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Charge Screening in the Higgs Phase of Chern-Simons Electrodynamics

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Though screened at large distances, a pointlike electric charge can still participate in a long-range electromagnetic interaction in the Higgs phase, namely, that with the Aharonov-Bohm field produced by a localized magnetic flux. We show that this follows from the fact that the screening charge, induced in the presence of a Higgs condensate, does not interact with the Aharonov-Bohm field. The same phenomenon occurs if a Chern-Simons term is incorporated in the action. This observation provides a physical basis for the recently proposed classification of the superselection sectors of this model in terms of a quasi-Hopf algebra.

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The question whether charges in the Higgs phase can be measured through Aharonov-Bohm (AB) scattering with magnetic fluxes [1] has received considerable attention in recent literature. In this context, it is important to understand that there are essentially two different definitions of the physical charge: (1) as a coupling constant for Coulomb interactions and (2) as a coupling constant for AB scattering [2]. In this Letter we explain why "Coulomb charges" are screened in the Higgs phase, while the "AB charges" are not [3].

In the Higgs medium, any charge q (even irrational fractions of e) is screened in the sense that there are no Coulomb fields around it at distances $\gg 1/M_A$ (with M_A the mass of the gauge fields obtained by the Higgs mechanism). So the electric flux $\mathbf{E}$ through any $(D-1)$-dimensional surface of radius $\gg 1/M_A$ in the Higgs medium disappears: $Q = \int \nabla \cdot \mathbf{E}\, d^D x = 0$. This is nothing more than the statement that electromagnetic fields are massive in the Higgs phase, i.e., that the Higgs effect does occur.

For the charge q to be measurable through AB scattering, it is necessary that the screening charge $q_{\text{induced}} = -q$, induced by the Higgs mechanism, does not take part in the AB interaction. We will argue that this is precisely what happens. To proceed, it turns out that this effect is specific for screening by a Higgs condensate. It is not a common feature of any screening mechanism. For example, Debye screening by a real plasma, or the (sometimes partial) screening related to vacuum polarization as described by the renormalization group (Gell-Mann-Low β function), is not of this type. We expect that they screen both Coulomb and AB interactions [4]. Even more peculiar is the screening related to the Chern-Simons (CS) term [5]. It leads to complete screening of the Coulomb interaction and partial screening (by a factor 1/2) of the AB interaction (see for instance Ref. [6] and the text below).

Our analysis extends to the Higgs phase of CS electrodynamics. Here it gives rise to a new observation. In the normal phase of CS electrodynamics [5] the Coulomb fields of external charges q will be screened by generating a magnetic flux $\phi = -q/\mu$ (with μ the topological mass), and vice versa. So charges and fluxes are identified. However, in the Higgs phase ϕ is quantized, so this CS screening mechanism could *not* be effective for arbitrary q. Here the Higgs condensate brings salvation again; it screens the Coulomb fields of both q and $\mu\phi$, thus removing the aforementioned identification of charges and fluxes.

The model in which we discuss these phenomena is the $(2+1)$-dimensional theory, governed by the Lagrangian

$$\begin{aligned}\mathcal{L} &= \mathcal{L}_{\text{ed}}(\mu) + \mathcal{L}_{\text{Higgs}} + \mathcal{L}_{\text{matter}} \\ &= -\frac{1}{4}F^{\kappa\nu}F_{\kappa\nu} + \frac{\mu}{4}\varepsilon^{\kappa\nu\lambda}F_{\kappa\nu}A_\lambda \\ &\quad + |\mathcal{D}_\kappa\Phi|^2 - V(|\Phi|^2) + \mathcal{L}_{\text{matter}}. \end{aligned} \tag{1}$$

The first two terms $[\mathcal{L}_{\text{ed}}(\mu)]$ describe what is known as CS electrodynamics [5]. In the next two terms ($\mathcal{L}_{\text{Higgs}}$) a complex scalar field Φ with global $U(1)$ charge Ne is minimally coupled to this gauge theory. In our conventions $\mathcal{D}_\kappa\Phi \equiv (\partial_\kappa + iNeA_\kappa)\Phi$. To proceed, we assume that the potential $V(|\Phi|^2)$ takes the form

$$V(|\Phi|^2) = \frac{\lambda}{4}(|\Phi|^2 - v^2)^2, \tag{2}$$

which spontaneously breaks the gauge symmetry $U(1) \to Z_N$, and leads to a condensate of the charged Higgs field Φ. In this so-called Higgs phase the length scale is set by $1/M_H$, with $M_H = \sqrt{2\lambda}\,|\langle\Phi\rangle| = \sqrt{2\lambda}v$ the mass of the Higgs boson. We have introduced additional charged matter fields ($\mathcal{L}_{\text{matter}}$) in order to be able to discuss all conceivable charge sectors in the Higgs phase. We do not further specify these matter fields. We only assume that they are very massive, so that we can discuss the associated global $U(1)$ charges, denoted by q, as external [7].

