Strategic growth options

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Abstract

We provide a strategic rationale for growth options under uncertainty and imperfect competition. In a market with strategic competition, investment confers a greater capability to take advantage of future growth opportunities. This strategic advantage leads to the capture of a greater share of the market, either by dissuading entry or by inducing competitors to 'make room' for the stronger competitor. As a result of this strategic effect, payoffs are in a rough sense more convex than in the case of no investment in growth option. When the strategic advantage is strong, increased uncertainty encourages investment in growth options: higher uncertainty means more opportunity rather than simply larger risk. If the strategic effect is weak the reverse is true. On the other hand an increase in systematic risk discourages the acquisition of growth options. Our results contradict the view that volatility is a strong disincentive for investment.

JEL Classification: G31

Introduction

The goal of this paper is to investigate the decision to make an irreversible investment under imperfect competition and uncertainty. The real option literature has led to a significant advance in understanding the valuation of investment relative to the static NPV approach. (See Dixit and Pindyck (1994) for an excellent survey). However, real option analysis has often been based on two very specific assumptions: (a) the firm has a monopoly over an investment opportunity and (b) the product market is perfectly competitive. As a result, investment does not affect either prices or market structure.

On the other hand it is often recognized that early investment is associated with a greater ability to expand in the future; however, usually these 'growth options' have been introduced as exogenous technical advantages and modeled in an ad hoc fashion. This paper seeks to contribute to the theoretical foundations of growth options by developing the strategic advantage gained by investment vis-à-vis competitors.

The classic real option framework is an accurate description of investment decisions such as the development of a fully owned natural resource, for which monopolistic access to he investment opportunity is secure and the impact on market structure is minimal. However, when there are other potential competitors, not investing may lead some other producer to seize the opportunity. Moreover, under imperfect competition the commitment of an irreversible investment typically has strategic preemptive effects; immediate action may discourage entrants and enhance market share and profits (Gilbert, 1989). In the early contribution by Dixit (1979), investment confers a future cost advantage vis-à-vis potential entrants, creating a strong pre-emptive effect.[1] Interestingly, most of the subsequent literature has not considered the impact of exogenous uncertainty on this strategic decision.[2]

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[1] treatments of the investment decision described costs of postponement in terms of the higher adjustment cost of rapid construction of capacity. Hartman (1972) shows that under convex adjustment costs, higher uncertainty increases the optimal amount of investment even under perfect competition. See also Abel (1984) and Pindyck (1988, 1991).

We interpret the strategic value of initial investment as the acquisition of growth opportunities relative to competitors, which will be exercised if market conditions are favorable. We then contrast the value of acquiring such strategic growth options against the alternative of no investment, and offer some novel insight on the impact of uncertainty on the attractiveness of strategic investment.

In this paper an initial investment results in the acquisition of "capability" which allows the firm take better advantage of future growth opportunities. Specifically, in our model an initial investment in a growth option reduces future production costs, so that expansion can take place at a lower cost than for competitors without growth options. Examples of strategic investment leading to future comparative advantages may be research into building a technological advantage, an advertising campaign leading to identification and name recognition by consumers, and organizational and logistic planning leading to lower costs in building production capacity. The acquisition of this strategic advantage endogenously leads to the capture of a greater share of the market, either by dissuading entry or by inducing competitors to 'make room' for the stronger competitor. This is particularly valuable in states of high demand when profits per unit of output are higher. Thus strategic investment in conditions of uncertainty can be viewed as a commitment to a more aggressive future strategy.

Our main contribution is to show that, contrary to the result found in the real options literature, the effect of uncertainty on the relative value of the strategic growth options is ambiguous under imperfect competition. An important difference under imperfect competition is that profits are convex in demand, since oligopolistic firms respond to better market conditions by increasing both output and prices. Thus

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[2] While considerable work has been done on the role of asymmetric information (see Tirole (1988)), the only treatment of uncertain market conditions we are aware of is the partial equilibrium approach of Appelbaum and Lim (1985), who focus on production rather than investment as a form of precommitment. Their ad hoc revenue function drives entirely their results.

expected cash flows increase with volatility, as high marginal revenues at higher levels of demand more than compensate for low revenues at low levels of demand.

When strategic investment has a significant preemptive effect, it leads to higher market share, and thus a greater (relative) convexity of *ex post* profits relative to the case of no investment. As a result, even though the value of not investing increases with rising uncertainty, the value of the growth option increases even more. When instead the investment confers only a modest strategic advantage, the potential profit gain is less significant relative to the cost of the investment; an increase in volatility will increase the value of not investing and thus raise the threshold for investment in the growth option. For intermediate levels of strategic advantage, increased uncertainty favors strategic investment unless the probability shift results in a much higher likelihood of entry. Finally, since maximum losses are bounded above by the initial investment, at a very high level of uncertainty a further increase favors strategic investment.

