear that the cooling break energy will increase with jet height, suggesting that only solutions in which acceleration occurs close to the base of the jet (preliminary fits to broadband spectra from both Sgr A* and A0620 indicate an approximate range, \( z_{acc} \sim 5-20 \ r_g \)) will contain a synchrotron cooling break in the observed optically thin spectrum (at energies ranging from the IR and higher), assuming re-acceleration of the electrons in each zone.

### 2.4.2 Injection of a mixed particle distribution

The kinetics of the particles close to the black hole imply that the particle distribution will likely be mixed, i.e. some fraction of the particles will be non-thermal, with the bulk of the particles being thermal, and this is shown through previous modelling of Sgr A*’s accretion flow (e.g. Yuan et al. 2003; Dibi et al. 2014). As such we modify our model in order to allow for the possibility that a mixed particle distribution is injected at the base of the jet. The thermal particles follow a Maxwell-Jüttner distribution (as in the pure thermal case) at temperature \( T_e \), where the electrons are assumed to remain relativistic (\( \gamma_{min,th} = 1 \)). Those electrons presumed to have been accelerated prior to injection are distributed as \( dN = CE^{-p}dE \) between the limits \( \gamma_{min,nth} = 2.23 \ kT_e/m_e c^2 \) and \( \gamma_{max,nth} = \Delta_{fac} (2.23 \ kT_e/m_e c^2) \), where \( \Delta_{fac} \) is a fixed parameter, varying only on a case-by-case basis (see Section 2.6). The fraction of electron energy density injected into the non-thermal tail is also a model parameter, \( \epsilon_{nth} = U_{nth}/U_e \), where \( U_{nth} \) is the non-thermal electron energy density, given by \( U_{nth} = \int_{\gamma_{min,nth}}^{\gamma_{max,nth}} CE^{1-p}dE \). Thus \( \epsilon_{nth} \) can be related to the commonly prescribed
\[ \epsilon_e = \frac{\epsilon_{nth}}{(1 + k)} \]

which parameterises the energy given to electrons via shocks (Sari et al. [1998]), though here we assume only that some energy is given to electrons via an unspecified acceleration process. This full distribution is then cooled both due to the jet acceleration and synchrotron emission, and we assume there is no further particle acceleration elsewhere in the jet (i.e. \( z_{acc} \) becomes an inactive parameter of \texttt{agnjet}).

In each zone we adjust the limits of the power-law distribution from \( \gamma = 1 \) to \( \gamma_{\text{max, nth}} \) in that zone in order to represent the thermalising of those non-thermal particles as they cool. The power-law distribution is then described by Equation \ref{eq:2.2}. In contrast to the previous case, here we simply allow the electrons to cool along the jet without re-acceleration.

### 2.5 Method

We perform the spectral fits using the multiwavelength data analysis package \texttt{ISIS} (Houck & Denicola [2000], version 1.2.6-32). \texttt{agnjet} is imported into \texttt{ISIS} and (along with other model components, such as absorption routines) forward-folded through the detector response matrices. \texttt{ISIS} also allows one to read in lower frequency data (i.e. radio through to optical) from ASCII files for simultaneous broadband fitting. Any fits shown in flux space display the "unfolded spectra," which are independent of the assumed spectral model. The model fits to the data are performed in detector space, thus all residuals are the difference between the data and forward-folded model counts, normalised by the uncertainty in that bin (standardised \( \chi^2 \) residuals).

#### 2.5.1 Fitting methodology

The individual fitting routines for Sgr A* and A0620 are as follows. The X-ray spectra obtained for Sgr A* consist of the brightest 3 of 39 total flares, as discussed in Section 4, selected based on peak count rate and total fluence (counts), ensuring we have enough photon statistics to perform our fits (see Section 5.2). The data comprise both 0th order (i.e. undispersed photons) and MEG (Medium Energy Grating) and HEG (High Energy Grating) ± first order flares. Due to photon pileup 0th order spectra cannot be background subtracted, and are instead modelled as the superposition of a quiescent emission model (representative of the diffuse background emission around Sgr A*; Wang et al. [2013]) plus \texttt{agnjet}, with a kernel accounting for pileup. The constraints of our modelling therefore tie to the thermal quiescent spectrum. It is noted that the flares show no notable emission lines (Wang et al. [2013]). The MEG and HEG ± first order spectra are background subtracted and fit with \texttt{agnjet} corrected for interstellar absorption and dust scattering. We bin all the X-ray flare spectra at \( S/N = 4 \), setting a minimum number of 5 channels so as to avoid spurious groupings of photon counts at adjacent energies, and we set the energy bounds at 2–9 keV. For the 0th order and 1st orders, respectively, the fit functions

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are \( T_{\text{new}} \times \text{dustscat} \times (\text{agnjet} + \text{bremss}) + \text{gaussian}(1) + \text{gaussian}(2) \) and \( T_{\text{new}} \times \text{dustscat} \times \text{agnjet} \), where \( T_{\text{new}} \) represents interstellar absorption (Wilms et al. 2000, with cross-sections from Verner et al. 1996), and \( \text{dustscat} \) accounts for dust scattering (Baganoff et al. 2003). The \( \text{bremss} \) model represents the quiescent continuum associated with Sgr A*’s accretion flow (Wang et al. 2013), with temperature \( kT \sim 3.5 \) keV, and the two gaussian lines represent the best fit emission lines at 2.48 keV and 6.7 keV respectively, the He-like S and Fe K\( \alpha \) lines (these are the strongest emission lines). We adopt \( M_{BH} = 4 \times 10^6 \, M_\odot \), \( D = 8 \) kpc, and set the jet inclination to \( \theta_i = 80^\circ \) in accordance with the orbital inclination \( (0.75^\circ \leq i \leq 0.85^\circ) \) inferred from both broadband modelling (Markoff et al. 2007) and MHD simulations of Sgr A*’s accretion flow (Mościbrodzka et al. 2009; Shcherbakov et al. 2012; Drappeau et al. 2013).

The X-ray spectrum of A0620 is binned at a minimum of 15 counts per bin within energy bounds 0.3–8 keV in agreement with Gallo et al. (2006), such that we can perform \( \chi^2 \) statistics on the multiwavelength spectra. Since the IR to UV spectrum is de-reddened prior to fitting, the fitting methodology is very simple. The X-ray spectra are fit with \( T_{\text{new}} \times \text{agnjet} \), and the rest by \( \text{agnjet} \) alone. We are not concerned with line features present in the quiescent spectrum of A0620, however we do model the spectrum of the stellar companion, which dominates the near-IR - UV spectrum (4500 K \( \leq T_{\text{star}} \leq 4900 \) K González Hernández et al. 2004; Froning et al. 2011). We allow the X-ray absorbing column \( N_H \) to vary given the uncertainty on its measured value. We adopt \( M_{BH} = 6.6 \, M_\odot \), \( D = 1.06 \) kpc, and \( \theta_i = 51^\circ \) for A0620 throughout.

For the fitting method we first make use of a fast \( \chi^2 \) minimisation algorithm, and then we further explore our parameter space using an ISIS implementation (Murphy & Nowak 2014) of the Markov Chain Monte Carlo (MCMC) method of Foreman-Mackey et al. (2013). This routine makes use of the principles of an affine-invariant ensemble sampler, setting up a distribution of ‘walkers’ in the probability density landscape. These walkers explore the landscape by accepting moves based on the probability ratio of the proposed and current positions in the parameter space. In all our MCMC runs each parameter range contains \( > 200 \) walkers, all initialised uniformly within 1\% of the values found from the pre-MCMC \( \chi^2 \) minimisation, and allowed to evolve within flat uniform prior distributions over the area of parameter space we are exploring. Each run is allowed to evolve for at least 3000 steps in order to ensure reasonable convergence of the chain.

### 2.6 Individual spectral fits

Figures 2.3 and 2.4 show separate spectral fits to the broadband spectrum of A0620 and Sgr A* respectively, covering 4 cases for each source; (a) thermal particle injection, SSC-dominated, (b) thermal particle injection, synchrotron-dominated, (c)
mixed particle injection, SSC-dominated, (d) mixed particle injection, synchrotron-dominated. The corresponding maximum likelihood estimates (MLEs) and their 90% confidence regions are shown in Table 2.2. Here we briefly discuss the results of all 4 cases of model-fitting. Since the acceleration efficiency, \( f_{sc} \), and the multiplication factor, \( \Delta_{fac} \), determine the cut-off in the non-thermal particle distribution (and thus the synchrotron spectrum), we choose to fix these values (\( f_{sc} \) in cases (a) and (b), \( \Delta_{fac} \) in cases (c) and (d)) at their extremes. This ensures that the X-ray spectrum is fit with the non-thermal synchrotron emission in our synchrotron-dominated fits (with no cut-off present within the observing band), and the non-thermal synchrotron spectrum cuts off below X-ray energies in the SSC-dominated fits. We also note that whilst we allow the acceleration region \( z_{acc} \) to be free during fitting, due to the discretised nature of the jet axis, though \( z_{acc} \) is constrained it is not fully resolved; this is further complicated by possible correlations between \( z_{acc} \) and the other model parameters.

2.6.1 A0620

Case (a): thermal particle injection, SSC-dominated

This case corresponds to pure thermal particle injection at the jet base, with the X-ray spectrum dominated by SSC emission from the electrons in the base of the jet. The physical state portrayed is close to that found by Gallo et al. (2007) in which an older version of agnjet is fit to a multiwavelength spectrum of A0620 (the spectral coverage of the data in their modelling was the same, but there were fewer data points in the optical/UV bands). The jet base is relatively compact, and the magnetic energy density is sub-equipartition with respect to the electrons. Such a model class coincides with those previously found to work well for BHBs in quiescence at low luminosities (\( L_X \sim 10^{-9} – 10^{-8} \), Plotkin et al. 2015). Another distinct property we notice in this model fit when compared to cases (b), (c) and (d), is that the radio and X-ray spectra are fit simultaneously with ease.

Case (b): thermal particle injection, synchrotron-dominated

The synchrotron-dominated fit shows broadly different physical specifications, with higher electron temperatures than those seen in case (a), a slightly less compact jet base, and again a roughly equipartition magnetic field with respect to the electrons. No cooling break exists within the limits of the non-thermal particle distribution. This is because in all synchrotron-dominated fits in which particle acceleration occurs only at \( z_{acc} \), the fits evolve to solutions in which the acceleration region is too high for efficient cooling to occur (\( B_{z_{acc}} \sim 0.14 B_0 \), where \( B_0 \) is the jet-base magnetic field strength).
2.6 Individual spectral fits

Case (c): mixed particle injection, SSC-dominated

Here again the X-ray spectrum is SSC-dominated, except the injected distribution carries a fraction $\epsilon_{nth}$ of non-thermal energy. This fraction is poorly constrained here since it influences the synchrotron emission more than the SSC emission, but it must nonetheless be a small fraction ($< 23\%$). As in case (a) the electrons are constrained to fairly low temperature and the jet base is compact and well constrained. The magnetic field is sub-equipartition, and the jet power is high and statistically distinguishable from the ranges found for cases (a) and (b), albeit not well constrained. It is not obvious that this is a physical difference, it is more likely that the method by which we divide energy between thermal and non-thermal particles in each case causes a systematic change to the injected power.

Case (d): mixed particle injection, synchrotron-dominated

If again we assume a small fraction of the particles present at the jet base are non-thermal, we can explain the X-ray spectrum with synchrotron emission from those non-thermal particles, provided non-thermal particles carry just a small fraction of the total particle energy ($< 5.2\%$, see Table 2.2). The particles must however have been accelerated quite efficiently, with their cut-off extending to $10^4 \gamma_{min}$, and a cooling break between UV/X-ray energies at $\sim 10^{17}$ Hz. Again we find the jet power must be systematically higher when we inject this mixed distribution of particles in comparison to the pure thermal case, which again may simply be due to how we divide the energy between the particles. There is an increase in the upper limit on the electron temperature when compared with the SSC-dominated fit (inspection of the probability distribution reveals a bi-modal behaviour), and the system also tends towards a slightly super-equipartition magnetic field with respect to the electrons.
Figure 2.3: Spectral fits to the individual broadband spectrum of A0620. The four panels show cases (a) - (d) of our model fits to A0620. Orange diamond data points show the radio - FUV spectrum loaded into ISIS as flux density measurements. The X-ray spectrum is shown with purple circles. In each plot the absorbed model fit is indicated in thick black. In cases (a) and (b) the unabsorbed model components shown are pre-acceleration (thermal) synchrotron emission (blue dot-dashed line), post-acceleration synchrotron emission (red dashed line), SSC (green three-dot-dashed line), the blackbody spectrum of the stellar companion (black short-dashed line), and the total spectrum of agnjet (grey solid line). In cases (c) and (d) we do not include an explicit acceleration zone and thus there is no ‘post-acceleration’ spectrum. Instead the synchrotron spectrum emitted by the full thermal + non-thermal electron distribution is shown with the blue dot-dashed line, and the SSC spectrum is shown by the green three-dot-dashed line. The bottom panels of each plot show the standardised residual photon counts.
2.6 Individual spectral fits

Figure 2.4: Spectral fits to the individual broadband spectrum of Sgr A*. The first 4 panels show cases (a) - (d) of our model fits to Sgr A*. Radio and IR observations are indicated by orange diamonds, and X-ray 1st-order grating spectra shown with purple circles. The mid-IR 3σ upper limit is shown as a downward pointing arrow. In each plot the absorbed model fit is indicated in thick black. In cases (a) and (b) (top panels) the unabsorbed model components shown are pre-acceleration (thermal) synchrotron emission (blue dot-dashed line), post-acceleration synchrotron emission (red dashed line), SSC (green three-dot-dashed line), the blackbody spectrum of the stellar companion (black short-dashed line), and the total spectrum of \texttt{agnjet} (grey solid line). In cases (c) and (d) (middle panels) we do not include an explicit acceleration zone and thus there is no ‘post-acceleration’ spectrum. Instead the synchrotron spectrum emitted by the full thermal + non-thermal electron distribution is shown with the blue dot-dashed line, and the SSC spectrum is shown by the green three-dot-dashed line. The bottom panels of each plot show the standardised residual photon counts.
Figure 2.5: 0th-order flare (purple circles) and quiescent (red squares) X-ray spectra of Sgr A* and the overplotted (black) model fit, representative of all model fits to Sgr A*. 
Table 2.2: Fitted parameters for synchrotron-and-SSC-dominated individual spectral fits to A0620 and Sgr A*. Shown are 4 model cases for fits to each source, (a) thermal particle injection, SSC-dominated, (b) thermal particle injection, synchrotron-dominated, (c) mixed particle injection, SSC-dominated, (d) mixed particle injection, synchrotron-dominated. Confidence limits are at the 90% level, a result of our MCMC exploration of the posterior distributions of the parameters. The resultant MLEs are given by the median point of each posterior distribution - this proves to be a good measure of the converged best fit of the MCMC routine. The final column shows the resultant $\chi^2$ and the degrees of freedom (DoF). From left-to-right the following parameters are shown: $N_H$, the Hydrogen column density along the line-of-sight to the source, $N_j$, the jet power, $p$, the spectral index of the power-law-distributed electrons, $T_e$, the temperature of the electron distribution (Maxwell-Jüttner), $z_{\text{acc}}$, the location of acceleration in the jet (only applicable when a pure thermal particle distribution is injected at the base, cases (a) and (b)), $r_0$, the radius of the jet-base nozzle, $h_{\text{ratio}}$, the ratio of the nozzle height $h_0$ to the jet-base radius $r_0$, $k$, the energy partition factor, and $\epsilon_{\text{nth}}$, the fraction of energy density in non-thermal electrons. Jet-base electron densities ($n_e, 0$) and magnetic field strengths ($B_0$) are shown for the corresponding MLEs.

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Notes. / Frozen parameter
2.6.2 Sgr A*

In all fits to Sgr A* the mid-IR upper limit ($I_{8.6\mu m} < 58\,mJy$, Haubois et al. 2012) sets a rough flat prior on the upper limits of $T_e$ and $k$. In a mildly sub-equipartition regime ($k \sim 0.1–0.5$) we find an approximate upper limit on the electron temperature of $T_e \sim 3 \times 10^{11} \text{K}$ (and poor fits to the X-ray flare spectrum). Thus, in this regime, the brightest X-ray flares detected around Sgr A* cannot be explained by SSC emission as long as the plasma remains in equilibrium (see however Dibi et al. 2014). To achieve higher temperatures and thus satisfactorily model the X-ray flares, we require a much more sub-equipartition flow ($k \sim 10^{-3}$). Figure 2.4 (panels (a) and (c)) shows SSC-dominated fits in which the jet energy partition has fallen to values of $k \sim 10^{-3}$, and in this case, we find that an electron temperature of $T_e > 4 \times 10^{11} \text{K}$ provides a good fit to the full spectrum without violating the mid-IR upper limits.

Case (a): thermal particle injection, SSC-dominated

Here we require an electron temperature $T_e > 4.6 \times 10^{11} \text{K}$, with a highly sub-equipartition magnetic energy density in order to model the X-ray flare spectrum of Sgr A*. The size of the acceleration region is roughly consistent with the flare timescales, though too large when considering the brightest flares; Neilsen et al. 2013; Barrière et al. 2014). The NIR fluxes weakly constrain the electron power law index (producing non-thermal synchrotron emission, consistent with those found by Bremer et al. 2011): $\alpha \sim 0.7 \pm 0.4$, thus $p \sim 2.4 \pm 0.8$. We do not see any strong correlation between $T_e$ and $k$ in this local fit landscape, implying that once we go to very sub-equipartition conditions, the electron temperature must always be high.

Case (b): thermal particle injection, synchrotron-dominated

In this case we are able to model the X-ray flare spectrum with uncooled non-thermal synchrotron emission, in which the magnetic field is in rough equipartition with the electrons. Particle acceleration occurs at a region ($z_{acc}$) that is roughly consistent with the range of flare timescales, though again not consistent with the timescales of brightest flares (Barrière et al. 2014); solutions like this in which particle acceleration occurs further out in the jet are thus unlikely.

Case (c): mixed particle injection, SSC-dominated

Assuming we have a fraction $\epsilon_{nth}$ of non-thermal energy in non-thermal electrons in our jet also allows us to successfully model the Sgr A* X-ray flare spectrum in the SSC-dominated case (given a highly sub-equipartition magnetic energy density), though it should be noted that the thermal synchrotron spectrum reaches the mid-IR upper limit. The NIR spectrum is fit partially with the thermal synchrotron turnover, but is dominated by synchrotron emission from the non-thermal tail of electrons. We
notice that the fit also tends towards a more compact jet base and a high jet power, which is an attempt to boost the SSC flux to match the X-ray spectrum (as inverse Comptonisation depends strongly on the electron density), though as mentioned in Section 2.6.1 the systematic increase in power when injecting a mixed particle distribution may be a natural consequence of how we divide energy between the particles.

Case (d): mixed particle injection, synchrotron-dominated

A scenario in which particle acceleration occurs within a few $r_g$ of the black hole favours the flare timescales of Sgr A*, and such solutions are preferred in other more detailed modelling of the accretion flow close to the black hole (Yuan et al. 2003; Dibi et al. 2014). Our synchrotron-dominated fit within this scenario explains both the IR and X-ray flare spectra, with slightly super-equipartition conditions at the jet base. A small fraction of the electron energy is in non-thermal electrons, producing a hard non-thermal synchrotron tail that fits the IR emission, and the X-ray spectrum is then well modelled by synchrotron emission from the cooled electrons, with a break at $\sim 10^{16}$ Hz. It should however be noted that the thermal synchrotron flux sits at the mid-IR upper limit.

2.7 Joint Fitting

As seen in the individual spectral fits (Section 2.6), the ranges of the potential scaling parameters (which we shall now discuss) are comparable for all models, which leads well into exploring joint fits with these parameters tied. The technique used when fitting agnjet jointly to A0620 and Sgr A* follows the same logic as with individual fits. Each data set is loaded into ISIS, and the model definitions are split as described in Section 2.6 except we now require almost the same model for both Sgr A* and A0620.

Following the theoretical prescription of Markoff et al. (2003) and Heinz & Sunyaev (2003), we assume that scale invariance manifests itself in geometric quantities such as $r_0$, $h_0$, and $z_{acc}$, and so we choose to tie these together for both sources. We tie a further 2 parameters by presuming that the division of energy and the acceleration mechanisms are roughly coincident at similar accretion rate in the sources that fit on the FP ($k$ and $p$); both these parameters are key to understanding whether there are fundamental differences in the energy partition of sources at very quiescent levels, and also whether spectral indices may differ (Russell et al. 2013b; Plotkin et al. 2015). By tying these 5 parameters together, we both explore the extent to which black holes can be treated as scale-invariant, as well as potentially breaking some of the model degeneracies. Parameters that we expect to depend explicitly on cooling and the physical values of the electron density and magnetic field (e.g. $T_e$) would not be expected to scale with black hole mass, and are thus left to vary accordingly.
We effectively equate the $f_{sc}$ parameter for both sources by freezing its value at its extremes in the SSC/synchrotron-dominated cases as described in Section 2.6. We refer the reader to Table 2.3 for all parameter values mentioned in the following presentation of the results. We now discuss the 4 cases of joint fits just as in Section 2.6, that cannot be ruled out relative to one another in terms of their goodness-of-fit, but we discuss which areas of parameter space are less favourable given what we know about the behaviour of Sgr A* and A0620.

2.7.1 Case (a): thermal particle injection, SSC-dominated

Here, as shown in Figure 2.6, we achieve good fits to the data given a very subequipartition magnetic field, but the electron temperature still needs to be high in order to produce the required X-ray flux via pure SSC emission. The fit to A0620 sees a statistically significant decrease in the jet-base radius compared with individual spectral fits, whilst this value is consistent with single fits of this case to Sgr A* (see section 2.6). This property applies independently of the type of model we are fitting (though SSC-dominated fits, i.e. cases (a) and (c), give slightly more compact jet bases). The electron temperature of A0620’s inner accretion flow also decreases, and the particle acceleration zone ($z_{acc}$) drops compared with individual fits. A significant change is seen for the jet power of A0620, which has to increase to account for the drop in the energy partition parameter (and thus a reduced magnetic field strength), an effect displayed clearly in Figure 2.10 showing the two-dimensional confidence contours of parameters in which we see correlations.

2.7.2 Case (b): thermal particle injection, synchrotron-dominated

Here we see that the result of tying $z_{acc}$ during synchrotron-dominated fits is to push its value higher to allow the A0620 X-ray spectrum to be modelled sufficiently, and this also results in tighter constraints on its value. Solutions in which particle acceleration occurs so distant from the black hole are unlikely to be the source of bright X-ray flares given the timescale constraints on Sgr A*’s X-ray variability (Neilsen et al. 2013; Barrière et al. 2014); we expect the flare emission to be originating from within $\sim 5 \, r_g$ of the black hole during the brightest flares. One notices a few correlations in the 2D contours shown in Figure 2.10 including a weak but persistent positive correlation between $N_j$ and $k$ for Sgr A* and A0620 during these synchrotron-dominated states. This implies that by providing energy to the magnetic field in Sgr A*, the drop in electron density is enough to force an increase in the injected power; fits to Sgr A*, even in the synchrotron-dominated case, are very sensitive to electron density.
2.7.3 Case (c): mixed particle injection, SSC-dominated

Figure 2.8 shows, similar to case (a), that the mid-IR constraints on the jet-base electron temperature result in a significantly reduced partition factor. This is consistent with the case (c) single fit to Sgr A*, but significantly lower than single fits to A0620. The electron temperature of Sgr A* is high, even with such a sub-equipartition flow, and the mid-IR limits are almost surpassed. The electron temperature of Sgr A* remains consistent with single fits, but we see a statistically significant increase in the electron temperature of A0620, again reflecting the evolution to a highly sub-equipartition magnetic field (reflected in the jet-base magnetic field values shown in Table 2.3). We also note that whilst we see a correlation between \( r_0 \) and \( T_e \) in single fits to A0620, this correlation is not present in our joint fits, a possible indication that the joint fitting approach indeed reduces some of the physical degeneracy. The value of \( \epsilon_{nth} \) is statistically indistinguishable between Sgr A* and A0620, though we note that for A0620 its value is constrained to lower fractions than those in the fit to Sgr A*. There is a \( k-r_0 \) weak anti-correlation introduced by performing this joint fit, which may reflect the relative importance of the energy partition over the electron temperature (this is evident also from the weak constraints we are able to place on \( T_e \)). We also note that, as shown in Figure 2.8, the X-ray spectrum of A0620 is dominated by the synchrotron emission from the non-thermal tail injected at the base of the jet, indicating that in this case the mass-scaling changes the phenomenology of this class of fit to A0620.

2.7.4 Case (d): mixed particle injection, synchrotron-dominated

Here we see a fast-cooling dominated synchrotron spectrum, produced by a population of accelerated electrons close to the black hole, consistent with the observed timescales of the flares (Barrière et al. 2014). Russell et al. (2013b) indicate that BHBs that decline into quiescence seem to exhibit a cooling-break evolution down to UV energies. We find cooling breaks for both Sgr A* and A0620 at \( \sim 10^{16} \) Hz and \( \sim 10^{17} \) Hz respectively. Whilst the values of \( p \) and \( \epsilon_{nth} \) remain consistent with single fit values for both Sgr A* and A0620, other parameters show statistically significant changes. Sgr A*'s jet power is decreased, \( T_e \) and \( r_0 \) drop in the A0620 fit, and Sgr A*'s inner flow goes to slightly lower energy partition, decreasing the jet-base magnetic field. We note that just as in case (c), the \( T_e - r_0 \) correlation in A0620 fits is removed when fitting jointly, however Figure 2.10 shows that a correlation is introduced between \( T_e \) and \( N_j \) for both Sgr A* and A0620.
Figure 2.6: Case (a): Joint spectral model fits to both Sgr A* and A0620 spectrum, with thermal particles injected at the jet base, and SSC emission dominating the X-ray spectra. The top panel shows the model fit to Sgr A*, and the bottom panel shows the fit to A0620−00. In the Sgr A* spectrum is orange diamonds show radio - IR data loaded into ISIS as flux density measurements, with the X-ray spectrum in purple points. In A0620 spectrum the Orange diamonds show the radio - FUV spectrum loaded into ISIS as flux density measurements, and the 1st-order grating X-ray spectrum is shown with purple circles. The model fit in both panels is indicated in black. The unabsorbed model components shown are pre-acceleration (thermal) synchrotron emission (blue dot-dashed line), post-acceleration synchrotron emission (red dashed line), SSC (green three-dot-dashed line), the blackbody spectrum of the stellar companion (black short-dashed line), and the total spectrum of agnjet (grey solid line). Parameters $r_0$, $h_{ratio}$, $z_{acc}$, $p$ and $k$ are tied between both models.
Figure 2.7: Case (b): Joint spectra fit to both Sgr A* and A0620 spectrum, with thermal particles injected at the jet base, and non-thermal synchrotron emission from particles accelerated at $z_{acc}$ dominating the X-ray spectra. The top panel shows the model fit to Sgr A*, and the bottom panel shows the fit to A0620−00. In the Sgr A* spectrum is orange diamonds show radio - IR data loaded into ISIS as flux density measurements, with the X-ray spectrum in purple points. In A0620 spectrum the Orange diamonds show the radio - FUV spectrum loaded into ISIS as flux density measurements, and the 1st-order grating X-ray spectrum is shown with purple circles. The model fit in both panels is indicated in black. The unabsorbed model components shown are pre-acceleration (thermal) synchrotron emission (blue dot-dashed line), post-acceleration synchrotron emission (red dashed line), SSC (green three-dot-dashed line), the blackbody spectrum of the stellar companion (black short-dashed line), and the total spectrum of $\text{agnjet}$ (grey solid line). Parameters $r_0$, $z_{acc}$, $p$ and $k$ are tied between both models, and $h_{ratio}$ is fixed.
Figure 2.8: Case (c): Joint spectra fit to both Sgr A* and A0620 spectrum, with a mixed thermal/non-thermal distribution of particles injected at the jet base, and SSC emission dominating the X-ray spectra. The top panel shows the model fit to Sgr A*, and the bottom panel shows the fit to A0620−00. In the Sgr A* spectrum is orange diamonds show radio - IR data loaded into ISIS as flux density measurements, with the X-ray spectrum in purple points. In A0620 spectrum the Orange diamonds show the radio - FUV spectrum loaded into ISIS as flux density measurements, and the 1st-order grating X-ray spectrum is shown with purple circles. The model fit in both panels is indicated in black. The unabsorbed model components shown are pre-acceleration (thermal) synchrotron emission (blue dot-dashed line), post-acceleration synchrotron emission (red dashed line), SSC (green three-dot-dashed line), the blackbody spectrum of the stellar companion (black short-dashed line), and the total spectrum of $\text{agnjet}$ (grey solid line). Parameters $r_0$, $h_{ratio}$, $p$ and $k$ are tied between both models, and $z_{acc}$ is a deactivated parameter since particles are not re-accelerated.

Note: The model fit to A0620 has driven to being synchrotron-dominated.
Figure 2.9: Case (d): Joint spectra fit to both Sgr A* and A0620 spectrum, with a mixed thermal/non-thermal distribution of particles injected at the jet base, and non-thermal synchrotron emission from the injected non-thermal particles dominating the X-ray spectra. The top panel shows the model fit to Sgr A*, and the bottom panel shows the fit to A0620−00. In the Sgr A* spectrum is orange diamonds show radio - IR data loaded into ISIS as flux density measurements, with the X-ray spectrum in purple points. In A0620 spectrum the Orange diamonds show the radio - FUV spectrum loaded into ISIS as flux density measurements, and the 1st-order grating X-ray spectrum is shown with purple circles. The model fit in both panels is indicated in black. The unabsorbed model components shown are pre-acceleration (thermal) synchrotron emission (blue dot-dashed line), post-acceleration synchrotron emission (red dashed line), SSC (green three-dot-dashed line), the blackbody spectrum of the stellar companion (black short-dashed line), and the total spectrum of agnjet (grey solid line). Parameters $r_0$, $h_{\text{ratio}}$, $p$ and $k$ are tied between both models, and $z_{\text{acc}}$ is a deactivated parameter since particles are not re-accelerated.
Figure 2.10: Two-dimensional contours of parameters of interest in joint fits to Sgr A* and A0620, covering all 4 cases, (a) - (d), shown consecutively from the top row of panels (case (a)) to the bottom row of panels (case (d)). Blue solid lines show contours at 0.68/0.90/0.95 confidence for Sgr A* fitted parameters, and red dotted lines show the equivalent for A0620 parameters. Black solid lines show contours either for joint-fitted parameters or comparisons between a parameter applying to each source individually. The crosses indicate the MLEs.
Table 2.3: Fitted parameters for Synchrotron-and-SSC-dominated joint spectral fits to A0620 and Sgr A*. Shown are 4 model cases for fits, (a) thermal particle injection, SSC-dominated, (b) thermal particle injection, synchrotron-dominated, (c) mixed particle injection, SSC-dominated, (d) mixed particle injection, synchrotron-dominated. Confidence limits are at the 90% level, a result of our MCMC exploration of the posterior distributions of the parameters. The resultant MLEs are given by the median point of each posterior distribution - this proves to be a good measure of the converged best-fit of the MCMC routine. The final column shows the resultant $\chi^2$ and the degrees of freedom (DoF). From left-to-right the following parameters are shown: $N_H$, the Hydrogen column density along the line-of-sight to the source, $N_j$, the jet power, $p$, the spectral index of the power-law-distributed electrons, $T_e$, the temperature of the electron distribution (Maxwell-Jüttner), $z_{acc}$, the location of acceleration in the jet (only applicable when a pure thermal particle distribution is injected at the base, cases (a) and (b)), $r_0$, the radius of the jet nozzle, $h_{ratio}$, the ratio of the nozzle height $h_0$ to the jet-base radius $r_0$, $k$, the energy partition factor, and $\epsilon_{nth}$, the fraction of energy density in non-thermal electrons. The parameters $r_0$, $h_{ratio}$, $z_{acc}$, $p$, and $k$ are tied together where applicable (in cases (c) and (d) there is no acceleration zone in the jet, and thus $z_{acc}$ is null.) Jet-base electron densities ($n_{e,0}$) and magnetic field strengths ($B_0$) are also shown for the corresponding MLEs.