Let us first recall that pure CS electrodynamics $\mathcal{L}_{\text{ed}}(\mu)$ with $\mu \neq 0$ is known to possess only massive particles in the spectrum: one-component massive photons [5] with mass μ. On the other hand, the photons of pure Higgs electrodynamics $\mathcal{L}_{\text{ed}}(0) + \mathcal{L}_{\text{Higgs}}$ in the Higgs phase are also massive. This time they are endowed with two components, and they carry the mass $M_A = Ne\sqrt{2} \mid \langle\Phi\rangle \mid = Ne\sqrt{2}v$. These theories differ radically from massive electrodynamics with explicitly broken gauge invariance,

$$\mathcal{L}_{\text{med}} = -\frac{1}{4}F^{\kappa\nu}F_{\kappa\nu} + \frac{m^2}{2}A^\nu A_\nu. \tag{3}$$

Despite having only massive particles, the former models allow for the long-range AB interaction [1], which is absent for $\mathcal{L}_{\text{med}}$ [8]. One may think of the AB interaction in pure CS electrodynamics $\mathcal{L}_{\text{ed}}(\mu)$ with $\mu \neq 0$ as being mediated by certain "discrete" degrees of freedom [9], which are close analogs of the by now well known discrete states of the $c = 1$ string [10].

In the Higgs phase of the model $\mathcal{L}_{\text{ed}}(0) + \mathcal{L}_{\text{Higgs}}$ the AB interaction survives, because the field, which is acquiring mass due to the spontaneous breakdown, is actually the gauge invariant combination

$$\tilde{A}_\kappa \equiv A_\kappa + \frac{1}{Ne}\partial_\kappa \text{Im}\ln\langle\Phi\rangle, \tag{4}$$

rather than A_μ itself. Instead of (3), we find in the Higgs phase

$$-\frac{1}{4}F^{\kappa\nu}F_{\kappa\nu} + \mid \mathcal{D}_\kappa\Phi \mid^2 \longrightarrow -\frac{1}{4}F^{\kappa\nu}F_{\kappa\nu} + \frac{M_A^2}{2}\tilde{A}^\kappa\tilde{A}_\kappa. \tag{5}$$

Thus $\tilde{A}$ indeed has a finite ($\sim 1/M_A$) correlation length. This does not immediately imply that A should also fall off exponentially. It can instead remain pure gauge. Such AB fields, which are locally pure gauge, can be globally nontrivial, however. This is the case around topological defects of the Higgs condensate of the characteristic size $\sim 1/M_A$, corresponding to magnetic vortices [11] labeled by $\pi_1(U(1)) \simeq Z$. These vortices carry a magnetic flux ϕ which is quantized by the requirement that the Higgs condensate is single valued outside the core of the vortex; thus [12]

$$\phi = \frac{2\pi}{Ne} \times \text{integer}. \tag{6}$$

It is well known that the purely quantum mechanical AB interaction [1,3] leads to nontrivial elastic scattering of charges q' and vortices ϕ; i.e., it gives rise to a diffraction-like effect, which is of course observable [13]. The crucial ingredient in the corresponding cross sections is the phase $\exp iq'\phi$. If there were no magnetic vortices (with their AB fields) in the Higgs phase, electric charges q would be unobservable at *large* distances (of course they can always be seen in scattering processes with high energies $\geq M_A$, i.e., at short distances). But even at *low* energies any two states with $U(1)$ charges q_1 and q_2, such that $q_1 - q_2 = Ne \times$ integer, are in fact indistinguishable, because the fluxes which can be used to distinguish them are constrained by Eq. (6). This observation gives rise to the low-energy classification of the superselection sectors of this model by means of (q, ϕ), with

$$q = e \times (\text{ integer mod } N), \tag{7}$$

$$\phi = \frac{2\pi}{Ne} \times (\text{ integer mod } N). \tag{8}$$

Together with its less trivial generalization to the patterns of symmetry breaking $G \rightarrow H$ with non-Abelian discrete groups H [14], this classification has been further investigated in Refs. [3] and [15], while in Refs. [16] and [17] the underlying symmetry algebra was subsequently identified as the Hopf algebra $D(H)$.

