Studies on the semantics of questions and the pragmatics of answers
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STUDIES ON THE SEMANTICS OF QUESTIONS
AND THE PRAGMATICS OF ANSWERS

ACADEMISCH PROEFSCHRIFT

ter verkrijging van de graad van
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hoogleraar in de Faculteit
der Wiskunde en Natuurwetenschappen,
in het openbaar te verdedigen
in de Aula van de Universiteit,
(tijdelijk Roetersstraat 15)
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door

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en

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STUDIES ON THE SEMANTICS OF QUESTIONS AND THE PRAGMATICS OF ANSWERS
Promotores: prof. dr. R.I. Bartsch
          prof. dr. J.F.A.K. van Benthem
This book contains six studies on different subjects in the theory of questions and answers. They were written over a period of several years. Yet, we trust that they present a coherent view.

Except for the first paper, which being an introduction was written last, the papers appear in chronological order. The second paper was written in 1980, the third in 1982, and the fourth in 1983. These three papers have been published, and they are included here with permission of the copyright holders, which is gratefully acknowledged. Except for some minor corrections, they appear here as they were published. The remaining three papers were written specially for this volume, in 1984. There are some minor discrepancies in content and terminology between the earlier papers and the later ones. These are pointed out in the preliminary remarks. The later papers, like the earlier ones, were written as separate, independent papers. This has caused some overlap, which is the only excuse we have for the volume of this volume.

Our interest in the subject of questions and answers is a derivative of our main interest, which is the pragmatics of natural language, in particular the epistemic aspects thereof, and the role it plays in a general theory of meaning and understanding. It was some years ago that, while we were discussing the pragmatics of assertions, Simon Dik raised the problem of questions, and started us thinking about that subject. But in order to get a proper pragmatics, one needs a proper semantics, and so one thing starts another.
As the papers show, the enterprise in which we are engaged is one which does not eschew going into details. It bespeaks an attitude towards general philosophical claims that they can be, and sometimes need to be worked out in 'unphilosophical' detail in order to get a better idea of their contents and tenability. In this sense, formal semantics can also be viewed as the execution of a philosophical program. Quite generally, we think that this is a valuable and fruitful way to view the relationship between philosophy and science. And it depends on the actual division of labour what is classified as what.

Following good custom, we would like to express our gratitude here to all who have helped. Simon Dik, Johan van Benthem, Renate Bartsch, and Teun van Dijk initiated us in the ways and means of this profession, and encouraged and helped us getting started. Renate Bartsch and Johan van Benthem have been patient and careful supervisors ever since. Theo Janssen and Fred Landman helped us by their never-failing willingness to discuss problems and criticize our solutions, and by letting us share their knowledge and insights. Together with Renate Bartsch, Dick de Jongh and Frank Veltman, they provide an environment that is stimulating and pleasant to work in. Various other people have commented on earlier versions of the material as well. Of those who are mentioned in the papers themselves, we owe special thanks to Peter van Emde Boas, for his piercing and useful criticisms. We are grateful to Marjorie Pigge for performing a fine job typing and retyping various versions of various manuscripts. Finally, each of the authors would like to thank the other.

Amsterdam
October 1984

Jeroen Groenendijk
Martin Stokhof
PRELIMINARY REMARKS

The second, third and fourth paper are published papers, and they have been included in the present volume without any essential changes. The main purpose of these remarks is to indicate how they are related to, and at which points they deviate from, or are revised in, the other papers, which were written later.

Sections 1, 2 and 3 of II, 'Semantic analysis of wh-complements', present the core of our semantic analysis of wh-complements and interrogatives. The latter are not within the scope of II, but in section 1 of V, 'Questions and linguistic answers', the analysis of wh-complements it contains is adopted for the analysis of interrogatives as well.

Section 5 of II deals with certain aspects of coordination. Coordination of interrogatives is treated in more depth and detail in VI, 'Coordinating interrogatives'. This holds also for the scope phenomenon discussed in section 6.1 of II. The analysis given there, is criticized and replaced by a different one in VI.

A more specific remark concerns the use of Ty2, the language of two-sorted type theory, as a translation medium, instead of PTQ's IL. In section 6.2 of II it is asserted that the increase in expressive power Ty2 has over IL is really needed for a statement of the semantics of interrogatives. This claim has been refuted by Zimmermann, in his paper 'Comments on an article by Groenendijk & Stokhof', which is to appear in Linguistics and Philosophy. Zimmermann shows that all semantic operations we use in II, can be formulated in IL as well, be it in a much less elegant and perspicuous way.

In the same paper, Zimmermann proves the conjecture made
in section 3.8 of II, that in order to obtain so-called 'de dicto' readings of interrogatives in a compositional way, the intermediary level of abstracts is necessary. Further empirical motivation for the level of abstracts is provided in V, where it is argued that it plays an essential role in the derivation and interpretation of linguistic answers.

The third paper, 'Interrogative quantifiers and Skolem-functions', deals with the analysis of so-called 'functional readings' of interrogatives. Within the volume as a whole, III has a rather isolated position. Functional readings are distinguished from so-called 'pair-list readings'. The analysis of the latter that is used in III, is that presented in II. As remarked above, VI contains a better and more thorough analysis of this phenomenon. However, the argumentation in III concerning the non-identity of functional and pair-list readings is independent of this.

One of the conclusions of III is that the syntactic analysis of functional readings presented there, though effective, is not very elegant. In note 39 of V, some suggestions are made how to improve upon it. The matter is once more touched upon in note 51 of VI.

The fourth paper, 'On the semantics of questions and the pragmatics of answers', has a central position. It connects the semantics of interrogatives with pragmatic notions of answerhood. The definitions of these notions in IV reappear in section 4 of V. There they are stated in a slightly different form, but their contents remain essentially the same.

The last remark concerns terminology. Being written over an extended period, the papers inevitably show discrepancies in terminology. Most of these will not cause confusion. One shift in terminology needs to be mentioned. In II and III, 'question' is used as 'interrogative' is used in the other papers, viz. to refer to linguistic objects. In IV, V and VI, 'question' refers to the specific semantic content we assign to interrogatives, in I it stands for the semantic interpretation of interrogatives in general.
I

PROBLEMS AND PROSPECTS
IN THE THEORY OF QUESTIONS
1. The importance of studying questions

Of course, the semanticist's first answer to the perennial question 'Why?', is the same as that of the mountaineer. Questions and answers exert a fascination that some simply find impossible to resist.

But it seems that, in this particular case, there are also more principled reasons to consider the study of questions and answers a topic of special importance. And this holds especially for those who are working in what has become known as 'formal', or 'logical', semantics.

The enterprise of formal semantics is to try to understand the meaning of language, and of what lies behind it, by studying it with exact means. In this strand of thinking, the applicability of logical and mathematical techniques, in a certain sense, constitutes a criterion of adequacy, a measure of success. To the extent that we do not succeed in building a formal model of some domain, we are considered not to understand, in a cognitive sense of the word, what is going on.

The application of notions and methods derived from logic, more in particular from model-theoretic semantics, raises some important, perhaps even crucial questions. Logic deals, or so it seems, with just one aspect of natural language. Perhaps it is the most important aspect, or maybe that is not even true. But this does not really matter. The point is that the scope of logic as a theory of language, has seemed to many to be restricted in principle.

The assumed restriction, is, of course, that to descriptive language, or, perhaps more broadly, assertive language. From a logical point of view, this restriction is a natural
and a sound one. After all, logic as a theory of inference has little place for all that does not play a role in formal or informal reasoning. Consequently, for many it seemed that from the logical perspective, language can be identified with description, that asserting is the only relevant function of language, and that meaning exists only in virtue of this function and can be explained solely in terms of it.

This position is advocated today especially by those who uphold that natural language meaning is sui generis, and that the ways and means of formal, logical semantics can never be fruitfully applied to (all of) it. The very existence of non-descriptive language, and questions are, of course, a prime example, is taken to show that logical semantics, restricted as it is assumed to be, in principle will fall short of providing an adequate theory of meaning for natural language.¹

In view of this, questions form an outstanding challenge to the formal semanticist. If he succeeds to give a descriptively and explanatory adequate account of the semantics of interrogative sentences, he will, perhaps, be able to shake off the odium of being a myopic formalist with no real feeling for the intricacies and endless varieties of natural language.

So, here we come up against the great importance that lies behind the study of questions for the formal semanticist. Few would deny that, studying the semantics of indicatives, he has developed useful notions and has gained important insights. Should he succeed to come up with an analysis of interrogatives in which these notions and insights are equally helpful and illuminating, this would lend support to the claim that he has succeeded to uncover some fundamentals of language in general. It would support the wider applicability, and hence the general importance, of what was developed with the eye to a smaller area. And it would give us another reason to remain faithful to our gut feeling that, pace Wittgenstein, systematic and explanatory theories about language in general can be developed.

Of course, we do not want to suggest that those who have
concerned themselves with questions and answers, have done so for the reason just indicated. Most, if not all, of them have been motivated mainly by their fascination with the subject as such. And this, to be sure, is as good a reason as any. However, such considerations as expressed above, may serve to emphasize the great external importance of the results obtained in the area.

Besides this external importance, and the evident inherent significance of the subject, there seems to be good reason to suppose that the study of questions and answers might occupy a central position in the field of formal semantics and pragmatics of natural language. Let us indicate, very briefly, some of the reasons for thinking this to be the case.

Having been restricted to the study of sentence semantics for a long time, recent developments in formal semantics have shown an increasing interest in more comprehensive units of language, such as discourses. Question-answer sequences form a basic type of discourse, one of which the structural properties seem to be reasonably well-defined, and therefore, one which seems to be a promising starting point.