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<td>$12.3_{-0.7}^{+0.7}$</td>
<td>$17_{-2}^{+2}$</td>
<td>$\ldots$</td>
<td>$41.7_{-0.8}^{+0.3}$</td>
<td>$\ldots$</td>
<td>$\ldots$</td>
<td>$\ldots$</td>
<td>$\ldots$</td>
<td>$\ldots$</td>
<td>$5_{-3}^{+4}$</td>
<td>$\ldots$</td>
</tr>
<tr>
<td>A0620</td>
<td>$0.14_{-0.03}^{+0.09}$</td>
<td>$200_{-20}^{+40}$</td>
<td>$\ldots$</td>
<td>$17_{-5}^{+2}$</td>
<td>$\ldots$</td>
<td>$\ldots$</td>
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<td>$\ldots$</td>
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<td>$&lt; 3$</td>
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<tr>
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<td>$\ldots$</td>
<td>$2.02_{-0.03}^{+0.06}$</td>
<td>$0.85_{-0.03}^{+0.05}$</td>
<td>$0.002_{-0.001}^{+0.001}$</td>
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<td>$\ldots$</td>
</tr>
<tr>
<td>Sgr A*</td>
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<td>$3.4_{-0.8}^{+0.7}$</td>
<td>$\ldots$</td>
<td>$21_{-3}^{+3}$</td>
<td>$\ldots$</td>
<td>$\ldots$</td>
<td>$\ldots$</td>
<td>$\ldots$</td>
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<td>$1.2_{-0.9}^{+1.2}$</td>
<td>$\ldots$</td>
</tr>
<tr>
<td>A0620</td>
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<td>$120_{-30}^{+30}$</td>
<td>$\ldots$</td>
<td>$2.1_{-0.4}^{+0.4}$</td>
<td>$\ldots$</td>
<td>$\ldots$</td>
<td>$\ldots$</td>
<td>$\ldots$</td>
<td>$\ldots$</td>
<td>$0.5_{-0.3}^{+0.3}$</td>
<td>$\ldots$</td>
</tr>
<tr>
<td>Joint</td>
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<td>$2.1_{-0.2}^{+0.1}$</td>
<td>$\ldots$</td>
<td>$3.7_{-0.3}^{+0.4}$</td>
<td>$1.08_{-0.07}^{+0.12}$</td>
<td>$4_{-2}^{+2}$</td>
<td>$\ldots$</td>
<td>$\ldots$</td>
<td>$\ldots$</td>
<td>$381/218$</td>
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</table>

Notes. $^f$ Frozen parameter
2.8 Discussion and Conclusion

We have shown that mass-scaling in weakly accreting black holes (the FP) can be exploited to break degeneracies in spectral modelling. This has been explored for higher luminosity, hard-state-like sources by Markoff et al. (2015); here we extend this study for the first time to the most quiescent sources. We find that the majority of key parameters can be tied in the modelling of Sgr A* and A0620, namely the jet-base radius $r_0$, the distance from the black hole along the jet axis at which particle acceleration occurs $z_{acc}$, the height of the jet nozzle $h_0/r_0$, the partition of energy between the magnetic field and electrons $k$, and the spectral index of the accelerated electron distribution $p$. This new approach reduces the strength of some parameter correlations (i.e. degeneracy) and better constrains those tied parameters, leading to more informative distinctions between different model classes.

Our results imply that if the X-ray spectrum of Sgr A* during the brightest flares is dominated by SSC emission from a purely thermal population of electrons, its jet-base magnetic field must be significantly sub-equipartition, and the electron temperatures push to high values, giving rise to mid-IR fluxes close to the upper limit obtained by Haubois et al. (2012). If indeed A0620 and Sgr A* can be related via their energy partition, then we would expect sources accreting at $l_X \sim 10^{-9}–10^{-8}$ to be highly sub-equipartition.

An alternative physical scenario in which the X-ray spectra of A0620 and Sgr A* are dominated by synchrotron emission from an injected non-thermal distribution of electrons is consistent with what we know about the variability of Sgr A* (as opposed to the solutions we find in which particle acceleration occurs in regions $> 5 r_g$ from the black hole Barrière et al. 2014), and can also explain the broadband spectra of both sources. In this scenario, a small fraction ($\sim$ a few %, statistically consistent for both Sgr A* and A0620) of the injected electron energy density is in non-thermal electrons, and these electrons experience a cooling break at $\sim 10^{16} \text{ Hz}$ and $\sim 10^{17} \text{ Hz}$ for Sgr A* and A0620 respectively. These results are consistent with previous modelling of Sgr A* that indicates a cooling break in the synchrotron spectrum between IR and X-ray energies (Dibi et al. 2012, 2014). We therefore find that the suggestion of Markoff (2010b) and Plotkin et al. (2015) that quiescent BHBs enter a regime of inefficient particle acceleration is an inherently degenerate one. The effect of inefficient acceleration (and therefore SSC-dominated X-ray states) can be subsumed by efficient particle cooling within the peak synchrotron emission zones. This interpretation is consistent with the findings of Russell et al. (2013b) that as BHBs evolve to quiescence their cooling breaks evolve to lower frequencies such that the break may be observed in the UV band. We also note that recent single-zone modelling of the inner accretion flow of Sgr A*, with the goal of reproducing the observed IR/X-ray flare distributions, shows that particle acceleration as well as density and magnetic field changes are key to producing the flares (Dibi et al. 2016).
Both these scenarios are consistent with a matter-dominated disc-jet system, in which the jet magnetisation is low (Blandford & Payne 1982), as opposed to a Poynting-flux dominated jet in which we should expect high magnetisations (Blandford & Znajek 1977). As discussed in section 4.3, we are unable to properly describe a highly magnetised jet with our model, since we explicitly assume \( U_B \leq n m_p c^2 \). However, we note that a scenario in which the jet has a Poynting-dominated spine surrounded by a matter-dominated outer sheath may still be consistent with the conditions we find (Hawley & Krolik 2006; Mońcibrodzka et al. 2016).

Achieving efficient particle acceleration, whether via internal shocks or magnetic reconnection, is an ongoing area of study, with some recent progress coming from PIC simulations of both such mechanisms (e.g. Sironi & Spitkovsky 2011, 2014; Sironi et al. 2013, 2015). In the shock-acceleration scenario (assuming quasi-perpendicular shocks), it proves difficult to accelerate electrons to high energies unless the pre-shock conditions are at low magnetisation. Conversely in the magnetic reconnection scenario, electrons may be accelerated to high energies if the inner regions are highly magnetised. We propose that at the most quiescent levels \( (l_X \sim 10^{-9} - 10^{-8}) \) accreting black holes struggle to achieve the structures necessary for efficient particle acceleration (as represented by the acceleration regions at \( z > z_{acc} \)), but that there are still likely a small fraction of non-thermal radiating particles at the jet base (in the fast-cooling regime) of the black hole produced via another acceleration mechanism (Yuan et al. 2003). We also propose that such sub-equipartition conditions at the jet base favour brighter SSC emission in the X-rays, which reinforces the findings of Plotkin et al. (2015) that a switch to quiescence in BHBs is associated with a compact jet base (a few gravitational radii) and sub-equipartition magnetic fields with respect to the electrons. These sub-equipartition magnetic fields may coincide with the production of weakly magnetised outflows as opposed to highly collimated jets.

We can understand more about what each mechanism (synchrotron or SSC) predicts in terms of the emission timescales of Sgr A*’s daily flares, in particular the connection between X-ray and IR emission, by considering the timescales for particle acceleration in weakly relativistic outflows. An electron with a Lorentz factor \( \gamma \) gyrating in a magnetic field \( B \), will have a peak synchrotron frequency of

\[
\nu = \frac{q \gamma^2 B}{2 \pi m_e c} \quad \Rightarrow \quad \gamma \approx 3 \times 10^4 \left( \frac{B}{100 \text{ G}} \right)^{-1/2} \left( \frac{h \nu}{1 \text{ keV}} \right)^{1/2}.
\]

From Equation 2.4 we can see that electrons with energies \( \sim 15 \text{ GeV} \) will be capable of radiating X-rays in a 100 G strength magnetic field.

The time it takes for an electron to be accelerated to an energy capable of radiating at a given frequency \( \nu \) in diffusive shock acceleration (DSA) is (e.g., Caprioli & Spitkovsky 2014)

\[
t_{acc} = \frac{6 D(\epsilon)}{\nu^2},
\]

\( t_{acc} \) is the acceleration time.

\[
\frac{D(\epsilon)}{\nu^2} = \frac{4 \pi^2 m_e c^3}{e^2} \left( \frac{m_e c^2}{\epsilon} \right)^{-1} \left( \frac{1}{\nu} \left( \frac{100 \text{ G}}{B} \right) \right)^{-1/2}.
\]

\( D(\epsilon) \) is the diffusion coefficient.
where the $v_s$ is the velocity of the upstream flow in the downstream reference frame relative to the shock. If we assume that the particle acceleration happens in the Bohm diffusion limit, the diffusion coefficient is (e.g., Jokipii 1987; Amato 2015):

$$D_B(\epsilon) = \frac{c r_L}{3}; \quad r_L = \frac{\gamma mc^2}{q_B}.$$  \hspace{1cm} (2.6)

Combining Equations (2.4), (2.5), and (2.6) yields the following acceleration time:

$$t_{\text{acc}} = \frac{2 c^2 r_L}{v_s^2 c} \approx 1.3 \times 10^{-4} \left( \frac{B}{100 \, \text{G}} \right)^{-3/2} \left( \frac{h \nu}{1 \, \text{keV}} \right)^{1/2} \left( \frac{v_s}{0.5 c} \right)^{-2} \text{s.} \tag{2.7}$$

We assumed shock acceleration to derive Equation (2.7), but a derivation assuming magnetic reconnection would yield a similar result for the minimum acceleration time, (e.g. Kumar & Crumley 2015):

$$t_{\text{acc}} = \frac{1}{\epsilon_0} \frac{r_L}{c} \sim 2 \times 10^{-4} \left( \frac{B}{100 \, \text{G}} \right)^{-3/2} \left( \frac{h \nu}{1 \, \text{keV}} \right)^{1/2} \left( \frac{\epsilon_0}{0.1} \right)^{-1} \text{s.} \tag{2.8}$$

$\epsilon_0 = E/B_0 \leq 1$ is the reconnection rate, and simulations find an $\epsilon_0 \sim 0.1$ (e.g. Kagan et al. 2013). The fastest particles can be accelerated roughly the same whether the particles are accelerated via shocks or magnetic reconnection in a mildly relativistic outflow.

During a flare, $t_{\text{acc}}$ is the smallest time we should expect the X-rays to lag the IR photons (assuming the IR photons are produced by electrons at the initial stage of acceleration). The lag time is so small we should expect the infrared and X-rays to be simultaneous. For example the X-ray lags on minute timescales (with respect to IR emission) reported by Yusef-Zadeh et al. (2012)—who argue an inverse Compton origin for the X-ray flares—are unlikely to be explained by delayed particle acceleration and subsequent synchrotron emission in both bands. If further studies of the coupling between the IR/X-ray variability of Sgr A* indicated that the IR lags the X-ray, synchrotron emission would not be able to explain the observed time lag. This calculation does not, however, take into account synchrotron cooling of the electrons during the acceleration process, which is one of our predicted scenarios. Also we consider here only the time to accelerate particles, not the time delays we may expect for shocks or magnetic reconnection events to develop in a relativistic outflow, which is an interesting further point to explore in the context of Sgr A*'s IR/X-ray variability. For example, as discussed in section 4.1 studies of Sgr A*’s
X-ray variability have allowed inferences regarding the particle acceleration process, with many finding magnetic reconnection to be a viable process, likely in a fast-cooling regime (Neilsen et al. 2013; Li et al. 2015; Dibi et al. 2016)—we note that this supports our synchrotron-dominated model scenario.

Our new joint-fitting technique strongly favours a scenario in which the most sub-Eddington accreting black holes (quiescence down to $l_X \sim 10^{-9} - 10^{-8}$) have very compact jet bases, on the order of a few gravitational radii. This holds regardless of whether the emission is dominated by non-thermal synchrotron from a population of accelerated electrons (accelerated within a few $r_g$ of the black hole), or a high-density sub-equipartition flow which produces dominant SSC emission (or potentially a mixture of these two processes). We find that the particle acceleration component in the outer jet recedes, leaving evidence for another kind of weak acceleration in the inner accretion flow (Yuan et al. 2003), or an inverse Compton-dominated jet base. We echo the statements made by Plotkin et al. (2015) that further well-sampled SEDs of quiescent BHBs are required to draw more precise conclusions regarding the accretion/jet dynamics and local conditions, in particular breaking the degeneracy between synchrotron or SSC domination in such weakly accreting systems, and what is driving the change from one regime to another. In addition to this, the efforts of the Event Horizon Telescope (EHT) (Doeleman et al. 2008) to probe the inner regions of Sgr A*’s accretion flow will shed light on local plasma conditions, and may reveal more about the plausibility of these model scenarios. A parallel strategy is to improve our modelling, and in the near future we will implement self-consistent, relativistic MHD flow solutions to reduce the free parameters in our jet modelling, in particular the geometrical quantities and their relationship to the internal properties (e.g. Polko et al. 2010, 2013, 2014). This will be presented in an upcoming paper (Ceccobello et al., in prep).

In the future we hope to build further upon our results here based upon more recent broadband observations (X-ray/radio/optical-IR) of A0620 (e.g. MacDonald et al. 2015) that may give more insight into the outflow structure of A0620. Such insights would come primarily from the radio spectrum of A0620, since in our modelling we are limited by the lack of radio coverage (with only the 8.5 GHz flux).
Using mass-scaling to study accretion

<table>
<thead>
<tr>
<th>$\nu$ (Hz)</th>
<th>$I_\nu$ (mJy)</th>
<th>Instrument</th>
</tr>
</thead>
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<tr>
<td>$8.50 \times 10^9$</td>
<td>$0.051 \pm 0.007$</td>
<td>VLA$^a$</td>
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<tr>
<td>$1.25 \times 10^{13}$</td>
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<td>$Spitzer^a$</td>
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<td>$0.38 \pm 0.04$</td>
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<td>$1.3 \pm 0.2$</td>
<td>ANDICAM (CTIO)$^b$</td>
</tr>
<tr>
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<td>$1.7 \pm 0.2$</td>
<td>$SMARTS^a$</td>
</tr>
<tr>
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<td>$1.4 \pm 0.1$</td>
<td>ANDICAM (CTIO)$^b$</td>
</tr>
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<td>STIS ($HST^b$)</td>
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**Notes.**

a Data taken from Gallo et al. (2006, 2007).

b Data taken from Froning et al. (2011)
Table 2.5: Sgr A* radio-to-submm spectrum

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<th>Instrument</th>
<th>$\nu$ (GHz)</th>
<th>$I_\nu$ (mJy)</th>
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<td>218</td>
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<td>JCMT$^a$</td>
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<td>500</td>
<td>4000 ± 1200</td>
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</tr>
<tr>
<td>97</td>
<td>2403 ± 186</td>
<td>ALMA$^e$</td>
<td>666</td>
<td>3000 ± 1000</td>
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<td>105</td>
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<td>ALMA$^e$</td>
<td>850</td>
<td>7000 ± 2000</td>
<td>CSO-JCMT$^b$</td>
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</tbody>
</table>

The jet-disk symbiosis without maximal jets: 1-D hydrodynamical jets revisited

My reality needs imagination like a bulb needs a socket. My imagination needs reality like a blind man needs a cane.
—Tom Waits

P. Crumley, C. Ceccobello, R. M. T. Connors, Y. Cavecchi

Astronomy & Astrophysics, 2017, 601, A87

Abstract

In this work we discuss the recent criticism by Zdziarski (2016) of the maximal jet model derived in Falcke & Biermann (1995). We agree with Zdziarski that in general a jet’s internal energy is not bounded by its rest-mass energy density. We describe the effects of the mistake on conclusions that have been made using the maximal jet model and show when a maximal jet is an appropriate assumption. The maximal jet model was used to derive a 1-D hydrodynamical model of jets in agnjet, a model that does multiwavelength fitting of quiescent/hard state X-ray binaries and low-luminosity active galactic nuclei. We correct algebraic mistakes made in the derivation of the 1-D Euler equation and relax the maximal jet assumption. We show that the corrections cause minor differences as long as the jet has a small opening angle and a small terminal Lorentz factor. We find that the major conclusion from the maximal jet model, the jet-disk symbiosis, can be generally applied to astrophysical jets. We also show that isothermal jets are required to match the flat radio spectra seen in low-luminosity X-ray binaries and active galactic nuclei, in agreement with other works.
3.1 Introduction

In this work we reexamine the maximal jet model of accreting black-holes with disks and jets derived in Falcke & Biermann (1995). The idea behind a maximal jet is that there is a strict upper limit on the power carried by a jet in terms of its mass flux. As first pointed out by Zdziarski (2016), the maximal jet is the result of an erroneous conclusion that the internal energy density of a gas must be less than or equal to the rest-mass energy density of the gas. The maximal jet model was used to link the power carried by the jet to the power in the accretion disk, arriving at the highly influential and widely used “jet-disk symbiosis.” The main result of the jet-disk symbiosis states that the total power in the jet, $L_j$, can be related to the mass accretion rate of the disk, $\dot{M}_{\text{disk}}$, through the jet’s Lorentz factor $\gamma_j$ and an efficiency, $\eta < 1$.

$$L_j = \eta \gamma_j \dot{M}_{\text{disk}} c^2.$$  

Without appealing to a maximal jet, we argue in this paper that the jet-disk symbiosis is reasonable for astrophysical accreting black-holes in general. In fact, it is reasonable to estimate the power of the jet to be $\lesssim \eta \dot{M}_{\text{disk}} c^2$, as long as one takes $\eta \lesssim a$ few, as opposed to being strictly less than one. It is intuitive why this conclusion should hold in a Blandford-Payne type jet where the disk itself is powering the jet (Blandford & Payne 1982), but the conclusion should also hold for jets that are powered by the black hole’s rotational energy via the Blandford-Znajek mechanism (Blandford & Znajek 1977). In the Blandford-Znajek mechanism, the jet power is proportional to the poloidal magnetic flux at the black hole, and the magnetic flux that can be carried to the black hole is ultimately limited by the mass-accretion rate (Narayan et al. 2003; Tchekhovskoy et al. 2011). This explains why the jet-disk symbiosis has been such a successful concept.

However, the maximal jet conclusion results solely from an algebraic mistake in Falcke & Biermann (1995) and cannot be applied broadly to accreting black-holes with jets. We argue in this work that if the jet is efficiently accelerated and has a small terminal Lorentz factor, the initial enthalpy should be roughly equal to the rest-mass energy, in agreement with a maximal jet. A jet with a large terminal Lorentz factor will start with a large enthalpy, but if the jet is efficiently accelerated, the internal energy of the jet will be approximately equal to the rest-mass energy density after the jet has reached its final Lorentz factor (Vlahakis & Königl 2003; McKinney 2006). There are some cases when this approximation breaks down. For a steady-state, axisymmetric, magnetically-accelerated outflow to be efficiently accelerated, it must stay causally connected in the transverse direction. However, it is unclear if this requirement is an actual impediment to the magnetic acceleration of astrophysical jets, or an artifact of the symmetries imposed. (For a concise review of magnetic acceleration of jets, see Komissarov 2011.) A radiation or thermal pressure-driven jet will be efficiently accelerated even if it is conical and free-streaming. We assume the jet opening angle is small enough to ensure it remains in causal contact with the
external medium throughout this paper.

In addition, we correct algebraic errors made in the derivation of the jet’s Lorentz factor as a function of distance in Falcke (1996). This Lorentz factor profile is used to calculate the dynamics of a jet in agnjet, an outflow dominated model of low-luminosity accreting black holes that has been applied to several different low-luminosity active galactic nuclei and X-ray binaries (Markoff et al. 2005, 2008; Maitra et al. 2009; van Oers et al. 2010; Markoff et al. 2015; Plotkin et al. 2015; Connors et al. 2017). The physics behind agnjet is presented in detail in Markoff et al. (2005) & Maitra et al. (2009). We characterize how the changes to the Lorentz factor profile effect the resulting spectral energy distribution calculated by agnjet. We find that the aforementioned algebraic mistakes have a negligible effect on the radiation from the outflow as long as the jet is roughly isothermal, has a small Lorentz factor, and is launched with an aspect ratio of order unity.

Our paper is organized as follows: in Section 3.2 we outline the mistake made when the maximal jet was derived. Then, we argue why the main conclusion of the maximal jet model, the jet-disk symbiosis, still holds. We also describe the systems for which the maximal jet model can be applied. In Section 3.3 we re-examine 1-D pressure-driven jets, relaxing the maximal jet requirement. In Section 3.4 we make the dynamics of agnjet self-consistent, and find the combined effects of all the changes to the dynamics on the calculated spectral energy distribution are small. We end by summarizing and discussing our results.

3.2 Bernoulli’s equation and Maximal Jets

The total power of an axisymmetric, conical jet at height $z$ from the launching point with an opening angle $\theta$ is equal to the jet’s Lorentz factor $\gamma_j$ times the enthalpy flux,

$$L_j = \gamma_j^2 \beta_j c \omega \pi z^2 \sin^2 \theta,$$

where $\omega$ is the enthalpy. In ideal magneto-hydrodynamics (MHD), a baryonic jet with a co-moving number density of protons $n$, has an enthalpy given by

$$\omega = n m_p c^2 + U_j + P_j = n m_p c^2 + U_{th} + P_{th} + \frac{B^2}{4\pi}.$$  \(3.2\)

$U_j, P_j$ are the total energy density and pressure of the jet, which can be broken down into a gas component ($U_{th}, P_{th}$) and a magnetic component ($U_B = P_B = B^2/8\pi$). \(^1\) The jet’s gas pressure can be related to the internal energy of the gas via the adiabatic index, $\Gamma$, $P_{th} = (\Gamma - 1) U_{th}$. We define the magnetization parameter, $\sigma$,

\(^1\)The gravitational potential energy and the radiation pressure and energy density contributions to the enthalpy are neglected in this work.
as $B^2/(4\pi nm_pc^2)^2$. The enthalpy then simplifies to

$$\omega = nm_pc^2 \left[ 1 + \sigma + \frac{\Gamma U_{\text{th}}}{nm_pc^2} \right] \quad (3.3)$$

In Falcke & Biermann (1995), the authors assume that the magnetic fields are isotropically turbulent, and therefore can be treated as an ideal gas with adiabatic index $\Gamma$. The authors then write the enthalpy in terms of the total jet internal energy density $U_j$ as

$$\omega \sim nm_pc^2 + \Gamma U_j \quad (3.4)$$

The authors then re-write the enthalpy in terms of the sound speed, $\beta_s$, adiabatic index, and density. The sound speed is

$$\beta_s^2 = \frac{\Gamma P_j}{\omega} = \frac{\Gamma(\Gamma - 1)U_j}{\omega} \quad (3.5)$$

(see e.g. Königl 1980). From Equations (3.4) & (3.5), one can derive the well known result that the maximal sound speed is $\sqrt{\Gamma - 1}$. Substituting Equation (3.5) into Equation (3.4) and solving for $\omega$ yields

$$\omega = \frac{nm_pc^2}{1 - \beta_s^2/(\Gamma - 1)} \quad (3.6)$$

Compare the above equation to the equivalent equation in Falcke & Biermann (1995). They give a formula that is the approximation when $\beta_s$ is small,

$$\omega \approx nm_pc^2 \left( 1 + \frac{\beta_s^2}{\Gamma - 1} \right) \quad \text{when} \quad \beta_s \ll \sqrt{\Gamma - 1}. \quad (3.7)$$

The mistake is that Falcke & Biermann (1995) then use Equation (3.7) to argue $\omega$ must be less than $2nm_pc^2$ because $\beta_s$ must be less than $\sqrt{\Gamma - 1}$. Clearly this is wrong, because when using the correct formula, Equation (3.6), $\omega$ diverges to infinity as $\beta_s \to \sqrt{\Gamma - 1}$. This mistake was first pointed out by Zdziarski (2016).

Since the total jet power is equal to the Lorentz factor times the enthalpy flux, the mistake leads to a maximal jet power $L_j \leq 2\gamma_j \dot{M}_j c^2$, where $\dot{M}_j$ is the mass flux through the jet. Protons are not created at the jet base, so Falcke & Biermann (1995) argue $\dot{M}_j \leq \dot{M}_{\text{disk}}$, where $\dot{M}_{\text{disk}}$ is the mass accretion rate of the disk. Therefore, Falcke & Biermann (1995) conclude $L_j \sim \eta \gamma_j \dot{M}_{\text{disk}} c^2$, $\eta \leq 1$, i.e., the jet-disk symbiosis. The mass flux through the jet could be larger than $\dot{M}_{\text{disk}}$ if the jet has significant baryon loading from winds. If the disk drives a strong wind, the disk wind may be able to increase the baryon loading, but it is generally thought that in X-ray binaries

\footnote{Our $\sigma$ is the same as the more standard magnetization parameter $\sigma$ in the “force-free” MHD regime. We neglect the gas pressure contributions to $\sigma$ to simplify the equations.}
a strong disk wind is associated with the soft states, i.e., when the jet is not present (Neilsen & Lee 2009). New evidence shows simultaneous winds and jets in the high state, but there is no evidence that shows a strong wind during the low hard-state (Muñoz-Darias et al. 2016; Kalencz et al. 2016; Muñoz-Darias et al. 2017). Stellar winds could also play a role in increasing $\dot{M}_j$ in AGN or high-mass X-ray binaries (Komissarov 1994). At any rate, the baryon loading would only significantly increase the power carried by the jet when the power of the intercepted baryons exceeds that of the jet.

It is a reasonable approximation that the power of the jet does not significantly exceed $\dot{M}_{BH}c^2$ even if the power in the jet is supplied by the spin of the black hole via the Blandford-Znajek mechanism. In the Blandford-Znajek mechanism, the power extracted is proportional to the magnetic flux carried to the black hole (Blandford & Znajek 1977). The magnetic flux carried to the black-hole is proportional to $\dot{M}_{BH}$, and GRMHD simulations find that the resultant jet power exceeds $\dot{M}_{BH}c^2$ by a factor of at most $\sim 3$ (Tchekhovskoy et al. 2011). However, there is no general requirement that the resultant jet’s power is dominated by the jet’s kinetic energy throughout the jet. For instance, in a Poynting-dominated jet, the power is $L_j = \gamma_j\dot{M}_j c^2 (1 + \sigma)$ and $\sigma$ can be $\gg 1$. Clearly, the enthalpy flux can be much larger than the mass flux, but in the next subsection, we will argue from completely different grounds than Falcke & Biermann (1995) that while a jet can start with any initial enthalpy, the jet will reach an $\omega/nm_p c^2 \sim 2$ at the modified magneto-sonic fast point if it is efficiently accelerated. Furthermore, in efficiently accelerated jets that have mildly relativistic terminal Lorentz factors, i.e., $\gamma_j \beta_j \sim 1$, the initial enthalpy will not exceed the rest mass energy density by a significant amount. The reason why this is true can be seen most easily via Bernoulli’s equation.

### 3.2.1 Bernoulli’s equation

In a steady-state, conservative jet, the total power carried by the jet is constant along the jet. Dividing the power by another conserved quantity, the particle number flux through a cross-section of the jet, one re-derives the relativistic Bernoulli’s equation (see e.g. Königl 1980):

$$\frac{\omega}{n} = \text{constant.}$$

(3.8)

Bernoulli’s equation is simply a statement that the energy of the fluid per particle must not change as the particles travel along a streamline, as long as particles are not created or destroyed. In this paper we are only considering self-similar jets with no gradient of the pressure in the toroidal direction of the jet. If a jet is launched with an initial Lorentz factor $\gamma_0$ with an initial enthalpy per particle of $\omega_0/n_0$, then because $\omega/n \geq m_p c^2$, it is clear from Eq (3.8) and (3.3) there is a maximal Lorentz
factor:

$$\gamma_{\text{max}} = \gamma_0 \frac{\omega_0}{n_0 m_p c^2} = \gamma_0 \left[ 1 + \sigma_0 + \frac{\Gamma U_{\text{th},0}}{n_0 m_p c^2} \right].$$

(3.9)

If a jet achieves $\gamma_{\text{max}}$, it means it has converted 100% of its Poynting and thermal energy into bulk kinetic energy. Doing so in a steady-state, Poynting-dominated jet requires that the jet be causally connected in the transverse direction. If the jet goes out of causal contact in the transverse direction, the magnetic pressure may not be able to accelerate the jet further. If the flow is completely spherical with $\sigma_0 \gg 1$, the outflow reaches a terminal Lorentz factor $\sim \sigma_0^{1/3}$ (Goldreich & Julian 1970). If instead the jet has an opening angle $\theta_j$, the terminal Lorentz factor is $\gamma_f = \min (\sigma_0, \sigma_0^{1/3} \theta_j^{-2/3})$ (Kumar & Zhang 2015). When the jet is thermally or radiatively driven, equation (3.9) should hold regardless of the jet’s geometry. Vlahakis & Königl (2003); Komissarov et al. (2007) show that in some Poynting dominated jets, electromagnetic fields will naturally self-collimate the jet and ensure that the jet remains causally connected up to the modified magneto-sonic fast point. In doing so the jet reaches rough equipartition between kinetic flux and Poynting flux, and the jet reaches a final Lorentz factor equal to $\sim \frac{1}{2} \omega_0/n_0 m_p c^2$. Therefore, using Bernoulli’s equation, the jet should have $\omega/nm_p c^2 \sim 2$ at the modified magneto-sonic fast point, right after the jet has finished accelerating. Furthermore, $\omega_0/n_0 m_p c^2 \approx 2$ is likely to be a good assumption for mildly-relativistic outflows with $\gamma_j \beta_j$ of order unity, because if $\omega_0 \gg n_0 m_p c^2$ the jets would reach $\gamma_j$ that are too large. Mildly relativistic jets are expected to occur in quiescent/hard state black-hole x-ray binaries (Gallo et al. 2003; Miller-Jones et al. 2007). A mildly relativistic jet may be launched by the super-massive black-hole at the center of our galaxy, Sgr A* (Falcke et al. 2009; Brinkerink et al. 2013).

Therefore, $\omega_0/n_0 \sim 2 m_p c^2$ should be a fine assumption as long as we restrict ourselves to mildly relativistic outflows with small enough opening angles. In jets with large Lorentz factors, $\gamma_j \beta_j \gg 5$, like blazars, BL Lacs, relativistic tidal disruption events, and gamma-ray bursts, $\omega_0/n_0 \gg 2 m_p c^2$, but if the jet is accelerated efficiently, at the modified magneto-sonic fast point $\omega/nm_p c^2 \sim 2$. The previous jet model of agnjet, and the one described in this paper, are not capable of reproducing the jets in these objects.

In the process of examining the maximal jet model, we have discovered additional minor algebraic mistakes in the Lorentz factor profile derived in Falcke (1996) and used in agnjet. In the rest of the paper, we show that these mistakes are minor

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3 The requirement that Poynting-dominated jets remain causally connected in the transverse direction can be relaxed if the outflow is impulsive (Granot et al. 2011).

4 The jet is launched with a sub-relativistic velocity, $\gamma_0 = 1$.

5 Although see Heinz & Merloni (2004); Miller-Jones et al. (2006) regarding the difficulties in placing a strong upper limit on the Lorentz factor of X-ray binaries.

6 If the highly relativistic jet is comprised of electron-positron pairs, then $\omega_0/n_0 \gg 2m_e c^2$. 

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and do not affect the conclusions drawn from fitting the model to low Lorentz factor sources. We then show that isothermal or nearly isothermal jets are required to have a flat radio spectrum.

### 3.3 Pressure-Driven Conical Jets

In this section we reproduce with minor corrections the derivation of the one-dimensional propagation of a quasi-isothermal hydrodynamic jet from Falcke (1996). We find that the corrected quasi-isothermal acceleration profile agrees within $\sim 20\%$ with the result in Falcke (1996), and the corrected Lorentz factor profile is much closer to the profile in a perfectly isothermal flow (see Figure 3.1).

The one-dimensional propagation of a supersonic jet in the $z$ direction follows the Euler equation given in Pomraning (1973); Falcke et al. (2009):

$$\gamma_j \beta_j n \frac{\partial}{\partial z} \left\{ \gamma_j \beta_j \frac{\omega}{n} \right\} = - \frac{\partial P_j}{\partial z} \quad (3.10)$$

Following Falcke & Biermann (1995), we assume that the jet is well-described by a fluid with adiabatic index 4/3 and use the enthalpy from Eq (3.4)

$$\omega = n m_p c^2 + \Gamma U_j, \quad (3.11)$$

so Eq (3.10) becomes:

$$\gamma_j \beta_j n \frac{\partial}{\partial z} \left\{ \gamma_j \beta_j \left( m_p c^2 + \frac{\Gamma U_j}{n} \right) \right\} = -(\Gamma - 1) \frac{\partial U_j}{\partial z}. \quad (3.12)$$

Particle number conservation along the jet forces the number density to be

$$n_j = n_0 \left( \frac{\gamma_j \beta_j}{\gamma_0 \beta_0} \right)^{-1} \left( \frac{z}{z_0} \right)^{-2} \left( \frac{\sin \theta}{\sin \theta_0} \right)^{-2}, \quad (3.13)$$

where $\theta$ is the opening angle of the jet (i.e. the cross-sectional radius of the jet divided by the height of the jet).