These results are confirmed in the case when more firms have access to the investment opportunity and is robust to various specifications of strategic advantages gained by the investment (Kulatilaka and Perotti, 1991). However, when a systematic risk component to the volatility is introduced, we find that an increase in systematic risk discourages strategic investment, as it leads to higher risk exposure.

The rest of the paper is organized as follows. Section 1 examines a simple benchmark model of monopoly investment under uncertainty. This section allows us to introduce several key features arising from market power. Section 2 studies the more general case of imperfect competition and the strategic effect of investment in capability. We explore the impact of uncertainty on the value of a strategic growth option and obtain a closed-form solution for the case of a log-normally distributed demand. Section 3 considers the possibility of simultaneous strategic investment and the effect of systematic risk. The last section offers some conclusive remarks.

1. A Benchmark Model: Monopoly Investment in a Growth Option

We consider as a benchmark the extreme but simple case of a firm with monopoly in both the investment opportunity and in the product market. In this context the growth option has no strategic effect. We progressively relax these assumptions later.

At time 0, a single firm (denoted by *M*) has the opportunity to make an initial irreversible investment of amount *I*, which confers a capability for more efficient (specifically, lower cost) production.[3] This will turn out to equivalent to the purchase of a growth option. Until time 1, when the market opens, there is uncertainty over the scale of future demand. We assume that the demand for the good is linear in prices and increasing in the random variable θ. Let P(Q) be the inverse demand function expressing the market price as a function of total supply Q:

$$P(Q, \theta) = \theta - Q$$

where θ is distributed on $(0,\infty)$, with expected value $E_0[\theta]\equiv\theta_0 > 0$. Uncertainty is fully resolved at time 1 prior to production.

If no initial investment is made the firm will produce only when the market is profitable, producing at a unit cost of K. The firm will choose an output level $Q_M^N = \frac{1}{2}(q - K)$ with associated profits $p_M^N = \frac{1}{4}(q - K)^2$; it will not produce if $\theta < \theta^*_M \equiv K$.

In contrast, an initial investment reduces the future unit cost to κ (where $\kappa<K$), due to learning, logistic and product development improvements. If $\theta < \kappa$ the firm will not produce; else, it will choose an output level $Q_M^I = \frac{1}{2}(q - k)$ with associated profits $p_M^I = \frac{1}{4}(q - k)^2$. Thus Q^N_M is less than Q^I_M because investing in the growth option enhances the incentive to expand production.

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[3] Our view of investment in growth opportunities is fairly general. Analogous results would obtain if the investment would result in greater quality or consumer appeal.

Notice that these payoff functions are continuous, monotonically increasing, and convex in θ. An increase in θ has a more than proportional effect on payoffs because a firm with market power respond to higher demand by increasing both output and prices.

We initially assume no systematic risk (or risk neutrality) and a zero interest rate. Then the net present value of the capability investment is:

$$V_M^I \equiv E_0[p_M^I] - I = E_0\left[\tfrac{1}{4}(q-k)^2 \middle| q \geq k\right] prob(q \geq k) - I$$

The correct investment criterion calls for a comparison between the net present values of making the investment with the NPV of not making the investment. The latter value is

$$V_M^N \equiv E_0[p_M^N] = E_0\left[\tfrac{1}{4}(q-K)^2 \middle| q \geq K\right] prob(q \geq K)$$

To solve for the optimal investment decision we define the ex post <u>net</u> gain to investment (the relative value of investment) as $\Delta^M(\theta) \equiv p_M^I$ - I - p_M^N. Then the net present value of the decision to acquire the growth option (relative to not investing) is the expectations of Δ^M: $G(\theta_0) \equiv E[\Delta^M(\theta)]$. The level of expected demand $\theta_0 = \Theta^M$ such that $G(\Theta^M) = 0$ is then a point of indifference.

It is easy to show that a unique value for Θ^M exists under some simple regularity conditions, namely for the set of distributions with a strictly positive support on θ where higher mean implies first order stochastic dominance (i.e., for which given two random variables x_1, x_2, $E(x_1) > E(x_2) \Leftrightarrow x_1$ first order stochastically dominates x_2).

Proposition: Strategic investment is optimal when θ_0 exceeds the unique expected demand threshold Θ^M.

Proof: From the definition of *ex post* profit functions we know that $\lim_{q_0 \to 0} G(q_0) = -I < 0$ and $\lim_{q_0 \to \infty} G(q_0) = \infty$. It is sufficient to show that $d\{G(\theta_0)\}/d\theta_0 > 0$; then uniqueness is established by the intermediate value theorem. Since Δ is an increasing and differentiable function of θ, the condition $d/d\theta_0 \{E\,\Delta[\theta(\theta_0)]\} > 0$ is satisfied for all distributions of θ under consideration. Since $G(\theta_0) > 0$ for all $\theta_0 > \Theta^M$, in this range investment in the growth option has a higher NPV than the alternative of not investing. ■

Figure 1 plots π^I_M - I, π^N_M , and Δ^M against θ for the case where κ=0.