From our point of view, the prime importance of question-answer sequences as a discourse type, lies in the fact that these interactions constitute a discourse which explicitly aims at information exchange. The importance of the notion of information, not only for pragmatics, but also for semantics, is acknowledged increasingly. Notions of (partial) information, and of information growth, have proved to be helpful, if not essential, for giving an adequate account of the semantics of various constructions and expressions in natural language. And, recently, some have even pleaded for an essentially informational perspective on meaning in natural language, as such.

As is to be expected, the notion of information, and that of information exchange, has played a prominent role in pragmatics from the very start. To give a simple example, those who take a pragmatic view on presuppositions, account for them in terms
of the opposition between 'old' and 'new' information, a distinction which is also considered to be relevant for the analysis of topic/comment, and the like. Also, the entire theory of conversational maxims, initiated by Grice, and developed into an essential part of a theory of natural language meaning by him and others, makes essential use of the notions of information and of information exchange.

Despite the central role these notions play, their exact content, and their precise analysis, still calls for further study. Especially, this holds for partialness of information, for information growth, and for 'embedded' information. It seems reasonable to expect that the study of questions and answers, which is intimately related to such notions, can contribute to a better understanding of them.

Let us conclude with pointing out a specific topic in pragmatics that, we feel, an adequate theory of questions and answers can contribute to significantly. A notoriously difficult, but quite essential maxim proposed by Grice, is the Maxim of Relation. Relevance, it seems, is essentially tied to what a conversation is about, to what the topic of a conversation is. And a topic of conversation may very well be thought of as a (set of) questions. This is obvious for discourses which consist of explicit question-answer sequences, but seems to hold also for types of conversation that are not explicitly concerned with information exchange. Even if in some discourse, no question is explicitly raised, it still plays an important role at the background, viz. as the topic that makes the discourse a coherent whole, rather than a random sequence of assertions. The topic, i.e. an explicitly or implicitly raised question, is what defines the relevance of the assertions in a discourse for each other.

One might indeed go one step further, and uphold that the notion of an assertion as such, is intelligible only given the complementary notion of a question. If we did not have any questions, we would not have any need for assertions either. The study of questions is important for the study of assertions, and vice versa. Neither one is fundamental in the sense
that the other is a derivative of it. Each can be understood only in the context of the other.
2. Some general constraints on a theory of questions and answers

Our purpose in this section, is to formulate some methodological constraints on a theory of questions and answers. These will be helpful in evaluating existing proposals, and as ordering principles in stating the major empirical issues. For the larger part, these constraints follow from, or are at least intimately related to, basic principles, or prejudices if you like, of the enterprise of logical semantics for natural language. It may therefore be useful to state some of these in a nutshell.

2.1. Framework principles

2.1.1. Compositionality, syntax and semantics

A fundamental principle, adhered to, implicitly or explicitly, by many who work in the formal semantics framework, is that of compositionality, or 'Frege's principle' as it is sometimes referred to. What it basically amounts to, is that it makes good sense to assume that meaning is a matter of composition, that the meaning of larger linguistic units is determined, in a systematic way, by the meanings of their parts. If this idea is to be made to work in an explicit theory, we need a syntax which tells us what the parts of a given linguistic expression are. In many respects, such a syntax may follow its own autonomous ways. But, if it is to serve our semantic purposes as well, it has to be designed in such a way that the syntactic operations can be matched by semantic ones, and that,
conversely, every semantic operation has a syntactic counterpart. As a consequence, every structural semantic ambiguity has to be the result of a corresponding derivational syntactic ambiguity.6

This means that compositionality imposes certain requirements on the content of a syntactic theory, i.e. that it contain a semantically motivated level of derivational structure, and that in this sense syntax is not autonomous. On the other hand, those parts of syntax for which an independent, purely syntactic, motivation can be given, should be respected by semantics. Assuming that, unlike derivational structure, constituent structure can and should be motivated on purely syntactic grounds, this means that semantic interpretation should respect constituent structure. In other words, syntactic units, constituents, should be considered semantic units as well. Adherence to such a principle seems reasonable enough. What it basically amounts to, is the belief that units of form are also units of content, that form and content are systematically related.7

Two remarks are in order. First of all, it should be stressed that principles of this kind are methodological principles, and not empirical hypotheses. They serve as guide-lines in developing and organizing a particular kind of grammar. Secondly, as far as compositionality is concerned, one need not believe that all of interest that can be said about meaning in natural language, can be said in a compositional semantic theory. Compositionality may have its limits. It may very well be that other principles are active as well. What is presupposed by those who adhere to compositionality, is that it leads to well-defined semantic theories that account for important, central aspects of natural language meaning and understanding.8

For example, with many other semanticists, we believe that an overall theory of meaning should encompass a pragmatic theory over and above a compositional semantic theory.9 Such a pragmatics may have principles of its own, such as the general principle of cooperation, on which the Gricean con-
versational maxims are founded. A Gricean theory starts from the assumption that a logical semantics provides an adequate basis for accounting for conventional aspects of meaning, and that other aspects of meaning can be explained in terms of the conversational principle that in using expressions, given their conventional meaning, language users behave in a cooperative way.  

2.1.2. Descriptive and explanatory adequacy

The principle of compositionality embodies a certain view on the structure of a semantic theory, but as such it does not tell us what kind of things meanings are, let alone what the meaning of some concrete linguistic expression is. Doing the latter, i.e. assigning a proper meaning to (categories of) expressions in some domain of investigation, is, of course, the first requirement a descriptive semantic theory should meet. We want it to be at least descriptively adequate. But it is a first requirement only. We are not satisfied with a semantic theory that operates as a black box, assigning meanings to expressions. We want the theory to do this in a certain way, we want it to be explanatory adequate as well.

To be sure, the notion of explanation, especially in semantics, is a notoriously difficult one. There seems to be no general agreement yet on what constitutes an explanation, and hence on what makes a theory explanatory adequate. Still, we are confident that what will be said here about requirements an explanatory adequate theory should meet, is acknowledged, be it only implicitly, by the majority of those who are working in formal semantics.

Logical semantics is first and foremost interested in structural aspects of meaning. Descriptive adequacy thus means that a theory should associate with (categories of) expressions, semantic objects of a proper type, and having such a structure that relations between semantic objects
are accounted for. To the extent that this is done in a systematic way, the theory gains explanatory power. This requirement of being systematic has at least two sides. First of all, compositionality presupposes a certain amount of system in the types of semantic objects that will be used. Secondly, and more importantly, it seems natural to require of a semantic theory that deals with a certain domain of phenomena, that it account for such phenomena as occur elsewhere too, by using general principles, notions and operations, which can be applied outside the particular domain of the theory as well.

Let us try to make this a little more concrete. An example of a semantic relation that can be found in every descriptive domain, is the relation of entailment. Whatever concrete phenomena some particular analysis deals with, the relation of entailment will be one of the most fundamental relations that the analysis will have to account for. Descriptive adequacy requires only that the analysis give a correct account of whatever entailments hold in its descriptive domain. But, explanatory adequacy is achieved if this account is based on a general notion of entailment, one that applies in other domains equally well. In fact, the semantic framework one uses brings along a general definition of entailment. For example, if the framework is based on set theory, entailment will basically be inclusion. Hence, whenever some analysis in this framework is to account for the fact that one expression entails another, it should do so by assigning them meanings in such a way that the meaning of the one is included in the meaning of the other.

Another example that illustrates this point, is provided by the operations of coordination. Coordination, too, is to be found in all kinds of categories. Hence, the explanatory adequacy of an analysis that deals with coordinations of expressions of some particular category, is greatly enhanced if the account it gives is based on general semantic operations associated with the coordination processes. Again, the semantic framework defines these operations. If the frame-
work is based on set theory, conjunction and disjunction of expressions in whatever category, will have to be interpreted as intersection and union, respectively.\textsuperscript{12}

Living up to these standards is, of course, not the only measure of explanatory adequacy. But, we feel, these requirements are really basic ones. They give us useful tools to compare theories with each other, and to evaluate them.

2.2. Domain principles

In what follows we will discuss three general constraints on a theory of questions and answers, which to a large extent are derivatives of general framework principles, such as discussed above, but which are specific for the particular empirical domain such theories range over. These constraints have been formulated by Belnap, and our discussion of them leans heavily on his work.\textsuperscript{13}

2.2.1. The equivalence thesis

A first constraint that Belnap formulates, he calls the 'equivalence thesis'. Observing that interrogative sentences ('direct questions') and wh-complements ('indirect questions'), by and large, come in pairs, he requires that the semantics of the two should be treated equivalently. Belnap views the relation between interrogatives and wh-complements as analogous to that between indicative sentences and sentential complements, i.e. as the relation between what he calls a 'stand-alone' form and an 'embedded' form. Treating the semantics of the two equivalently, does not necessarily mean making them equivalent, but assigning them meanings which can be related to each other in a systematic way.

Obviously, the equivalence thesis is related to the general framework principle of compositionality. At least in such languages as English, Dutch, German, and French, in
which wh-complements clearly appear as noun-phrase-like forms of interrogatives, compositionality requires that the meaning of the former is derived from the meaning of the latter. For such languages, compositionality implies the equivalence thesis.

The equivalence thesis not only serves to evaluate theories which analyze both interrogatives and the corresponding wh-complements, it also allows us to do so with theories which analyze only one of these constructions. For, of some theories which deal with interrogatives, or wh-complements, only, it can be seen beforehand that they cannot be extended to a theory which deals with both and, at the same time, complies with the equivalence thesis.

Further, it has some descriptive implications as well. Among other things, it predicts that interrogatives and wh-complements exhibit the same kind of ambiguities. In this sense, the equivalence thesis also helps to structure the domain of relevant phenomena.