The jet is launched at an initial height of $z_0$, and is assumed to be traveling at the sound speed. The sound speed in the jet’s rest frame, $\beta_s$, is

$$\beta_s^2 = \frac{\Gamma P_j}{\omega} = \frac{\Gamma(\Gamma - 1) U_j}{n m_p c^2 + \Gamma U_j}. \quad (3.14)$$

Instead of requiring $U_j = n m_p c^2$ throughout the jet, we introduce a new parameter $\zeta$, which is the ratio between the initial internal energy of the jet and the initial rest-mass energy density, i.e., $U_0 = \zeta n_0 m_p c^2$. $\zeta = 1$ corresponds to the maximal jet conditions in Falcke (1996). The initial sound speed at the base of the jet is

$$\beta_{s0} = \sqrt{\frac{\zeta \Gamma(\Gamma - 1)}{1 + \zeta \Gamma}}. \quad (3.15)$$
Figure 3.1: This figure shows the difference between the Lorentz factor profile derived in Falcke (1996) and used in \textit{agnjet} c.f. Markoff et al. (2005) (yellow solid line), the derived Lorentz factor profile after correcting for algebraic mistakes (eq 3.18, red solid line), the 1-D Euler equation in a conical jet assuming continual heating of the jet by an outside source such that the jet is isothermal, \( U_j = n m_p c^2 \) (green dashed line), and the Euler equation in an adiabatic jet, where \( U_j = n_0 m_p c^2 (n/n_0)^\Gamma \) (lilac solid line). If the jet follows Bernoulli’s equation, \( \gamma_{\text{max}} \beta_{\text{max}} \approx 2.39 \) when \( \gamma_0 \beta_0 \approx 0.485 \) and \( \Gamma = 4/3 \)—precisely the terminal value in the adiabatic jet Lorentz factor profile.

For \( \zeta = 1, \Gamma = 4/3 \) the sound speed is \( \approx 0.43 \). Since the jet is assumed to be launched traveling at the sound speed, the initial velocity of the jet is

\[
\gamma_0 \beta_0 = 1/\sqrt{\beta_{s0}^2 - 1} = \sqrt{\frac{\zeta \Gamma (\Gamma - 1)}{1 + 2 \zeta \Gamma - \zeta^2}}, \tag{3.16}
\]

For \( \Gamma = 4/3, \zeta = 1, \gamma_0 \beta_0 \approx 0.485 \).

The jet Lorentz factor profile from Falcke (1996) can be derived by treating the jet as a conical jet \((\theta = \theta_0)\) and using particle number conservation to get a \( z \) dependence on the density (see Eq 3.13). To fix the \( z \) dependence on the internal energy, we need a prescription of how the temperature changes with a change in density. If the jet is isothermal, \( T_j \) is constant and \( U_e \propto n \). If the jet is adiabatic, \( U_e \sim nkT_j \propto n^\Gamma \), and therefore \( T_j \propto n^{\Gamma - 1} \propto (\gamma_j \beta_j)^{1-\Gamma} z^{2-2\Gamma} \). Falcke (1996) assumes that the gas is only able to do \( PdV \) work in the \( z \)-direction, hence the only adiabatic losses are due to the jet’s acceleration. We call this assumption quasi-isothermal. If the jet is quasi-isothermal, the temperature \( T_j \), is proportional to \((\gamma_j \beta_j)^{1-\Gamma}\). It is difficult to understand how exactly the gas is prevented from doing \( PdV \) work in the lateral direction. It is far more realistic to assume there is continuous particle acceleration to
counteract the adiabatic losses due to the expansion, as Blandford & Königl (1979) use to explain their isothermal jet model. For a heating mechanism to recover the quasi-isothermal temperature dependence, it means it must be capable of compensating the large adiabatic losses due to the lateral expansion, but not the comparatively small adiabatic losses from the acceleration. Why the heating mechanism would do this is unclear. We retain \( T_j \propto (\gamma_j \beta_j)^{1-\Gamma} \) here for historical reasons, and we note that when using the correct Euler equation, the difference between the quasi-isothermal case and the isothermal case is negligible for the small Lorentz factors achieved in jets with \( U_{j0} \sim n_0 m_p c^2 \) and \( \Gamma = 4/3 \), as assumed in this work.

In a quasi-isothermal jet, \( U_j \) is

\[
U_j = \zeta n_0 m_p c^2 \left( \frac{\gamma_j \beta_j}{\gamma_0 \beta_0} \right)^{\frac{\Gamma - 1}{\Gamma}} \left( \frac{z}{z_0} \right)^{-2}.
\]

(3.17)

Substituting Eqs (3.17) and (3.13) into Eq (3.10), and assuming the jet is launched with an initial \( \gamma_0 \beta_0 \) equal to the sound speed (Eq 3.16), the 1-D Euler equation that results is

\[
\left\{ \gamma_j \beta_j \frac{\Gamma + \xi}{\Gamma - 1} - \Gamma \gamma_j \beta_j - \frac{\Gamma}{\gamma_j \beta_j} \right\} \frac{\partial \gamma_j \beta_j}{\partial z} = \frac{2}{z};
\]

(3.18)

\[
\xi = \frac{1}{\zeta} \left( \frac{\gamma_j \beta_j}{\gamma_0 \beta_0} \right)^{\frac{\Gamma - 1}{\Gamma}}; \quad \gamma_0 \beta_0 = \sqrt{\frac{\zeta \Gamma (\Gamma - 1)}{1 + 2 \zeta \Gamma - \zeta \Gamma^2}}.
\]

(3.19)

The above equation should reduce to the jet Lorentz factor profile used in Falcke (1996); Markoff et al. (2005) when \( \zeta = 1 \). However, it differs from Eq (2) in Falcke (1996):

\[
\left\{ \gamma_j \beta_j \frac{\Gamma + \xi}{\Gamma - 1} - \Gamma \gamma_j \beta_j \right\} \frac{\partial \gamma_j \beta_j}{\partial z} = \frac{2}{z};
\]

(3.20)

\[
\xi = \left( \gamma_j \beta_j \frac{\Gamma + 1}{\Gamma (\Gamma - 1)} \right)^{1-\Gamma}
\]

(3.21)

The difference between our equation and the equation in Falcke (1996) can be accounted for as follows: the \(-\Gamma \gamma_j \beta_j \) term in Eq (3.18) results from a neglected \( \frac{\partial}{\partial z} (U_j/n) \) term, the difference in the exponent in \( \xi \) results from an arithmetic error, and finally the difference in the inside of the parenthesis of \( \xi \) terms is from setting \( \gamma_0 \beta_0 = \beta_s^2 \) instead of using the proper value given in Eq 3.16. The difference between the solutions of Eqs (3.18) and (3.20) are small and shown in Figure 3.1. In Figure 3.1, we also include solutions to the 1-D Euler equations when the jet is isothermal (\( T_j = \) constant, i.e., Eq 3.20 with \( \xi = 1 \)) and adiabatic (\( T_j \propto (\gamma_j \beta_j)^{1-\Gamma} z^{2-2\Gamma} \), see Eq 3.25).

The above quasi-isothermal and isothermal solutions do not conserve energy, nor do they follow the Bernoulli equation. The violation of Bernoulli’s equation is clear by
looking at maximal Lorentz factor on any pressure driven jet that conserves energy, with \( U_0 = \zeta n_0 m_p c^2 \), Eq (3.9) becomes

\[
\gamma_{\text{max}} = \gamma_0 (1 + \Gamma \zeta),
\]

(3.22)

or \( \gamma_{\text{max}} = 7\gamma_0/3 \) for a relativistic gas starting with equal parts internal energy density and rest mass energy density, \( U_0 = n_0 m_p c^2 \). All solutions except for the adiabatic solution eventually reach a \( \gamma_j \) that exceeds \( \gamma_{\text{max}} \), which is easily seen in Figure 3.1.

The total amount of heating needed to explain the solution in a quasi-isothermal jet is equivalent to the increase in the jet power. Using Eq (3.1) for the jet’s total power, the increase of power in a quasi-isothermal jet is

\[
\frac{L_j}{L_0} = \frac{1}{1 + \Gamma} \frac{\gamma_j}{\gamma_0} \left[ 1 + \Gamma \left( \frac{\gamma_j \beta_j}{\gamma_0 \beta_0} \right)^{1-\Gamma} \right].
\]

(3.23)

The required heating to power a quasi-isothermal jet is shown in Figure 3.2. The heating must come from some internal process in the jet, but the heating mechanism is not capable of being captured by our time-independent, laminar flow treatment in this work. One possibility is that heating originates from internal shocks (Malzac 2013). Internal shocks would do more than just heat the gas, they would also change the momentum and hence the dynamics. Magnetic reconnection can convert magnetic energy into thermal energy to keep the jet’s electrons isothermal, but reconnection would not increase the total power carried by the jet.

The jet will conserve energy if instead of assuming \( T_j \propto (\gamma_j \beta_j)^{1-\Gamma} \), we use an adiabatic jet where \( U_j \propto n^\Gamma \), or \( T_j \propto (\gamma_j \beta_j)^{1-\Gamma} z^{2-2\Gamma} \). In an adiabatic conical jet the \( z \) dependence of the internal energy is:

\[
U_j = \zeta n_0 m_p c^2 \left( \frac{\gamma_j \beta_j}{\gamma_0 \beta_0} \right)^{-\Gamma} \left( \frac{z}{z_0} \right)^{-2\Gamma},
\]

(3.24)

and the full 1-D Euler equation is

\[
\left\{ \gamma_j \beta_j \frac{\Gamma + \xi}{\Gamma - 1} - \Gamma \gamma_j \beta_j - \frac{\Gamma}{\gamma_j \beta_j} \right\} \frac{\partial \gamma_j \beta_j}{\partial z} = \frac{2\Gamma}{z} \left( 1 + \gamma_j^2 \beta_j^2 \right); \]

(3.25)

\[
\xi = \frac{1}{\zeta} \left( \gamma_j \beta_j \sqrt{\frac{1 + 2(\Gamma - \zeta^2)}{\zeta \Gamma (\Gamma - 1)}} \right)^{\Gamma-1} \left( \frac{z}{z_0} \right)^{2(\Gamma-1)}.
\]

(3.26)

The solution to the adiabatic 1-D Euler equation when \( \zeta = 1 \) is shown in Figure 3.1. Unlike the quasi-isothermal case, the jet reaches the maximal Lorentz factor equal to that predicted by the Bernoulli equation \( \gamma_0 (1 + \Gamma) \).
3.4 Radiation, Collimation, and a more Self-Consistent agnjet

The agnjet model was developed in Markoff et al. (2005); Maitra et al. (2009) as way of fitting multi-wavelength spectra of black-hole jets. The model is described in full detail in the aforementioned papers, but we will give a brief description here. In agnjet the gas is assumed to be moving at a velocity equal to the sound speed though a nozzle with constant radius \( r_0 \) that ends at \( z_0 \). At \( z_0 \), the jet is allowed to expand at constant velocity equal to the sound speed \( \gamma_0 \beta_0 \) in the cross-section radial direction, and weakly accelerated in the \( z \) direction by the jet’s pressure, i.e., it follows the 1-D Euler equation in the \( z \) direction. Electrons initially have a Maxwell-Juttner thermal distribution in the nozzle and the jet, and a fraction of the electrons are accelerated into a non-thermal population at a height \( z_{\text{acc}} \). The electrons radiate via synchrotron and inverse Compton processes.

The assumption of a lateral expansion at constant speed while accelerating vertically results in a jet that is self-collimating. The effect of the self-collimation on the dynamics was not previously considered in agnjet. We show here that the effect is

\[ \gamma_0 \beta_0 = \gamma_0 \beta_0 \]
small as long as the initial cross-sectional radius of the jet is roughly equal to the launch height.

The cross-sectional radius of the jet, $r$, is assumed to follow

$$ r = r_0 + (z - z_0)\gamma_0\beta_0 / (\gamma_j\beta_j), $$

and conservation of number density of particles is

$$ n = n_0 \left( \frac{\gamma_j\beta_j}{\gamma_0\beta_0} \right)^{-1} \left( \frac{r}{r_0} \right)^{-2}. $$

For the quasi-isothermal case, i.e., what is currently used in agnjet, the internal energy profile is

$$ U_j = \zeta n_0 m_p c^2 \left( \frac{\gamma_j\beta_j}{\gamma_0\beta_0} \right)^{-1} \left( \frac{r}{r_0} \right)^{-2}. $$
(ζ = 1 in agnjet). With the cross-sectional radius assumed in Equation (3.27), the 1-D Euler equation becomes:

\[
\left\{ \gamma_j \beta_j \left( \frac{\Gamma + \xi}{\Gamma - 1} - \Gamma \gamma_j \beta_j \right) - \frac{\Gamma}{\gamma_j \beta_j} \right\} \times \frac{\partial \gamma_j \beta_j}{\partial z} = \frac{2 \gamma_0 \beta_0}{r_0 \gamma_j \beta_j + \gamma_0 \beta_0 (z - z_0)} \quad \text{; (3.30)}
\]

and as before,

\[
\xi = \frac{1}{\zeta} \left( \frac{\gamma_j \beta_j}{\gamma_0 \beta_0} \right)^{\Gamma - 1} \quad \text{; \quad} \gamma_0 \beta_0 = \sqrt{\frac{\zeta \Gamma (\Gamma - 1)}{1 + 2 \zeta \Gamma - \zeta \Gamma^2}} \quad \text{. (3.31)}
\]

There is now another free parameter in the velocity profile: the initial aspect ratio of the jet: \(r_0/z_0\). We show the effects of this new parameter and compare our results to a quasi-isothermal conical jet in Figure 3.3.

To divvy up the total internal energy into an electron energy density and magnetic energy density, agnjet uses a free parameter \(k\), defined as the ratio of the magnetic energy density to the electron energy density. The dependence of the magnetic field on the height is

\[
B = \sqrt{\frac{8\pi k U_j}{1 + k}} \quad \text{. (3.32)}
\]

\(U_j\) has a height dependence that is different if the jet is assumed to be isothermal, quasi-isothermal, or adiabatic. In the isothermal jet, \(B \propto r^{-(\gamma_j \beta_j)^{-1/2}}\), which is slightly slower than the expected dependence if there is flux conservation of a toroidal magnetic field (\(\propto r^{-(\gamma_j \beta_j)^{-1}}\)). In the adiabatic or quasi-isothermal case, the magnetic field decreases faster than in an isothermal jet. The electron’s characteristic Lorentz factor \(\gamma_e\) has the same distance and bulk Lorentz factor dependence as the temperature. \(\gamma_{e,0}\) is a free parameter. The density profile of the electrons is determined by number conservation, but the initial number of electrons and positrons is fixed by requiring that \(U_{j,0} = n_p m_p c^2\),

\[
\frac{n_e}{n_p} = \frac{1}{1 + k} \frac{m_p}{\gamma_{e,0} m_e} \quad \text{. (3.33)}
\]

The above relationship will break down if \(k\) is too large, and it results in a non-physical, electrostatically charged jet with \(n_e < n_p\). \(k\) must satisfy the following inequality

\[
k + 1 \lesssim 110 \left( \frac{T_{e,0}}{10^{11} \text{ K}} \right)^{-1} \quad \text{. (3.34)}
\]

If agnjet requires a \(k\) large enough to violate the above inequality to fit the spectrum of an object, the \(k\) is inconsistent with the model of agnjet. In this case, it likely
 means that the jet is Poynting dominated, i.e., $U_B > U_p$, a scenario which is not considered in agnjet. We note that versions of agnjet prior to 2014 did not use eq (3.33) to set the number of electrons and positrons, and instead simply set $n_e = n_p$. In the earlier version, agnjet was only self-consistent if $1 + k$ was equal to the right hand side of eq (3.34). If we force $n_e = n_p$, as in the earlier version of agnjet, when $1 + k < 110 \left[ T_{e,0}/(10^{11} \text{ K}) \right]^{-1}$, the difference from a self-consistent solution can be estimated by solving the Euler equation in eq (3.30) with a $\zeta = (1 + k) \left[ T_{e,0}/(10^{11} \text{ K}) \right] /110$. The differences are likely to be minor even if $\zeta \ll 1$ because the jet is not accelerated very much even when $\zeta = 1$. If in the previous version of agnjet a solution was found with $1 + k > 110 \left[ T_{e,0}/(10^{11} \text{ K}) \right]^{-1}$, the jet is Poynting dominated, and its dynamics are not correctly calculated by agnjet.

We plot an example multiwavelength spectral-energy distribution using agnjet in Figure 3.4. In the figure it is clear that the quasi-isothermal, isothermal, and previous agnjet with minor errors all give roughly the same result: a nearly flat, self-absorbed synchrotron spectrum at frequencies below the thermal bump, and a Compton hump in the X-rays. In Figure 3.4 we assume the jet to be self-collimating when calculating the dependence of the internal energy of the jet with height, but we use the velocity profiles from Figure 3.1. Unlike the models with a constant or nearly-constant temperature throughout the jet, the adiabatic model shows a steep
rise from the self-absorbed radio emission to the thermal synchrotron bump. We find that to have a flat radio spectra the jet must be kept at a nearly constant temperature, a similar conclusion as found by Blandford & Königl (1979) when fitting the radio cores of AGNs or as found by Mościbrodzka & Falcke (2013) when fitting the radio spectrum of Sgr A*, or as found by Pe’er & Casella (2009) when fitting the radio emission of X-ray binaries.

### 3.5 Summary and Discussion

In this work, we have re-analyzed the hydrodynamical jets derived in Falcke (1996). When deriving the Lorentz factor profile, Falcke (1996) used the maximal jet model from Falcke & Biermann (1995), that contains an algebraic error. The maximal jet assumption has two main conclusions—the jet’s power is dominated by its kinetic power, and the jet power must be less than the mass accretion rate of the disk onto the black hole times the bulk Lorentz factor of the jet. We argued in this letter that the second conclusion is likely true in general for astrophysical jets. Even in jets that extract their energy from the black-hole spin, the energy in the jet does not greatly exceed $\dot{M}_{\text{BH}}c^2$ because the amount of magnetic flux that can be carried to the black hole depends on the mass accretion rate. The maximal efficiency of jet production found in GRMHD simulations is $\lesssim 300\%$ (Tchekhovskoy et al. 2011; Nemmen & Tchekhovskoy 2015). Furthermore, while it is true that the jet’s power need not be dominated by its kinetic power, we argued that this is a good approximation in a jet with a sufficiently small opening angle and a small terminal bulk Lorentz factor. X-ray binaries and low-luminosity AGN likely host such jets, so models based on these assumptions are well-founded for such objects. In addition, we corrected minor algebraic mistakes made in the derivation of the Lorentz factor profile from Falcke (1996). The effects of correcting these errors are to make the quasi-isothermal jet behave more similarly to a jet where the temperature is kept completely constant.

The Lorentz factor profile from Falcke (1996) was used in the agnjet model described in Markoff et al. (2005) & Maitra et al. (2009). agnjet has been used to fit the multiwavelength spectra of outflow dominated x-ray binaries and nearby low-luminosity active galactic nuclei. In agnjet, the jet is assumed to be self-collimating, but the collimation was not self-consistently applied to the jet’s dynamics. We show that the effects of the self-collimation are negligible for quasi-isothermal jets as long as the aspect ratio of the jet at the launching point is of order unity. We examined the effects of the new Lorentz factor profile on the spectral energy distribution calculated by agnjet. We find there is very little difference between the assumed quasi-isothermal jet and a completely isothermal jet, however, we find a large difference in the spectral energy distribution between an adiabatic jet and a jet where the temperature is kept roughly constant. We find that isothermal jets are required to
match the flat radio spectra seen in hard/quiescent state XRB and low-luminosity AGN. We do not self-consistently account for how the jet is kept hot. If the jet is heated internally, the heating mechanism would change the Lorentz factor profile from the one calculated in this work. If the gas is shock heated, the shocks will change the jet’s momentum, and if the jet is Poynting dominated, the dissipation of the magnetic field to heat the particles will change the magnetic pressure gradient.

Finally, we note that in addition to this work, there is an ongoing effort to derive a MHD-consistent jet model that will be capable of calculating the jet properties self-consistently, e.g. Polko et al. \cite{polko2010, polko2014}, Ceccobello et al. \textit{in prep.} Such models will be able to address many of the shortcomings of the models described in this work.
Inflows and outflows: What can we learn from the evolving spectral and timing properties of GX 339-4?

Some people say the glass is half empty or the glass is half full, but to me that’s irrelevant ’cause I’m having another drink.

— Sean Lock

R. M. T. Connors, D. van Eijnatten, C. Ceccobello, S. B. Markoff, V. Grinberg, L. Heil

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Abstract

GX 339–4 is a low-mass X-ray binary that has been a key focus of accretion studies due its high duty cycle, going into outburst on average once every two years. Tracking of its radio, IR and X-ray flux during multiple outbursts reveals a regular flux correlation between the three bands. Whilst it is clear that the radio emission originates in a compact, self-absorbed, quasi-isothermal jet, the origin of the X-ray emission is still debated: jet or corona? We fit 20 quasi-simultaneous radio, IR, optical and X-ray observations of GX 339–4 covering 3 separate outbursts in 2005, 2007, 2010–2011, with a composite corona + jet model, with inverse Compton emission from both regions contributing to the X-ray emission. Using a novel identifier of the X-ray variability properties known as power-spectral hue, we attempt to explain both the spectral and timing characteristics with the model. We find in most fits that whilst the bulk of
the X-ray flux can be explained by Inverse Compton scattering in a corona, constraints imposed on jet emission by radio and IR-optical data imply a non-negligible contribution from inverse Compton scattering in the jet, ranging from 10–50% of the integrated 10–40 keV flux. This non-negligible jet contribution to the X-ray spectra of LMXBs should be considered in disc reflection modelling. We also find that the jet in GX 339–4 appears more compact during early stages of its outburst decay, with cooler electrons, coincident with narrower broadband X-ray variability. We discuss current improvements being made to our treatment of the jet dynamics in our model in order to develop more self-consistent broadband spectral fitting.
4.1 Introduction

Accreting black holes are found to exist across a wide range of masses, from the stellar-mass remnants of stars (discoverable as the primaries of binary systems; X-ray binaries), to their supermassive ($10^6$–$10^{10} M_\odot$) analogues at the centres of galaxies (active galactic nuclei; AGN). Despite the disparity in mass, size-scale, and local environment, there is growing evidence that the physical nature of accretion flows around stellar-mass and supermassive black holes is mass-invariant, at least in the innermost regions (e.g., Merloni et al. 2003; Falcke et al. 2004; Körding et al. 2006b; Plotkin et al. 2012). This apparent scale-invariant property of accretion has led to the hypothesis that the diversity of AGN types arises due to the combination of observer viewing angle (Urry & Padovani 1995) and the evolving states of AGN (e.g., Körding et al. 2006b), akin to the changes we see occurring in X-ray binaries (see, e.g., Nowak 1995; van der Klis 1995; Remillard & McClintock 2006a; Belloni 2010). However, tracking the long timescale evolution of individual AGN to observe such state changes is not possible due to the orders-of-magnitude difference in dynamical times compared to those of X-ray binaries. We can instead further our understanding of the evolving properties of X-ray binaries, determine the physical conditions under which state-changes occur, and then see if it can account for the phenomenology observed in different AGN types.

X-ray binaries in which the secondary companion is a low-mass star (low-mass X-ray binaries; LMXBs) exhibit a wealth of transient phenomena, as observed across a broad range of the electromagnetic spectrum. Since the first discovery of such an object as an X-ray point source (Giacconi et al. 1962), we have up to now developed a broad picture of the behaviour of LMXBs as they evolve through states, primarily via the spectral and time-variability domains (see, e.g., Nowak 1995; van der Klis 1995; Remillard & McClintock 2006a; Belloni 2010). New transient sources are still being continually discovered (see, e.g., Tetarenko et al. 2016, for a recent catalogue of known sources).

The spectral classification of LMXB states can be loosely divided into two categories: soft and hard. In soft LMXB states the X-ray spectrum is dominated by a multi-temperature blackbody component, attributable to optically thick emission originating from a thin accretion disc (Shakura & Sunyaev 1973). In hard LMXB states we instead see a spectrum dominated by hard power-law emission which has a more ambiguous origin. Some models adopt either static or inflow geometries, whilst others place the emission region within the outflow/jet. Inflow/static models include inverse Compton (IC) or synchrotron self-Compton (SSC) scattering within an optically thin ‘corona’ or radiatively inefficient accretion flow (RIAF) in the inner regions of the accretion flow (Lightman & Eardley 1974; Eardley et al. 1975; Shapiro et al. 1976; Haardt & Maraschi 1993; Narayan & Yi 1994, 1995a; Esin et al. 1997). Outflow models instead propose either IC or optically thin synchrotron from within a
Studying the inflow/outflow geometry of GX 339-4

jet/outflow (Stirling et al. 2001; Fender 2001; Markoff et al. 2003; Yuan & Cui 2005). Understanding the interplay between these spectral components, determining which is the dominant mechanism at play, and explaining the connection between the accretion disc, corona and jet, is a key focus of recent targeted multiwavelength observing campaigns on LMXBs (e.g. Corbel et al. 2000, 2003; Gandhi et al. 2003, 2010; Miller-Jones et al. 2012; Russell et al. 2013a; Corbel et al. 2013; Russell et al. 2014).

Indeed targeted observing campaigns focused on LMXBs as they evolve through their outbursts have led to an empirical correlation between their X-ray and radio fluxes (e.g. Hannikainen et al. 1998; Corbel et al. 2000, 2003; Gallo et al. 2003; Corbel et al. 2008; Miller-Jones et al. 2011; Corbel et al. 2013; Gallo et al. 2014), and these correlations have been extended into the optical/NIR bands (e.g. Russell et al. 2006). The radio/X-ray correlation has been observed to cover several orders of magnitude in luminosity in the low hard states of some sources, such as GX 339−4 (Corbel et al. 2000, 2003, 2013), and V404 Cygni (Corbel et al. 2008), and the sources track the same correlation over different outbursts. So the radio/X-ray correlation is in a sense locked in as LMXBs evolve through their hard states. These correlations indicate that the allocation of power between the physical components in LMXBs, as a function of accretion rate, is an intrinsic property of hard state LMXBs. In order to draw robust conclusions about the drivers of state changes and the nature of jet launching, we need to be able to reliably identify the source of the X-ray emission.

Further information on the structure and evolution of LMXBs comes from studies in the time-variability domain. Fourier-domain analyses of the X-ray light curves of LMXBs have led to a complementary state classification, with low-rms variability observed in the soft states, and high-rms variability in hard states. In addition, quasi-periodic oscillations (QPOs)—sharp, narrow features (rms ~ 1–20%, $\nu/\Delta \nu \geq$ a few, e.g., Nowak 2000) in the power-spectra of LMXBs—have now been observed in multiple sources during outburst (Miyamoto et al. 1993; van der Klis 1995; Takizawa et al. 1997; Wijnands & van der Klis 1999). The broadband rms noise observed in LMXBs is attributed to fluctuations in the mass accretion rate propagating through the disc, roughly in adherence to the viscous flow timescale (Lyubarskii 1997; Uttley & McHardy 2001). QPOs observed in LMXBs have been interpreted both as precessing structures in the inner regions of the accretion flow due to misalignment of the accretion flow and black hole (or neutron star) spin (Lense-Thirring precession (Stella & Vietri 1998; Fragile et al. 2001; Schnittman et al. 2006; Schnittman et al. 2003; Ingram et al. 2009; Ingram & Done 2011), and as wave modes in the accretion flow (e.g. Wagoner et al. 2001).

Aligning these models of the accretion flow and how they interact with the jet/outflow in LMXBs requires a combination of broadband (radio-to-X-ray) timing and spectral information; these two pictures are seldom treated in unison however, unfortunately. Heil et al. (2015) developed a novel state classification method for LMXBs (including those systems with a neutron star as the primary) which characterises the shape of
4.1 Introduction

Figure 4.1: **Left**: A conceptual diagram representing an LMXB power spectrum divided into frequency bins in log space. Two power-colour ratios are defined as \( PC_1 = C/A \), \( PC_2 = B/D \), where A, B, C, and D are the integral power across the defined frequency bands. **Right**: A power-colour hue diagram adapted from Heil et al. (2015). The angular position in degrees is defined as the hue, and the corresponding states are indicated roughly as soft from \( 300^\circ - 20^\circ \), hard from \( 340^\circ - 140^\circ \), Soft intermediate (SIMS) from \( 220^\circ - 300^\circ \) and hard intermediate (HIMS) from \( 140^\circ - 220^\circ \). Soft and hard states overlap in the top left of the diagram because their power-spectra have a similar shape, though the normalisations are different—hard states have stronger broad-band variability than soft states. A LMXB will start from the top left of the diagram, follow a clockwise path during outburst back to its original position, and then move anti-clockwise through outburst decay back towards the hard state.

the power spectrum of their light curves through the course of an outburst, analogous with the well-known hardness intensity diagram of LMXB states (HID; Homan et al. 2001; Homan & Belloni 2005, Belloni 2004; Belloni et al. 2005). A single variable, the power-spectral ‘hue’, encodes the relation between two ratios of integrated power across individual frequency bands in Fourier space (see Figure 4.1). One can use this information to track spectral modelling parameters alongside timing characteristics, and since the timing characteristics represent complementary changes in the system configuration over time, we would expect to see some consistency between the physical state of the inner accretion flow/jet and the hue.

4.1.1 This paper

In this paper we combine the analysis of Heil et al. (2015) with broadband spectral information to build a consistent picture of the evolution of the jet/accretion flow of GX 339–4, by probing the dominant spectral components and comparing model parameters with the evolution of its variability. We focus in particular on the relative dominance of the jet base and corona in the X-ray band. In Section 4.2 we present the radio-IR-optical-X-ray data compilation we use for model-fitting. In Section 4.3 we briefly discuss the outflow-dominated model used in our fits. In Section 4.4 we
present our spectral-modelling method and the results of fits to X-ray and broadband (radio, IR-optical, X-ray) spectra, as well as the key parameter trends with variability properties of GX 339–4. In Section 5.5 we discuss the significance of these parameter trends and comparisons with previous modelling of the broadband spectra GX 339–4. In Section 4.6 we summarise our results and conclude.

4.2 GX 339–4: physical characteristics and data selection

GX 339–4 has been one of the most intensely studied LMXBs since its discovery in 1973 (Markert et al. 1973), due primarily to its short X-ray duty cycle, a characteristic of its long orbital period of $P_{\text{orb}} \sim 1.7$ days (Hynes et al. 2003); an approximate calculation of the duty cycle of a LMXB with an irradiated accretion disc gives $t_o/(t_o + t_q) \sim 1/(1 + P_{\text{orb}}^{-1})$, where $t_o$ is the outburst time, $t_q$ is the quiescence time (King & Ritter 1998). As such we have extensive spectral and timing information of GX 339–4 covering multiple outbursts (7 with simultaneous radio/X-ray coverage; see Corbel et al. 2013), making it the ideal candidate for studies of how spectral properties (and the physical mechanisms behind them) track the time variability behaviour in LMXBs.

One caveat of conducting such studies on GX 339–4 is the lack of accuracy achieved in determining its physical properties. The most heavily cited and utilised mass function measurement is that obtained by Hynes et al. (2003) of $5.8 \pm 0.8 \, M_\odot$, and a later estimate included a lower limit of $7 \, M_\odot$ (Muñoz-Darias et al. 2008). More recent near-infrared detections of absorption lines from the donor star of GX 339–4 indicate a mass function of $\sim 1.91 \pm 0.08 \, M_\odot$ (Heida et al. 2017). Distance has also been difficult to determine, with early estimates finding a broad range from 6–15 kpc (Hynes et al. 2004), and best estimates giving $\sim 8$ kpc (Zdziarski et al. 2004), based on a comparison of the redshifted spectral lines seen in GX 339–4 with those of stars in the Galactic bulge region at $D = 8 \pm 2$ kpc, and the high peculiar velocity of GX 339–4 ($v \sim 140$ km s$^{-1}$; Hynes et al. 2004). One of the most elusive physical properties of GX 339–4 has been the orbital inclination. With almost no model-independent consensus, we mostly rely on modelling of accretion disc reflection in the X-ray spectra to determine the inclination. The extremely model-dependent method of reflection modelling has derived inclination estimates over a large range: $15^\circ$–$50^\circ$ (Miller et al. 2006; Reis et al. 2008; Done & Díaz Trigo 2010; Plant et al. 2014, 2015; García et al. 2015b; Parker et al. 2016). These values are not wholly reliable for two reasons: 1) these are model-dependent estimates that are degenerate with other key parameters of the reflection models, and 2) the disc inclination may not be equal to the orbital inclination of the binary (see, e.g., Wijers & Pringle 1999; Maccarone 2002; Begelman et al. 2006).
GX 339–4 has a compact radio jet during the hard state (Fender 2001), and the emission from this jet dominates up to IR (Corbel & Fender 2002) and possibly optical frequencies (Gandhi et al. 2008, 2010, 2011; Casella et al. 2010). Correlations between the optical/IR/X-ray light curves during various GX 339–4 outbursts indicate a physical connection between the regions near the black hole and the self-absorbed regions of jets at $\sim 10^3–10^4 \, r_g$. Recently Kalamkar et al. (2016) reported the first detection of a QPO at IR frequencies in an LMXB with their Very Large Telescope/ISAAC observations of GX 339–4 (0.08 Hz). They found the IR QPO to be at roughly half the frequency of the X-ray QPO (0.16 Hz), at the same frequency as the previously discovered optical band QPO (Motch et al. 1982; Gandhi et al. 2010), with IR photons lagging X-ray photons by $\sim 100 \, \text{ms}$. This lag between high and low frequencies could be interpreted as variations propagating through the jet, giving rise to delayed IR variability. The existence of the QPO in multiple bands implies a structural connection between the corona and the jet.