We now examine the impact of greater uncertainty on the relative attractiveness of investment. We define an increase in uncertainty over θ as one which does not affect its mean, thus adopting the concept of a mean-preserving spread as in Rothschild and Stiglitz (1970). Note that both π^I_M and π^N_M are convex and differentiable in θ; so by Jensen' s inequality the impact of a mean-preserving spread increases their expected value. Similarly, if the net gain function Δ^M were convex or concave, the net effect of increasing uncertainty would be unambiguous. However, the net gain function has two kinks; at the points κ and K. The first kink occurs at the level of demand when a monopolist which invested in growth option starts production; the second is the level at which a monopolist which did not invest starts production.

Therefore, the impact of greater uncertainty on the relative value of investment is uncertain because these entry points create discontinuities in the rate of change in marginal profits. As the next section confirms, this ambiguity is always present under imperfect competition since entry has always an impact on price and thus marginal profitability.

2. Strategic Growth Options Under Imperfect Competition

We now consider, within the framework of the model in the previous section, the possibility of a competitor entering at time 1. Now investment will have a strategic effect.

Firm 1 has a monopoly over the investment opportunity. (We relax this assumption later.) Firm 1 chooses whether to make a strategic investment at time 0, anticipating its impact on future market structure. A second firm (firm 2) may choose to enter the market at time 1, with a unit production cost of K. We assume that if both firms produce, the market outcome is Cournot competition.[4]

[4] This is consistent with the theoretical work by Kreps and Scheinkman [1983] on capacity choice followed by price competition.

Consider first the case where firm *1* makes no initial investment, so that *ex post* it has no strategic advantage vis-à-vis the competitor. If both firms choose to produce, they face the same production cost K. As long as $\theta \geq K$, the outcome is a symmetric Cournot equilibrium in which they share the market equally; each firm produces an amount $Q^N_1 = (\theta - K)/3$, which yield profits equal to $\pi^N_1 = \frac{1}{9}(q - K)^2$. If $\theta < K$ neither will produce, as the marginal revenue falls below cost. Hence, $\theta^* \equiv K$ can be interpreted as the symmetric Cournot entry point, below which no production takes place.

If instead firm 1 invests I at time 0, market interaction is affected by its strategic advantage, which is acknowledged by firm *2* when making its output decision. It is easy to see that if both firms produce, firm *1* will choose an output level $Q_1^I = \frac{1}{3}(q + K - 2k)$ with associated profits $\pi_1^I = \frac{1}{9}(\theta + K - 2\kappa)^2$. It is now optimal for firm *2* to choose a lower quantity, $Q_2^I = \frac{1}{3}(q - 2K + k)$, yielding profit, $p_2^I = \frac{1}{9}(q - 2K + k)^2$. Moreover, the Cournot entry point for the competitor is now higher and is defined by $\theta^{**} \equiv 2K - \kappa$.

Note the source of the market share gained by firm 1 when it invests at 0. Q^I_2 is less than Q^I_1 for various related reasons: first, because firm 2 faces a higher production cost; second, because it recognizes firm 1's greater incentive to expand production (the post-entry dissuasion effect of strategic investment). Finally, firm 2 does not enter unless $\theta > \theta^{**}$, which is higher than θ^*. This is the entry-dissuasion effect. As a result, firm 1 acts as a monopolist for $\theta^{**} > \theta > \kappa$, charging a price ½(θ-κ) and earning profits ¼$(\theta - \kappa)^2$. In general, the cost advantage derived from strategic investment increases firm *1*'s market share and profits for all θ. We conclude that strategic investment can be seen as offering an enhanced market share.[5]

In summary, the ex-post gross profits of firm *1* under not investing (π^N_1) and investment (π^I_1) are given by:

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[5]We find it useful to decompose the growth option gained by strategic investment in two components. First, it results in a lower "unit exercise price" for future expansion. In addition, the optimal output Q_1^I, "the number of unit production options that are optimally exercised", also increases, as other firms choose to limit their own output to make room for the stronger competitor.

$$p_1^N = \begin{cases} 0 & if\ q < q^* \\ \frac{1}{9}(q-K)^2 & if\ q \geq q^* \end{cases} \qquad p_1^I = \begin{cases} 0 & if \quad q \leq k \\ \frac{1}{4}(q-k)^2 & if \quad k < q < q^{**} \\ \frac{1}{9}(q+K-2k)^2 & if \quad q \geq q^{**} \end{cases}$$

As before, these payoff functions are continuous and monotonically increasing in θ, and profit increases more than proportionately with demand. Note that profits rise faster with demand if firm 1 has invested at time 0. As little generality is gained by a positive κ, we henceforth normalize its value to 0.