2.2.2. The independent meaning thesis

The independent meaning thesis is related, on the one hand, to the equivalence thesis, and hence to compositionality, and, on the other hand, to the requirement that semantics should respect constituent structure. This thesis says that interrogatives and wh-complements should be assigned a meaning of their own.

The relation with the equivalence thesis is the following. The latter actually puts a ban on all so-called 'paraphrase' theories, i.e. theories which try to define the meaning of an interrogative by way of some indicative paraphrase. Such paraphrases always contain the corresponding wh-complement. Given the equivalence thesis, this cannot work. Hence, interrogatives should be assigned a meaning of their own.

Considerations concerning the relation between constituent structure and semantic interpretation, lead to the same
conclusion. Clearly, interrogatives form a natural syntactic unit. There seem to be no syntactic reasons whatsoever not to regard them as a separate syntactic category. So, interrogatives should be assigned a meaning directly, as they appear, without recourse to syntactically unmotivated levels of analysis.

The same holds for wh-complements. As various simple syntactic tests show, they form a separate constituent of the larger expressions in which they occur. They can be preposed, referred to anaphorically, coordinated, and so on. Consequently, wh-complements, too, should be assigned a meaning of their own in a direct way, a meaning which, moreover, should be derived from that of the corresponding interrogatives, in keeping with the equivalence thesis.

2.2.3. The answerhood thesis

A last, but important, constraint is Belnap's answerhood thesis. His formulation of it, reads as follows: "The semantic representation of a question, whether direct or indirect, should give us enough information so as to determine which propositions count as possible answers to it.".

Concerning Belnap's formulation, the following has to be noticed. Belnap describes a possible answer as follows: "An answer with neither too much not too little information". In his interpretation, what constitutes a possible answer is determined completely by the semantic content of the interrogative. For ordinary interrogatives, a unique answer is the result. Clearly, Belnap's notion of an answer does not coincide with the intuitive one. It seems natural to consider many things as possible, partial, complete answers to an interrogative. What Belnap calls an answer, is what we will call a standard semantic answer. If we interpret Belnap's thesis with this in mind, it seems a fair and natural requirement on an analysis of interrogatives. There is little to be gained by an account of questions that remains silent
about answers. An interesting analysis is one which assigns interrogatives a meaning from which the standard semantic answers can be obtained.

In our opinion, the requirement that the answerhood thesis makes is to be supplemented by another one, viz. that the notion of standard semantic answer that a theory characterizes, should be such that it forms a suitable basis for a theory of answerhood in general. There are many more kinds of answers than just the standard semantic ones, and all these are related to each other in systematic ways. The notion of standard semantic answer that a theory provides through the semantic object it assigns to interrogatives, should be such as to allow an account of this to be based upon it.

Belnap contrasts his interpretation of the answerhood thesis with the (hypothetical) position that what constitutes an answer cannot be characterized systematically, i.e. that no systematic theory about the question-answer relationship is possible. Like Belnap, we do not agree: the question-answer relationship is an important fact that needs to be accounted for. But we disagree as to the role the semantic interpretation of interrogatives can and should play in this. Whereas Belnap seems to think that the semantic analysis of interrogatives should say all there is to say about possible answerhood, we merely require it to play an essential role as part of an overall theory. For, we feel that there is far more systematics outside the realm of the purely semantical than, apparently, is dreamt of in Belnap's philosophy. His conception of the question-answer relationship fits those theories which assume that questions can be answered in some (one) ways, but not in all. Contrary to this, we would like to uphold that, in principle, any question can be answered in any way. Of course, not all propositions will answer all questions all of the time, but any proposition may answer any question some of the time. And it is the task of the theory of questions and answers to tell which propositions answer which questions when.

The answerhood thesis seems to be connected with the general
constraint that entailment be accounted for in a general way. This can be argued for as follows. Entailment is essentially inclusion of meaning. If we apply this view to interrogatives, it seems natural to consider one interrogative entailing another as every proposition giving an answer to the first also giving an answer to the second. And this squares with the answerhood thesis, which requires that the semantic interpretation of an interrogative determine what its standard semantic answers are.
3. Some empirical issues in the theory of questions and answers

In this section, we will give a brief sketch of several empirical issues, against the background of the general principles discussed above. Our main purpose in doing so, is to show in what way such theoretical considerations, implicitly or explicitly, guide us in focussing on some phenomena rather than on others. At the background these principles help to determine the relative importance of issues, their interrelations, and so on. Also, they indicate in which direction a proper analysis of the phenomena is to be looked for.

The issues raised here are the main subjects of the papers to follow, and also play an important role in the works of others in the formal semantics tradition on questions and answers, on which these papers build and by which they are inspired. This is not to say that these authors will always view these matters in the same way as we will present them. But, by and large, they are concerned with the same topics.

Two caveats should be added. First of all, the phenomena we will discuss are those which are relevant from the point of view of a formal semantics, and, to some extent, a formal pragmatics of questions and answers. Outside this field, there are certainly lots of interesting and important phenomena pertaining to questions and answers as well. And the ultimate theory should deal with these too. However, throughout we will just be concerned with questions of formal semantics, and will restrict ourselves to the kind of answers that are given in this framework.

Secondly, empirical issues are only mentioned in this
section, they are not discussed in detail. For such discussions, the reader should turn to the papers to follow, and to the literature that is referred to there. One exception to this rule is the discussion of interrogatives and presuppositions in section 3.3. Since hardly anything is said about this topic in the other papers, we discuss it in some detail here.

3.1. The semantics of interrogatives and wh-complements

In view of the independent meaning thesis, a central task for a semantic theory of interrogatives and wh-complements, is to decide upon the kind of semantic object that is an adequate formal representation of the meaning of such expressions.

Generally, two aspects of this problem can be distinguished. First of all, it should be decided of what type, or types, these objects should be. Such decisions are made within the context of a specific semantic framework which determines a range of available types. Secondly, given some type, or types, of objects that are suitable representations of meanings, a further problem is to determine which particular objects within that type qualify. One has to find out which specific properties these objects are to have.

The usual heuristics is to consider structural semantical relations. For these, in general, give important clues concerning the type of semantic object one is after. The structural relations one may take into consideration, may either be relations between expressions of the kind that is being studied, or they may be relations between such expressions and others. Especially, if the semantic type of these other expressions is (supposed to be) known, this provides valuable information.

Important structural semantic relationships concern e.g. entailment, coordination and functional application. In the light of the framework principle that throughout all categ-
ories, these should be dealt with in a uniform way, a way that is determined by the framework in which the analysis takes place, the existence and non-existence of these relations gives direct indications of the type of semantic object that is involved.

In the analysis of interrogatives and wh-complements, it seems attractive to start looking at relationships which involve indicative sentences, of which the semantic properties are most familiar. More concretely, the existence of systematic entailment relations involving indicative sentences with wh-complements, and sentences with sentential complements, gives important clues concerning the type of semantic object that is to be associated with wh-complements, and hence, in view of the equivalence thesis, with interrogatives.

Such entailment relations can be taken as a starting point. Two simple examples are the following valid arguments:

(1) John knows whether Mary walks in the garden
Mary doesn't walk in the garden
________________________________________________________________________
John knows that Mary doesn't walk in the garden

(2) John knows who walks in the garden
Mary walks in the garden
________________________________________________________________________
John knows that Mary walks in the garden

The existence of entailments such as these indicate that there is an intimate relation between the type of semantic object that is associated with sentential complements and that of wh-complements.

This point is underscored by the observation that the two types of complements can occur in coordinate structures, as e.g. in (3):

(3) John knows that Peter left for Paris, and also whether Mary went with him, and when he will be back
A consideration having to do with functional application, and hence with compositionality, makes the same point. As (1) and (2) show, both sentential complements and wh-complements can occur as argument of the same function, the verb know. Of course, this does not hold in general, as is shown by the existence of verbs such as inquire, which take only wh-complements, and verbs such as believe, which only take sentential ones. Though this is primarily a matter of lexical semantics, it also indicates that the semantic objects associated with sentential complements and wh-complements have different properties.

So, structural semantic relations suggest a close association between the type of semantic object that corresponds to wh-complements, and given the equivalence thesis, to that of interrogatives, and the type of semantic object that corresponds to sentential complements.

Such an association squares with the answerhood thesis. For it tells us that the semantic interpretation of an interrogative should characterize a notion of semantic answerhood. As such, it also points into the direction of the existence of a relation between the semantic interpretation of interrogatives, and that of indicative sentences. The semantic content of an answer, the information it gives, is the semantic content of an indicative sentence, i.e. a proposition, or whatever is the equivalent of that in the semantic framework that is used.

The examples given above, also show that structural semantic relations may give certain indications concerning specific properties of semantic objects that are to serve as interpretation of interrogatives and wh-complements. For example, compare (1) with (4):

(4) John knows whether Mary walks in the garden
Mary walks in the garden
John knows that Mary walks in the garden
The contrast between (1) and (4) shows that situation-dependency is an important property of wh-complements. Depending on what is actually the case in a given situation, the wh-complement entails a different that-complement. Again, this squares with the answerhood thesis, since what constitutes a true answer to an interrogative will depend on the situation as well.

Other hints concerning specific properties of interrogatives and complements are given by relations of interrogatives to one another. Consider (5):

(5) Who walks?  
   ---------------  
   Does John walk?

The first interrogative in (5) entails the second. Given the answerhood thesis, entailment of interrogatives can be described in terms of answerhood. One interrogative entails another if every complete answer to the first, also gives a complete answer to the second. So, in view of the validity of such examples as (5), a complete answer to a who-interrogative, must give us an answer to every corresponding yes/no-interrogative. This means that a complete answer to such an interrogative must give an exhaustive specification of the individuals that have the property the extension of which the interrogative asks for. In other words, interrogatives are requests for such exhaustive specifications.