### 4.2.1 Data

We combine data from 20 separate quasi-simultaneous (all observations within 24-hrs of one another), broadband observations, covering the radio, near-IR, and X-ray bands. Here we describe how the data were collected and reduced.

### 4.2.2 X-ray data

Data from RXTE’s proportional counter array (Jahoda et al. 2006, PCA) and High-Energy Timing Experiment (Rothschild et al. 1998, HEXTE) were extracted using HEASOFT 6.16 following the standard procedure as described, e.g., in Grinberg et al. (2013), in particular discarding data within 10 minutes of the South Atlantic Anomaly passages.

For PCA, we use data from the top xenon layer of proportional counter unit (PCU) 2 only since these data are best calibrated. We apply PCACORR calibration tool (García et al. 2014b) to further improve the data quality. No HEXTE data are available for over half of our observation (spectra 11–21, Table 4.2) due to the failure of the rocking mechanisms of both HEXTE clusters late in the RXTE mission lifetime. We extract cluster A and B data where available. We refrain from using the HEXTECORR calibration tool (García et al. 2016) on the HEXTE B data as the improvement would only be marginal given the data quality.

The PCA light curves are used to calculate the power-spectral hue of each observation (shown in Tables 4.1 and 4.2), following the method of Heil et al. (2015).
4.2.3 Radio/IR-Optical data

We select radio fluxes of GX 339−4 covering a 15-year period (1997–2012) resulting from observations made with the Australian Telescope Compact Array (ATCA) Corbel et al. (2013), choosing only those observations falling within a 24-hr window of the corresponding X-ray observations. We then include optical and near-infrared fluxes resulting from observations of GX 339−4 made with the SMARTS 1.3 m telescope from 2002–2010, covering the V, J, I and H bands (Buxton et al. 2012). The magnitudes in all four bands are de-reddened assuming $N_H = 5 \pm 1 \times 10^{21} \text{ cm}^{-2}$ (Kong et al. 2002a), giving $E(B−V) = 0.94 \pm 0.19$ (Predehl & Schmitt 1995), such that $A_V = 2.9 \pm 0.6$ (Cardelli et al. 1989). The flux density values quoted in Tables 4.1 and 4.2 are the de-reddened flux densities given by Buxton et al. (2012). We reject SMARTS observations that fall outside the 24 hour window of the pre-selected quasi-simultaneous radio and X-ray observations. This selection criterion leaves us with 20 separate broadband quasi-simultaneous spectra of GX 339−4, covering the decay of its 2005 outburst, the peak and decay of its 2007 outburst, and the rise and decay of its 2010 outburst. A full description of the data is shown in Tables 4.1 and 4.2.

Selecting quasi-simultaneous data with a 24-hr time-window coincidence across radio/NIR/optical/X-ray bands in this way optimises the trade-off between the quantity of data we require for our modelling, and the information lost by neglecting source variability on short timescales. Gandhi et al. (2011) show that the mid-IR spectral slope is variable on timescales of ∼ 20 minutes. We must therefore consider the uncertainties in the overall flux and spectral slope incurred by grouping data over the 24-hr time window, and the effect this has on estimates of our best-fit parameters (see Section 4.4).
Table 4.1: The broadband quasi-simultaneous data of GX 339–4. Shown are radio, optical and near infrared fluxes along with X-ray spectral data fluxes for each MJD of the observation.

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<th>$F_R$ [mJy]</th>
<th>$F_{\text{NIR-opt}}$ [mJy]</th>
<th>ObsID</th>
<th>$F_X$ [$10^{-10}$ erg s$^{-1}$ cm$^{-2}$]</th>
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<th>Hue [$^\circ$]</th>
<th>HEXTE?</th>
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<td>1.7 ± 0.1</td>
<td>11 ± 1, 10 ± 1</td>
<td></td>
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</tr>
<tr>
<td>7</td>
<td>54135</td>
<td>19.5 ± 0.3</td>
<td>120 ± 40, 72 ± 7</td>
<td>92035-01-02-02</td>
<td>104.2 ± 0.1</td>
<td>0.56</td>
<td>87 ± 5</td>
<td>B only</td>
</tr>
<tr>
<td></td>
<td></td>
<td>17 ± 1</td>
<td>67 ± 6, 65 ± 6</td>
<td></td>
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</tr>
<tr>
<td>8</td>
<td>54258</td>
<td>2.6 ± 0.2</td>
<td>16 ± 5, 12 ± 1</td>
<td>92704-03-26-00</td>
<td>3.02 ± 0.05</td>
<td>0.60</td>
<td>22 ± 10</td>
<td>No</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2.6 ± 0.2</td>
<td>10 ± 1, 10 ± 1</td>
<td></td>
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</tr>
<tr>
<td>9</td>
<td>54335</td>
<td>3.3 ± 0.05</td>
<td>26 ± 8, 20 ± 2</td>
<td>93409-01-05-03</td>
<td>7.33 ± 0.05</td>
<td>0.62</td>
<td>18 ± 6</td>
<td>B only</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2.95 ± 0.07</td>
<td>17 ± 2, 20 ± 2</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>10</td>
<td>55240</td>
<td>6.17 ± 0.06</td>
<td>47 ± 15, 29 ± 3</td>
<td>95409-01-06-00</td>
<td>17.53 ± 0.05</td>
<td>0.67</td>
<td>347 ± 3</td>
<td>No</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5.9 ± 0.1</td>
<td>31 ± 3, 31 ± 3</td>
<td></td>
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</table>
Table 4.2: Continuation of Table 4.1 showing the broadband quasi-simultaneous data (spectra 11–20) of GX 339–4. Shown are radio, optical and near infrared fluxes along with X-ray spectral data fluxes for each MJD of the observation.

<table>
<thead>
<tr>
<th>#</th>
<th>MJD</th>
<th>$F_R$ [mJy]</th>
<th>$F_{NIR-opt}$ [mJy]</th>
<th>ObsID</th>
<th>$F_X$ [10^{-10} \text{ erg s}^{-1} \text{ cm}^{-2}]$ [2–20 keV]</th>
<th>HR</th>
<th>Hue</th>
<th>HEXTE?</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(−245000)</td>
<td>5.5 GHz</td>
<td>8.8 GHz</td>
<td>V, I J, H</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>55260</td>
<td>7.2 ± 0.1</td>
<td>65 ± 21, 44 ± 4</td>
<td>95409-01-08-03</td>
<td>29.94 ± 0.06</td>
<td>0.65</td>
<td>359 ± 2</td>
<td>No</td>
</tr>
<tr>
<td></td>
<td></td>
<td>7.3 ± 0.1</td>
<td>37 ± 4, 39 ± 4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>55263</td>
<td>8.24 ± 0.05</td>
<td>62 ± 20, 39 ± 4</td>
<td>95409-01-09-01</td>
<td>33.4 ± 0.1</td>
<td>0.65</td>
<td>358 ± 3</td>
<td>No</td>
</tr>
<tr>
<td></td>
<td></td>
<td>8.1 ± 0.1</td>
<td>40 ± 4, 39 ± 4</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>13</td>
<td>55271</td>
<td>10.2 ± 0.1</td>
<td>92 ± 29, 54 ± 5</td>
<td>95409-01-10-03</td>
<td>41.53 ± 0.07</td>
<td>0.63</td>
<td>15 ± 3</td>
<td>No</td>
</tr>
<tr>
<td></td>
<td></td>
<td>11.3 ± 0.1</td>
<td>50 ± 5, 47 ± 5</td>
<td></td>
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<tr>
<td>14</td>
<td>55277</td>
<td>13.8 ± 0.1</td>
<td>99 ± 32, 58 ± 6</td>
<td>95409-01-11-02</td>
<td>57.0 ± 0.2</td>
<td>0.61</td>
<td>19 ± 13</td>
<td>No</td>
</tr>
<tr>
<td></td>
<td></td>
<td>15.45 ± 0.06</td>
<td>48 ± 5, 48 ± 5</td>
<td></td>
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</tr>
<tr>
<td>15</td>
<td>55280</td>
<td>15.56 ± 0.05</td>
<td>88 ± 28, 56 ± 5</td>
<td>95409-01-11-03</td>
<td>63.7 ± 0.2</td>
<td>0.60</td>
<td>45 ± 10</td>
<td>No</td>
</tr>
<tr>
<td></td>
<td></td>
<td>18.59 ± 0.05</td>
<td>54 ± 5, 51 ± 5</td>
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</tr>
<tr>
<td>16</td>
<td>55290</td>
<td>18.8 ± 0.1</td>
<td>N/A, 54 ± 5</td>
<td>95409-01-13-00</td>
<td>83.2 ± 0.1</td>
<td>0.58</td>
<td>38 ± 26</td>
<td>No</td>
</tr>
<tr>
<td></td>
<td></td>
<td>21.1 ± 0.2</td>
<td>56 ± 5, 52 ± 5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>55605</td>
<td>4.45 ± 0.04</td>
<td>7 ± 2, 4.2 ± 0.4</td>
<td>96409-01-07-03</td>
<td>6.32 ± 0.05</td>
<td>0.60</td>
<td>95 ± 17</td>
<td>No</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4.17 ± 0.05</td>
<td>2.7 ± 0.3, 2.0 ± 0.2</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>18</td>
<td>55608</td>
<td>4.07 ± 0.04</td>
<td>9 ± 3, 5.3 ± 0.5</td>
<td>96409-01-07-02</td>
<td>5.02 ± 0.05</td>
<td>0.56</td>
<td>138 ± 7</td>
<td>No</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3.87 ± 0.05</td>
<td>3.5 ± 0.3, 2.8 ± 0.3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>19</td>
<td>55610</td>
<td>3.9 ± 0.1</td>
<td>10 ± 3, 6.1 ± 0.6</td>
<td>96409-01-07-04</td>
<td>4.05 ± 0.05</td>
<td>0.63</td>
<td>80 ± 18</td>
<td>No</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4.0 ± 0.1</td>
<td>4.6 ± 0.4, 4.1 ± 0.4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>55618</td>
<td>2.54 ± 0.04</td>
<td>16 ± 5, 11 ± 1</td>
<td>96409-01-09-00</td>
<td>1.84 ± 0.03</td>
<td>0.57</td>
<td>14 ± 15</td>
<td>No</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2.95 ± 0.05</td>
<td>9.5 ± 0.9, 8.6 ± 0.8</td>
<td></td>
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</tr>
</tbody>
</table>
4.3 The model

We use a semi-analytical, zonal jet model (see Markoff et al. 2005; Maitra et al. 2005; Connors et al. 2017). To calculate the dynamics, we assume the LMXB launches a roughly isothermal jet that is accelerated to mildly relativistic velocities by internal pressure (Falcke & Biermann 1995; Crumley et al. 2017). We make two key improvements upon previous implementations of the model.

Firstly, the calculations of IC emission within the jet has been improved to include multiple scattering events rather assuming a single-scattering treatment. This allows us to treat cases in which the flux contribution from higher IC scattering orders may be significant. In LMXBs we generally expect the IC-emitting regions to remain optically thin (Haardt & Maraschi 1993; Done et al. 2007). However, even as the IC region approaches $\tau \sim 1$, the higher-energy emission may become relevant. Details can be found in a forthcoming paper (Ceccobello et al., in prep.).

Secondly, we have altered our method of defining the jet zones to improve the treatment of IC scattering in the first few zones of the jet. In all previous implementations of the model, a log scale was used between $z_{min}$ and $z_{max}$, where $z_{min} \sim 0.3 \ r_0$ and $z_{max}$ is a model parameter. Instead we now enforce $\Delta z = 2 \ r$ in all zones where IC-scattering significantly contributes to the spectrum ($z_{cut} = 100 \ r_0$), and space the remaining zones logarithmically up to $z_{max}$. In this way, we treat the input photon distribution for IC scattering as roughly isotropic without incurring any resolution-dependent errors, and without losing too much resolution in the effects of the jet profile at low heights.

We refer the reader to Connors et al. (2017) for a description of the model prior to the two changes discussed above, and to Table 4.3 for a description of the key physical parameters of the model. The model has the name agnjet, reflective of its applications to mildly-relativistic ($\gamma_j \sim$ a few) jets in Active Galactic Nuclei (Markoff et al. 2008, 2015; Prieto et al. 2016; Connors et al. 2017; Crumley et al. 2017; van Oers et al. 2017).

4.4 Spectral fits

We perform all spectral fits in this work using the multiwavelength data analysis package ISIS (Houck & Denicola 2000), version 1.6.2-40. All models are forward-folded through the detector response matrices; when fitting to X-ray spectra this corresponds the Proportional Counter Array (PCA) and High Energy X-ray Transmission Spectrometer (HEXTE) instrument responses, whereas data at all other wavelengths is assigned a "dummy" response equivalent to a detector of effective area = 1 m$^2$. Data at wavelengths outside the X-ray band are loaded into ISIS as flux measurements (shown in Tables 4.1 and 4.2). We bin PCA spectra at a minimum signal-to-noise
Table 4.3: A list of the main input parameters of the agnjet model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_j$</td>
<td>the normalised jet power, in units of $L_{Edd}$.</td>
</tr>
<tr>
<td>$r_0$ and $h_0$ ($r_g$)</td>
<td>the radius and height (length) of the jet nozzle. The height is fixed at $h_0 = 2r_0$, such that the nozzle is a cube.</td>
</tr>
<tr>
<td>$\Theta_e (kT_e/mc^2)$</td>
<td>the electron temperature of the input distribution.</td>
</tr>
<tr>
<td>$\beta_e$</td>
<td>the ratio of electron to magnetic energy density, $U_e/U_B$, at the base of the jet.</td>
</tr>
<tr>
<td>$p$</td>
<td>the power-law index of the accelerated electron distribution.</td>
</tr>
<tr>
<td>$z_{acc}$ ($r_g$)</td>
<td>the distance from the black hole along the jet axis where particle acceleration begins.</td>
</tr>
<tr>
<td>$f_{nth}$</td>
<td>the fraction of particles accelerated at $z_{acc}$ into a power law distribution.</td>
</tr>
<tr>
<td>$f_{sc}$</td>
<td>$\beta_{sh}^2/(\lambda/R_{gyro})$ where $\beta_{sh}$ is the shock speed relative to the plasma, $\lambda$ is the scattering mean free path in the plasma at the shock region, and $R_{gyro}$ is the gyroradius of the particles in the magnetic field. In reality we do not require a shock so this parameterisation can generally be seen as a measure of the particle acceleration efficiency.</td>
</tr>
</tbody>
</table>

ratio of $S/N = 4.5$, between energy limits of 3–45 keV or 3–20 keV depending on the quality of the counts data in the highest energy bins. A systematic error of 0.1% is added to the PCA counts based on the improved calibration tool PCACORR (García et al. [2014b]). We include HEXTE A/B spectra for the observations indicated in Tables 4.1 and 4.2 and bin each at minimum signal-to-noise $S/N = 4.5$ between energy limits 20–200 keV. We fix the observational characteristics of GX 339–4 at distance $D = 8$ kpc (Zdziarski et al. [2004]), inclination $i = 40^\circ$, and mass $M_{BH} = 9.7 M_\odot$ adopting the mass function of Heida et al. (2017) (see Section 4.1). At each stage of the fitting process we use the ISIS implementation of Markov Chain Monte Carlo (MCMC) parameter exploration routine (Murphy & Nowak 2014), based on the popular routine, emcee, developed by Foreman-Mackey et al. (2013). In each case we initialize $50 \times n_{fp}$ walkers per free parameter, where $n_{fp}$ is the number of free parameters, and we run the MCMC chain until it has converged—we judge convergence as the point beyond which changes to the posterior probability distribution functions of the parameters are minimal.

4.4.1 X-ray spectral fits

Before exploring broadband model fits to the quasi-simultaneous data of GX 339–4, we first fit phenomenological models to the available X-ray spectra in order to place
prior constraints on key parameters, allowing us to reduce the uncertainties in our broadband fits. These include the Interstellar absorption cross-section, $n_H$, the energy of the Gaussian iron emission line resulting from disc reflection, $E_{\text{line}}$, and its corresponding line width, $\sigma_{\text{line}}$. We fix the value of $n_H = 5 \times 10^{21}$ cm$^{-2}$ based on previous X-ray spectral modelling of GX 339–4 (Shidatsu et al. 2011; García et al. 2015b; Parker et al. 2016), and extinction cross-section applied to the IR-optical data (Kong et al. 2002a). We consider 3 model classes, assigned according to the breadth of X-ray band coverage and the number of X-ray counts in the spectra: model X1) $\text{tbabs} \times (\text{powerlaw} + \text{gaussian})$, model X2) $\text{tbabs} \times \text{reflect(}\text{powerlaw} + \text{gaussian}\text{)}$, model X3) $\text{tbabs} \times \text{reflect(}\text{(powerlaw} + \text{gaussian)} \times \text{highecut}\text{)}$. The reflection convolution model $\text{reflect}$ is from Magdziarz & Zdziarski (1995), and we adopt this in preference to more recent reflection models ($\text{RELXILL}$; Dauser et al. 2014; García et al. 2014a, $\text{REFLIONX}$; Ross et al. 1999; Ross & Fabian 2005) since it convolves an arbitrary input spectrum, whereas the more recent models rely on robust model tables that are expensive to produce (however see van Eijnatten et al., in prep.). The absorption model $\text{tbabs}$ is described in Wilms et al. (2000). We adopt the solar abundances of Wilms et al. (2000) and set the photo-ionisation cross-sections according to Verner et al. (1996). Model X1 is most likely to provide a sufficient fit to those X-ray spectra with low source counts, model X2 (a reflected power law) will apply when source counts are high enough to distinguish a break in the spectrum at $E \sim 10$ keV, characteristic of a reflected X-ray spectrum, and model X3 applies to only one spectrum for which we have enough source counts up to $\sim 100$ keV to require a cut off in the spectrum. We perform Markov Chain Monte Carlo (MCMC) parameter exploration to each X-ray dataset in order to characterise the posterior probability distribution functions (PDF) of $E_{\text{line}}$ and $\sigma_{\text{line}}$. We fix $E_{\text{line}}$ and $\sigma_{\text{line}}$ based on these fits, and carry those values forward to our broadband spectral modelling described in Section 4.4.2.

Figure 4.2 shows the evolution of $\Gamma$, the power-law spectral index, against both the power-spectral hue and the unfolded data luminosity (assuming $D = 8$ kpc), showing a clear dichotomy between the more luminous X-ray spectra of the 2010 outburst rise and 2007 single observation of its outburst, with the observations in the decay phases of 2005, 2007, and 2011. The spectrum appears to slightly soften with increasing luminosity/hue. The increase in power-spectral hue is coincident with increasing luminosity as the source evolves through its outburst, but with two distinct trends depending on whether the source is in the rise or decay of an outburst (see Figure 4.3). Many previous works find $\Gamma$ to be mostly constant during the rising hard state of GX 339–4 (Wilms et al. 1999; Zdziarski et al. 2004; Plant et al. 2014; García et al. 2015b), so the fact that we see a slight positive correlation may be because most of our observations are in the low/hard state ($L_X/L_{\text{Edd}} \leq 0.01$). However, such a trend does agree with the broader trends seen in multiple LMXBs (see e.g. Remillard & McClintock 2006a), and is coincident with the narrowing and strengthening of
4 Studying the inflow/outflow geometry of GX 339-4

Figure 4.2: The power-law spectral index ($\Gamma_{pl}$, derived from initial spectral fits to all 20 X-ray spectra) against the power-spectral hue (left) and unfolded data luminosity (right) between 2–10 keV. The spectrum softens slightly with both the increasing luminosity and changes in the shape of the power spectrum. The key shows how the data are divided into the year of the observation.

broadband X-ray variability (Heil et al. 2015), and brighter radio jets (Fender 2006).

4.4.2 Broadband spectral modelling

Next, we model all 20 of the broadband spectra energy distributions (SEDs) of GX 339−4 with two more physical scenarios: B1) an absorbed, reflected jet component + gaussian line: $tbabs \times reflect(agnjet+gaussian)$ and B2) the sum of absorbed, reflected jet and coronal components + Gaussian line, thus only distinguishable from model B1 by the additional corona, $tbabs \times reflect(agnjet+nthcomp+gaussian)$, where $agnjet$ represents the jet component, and $nthcomp$ represents a spherical corona in the inner regions of the accretion flow (Zdziarski et al. 1996; Życki et al. 1999). Figure A shows a diagram of the setup which represents spectral model B2. Whilst $agnjet$ does in fact include a coronal-like jet base (Markoff et al. 2005), its treatment of synchrotron self-Compton is purely relativistic, allowing only for photon-scattering electrons at $\Theta_e \equiv kT_e/m_ec^2 \geq 1 \ (kT_e \geq 511 \text{ keV})$—this is due to the expectation that energy is dissipated to the electrons quite rapidly within the jet, giving rise to high synchrotron fluxes in the radio bands in regions further out along its axis;
conservation arguments suggest similarly hot electrons ($\Theta_e > 1$) at the base of the jet (see e.g. Markoff et al. 2005). Popular models for the X-ray spectra observed in LMXB hard states typically include `nthcomp` in which a thermal population of electrons at roughly $\Theta_e \sim 0.02$–0.2 IC scatter the soft blackbody component of the accretion flow with seed photons temperatures in the range $kT_{BB} \sim 0.01$–1 keV, set by the inner disc temperature (see e.g. Haardt & Maraschi 1993; Done et al. 2007 and references therein). Thus in our model-fitting treatment we choose to test a combination of both emission components in order to determine the relevant importance of each during an evolving outburst.

We fix the location where particle acceleration starts to $\log_{10}[z_{acc}] = 3.5$ since this is the approximate location at which a non-thermal population of electrons is generated in GX 339–4, according to the location of the variable self-absorption spectral break (Markoff et al. 2003; Gandhi et al. 2011). If we interpret a time lag of 100 ms (Kalamkar et al. 2016) as being caused by the delay of plasma flow through the jet, this would imply a distance scale of $z \geq 0.1s \times \gamma_j \beta_j c \sim 10^3 r_g$. This distance is conservative given that the jet is assumed to travel at constant velocity—the jet could accelerate efficiently along its axis, as is the case with `agnjet`. It is preferable to keep $z_{acc}$ fixed at this value since the data coverage provides limited constraints on its value, and the self-absorption break is variable on timescales shorter than 24 hours (Gandhi

\[ \text{Figure 4.3: The Eddington-scaled X-ray luminosity of all X-ray spectra against the power-spectral hue derived from the light curves. The key shows how the data are divided into the year of the observation.} \]
Figure 4.4: Diagram of a corona + jet model, representing spectral model B2. The thin disc is truncated to radii on the order of 2–10 $r_g$, and an optically thin compact corona exists within the inner accretion flow, with electron temperatures $kT_e \sim 10–100$ keV. The plasma from the corona is advected into the base of the jet. The electrons inside the jet are relativistic, with $kT_e \geq 511$ keV, and the plasma has bulk motion in the z-direction with $\gamma_j \sim$ a few. The observer sees direct emission from the jet in the form of synchrotron, SSC, and IC scattering disc photons, as well as direct IC emission from the corona, and direct blackbody emission from the disc. Both the jet and coronal X-ray components will irradiate the disc, resulting in a reflected X-ray spectrum and reprocessed optical emission.

et al. (2010). We fix $f_{nth} = 0.1$, based on current studies of particle acceleration across mildly-relativistic shocks (e.g. Sironi & Spitkovsky 2011, Crumley et al. 2017, in preparation). We either fix $f_{sc} = 10^{-6}$ or allow it to vary freely between $10^{-6}$–$10^{-1}$ in order to constrain the contribution of optically-thin non-thermal synchrotron emission to IR/optical frequencies, and to include the possibility of non-thermal IC scattering, whilst also ensuring no significant direct contribution of optically-thin synchrotron to the X-ray spectrum. This choice to suppress the X-ray synchrotron contribution is motivated by our objective to constrain the jet IC contribution to the X-ray spectrum of GX 339–4, and to limit degeneracies in tracking the jet properties in outburst. The fundamental parameters of interest in agnjet are the normalised jet
4.4 Spectral fits

power, $N_j$, the radius of the jet base, $r_0$, the dimensionless initial electron temperature, $\Theta_e$, the ratio of energy density between the electrons and magnetic field at the jet base, $\beta_e$, and the power-law spectral index of the accelerated electrons, $p$, all of which remain free parameters in our fits, since the non-thermal synchrotron component contributes very little direct emission to the optically-thin portion of the observed spectrum. However, the acceleration of electrons at $z_{acc}$ increases the energy density in the electrons and thus boosts the emission further out in the jet, particularly in the self-absorbed region regions; hence $p$ is anti-correlated with both $N_j$ and $\Theta_e$.

We set the input photon distribution of nthcomp as a multi-temperature disc blackbody, and tie the disc temperature $T_{BB}$ to the multi-temperature disc component within agnjet. The coronal electron temperature, $T_{e,cor}$, and spectral index, $\Gamma_{cor}$ are free parameters of the model. As discussed in Section 4.4.1, we fix the centroid energy of the Gaussian iron line, $E_{line}$, according to the initial fits to each individual X-ray spectrum, after having fully explored the parameter distributions using MCMC parameter exploration.

After achieving good fits to all 20 broadband spectra, we initialise MCMC walkers around the maximum likelihood estimates of parameters in model B2, allowing the parameter search to explore the contributions of agnjet and nthcomp. Each MCMC chain is allowed to run for $\geq 3000$ steps such that the resultant posterior PDFs of the model parameters show coverage of the broad range intrinsic to models B1 and B2, and there is no longer significant evolution in those PDFs.

Figures 4.5–4.8 show model fits to all 20 broadband spectra of GX 339−4. Specifically, Figures 4.5 and 4.6 show fits to spectra 1–10 and 11–20 respectively, in which IC emission from agnjet is the dominant X-ray spectral component. Figures 4.7 and 4.8 show fits to the same spectra in which the coronal IC emission of nthcomp dominates the X-ray spectra. The first thing we notice, is that model B2 (due to the additional presence of a corona, i.e., nthcomp) provides a much better fit to the X-ray spectra than model B1 in each case, due to the lower electron temperatures treated in the model: $kT_{e,cor}/mc^2 \sim 0.02$–0.4. We find optical depths in the corona from all our fits to be in the range $\tau \sim 0.1$–1, assuming the corona has a spherical geometry. The electrons at the base of the jet in agnjet are strictly relativistic, and they remain quasi-isothermal throughout the jet. The optical depth in the jet base ranges between $\tau \sim 10^{-4}$–10$^{-2}$. These conditions give rise to an emergent IC spectrum that is not a power law, but instead has significant curvature. This is a distinguishing feature of Comptonisation with low optical depth ($\tau \ll 1$) and high electron temperature ($\Theta_e \geq 1$). In order to counter-act the spectral curvature of the IC emission in agnjet, the reflection fraction ($R_f$) systematically increases (see Figure 4.9). This increase in $R_f$ is in disagreement with values derived from simpler X-ray spectral fits, and it is unlikely that a curved IC spectrum from the jet conspires with reflection to reproduce stable power-law spectra over time. The iron line in fits of model B1 is also grossly

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Figure 4.5: Broadband spectra 1–10 of GX 339-4, fit with model B1 (jet IC-dominated X-ray spectra): $tbabs \times reflect(agnjet+gaussian)$. Radio data are marked with dark green squares, IR-optical data with orange triangles, and RXTE-PCA, HXT A, and HXT B with blue, dark blue, and purple circles respectively. The unfolded model fits to the data are shown in thick black. The individual components of $agnjet$ are shown as follows: pre-acceleration optically thin thermal synchrotron (blue dot-dot-dashed lines), post-acceleration synchrotron (red dot-dashed lines), IC (green dashed lines), and multi-temperature disc blackbody (black dotted lines). The orange curves show the reflection spectra generated by convolving the continuum with the $reflect$ model, plus the gaussian iron line.
4.4 Spectral fits

Figure 4.6: Broadband spectra 11–20 of GX 339–4, fit with model B1 (jet IC-dominated X-ray spectra): \texttt{tbabs} \times \texttt{reflect(agnjet+gaussian)}. Radio data are marked with dark green squares, IR-optical data with orange triangles, and \textit{RXTE}-PCA, HXT A, and HXT B with blue, dark blue, and purple circles respectively. The unfolded model fits to the data are shown in thick black. The individual components of \texttt{agnjet} are shown as follows: pre-acceleration optically thin thermal synchrotron (blue dot-dot-dashed lines), post-acceleration synchrotron (red dot-dashed lines), IC (green dashed lines), and multi-temperature disc blackbody (black dotted lines). The orange curves show the reflection spectra generated by convolving the continuum with the \texttt{reflect} model, plus the gaussian iron line.
Figure 4.7: Broadband spectra 1–10 of GX 339−4, fit with model B2 (coronal IC-dominated X-ray spectra): \texttt{tbabs} × \texttt{reflect(agnjet+nthcomp+gaussian)}. Radio data are marked with dark green squares, IR-optical data with orange triangles, and \texttt{RXTE-PCA}, HXT A, and HXT B with blue, dark blue, and purple circles respectively. The unfolded model fits to the data are shown in thick black. The individual components of \texttt{agnjet} are shown as follows: pre-acceleration optically thin thermal synchrotron (blue dot-dot-dashed lines), post-acceleration synchrotron (red dot-dashed lines), IC (green dashed lines), and multi-temperature disc blackbody (black dotted lines). The brown triple-dot-dashed line shows the model \texttt{nthcomp}, the thermal Comptonising corona. The orange curves show the reflection spectra generated by convolving the continuum with the \texttt{reflect} model, plus the gaussian iron line.
Figure 4.8: Broadband spectra 11–20 of GX 339−4, fit with model B2 (coronal IC-dominated X-ray spectra): $\text{tbabs} \times \text{reflect(agnjet+nthcomp+gaussian)}$. Radio data are marked with dark green squares, IR-optical data with orange triangles, and RXTE-PCA, HXT A, and HXT B with blue, dark blue, and purple circles respectively. The unfolded model fits to the data are shown in thick black. The individual components of $\text{agnjet}$ are shown as follows: pre-acceleration optically thin thermal synchrotron (blue dot-dot-dashed lines), post-acceleration synchrotron (red dot-dashed lines), IC (green dashed lines), and multi-temperature disc blackbody (black dotted lines). The brown triple-dot-dashed line shows the model $\text{nthcomp}$, the thermal Comptonising corona. The orange curves show the reflection spectra generated by convolving the continuum with the $\text{reflect}$ model, plus the gaussian iron line.
over-fit, as can be seen in the residuals of all plots show in Figures 4.5 and 4.6. Again this over-fitting is in disagreement with our initial phenomenological fits to the X-ray spectra. We also notice in fits of model B2 (see Figures 4.7 and 4.8) that the presence of non-negligible IC emission from the jet (we find that the jet contributions a range of 10–50% of the continuum flux in the 10–40 keV band) acts to skew the shape of the model coronal spectrum. If the jet contributes to the X-ray spectrum, the corona may either have a softer or harder spectral shape than would be concluded if the jet were to be ignored.

4.4.3 Global trends

Figures 4.9–4.12 show the trends of reflection fraction $R_f$, and jet parameters $N_j$, $r_0$, $\beta_e$, and $\Theta_e$ respectively as a function of $L_X/L_{Edd}$ and power-spectral hue, in both models B1 and B2. Table 4.4 shows the numerical values of the best-fit parameters of model B2, and their confidence limits (we show only the best-fit values of model B2 as it achieves better fits to all 20 broadband spectra). The normalised jet power, $N_j$ (Figure 4.10) increases with increasing $L_X$, and this is a clear trend despite the uncertainty on its value. We see that given the similar X-ray luminosities during the
2005 and 2011 outburst decays, \( N_j \) remains roughly constant as the hue decreases, until the source progresses further into the low hard state in the latter stages of outburst decay, at which point \( N_j \) decreases.