To understand the trade-off in the optimal investment problem we investigate the characteristics of the ex post <u>net</u> gain to strategic investment, defined as $\Delta \equiv \pi_1^I - I - \pi_1^N$. Figure 2 plots the function Δ.

We note that Δ exhibits two kinks. The first kink occurs at $\theta^* = K$ and corresponds to the beginning of production in the no-investment, symmetric Cournot case. This is the same as the kink in the monopoly case: at this point marginal profitability is continuous but not differentiable.[6] The relative gain to investment between θ^* and θ^{**} remains convex in θ but profits rise at a lower rate.

A more dramatic discontinuity occurs in the strategic investment case when demand is above the higher entry threshold θ^{**}. Here entry by the competitor results in a drop in the rates of increase of both price and output of the investing firm and creates a discontinuity in marginal profitability. In summary, the net gain function is piece-wise convex in the region of entry dissuasion ($\theta \leq \theta^{**}$) and is linear for $\theta > \theta^{**}$.

Table 1: Net Gain Function: $\Delta = \pi_1^I - I - p_1^N$

Range	Δ	$\partial\Delta/\partial\theta \equiv \Delta$\`	$\partial^2\Delta/\partial\theta^2 \equiv \Delta$\`\`
$\theta < \theta^*$	$\frac{\theta^2}{4} - I$	$\frac{q}{2}$	1/2
$\theta^* < \theta < \theta^{**}$	$\frac{5q^2}{36} + \frac{2qK}{9} - \frac{K^2}{9} - I$	$\frac{5\theta}{18} + \frac{2K}{9}$	5/18
$\theta > \theta^{**}$	$\frac{4qK}{9} - I$	$\frac{4}{9}K$	0

[6] Note that for a range of demand beyond this point the relative gain to investment is higher than for the monopoly case. This reflects the fact that not investing leads to a lower payoff because of loss of market share to competition, so in this range there is a greater incentive for investment. When demand is quite high the monopolist benefits comparatively more from the cost advantage, while the relative market share gain declines for the investing firm.

The optimal investment decision facing firm *1* at time 0 requires comparing the relative NPV of making the strategic investment.

$$V_1^N \equiv E_0[p_1^N] = E_0\left[\tfrac{1}{9}(q-K)^2 \middle| q \geq q^*\right] prob(q \geq q^*)$$
$$V_1^I \equiv E_0[p_1^I] - I = E_0\left[\tfrac{1}{9}(q+K)^2 \middle| q \geq q^{**}\right] prob(q \geq q^{**}) + E_0\left[\tfrac{1}{4}q^2 \middle| q^{**} > q\right] prob(q^{**} > q) - I$$

Let $H(\theta_0) \equiv E(\Delta)$ denote the *expected net gain* from strategic investment, $V_1^I - V_1^N$. The threshold value of *expected demand* such that firm *1* is indifferent whether to make the strategic investment or not is denoted by Θ and defined by $H(\Theta) \equiv 0$.

Under the regularity conditions defined in the Proposition 1, it is easy to show that strategic investment is optimal when θ_0 exceeds the unique threshold Θ.

The proof follows that of Proposition 1. Although in this case Δ non-differentiable at θ^{**}, this does not alter the result as it occurs only at a countable number of points.

We now consider the impact of a mean-preserving spread on the relative value of investment.

From the valuation expression it is immediate to see that the value of acquiring the growth option is strictly increasing with the uncertainty over θ. This is a result of convexity of profits created by market power and Jensen’s inequality. This suggests that as uncertainty in demand increases the incentive to invest will unambiguously increase. However, as with the real options literature, the NPV of not investing is also increasing with increasing demand uncertainty. Therefore, it is the shape of the difference in ex post marginal profits (i.e., the curvature of Δ) which will determine the overall effect of uncertainty.

As we saw also in the benchmark monopoly case, since the net gain function Δ is neither convex or concave the net effects of increasing uncertainty is ambiguous. The intuition is that the rate of change in marginal profits is different to the left and right of a kink. Thus when a mean-preserving spread shifts around probability mass to more extreme values, the position of the expected level of θ relative to the kink matters a great deal.

In the benchmark case this is easy to see. Figure 1 (where the first kink disappears because κ is set to zero) indicates that Δ^M (the NPV of investing over non investing) is convex for $\theta<\theta^*$ and linear for $\theta>\theta^*$. When the threshold Θ^M is in an area where Δ^M is convex in θ, more uncertainty will increase the incentive to invest; viceversa when the threshold lies above θ^*, where the loss of profit on the downside (where Δ^M is convex in θ) is larger than the potential gain on the upside (where Δ^M is linear in θ).