Again, the analogous phenomenon can be observed with wh-complements. (6) is a valid argument:

(6) John believes that only Bill walks in the garden  
    Bill and Mary walk in the garden  
    ---------------------------------  
    John doesn't know who walk in the garden

An indication of the exact extent to which the specification that an interrogative asks for, should be exhaustive, is given by the fact that, unlike (5), (7) is not valid:
(7) Which men walk in the garden?
    Which men do not walk in the garden?

Neither one of the interrogatives entails the other, for a complete answer to the one gives a complete answer to the other, only for someone who knows who the men are. So, what is a valid argument, is (8):

(8) Which men walk in the garden?
    Who are the men?
    Which men do not walk in the garden?

And again, there is an analogue in terms of complements. Consider (9):

(9) John knows which men walk in the garden
    John knows which men do not walk in the garden

This argument is not valid, and becomes so only if we add the following premis:

(10) John knows who the men are

These examples indicate another important property of semantic objects to be associated with interrogatives and wh-complements. They show that to know the answer to a certain question, may involve a certain amount of de dicto knowledge. In order to know which men walk in the garden, one needs to know of every man that walks in the garden, that it is a man and that he walks in the garden. An exhaustive specification of this de dicto nature, is what an answer should express, and hence what an interrogative asks for. The few examples illustrate how observations concerning structural semantic relations, most prominent among them being the entailment relation, can guide us in our attempts to formulate a proper semantic analysis of interrogatives.
and wh-complements. They give strong indications concerning the type of semantic object that will be an adequate representation of the meaning of these expressions, suggesting that this type is of a propositional nature. Also, they indicate that there is a uniform semantic type for all interrogatives and complements. Further, such observations as made above, also give us valuable clues concerning the more specific properties of the relevant semantic objects. Prominent among these, we consider to be the situation dependency of interrogatives and wh-complements, and their de dicto and exhaustive nature.

And this is what makes these issues into important empirical issues, that any semantic theory should account for. Precisely because these phenomena tell us what type of object to look for, and what specific properties it should have, they are of central importance. It should be borne in mind that it are the general framework principles that tell us, beforehand, what kind of phenomena we should direct our attention to. In this sense, their importance should not be underestimated.

It is again a framework principle, viz. that of compositionality, that suggests that it is important to look out for ambiguities. Coming up with the right semantic object, is only one half of what a proper semantic theory should do. The other half is to show how the proper objects can be associated with expressions in a systematic fashion. In the formal semantics framework, adherence to compositionality means that one should show how the right semantic interpretation can be derived compositionally from the interpretations of the parts. Then, ambiguities become an important phenomenon. For, every structural, i.e. non-lexical, ambiguity, is to correspond to a different derivational structure. And that means that ambiguities can give good indications as to how expressions are to be derived, and how meanings are to be composed.

For this reason, discussions of ambiguities, and how to account for them, are a prominent subject in many papers
in formal semantics of natural language, and papers on the semantics of questions, including the ones to follow, are no exception to this rule.

Of course, the ambiguities that count, are those that are specific for interrogatives and wh-complements, i.e. those that do not occur also in analogous indicative constructions. A simple example is provided by the following sentence:

(11) Which student did every professor recommend?

This interrogative is threefold ambiguous. As is generally the case with interrogatives, the ambiguity shows in the different ways in which (11) can be answered:

(12) John.
(13) Professor Jones, John; professor Williams, William; professor Peters, Peter ...
(14) His best one.

The difference between the first two readings, evidently is one of scope. Which reading results, which type of answer is called for, depends on the relative scope of the wh-phrase and the term.

That the third reading is really a distinct one, and cannot be identified with an arrangement of scopes, is shown by the fact that (15) can be answered by (16), and not by (17):

(15) Which student did no professor recommend?
(15) *Professor Jones, John; professor Williams, William; professor Peters, Peter ...
(16) His worst one.

Two other examples of ambiguous interrogatives, which by being ambiguous tell us a lot about how interrogatives should be derived and what their proper semantic interpretation is, are (17) and (18):
(17) What did two of John's friends give him for Christmas?
(18) Where do they have all books written by Nooteboom in stock?

The first, perhaps less likely, reading of (17) inquires after the nature of some present that John got twice. The more likely reading is the one which asks to specify for two of John's friends what each of them gave John for Christmas. Notice, that on this reading, the interrogative leaves the addressee a choice. She may pick any two friends of John's, and answer for each of them the question what he or she gave him. The particular importance of this type of reading, is that it shows that interrogatives may have more than one complete semantic answer.

Interrogative (18) illustrates a similar point. Depending on the context, it may be given an interpretation on which it asks for an exhaustive listing of all decent bookshops, or it may be taken to ask to mention some bookshop where I can buy Nooteboom's oeuvre.

These few examples may serve to show that an account of ambiguities is an important empirical issue in the theory of questions, not because they are always that interesting per se, but because they reveal important properties of the semantic objects to be associated with interrogatives, and of the way in which these are to be composed.

3.2. Questions and answers

The phenomena indicated in the previous section all concern the semantic interpretation of interrogatives and wh-complements as distinct kinds of linguistic expressions. As such, they are, of course, of central importance, but clearly, they do not constitute the whole story. Interrogatives express questions, and where there are questions, there are, fortunately, also answers. And a satisfactory theory of interrogatives will have to deal with them as well.
This is what the answerhood thesis says. This principle states that the semantic interpretation of interrogatives should tell us what the answers are that can be given to the questions expressed by these interrogatives. In our discussion of the answerhood thesis in section 2.2.3, we expressed our opinion that it should be taken in a broad sense. It is not sufficient that the semantic object associated with an interrogative determines some notion of answer, it should be a notion on which a systematic theory of answerhood can be founded.

This opinion is based on our conviction that, although it is possible and meaningful to study the semantics of interrogatives in isolation, the ultimate test is whether the results that are obtained that way, can be extended into a wider theory, one that takes into account the ways and purposes for which interrogatives are used. If one takes a closer look at that, one sees that pragmatics is involved in an essential way. Questions signal gaps in one's information, and are used to get these gaps filled. And answers are attempts to fill in such gaps. The relationship between questions and answers cannot be viewed properly without taking this informational perspective into account.

If one considers in some more detail various phenomena concerning the relations between questions and answers, one observes on the one hand a great variety, and on the other hand a clear system. In this, the notion of available information plays an essential role. Hence, a purely semantically defined notion of answerhood, whatever it covers, cannot be adequate. Either it is too restricted, excluding all kinds of normal cases, or it will be too liberal, accounting for the variety, but not for the systematic relationships. For that reason, we do not interpret the answerhood thesis as a requirement to construe the semantic interpretation of interrogatives in such a way that it tells us all about answers. This it will never be able to do. Our interpretation is that the semantics should give us a good fundament to base a pragmatics on.
As a simple example of the variety of answers that can be given, consider the following:

(19) Whom did John kiss at the party last night?
(20) Mary.
(21) The girl from next door.
(22) A redhead.

The three answers (20), (21), and (22) have clearly different semantic characteristics, yet they may all serve as answers to the same interrogative (19).

The first answer, (20), in a certain sense is a model one. It indicates who the person that John kissed last night was by giving a name, i.e. by using a rigid identification, one that is tied uniquely to one and only one person. It is a standard answer, one that is supposed to work in all cases for all questioners.

The second answer is typically not of that chosen semantic kind. Descriptions are not uniquely tied to one and the same referent all of the time, as names are. Yet, it is easy to think of a situation in which it is a good, complete answer to (19). And it is also easy to see what aspect of that situation is responsible for that: available information. If I know who the girl from next door is, (21) answers my question completely. But if I don't, it doesn't.

These are two simple, but important facts that a theory of questions and answers should account for. First of all, there exists a kind of answer that is standard, that uses designations that are semantically rigid, and that hence does not depend on available information, or at least is not supposed to depend on that. Secondly, non-standard answers may be as good as standard ones, given a suitable information structure. So, there are at least two major classes of answers, semantic ones and pragmatic ones.

Another opposition within the totality of answers is illustrated by the third answer to (19), (22). This answer differs from the former two in that it is indefinite. Whereas (20)
and (21) each in their own way are definite identifications of one individual, this does not hold for (22). Without any specific assumptions about available information, (22) will not be a complete answer to (19), but only a partial one. It gives some information, e.g. that John didn't kiss Suzy, who is a brunette, but it does not identify the one that John kissed. Unless of course some, in this case rather specific, information is available, such as that only one redhead attended the party, viz. Jane. In that case (22) is a complete answer.

This simple example illustrates another major opposition in the totality of answers, that between partial answers and complete answers. It also illustrates that what answer results, depends in general on two factors: the semantic characteristics of the linguistic expressions involved, and the information that is available.

A general theory of answerhood hence has to build on two notions: semantic interpretation on the one hand, and information of speech participants on the other. The role of the semantic interpretation of the interrogative in this then seems to be to characterize the information-independent notion of a standard semantic answer. Starting from that, the theory will develop other notions of answerhood such as hinted at above, give an account of their systematic interrelations, and show how semantic characteristics of linguistic expressions are related to various notions of answerhood.

Answers form an important empirical issue also in another way. The relationship between interrogatives and linguistic answers has some particular problems to offer, the solution of which in its turn bears on the syntactic and semantic derivation of both.

The first phenomenon that a theory of interrogatives and linguistic answers should come to grips with, is that linguistic answers typically come in two varieties. They may have the form of a constituent, or they may consist of a full sentence. There has been some debate in the literature
about the relation between the two. Some hold that constituent answers are primary, others that sentential ones are, and others again do not care. To us, the relevant empirical issue seems to be that both exist, and are systematically related. The most striking aspects of the relation between interrogatives and linguistic answers, apply to both varieties.