The jet-base radius (see Figure 4.11) is poorly constrained across all fits, and has a broad range from 10s to 100s of \( r_g \). There is tentative evidence for lower values of \( r_0 \) during the 2011 outburst decay in model B1, likely due to the degeneracy inherent between \( N_j \) and \( r_0 \). A decrease in \( N_j \) is constrained by decreasing radio and IR-optical flux, and this independent constraint on \( N_j \) is accounted for by a decrease in \( r_0 \) in order to fit the X-ray spectrum. However, fitting with model B2 shows that when IC scattering in the corona dominates the X-ray spectrum, the jet base can become large, such that only the conditions at regions further out in the jet (\( z \sim 10^3 \ r_g \)) are constrained by the broadband spectrum.

Despite the systematically higher values of \( \Theta_e \) when the jet IC emission dominates the X-ray spectrum (see Figure 4.12), in both cases the trends are similar: \( \Theta_e \) decreases slightly with increasing power-spectra hue. This is because \( \Theta_e \) is not solely constrained by the X-ray spectrum. As shown by Gandhi et al. (2008), the rapid optical variability of GX 339–4 at the onset of the hard state is best explained as
originating in the jet, likely as optically-thin synchrotron emission (thermal)—though we note that at higher luminosities disc reprocessing may provide a more significant contribution. Thus the hardening of the optical spectra in all 20 of our datasets is modelled by thermal synchrotron emission from the optically-thin regions of the jet. In addition to the contribution of synchrotron emission to the radio flux at larger distances in the jet, these two factors act to constrain $\Theta_e$. In addition, $\Theta_e$ appears lower during the early stages of the 2011 outburst decay than in all other fits. This constraint is determined by the lower optical fluxes in the spectra.

We see no clear global correlation between $\beta_e$, the partition of energy between electron and magnetic energy density (see Figure 4.13), and the power-spectral hue, or $L_X$, except for an apparent increase at the highest hue values, i.e., in harder states. Its value ranges between $\sim 0.02$–1 across all the fits, with most fits yielding $\beta_e \sim 0.1$. The trend in the fitting process is for $\beta_e$ to be pushed to values $< 1$, i.e. a magnetically-dominated jet base, which is due to increases in $N_j$, the jet power. $N_j$ increases in accordance with the increase in radio flux irrespective of the jet’s X-ray contribution, and $\beta_e$ in theory decreases in order to reduce the electron density in the jet base (lower electron densities lead to a lower IC flux from the jet). $\beta_e$ is
Figure 4.12: The maximum likelihood estimates of the temperature of the electrons in the jet base, $\Theta_e$, as a function of Eddington-scaled X-ray data luminosity (left) and power-spectral hue (right). Filled squares show the parameters derived from fits of model B1, and hollow circles fits of model B2, where the key applies to both panels.

4.4.4 Coronal parameter trends

Figure 4.14 shows the trends of the spectral index of the IC power-law in the corona, $\Gamma_{\text{cor}}$, and the coronal electron temperature, $kT_{e,\text{cor}}$, with $L_X$ and hue. There is no observable trend between coronal electron temperature with increasing $L_X$ and hue, any potential correlation is likely quenched by the fact that in most of the 20 GX 339−4 spectra the X-ray spectral coverage and photons counts are insufficient to constrain the cut-off energy, and the jet IC spectrum introduces significant scatter due to its high fractional contribution to the X-ray flux. There is a correlation between $\Gamma_{\text{cor}}$ and hue and $L_X$ during each outburst rise/decay. Whilst a trend is expected based on our initial fits to the X-ray spectra (Section 4.4.1), there is added scatter in the the slope again caused by the non-negligible contribution from IC emission in the jet base. This paradigm of mixed jet/coronal emission in the X-ray agrees
Studying the inflow/outflow geometry of GX 339-4

Figure 4.13: The maximum likelihood estimates of the plasma partition factor, \( \beta_e \), as a function of Eddington-scaled X-ray data luminosity (left) and power-spectral hue (right). Filled squares show the parameters derived from fits of model B1, and hollow circles fits of model B2, where the key applies to both panels.

qualitatively with previous modelling of GX 339–4 and Cyg X-1 by Nowak et al. (2005) and Wilms et al. (2006), in which both find strong correlations between the harder X-ray emission (> 10 keV) and their radio fluxes, and evidence for multiple contributions in the X-ray band.
4.4 Spectral fits

Figure 4.14: The maximum likelihood estimates of the photon index ($\Gamma_{\text{cor}}$; top) and electron temperature ($kT_{e,\text{cor}}$; bottom) of nthcomp as a function of Eddington-scaled X-ray data luminosity (left) and power-spectral hue (right). The data are colour-coded according to observation year and thus track separate outbursts.
Table 4.4: The maximum likelihood estimates and confidence limits of fit-parameters of model B2 to all 20 broadband spectra of GX 339−4. From left to right: (1) spectrum number, (2) $N_j$, the normalised jet power, (3) $p$, the power law index of acceleration jet electrons, (4) $r_0$, the jet base radius, (5) $\Theta_e$, the electron temperature in the base of the jet, (6) $\beta_e$, the ratio of electron to magnetic energy density in the jet, (7) $T_{in}$, the inner disc temperature (and thus the temperature of the seed photon distribution for IC scattering in both the jet and corona), (8) $\Gamma_{cor}$, the photon index of the thermal Compton spectrum in the corona, (9) $kT_{e,cor}$, the electron temperature in the corona, (10) $R_f$, the reflection fraction, (11) goodness-of-fit.

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4.4.5 Joint broadband modelling

We jointly model selected datasets in order to better constrain the trends of key parameters as a function of the hue. For example, we should expect the size scales of the jet to evolve with the power-spectral hue, since the timescales of variability depend on the compactness of the X-ray emitting region, and these variability timescales are seen to propagate through the jet (Gandhi et al. 2010; Kalamkar et al. 2016). Figures 4.15 and 4.16 show joint broadband spectral fits to selected datasets from the 20 broadband spectra of GX 339–4. Figure 4.15 shows a joint fit to observations of GX 339–4 during the decay of its 2005 outburst, the decay of its 2007 outburst, and the rise of its 2010 outburst, where the power-spectral hue is indistinguishable between all 4 datasets (in the range $\sim 5^\circ$–$30^\circ$). We are able to achieve a good fit by tying $r_0$, $\Theta_e$, $p$, and $\beta_e$, re-affirming the findings from individual fits that the jet base radius, electron temperature, and division of energy between electrons and magnetic field are indistinguishable between outburst rise and decay, assuming a dominant coronal component in the X-rays. However, as shown in Figure 4.16, tying the same parameters, with exception to the electron temperature, leads to much lower electron temperatures during the 2011 outburst decay, where the value of the power-spectral hue is higher ($\sim 100^\circ$).

4.5 Discussion

Previous modelling of GX 339–4 with older versions of agnjet proposed a significant contribution in the X-ray from optically-thin non-thermal synchrotron emission, either dominating the full observable X-ray band, or solely the soft band, with jet IC dominating the harder emission (Markoff et al. 2003; Maitra et al. 2009). Here we have instead considered the case in which synchrotron emission is suppressed and the jet’s X-ray contribution is almost entirely dominated by thermal SSC, with some contribution from up-scattered disc photons, and non-thermal SSC originating from larger distances in the jet ($z \sim 10^{3.5} r_g$). There can also be contributions in the X-ray from synchrotron-emitting non-thermal electrons in the jet base or inner accretion flow, given that both are collisionless, turbulent regions in which particle acceleration can occur. Connors et al. (2017) explore this scenario in modelling of Sgr A*, the Galactic centre black hole, and A0620-00, a LMXB in quiescence, and though they both have significantly lower X-ray luminosities than GX 339–4 ($L_X/L_{Edd} \sim 10^{-9}$), such a scenario cannot be ruled out in the case of GX 339–4. However, the millisecond-to-second timescale hard X-ray lags observed in the hard state of GX 339–4 do not favour particle acceleration as being responsible for the delayed hard X-ray emission.

As shown in Section 4.4.1 there are no obvious distinctions between the rise and decay of outburst in the data based on individual fits to the X-ray spectra: all outbursts, whether in decay or rise, tend to show softening spectra with $L_X$ and
Figure 4.15: Joint spectral fits of model B2 to observations 3, 5, 9 and 14 (see Tables 4.1 and 4.2). We tie the jet base radius, power law spectral index, partition factor and electron temperature between all 4 datasets and find $r_0 = 60 \, r_g$, $p = 1.8$, $\beta_e = 0.17$ and $\Theta_e = 1.9$. Radio data are marked with dark green squares, IR-optical data with orange crosses, and RXTE-PCA, HXT A, and HXT B with blue, dark blue, and purple circles respectively. The unfolded model fits to the data are shown in thick black. The individual components of agnjet are shown as follows: pre-acceleration optically thin thermal synchrotron (blue dot-dot-dashed lines), post-acceleration synchrotron (red dot-dashed lines), IC (green dashed lines), and multi-temperature disc blackbody (black dotted lines). The brown triple-dot-dashed line shows the model nthcomp, the thermal Comptonising corona. The orange curves show the reflection spectra generated by convolving the continuum with the reflect model, plus the gaussian iron line.
Figure 4.16: Joint spectral fits of model B2 to observations 1, 2, 17, and 19 (see Tables 4.1 and 4.2). We tie the jet base radius, power law spectral index, and partition factor between all 4 datasets and find $r_0 = 90$ $r_g$, $p = 1.8$, and $\beta_e = 0.1$. The electron temperatures are free to vary between datasets, and we find $\Theta_e = 1.8$, 2.3, 1, and 1 for datasets 1, 2, 17, and 19 respectively. Radio data are marked with dark green squares, IR-optical data with orange crosses, and RXTE-PCA, HXT A, and HXT B with blue, dark blue, and purple circles respectively. The unfolded model fits to the data are shown in thick black. The individual components of $\text{agnjet}$ are shown as follows: pre-acceleration optically thin thermal synchrotron (blue dot-dot-dashed lines), post-acceleration synchrotron (red dot-dashed lines), IC (green dashed lines), and multi-temperature disc blackbody (black dotted lines). The brown triple-dot-dashed line shows the model $\text{nthcomp}$, the thermal Comptonising corona. The orange curves show the reflection spectra generated by convolving the continuum with the $\text{reflect}$ model, plus the gaussian iron line.
power-spectral hue. However, we see a tendency for smaller jet base radii \((r_0)\), lower electron temperatures, and a higher proportion of the jet’s internal energy density in the electrons (with respect to the magnetic field) in the decay of the 2011 outburst of GX 339–4, with respect to all other datasets. These differences appear to occur at higher values of the hue \((> 100^\circ)\). This is due both to the degeneracy between the jet power and the electron energy density at the jet base, and the lower optical fluxes which give rise to lower thermal synchrotron flux with respect to the X-ray flux. As the jet power increases, the resulting electron energy density will increase and violate the X-ray constraints unless the jet base radius is increased accordingly. Since the jet power is found to be lower during the 2011 decay (and indeed the 2005/2007 decays) than in the 2010 rise, solutions with a more compact jet base are permitted. Coronal models in the context of spectral softening in LMXBs predict a progressively recessing optically-thick disc, leading to increased cooling in the corona, and thus a lower temperature and a softer spectrum (Haardt & Maraschi 1993; Ibragimov et al. 2005). The X-ray power-law spectral index is determined by the ratio of coronal heating to cooling, \(L_h/L_s\), and optical depth of the coronal region, \(\tau\). The hard state is associated with \(L_h/L_s \gg 1\) (low soft-photon flux, high coronal luminosity), whereas as the disc recedes toward the ISCO \(L_h/L_s \leq 1\). Thus as the disc moves inwards during outburst, the electron temperature in the corona/jet base decreases, and \(\Gamma_{pl}\) increases. This interpretation has yet to explain why transitions between the dominant optically-thin inner flow and optically-thick accretion disc occur over a broad range of X-ray luminosity in LMXBs (Done & Gierliński 2003, \(L_X/L_{Edd} \sim 0.003–0.2\), with variations within the same source, and a tendency for sources to transition at higher luminosities in outburst rise than in decay (Nowak 1995; Maccarone & Coppi 2003). Modelling the broadband spectra of GX 339–4 allows more robust determination of the differences in the jet properties between the precipice of outburst rise and the onset of outburst decay. We require broadband observations of the latter stages of the rise of an outburst in order to test whether the parameter trends we have derived here during the 2011 outburst decay are present during the higher-luminosity transition.

Spectral and timing studies of LMXBs focus heavily on the location of the accretion disc inner edge in the low/hard state, and GX 339–4 is one of the most controversial subjects of these studies. Whilst most agree that in the brightest hard states the disc extends down to the ISCO (e.g. Gierliński & Done 2004; Penna et al. 2010, Miller et al. 2006) claim the disc in GX 339–4 sits at the ISCO throughout the low/hard state. Done et al. 2007 strongly contest this and instead claim the disc is significantly truncated and gradually moves inwards during the rise of an outburst, with an ADAF at \(r < r_{in}\). Done et al. 2007 also argue, in discussing the strengths and weaknesses of the many LMXB hard-state spectral models, that for jet emission to successfully explain the trend of increasing reflection fraction \((R_f)\) with X-ray power-law spectral slope \((\Gamma_{pl})\), the bulk velocity of the jet \((\beta_j)\) must decrease with increasing luminosity. This disagrees with fundamental observations of LMXB jet formation and accretion processes.
4.6 Summary and conclusions

The main results of our comprehensive modelling of 20 broadband spectra of GX 339–4 can be summarised in the following points:

- The jet base is more compact during the 2011 outburst decay of GX 339–4 as the source evolves to power-spectral hue \( \geq 100^\circ \), which is coincident with the broadband X-ray variability strengthening and becoming narrower in frequency. At hue \(< 100^\circ \), we see no clear distinctions in the jet physics between outburst rise and decay. We reinforce this result by jointly fitting the same model to datasets from different epochs, both where the hue is similar, and over epochs in which it evolves. A more compact jet base or corona leads to higher X-ray luminosities, but its connection to the variability properties is less certain. Our results point to a way of constraining the geometrical changes by linking the evolving X-ray variability in the inner regions to the plasma conditions further out in the jet.

- Even if the corona dominates the X-ray spectrum of GX 339–4 in the low/hard state, there will still likely be a non-negligible contribution from jet IC photons. We find ratios of jet-to-corona continuum flux of \( \sim 10\text{--}50\% \) in the 20–40 keV band across all fits. However we note that this conclusion is strongly model-dependent. The jet (agnjet) electrons are treated relativistically, and a treatment which includes cooler electrons, and thus produces IC spectra with less curvature, would likely reduce the difference in spectral shape between the

radio cores (Fender 2006). However, this relied on a very basic treatment of a jet, in which the outflowing plasma moves with constant velocity, and the particle distribution is an ideal power law. The complicated physical connection between the bulk flow properties and dissipation of energy into the radiating electrons in jets means there may be other scenarios in which the correlation between \( R_f \) and \( \Gamma_{pl} \) can be realised without violating requirements on the jet dynamics. In agnjet the electrons energies are in a Maxwell-Jüttner distribution with initial temperatures \( \Theta_e \geq 1 \), and remain quasi-isothermal, cooling only in proportion to the jet acceleration in the z-direction (\( T(z) = T_0[\gamma_j(z)\beta_j(z)]^{1-\Gamma} \)), where \( \Gamma = 4/3 \) is the adiabatic index. The electrons in the outer regions of the jet must remain hot (\( \Theta_e \geq 1 \)) to reproduce the radio spectral index (and in our modelling particle acceleration occurs, so further energy has been dissipated into the electrons), but the electrons in the jet base may have low initial temperatures typical of coronae (\( \Theta_e \sim 0.2 \)), and heating can occur rapidly due to turbulence, thermal conduction or magnetic reconnection (Quataert & Gruzinov 2000; Johnson & Quataert 2007; Sironi 2015; Rowan et al. 2017).
corona and jet base IC emission in our modelling (and in fact may return to a scenario where the ‘corona’ is the base of the jet).

- The division of energy density between electrons and magnetic field in the jet ($\beta_e$) ranges between 0.01–1 in all fits, remaining statistically indistinguishable as the jet evolves through outburst rise and decay. The fact that the jet appears to remain magnetically-dominated could indicate that the jet energetics are not necessarily dependent on the geometry, but instead this is determined by the conditions of the inner accretion flow (and the jet plasma conditions are driven by the conditions in the inflow).

For the first time we combine a thermal IC-scattering corona (nthcomp: Zdziarski et al. 1996; Życki et al. 1999) and a jet in broadband spectral modelling of GX 339−4, with two fundamental differences to the jet IC treatment of agnjet: the input soft-photon distribution for the jet scattering in agnjet is dominated by thermal synchrotron photons, and the electrons are strictly relativistic ($\Theta_e \geq 1$, $kT_e \geq 511$ keV), whereas the input photons of the the corona in nthcomp are disc blackbody photons at $T_{BB} \sim 0.01–1$ keV, and the electrons are typically on the order of $kT_e \sim 10–200$ keV. Only the cooler coronal component of nthcomp can successfully reproduce the X-ray spectra of all 20 GX 339-4 datasets we modelled, and this is due to precisely the two identified model discriminants. The hotter electrons in the jet fail to reproduce a single power-law across the full 2–150 keV X-ray band, showing multiple Compton scattering humps, and significant curvature. Therefore, if the X-ray spectrum of GX 339−4 in the low/hard state results from IC scattering, whether the scattering is confined to the corona, jet, or occurs in both, the electrons must be primarily at sub-relativistic energies, and are likely thermal. We highlight the allusions made by Nowak et al. (2005) and Wilms et al. (2006) to the presence of multiple hard X-ray components in the low/hard state. We suggest that a non-negligible contribution to the X-ray flux (~ 10–50%) can come from IC scattering (predominantly SSC) in the base of a mildly-relativistic jet, and we have shown that such a scenario can be achieved with realistic jet parameters that explain the radio and IR-optical emission in the outer regions of the jet. The large jet-base radii in our solutions is degenerate with the temperature of the electrons, such that if the electrons are cooler the jet base can become significantly more compact ($\leq 10 r_g$), and thus can influence the observed reflection spectrum.

Determining the contribution of jet emission in the X-ray still remains a difficult task in the modelling of LMXBs. The jet contribution must be quantified in order to better constrain the fraction of hard X-ray emission reflected off LMXB accretion discs (Ross et al. 1999; Ross & Fabian 2005; Dauser et al. 2010; Garcia et al. 2014a), since if a significant fraction of the X-rays are beamed away from the disc, the emissivity profile along the disc is affected, and therefore the reflection fraction changes (Dauser et al. 2013; Wilkins & Gallo 2015). We will address the importance of the
jet contribution to X-ray disc reflection in a forthcoming paper (van Eijnatten et al., in preparation, and chapter 5).

The results of all our modelling here suffer from the limitation that the plasma conditions which determine the spectrum of the jet are disconnected from the jet dynamics. The velocity, particle density, and magnetic field profiles are pre-calculated dynamical quantities in the model, and the broadband spectrum follows from the free parameters. An improved treatment would involve reducing the number of free parameters by physically linking them with the jet dynamics. We are currently working on a model that achieves this, by solving the relativistic magnetohydrodynamic equations for the jet dynamics (Ceccobello et al., in press).

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Towards a self-consistent model of jet-disc reflection in X-ray binaries

You know you must be doing something right if old people like you.
—Dave Chappelle

D. van Eijnatten, R. M. T. Connors, S. B. Markoff, J. Garcia, T. Dauser

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Abstract

Irradiation of an accretion disc by a central source above the black hole can lead to reflection features. The irradiation component is often modelled as a power-law-emitting point source with a reflection fraction parameter (the ‘lamppost’ geometry) to normalise the continuum and reflection spectra. More recent reflection modelling of accreting black holes reveals some evidence for an irradiating component in bulk relativistic motion with respect to the disc. We introduce a self-consistent treatment of reflection of jet emission. We take an existing model for the broadband spectra of both active galactic nuclei and black hole X-ray binaries, and create an interface with existing reflection simulations. This treatment has an advantage over current reflection models because the normalisation of the irradiation spectrum is set by physical jet parameters, and does not require a parameterised treatment of the reflection fraction. Instead the reflection fraction can be calculated based on the physical jet parameters. We calculate the flux on the disc for a large set of jet parameters, and use these incident spectra to generate a grid of reflected spectra to use in fitting.
The disc ionization parameter falls in the range $1 < \xi < 4$ for the full grid of jet parameters, such that the jet will always produce reflection features in an observed spectrum if its flux contribution is non-negligible. We find a very steep emissivity profile, up to values of $\epsilon > 6$ at inner disc radii. This steepness is mostly caused by light bending effects close to the black hole. The grid produces low reflection fractions of 0.19–0.28 since a significant portion of the jet emission is beamed away from the disc. This constraint on the reflection fraction can provide a way to put limitations on the normalisation of reflection spectra in future modelling.
5.1 Introduction

Modelling the disc reflection spectrum of X-ray radiation from the inner regions of accretion flows in black hole binaries (BHBs) and active galactic nuclei (AGN) provides a tool for measuring black hole spin (e.g., BHBs: Fabian et al. 1989; Miller et al. 2008; Reis et al. 2008; Duro et al. 2011; Miller et al. 2013; Tomsick et al. 2014; AGN: Fabian & Vaughan 2003; Fabian et al. 2009; Dauser et al. 2012; Risaliti et al. 2013; Marinucci et al. 2014), and for estimating the location of the inner edge of the thin accretion disc (e.g., Fürst et al. 2016a,b; Basak & Zdziarski 2016; Basak et al. 2017; Walton et al. 2017). Both are fundamental to our understanding of jet launching (i.e. due to the spin angular momentum of the black hole Blandford & Znajek, 1977, or rotation of the accretion flow Blandford & Payne, 1982) and the evolution of the structures accreting black holes. However, constraints on the disc inner radius and black hole spin using reflection modelling are limited by our lack of understanding of the geometry of the X-ray irradiator. Reflection modelling can only progress if we improve calculations of the irradiation spectrum and geometry and its effects on the reflection spectrum.

If a strongly smeared iron line is present in the X-ray spectrum, the primary irradiation must lie within $\sim 10$ gravitational radii \(r_g\) of the black hole (Fabian et al. 2014), as well as the disc’s inner edge, depending on the inclination of the system (Brenneman & Reynolds 2006). The principle assumptions in question are whether this region is static (i.e. a classical Comptonising corona, e.g., Lightman & Eardley 1974; Shapiro et al. 1976; Haardt & Maraschi 1993) or in bulk relativistic motion directed away from the black hole (e.g., Beloborodov 1999; Malzac et al. 2001; Markoff et al. 2001a), and the shape of the irradiation spectrum. If instead an unbroadened iron line is detected in the spectrum, a degeneracy exists between the determination of black hole spin, the disc inner edge, and the extent and Lorentz factor of the irradiating source (Dauser et al. 2013). This degeneracy can lead to incorrect low spin measurements, or over-predictions of disc truncation.

The canonical accretion disc prescription of Shakura & Sunyaev (1973) predicts an emissivity profile of the irradiation along the disc of $I(r) \propto r^{-3}$. Reflection modelling of both BHBs and AGN predicts steeper emissivity profiles (see. e.g., Wilms et al. 2001; Dauser et al. 2012; Miller et al. 2013; Risaliti et al. 2013), and such steep profiles are naturally produced by the now commonly adopted ‘lamppost’ geometry (Martocchia & Matt 1996), in which the irradiating source is point-like and located some height above the black hole horizon, perpendicular to the disc. However, recent reflection modelling predicts a more extended geometry for the irradiator (e.g., Wilkins & Gallo 2013).

Reflection models have progressed substantially since the earliest considerations of X-ray reprocessing and its resultant line and continuum features (Ross et al. 1978; Ross 1979; Lightman & Rybicki 1981; Lightman et al. 1981). Two prevailing models
regularly applied to reflection spectra are REFLIONX (Ross & Fabian 1993, 2005) and relxill (García et al. 2014a). For a review of reflection modelling see Fabian & Ross (2010) and Dauser et al. (2016b). relxill combines the reflection code XILLVER (García & Kallman 2010; García et al. 2013) with the relativistic smearing code RELLINE (Dauser et al. 2010) to self-consistently treat the angular-dependence of irradiation, reflection, and smearing, and calculate the resultant reflection spectrum. This treatment of the angular-dependence distinguishes relxill from previous models, making it the best reflection model currently available.

As discussed by Dauser et al. (2013), assumptions regarding the geometry of the X-ray irradiating source are important in calculations of reflection. The degeneracy lies in whether the source is point-like or extended (i.e. a super-position of multiple lampposts), and whether the source is in relativistic motion (akin to the base of a jet, e.g., Beloborodov 1999; Malzac et al. 2001; Markoff et al. 2001a). As Dauser et al. (2013) show, the effects of relativistic beaming and changes to the distribution of irradiation impinging on the accretion disc result in decreased line-broadening effects on the emergent reflection spectrum. In essence, if one measures a broad Iron-line feature in the spectrum it is likely that the irradiating source is compact and close to the black hole (within a few $r_g$). However, if one measures a narrow Iron-line feature, this may either indicate an identically compact irradiating region within a system in which the accretion disc is truncated to several $r_g$, or an elongated irradiating structure in which significant emission can be observed from heights beyond a few $r_g$.

We must therefore determine the irradiating geometry with more accuracy in order to properly interpret the results of X-ray reflection spectral modelling of BHBs and AGN.

The spectral shape of the irradiation in reflection models is also a key assumption taken for granted. Reflection models are starting to progress beyond the simplified power law irradiation spectrum. relxill has been combined with the coronal thermal Comptonisation model nthcomp (Zdziarski et al. 1996; Życki et al. 1999), which provides an improved treatment of the spectral shape of the inverse Compton (IC) scattering spectrum. As shown by García et al. (2015b), relxill is capable of constraining the cut-off energy of the primary continuum up to X-ray energies well beyond the observable band of NuSTAR (Harrison et al. 2013), due to the effects of high-energy emission on the ionisation structure of the disc. This shows that reflection modelling can place strong constraints on the primary spectrum, assuming it resides in a single location (i.e. a lamppost). We propose that a more self-consistent treatment of reflection requires further testing of the interplay between the geometry and spectral shape of the primary irradiating source. Such tests may point to ways to discriminate between coronal and jet X-ray emission in accreting black holes.

In this paper we present a new grid of X-ray reflection spectral models (based on the physical jet parameters of a BHB) in which the source of irradiating X-rays are IC scattered photons from a mildly relativistic jet, using the codes XILLVER (García
5.2 The jet model

The model we adopt as our X-ray irradiating source, \texttt{agnjet}, is a semi-analytical, mildly-relativistic, jet. A full description of the outflow dynamics and radiative mechanisms is given by Markoff et al. (2005), Crumley et al. (2017), and Connors et al. (2017, Chapter 2). Here we give a brief description of the elements of the model most relevant to the emission as seen by the accretion disc (the irradiation spectrum). Whilst \texttt{agnjet} has multiple parameters which are relevant for its broadband emission from radio to X-rays (the most influential of which are discussed in detail by Connors et al. 2017, Chapter 2), IC emission in the lower regions of the jet \((\lesssim 10^3 \ r_g)\) is governed principally by 4 parameters: \(N_j\), the jet power in units of \(L_{\text{Edd}}\), \(r_0\), the jet-base radius, \(\Theta_e \equiv kT_e/m_e c^2\), the electron temperature, and \(\beta_e \equiv U_B/U_e\), the partition of energy density between the electrons and magnetic field. Here \(T_e\) is the electron temperature in units if Kelvin, \(k\) is the Boltzmann constant, \(m_e c^2 = 511\) keV is the electron rest mass, \(U_B = B^2/8\pi\) is the magnetic energy density, and \(U_e\) is the electron energy density. As discussed in Crumley et al. (2017), \texttt{agnjet} captures the dynamics of a mildly-relativistic jet, limited to Lorentz factors \((\gamma_j)\) of a few.

The photon field for IC scattering in the jet has two components: thermal + non-thermal synchrotron photons emitted within the jet, and external multi-temperature disc blackbody photons. Thus the resulting IC spectrum may become double-peaked if the disc photon energy density is high enough, or if the plasma is dense enough for significant scattering events to occur. Since the peak frequency of thermal synchrotron emission in the jet rest frame is set by the critical frequency \(\nu_c \propto \gamma_e^2 B\) (in the range \(10^{13} - 10^{14}\) Hz with typical BHB magnetic fields of \(\sim 10^5\) G and \(\Theta_e \sim \text{a few}\)), we expect an observed synchrotron self-Compton (SSC) peak at \(\nu_{\text{ssc}} \sim \Theta_e^2 D\nu_c \sim 10^{16} - 10^{18}\) Hz, governed by the electron temperature \(\Theta_e\). The disc photons incident on the jet emit at a rest-frame peak frequency \(\nu_{\text{disc}} \sim 3kT_{\text{in}}/h \sim 10^{15} - 10^{17}\) Hz, assuming typical disc temperatures of \(T_{\text{in}} \sim 1 - 100\) eV. The peak frequency of disc blackbody emission in the jet’s rest frame is lower by a factor of the Doppler boost \(1/|\gamma_j(1 + \beta_j \cos(\theta))|\), where \(\theta\) is the incident angle between the normal to the disc and the light travel path towards the jet spine. Since the jet has relatively low bulk flow velocity at the base
Figure 5.1: A schematic of the geometry. The black hole is in the lower right of the diagram. The jet and its zones is given in blue and the point sources used in the lamppost approximation are shown as black dots. The grey triangles indicate the opening angle of the beaming at this location in the jet. Along the jet are schematics of spectra emitted by the jet zones. The accretion disc and its zones are given in red. Along the disc are schematics of spectra received by disc annuli. An example of a reflected spectrum is shown in the upper left. The black vertical line indicates the Fe-Kα line at 6.7 keV. The shaded gray area indicates the “Compton hump” between 10 and 40 keV. The first arrow indicates a transformation from the jet frame to the disc frame. The second arrow indicates the reprocessing of jet emission in the disc resulting in the reflected spectrum. This diagram is strictly qualitative, the amount of zones or the opening angle of the beaming factors do not represent agnjet or XILLVER quantitatively.
5.3 Disc reflection of jet emission

Radiation close to a black hole is subject to several relativistic effects. For self-consistent disc reflection from jet emission, we have to apply these effects twice: once when the radiation is travelling from the jet towards the disc and again when the radiation is travelling from the disc towards the observer. The former will alter the spectrum that is the seed for the reflected spectrum and the latter will alter the way this reflected spectrum appears to an observer. This allows us to calculate the exact flux that irradiates the disc. Firstly, we will discuss the relativistic effects we apply to the irradiation from the jet towards the disc. Then, we will discuss how we apply these effects to \textit{agnjet}. Finally, we will discuss how we can use our results to generate reflection spectra and we make a comparison to current frequently-used reflection models.

5.3.1 Relativistic effects pre-reflection

We treat the relativistic effects on radiation as it travels towards the disc analogously to Dauser et al. (2013). We use the positions and velocities (see figure 5.2) of our jet zones that are treated as moving point sources to calculate how the radiation emitted by each zone is affected by relativistic effects. A key difference here is that unlike the case for a pure power law continuum, a blueshift/redshift cannot be treated as a change in normalization. As such, energy shifts have a distinctly different effect than light bending or beaming. For example, blueshifts increase the number of high-energy photons. The normalization however, impacts how much each jet zone contributes to the incident spectrum.

The gravitational redshift/blueshift in the lamppost geometry is given by

\[
g = \frac{(r\sqrt{r} + a)\sqrt{h^2 - 2h + a^2}}{\sqrt{r\sqrt{r^2 - 3r + 2a\sqrt{r}\sqrt{h^2 + a^2}}} - \sqrt{h^2 - 2h + a^2}}
\] (5.1)
where \( r \) is the radius of the disc annulus, \( h \) is the height of the jet zone and \( a \) is the dimensionless spin parameter \( J/M \), where \( J \) and \( M \) are the black hole’s angular momentum and mass, respectively (Dauser et al. 2013). Disc radii near the innermost stable circular orbit (ISCO) receive exclusively blueshifted photons from the jet, up to a factor of \( E_i/E_e \sim 4 \), where \( E_i \) and \( E_e \) are the incident and emitted photon energies respectively. Further out in the disc, the emission from lower jet zones (\( h \ll r \)) is redshifted and emission from higher jet zones slightly blueshifted, see figure 5.3.