In the case of imperfect competition, the effect of entry by competitors creates a second kink. Table 1 shows that at the entry point θ^{**} the marginal net gain from investment drops discontinuously. When expected demand is around this point, the downside losses from a spread in probability mass will exceed the upside gain as the profits are convex for $\theta < \theta^{**}$ and linear above this range. For values of θ below θ^{**} the payoff to strategic investment is piece-wise convex; thus in this range a mean preserving spread encourages investment.

Therefore, the direction of the impact of uncertainty on the relative value of investment is critically dependent on the magnitude of the strategic advantage, measured by the cost advantage K. When K is large, the entry dissuasion range of demand ($\theta < \theta^{**}$) is larger, and thus the convex area of Δ is larger. As a result, the impact of higher uncertainty tends to favor investment. The opposite is true for a small K.

The conclusion is that in the case of strategic investment with strong preemptive effects, higher uncertainty will tend to decrease Θ and encourage investment; the reverse is true in the case of a weak strategic effect. In general the result is ambiguous and its exact impact depends both on the nature of the market structure ***and*** the nature of the distribution.

An Example: Lognormal distribution

We now examine the behavior of the value functions and the strategic investment threshold under the assumption that θ is log- normally distributed with expected value $E_0(\theta) = \theta_0$ and variance σ^2, i.e., $ln(\theta/\theta_0) \sim N(-½\sigma^2, \sigma^2)$. By construction, an increase in σ will not affect the expected value of θ and would, therefore, amount to a mean-preserving spread. In all our simulations, I is normalized to 1, and κ is normalized to zero. Then the value functions have the following analytical solutions:

$$V^N(q_0, s) = \tfrac{1}{9}\left[q_0^2 e^{s^2} N(d_1) - 2Kq_o N(d_2) + K^2 N(d_3)\right]$$

and

$$V^I(q_0, s) = \tfrac{q_o^2 e^{s^2}}{4}\left[1 - \tfrac{5}{9} N(d_4)\right] + \tfrac{1}{9}\left[2Kq_o N(d_5) + K^2 N(d_6)\right] - I$$

where

$$d_1 = \frac{2ln(\frac{q_o}{K}) + 3s^2}{2s}, \quad d_2 = d_1 - s, \quad d_3 = d_2 - s,$$

$$d_4 = \frac{2ln(\frac{q_o}{2K}) + 3s^2}{2s}, \quad d_5 = d_4 - s, \quad d_6 = d_5 - s.$$

Table 2 summarizes our simulations on the effect of a mean-preserving increases in volatility on the threshold point, Θ. This effect depends on the relative impact of higher uncertainty on V^I and V^N. The table clearly indicates that the net effect depends on K. A high value of K implies that strategic investment has a strong entry dissuasion effect and a marked market share advantage in a contested market.

At high levels of K, strategic investment is preferred even for low expected values of demand, and entry dissuasion is strong so that the competitor's entry threshold θ^{**} is relatively high. Figure 3 plots the expected values of strategic investment and no investment for such a case at two levels of uncertainty. As uncertainty increases, the value of V^N unambiguously rises; but so does V^I. The overall effect in this case is that the threshold level Θ drops with higher uncertainty. In this case, higher uncertainty leads to greater upside opportunities which outweigh the higher downside risk..

At low level of strategic advantage (low K), the entry dissuasion region is small, and the market share gain is also limited. Then strategic investment is justified only under high expected demand, when entry is almost certain. In this case growth option is less valuable. Interestingly, as volatility increases without bounds, strategic investment again is favored, as the break-even point starts falling. This is the effect of bounded losses under strategic investment.

At intermediate levels of *K*, the impact of uncertainty depends on the position of the break-even point relative to the ex post entry threshold, θ^{**}, and is in general ambiguous.

These results are consistent with our earlier general conclusions. The choice of not investing becomes more valuable under higher volatility, reflecting a lower risk exposure; however, uncertainty *may be* favorable to investment, when the extra profits from strong market share due to the deterrence or *commitment effect* outweigh its downside risks. The ambiguity of our results arises from the very essence of imperfect competition. Strategic growth options do not exhibit the continuous features of expansion options under perfect competition: now an individual firm's investment decision has a significant impact on market structure and thus on the market price.

3. Extensions

Simultaneous Strategic Entry

We now extend the context of the basic model to the case when neither firm enjoys a monopoly on pre-emptive investment. In other words, both firms are able to invest at time 0 in order to reduce future capital investment costs to κ.[7]

The payoff to investment by one firm depends on the competitor's investment decision and its ex post output decision. Since the two firms decide simultaneously, neither can condition its strategy on the other's decision. The final market outcomes are monopoly, asymmetric or symmetric cost Cournot equilibrium and no production, respectively. Each firm's belief about the other's strategy now plays a key role. In order to model ex ante identical firms, we ignore equilibria driven by asymmetric beliefs and focus only on symmetric equilibria.