The most important phenomenon to be observed, is that the interpretation of a linguistic answer depends on the context of an interrogative. Consider the following examples:

(23) Who walk in the garden?
(24) Which men walk in the garden?
(25) John and Bill.
(26) John and Bill walk in the garden.

The interpretation of both the constituent answer (25) and the sentential answer (26) depends on the context of the interrogative. As answers to (23), (25) and (26) convey that John and Bill are the ones that walk in the garden. As answers to (24), they express that John and Bill are the men that walk in the garden. As answers to (23), they would not be true and complete if Mary walks there too, as answers to (24) this would not affect their being true and complete.

This has two consequences. It indicates that the derivation and interpretation of linguistic answers needs the syntactic and semantic structure of the interrogative. And it tells us something about the semantic analysis of interrogatives as well: at some level, it should contain a syntactic and semantic unit that can be used in the syntactic and semantic derivation of linguistic answers.

This concludes our discussion of the second area in the empirical domain, centered around the question-answer relation. By the answerhood thesis, it is firmly linked to the first area, concerning the semantics of interrogatives and wh-complements. In characterizing a notion of a standard semantic answer, semantics provides the basis for an overall theory of answerhood, which has to take into account the pragmatic function of question-answering.
3.3. **Interrogatives and presuppositions**

Several proposals for the semantic analysis of interrogatives take presuppositional phenomena to be an integral part of their empirical domain. Others have argued that one need not do so. In our own proposal, as it is developed in the papers to follow, the phenomenon of presuppositions is largely ignored. Not because we think it to have no significance at all, but because we believe it to lie outside the realm of semantics proper. The present section is meant to provide some arguments for this position.

In discussing interrogatives and presuppositions, we are not concerned with presuppositional phenomena that interrogatives share with indicative expressions. Consider e.g. (27) and (28):

(27) When did John stop smoking?

(28) John stopped smoking

The interrogative (27) and the indicative sentence (28) share the presupposition that John has smoked. It may safely be assumed that any correct analysis of this presupposition of (28), can be made to work for (27) as well.

What we are interested in here, is whether there are presuppositional phenomena which are specific for the use of certain wh-terms, or for certain interrogative constructions, and if so, what their nature is. Two relevant examples are (29) and (30):

(29) To whom is John married?

(30) Do you want coffee or do you want tea?

It is often assumed that the interrogative (29), c.q. the one who uses it, presupposes that John is married to someone.
This existential presupposition is then associated with the lexical meaning of the wh-term who. The interrogative (30) is sometimes associated with two presuppositions: that the addressee wants coffee or tea, and that he does not want both. The alternative interrogative construction is taken to presuppose that exactly one of the alternatives will prove to be the case. So, besides an 'existential' presupposition, a uniqueness presupposition is observed.

Singular forms of wh-terms, such as who or which book, are also often assumed to carry a uniqueness presupposition, besides an existential one. So, (31) would presuppose that only one person, and (32) that only one book, is involved:

(31) Who has made this mess?
(32) Which book did you bring back to the library?

Uniqueness is considered to be more strongly involved with wh-terms containing the wh-determiner which, than with such wh-terms as who. This even in case the latter occurs as the subject of a verb in the singular form, as is the case in (31).

The controversy about the nature of presuppositional phenomena, and hence about the proper way to account for them, has not yet been settled. The various positions that have been taken in the past, all still have defenders today. This is not to say that no progress has been made. The strongpoints and weaknesses of the different approaches are much clearer than they were in the past, more empirical material is brought under attention, and the various proposals have been worked out more explicitly.21

This is more true for presuppositions of indicatives, then for those of interrogatives. But, in case of the latter, the two main views on the nature of presuppositions, the semantic and the pragmatic view, have their proponents too.

From the semantic point of view, presupposition failure in case of an interrogative, results in its failing to have a (true or false) answer. In case an interrogative has a
certain presupposition, and is used in a situation in which this presupposition is false, the interrogative cannot be answered, but has to be rejected. An appropriate response to the question in such a situation, would not be an answer, but a mere reply. This corresponds to an indicative sentence lacking a truth value, in case one of its presuppositions is not fulfilled. And this parallel is a rather direct consequence of the answerhood thesis, which tells us that where semantics states truth conditions for indicatives, it states answerhood conditions for interrogatives. A semantic analysis of presuppositions, characterizes them as a kind of preconditions in both cases.

On the pragmatic view, presuppositions of interrogatives are reflections of certain expectations the questioner has about the answer. On this view, failure of presupposition does not imply failure of answerability, it just means that the answer will contravene expectations on part of the questioner.

Perhaps it is useful to point out that one need not choose between these two views, in the sense that one has to regard all presuppositional phenomena to belong to one and the same class. It is not a priori impossible that some presuppositions are semantical, and others are pragmatical. What would distinguish between the two in case of an interrogative, would be that failure of the former would result in unanswerability, whereas failure of the latter would not.

A main problem is, that this distinction presupposes a clear observational difference between answers and mere replies. Though there certainly are cases on which there is general agreement, the notions of answer and reply are too theory dependent for a systematic classification of presuppositional phenomena to be based upon them. As the literature shows, presuppositions of interrogatives, as such, seem to belong to the large class of phenomena, the status of which is debatable. We, for our part, tend to believe that only those presuppositions which interrogatives share with indicatives, constitute clear cases in which failure results in
unanswerability. A typical example is the presupposition of the interrogative (27), discussed above. It seems that (33) can be characterized indisputably as a rejection of the question:

(33) John never smoked in the first place

The other cases of presuppositions, those which are connected with the use of certain wh-terms and certain interrogative constructions, are far less clear. The existential and uniqueness presuppositions can often, at least partly, be related to the meaning of other components of the interrogative, or to certain aspects of the context. Many examples of interrogatives that do carry a presupposition can be contrasted with similar ones that do not. Consider the following three interrogatives:

(34) Who is that?
(35) To whom is John married?
(36) Who is coming with me?

Clearly, (34) has an existential presupposition, in particular if that is used demonstratively. But it seems that in this case, the presupposition is triggered by the use of the demonstrative, rather than by the wh-term. For, consider (35). In this case, it is not clear why the answer To nobody, could not be regarded as a satisfactory answer to the question, rather than as a mere reply that rejects the question. This is even more clear in case of (36), in which Nobody seems perfectly allright as an ordinary answer. The existential presupposition: , as an expression of the expectation of the questioner, is stronger in case of (35) than in case of (36).

As for the uniqueness presuppositions, it appears that they too, should be regarded as a suggestion, an expectation, on part of the speaker. Their occurrence cannot be tied to specific aspects of the grammatical form of an interrogative,
viz. to it having a singular, c.q. a plural form. Other factors, grammatical and non-grammatical, seem to be involved. The following pair of examples illustrates this:

(37)(a) Who is in favour of the proposal?
(b) Who are in favour of the proposal?

In our opinion, (37)(a) and (b) are both neutral with respect to uniqueness: neither one carries a suggestion to the effect that there is only one, c.q. that there is more than person in favour of the proposal. This holds most clearly in a situation in which (37)(a) or (b) is used by a chairman, as part of a voting procedure. Notice, by the way, that in this case the existential presupposition is absent as well. Since chairmen are supposed not to give expression to their personal expectations in conducting formal procedures, and since both interrogatives seem to be quite appropriate phrases to be used by them in performing such procedures, the conclusion seems warranted that these interrogatives do not carry a (non-) uniqueness or existential presupposition. For, if they would, it would be inappropriate for the chairman to use them. One could say that it is the context of a person acting in such an official capacity, that cancels such suggestions, if any there are.

As can be observed by comparing (37)(a) and (b) with the pair (38)(a) and (b), the facts are slightly different for interrogatives with such wh-terms as which member(s):

(38)(a) Which member is in favour of the proposal?
(b) Which members are in favour of the proposal?

It seems that, whereas the plural form of (38)(b) is neutral with respect to (non-) uniqueness, the singular form (38)(a) does carry a uniqueness suggestion. This is reflected by the observation that a chairman will tend to use (38)(b) in a voting procedure, and not (38)(a).

That in these cases as well, non-grammatical, contextual
factors play a role, becomes clear if one compares the following two pairs of interrogatives:

(39)(a) Which member of the cabinet voted against the proposal?
(b) Which members of the cabinet voted against the proposal?

(40)(a) Which member of the cabinet leaked the information to the press?
(b) Which members of the cabinet leaked the information to the press?

It seems that, whereas of (39)(a) and (b), the plural form (b) is the neutral one, in that it carries no suggestion as to the actual number of people involved, the reverse holds for (40)(a) and (b). Of the latter two, the singular form (a) seems to be neutral, and the plural form (b) marked.

Perhaps, this can be explained along the following lines. In some sense, the 'normal' situation that calls for voting, is one that involves two 'pluralities': those who are in favour, and those who are against. Only one person holding a position that is opposed to that of all the others is a marked case, though certainly not excluded. This suggests that if the number of people who voted in a certain way is not known, the question as to their identity (or after their number, as in 'How many ...?'), should be phrased in the plural form. Only if it is (supposed to be) known that only one such person is involved, the singular form is appropriate.

On the other hand, leaking a certain piece of information, typically seems to be an individual activity, though certainly, several people could be involved in it as well. Therefore, it seems that the 'normal', the neutral and unmarked situation, calls for the singular form. The plural form seems to be appropriate only if it is suspected that more than one person is involved.

These considerations once more seem to warrant the conclu-
ion that there is no clear grammatical relation between singular and plural forms of wh-phrases on the one hand, and existential and uniqueness presuppositions on the other. Rather, it seems that these presuppositions arise from the interplay of the way in which properties of certain types of activities are conceptualized, and certain expectations about the actual situation.