What remains to be discussed is *why* all the states (q, ϕ), as listed in Eqs. (7) and (8), can indeed be distinguished by the AB interaction. Let us recall that the generic AB field is a pure gauge $A_\mu = g^{-1}\partial_\mu g$ with $g \in U(1)$. In a non-simply-connected domain, one may have that $\oint A_\mu dx^\mu \neq 0$ along a noncontractable loop, and in fact $\oint A_\mu dx^\mu = \phi$ if the contour goes around a vortex once. The interaction of a pointlike charge q' (moving along the world line γ) with the electromagnetic field can be written as $q' \int_\gamma A_\mu dx^\mu$. In the first-quantization formalism the phase factor arising when the charge is carried around the vortex therefore equals $\epsilon\{\phi, q'\} = \exp iq'\phi$. However, in the Higgs phase the charge q' is completely screened at distances $\gg 1/M_A$. This means that it is surrounded by a "cloud" with a total electric charge exactly equal to $-q'$. This cloud serves to cancel the contribution of q' to the Coulomb field. From a physical point of view, this leads to a potential embarassment, the problem being that the cloud will also be carried around the vortex, and consequently, it seems, that the factor $\epsilon\{\phi, q'\}$ will be *unobservable*, because it should be multiplied by $\epsilon_{\text{induced}}\{\phi, -q'\} = \epsilon\{\phi, q'\}^{-1}$. By this line of reasoning, we are led to conclude that for a physical (i.e., dressed) charge, the AB effect would be absent in the Higgs phase.

Remarkably enough, the reasoning just given turns out to be incorrect. While the Coulomb interaction is exponentially damped by the Higgs mechanism, the AB interaction is not. In fact $\epsilon_{\text{induced}} \equiv 1$, and the reason for this is that the (induced) screening charge density is *not* an ordinary electric charge density, but rather the field $-M_A^2\tilde{A}_0$. This is clear from Gauss's law, implied by Eqs. (1), (4), and (5),

$$\nabla\cdot\mathbf{E}(\mathbf{x}) = q\delta(\mathbf{x}) - M_A^2\tilde{A}_0(\mathbf{x}) = q\delta(\mathbf{x}) + q_{\text{induced}}(\mathbf{x}), \tag{9}$$

with $E^i \equiv F^{i0}$. The associated induced current is [18]

$$j^\kappa_{\text{induced}} = -M_A^2\tilde{A}^\kappa. \tag{10}$$

In order for this current to interact with the AB field (produced by some remote vortex), there should be a term in the Lagrangian of the form $-j^\kappa_{\text{induced}}A_\kappa = M_A^2\tilde{A}^\kappa A_\kappa$. Instead we only encounter the term $\frac{1}{2}M_A^2\tilde{A}^\kappa\tilde{A}_\kappa$ in Eq. (5). In other words, q_{induced} couples to $\tilde{A}$ rather than to A, and thus does *not* feel AB fields related to remote vortices, which have nonvanishing A

component but strictly vanishing $\tilde{A}$ at large distances. Thus our conclusion is that for a physical particle in the Higgs phase the AB interaction is sensitive to the *unscreened* charge only, while ordinary interactions, like the Coulomb one, are completely screened. The phase factor associated with the AB interaction of two localized states (q,ϕ), and (q',ϕ') is therefore given by

$$\epsilon\{(q,\phi),(q',\phi')\} \equiv \epsilon\{\phi,q'\}\epsilon\{\phi',q\} = \exp i(q'\phi + q\phi'). \tag{11}$$

Let us now turn to the situation where the CS term is present. The masses of the electromagnetic fields are modified, and become [19]

$$M_{\pm} = \sqrt{M_A^2 + \frac{1}{2}\mu^2 \pm \mu\sqrt{M_A^2 + \frac{1}{4}\mu^2}}, \tag{12}$$

where + and − stand for two different components of the photon [20]. The AB field carries no energy, and is still present at large distances. Gauss's law is affected though; it now reads

$$\nabla\cdot\mathbf{E}(\mathbf{x}) = q\delta(\mathbf{x}) + \mu B(\mathbf{x}) - M_A^2\tilde{A}_0(\mathbf{x}). \tag{13}$$

The new term, $\mu B \equiv \mu F^1{}_2$, implies that the magnetic field of a vortex generates an additional electric charge density $\mu B(\mathbf{x})$, the total extra charge being $\int \mu B d^2x = \mu\phi$. This extra charge is added to q, and will therefore be completely screened at distances $\gg 1/M_-$ by the screening charge, corresponding to $-M_A^2\tilde{A}_0$. Note that in this CS Higgs phase, the Coulomb fields of an external charge are still screened by the Higgs condensate, and not by assigning a flux $\phi = -q/\mu$ to it as in the normal phase of CS electrodynamics [5]. In the latter case, the fractional "screening" fluxes would render the Higgs condensate multivalued. Thus in the Higgs phase of CS electrodynamics, the identification $q = -\mu\phi$ of charges and fluxes is lost, and they become independent quantum numbers.