In the rest frame of the jet, the size of an annulus of the disc is smaller than in the disc’s rest frame. The magnitude of this effect depends on the orbital velocity of the disc and the position of the annulus. This contraction effect results in a reduced flux on the annulus when compared to the same geometry in Euclidean space. The area of an equatorial disc annulus in the frame of a stationary observer is given by

\[
A_{\text{obs}}(r,dr) = 2\pi \frac{\rho}{r} \sqrt{r^2 + a^2 + \frac{2a^2 r}{\rho^2}} dr
\]  

(5.2)
5.3 Disc reflection of jet emission

Figure 5.3: The combined effects of the processes given in section 5.3.1 are shown. $N_i/N_e$ refers to the ratio of incident photons on a disc annulus and the total amount of emitted photons by the jet segment, given by the combined effects of light bending, length contraction and beaming. $E_i/E_e$ is the fractional difference of the energy of a photon when it is received by the disc versus when it was emitted by the jet segment in their respective rest frames, given by the combined effect of gravitational and Doppler shifts. The effects are shown for disc radii within $5R_G$ for each jet zone below $500R_G$. At the lowest jet zones a large proportion of photons are directed towards the accretion disc, whilst also experiencing more redshift than photons emitted from zones beyond $\sim 10r_g$ (this explains the dark blue curve as the bottom of the top panel).

in Boyer-Lindquist coordinates (Wilkins & Fabian 2012) where $\rho = r^2 + a^2 \cos^2 \theta$ and $\Delta = r^2 - 2r + a^2$. We assume the disc to be thin, such that its orbital velocity is equal to the Keplerian velocity (Shakura & Sunyaev 1973). This results in the following Lorentz factor for a stationary observer (Bardeen et al. 1972):

$$\gamma = \frac{r^{1/2}(r^{3/2} + a)}{r^{1/4}(r^{3/2} - r^{1/2} + 2a)^{1/2}(r^3 + r + 2)^{1/2}}$$

The area in the disc’s own frame is now given by $A_{disc} = \gamma A_{obs}$. This thus reduces the photon flux irradiating the disc, since the same number of photons are now distributed over a larger area.
In the gravitational potential of a black hole, light travels on curved paths. For an isotropic source above an accretion disc this results in more than half of the emitted flux impinging on the disc. For our calculation of the reflected spectrum we only consider photons that directly hit the disc after being emitted by the jet beam between the observer and disc, i.e. we leave out photons emitted by the jet beam on the opposite side of the disc and reflected photons whose paths are curved towards the disc once more. The bending of light both affects the photon flux irradiating the disc and the angle of incidence at every radius. We use the publicly available tabulated results of ray-tracing in the lamp-post geometry provided in the context of Dauser et al. (2013) to calculate incident angles and fluxes.

In the \texttt{agnjet} nozzle, the flow travels at a constant velocity $\beta$ perpendicular to the accretion disc and away from the black hole with $\beta \sim 0.4$. When the flow leaves the nozzle (at $h = 2 \, r_0$) it starts expanding and accelerating up to $\beta \sim 0.9$ at $500 \, R_G$ and to higher velocities beyond (see figure 5.2). The Doppler factor is given by

$$D = \frac{1}{\gamma (1 + \beta \cos \theta_e)} = \frac{1}{\gamma \left( \frac{\sqrt{h^2 + a^2 - \Delta(q^2 + a^2)}}{h^2 + a^2} \right)} \quad (5.4)$$

where $\theta_e$ is the emission angle (Dauser et al. 2013). Photons originally emitted away from the disc are blueshifted and photons originally emitted towards the disc are redshifted.

For reasons analogous to those given in the paragraph above, the photon flux emitted by each zone of the jet is altered by a factor $\propto D^2$. The results in most of the jet flux being "beamed" away from the disc.

Applying all these effects to \texttt{agnjet} allows us to calculate the exact flux impinging on each disc annulus if we know the height, velocity and spectrum of each lamp-post (zone). Before we consider the spectrum, combining the special relativistic effects with light bending already produces some interesting effects: Although most of the photons are blueshifted and beamed away from the disc, most of the photon trajectories are bent towards the disc for low jet zones, whilst redshifted and debeamed radiation is lost to the black hole. Due to these beaming and Doppler effects, the lowest jet zones dominate the incident flux on the inner disc radii, the area where reflection can be used to constrain black hole spin (Brenneman & Reynolds 2006). The relativistic effects in this area of the disc can be seen in figure 5.3.

SSC spectra produced by the jet are given in figure 5.4. After applying the effects described in section 5.3.1, we can calculate the spectrum that irradiates each disc segment, see figure 5.5.

In a Euclidian geometry, the irradiance of the disc by a single lamp-post is given by $I \propto r^{-3}$ (Shakura & Sunyaev 1973). In a Schwarzschild or Kerr geometry, this
5.3 Disc reflection of jet emission

A simple power law is modified to $I \propto r^{-\epsilon(r)}$. In previous modelling of reflection in BHBs and AGN, $\epsilon > 3$ was found (Dauser et al. 2012; Miller et al. 2013; Risaliti et al. 2013), and values up to $\epsilon \sim 6, 7$ are predicted at inner disc radii by lamp post reflection models (see, e.g., Martocchia & Matt 1996; Dauser et al. 2013). At infinity the so-called emissivity profile should converge to $\epsilon = 3$. We find that our jet model is consistent with these predictions (see Figure 5.6). As the emissivity profile is so steep, we only consider the spectrum incident on the innermost radii of the disc for reflection, since generating reflected spectra at each radius would be very computationally expensive.

5.3.2 Relativistic effects post-reflection

Depending on the inclination of the system, the differential rotation of the disc and gravitational potential of the black hole further alter the flux and energy of the re-
Modelling disc reflection of jet emission

The incident spectra for 8 disc annuli are shown for the typical parameters $N_j = 0.001 \dot{M}_{\text{Edd}} c^2$, $\Theta_e = 1.5$, $r_0 = 20 R_G$ and $\beta_e = 1$.

Figure 5.5: The incident spectra for 8 disc annuli are shown for the typical parameters $N_j = 0.001 \dot{M}_{\text{Edd}} c^2$, $\Theta_e = 1.5$, $r_0 = 20 R_G$ and $\beta_e = 1$.

5.3.3 Reflection effects

X-rays incident on the accretion disc can interact with free electrons in the plasma and can be down-scattered via Compton scattering or interact with atoms to produce absorption and emission features, which all depend on the ionization state of the disc. We treat this reflected spectrum with the model XILLVER (García & Kallman 2010; García et al. 2011). As discussed in Section 5.3.1, we use the spectrum from the first
5.3 Disc reflection of jet emission

Figure 5.6: The emissivity profile for the typical parameters $N_j = 0.001 \dot{M}_{\text{Edd}} c^2$, $\Theta_e = 1.5$, $r_0 = 20 R_G$ and $\beta_e = 1$. The emissivity index $\epsilon$ is given by $I \propto r^{-\epsilon}$ where $I$ is the intensity of the incident radiation. In Newtonian gravity, $\epsilon = 0$ at every radius. The inner disc radii are more heavily irradiated than radii further out, as is consistent with previous results (Dauser et al. 2013). The zoomed-in panel shows the unintuitive relativistic effects affecting the emissivity profile within the inner few $r_g$ of the black hole. A dip in the profile can be seen at 1.24 $r_g$, due to photon loss to the black hole.

A common diagnostic tool for the ionization state of an accretion disc is the ionization parameter (Tarter et al. 1969):

$$\xi = \frac{4\pi F_X}{n_e}$$

(5.5)

where $\xi$ is the ionization parameter, $F_X$ is the flux between 1–1000 Ry and $n_e$ is the free electron density of the disc. The ionization parameter is a measure of the photo-ionization rate to the recombination rate. The XILLVER model is generated with a constant density of $n_e = 10^{15}$ cm$^{-3}$. We can calculate the ionizing flux at each disc annulus due to irradiation from the jet. For each parameter combination in our grid (see section 5.4) this constrains the ionization parameter to be in the range $1 < \xi < 4$ at inner disc radii, and decreasing outward. Our model will thus always produce a reflection feature (Matt et al. 1993, 1996).

A common normalizing parameter for reflected spectra is the reflection fraction ($R_f$; e.g., Magdziarz & Zdziarski 1995; García et al. 2014a, defined as the ratio of the photons that hit the disc to the photons that escape to infinity. A strength of our more self-consistent procedure is that we can actually calculate the reflection fraction rather than using it as a normalization constant in the model. We find that $0.19 < R_f < 0.28$. Usually $R_f > 1$ is found for lamppost models (Dauser et al. 2016a). For the lowest jet zone, a large amount of photons are lost to the black hole (Dauser et al. 2014). For the consecutive zones, the radiation is heavily beamed away at a
large distance away from the disc. This results in a lower reflection fraction. This is in line with Reynolds & Fabian (1997) and Beloborodov (1999) where special relativistic effects were used to reduce the reflection fractions commonly found in coronal models, which did not agree with observations (Dove et al. 1997). Using an earlier version of agnjet, irradiation by a jet was also proposed as a solution (Markoff & Nowak 2004). There $0.10 < R_f < 0.18$ was found.

**Comparison to relxill**

Several recent reflection studies employ instead of RELCONV and XILLVER the more recent model relxill (Ludlam et al. 2015; García et al. 2015b; Middleton et al. 2016). This model replaces the angle-averaged approach of the relativistic smearing kernel RELCONV with a self-consistent combination of the radiative transfer code XILLVER and RELCONV. Combining agnjet with a model that treats the reflection at each radius of the disc separately is something left to future research. Radially averaging, as done in this paper, has three effects. One, the incident spectrum is assumed the same for every radius. For this, we use the spectrum at the disc radius with the highest emissivity. Two, the incident radiation is assumed normal to the disc. And three, the relativistic smearing is radially averaged.

### 5.4 Dependence on jet parameters

We generate a grid of table models using the disc reflection code XILLVER (García & Kallman 2010; García et al. 2011), with 4 dominant free agnjet parameters, $N_j$ (0.001–0.01), $\Theta_e$ (1.5–20), $r_0$ (2–100 $r_g$) and $k$ (0.1–10). The grid is generated using the irradiation spectrum as seen by the accretion disc, the calculations for which are based on the work of Dauser et al. (2013), and described in Section 5.3. The model agnjet has parameters which are inherently source-specific, e.g., the black hole mass ($M_{\text{BH}}$), distance to the source ($D$), and inclination ($i$). We fix these to $M_{\text{BH}} = 5.5 \, M_\odot$, $D = 8$ kpc, and $i = 40^\circ$ based on previous broadband modelling of BHB GX 339–4 (see e.g., Markoff et al. 2003 and Connors et al., in prep., Chapter 4), a canonical source in which the connection between the radio and infrared emitting jet and its X-ray emission in the hard state (Corbel et al. 2000, 2003). Further model parameters have an effect on the resulting jet IC spectrum, most notably the inner disc temperature and radius ($T_{\text{in}}$ and $r_{\text{in}}$, parameterising the multi-temperature blackbody spectrum). We fix these to $T_{\text{in}} = 0.5$ keV and $r_{\text{in}} = 5 \, r_g$ respectively, again representing the typical disc conditions of BHBs in outburst in the hard state. Additional parameters upon which the emerging IC spectrum depends include the location along the jet axis at which electrons are accelerated ($z_{\text{acc}}$), the fraction of electrons accelerated at $z_{\text{acc}}$ ($f_{\text{nth}}$), and the power law index of the resulting distribution ($p$). As discussed in Section 5.2, if some portion of the electrons are accelerated into a
power law within the Comptonising region of the jet, the resultant non-thermal IC scattering will contribute to the irradiation spectrum, and thus effect the reflection. The decision to fix many of these parameters is based on the computational expense of generating the table of reflection models based on our parameter grid. The key parameters of \texttt{agnjet} are listed in table 5.1 along with the values used to generate the reflection models.
**Table 5.1:** The grid of parameters used to generate the table of reflection models for *xilver-jet*, and the relativistic smearing parameters used to generate the resultant relativistic reflection spectra. The black hole spin \((a)\) and inner and outer disc radii \((r_{in}, r_{out})\) are used to generate the irradiation spectrum in the disc frame, and these parameters are subsequently fixed in the model RELCONV to generate the relativistic reflection spectra.

<table>
<thead>
<tr>
<th>Model</th>
<th>Parameter</th>
<th>Description</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>agnjet</td>
<td>(M_{BH}) ([M_\odot])</td>
<td>mass of the black hole</td>
<td>5.5</td>
</tr>
<tr>
<td>agnjet</td>
<td>(D) ([\text{kpc}])</td>
<td>distance to the source</td>
<td>8</td>
</tr>
<tr>
<td>agnjet</td>
<td>(i)</td>
<td>the source inclination</td>
<td>40°</td>
</tr>
<tr>
<td>agnjet</td>
<td>(z_{acc}) ([r_g])</td>
<td>the location along the jet axis at which particle acceleration occurs</td>
<td>(10^3)</td>
</tr>
<tr>
<td>agnjet</td>
<td>(f_{sc})</td>
<td>the particle acceleration efficiency, parameterized assuming shock acceleration. The fractional scattering per particle interaction is given by (f_{sc} = \beta^2_{sh}/(\lambda/r_{gyro})), where (\lambda) is the mean free path of the particles, and (r_{gyro}) is their gyro-radius</td>
<td>(10^{-4})</td>
</tr>
<tr>
<td>agnjet</td>
<td>(f_{nth})</td>
<td>the fraction of particles accelerated into a power law at (z_{acc})</td>
<td>0.1</td>
</tr>
<tr>
<td>agnjet</td>
<td>(r_{in}) ([r_g])</td>
<td>the inner disc radius</td>
<td>5</td>
</tr>
<tr>
<td>agnjet</td>
<td>(T_{in}) ([\text{keV}])</td>
<td>the inner disc temperature</td>
<td>0.5</td>
</tr>
<tr>
<td>agnjet</td>
<td>(r_{out}) ([r_g])</td>
<td>the outer disc radius</td>
<td>(10^3)</td>
</tr>
<tr>
<td>agnjet</td>
<td>(N_J) ([L_{Edd}])</td>
<td>the normalised jet power</td>
<td>(10^{-3}–10^{-2})</td>
</tr>
<tr>
<td>agnjet</td>
<td>(\Theta_e) ([kT_e/mc^2])</td>
<td>the temperature of the relativistic electrons injected into the jet</td>
<td>1.5–20</td>
</tr>
<tr>
<td>agnjet</td>
<td>(r_0) ([r_g])</td>
<td>the radius of the jet nozzle</td>
<td>2–100</td>
</tr>
<tr>
<td>agnjet</td>
<td>(\beta_e) ([U_e/U_B])</td>
<td>the ratio of electron to magnetic energy density</td>
<td>0.1–10</td>
</tr>
<tr>
<td>XILLVER</td>
<td>(\log \xi)</td>
<td>the disc ionization, defined as (F_X/4\pi n_e), where (F_X) is the ionizing flux in the 1–1000 Ry band, and (n_e) is the disc free electron density</td>
<td>1–3</td>
</tr>
<tr>
<td>RELCONV</td>
<td>(\epsilon_1)</td>
<td>the index of the emissivity profile (I(r) \propto r^{-\epsilon}) at (r \leq r_{br})</td>
<td>3.5</td>
</tr>
<tr>
<td>RELCONV</td>
<td>(\epsilon_2)</td>
<td>the index of the emissivity profile (I(r) \propto r^{-\epsilon}) at (r \geq r_{br})</td>
<td>3</td>
</tr>
<tr>
<td>RELCONV</td>
<td>(r_{br}) ([r_g])</td>
<td>the break radius of the emissivity profile</td>
<td>10</td>
</tr>
<tr>
<td>RELCONV</td>
<td>(a)</td>
<td>the black hole spin</td>
<td>0.998</td>
</tr>
</tbody>
</table>
For the relativistic smearing model RELCONV we have to set an emissivity profile as described in section 5.3.2. We fix the break radius at $r_{\text{br}} = 10 \, r_g$, and the indices at $\epsilon_1 = 3.5$, $\epsilon_2 = 3$ based on the emissivity profile shown in Figure 5.6. The resultant reflection spectrum as a function of the input jet parameters, generated using the model agnjet + RELCONV(XILLVER-JET), is shown in Figure 5.7. As the normalised jet power ($N_j$) increases, the reflection spectrum hardens, due to the increase in harder jet IC emission irradiating the disc. The jet IC spectrum hardens significantly due to increased external IC scattering of disc photons and higher jet power (a higher jet power increases the overall energy budget of the jet, resulting in higher magnetic field strength and plasma density). The most striking feature of the dependency of reflection spectra on the jet parameters is the dramatic boost in the Compton hump as the jet becomes very compact ($r_0 \leq 5 \, r_g$). This strong dependency of the reflection on the size and location of the base of the jet is due to relativistic effects as discussed in Section 5.3. Photons from lower in the jet are blueshifted and primarily strike the inner regions of the disc, and a large proportion of photons are directed towards the disc as opposed to travelling to infinity. These two effects result in higher reflection fractions and significantly harder reflection spectra for a more compact jet which lies closer to the horizon. The reflection spectrum has a complicated dependence on the electron temperature, $\Theta_e$. The double-peaked jet IC spectrum shifts to higher energies as $\Theta_e$ increases, and the resultant reflection spectrum sees a change in the ratio of the 2 peaks in its shape. The Compton hump is purely a result of down-scattering irradiating photons above 10 keV (see García et al. 2013), so as the peaks of the multiple IC spectrum shift to higher energies, the reflection spectrum changes its shape similarly. The dependence of reflection on $\beta_e$ is similar to that of $N_j$ and $r_0$. As a higher proportion of the jet’s internal energy density is assigned to the electrons, the reflection spectrum hardens.

The shape of the reflection spectrum as a result of irradiation from agnjet differs significantly from that of a irradiating cut-off power law spectrum (see Figure 5.8). The electrons injected into the base of the jet are purely relativistic ($\Theta_e \geq 1$), and the jet base has optical depths in the range $\tau \sim 10^{-4}$–$10^{-2}$ for the range of parameters used to generate the reflection models. As a result, the jet IC spectrum is curved, and not power-law like. The emergent jet IC spectrum depends on the two photon fields scattered by the electron population (thermal/non-thermal synchrotron from the jet, and blackbody emission from the disc). However, as shown in Figure 5.7, the secondary peak in the reflection appears above 100 keV and thus would not be a detectable feature in X-ray observations of BHBs.
Figure 5.7: Reflection spectrum of XILLVER-JET as a function of the input parameters of agnjet. Each panel shows iterations of each of the jet parameters \((N_j, r_0, \Theta_e, \beta_e)\) used to generate the grid of reflection spectra, across their full range, where the default values are \(N_j = 0.005, r_0 = 25 r_g, \Theta_e = 5\) and \(\beta_e = 1\). The parameters of RELCONV are the same in all spectra, with \(a = 0.998, \epsilon_1 = 3.5, \epsilon_2 = 3,\) and \(r_{br} = 10 r_g\). The disc extends down to \(r_{in} = 5 r_g\) and up to \(r_{out} = 1000 r_g\), and the inclination to the line of sight is \(i = 40^\circ\). Dotted lines show the associated irradiating spectrum as seen by the observer.
5.5 Discussion

We calculate a grid of reflection models where IC emission from a mildly-relativistic jet (with initially velocity of $\beta = 0.4c$, and $\beta \sim 0.8c$ at 100 $r_g$) is the primary continuum irradiating the disc. Two distinguishing features differentiate our reflection model from previous models (e.g., Ross et al. 1999; Ross & Fabian 2005; García et al. 2014a). Firstly, the irradiation flux is set by the physical parameters of the jet, and thus the continuum component requires no renormalisation when fitting our reflection model to observed spectra. Secondly, the irradiation spectrum has significant curvature, and thus results in a reflection spectrum with a curved, double-peaked Compton hump at energies $> 10$ keV.

We calculate the reflection fraction for the range of jet parameters used to generate the reflection models, and find values in the range $0.19 < R_f < 0.28$, slightly higher than the range found previously with a previous version of the jet model (Markoff & Nowak 2004). The higher values we obtain here are mostly due to the inclusion of relativistic effects in the inner 10 $r_g$ of the jet-disc interface. In these inner regions, light-bending and gravitational energy shifts result in more high-energy photons strik-
Modelling disc reflection of jet emission

Determining the relative contribution of the jet and corona to the X-ray spectra of BHBs can better our understanding of the resultant reflection spectrum and explain the reflection fractions derived from spectral modelling (e.g., Dove et al. 1997). Recent modelling of the reflection spectra of BHBs in which the irradiating continuum has a lamppost geometry reveals a persistent degeneracy between the location of the inner edge of the accretion disc and the height of the lamppost (e.g., Fürst et al. 2016a,b; Basak & Zdziarski 2016; Basak et al. 2017). The reflection fraction is predicted to decrease both when the irradiating source is further from the disc, and when the disc is truncated further from the black hole. The additional degeneracy between these model parameters and the spin of the black hole adds further uncertainties (Dauser et al. 2013). The further complication of multiple irradiating components, in particular the possible stratification of the electron temperature within $h \sim 2-100 r_g$ of the black hole (Connors et al. 2017, in prep., and Chapter 4) should be considered as an additional element in disentangling reflection from the irradiating X-ray continuum.

We are currently improving our grid of reflection models to enable a physical treatment of the normalisation in reflection modelling. Given that we can directly calculate the irradiating flux from the jet based on physical parameters, the normalisation of the reflection component of XILLVER-JET can be calculated for any given disc inclination, removing the need for a single normalisation constant for the continuum and reflected components.

Acknowledgements

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Searching for Primordial Black Holes in the radio and X-ray sky

All you have to decide, is what to do with the time that is given to you.
— Gandalf


*Physical Review Letters, 2017, 118, 241101*

Abstract

We model the accretion of gas onto a population of massive primordial black holes in the Milky Way, and compare the predicted radio and X-ray emission with observational data. We show that under conservative assumptions on the accretion process, the possibility that $\mathcal{O}(10) M_\odot$ primordial black holes can account for all of the dark matter in the Milky Way is excluded at $5\sigma$ by a comparison with a VLA radio catalog at 1.4 GHz, and at $\simeq 40\sigma$ by a comparison with a Chandra X-ray catalog (0.5–8 keV). We argue that this method can be used to identify such a population of primordial black holes with more sensitive future radio and X-ray surveys.
6 Searching for primordial black holes

6.1 Introduction

The first direct detection of a gravitational wave signal, announced by the LIGO collaboration earlier this year Abbott et al. (2016b) demonstrated the existence of \( \sim 30 M_{\odot} \) black holes (BHs), prompting the suggestion (Bird et al. 2016; Clesse & García-Bellido 2016) that these objects are primordial black holes (PBHs) that may account for all of the dark matter (DM) (Jungman et al. 1996; Bertone et al. 2005; Bertone 2010) in the Universe. The connection between PBHs and DM has been extensively studied in the past (see, e.g., Ivanov et al. 1994; Khlopov 2010; Carr et al. 2016; Blais et al. 2002; Afshordi et al. 2003; Frampton et al. 2010), and a number of constraints exist on the cosmic abundance of PBHs over a very wide mass range (see the discussion below, and, e.g., Carr et al. (2016) for a recent review).

In this Letter, we consider for the first time, in the context of PBH searches, the X-ray and radio emission from the Galactic Ridge region produced by the accretion of interstellar gas onto a population of \( \sim 10^{9} M_{\odot} \) PBHs in the Milky Way. Given current estimates of the bulge mass (Portail et al. 2015), if PBHs constitute all of the DM, there should be \( \mathcal{O}(10^{9}) \) such objects within 2 kpc from the Galactic center (GC). Since the inner part of the bulge contains high gas densities (Ferrière et al. 2007), a significant fraction would inevitably form an accretion disc and emit a broad-band spectrum of radiation. We show (fig. 6.1) that radio and X-ray data in the Galactic Ridge region rule out, at 5 and 40\(\sigma\) respectively, the possibility that PBHs constitute all of the DM in the Galaxy, even under conservative assumptions on the physics of accretion.

Our limits arise from a realistic modelling of the accretion process, based on the observational evidence for inefficient accretion in the Milky Way today (Perna et al. 2003; Pellegrini 2005), and corroborate, with a completely independent approach, the exclusion of massive PBHs as DM candidates.

6.2 Accretion onto black holes

We should expect the accretion rates, \( \dot{M} \), of a Galactic population of PBHs accreting from interstellar gas to be well below the Eddington limit \( \dot{M}_{\text{Edd}} \). Even under the unrealistic assumption of Bondi-Hoyle-Lyttleton accretion (Hoyle & Lyttleton 1939; Bondi & Hoyle 1944), and typical velocities as low as \( \sim 10 \) km/s, the accretion rate would definitely be sub-Eddington: \( \dot{M} \sim 10^{-5} \left( \frac{n_{\text{gas}}}{\text{cm}^{-3}} \right) \dot{M}_{\text{Edd}} \).

BHs accreting at \( \dot{M} < 0.01 \dot{M}_{\text{Edd}} \equiv \dot{M}_{\text{crit}} \) are radiatively inefficient, such that the luminosity scales non-linearly with \( \dot{M} \) (Heinz & Sunyaev 2003). The prevailing physical pictures adopted to explain the weak emission properties are advection-dominated accretion in which the gas cooling timescales greatly exceed the dynamical timescales (Narayan & Yi 1994), and mass loss from the disc or internal convective flows, such
6.2 Accretion onto black holes

Figure 6.1: Upper limits on the fraction of DM in PBHs of a given mass $M$, arising from the non-observation of bright X-ray (blue shaded regions) and radio (red) BHs candidates at the GC. We assume a conservative value of $\lambda$, regulating the departure from Bondi accretion rate: $\lambda = 0.02$. The dotted grey line corresponds to $30 M_{\odot}$ PBH, the hatched grey region is unphysical ($f_{DM} > 1$).

that the accretion rate itself has decreased once gas reaches the inner edge of the disc (Blandford & Begelman 1999; Quataert & Gruzinov 2000). It is likely that both mechanisms are at play, a view supported by both radio and X-ray constraints on the gas density around Sgr A*, the supermassive BH at the center of the Galaxy, the least luminous accreting BH observed to date (in Eddington units), and thus a well-studied source from the point of view of weak accretion physics Bower et al. (2003); Marrone et al. (2007); Wang et al. (2013). We compute the accretion rates and the radiative efficiencies of a Galactic population of PBHs in the low-efficiency limit, following the formalism presented in Maccarone 2005; Fender et al. 2013. We take into account the findings of previous studies regarding accretion of interstellar gas onto isolated black holes (Fujita et al. 1998; Armitage & Natarajan 1999; Agol & Kamionkowski 2002).

We model the radiative efficiency $\eta$, defined by the relation for the bolometric luminosity $L_B = \eta \dot{M} c^2$, as $\eta = 0.1 \dot{M} / \dot{M}_{crit}$ for $\dot{M} < \dot{M}_{crit}$ (if we were to assume instead efficient accretion above the critical rate, $\dot{M} > \dot{M}_{crit}$, then we would have
a constant $\eta = 0.1$). As already discussed, all our sources fall below this critical accretion rate, such that they are all inefficient accretors: this means the luminosity scales non-linearly with accretion rate, $L \propto \dot{M}^2$.

We parameterize the accretion rate as $\dot{M} = \lambda \dot{M}_{\text{Bondi}}$, such that

$$\dot{M} = 4\pi \lambda (GM_{BH})^2 \rho \left( v_{BH}^2 + c_s^2 \right)^{-3/2}$$

(6.1)

where $G$ is the gravitational constant, $v_{BH}$ is the velocity of the BH, and $c_s$ is the sound speed of the accreted gas, which is below 1 km/s in cold, dense environments.

An important element that needs consideration is the temperature of the accreted gas due to radiative pre-heating (Maccarone 2005). Photoionising radiation will lead to an ionisation bubble surrounding the source, known as the Strömgren sphere Strömgren (1939), with a characteristic radius, $R_s$. In the following, we assume that the gas around the BH is fully ionized – and therefore, we set $c_s = 10$ km/s – if the timescale for the ionization of the Strömgren sphere is shorter than the timescale associated with the incoming flux of fresh, unprocessed material. We show a full calculation of the size of the Strömgren sphere in Appendix A.

Regarding $\lambda$, we choose a reference value of 0.02. Given the degeneracy between $\lambda$ and the angular momentum and temperature of the accreted gas, this value is consistent with isolated neutron star population estimates and studies of active Galactic nuclei accretion (Perna et al. 2003; Pellegrini 2005; Wang et al. 2013). We present additional considerations of the relationship between $\lambda$, $v_{bh}$ and the pre-heating of accreted gas in Appendix A.

This prescription is the same as that adopted by Fender et al. (2013); however, we consider $M_{BH} = 30 M_\odot$, and rescale the value of $\dot{M}_{\text{crit}} = 0.01 \dot{M}_{\text{Edd}}$ used in that work across the full 10–100 $M_\odot$ mass range.

We convert bolometric luminosity to X-ray luminosity via the approximate factor $L_X \simeq 0.3 L_B$ following Fender et al. (2013).

Motivated by the results presented in Fender (2001) and by the discussion in Mac- carone (2005); Fender et al. (2013), we assume the presence of a jet—thus requiring a system with a surplus of angular momentum, or a dynamically important magnetic field combined with a spinning black hole—emitting radio waves in the GHz domain with an optically-thick, self-absorbed, almost flat spectrum, whilst the X-ray emission is non-thermally dominated, originating from optically thin regions closer to the BH. In order to convert the X-ray luminosity into a GHz radio flux, we adopt the universal empirical relation discussed, e.g., in Plotkin et al. (2012), also known as the fundamental plane (FP), which applies for a remarkably large class of compact objects of different masses, from X-ray binary systems to active Galactic nuclei. We calculate the X-ray luminosity in the 2–10 keV band in accordance with the FP, assuming a hard power-law X-ray spectrum with photon index $\alpha$, and a typical range for hard state X-ray binaries of 1.6–2.0 (see Hong et al. 2016). We extrapolate this power-law
spectrum into the 0.5–8 keV and 10–40 keV bands in order to also make comparisons with Chandra and NuSTAR catalogs. We then use the FP relation to calculate the 5 GHz radio flux from the 2–10 keV X-ray flux and assume a flat radio spectrum, such that $F_{5\text{GHz}} = F_{1.4\text{GHz}}$, allowing direct comparison with the 1.4 GHz source catalog from a VLA survey of the GC region.

6.3 Primordial black hole population

In order to derive a bound from X-ray and radio data, we set up a Monte Carlo simulation for each PBH mass, assuming a delta mass function. We populate the Galaxy with PBHs following the Navarro-Frenk-White (NFW) distribution (Navarro et al. 1996) (other more conservative choices are discussed below). We implement the accurate 3D distribution of molecular, atomic, ionized gas in the inner bulge presented in (Ferrière et al. 2007); that distribution includes a detailed model of the 3D structure of the Central Molecular Zone (CMZ), a 300 pc wide region characterized by large molecular gas density and centered on the GC, i.e. in the region where the largest density of PBHs is expected.

For each PBH, the velocity is drawn randomly from a Maxwell-Boltzmann distribution. The characteristic velocity of the distribution is position-dependent. The velocity distribution at a given radius is a crucial ingredient, because the accretion rate scales as $v^{-3}$, Equation 6.1. In order to derive such a distribution, we consider the recent state-of-the-art model for the mass distribution in the Milky Way described in McMillan (2016), where 6 axis-symmetric components are taken into account (bulge, DM halo, thin and thick stellar discs, and HI and molecular gas discs). We then assume that the velocity distribution at a distance $R$ from the GC is a Maxwell-Boltzmann with $v_{\text{mean}} = v_{\text{circ}}(R) = \sqrt{(GM(<R))/R}$. Under the assumption of isotropic orbits, an exact computation of the phase-space density could be performed by means of the Eddington formalism (Eddington 1915), as done, e.g., in Fornasa & Green (2014). We checked that our simple approach is equivalent in the low-velocity tail, up to $v \simeq 40 \text{km/s}$. Since our results depend only on PBHs with velocities $\lesssim 10 \text{km/s}$ (see below), we can safely neglect the high-velocity tail and adopt the simple formalism described above.

Given the mass, position and velocity of each PBH (and the gas density), we compute accretion rate, X-ray, and radio emission adopting the prescriptions discussed in the previous section.

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1 We verified that, in the high-resolution Aquarius N-body simulations, the anisotropy parameter $\beta = 1 - \sigma_t/\sigma_r$ is consistent with 0 in the whole range of radii we are interested in, therefore the assumption of isotropic orbits is solid.

2 M. Fornasa, private communication.
6.4 Radio BH candidates

The 1.4 GHz source catalog from a VLA survey of the GC region (Lazio & Cordes 2008) contains 170 sources in a $1^\circ \times 1^\circ$ region centered on the GC. The minimum detectable flux for this catalog is $\sim 1$ mJy.

In order to compare our predictions to the observations, we carry out a data analysis on the VLA catalog and check if there can be any BH candidate among the detected sources.

If any of these sources are accreting BHs, their X-ray and radio emissions should be co-located. We therefore compare the radio catalog with the X-ray point source catalog from Muno et al. (2009), which contains 9017 sources detected by Chandra in the 0.5 – 8 keV band in a $2^\circ \times 0.8^\circ$ band centered on the GC, and search for all sources in both catalogs that have positions within $10''$ of each other.