The discussion of these examples also makes clear that in all these cases, the relevant presuppositions are 'speaker presuppositions': they concern certain expectations that the questioner has. If such expectations fail to come out true, the result is not that the question cannot be answered, that is has no (true) answer. The one who responds to the question in such a situation, does not reject the question, but answers to it. Though he may explicitly indicate, that his answer goes against the expectations of the questioner. In this respect, there is a fundamental difference between the responses (33) to (27), and (42) to (41):

(27) When did John stop smoking?
(33) John never smoked in the first place
(41) Which member of the cabinet voted against the proposal?
(42) (actually there were two,) Brinkman and de Ruyter

Clearly, (33) is a rejection of the question posed by (27), it cannot be continued with 'last month' consistently. On the other hand, (42), with or without the qualification, does present an answer to (41). That the 'presuppositions' of (27) and (41) have a different status, can also be seen from the fact that whereas (33) cannot be continued in a way that would count as an answer, the qualification in (42), directed against the uniqueness expectation expressed by (41), has to be continued, either in a way that answers the question, or by saying that one is unable to provide an answer ('Actually, there were two, but I don't know which ones').
From this discussion, it is safe to conclude that presuppositions particular to interrogatives, are an interesting phenomenon, revealing dazzling subtleties of language and its use. We also hope to have shown that, despite their intrinsic interest, presuppositions in this area are, by and large, a non-grammatical matter, and that one is justified in ignoring them in a semantic analysis for the time being. However, it goes without saying, that in the end, they deserve proper attention of their own.

3.4. Conclusion

It was our aim in this section, to sketch some elements in the empirical domain of the theory of questions and answers. We explicitly did so from the perspective of formal semantics. An empirical domain of a certain theory is not something that is just there, but its contents and structure are at least partially determined by one's theoretical framework.

We tried to motivate a particular choice from the chaotic totality of potentially relevant phenomena, by linking them to principles underlying logical semantics in general, and the semantic analysis of interrogatives and answers in particular. In doing so, we hope to have shown that it is not a matter of pure accidence that these are empirical issues that most studies in the semantics of questions and the semantics and pragmatics of answers, carried out within the tradition of logical grammar, are directed towards.
4. Three approaches to the theory of questions and answers

4.1. A general characterization

Now that we have sketched the contours of the empirical domain of the theory of interrogatives and the question-answer relation, and have formulated a few general theoretical constraints which such a theory should meet, we will turn to a short discussion of the three main approaches that can be distinguished in this field. As we do throughout, we thereby restrict ourselves to those theories and analyses which are developed within the wider framework of formal semantics. This restriction is met by quite a number of interesting descriptive and theoretical studies, more than can actually be discussed in any detail in this context. But fortunately, not all theories constitute radically different approaches to the syntactic, semantic and pragmatic analysis of interrogatives and of the question-answer relation. It seems that we can distinguish, overall, three main approaches, three main views on what the basic characteristics of interrogatives and answers in natural language are.

Rather than discussing any particular details of any particular theory, we will give a general characterization of these three approaches, i.e. of what particular theories within one approach have in common. It will turn out that each of these three approaches, explicitly or implicitly, concentrates on a specific part of the domain of empirical issues which we outlined in the previous section. And, as is to be expected, in the area that it treats lie its strongpoints, and often what it does not deal with contains its weaknesses. The general constraints which we discussed in section 2, in
effect connect various subfields of the empirical domain, as we saw above. They allow us to extrapolate beyond the boundaries of what a theory explicitly treats, and thus give a view of what a theory would say about phenomena it does not deal with explicitly. So, together, the empirical domain and the theoretical constraints will be of much help to us in getting a clear picture of what are the merits and what are the flaws of the three main views on interrogatives and answers that we will discuss.

A note of warning must be issued at this point. The discussion of empirical issues presented above was not entirely free of theoretical and other biases, such discussions never are, and never can be. So any conclusions that will be reached on the basis of them will be biased to a certain extent as well. This certainly holds for what we take for granted right from the start, viz. that the ways and means of formal semantics, and those of formal pragmatics for that matter too, can be and should be extended from their homeground to larger domains. Anyone who for philosophical or other reasons does not agree, will not agree with our discussion of the problems and prospects of such theories either.

The three main approaches to the theory of interrogatives and answers are often referred to as the categorial approach, the propositional approach and the imperative-epistemic approach. Although, as their names indicate, these three approaches start from distinct underlying principles, these starting points are seldom discussed, and even more seldom argued for explicitly. Apparently, the excitement lies in developing and applying a certain view, in using it in description and explanation of empirical phenomena, and not in discussing its merits out of the blue. Yet, some remarks can be found that indicate a line of reasoning, and some rational reconstruction of motives is possible as well.

On the categorial view, the main semantic property of an interrogative is that it is in some sense an incomplete object, something that needs to be augmented, that something else needs to be added to. This 'something else' is, of course, an answer.
Different types of interrogatives, it is observed, call for different types of answers. And this means, so it is assumed, that different types of questions belong to different syntactic categories, and hence stand for semantic objects of different types as well. The support adduced for this point of view is mainly empirical, and not theoretical. Observations are made, in this case primarily concerning the syntactic status of the linguistic expressions involved, and from these the conclusion is drawn.

On the propositional view the main point is that interrogatives and answers are to be analyzed in terms of propositions. This idea can be developed in various ways. The main implicit or explicit motivation for the propositional view seems to be twofold. First of all, it is observed, and this is really rather uncontroversial, that answers to interrogatives convey information, and that interrogatives may be used to express requests for information. This leads naturally to the notion of a proposition, the formal semanticist's main tool for dealing with the informational content of linguistic expressions. So one rather obvious reason for upholding the propositional view has to do with the content of interrogatives and answers. Another type of motivation for analyzing all interrogatives in terms of propositions that can be found in the literature, is of a formal rather than of a material nature. It has to do with the overall simplicity of the resulting semantic theory. Observations concerning embedding, coordination, and the like, are taken to show that, despite surface syntactical differences, interrogatives do form a uniform class. Assigning them to the same syntactic category and the same semantic type, to be defined in terms of the notion of a proposition, is assumed to lead to a simplified analysis.

Proponents of the imperative-epistemic view on interrogatives and the question-answer relation concentrate on yet another aspect, viz. the way in which interrogatives function, the purpose for which they are used. It is observed that, at least under normal circumstances, the utterance of an interrogative is meant as a request for information, as an exhortation
of the addressee to bring about a certain epistemic state in the one who asks the question. Hence, it is concluded, interrogatives ought to be analyzed as such imperatives. The semantic interpretation of interrogatives can be stated in terms of such imperative-epistemic paraphrases. It can be noticed that in this case too, the starting point of the entire approach is argued for not so much on theoretical grounds, but on the basis of empirical observations. Here a correct observation concerning the way in which interrogatives (normally) are used, is exalted to a principle on which the semantic content of interrogatives should be based.

From these rough characterizations it will already be clear that in a certain sense all three approaches can be said to deal with the analysis of interrogatives from the perspective of the question-answer relationship. But each seems to focus on a different aspect of it. For categorial theories the relation between interrogatives and answers as linguistic, syntactic expressions is of central importance. Propositional theories, on the other hand, argue more from the semantic content of answers. And in the imperative-epistemic approach the pragmatic viewpoint dominates.

So, theories within the different approaches not only have different starting points, they also tend to deal with different sets of phenomena, with different parts of the empirical domain. This will become even more clear in what follows, where we will take a closer look at the three approaches, and will confront them with some of the phenomena and constraints discussed earlier.

4.2. The categorial approach

Under the general heading 'categorial', various theories may be grouped together which, despite obvious differences in details of implementation and even some differences in their respective aims, share a particular, distinct view on how interrogatives and answers should be analyzed. The main
proponents of this kind of theory are Hausser, Tichy and Scha, and their proposals are the ones that we will mainly draw upon in our characterization.27

Common to categorial theories, as the phrase 'categorial' indicates, is the view that one should pay due attention to the categories of interrogatives and their answers. Straightforwardly opposing propositional theories, which aim at a uniform analysis, the proponents of categorial theories uphold that no uniform syntactic category of interrogatives, nor of answers, exists. Rather, they claim, a satisfactory account of interrogatives and answers requires that we respect their categorial diversity. For it is through relationships between their respective categories that relations between different kinds of interrogatives and their answers can be accounted for. In categorial theories, interrogatives and answers are first and foremost studied as linguistic objects, as specific kinds of syntactic and semantic constructions one finds in the language. They therefore tend to focus, at least at the outset, on structural, often surface syntactical, properties of interrogatives and answers. Investigation of these properties then leads to the idea that relations between interrogatives and answers are to be accounted for in terms of categorial links that hold between them.

On the basis of such observations regarding structural properties, all categorial theories subscribe to some version of the following general principle:

(C) The syntactic category and the semantic type of an interrogative are determined by the category and type of its characteristic constituent answers

The various argumentations one can find in the literature in support of (C) all have in common that they exploit the differences that exist between two kinds of characteristic linguistic answers, viz. constituent answers and sentential answers.

Consider the following examples:
(1) Whom did John kiss?
(2) What happened in the kitchen last night?
(3) Mary.
(4) John kissed Mary.

There is a clear difference between the constituent answer (3) and the sentential answer (4). E.g. (3) can be used to answer (1), but it cannot be used to answer (2). Sentence (4) on the other hand can be used as an answer both to (1) and to (2). Evidently, constituent answers are closely tied to certain types of interrogatives, whereas the tie between sentential answers and interrogatives seems much looser.