The most significant effect of the CS term occurs in the phase factor $\epsilon\{(q,\phi),(q',\phi')\}$. In fact, we may think of every physical state (q,ϕ) as being composed of *three* parts: the pointlike global $U(1)$ charge q, the vortex ϕ, and the screening charge $q_{\text{induced}} = -q - \mu\phi$ (all concentrated in the domain of radius $\sim 1/M_-$). In the AB field, produced by some remote vortex ϕ', (a) q is coupled to the total flux ϕ', (b) q_{induced} does not couple at all, and (c) ϕ is coupled to $\frac{\mu}{2}\phi'$. This can be seen from the form of the Lagrangian density (1), rewritten in terms of $\tilde{A}$,

$$-\frac{1}{4}F^{\kappa\nu}F_{\kappa\nu} + \frac{1}{4}\mu\varepsilon^{\kappa\nu\lambda}F_{\kappa\nu}A_\lambda + (Ne)^2\chi^2\tilde{A}^\lambda\tilde{A}_\lambda - V(\chi^2) + (\partial^\kappa\chi)(\partial_\kappa\chi) + \mathcal{L}_{\text{matter}}, \tag{14}$$

where we substituted $\Phi(x) = \chi(x)e^{i\sigma(x)}$ for the Higgs field, with real valued $\sigma(x)$ and gauge invariant $\chi(x) = |\Phi(x)|$, so $\tilde{A}_\kappa \equiv A_\kappa + \frac{1}{Ne}\partial_\kappa\sigma$. The fluctuations of χ around $\langle\chi\rangle = v$ describe neutral Higgs bosons with mass M_H. Now let us assume that we are in the Higgs phase, i.e., χ takes the vacuum expectation value v everywhere, except for a finite number of points where the vortices are located. We first note that $\exp(i\sigma/N)$ is not single valued in the presence of a vortex; it is therefore not a well defined gauge transformation. For this reason we cannot simply replace A by $\tilde{A}$ in the matter term. This implies statement (a). The fact that $\tilde{A}$, rather than A, appears in the interaction term with the Higgs sector leads to the statement (b) above. Finally, we observe that the coefficient in front of the CS term differs by a factor of 2 in the Lagrangian (14) and in Gauss's law (13). This difference is the origin of the factor $\frac{1}{2}$ in (c) (see for instance Refs. [6,17]).

From (a)–(c) it follows that the total effect of the AB interaction of the two states (q,ϕ) and (q',ϕ') when $\mu \neq 0$ is the phase factor [17]

$$\exp i\left\{\left(q + \frac{\mu}{2}\phi\right)\phi' + \left(q' + \frac{\mu}{2}\phi'\right)\phi\right\} = \exp i(q\phi' + q'\phi + \mu\phi\phi'). \tag{15}$$

This nontrivial factor arises in the amplitudes, although the total electric charge Q of every localized state (q,ϕ), if measured by the Coulomb field at distances $\geq 1/M_-$, vanishes:

$$Q \equiv q + \mu\phi + q_{\text{induced}} = 0. \tag{16}$$

In Ref. [17], Eq. (15) formed the starting point for the study of the interchange and fusion properties of the states (q,ϕ) in the case where $\mu \equiv pe^2/4\pi$ with $p \in Z$. It was shown that the underlying symmetry algebra in the absence of a Chern-Simons term, which is the Hopf algebra $D(Z_N)$ [16], is deformed by a nontrivial 3-cocycle on the unbroken discrete group Z_N for nonvanishing values of μ. The generalization to non-Abelian unbroken gauge groups turns out to be straightforward in this algebraic approach.

To conclude, we have described a mechanism, which makes the fact that electric and magnetic fields are massive in the Higgs phase consistent with the observability of unscreened charges at large distances through long-range Aharonov-Bohm interactions. The key observation here is that the charge that screens the Coulomb fields in the Higgs phase does not couple to the Aharonov-Bohm fields. This mechanism allows for smoothly switching on the Chern-Simons term in the Higgs phase in (2+1)-dimensional space-time, and therefore provides a solid basis for the discussion of the statistical properties of the states presented in Ref. [17].

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[13] If the explicit gauge symmetry breaking term $\frac{m^2}{2}A^2$ in (3) is introduced with $1/m \gg 1/M_A$, then the AB field, which was pure gauge and carried no energy for vanishing m, will acquire a nonvanishing energy. This implies that vortices will be attracted to each other, and therefore will be confined at distances $\sim 1/m$, forming "meson"- and "baryon"-like structures of a vortex antivortex pair or N vortices, respectively.

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