We find 24 sources in both the X-ray and radio catalogs within $10''$ of each other.

\footnote{This is a very conservative separation. The positional accuracy of Chandra is $< 1''$. For the VLA, the positional accuracy is typically a small fraction of the synthesized beam, $2''.4 \times 1''.3$ for the survey in Lazio & Cordes (2008), taken in A configuration. A separation of $10''$ is chosen in Lazio & Cordes (2008) to search for positional coincidences in other radio catalogs; we therefore also adopt $10''$ as the maximum allowed separation.}
If we assume that these sources are accreting BHs, then their X-ray and radio fluxes should lie on the FP, as explained above. So, we use the FP (considering masses from 10 to 100 $M_{\odot}$) to predict the X-ray flux from the radio flux of each of these objects (24 in the very conservative case, 9 if we exclude likely foreground sources).

We find that the predicted X-ray fluxes are substantially larger ($\sim 3 - 7$ orders of magnitude) than the flux reported in the catalog from Munoz et al. (2009) in the whole mass range we consider. We therefore conclude that none of the 24 (or 9 likely Galactic) VLA sources with overlapping positions lie on the FP, and therefore, given the assumptions described above regarding the presence of a jet, we have no BH candidate in our sample.

### 6.5 X-ray BH candidates

Hard X-ray emission ($> 10$ keV) suffers from far less Galactic absorption than soft X-ray emission and is therefore a good band to search for emission from accreting BHs.

We consider sources in the Chandra catalog (Munoz et al. 2009) in the $0.5 - 8$ keV band, and those detected by NuSTAR in the $10 - 40$ keV band (Harrison et al. 2013). For Chandra (NuSTAR), we consider a small region-of-interest (ROI) including the high-density region of the Galactic Ridge: $-0.9^\circ < l < 0.7^\circ; -0.3^\circ < b < 0.3^\circ$ ($-0.9^\circ < l < 0.3^\circ; -0.1^\circ < b < 0.4^\circ$). There are 483 likely Galactic X-ray sources in the Chandra catalog above a flux threshold of $2 \times 10^{-6}$ ph cm$^{-2}$ s$^{-1}$ and 70 NuSTAR sources. Since in all cases the corresponding radio flux predicted with the FP would be $3 - 7$ orders of magnitude below the detection threshold of the VLA survey in Lazio & Cordes (2008), we cannot draw any conclusions on the nature of these X-ray sources. Therefore, we consider all of them in our analysis as potential BH candidates (we only remove $\sim 40\%$ of the detected NuSTAR sources that are thought to be cataclysmic variables Hong et al. 2016).

### 6.6 Results

The main result of the Letter is presented in fig. 6.1. We display the $2\sigma$, $3\sigma$, and $5\sigma$ constraints on the DM fraction as a function of the PBH mass.

The upper limits are derived as follows. We perform $O(100)$ Monte Carlo simulations for 10 reference values of the mass in the $10 - 100 M_{\odot}$ interval, assuming a DM fraction $f_{DM} = 1$. We determine the mean and standard deviation of the distributions of the predicted number of PBHs with radio fluxes above the VLA threshold and

---

4"Likely Galactic" sources are defined in Munoz et al. (2009) based on their hardness ratios. The exposure across the Chandra survey region is variable and the flux threshold used here is a compromise between maximizing the ROI and the completeness, per Munoz et al. (2009).
with X-ray fluxes exceeding the Chandra (nuSTAR) threshold, in the corresponding ROIs. We verify that the number of bright PBHs is compatible with Poisson statistic and the average predicted number scales linearly with $f_{\text{DM}}$. We derive the radio and X-ray bounds by comparing the number of predicted PBHs with the number of BH candidates derived from the analysis of radio and X-ray catalogs described in the previous section. For the X-ray bound, we show the result obtained with the more sensitive Chandra catalog. The NuSTAR bound is slightly weaker: It allows us to exclude at $2\sigma$ values of $f_{\text{DM}}$ as low as 0.4 (for $30\, M_\odot$).

In fig. 6.2 we show the PBHs detectable by VLA at 1.4 GHz assuming a PBH mass of $30\, M_\odot$ and DM fraction equal to 1, for one specific Monte Carlo realization. This scenario predicts, on average, $40 \pm 6$ sources above the VLA flux threshold for $30\, M_\odot$ and, thus, it is excluded by more than $5\sigma$ from radio observations. However, it is important to understand where the constraining power comes from: The PBHs above the detection threshold, and thus the ones with the larger X-ray flux, lie in the very inner region of the Galaxy where the column gas density is the highest and show very small velocities, in the range $\sim 0.3 - 3$ km/s. Therefore, the constraints arise from the very low velocity tail of the distribution and from regions correlated with very high column densities, e.g. CMZ, as already mentioned above.

6.7 Discussion and conclusions

In this Letter we derive new, strong constraints on the hypothesis that PBHs comprise all of the DM in the Universe. In particular, we find that PBHs with $M \approx 30\, M_\odot$, that could be responsible for the gravitational waves detected by LIGO, contribute less than 20% to the whole DM density.

In the mass window $10 - 100\, M_\odot$, our constraints are competitive with (and even stronger than) those arising from the study of microlensing events with the MACHO project (Alcock et al. 2001) (for $\gtrsim 15\, M_\odot$) and with those from halo wide binaries (Quinn et al. 2009, Monroy-Rodríguez & Allen 2014) (for $\gtrsim 60\, M_\odot$). For $M \gtrsim 10\, M_\odot$, they are also comparable or stronger than the constraints from the survival of central star clusters in faint dwarf galaxies, in particular in Eridanus II (Brandt 2016; Li et al. 2016). Even more stringent constraints arise in principle from the analysis of the Cosmic Microwave Background (CMB) (Ricotti et al. 2008). However, those arising from the analysis of spectral distortions (based on FIRAS data) turned out to be much weaker than originally thought (Clesse & García-Bellido 2016), while the ones based on the study of CMB anisotropies (see also the recent results by Chen et al. 2016), are based on assumptions on the accretion of gas on PBHs in the early Universe that are still under debate, as the modelling of the accretion process is based on theoretical arguments, and not directly supported by observations as in our case (see also the discussion in Clesse & García-Bellido 2016).
In contrast with Ricotti et al. (2008), in fact, we adopt a very conservative prescription, compatible with current astronomical observations, for both the accretion rate and the radiative efficiency, setting the ratio of the actual accretion rate to the Bondi rate, $\lambda$, equal to 0.02. We remark that $\lambda$ probably follows some distribution and is also likely degenerate with $c_s$ and $v_{BH}$—future studies are required to disentangle these. Moreover, we exploit for the first time in this context the empirical FP relation between radio and X-ray emission, which has been observed on a wide class of sources in a large mass range, from X-ray binaries to active Galactic nuclei. By adopting such a relation, we are able to predict the expected radio and X-ray luminosities of a population of PBHs in the Galaxy compatible with the DM phase-space distribution, as well as to look for BHs candidates in radio and X-ray catalogs. We set upper limits on the DM PBH fraction using both radio (VLA) and X-ray (Chandra and NuSTAR) point-like source catalogs, by comparing the number of expected PBHs above observational thresholds and the observed number of BH candidates in a very narrow region about the GC.

These bounds are robust with respect to the modelling of the full velocity distribution, since the predicted number of bright PBHs only depends on the very low-velocity tail ($< 10$ km/s) where we checked the agreement among different numerical/analytical methods. Moreover, our limits are independent of the details of the gas distribution (we checked that the bound is still present even with a naive modelling of the CMZ as a sphere with uniform density compatible with the mass constraints provided in Ferrière et al. (2007)). They are also not affected significantly by a shallower DM profile as proposed e.g. in Calore et al. (2015); however, assuming an even flatter profile like the Burkert one (an extremely conservative assumption for our Galaxy), the bound is present only under the assumption of Bondi accretion.

We recall that our limits hold for a narrow mass function; a detailed study of the impact of different mass distributions is beyond the scope of the present paper and postponed to a future work.

Although our radio and X-ray bounds vanish for $\lambda \lesssim 10^{-2}$, future instruments will be able to verify better the accretion model as well as the PBH DM fraction. In particular, given the significant increase in sensitivity of future radio telescopes, we expect an important part of the yet-allowed parameter space to be probed by upcoming facilities such as MeerKAT and, later, SKA. Using the radiometer equation (Dewey et al. 1984), the minimum (1σ) detectable radio flux is $S_{\nu,\text{rms}} = (T_{\text{sky}} + T_{\text{rx}})/(G \sqrt{2 T_{\text{obs}} \Delta \nu})$. For SKA1-MID (1.4 GHz), we assume gain $G = 15$ K/Jy, receiver temperature $T_{\text{rx}} = 25$ K, sky temperature towards the GC $T_{\text{sky}} = 70$ K, and bandwidth $\Delta \nu = 770$ MHz (Calore et al. 2016).

For one-hour exposure, the instrumental detection sensitivity of SKA1-MID turns out to be $\sim 2.7 \mu$Jy (significantly above the source confusion limit), which would give O(2000) detectable PBHs for our reference value $\lambda = 0.02$ ($f_{DM} = 1$, and $M = 30M_\odot$).

SKA will therefore be able to either place a very strong constraint in absence of
BH candidate detection, or detect a subdominant population of PBHs (although the expected population of astrophysical BHs becomes comparable with the primordial one for DM fractions lower than $\sim 10^{-3}$). With an even longer exposure ($\simeq 1000$ h, 85 nJy sensitivity), achievable for dedicated deep field continuum observations, such strong constraints can be placed by SKA even under the assumption of extremely low values of $\lambda$, $\mathcal{O}(10^{-3})$.

Interestingly, our procedure can also be applied in order to extend work on searches for astrophysical BHs in the Galaxy (Agol & Kamionkowski 2002; Macaron 2005; Fender et al. 2013), adopting the realistic spatial and velocity distributions expected for those objects. Our formalism has the potential to characterize this guaranteed population of objects in future analyses.
In Section 6.2 of chapter 6 we present a simple prescription for accretion of interstellar gas onto a population of primordial black holes (PBHs) based on the Bondi-Hoyle-Littleton formula (Hoyle & Lyttleton 1939; Bondi & Hoyle 1944), Equation 6.1. In this prescription the parameter $\lambda$ contains all our ignorance with regards to how the accretion flow is modified from the Bondi radius ($R_B = 2GM/c_s^2$) down to the Schwarzschild radius of the black hole ($R_s = 2GM/c^2$). In addition to this uncertainty, many previous considerations of accretion onto isolated black holes have raised the issue of the pre-heating of interstellar gas (e.g., Shapiro 1973; Ostriker et al. 1976; Agol & Kamionkowski 2002; Ricotti et al. 2008; Park & Ricotti 2012). If the ionising flux emitting by the accreting PBH is sufficient to ionise the gas beyond $R_B$, the sound speed of the gas will increase, modifying the inverse-cubed velocity term in Equation 6.1, $(v_{BH}^2 + c_s^2)^{-3/2}$. As shown in chapter 6, the detection probability of a population of PBHs in the Galactic centre region rises sharply for black holes travelling at $v_{BH} \leq 10$ km s$^{-1}$, assuming the surrounding interstellar gas is cold ($c_s \sim 1$ km s$^{-1}$). If instead the gas surrounding any given PBH is pre-heated by the ionising flux emitted from the gas as it accretes, a lower bound of $c_s \sim 10$ km s$^{-1}$ is placed on the sound speed of the gas, based on the ionisation temperature of Hydrogen, $T \sim 10^4$ K. Here I present an analytical treatment of the size of the ionised bubble surrounding an accreting PBH in the Galactic central molecular zone (CMZ).

A.1 The Strömgren sphere

The standard relation for the Strömgren radius is obtained by equating the ionisation and recombination rates ($\dot{N}_R = \dot{N}_I$) and solving for $R_S$. The recombination rate per unit volume is given by $\dot{N}_R = n_e n_{HII} \alpha_H(T)$, where one assumes $n_e \sim n_{HII}$, such that $\dot{N}_R = n_{HII}^2 \alpha_H(T)$. The ionisation rate is the emission rate of Hydrogen-ionising
photons \(E_{ph} > 13.6\text{eV}\), which we define here as \(S_*\). We multiply the recombination rate per unit volume by the volume of the region, characterised by \(R_S\), and rearrange for \(R_S\), giving (Strömgren 1939)

\[
R_S = \left[ \frac{3S_*}{4\pi n_H^2 \alpha_H(T)} \right]^{1/3} \tag{A.1}
\]

where the case B recombination coefficient, \(\alpha(T)\), is a weak function of gas temperature, and a table of its values can be found in Osterbrock (1989). For a gas at \(T = 10^4\ \text{K}\), \(\alpha(T) = 2.59 \times 10^{-13}\ \text{cm}^3\text{s}^{-1}\). Equation (A.1) describes the size of the Strömgren sphere for a stationary source of isotropic ionising flux. Since we treat sources in motion in chapter 6 a proper treatment must include the effects of a moving source.

A.2 The effective Strömgren sphere of a moving source

The effective Strömgren sphere of a moving source (be it a star or an arbitrary illuminating source) differs from the standard relation if one considers the space velocity of the source to be much greater than the sound speed of the traversed gas \((v_\ast \gg c_s)\) (Raga et al. 1997); first and foremost, the ionisation surface will no longer be spherical.

For consistency from here on, I consider a 30 solar mass black hole traveling through the Central Molecular Zone (CMZ—assume density roughly \(n_H \sim 10 \text{cm}^{-3}\)) of the Galaxy at a conservative velocity of \(v_{\text{BH}} \sim 10\ \text{km s}^{-1}\), where the characteristic temperature of the region is roughly 100 K, consistent with a gas sound speed of \(c_s \sim 1\ \text{km s}^{-1}\).

The following calculation (adapted from Raga et al. (1997)) only applies to a highly supersonic flow, such that we can approximate two separate regions of homogenous density, upstream and downstream from the shock. We can solve this problem from the reference frame of the black hole, with gas travelling towards it at \(v_{\text{BH}}\). The rate of ionising photons emitted by the accreting black hole in a unit solid angle, \(\Delta\Omega\) (for simplicity we assume the emission in the rest frame of the source is isotropic), must balance the recombination rate of the gas it is ionising, but it must also balance the inflowing neutral gas due to its motion. This leads to the following balance equation:

\[
\frac{S_{\text{BH}}}{4\pi} \Delta\Omega = \dot{N}_R(\theta) + \dot{N}_{\text{in}}(\theta) \tag{A.2}
\]

Notice that these all have an angular dependence, such that \(\Delta\Omega = 2\pi(1 - \cos \theta)\). The geometry of this scenario is such that we have two radii to consider, the spherical
radius $R$, and the cylindrical radius $r$, defined such that $r = R \sin \theta$. The total number of recombinations per unit time is given by

$$\dot{N}_R(\theta) = n_H^2 \alpha_H V(\theta); \quad V(\theta) = \frac{2\pi}{3} \int_0^\theta R^3 \sin \theta d\theta,$$

where $V(\theta)$ is the ionised volume. The inflow rate of neutral atoms is given by

$$\dot{N}_\text{in}(\theta) = \pi r^2 n_H v_{\text{BH}}$$

There is no general analytic solution to Equation (A.2) but one can consider the limiting case that $v_{\text{BH}}$ is large, since with increasing velocity $\dot{N}_\text{in}$ will increase linearly, whereas $\dot{N}_R$ will actually decrease since the effective ionisation sphere is getting smaller, leading to $\dot{N}_\text{in} \gg \dot{N}_R$. Now one can find an analytic solution, which is given by

$$r(\theta) = \sqrt{\frac{S_{\text{BH}}}{2\pi n_H v_{\text{BH}}}} (1 - \cos \theta).$$

The $\theta$-dependence here just leads to variations between $\pi$ and $4\pi$ in the denominator, so let’s just consider the on-axis separation radius ($\theta = 0$), where one then obtains the following solution for the effective Strömgren radius:

$$R_0 = \sqrt{\frac{S_{\text{BH}}}{4\pi n_H v_{\text{BH}}}}$$

One can now check whether our assumption that $\dot{N}_\text{in} \gg \dot{N}_R$ really holds, and this is done by checking that $R_0 \ll R_S$, i.e. that the recombination rate only becomes important far away from the shock region. What this really leads to is the requirement that $v_{\text{BH}}$ be large enough to satisfy the following relation (where now we must distinguish the irradiating photon emission rate between the cases $v_{\text{BH}} \gg c_s$ and vice versa, using $S_0$ and $S_s$ for each case respectively):

$$v_{\text{BH}} \gg S_0 \left( \frac{n_H}{4\pi} \right)^{1/3} \left( \frac{3S_s}{\alpha_H(T)} \right)^{-2/3}$$

One can see that Equation (A.7) reduces to something further after substituting in the expressions for $S_0$ and $S_s$, and we show this calculation in the following section.
A.3 Application to primordial black holes within the CMZ

We can now apply this to a primordial black hole of 30 solar masses travelling at \( \sim 10 \text{ km s}^{-1} \) in the CMZ. First we consider the emission rate of ionising photons, which we can do by assuming a photon index of 1.7 (representative of XRBs in the hard state; e.g., Hong et al. 2016), between the energy limits of \( E_0 = 13.6 \text{ eV} \) and \( E_1 = 100 \text{ keV} \) (again representative of the typical high-energy cut-offs seen in hard state XRBs). This allows us to relate the luminosity in this energy range (\( L \)) to the photon arrival rate (\( S_{\text{BH}} \)). This luminosity in ergs s\(^{-1}\) is given by

\[
L_X = \int_{E_1}^{E_0} S_{\text{BH}} \, dE
\]

where \( dS_{\text{BH}} = N E^{-0.7} \, dE \), such that after rearranging for \( N \), performing the integrals, and then rearranging for \( S_{\text{BH}} \), we have the photon emission rate:

\[
S_{\text{BH}} = 0.3 \frac{L}{0.7 [E_1^{0.3} - E_0^{0.3}]} \left[ \frac{1}{E_0^{0.7}} - \frac{1}{E_1^{0.7}} \right] = 1.5 \times 10^9 L \text{ photons s}^{-1}, \tag{A.8}
\]

where we have substituted in \( E_0 = 13.6 \text{ eV} \) and \( E_1 = 100 \text{ keV} \) (after converting to ergs) to arrive at the final expression. We can then derive our expression for \( L \) using the formula for the Bondi accretion rate, assuming \( v_{\text{BH}} \gg c_s \):

\[
L = \eta \lambda \dot{m}_{B} c^2 = \frac{(\eta \lambda) 4\pi G^2 M_{bh}^2 n_H m_H}{\dot{m}_{\text{BH}}^3}, \tag{A.9}
\]

where \( \lambda \) is the ratio between \( \dot{m}_{\text{in}} \), the accretion rate at the inner disc radius, and \( \dot{m}_{B} \), which we assume to be \( \sim 0.01 \). The variable \( \eta \) is the radiative efficiency for any given accretion rate, which we characterise according to our empirical understanding of the switch to inefficiency in XRBs, such that

\[
\eta = 0.1 \left( \dot{m}_{\text{in}} / \dot{m}_{\text{crit}} \right) = 0.1 \left( \dot{m}_{\text{in}} / 0.01 \dot{m}_{\text{Edd}} \right) \tag{A.10}
\]

assuming the switch to inefficiency (where one can see that \( L \propto \dot{m}_{\text{in}}^2 \)) occurs around \( L \sim 0.01 L_{\text{Edd}} \). If we now substitute these expressions for \( \eta \) and \( \lambda \) into Equation A.9 (using \( L_{\text{Edd}} = 1.2 \times 10^{38} (M_{bh}/M_{\odot}) \) erg s\(^{-1}\) and \( L_{\text{Edd}} = 0.1 \dot{m}_{\text{Edd}} c^2 \)), and then substitute Equation A.9 into Equation A.8 and the result into Equation A.6, we arrive at the following relation for the effective Strömgren radius of an accreting black hole in motion:

\[
R_0 \approx 1.7 \times 10^{15} \left( \frac{\lambda}{0.01} \right) \left( \frac{M_{bh}}{30 M_{\odot}} \right)^{3/2} \left( \frac{n_H}{10 \text{ cm}^{-3}} \right)^{1/2} \left( \frac{v_{\text{BH}}}{10^6 \text{ cm s}^{-1}} \right)^{-7/2} \text{ cm}. \tag{A.11}
\]
This value has significance, since for a 30 solar mass black hole moving at $v_{\text{BH}} = 10 \text{ km s}^{-1}$, the Bondi radius is given by

$$R_B = 8 \times 10^{15} \left( \frac{M_{30}}{30 \, M_{\odot}} \right) \left( \frac{v_{\text{BH}}}{10^6 \, \text{ cm s}^{-1}} \right)^{-2} \text{ cm.} \tag{A.12}$$

This tells us that for the average conditions within the CMZ, the $(v_{\text{BH}}^2 + c_s^2)^{-3/2}$ term in the Bondi formula is entirely dominated by the space velocity of the black hole (since there will be no modification to $c_s$ due to the photoionisation). However this all assumes that $v_{\text{BH}} \gg c_s$. If the detection likelihood of a primordial black hole in this region is entirely dominated by those black holes with $v_{\text{BH}} \ll c_s$, this relation is no longer valid, and we regress to the usual formula for the Strömgren radius given by Equation A.1.

Another check we should now perform is that $v_{\text{BH}}$ satisfies the inequality given by Equation A.7. This gives

$$v_{\text{BH}} \gg 10^5 \left( \frac{\lambda}{0.01} \right)^{2/21} \left( \frac{M_{30}}{30 \, M_{\odot}} \right)^{1/7} \left( \frac{n_H}{10 \, \text{ cm}^{-3}} \right)^{1/7} \left( \frac{c_s}{10^5 \, \text{ cm s}^{-1}} \right)^{4/7} \text{ cm s}^{-1}. \tag{A.13}$$

This follows logically from the a priori condition we set that when the black hole is stationary $c_s \gg v_{\text{BH}}$, whereas when the black hole is in motion at approximately 10 km s$^{-1}$, $v_{\text{BH}} \gg c_s$. The value lines up almost perfectly with the initial requirements, so we can equate this to the general rule that these black holes must be traveling fast enough to induce a supersonic flow between themselves and the surrounding gas, causing the inflow rate of neutral atoms $N_{in}$ to far exceed the recombination rate $N_R$. The weak dependencies on $\lambda$, $M$, and $n_H$ are negligible. If $c_s \gg v_{\text{BH}}$ we regress to the standard formula for the Strömgren radius:

$$R_S = 1.6 \times 10^{17} \left( \frac{\lambda}{0.01} \right)^{2/3} \left( \frac{M}{30 \, M_{\odot}} \right) \left( \frac{c_s}{10^5 \, \text{ cm s}^{-1}} \right)^{-2} \text{ cm} \tag{A.14}$$

Whilst for our assumptions this is indeed still within the Bondi radius, the difference is small, just a factor of a few (if you replace $v_{\text{BH}}$ in Equation A.12 with $c_s$), which would be accounted for if we were to increase $\lambda$, i.e. the primordial black holes accrete all the originally captured gas. We also note that an increase in the radiative
efficiency ($\eta$) would account for the difference. However, here we have also assumed that all photons with energy $> 13.6$ eV will ionise a H atom. This is not necessarily the case, since the photoionisation cross-section is energy dependent, such that higher energy photons have a lower cross-section.

One can see how this will interplay with detection limits by calculating the luminosity of a source that is either moving at $v_{\text{BH}} = 10$ km s$^{-1}$ or stationary (i.e. the Bondi accretion rate is determined by the local sound speed of the gas):

\[ L_{\text{moving}} = 2.4 \times 10^{29} \left( \frac{\lambda}{0.01} \right)^2 \left( \frac{M}{30 \, \text{M}_\odot} \right)^3 \left( \frac{n_H}{10 \, \text{cm}^{-3}} \right)^2 \left( \frac{v_{\text{BH}}}{10^6 \, \text{cm s}^{-1}} \right)^{-6} \text{ erg s}^{-1}, \]

(A.15)

and,

\[ L_{\text{stationary}} = 2.8 \times 10^{32} \left( \frac{\lambda}{0.01} \right)^2 \left( \frac{M}{30 \, \text{M}_\odot} \right)^3 \left( \frac{n_H}{10 \, \text{cm}^{-3}} \right)^2 \left( \frac{c_s}{10^5 \, \text{cm s}^{-1}} \right)^{-6} \text{ erg s}^{-1}. \]

(A.16)

The huge difference in luminosities here comes about simply because of the velocity term, due to the power 6 inverse dependence.

An estimate of the likelihood that a given PBH is surrounded by an ionised (pre-heated) sphere of interstellar gas can be calculated by considering the ionisation timescale and the crossing time of a PBH through its own Bondi sphere. The characteristic ionisation timescale if given by

\[ t_{\text{ion}} \sim \frac{n_H V_s}{S_{\text{BH}}} = \frac{4 \pi n_H R_S^3}{S_{\text{BH}}}. \]

(A.17)

The crossing time is given by

\[ t_{\text{cr}} \sim \frac{R_B}{v_{\text{BH}}}. \]

(A.18)

If $t_{\text{ion}} < t_{\text{cr}}$ holds, then the ionisation process is effective, since the PBH has time to ionise the surrounding gas before the arrival of fresh neutral gas. Figure A.1 shows the number of PBHs detectable with NuSTAR as a function of $\lambda$, where we treat the two extreme cases in which none or all of the PBHs in the simulated population effectively pre-heat the accreted gas, as well as the canonical case in which the pre-heating of accreted gas is based on whether $t_{\text{ion}} < t_{\text{cr}}$ holds. Pre-heating significantly
A.3 Application to primordial black holes within the CMZ

Figure A.1: The number of predicted X-ray sources detectable by NuSTAR as a function of $\lambda$, shown with pre-heating of the interstellar gas for all sources (yellow line), no sources (blue line), and under the condition that only sources with ionisation timescales less than the Bondi crossing time successfully ionise the captured gas (red line).

reduces the number of detectable sources. This is because the inverse-cube velocity term suffers a lower bound at the lowest PBH velocities ($v \geq c_s$), since ionised gas will have a sound speed of $c_s \geq 10$ km s$^{-1}$. The effect weakens significantly when we base the pre-heating of gas on ionisation and crossing timescales.
Conclusions and outlook

Our pursuit to understand the complexities of accretion onto black holes is building towards the seemingly fundamental characteristics of black holes themselves: mass and spin angular momentum. As we continue to develop models and simulations for the infalling and outflowing gas in accretion flows and jets, and employ innovative observational tools and techniques to track the broadband spectral and time-variability evolution of accreting black holes across the mass scale, we edge nearer toward realising the true impact black holes have on our universe. However, the degeneracy that persists in the modelling of radiation within the X-ray-emitting regions in the neighbourhoods of black holes is a barrier we have yet to surpass. Breaking these modelling degeneracies is the key to unlocking the riddle of accretion and outflows. Here are the conclusions and future prospects resulting from the work presented in this thesis.

In this thesis I have presented work that attempts to break degeneracies in spectral modelling of radiative processes observed in accreting black holes, ranging from stellar-mass to supermassive. Chapters 2–5 approached this problem through the development and application of a semi-analytical, outflow-dominated model which self-consistently treats the dynamics of the jet. The treatment is based upon jets with low Lorentz factors ($\gamma_j \sim$ a few) and internal energy densities approximately equal to the rest mass energy in the jet. The power in the jet is given as a fraction of the Eddington luminosity of the accreting gas. The parameters defining all size scales are scale-invariant (defined in units of the gravitational radius of the black hole).

In Chapter 2 we applied this jet model to simultaneous broadband observations of BHB A0620–00 and the Galactic centre SMBH Sgr A*, and showed that key fitted physical properties of the jet are strikingly similar despite the 6 orders of magnitude difference in black hole mass. This was the first study to compare accretion occurring at such low luminosities $L/L_{\text{Edd}} \sim 10^{-9}$ around black holes with such a wildly varied...
mass. A0620−00 probes the most quiescent region of the FP, the relation between X-ray luminosity, jet radio core luminosity, and black hole mass. We discussed two possible scenarios for the conditions of the plasma responsible for the primary continuum in the X-ray: 1) a SSC-dominated spectrum produced in the jet just a few gravitational radii from the black hole, in which the bulk of the jet’s internal energy density is in the electrons, and 2) an optically-thin non-thermal synchrotron spectrum produced by a power-law tail of electrons in the jet base. The ability to distinguish between synchrotron and SSC emission in the inner regions of Sgr A*’s accretion flow/jet hinges on further simultaneous broadband observations of the Galactic centre. In particular, simultaneous observations of IR and X-ray variability of Sgr A* will allow us to test the most likely scenario for what produces the power law X-ray of Sgr A* during its flares. We showed a possible way to test this, by predicting the timescales for particle acceleration in idealised conditions of shock acceleration and magnetic reconnection. Better IR and X-ray statistics will shed light on the dominant emission mechanism, or perhaps show that the variability is polluted by variable contributions of synchrotron radiation and SSC scattering. When combined with the Event Horizon Telescope (Doeleman et al. 2008) observations already taking place, which looks to probe the event horizon of Sgr A* (as well as the nearby LLAGN, M87), soon we will be able to connect what we actually see in the peak emission frequencies (submm) of the inner accretion flow with the variability in the IR and X-ray, and finally break these modelling degeneracies. This is a unique opportunity to understand the conditions of optically-thin accretion flows at the lowest accretion rates.

In Chapter 3 we showed that the application of our jet model, agnjet, is valid for low-luminosity jets in which the Lorentz factor only reaches factors of a few. We addressed algebraic errors intrinsic to the calculation of the jet’s velocity profile, and show that the effects on the resulting jet spectrum are minimal after corrections are made. We conclude that despite the inaccuracies in the assumption of a physical limit to the jet’s internal energy density (i.e. maximal jets), the key facets of the jet model that the jet enthalpy flux is roughly twice its rest mass energy density, and it remains quasi-isothermal, are sound assumptions to make when modelling astrophysical jets in BHBs and LLAGN. This is strongly supported by the success of the model in achieving physically consistent best-fit parameters when modelling multiple hard-state BHBs and LLAGN.

In Chapter 4 we modelled 20 separate quasi-simultaneous observations (radio, IR, optical, and X-ray) of GX 339−4 over 3 separate outbursts, covering several outburst rises and decays. By calculating a single-parameter identifier of the shape of the power spectrum of X-ray variability in each observation, we were able to track how the physical geometry and plasma conditions from the inner regions of the accretion flow to the self-absorbed regions of the jet evolve with the changing X-ray variability properties. We showed that there is some evidence for a slightly more compact jet
base, lower electron temperatures, and sub-dominant magnetic fields (with respect to the electrons) during the early stages of outburst decay, in contrast to the latter stages. The earlier stages have narrower-peaked rms variability, associated with higher X-ray luminosity, and coincident with softer X-ray spectra. We showed that the high electron temperatures and low optical depths treated in the jet model result in curved IC spectra that require excessively high reflection fractions to successfully model the X-ray spectrum alone. We invoked a composite jet + corona geometry, where the key difference was the treatment of the plasma; the coronal model has lower electron temperatures and higher optical depths. Within the context of this composite model, thermal Comptonization in the corona dominates the X-ray spectrum, however there is a 10–50\% contribution to the flux from jet IC emission in the 10–40 keV band. This has important implications for disc reflection modelling, whereby lower reflection fractions are derived from reflection modelling of BHBs and AGN than one would expect from the covering fractions of coronal components that are compact and close to the disc. Our treatment of the X-ray-emitting regions as a two-component geometry highlights the need for a more complete treatment in which electron heating in the corona and jet should be considered in more detail. I propose that this is the next step in refining BHB jet models.

In Chapter 5 we presented a new self-consistent jet-disc reflection model. We generated a table of reflection models using the most state-of-the-art disc reflection code, XILLVER, using a grid of key physical jet parameters from our jet model. We calculated the incident flux on the disc for every combination of gridded parameters, taking into account general and special relativistic effects based on maximal black hole spin and the Lorentz profile and geometry of the irradiating jet. We found realistic ranges for both the ionization parameter and reflection fraction based on typical input parameters obtained from broadband modelling of BHBs (see, e.g., Chapter 4). We also obtained an emissivity profile similar to those obtained from lamppost reflection modelling. The advantage of calculating reflection models with irradiation spectra dependent upon physically motivated jet parameters is that we are able to calculate the reflection fraction prior to fitting to real data. Therefore we can rule out spectral fits that result in excessive reflection fractions in future modelling on purely energetic grounds.