[7] It is not essential that there are only two firms; however, if we increase the number of potential investors the expected value of both the waiting to invest and the strategic investment option would be zero.

There are three possible symmetric equilibria depending on the expected level of demand, q_0.

Define the following ex-ante payoff functions:

$$f_1 \equiv E_0\left[\tfrac{(q-k)^2}{9} \middle| q > k\right] prob(q > k) \quad \text{(Simultaneous investment)}$$

$$f_2 \equiv E_0\left[\tfrac{(q-k)^2}{4} \middle| 2K-k > q > k\right] prob(2K-k > q > k) \quad \text{(Strategic investment and no entry by competitor)}$$

$$f_3 \equiv E_0\left[\tfrac{(q+K-2k)^2}{9} \middle| q \geq 2K-k\right] prob(q > 2K-k) \quad \text{(Strategic investment and late entry by competitor)}$$

$$f_4 \equiv E_0\left[\tfrac{(q-2K-k)^2}{9} \middle| q > 2K-k\right] prob(q > 2K-k) \quad \text{(No investment while competitor invests)}$$

$$f_5 \equiv E_0\left[\tfrac{(q-K)^2}{9} \middle| q \geq K\right] prob(q > K) \quad \text{(No investment and ex post entry by both)}$$

As before, in what follows we set $\kappa = 0$.

Proposition 2: *The optimal investment policy is no investment by both firms when* $\theta_0 \leq \theta^R$, *randomized investment by each firm with probability* $y(\theta_0)$ when $\theta^R \leq \theta_0 \leq \theta^S$, *and simultaneous investment by both when* $\theta_0 \geq \theta^S$, *where:*

$\theta^R \equiv E_0(\theta)$ s.t. $I + \phi_5(\theta^R) = \phi_2(\theta^R) + \phi_3(\theta^R)$

$\theta^S \equiv E_0(\theta)$ s.t. $\phi_1(\theta^S) = \phi_4(\theta^S) + I$

In addition, the equilibrium probability of entry, $y(\theta_0)$, for $\theta^R < \theta_0 \leq \theta^S$ is given by

$$y(\theta_o) = -\frac{I + \phi_5 - \phi_2 - \phi_3}{\phi_1 + \phi_5 - \phi_2 - \phi_3 - \phi_4}$$

Proof: See Appendix.

The optimal investment strategy is similar to the basic model. Clearly, higher expected demand favors strategic investment and reduces the value of not investing. As before, the impact of volatility depends on the degree of advantage gained by exercising the strategic investment option. Under a strong strategic advantage, higher volatility shifts downward the threshold for investment. Intuitively, when strategic gain is significant there is a stronger pre-emptive effect; thus the threshold level of expected demand at which investment is preferred, Θ, is low.

In contrast, a weak strategic advantage means that the competitive gain of pre-emptive investment is weaker on the upside, and the area of deterrence is smaller; so the reduction in profitability on the downside is greater. Since the value of not investing *increases* with volatility, the local effect of greater uncertainty in this case will be to discourage investment.

The simulation results in Table 3 confirms these effects of uncertainty on the probability of investment, around a parameter range when a randomized investment strategy may be optimal for both firms.

Systematic Risk

If an increase in uncertainty over θ induces an increase in non-diversifiable risk, our results need to be qualified. Assume that in equilibrium, the risk premium associated with uncertainty in θ is proportional to its volatility. That is, the equilibrium rate of growth of θ equals $r + \lambda\sigma$, where λ is the market price of risk associated with θ and r is the risk-free rate of interest. Since the payoffs to the firm are non-linear functions of θ, the risk adjusted discount rate depends on the realization of θ which is unknown at time 0. Hence, we transform the valuation problem into its risk-neutral representation, and thereby achieve risk-adjustment via adjustment to probabilities rather than the discount rate.[8]

It can easily be shown that under a risk-neutral probability measure, θ would still be lognormally distributed but its rate of return will fall short of the risk free rate of interest by adjustment factor:

$$\ln\left(\frac{\theta}{\theta_0}\right) \sim N\left(-\tfrac{1}{2}\sigma^2 - 2\lambda\sigma, \sigma^2\right).$$

[8] See Cox, Ingersoll, and Ross [1985]. For instance, in a single factor asset pricing model $\lambda = \frac{\rho_{\theta,M}}{\sigma_M}(r_M - r)$, where $\rho_{\theta,M}$ is the correlation coefficient between θ and the market portfolio, and r_M and σ_M are, respectively, the rate of return and its standard deviation.