It is remarkable that though this observation is made by several authors, they do not draw the same conclusions from it. On the contrary, Hauser, for example, claims that sentential answers, which he calls 'redundant' answers, are not interesting for a theory of interrogatives and answers since unlike constituent answers of which the interpretation depends essentially on the context provided by the interrogative, they have an interpretation of their own. Schä, on the other hand, bases his preference for constituent answers precisely on the fact that sentential answers do need the context of an interrogative to be assigned their correct interpretation. He observes that (4) as an answer to (1) means something different from what it means in isolation, or from what it means as an answer to (2), viz. (5) and (6) respectively:

(5) Mary is the one whom John kissed.
(6) What happened in the kitchen yesterday is that John kissed Mary.

In fact, it seems that Schä is right. Especially if one takes the phenomenon of exhaustiveness into account, it is quite obvious that the interpretation of a sentential answer depends as much on the context provided by the interrogative as constituent answers do.

Tichy argues against what he calls the 'full-statement
theory of answerhood' on somewhat similar grounds. But his conclusions are more radical. Observing that on the full-statement theory (7) answers both (8) and (9), he concludes that the theory is simply false:

(7) Jimmy Carter is the president of the U.S.
(8) Who is the president of the U.S.?
(9) What is Jimmy Carter the president of?

For, he says: "It would plainly be absurd to say that (8) and (9) have the same right answer".\(^\text{31}\) This is certainly true, but rather misses the point. The only thing such examples show against a propositional theory is that, in assigning an interpretation to sentential answers, it must take into account the context of an interrogative.

Although the reasons for doing so are not always the same, all proponents of categorial theories focus on the relationship between interrogatives and constituent answers. The existence and non-existence of a categorial match between interrogatives and constituent answers, is taken to determine the syntactic category and the semantic type of interrogatives. The categorial definition of an interrogative is chosen in such a way that in combination with the category of the constituents it allows as answers, the category of sentences results. Thus, (10), (11), (12) and (13) are all assigned different syntactic categories:

(10) Who walks in the garden?
(11) Which man loves which woman?
(12) Where did John and Mary meet for the first time?
(13) Does John love Mary?

Each of these interrogatives has its own particular kind of constituent answers, e.g. those in (14), (15), (16) and (17) respectively:
(14) John.
(15) The tall one, the redhead; and the small one, Mary.
(17) No.

Clearly, each of these answers matches only one of the interrogatives. Hence, from the category of the constituent, the category of the interrogative is deduced. Consequently, (10) is regarded as denoting a property of individuals, (11) as denoting a relation between individuals, and so on.

The categorial match between interrogative and answer can be construed in various ways. Tichy and Scha construe it in terms of identity of extension, Hausser in terms of functional application. In the latter case there are two options: one could let the interrogative be the function of the answer, or vice versa.  

Which of all these possible ways of implementing the categorial view is taken depends on various factors, such as the kind of phenomena one is primarily interested in, what kind of constituent answers one wants to allow for, independent motivations for assigning a certain interpretation to interrogatives, and so on. 

From these basic characteristics of the categorial approach, it will be clear that categorial theories are mainly concerned with interrogatives and characteristic constituent answers. And, disregarding all kinds of criticisms of detail, it can be said that they are pretty successful in this specific area. They all account for the fact that constituent answers depend for their interpretation on the context provided by the interrogative. Moreover, their approach is flexible enough to take into account constituent answers of a wide variety of types. They are not restricted to just rigid, definite answers, but can account also for indefinite and non-rigid answers.

However, even in this area, some serious criticisms can be raised against the categorial approach. Since categorial theories concentrate on interrogatives and answers as linguistic expressions, and impose categorial fit as
virtually the only condition on their relation, the account of the question-answer relation that results is rather superficial. Apart from the fact that concentrating on categorically matching interrogative-answer pairs, they disregard other types of linguistic answers, the main problem is that the account that categorial theories offer does not lead to a proper theory of the question-answer relation. What one wants is first of all a systematic theory about different notions of answerhood. There are complete answers, partial answers, semantic answers and pragmatic answers, and so on, and these are all systematically related. And secondly, one would like to give an account of the systematic relationships that exist between semantic and pragmatic properties of constituent answers and such notions of answerhood. The categorial approach accounts e.g. for the fact that answers need not be rigid, but it does not tell us under what circumstances non-rigid answers can be equally good as rigid ones.

As a theory about interrogatives and answers as linguistic expressions, the categorial approach has certainly led to insights that should be incorporated in an overall theory of the question-answer relationship, but it does not in itself constitute such a theory. Nor can it be expected that the categorial approach can be extended to such a theory without a major modification of its starting point. For, a general theory of questions and answers will have to be based upon a general characterization of the notion of answerhood and the notion of a question. And that will be forthcoming only if one interprets interrogatives and answers in a uniform way, something that is quite alien to the spirit of the categorial approach.

This lack of a uniform interpretation of interrogatives within the categorial approach has serious drawbacks in other areas in the theory of interrogatives as well. As we saw in section 3.1, there are entailments between interrogatives, not only between interrogatives within the same category, such as e.g. in (18), but also between interrogatives that are assigned different categories within this
approach, such as the ones in (19):

(18) Which men walk?  
Which men talk?  
______________________________  
Which men walk and talk?  

(19) Who walks in the park?  
______________________________  
Does John walk in the park?

The notion of entailment between interrogatives, like that of entailment between expressions of any other category, should be an instance of a general definition that applies to all semantic objects one's framework acknowledges. Basically, this general definition defines entailment between any two objects of a certain type as inclusion of one in the other. 35

It is easy to see that any categorial theory will account at most for entailments that hold between interrogatives that are associated with the same type of semantic object. Hence, such theories can account for an example like (18). But all cross-categorial entailments are left unexplained, such as the quite basic entailment relation exemplified in (19).

The same problem reappears if we look at coordination of interrogatives. Consider (20) and (21):

(20) Who went out for a walk? And who stayed home?  
(21) Who went out for a walk? And did they take the dog along?

Like entailment, coordination of interrogatives should be an instance of a general rule that predicts what, for any category, coordination of elements in that category amounts to. Classifying constituent interrogatives and yes/no-interrogatives as belonging to different categories, as lies at the heart of the categorial approach, makes it impossible to account for such coordinated interrogatives as (21) in a standard way. So, a uniform semantic interpretation of interrogatives seems to be called for, not only for developing a
systematic theory of answerhood, but also for an adequate account of entailment and coordination.

This need for one type of semantic object that all interrogatives share, is underscored by another weakness of categorial theories, viz. the lack of a decent analysis of wh-complements. Most categorial theories do not even seriously attempt to develop a theory of wh-complements, and if they do, the result is generally poor. Assuming something like the equivalence thesis, it will be obvious that the categorial approach faces serious difficulties. Not only does the proliferation of categories of interrogatives lead to a similar proliferation of categories of wh-complements, and hence of complement embedding verbs, the systematic relationships that hold between wh-complements and sentential complements show once more that a satisfactory account of interrogatives that meets the equivalence thesis has to be based on a uniform semantic analysis.

From these considerations, we can draw the following conclusion. The view that the categorial approach takes,leads to a reasonably adequate account of the relation between interrogatives and constituent answers. In this area lie its main contributions to the theory of interrogatives as a whole. As for other parts of the empirical domain, among which are some which are quite essential to a formal semantic approach, the starting point seems to be too narrow, and does not lead to adequate results which are in agreement with theoretical constraints one would like to impose on semantic theories in general, and on analyses of interrogatives and the question-answer relation in particular.

4.3. The propositional approach

Common to all theories in the propositional approach is that they associate with interrogatives a semantic object that is defined in terms of the notion of a proposition. As was the case in the categorial approach, the theories within this
one differ in details of implementation, and sometimes even in the interpretation of what their main objective is. But, it seems that they all share three considerations regarding the way in which interrogatives should be analyzed. First of all, it is taken for granted that answers are essentially of a propositional nature. Answers convey information, and information is coded in propositions. Secondly, it is assumed that the notion of an answer should play a role in the characterization of the semantic object to be associated with interrogatives. And finally, there is a tendency to treat all interrogatives uniformly, i.e. to associate them all with one and the same kind of semantic object.

So, it seems that the gist of the propositional approach can be formulated in the following general principle:

(P) The semantic interpretation of an interrogative should give its answerhood conditions, i.e. it should determine which propositions count as its semantic answers.

It should be noted that neither this principle, nor the considerations that lead to it are always explicitly stated or argued for at the outset. But the principle does characterize the main examples of propositional theories, those of Hamblin, Karttunen, and Bennett and Belnap. And in each of them, some of these considerations can be found, be it sometimes only implicitly.

If we compare the principle (P) with the competing principle (C) underlying categorial theories, the difference in the initial perspective becomes clear. Categorial theories tend to start from considerations concerning surface syntactic properties, whereas propositional ones proceed from observations of a logical semantical nature. Consequently, they focus on different aspects, and, as we shall see, with regard to their strong and weak points they are mirror images.

The oldest, the best known, and the least understood propositional approaches are those of Hamblin, Karttunen and, Bennett and Belnap respectively. All three assign the same
type of semantic object to interrogatives, viz. a set of propositions. The interpretation they give of this object, however, differs. For Hamblin this set consists of the possible answers to the interrogative. Karttunen, on the other hand, takes the set of propositions denoted by an interrogative to consist of the true answers to it. As a matter of fact, the difference between Hamblin's interpretation and Karttunen's is marginal from a material point of view. This is obscured by the fact that the respective analyses are worked out in different frameworks. Karttunen's approach has some formal advantages however, that is why we will mainly use his interpretation.