In Chapter 6 I showed the results of a project in which I collaborated with researchers in the field of astroparticle physics to investigate the possibility of detecting a population of PBHs within our own Galaxy. This was motivated by recent claims that the binary black hole merger events detected by LIGO may in fact have been candidates for PBHs. The topic of PBHs and their contribution to the DM in the universe has always been a lingering idea, and the LIGO detection has sparked the debate again. We simulated a population of PBHs in the Galactic centre region (central molecular zone), estimated their spatial and velocity distributions, and calculated their expected accretion rates of interstellar gas in the surrounding medium. We made
use of the FP to determine the expected radio luminosities of the PBHs for the given
X-ray luminosities, and showed that based on the most recent surveys of the Galactic
centre region, we can rule out the possibility that PBHs can account for all of the DM
in the Galaxy at the $5\,\sigma$ and $\sim 40\,\sigma$ significance levels, based on VLA and *Chandra*
catalogues respectively. Our treatment of the accretion physics of PBHs was simple,
invoking Bondi-like accretion, with modifications due to the pre-heating of accreted
gas and the accretion flow structure as the gas travels toward the black hole. A
more detailed treatment will come in the future, with predictions of the cosmological
signatures PBHs would theoretically imprint on our universe.
Basak, R., Zdziarski, A. A., Parker, M., & Islam, N. 2017, ArXiv e-prints,
Bertone, G. 2010, Nature 468, 389
Bolton, J. G., & Stanley, G. J. 1948, Nature 161, 312
Boyarsky, A., Ruchayskiv, O., & Shaposhnikov, M. 2009, Annual Review of Nuclear and Particle Science, 59, 191
Bridle, A. H. 1967, The Observatory, 87, 263
Calore, F., Bozorgnia, N., Lovell, M., et al. 2015, Journal of Cosmology and Astropar-

Carr, B. J., Kohri, K., Sendouda, Y., & Yokoyama, J. 2010, Phys. Rev. D, 81, 104019
Chandrasekhar, S. 1960, Proceedings of the National Academy of Science, 46, 253


Clesse, S., García-Bellido, J. 2015, Phys. Rev. D, 92, 023524


Curtis, H. D. 1918, Publications of Lick Observatory, 13, 9
Dauser, T., García, J., & Wilms, J. 2016b, Astronomische Nachrichten, 337, 362

Dewey, R., Stokes, G., Segelstein, D., Taylor, J., & Weisberg, J. 1984, in Birth and


Einstein, A. 1915a, Sitzungsberichte der Königlich Preußischen Akademie der Wissenschaften (Berlin), Seite 844-847., ads


Einstein, A. 1915c, Sitzungsberichte der Königlich Preußischen Akademie der Wissenschaften (Berlin), Seite 778-786., ads


Fabian, A. C. 2012, ARA&A 50, 455


Fath, E. A. 1909, Popular Astronomy, 17, 504
Fender, R. 2006, Jets from X-ray binaries, ed. W. H. G. Lewin & M. van der Klis, 381
Ferrière, K., Gillard, W., & Jean, P. 2007, AAP, 467, 611
Fornasa, M., & Green, A. M. 2014, Phys. Rev. D, 89, 063531
Frampton, P. H., Kawasaki, M., Takahashi, F., & Yanagida, T. T. 2010, Journal of Cosmology and Astroparticle Physics, 1004, 023
Genzel, R., Eisenhauer, F., & Gillessen, S. 2010, Reviews of Modern Physics, 82, 3121
Harrison, C. M. 2017, *Nature Astronomy* 1, 0165


Jansky, K. G. 1933, Popular Astronomy, 41, 548


Jungman, G., Kamionkowski, M., & Griest, K. 1996, Phys. Rept., 267, 195


Kardashev, N. S. 1962, Soviet Ast., 6, 317


Königl, A. 1980, Physics of Fluids, 23, 1083

Kormendy, J., & Ho, L. C. 2013, ARA&A 51, 511
Lasota, J.-P. 2001, New A Rev. 45, 449
Lynden-Bell, D. 1969, Nature 223, 690
Markoff, S. 2010b, Proceedings of the National Academy of Science 107, 7196
Markoff, S., Nowak, M., Corbel, S., Fender, R., & Falcke, H. 2003, A&A 397, 645
McCray, R. 1979, Spherical accretion onto supermassive black holes, ed. C. Hazard & S. Mitton, 227
Montgomery, C., Orchiston, W., & Whittingham, I. 2009, Journal of Astronomical History and Heritage, 12, 90
Nemmen, R. S., & Tchekhovskoy, A. 2015, *MNRAS* 449, 316
Nowak, M. A. 1995, *PASP* 107, 1207
Osaki, Y. 1974, *PASJ* 26, 429
Osterbrock, D. E. 1989, *Astrophysics of gaseous nebulae and active galactic nuclei*
BIBLIOGRAPHY

Phinney, E. S. 1982, MNRAS 198, 1109
Reid, M. J. 1993, ARA&A 31, 345
Schmidt, M. 1963, Nature 197, 1040
Shaposhnikov, M. 2010, Sterile neutrinos, ed. G. Bertone (Cambridge University Press), 228


Shidatsu, M., Ueda, Y., Nakahira, S., et al. 2011, PASJ, 63, S803


Strömgren, B. 1939, Astrophys. J., 89, 526


177


Velikhov, E. P. 1959, Journal of Experimental and Theoretical Physics, 36, 1398


Webster, B. L., & Murdin, P. 1972, *Nature* 235, 37


Zel’dovich, Y. B. 1964, Soviet Physics Doklady, 9, 195


Contribution from co-authors

Chapter 1: **Introduction**
RC is the sole contributor to the introduction. I shall note here that in the following I have omitted Sera Markoff from an explicit description of her contribution, since her supervision was present throughout the projects.

Chapter 2: **Using mass-scaling to study accretion**
RC was responsible for all the model-fitting analysis, as well as key parts of the model development, and implemented and interpreted the results of the model-fitting to Sgr A* and A0620−00, individually and jointly. RC also ran the extensive MCMC parameter exploration techniques. Mike Nowak and Joey Neilsen provided the Chandra X-ray spectra compiled from observations of Sgr A*. Elena Gallo provided simultaneous radio, infrared, and optical flux measurements from observations of A0620−00, as well as an X-ray spectrum taken during the same observation. Cynthia Froning provided another set of flux measurements from simultaneous infrared, optical, and UV observations of A0620−00. Patrick Crumley calculated the timescales for particle acceleration in the jet as shown in the discussion and conclusions section. Chiara Ceccobello and RC worked together on model development and testing throughout the analysis. RC was the sole author of the manuscript itself (aside from the discussion contribution from Patrick Crumley), with co-authors giving guidance where appropriate.

Chapter 3: **Jet-disk symbiosis**
I have included this chapter in the thesis because it provides important details regarding changes to the model \texttt{agnjet}, implemented between Chapters 2 and 4. This chapter gives a full account of the physical basis for the success of the model and checks the validity of its assumptions, and thus is informative to the reader of this thesis. Patrick Crumley was the project lead and responsible for the bulk of the algebraic derivations shown, and the underpinning physi-
Chiarra Ceccobello implemented changes to the jet velocity profile based on the fixes made to the Euler solution, and tested the effects on the broadband spectrum of the jet. Yuri Cavecchi held extensive discussions with both Patrick Crumley and Chiarra Ceccobello regarding the physical treatment of the jet, informing the solutions shown in the manuscript. RC first brought attention to the mistake in the original work of Falcke & Biermann (1995), as outlined by Zdziarski (2016), and verified that the mistake was inherent to the model agnjet. RC was also responsible for testing the updated version of agnjet by performing broadband spectral fits to the same data shown in Chapter 2 and confirming that the model changes did not lead to statistically significant differences in the maximum likelihood estimates of key physical parameters. Patrick Crumley wrote the bulk of the paper. RC repeated the calculations as a cross-check.

Chapter 4: Studying the inflow/outflow geometry of GX 339-4
RC conducted all model-fitting analyses to X-ray and broadband spectra of GX 339–4 as well as the interpretation of results and a full discussion and conclusion. David van Eijnatten performed initial model-fits to a few of the broadband spectra of GX 339–4, and these guided the analyses of RC. Chiarra Ceccobello implemented the new treatment of multiple inverse Compton scattering within the Comptonizing region of the jet in the model agnjet, and continually assisted with model de-bugging and testing. Victoria Grinberg provided the PCA and HEXTE X-ray spectra of GX 339–4, and formulated the text describing the data reduction. Lucy Heil provided the power-spectral hue values for each observation in the X-ray, and compiled the simultaneous radio fluxes from the literature. RC then matched the infrared fluxes from the literature with the available radio/X-ray data. RC composed the full manuscript, excluding the section describing the PCA and HEXTE data reduction.

Chapter 5: Modelling disc reflection of jet emission
David van Eijnatten performed calculations of the disc-frame jet flux, which allowed us to generate the table of disc reflection spectra models. Javier Garcia generated the table of reflection models based on the irradiation spectra generated by David. Thomas Dauser assisted David van Eijnatten with the ray-tracing calculations of photons from the jet. RC co-supervised David van Eijnatten on the project, and performed the post-processing analysis of the reflection tables, exploring the physical interpretation of the reflection of jet emission in accreting systems. RC also performed broadband spectral modelling on GX 339–4 data and a full discussion of the importance of considering reflection of both coronal and jet emission in hard state BHBs. David van Eijnatten and RC wrote the paper together.
Chapter 6: Searching for primordial black holes

Daniele Gaggero took the lead on this project, and the remaining authors are listed in alphabetical order. Daniele implemented the MCMC code used to simulate our population of PBHs in the Galactic centre, and calculate the number of detectable sources based on a set of assumptions and input parameters. Gianfranco Bertone principally oversaw the project and sparked the initial idea and motivation for the project. Francesca Calore did the post-processing analysis on the population of detected black holes to produce the figure showing their spatial and velocity distributions in the Galactic centre (Figure 2 of the paper). RC provided the treatment of accretion physics and the calculation of X-ray and radio fluxes based on the Fundamental Plane of black hole activity. RC also calculated the influence of the pre-heating of interstellar gas in the vicinity of the PBHs, and gave re-calculated source luminosities on a case-by-case basis. This is shown in the appendix. Mark Lovell calculated the spatial and velocity distributions of the PBHs based on assumptions regarding the interactions of the PBHs with stars in the Galactic centre. Emma Storm explored the Chandra, NuSTAR, and VLA catalogues and provided the numbers of unidentified sources that could possibly be PBHs. Each author wrote the section corresponding to their contribution.
Summary

This thesis contains 6 separate chapters, including an introduction on the topic of black holes and why I chose to study how they accrete gas. The fundamental goal of this thesis is to use modelling techniques to break the degeneracies in broadband spectral fits to hard state black hole binaries (BHBs) and low-luminosity active galactic nuclei (LLAGN). Breaking modelling degeneracies is key if we are to disentangle the conditions of radiating plasmas in the inner regions of accretion flows and jets. It is within these inner regions that the X-ray emission originates from in the hard states of BHBs and in LLAGN, and the candidates for its emission include jets and coronae of differing geometrical nature and particle distributions (see Figure A). Additionally, I present interdisciplinary research I conducted during my PhD which broadened the scope of black hole accretion to that of primordial black holes (PBHs), using similar assumptions and modelling techniques. The chapters (excluding an introduction on the topic of black hole accretion and jets) are as follows:

- **Chapter 2**: In this chapter we apply the principle of scale-invariant black hole accretion to constrain outflows and particle acceleration in the lowest-luminosity accreting black holes: Sgr A* and A0620−00. We model the X-ray flare spectra of Sgr A* and a mean radio-to-submm and infrared spectrum with an outflow-dominated model (agnjet) in order to characterise the nature of its daily flare emission. We model an average broadband spectrum of A0620−00 in its quiescent state, and then conduct joint-comparative modelling alongside Sgr A* by tying model parameters between the two fits. We find that our scale-invariant jet model can explain the broadband spectra of both sources simultaneously with remarkable commonality in the jet physics. Fitting the sources jointly leads to improved parameter constraints, and we find two favoured physical scenarios for the nature of the X-ray emission: 1) a synchrotron self-Compton (SSC)-dominated state whereby the energy density of the jet plasma is dominated by the radiating particles, and 2) a power law synchrotron-dominated state in the...
Figure A: Diagram of a corona + jet model. X-ray emission is produced in the inner regions of the flow where an inherent degeneracy exists between a static or inflowing ‘corona’ of optically-thin gas, and a relativistic jet flow in the z-direction away from the accretion disc. Connecting the radio-emitting regions in the jet with the X-ray-emitting inner regions is a key goal among those of us studying black hole accretion.

fast-cooling regime in which the plasma in the inner regions of the jet is close to equipartition (energy density is divided evenly between the magnetic field and radiating particles).

- **Chapter 3** In this chapter we re-evaluate the assumption of *maximal jets* underpinning the *agnjet* model. We find that the assumption that the internal energy density of a jet can be physically bounded by its rest-mass energy density to be an incorrect one. We re-calculate the jet Lorentz profile by relaxing the *maximal jet* assumption, instead showing that the jet dynamics derived from the original treatment are valid if one assumes a small opening angle and small terminal Lorentz factor (i.e. that the jets typical of BHBs and LLAGN reach flow velocities of $\gamma_j$ only a few). We describe corrections made to minor algebraic errors in the original model and show that the errors incurred are minor. We also show that the jets in BHBs and LLAGN must be roughly isothermal in
order to reproduce the observed flat-to-inverted radio spectra.

- **Chapter 4**: In this chapter we model multiple quasi-simultaneous broadband observations of the BHB GX 339–4 with a view to tracking both the spectral and timing characteristics during outburst rise and decay. Despite the wealth of data showing the correlations between its radio, X-ray, infrared, and optical emission throughout multiple outbursts, we still have not confirmed the nature of the X-ray emission of GX 339–4. We fit the broadband spectra of GX 339–4 with agnjet and a model for the corona in the inner regions of the accretion flow, and present a physical interpretation in which the corona dominates the X-ray spectrum whilst the jet contributes $\sim 10\%-50\%$ to the $10\text{--}40$ keV X-ray flux.

- **Chapter 5**: In this chapter we present new self-consistent reflection models in which the irradiating continuum is inverse Compton (IC) emission from a mildly-relativistic jet. We calculate the incident flux on the accretion disc, taking into account general and special relativistic effects associated with the jet-disc interface and the strong gravity of the black hole. We show that our physical model treatment allows calculation of reflection fractions independently of spectral modelling, which can advance current reflection models which use the reflection fraction as a fitting parameter. We also discuss how the resultant reflection spectra depend upon the physical parameters of the jet, and compare those spectra with the most recent reflection models.

- **Chapter 6**: This chapter shows the interdisciplinary portion of my PhD. I worked together with members of GRavitational and AstroParticle Physics Amsterdam (GRAPPA) to simulate a search for PBHs in the Galactic centre in order to place constraints on the fraction of dark matter (DM) that could be comprised of PBHs. We use predicted source luminosities in the radio and X-ray (making use of the FP) to compare with up-to-date radio (Very Large Array) and X-ray (Chandra, NuSTAR) surveys of the Galactic centre, assuming a population of PBHs. By making some basic assumptions regarding the accretion rates of our simulated population of PBHs, and adopting the fundamental plane of black hole activity (FP) to make use of observational priors regarding the nature of their accretion flows and jets, we place the strongest reliable constraints yet on their contribution to the DM. I show additional calculations I performed on the pre-heating of gas that is accreting onto PBHs in Appendix A. These calculations show that whilst the pre-heating of accreted gas significantly reduces the accretion rates of our population of PBHs (which reduces the probability of detection), there is uncertainty in such a calculation due to the evolving nature of the accretion and ionisation of the gas as a function of the velocity of the PBH.
Nederlandse Samenvatting

Dit proefschrift bevat zes hoofdstukken. Het eerste hoofdstuk bevat de introductie tot zwarte gaten en een uitleg waarom ik besloot accreterende zwarte gaten te onderzoeken. Het overkoepelende doel van alle hoofdstukken is om met behulp van modelleringsmethoden ontaarding op te heffen in de fits aan dubbelsterren met een zwart gat (Engels: “black hole binary” of “BHB”) of actieve sterrenstelsels met een lage helderheid (Engels: “low-luminosity active galactic nuclei” of “LLAGN”). Model-ontaarding opheffen is zeer belangrijk als men de stralingsprocessen en plasmacondities in de accretieschijf en jet nabij het zwarte gat wilt kunnen onderscheiden. De Röntgen-straling in de harde staat van BHBs en LLAGNs vindt zijn origine in deze binnenste regionen en kan verklaard worden door verschillende vormen en distributies van deeltjes in zowel de accretieschijf als de jet (zie Figuur A). Ook presenter ik interdisciplinair onderzoek dat ik heb verricht gedurende mijn promotieonderzoek wat het domein uitbreidde met accreterende primordiale zwarte gaten (Engels: “PBHs”) met behulp van soortgelijke aannames en modelleringsmethoden. Nu volgt een samenvatting van elk hoofdstuk behalve het eerste hoofdstuk.

- **Hoofdstuk 2**: In dit hoofdstuk passen we het principe van schaal-invariante accretie van zwarte gaten toe om een grip te krijgen op de uitstroom en deeltjesversnelling in LLAGN Sgr A* en A0620-00. We modelleren het Röntgen-spectra van opflakkeringen van Sgr A* en een gemiddeld radio-tot-submm en infrarood spectrum met een model voor de uitstroom (agnjet) om de oorsprong van haar dagelijkse opflakkeringen te kunnen karakteriseren. Ook modelleren we een gemiddeld spectrum van A0620-00 in de stille staat. Vervolgens modelleren we beide zwarte gaten tegelijkertijd waarbij we sommige parameters gelijk aan elkaar gelijkstellen. We ontdekken dat ons schaal-invariante jet model beide spectra goed kan verklaren met veel opvallende veel overeenkomsten in de natuurkunde die de straling veroorzaakt. Het simultaan modelleren van beide zwarte gaten levert nauwkeurige parameter waarden op en dit leidt tot twee
7 Nederlandse Samenvatting

Figure A: Diagram van een corona + jet model. Röntgen emissie wordt geproduceerd in de binnenste regionen van de accretiestroom waar een inherente ontaarding is tussen de naar binnen stromende corona en een relativistische uitstroom loodrecht op de accretieschijf. Het verbinden van radio emissie in de jet met de Röntgenstraling in de binnenste regionen van de accretiestroom is zeer belangrijk voor wetenschappers die de accretie van zwarte gaten bestuderen.

aannemelijke situaties voor de mechanismen die de Röntgen-straling produceren: 1) een staat gedomineerd door Comptonverstrooiing van synchrotron straling (Engels: synchrotron “self-Compton” of “SSC”) waarbij de energie-dichtheid wordt gedomineerd door de stralende deeltjes en 2) een staat gedomineerd door synchrotron straling, gegeven door een machtsfunctie. In deze staat koelt het plasma snel en is de energie gelijk verdeeld tussen de magneetvelden en de stralende deeltjes (equipartitie).

• Hoofdstuk 3: In dit hoofdstuk kijken we opnieuw naar de aanname van maximale jets die het fundament is van agnjet. We ontdekken dat de aanname dat de interne energie-dichtheid gelimiteerd wordt door de rustmassa energie-dichtheid incorrect is. We versöpelen deze aannamen en berekenen het snelheidsverloop van de jet opnieuw. We tonen aan dat de oorspronkelijke resultaten nog gelden als men een kleine openingshoek van de jet en een lage terminale Lorentz factor
aanneemt. Beide aannames zijn correct voor BHBs en LLAGN. We omschrijven correcties van algebraische fouten in het originele model en laten zien dat deze correcties een klein effect hebben. We laten ook zien dat de jets in BHBs en LLAGN ongeveer isothermisch moeten zijn om het vlakke tot stijgende radio spectrum te verklaren.

- Hoofdstuk 4: In dit hoofdstuk modelleren we verscheidene quasi-simultane observaties over golflengtes van vele ordes van grootte van BHB GX 339-4 om zowel de spectrale als temporale eigenschappen te kunnen volgen gedurende de start en het einde van uitbarstingen. Ondanks een grote hoeveelheid data van meerdere uitbarstingen waaruit correlaties tussen radio, Röntgen, infrarood en optische emissie opgemaakt kunnen worden weet men nog steeds niet de natuurkundige origine van de Röntgen-straling. We modelleren spectra van GX 339-4 met agnjet en een apart model voor de corona in de binnenste regionen van de instroom. We geven een natuurkundige interpretatie waarbij de corona het Röntgen-straling spectrum overheerst en de jet $\sim 10^{-5}$ bijdraagt aan de $10^{-4}$ keV Röntgen-straling flux.

- Hoofdstuk 5: In dit hoofdstuk presenteren we vernieuwde modellen voor de reflectie van inverse Comptonverstrooiing (IC) door een lichtelijk relativistische jet. Met behulp van zowel speciale als algemene relativiteitstheorie berekenen we de flux van de jet die de accretieschijf bestraald. We laten zien dat de reflectie-fractie berekend kan worden en niet meer als normalisatie parameter uit het modelleren gehaald hoeft te worden. We laten ook zien hoe de gereflecteerde spectra veranderen met de belangrijkste jet parameters en vergelijken deze met andere reflectie modellen.

- Hoofdstuk 6: Dit hoofdstuk is de interdisciplinaire portie van mijn proefschrift. Ik werkte samen met leden van GRavitaional and AstroParticle Physics Amsterdam (GRAPPA) om een zoektocht voor PBHs in het centrum van de melkweg te simuleren. Het doel was het bepalen van limieten op de fractie van donkere materie (DM) die in PBHs kan zitten. We schatten de radio en Röntgen helderheden en vergelijken deze met overzichten van het galactisch centrum van de Very Large Array, Chandra en NuSTAR onder aanname van verschillende populaties van PBHs. Door simpele aannames te maken over de accretieniveau en de correlatie tussen de radio en Röntgen-straling helderheden kunnen we de sterkste limieten tot nu toe leggen op de bijdrage van PBHs aan DM. Ik laat berekeningen zien van het voor-verwarmen van het accreterende gas in Appendix A. Dit voorverwarmen verlaagt het accretieniveau (dus ook de kans op detectie) maar er is veel onzekerheid in deze berekeningen vanwege de feedback tussen het accretieniveau en ionizatiestaat als functie van de snelheid van het PBH.
Acknowledgements

True friends stab you in the front.
— Oscar Wilde

And so finally I arrive at the section of this thesis where the citations can remain in my head, and the things I write must be taken at face value. In addition, some parts of this have been written whilst under the influence of alcohol, I will let you figure out which parts. Let’s see if I can appall some of you....

I will start by thanking those who have had the most impact on my training as a scientist, and thus those principally responsible for making this thesis possible. The first person who deserves my thanks is my supervisor, Sera. Sera has provided me with a very broad and complete set of scientific skills, but most notably in my writing and ability to consider all the key elements in any work I do. Without this kind of training I would not have succeeded in producing this thesis, and I certainly would not be leaving Amsterdam to continue in science with a postdoc. In addition, Sera has been very supportive of me on a more personal level during the more difficult periods of my PhD, which I thank her for. Sometimes your personal issues can be a brick wall preventing any kind of professional progress, whether you are a researcher or in any other branch of work. It often takes patience from those who are there to support your progress to account for this, and Sera has always had that patience, so I thank her for that. I also say thank you to Sera for granting me the autonomy I needed to explore different avenues of work, since this was key for my motivation. It has sometimes been very difficult to keep myself focused on one project and see it through, and having the freedom to collaborate with other people and get inspiration from other areas has really helped.

During the course of my PhD Sera’s working group has evolved significantly, and every person who has been a part of the group deserves a mention, but in all honesty many students came through the group, and I have forgotten some of the names. The core group has included Salomé, Chiara, Adam, Patrick, Fe, Tom, Matteo, Koushik,
David, Stephen, Matthew, Casper, and there are a few more in the list, in particular Master’s and Bachelor’s students who have come and gone. Everyone in the group has contributed to the scientific growth of myself and everyone else, but I would like to thank a few people in particular.

First and foremost, Chiara, my second mum. If you have had the chance to interact with Chiara in either a professional or personal capacity, then you will know that she is an attentive, patient and considerate person. There is just no way I’d have managed to get through this PhD without Chiara, we have worked together on many aspects of one another’s science, and that mutual assistance has always kept me motivated. More than that, Chiara is a very close friend, and I have always been able to rely on her support anytime I’ve done something stupid.....those who know me will understand how often that might be.

Second I must mention Patrick. If you do not know Patrick, the best way to describe him is as a friendly giant who also happens to be a genius. It is a shame he is no longer at the API, because scientific discussion across different disciplines was rife with him around, due to his knack for understanding basically anything that is thrown at him and then coming back with something no one else has thought of. Patrick had a strong part to play in my work, he taught me a lot, mainly how to understand some of the fundamental physics involved in my work, and how to approach problems in a simple way to avoid getting too stuck. In addition, he was always light-hearted and fun to be around, whether discussing science or anything else.

Thirdly, there is Tom, who is a horrible person. I’ve written it, and I’m not deleting it, it is done. Send in your complaints. No in all seriousness, Tom has always been a good laugh and was certainly a dynamic addition to our working group. He started to occasionally appear at the API as a sort of random Australian, phantom, and whilst at first I didn’t question his presence, over time it became clear he could not be ignored. Since Tom started to work here he’s become very popular with everyone, and that’s because he’s a warm, welcoming person, to all those around him. You’ll be missed you mad ****!

Fourthly (yes it’s a long list), David. We started working together on a project for David’s Bachelor’s thesis, more than a year and a half ago now. I had initially thought I would simply show him a few things and then he would plod along and complete his thesis with little interaction between the two of us. It became clear early on that this would not happen. David is a very motivated student, and somebody who questions absolutely everything, so the more we worked together the less I realised I actually understood about my own work. David continued working closely with me well into his Master’s degree, and he is now a key contributor to this thesis, featuring on two of the chapters, leading one of them himself. I have to say thank you to him because I could not have completed my work without his efforts. Also, he’s quite a funny guy to interact with, everything is both light-hearted and die-hard at the same time. I wish him luck with whatever he does in future, and I know he’ll be especially good...
at it. Definitely become rich though.

Fifthly (that can’t be a word) I must mention Matteo, who has shared with me some of the burden and agony of broadband spectral modelling over the past year, and all that entails. I can’t remember too many days in which I didn’t see a silhouette of curly hair appearing in my peripheral vision. Chats with Matteo have always been useful, we’ve solved many problems together, and the progress we made has played a big part in my final year here, so a big thanks for that. And I wish him luck with the rest of his PhD, which will no doubt be a success.

Finally, there is Adam. Things didn’t go so well for Adam, and he had to leave us, unfortunately. Somehow he is still around every now and then, and it’s always an interesting experience when he’s here. We spent a large chunk of my early-to-mid PhD time as PhD brothers, which included an awesome 5 weeks in Texas. Most of that time was spent playing with aliens, and cycling around admiring Austin’s great views whilst nursing a banging headache from a hangover. Thanks to Adam for making PhD life everything but boring.

Ok I’m done giving praise to fellow group members for now. I must mention members of the GRAPPA, with whom I wrote a paper I really enjoyed working on. The final chapter of this thesis shows the work we did on the limits of the dark matter contribution from primordial black holes. This project began at a time when I was struggling a bit with my main work, so when Daniele approached me for some assistance in characterising an accreting system at very low accretion rates, I was happy to oblige. Over time I became more heavily involved in the project, and the dynamic nature of our discussions and the rapid progress we made as a team gave me a lot of motivation to push myself to the end of this PhD. So thank you to Daniele, Gianfranco, Francesca, Mark, Sera and Emma. Thanks especially to Daniele, who has also become a friend as a result of the project. I encourage all current and future APIs to interact more with members of the GRAPPA, they are a fun bunch who work on some very interesting topics with lots of crossover, and above all else, they’re not afraid to delve into topics that require cross-collaboration.

Now I’d like to mention the API as a whole. These past 4 years have been, by far, the best in my life, and API is the reason. Of course I’m not going to pretend everything was always sunshine and roses. I’ve had some very difficult times here, but also the best times. And during the difficult times, there is always an API round the corner to remind you that your problems are always solvable, whether it’s research-related, or personal. Whilst I would sincerely like to thank all of you, because it is all of you who make the API what it is, I will mention some names of people I’ve interacted with most over the 4 years, in as chronological a way as possible: Martin, Yuri, Theo, Chiara, Liliana, Danai, Stefano, Chris, Catia, Georgi, Tomas, Abbie, Daniele, Amruta, Rik, Manos, Alice, Nina, Kaustubh, Smriti, Gullo, Macla, Mathieu, Alicia, Frank, Rachel, Koushik, Vladimir, Claire…..and I’ve no doubt missed a few people, but if you’ve interacted with me a lot you are on this list. The secretaries
and institute managers over the years, Susan, Milena, Lidewijde, Annemarie, Vivian, and possible names I’m forgetting from the first year I was here. They’re brilliant, all of them, the department not only functions because of them, it also gets a regular injection of calm and well-needed spontaneous therapy. In addition, some interlopers, non-APIs who have become a part of a much bigger scientific web - Louise, Rachid, Lotte, and many others. There are however a few people I would like to spend a bit more time thanking:

Martin, who’s been at the API so long he’s become part of the foundations, so much so that if he left, the department would very likely crumble. Martin is a selfless person, and is actually the perfect example of a true friend, someone who will stab you in the front ;) He is the antithesis of a sycophant, and you should expect to be completely roasted if you want to know his opinion of you. Since I am rarely able to hide my faults from anyone, I’m too easy a target for Martin, but I didn’t expect that to lead to such an important friendship. You’ll be missed Martin, there’s no one quite like you.

Alice, my love. It’s been an interesting 3 years living together, plenty of drama, many late nights cycling home drunk and drinking tea whilst talking rubbish. Our greatest joint achievement has to be completing all 8 seasons of Dexter, and not being too mentally scarred. California will be a big and lonely place without you, my dear!

Gullo, thanks for all the chats during our long journeys to and from football, what will I do without your counsel ;) And man, the marathon, the pinnacle of madness, I can’t believe we managed to do that.

The ongoing PhD stress would have been impossible to handle if I wasn’t spending a large portion of my free time running around like a maniac and kicking a football. So thanks to all the guys at DVVA (Dario, Theo, Theo P, Bram, Ruud, Jasper, Gullo, and anyone else who joined over the past 4 years), and TOS Actief (too many names to mention, but you’re all quality, I’ll miss the team).

Before I leave the thanks from Amsterdam to elsewhere, I just want to mention another person who has been part of my life during this last year—Thao. No regrets, thank you, I’ll miss you.

Now I take a step back and acknowledge the importance of my British comrades, or as I like to call them, Brexit buddies. Thanks to everyone at Leicester who made it possible for me to even pursue a PhD here in Amsterdam. Professor Dick Willingale, and Dr. Graham Wynn, my academic tutor and Master’s supervisor respectively. Without their assistance I would have had neither the motivation nor the tools to make it this far. Thank you to my university friends, many of whom remain close friends to this day: Jeff, Josh, David, Katie, Crystal, Cameron. Their friendship has always meant a lot to me, and the time we all spent together either partying or trying to wrap our heads around incomprehensible physics problems will never be forgotten. A special mention to Jeff and Crystal, who’ve made regular trips out to visit me and have been a big part of my entire adult life. It will be sad to go so long without seeing
your faces.

Going back even further, I say thanks to my childhood friends, Tom, Haz, Antny, Sadler. It’s pretty impressive that such a degenerate bunch have managed to stay in touch and regularly wreak havoc upon the world for so long, but it’s happened nonetheless. These guys have been there for me for many years, and I can always rely on them to give me perspective on life by introducing fresh chaos. Trace...

Finally I thank my family, who have always been there for me in everything I’ve done. Their continued support has made it possible for me to live and work here without trouble, and every trip I have made back home to England has been a blessing, save for one. Thank you to all of you, Lilly, Ciaran, Monica, Ester, mum, Dan and Donovan. Every one of you is important to me, and I will miss you greatly when I leave the continent. It can perhaps be viewed as improper to highlight favourites within my family, but I have to make exception here. My sister Ester has always been very special to me, in particular over the past 4 years. Whenever I’ve travelled back to the UK I’ve been put up by her and her own little family — husband Dan, and twins Oscar and Darcie. Anytime I have needed a helping hand, or things have started to fall apart for me, Ester has been there, in full support. She is my best friend, the only continuity when I forget who I am. I will miss her, Dan, and the twins a great deal over the next few years, life will not be the same without them just around the corner.

My final thanks is reserved for the one person I had most wanted to witness the final product of my work, my dad, Michael Connors. It’s very difficult for me to properly articulate how my dad’s influence shaped me into who I am. I suppose the best quality he had, admittedly amongst some pitfalls, was that he made everyone feel good about themselves. He constantly reminded me of my worth, my potential, and that I could achieve a great deal if I worked hard. Managing to convey that whilst also instilling the value of enjoyment and to never take life too seriously is no easy task, but he did it. Thank you dad, for the many late drives, terrible detective soaps, ridiculous holidays, endless games of pool, and incomprehensible drunken conversations. I’ve only gone and bloody done it!

Riley, Raj, Novak, Wheels...