As a result of this risk adjustment, the valuation equations are modified as follows:

$$V^W(q_0, s) = \tfrac{1}{9}\left[q_0^2 e^{s^2 - 2ls} N(d_1) - 2Ke^{-ls} q_o N(d_2) + K^2 N(d_3)\right]$$

and

$$V^I(q_0, s) = \tfrac{q_o^2 e^{s^2 - 2ls}}{4}\left[1 - \tfrac{5}{9} N(d_4)\right] + \tfrac{1}{9}\left[2Ke^{-ls} q_o N(d_5) + K^2 N(d_6)\right] - I$$

where

$$d_1 = \frac{2ln(\frac{q_o}{K}) + 3s^2 - 2ls}{2s}$$

$$d_4 = \frac{2ln(\frac{q_o}{2K}) + 3s^2 - 2ls}{2s}$$

Note that the effect of risk-adjustment is equivalent to reducing θ_0, the expected value of θ. From Section 1 we know that a decrease in expected value, θ_0, will reduce $E(\pi^I)$ more than $E(\pi^W)$. Therefore, the impact of a higher risk premium will be to reduce the relative value of strategic investment. The intuition is that the strategic investment option requires the firm to bear more risk.

5. Concluding Remarks

This paper has proposed a reconciliation of the real options and the strategic approach to the optimal timing of investment. The real options literature assumes perfect competition and analyses the effect of exogenous uncertainty. In contrast, the strategic approach endogenizes market structure, however, it often ignores uncertainty, which is relevant for valuation even when the firm is risk neutral because of the option value of flexibility.

We show that the proper valuation of real investment must take into account both its strategic value (the pre-emptive effect of commitment) and the alternative value of not investing (a form of flexibility). We interpret the effect of strategic investment as lowering not just production costs but also the strike price of future expansion options. This is because choice also has a *strategic* influence on competitors' output decisions, inducing them to be less aggressive. This increases the investor's market share and, therefore, the value of its expansion option.

Our results on the effect of uncertainty on the valuation of strategic investment may be surprising in the light of current practice, which tends to view volatility as a strong disincentive for new investment.

In a richer intertemporal setting it would be possible to analyze how the *evolution* of strategic considerations should be contrasted with the opportunity cost of waiting to learn more about uncertain market conditions. We plan to do so in later research.

Empirically it may be true that real investment tend to fall when uncertainty rises, however, this is probably due to a concomitant decrease in expectations over market conditions. We intend to extend further our analysis on the valuation of investment in oligopolistic markets, as we are convinced that this line of research will not only improve our theoretical understanding of optimal investment timing, but may also contribute to modern capital budgeting practice and methodology.

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Appendix
Proof of Proposition

Consider the payoffs to strategic investment for firm *1*. Suppose first that it invests at time 0 while firm *2* does not. If demand next period is so low that late entry by firm *2* is not profitable, i.e. $\theta < 2K$, firm *1* will be a zero marginal cost monopolist. If $\theta > 2K$, firm *2* enters, but firm *1* earns higher payoffs as a low cost, higher market share Cournot competitor. Thus firm *1*'s expected payoff from solitary strategic investment at 0 is:

$$E_0\left[\tfrac{\theta^2}{4}|\theta < 2K\right] + E_0\left[\tfrac{(\theta+K)^2}{9}|\theta \geq 2K\right] - I$$

If instead firm *2* enters as well, they will share the market as Cournot competitors, and their expected payoff equals $E\left[\frac{\theta^2}{9}\right] - I$.

Consider now the payoff to not investing. If firm *1* does not invest and firm *2* does, firm *1*'s expected payoff equals the sum of zero when future entry is unprofitable $(\theta < 2K)$ and its profits as a high cost entrant otherwise:

$$0 + E_0\left[\tfrac{(\theta-2K)(\theta+K)}{9}|\theta \geq 2K\right]$$

Finally, if neither firm enters at 0, their *ex post* payoffs are zero if $\theta < K$ and equal to the high cost, symmetric Cournot equilibrium payoffs otherwise:

$$0 + E_0\left[\frac{(\theta-K)^2}{9}|\theta \geq K\right]$$

Note next that the relative payoff to investment by either firm increases monotonically with q_0, independently of the competitor's timing of investment. To see this, assume the other firm does not invest; the net gain from entering is

$$E_0\left[\tfrac{\theta^2}{4}|\theta < 2K\right] + E_0\left[\tfrac{(\theta+K)^2}{9}|\theta \geq 2K\right] + E_0\left[\tfrac{(\theta-K)^2}{9}|\theta \geq K\right] - I$$

which can be re-written as:

$$E_0\left[\tfrac{\theta^2}{4}|\theta< K\right]+E_0\left[\tfrac{5\theta^2-4K^2+8K\theta}{36}|K\leq\theta\leq 2K\right]+E_0\left[\tfrac{4K\theta}{9}|\theta\geq 2K\right]-I$$

In this expression, each term is positive and increasing in θ_0. If the other firm invests as well, the symmetric Cournot payoff is clearly increasing in θ_0.

The net gain equals zero at $\theta_0 = \theta^R$, the threshold value such that neither firm will invest . To verify that neither firm wishes to invest if the other firm is not expected to invest, let y = 0. Firm *1* will not invest if its net gain from strategic investment is negative. This occurs when

$$\varphi_2+\varphi_3-I<\varphi_5$$

which is satisfied for $\theta_0 < \theta^R$, since the net gain is monotonic in θ_0. The reasoning for firm *2* is analogous, so y = x = 0 is an equilibrium in this range. When θ_0 is below the threshold, neither firm invests even if were certain to gain a market advantage. A similar comparison of payoffs can be made when firm *2* does invest . The net benefit (loss) of strategic investment rises (falls) with θ_0 and equals zero exactly when θ_0 equals θ^S. When θ_0 is above this threshold, both firms invest.

To verify this, let y = 1. Firm *1* invests today with certainty if:

$$\varphi_1-I\geq\varphi_4$$

which is satisfied for all $\theta_0 > \theta^S$.

When θ_0 is in the intermediate region, both firms invest with positive probability. Let $x(\theta_0)$ be the probability of entry by firm *1* and $y(\theta_0)$ the corresponding probability for firm *2*. Firm *1* will randomize between investing and not investing as long as it is indifferent between the two choices.

This occurs if:

$$
yE_0\left[\tfrac{\theta^2}{9}\right] + (1-y)E_0\left[\tfrac{\theta^2}{4}\,|\theta \le 2K\right] + E_0\left[\tfrac{(\theta+K)^2}{9}\,|\theta \ge 2K\right] - I =
$$

$$
yE_0\left[\tfrac{(\theta-2K)^2}{9}\,|\theta > 2K\right] + (1-y)E_0\left[\tfrac{(\theta-K)^2}{9}\,|\theta \ge K\right]
$$

In the game with symmetric beliefs, the equilibrium value of $y(\theta_0)$ is equal to $x(\theta_0)$ as described in Proposition 2. It is also increasing in θ_0. To verify this, notice that the numerator of $y(\theta_0)$ as defined above is always negative in range $\theta^S > \theta_0 \ge \theta^R$, and equals the negative of the net payoff to strategic investment when the competitor does not invest, which we have shown to be increasing in θ_0. The denominator is also negative and larger in absolute value, since it equals the numerator plus ϕ_1 - I - ϕ_4 (the net gain to strategic investment when the competitors invests) which in this region is negative and decreasing in θ_0.Ü

Table 2

Sensitivity of Investment Threshold, Θ

Cumulative Volatility	**Capacity Cost Advantage: K-κ** 0.3	**0.8**	**1.06**	**1.3**	**1.8**	**3.0**
0.0	7.5000	2.8125	2.1214	2.0638	2.0047	2.0000
0.2	7.5000	2.8127	2.2046	2.0516	1.9785	1.9605
0.4	7.5000	2.8295	2.2677	2.0605	1.9139	1.8539
0.6	7.5000	2.8671	2.3117	2.0682	1.8494	1.7149
0.8	7.5002	2.9040	2.3380	2.0656	1.7887	1.5768
1.0	7.5017	2.9318	2.3495	2.0533	1.7306	1.4518
1.2	7.5058	2.9491	2.3494	2.0337	1.6749	1.3422
1.4	7.5124	2.9569	2.3408	2.0093	1.6221	1.2475
1.6	7.5201	2.9572	2.3266	1.9822	1.5729	1.1662
1.8	7.5272	2.9518	2.3090	1.9541	1.5277	1.0967
2.0	7.5328	2.9427	2.2895	1.9265	1.4868	1.0376
2.2	7.5364	2.9312	2.2697	1.9002	1.4503	0.9875
2.4	7.5381	2.9185	2.2503	1.8757	1.4181	0.9452
2.6	7.5379	2.9055	2.2319	1.8535	1.3900	0.9097
2.8	7.5363	2.8928	2.2150	1.8336	1.3657	0.8799

Table 3

Sensitivity of Probability of Investment

Expected Demand = 3.0

Cumulative volatility	**Cost Advantage: K-κ** 0.7	**0.8**	**0.9**	**1.0**	**1.1**
0.1000	0.00	0.23	0.56	0.75	0.87
0.2000	0.00	0.23	0.56	0.75	0.87
0.3000	0.00	0.23	0.55	0.75	0.87
0.4000	0.00	0.22	0.55	0.75	0.88
0.5000	0.00	0.21	0.54	0.75	0.89
0.6000	0.00	0.19	0.54	0.76	0.91
0.7000	0.00	0.17	0.54	0.78	0.94
0.8000	0.00	0.15	0.54	0.80	0.98
0.9000	0.00	0.13	0.55	0.83	1.00
1.0000	0.00	0.12	0.57	0.87	1.00
1.1000	0.00	0.11	0.59	0.92	1.00