The difference between Karttunen on the one hand, and Bennett and Belnap on the other, is very real. According to the latter, each proposition in the set denoted by an interrogative constitutes in itself a complete and true answer. Their concern is the existence of interrogatives which have more than one complete and true answer, interrogatives of the kind discussed in section 3.1. The propositions in the set Karttunen associates with an interrogative are partial true semantic answers. Only jointly, they constitute a complete and true semantic answer. Unlike Bennett and Belnap's scheme, Karttunen's analysis is only attuned to interrogatives which have a unique true and complete semantic answer at each index.

Since propositional theories assign a uniform semantic type to all interrogatives, it seems reasonable to expect that they do better where categorial theories fail, viz. in accounting for answerhood, and for entailment and coordination of interrogatives. This is true, but only to a certain extent. Consider answerhood first. To begin with, it should be noted that although the notion of a semantic answer figures prominently in the descriptions various theories give of the semantic interpretation of interrogatives, neither one of them provides a theory of answerhood that is worked out in any detail. But from their interpretation of interrogatives a relation of answerhood can readily be deduced.
In Karttunen's framework a sentence is a complete and true semantic answer to an interrogative if the proposition that the former expresses equals the conjunction of the propositions in the set denoted by the latter. For Bennett and Belnap a sentence is a complete and true semantic answer to an interrogative if the proposition it expresses is an element of the set denoted by the interrogative.

It might look as if for interrogatives which have a unique complete and true semantic answer, the results Karttunen and Bennett and Belnap get are the same, but this is not the case. There are some not unimportant differences, which, of course, are due to differences in the way in which interrogatives are derived. Let us illustrate this with an example:

(22) Which man walks in the garden?

In Karttunen's scheme, (22) denotes all true propositions which of an actual man say that that individual walks in the garden. So, if John, Bill, and Hilary are the men that walk in the garden, (22) denotes a set consisting of three propositions: that John walks in the garden, that Bill walks in the garden, and that Hilary walks in the garden. Notice that these propositions do not state of the individuals that they are men. They are de re characterizations, so to speak, of the men that walk in the garden. At this point there is a difference between Hamblin and Karttunen. If we take the true ones from Hamblin's possible answers, we would get, in this case the following three propositions: that John is a man and walks in the garden, that Bill is a man and walks in the garden, and that Hilary is a man and walks in the garden. So, Hamblin's propositions give de dicto characterizations. In view of the observations made in section 3.1. concerning (non-)entailment of interrogatives, and those concerning the dicto/de re ambiguity of wh-complements, which in view of the equivalence thesis are the same facts, it seems that one's framework should at least contain the possibility of de dicto characterizations.
Bennett and Belnap do get de dicto characterizations. They also differ from Karttunen in that they analyse (22) as having a uniqueness presupposition, so in the situation under discussion, (22) would not have a complete and true answer, it would denote the empty set. If we change the example to (23):

(23) Which men walk in the garden?

The result will be a singleton set of propositions, containing the proposition that the men that walk in the garden are John, Bill and Hilary. So it seems that, unlike Karttunen, to a certain extent, Bennett and Belnap build in exhaustiveness. (See section 3.1.)

From these remarks, it can be concluded that the account that propositional theories give of the answerhood relation, as far as this account can be deduced from the interpretation they assign to interrogatives, is a rather restricted one. Only answers that give rigid and definite characterizations are counted as semantic answers. Indefinite answers, non-rigid answers, partial answers, fall outside its scope, and so do pragmatic notions of answerhood. The only notion of answerhood they reckon with is that of, what we have called in section 3.2. a standard semantic answer. As such, this is not something to blame them for. Not only is the notion of a standard answer one that one would a theory of answerhood to characterize, also there seem to be no real obstacles for extending a propositional account to a full theory of answerhood.

More fundamental problems arise if we look at what happens with entailment and coordination in these propositional theories. The kind of semantic objects they assign to interrogatives is for all of these the same, and, moreover, is one that in principle makes it possible to apply the general definitions of entailment and coordination. However, if we apply these general definitions we find that even quite basic entailment-relations are not accounted for, and that simple coordinations come out wrong as well.
Since entailment is defined as inclusion, interpreting interrogatives as denoting sets of propositions implies that one interrogative entails another iff the denotation of the first is always included in the denotation of the other.\footnote{41}

Consider Karttunen's theory first. It is easy to see that such a basic entailment as holds between (24) and (25) is not predicted:

(24) Who walks in the garden?
(25) Does John walk in the garden?

Clearly, it does not hold that in all situations the set of propositions denoted by (24) is a subset of the set of propositions denoted by (25). A yes/no interrogative, such as (25), always denotes a singleton set, containing either the positive or the negative answer. And a who-interrogative like (24) will contain a proposition for every individual that satisfies the predicate. So, except for some marginal cases, no entailments between such constituent interrogatives and the corresponding yes/no-interrogatives are predicted.\footnote{42}

Similarly, a simple coordination such as (26) is assigned a wrong interpretation if we apply the standard definition of conjunction, which comes down to intersection:

(26) Whom does John love? And whom does Mary love?

Since the two sets denoted by the conjuncts of (26) are disjoint (or both empty), Karttunen's analysis predicts that (26) has no answers at all.

These considerations clearly indicate that the Karttunen framework simply assigns the wrong type of semantic object to interrogatives. In a sense, there is something inconsistent in describing an interrogative as determining at each index what its complete and true semantic answer is, and at the other hand letting its denotation be a set of propositions. The complete answer is the conjunction of these propositions. So, one would rather expect the type of interrogative
denotations to be that of propositions, instead of sets of propositions. And indeed, this would give better results. In Karttunen's case, the problem with conjunction would disappear.

However, the basic entailments of the kinds discussed above, would then still be left unaccounted for. And this suggests that even if we rephrase Karttunen's analysis so as to give the right type of semantic object, it still would give the wrong objects of that type. And this, in its turn, implies that there is something basically wrong also with Karttunen's account of answerhood, even if we restrict ourselves to the basic notion of standard semantic answers. Especially within the propositional approach, of which the starting point is that the semantic interpretation of an interrogative should give its answerhood conditions, entailment and answerhood are but two sides of the same coin. Entailment is inclusion of denotation, denotation determines answerhood, hence, one interrogative entailing another comes down to every proposition giving an answer to the first, also giving an answer to the second. Intuitively, (24) entails (25). And, indeed, this intuition seems to be no other than the one that in every situation in which we get a complete answer to (24), we also get a complete answer to (25). So, Karttunen's failure to account for entailments such as these, means that the interpretations he assigns to interrogatives do not, as the basic principle of the propositional approach requires, give their proper answerhood conditions.

Although the interpretation of the set of propositions that Bennett and Belnap assign to interrogatives as their denotation differs from that of Karttunen, the problems with entailment and coordination are structurally the same. For just consider interrogatives which do have a unique complete and true semantic answer, such as the examples discussed above: any two different such interrogatives will denote disjoint (unit) sets. This predicts that no two such interrogatives are related by entailment, which is obviously wrong, and that the conjunction of any two such interrogatives will
denote the empty set, i.e. has no answer, which is not right either. So, the same conclusions can be drawn as in Karttunen's case: the Bennett and Belnap theory assigns the wrong type of semantic object to interrogatives. Since they want to account for interrogatives which have more than one complete answer, it will not do in their case to simply form a simple propositional object from the sets they define. In this case, we have to look for a solution in another direction. 44

As for the treatment of wh-complements, propositional theories do fare better than categorial ones, first and foremost in that they assign them a uniform semantic type, thus avoiding the proliferation of types the categorial approach leads to. Further it can be remarked that, in view of the equivalence thesis, the same problems that occur with interrogatives will reappear with wh-complements. Notice that here too there is evidence that the type assigned is the wrong one. If in Karttunen's case we would proceed from sets of propositions to single propositions, we would gain a uniform analysis of both wh-complements and that-complements, which leads to a considerable simplification, at least. 45

One of the main weaknesses of the propositional approach is that its theories generally provide a poor basis for dealing with linguistic answers. As we saw in section 3.2, both sentential answers and constituent answers essentially need the context provided by the interrogative for their proper interpretation. Consider the simple example (27):

(27) Whom does John love? Mary.

In a propositional theory anyway, the constituent answer Mary in (27) should express a proposition. The natural way to achieve this is to combine the term phrase interpretation with a property. At the characteristic level of propositional theories, viz. that of (sets of) propositions, this property is not available. 46 In the propositional theories discussed here, there is a level of analysis, however, at which we can isolate a property. Both in Karttunen's and in Bennett and
Belnap's framework, the derivation of interrogatives starts from open sentences. These are turned into a kind of yes/no-interrogatives, which are further transformed into constituent interrogatives by introducing wh-terms. The open sentences define properties, but not in all cases this is the property which is needed to get the right interpretation of the linguistic answers. Compare (27) with (28):


In the theories under discussion, both interrogatives are derived from one and the same open sentence (29):

(29) John loves x

But the answers in (27) and (28) express different propositions. In (27) the answer expresses the proposition that Mary is the one whom John loves, whereas in (28) it expresses that Mary is the nurse that John loves.

These considerations show that the propositional theories of Karttunen, and Bennett and Belnap do not lead to a proper account of linguistic answers, but not of course that no propositional theory could. It seems reasonable to conclude that in order for a propositional theory to deal with the interpretation of linguistic answers adequately, it will have to 'look like' a categorial theory in important respects, at least at some level of analysis. This suggest that a more encompassing theory of interrogatives should combine the forces of both the categorial and the propositional approach. From the latter it should incorporate the propositional view on answerhood and the consequent uniform definition of the semantics of interrogatives in terms of answerhood conditions. From the former it should take over the categorial analysis as an underlying level from which linguistic answers can be derived, thus accounting for the fact that their interpretation depends on the interrogative. In that way, more kinds of answers than just the rigid and definite ones that
propositional theories allow for, can be brought within the scope of such a theory. Of this enriched domain of answers one wants a systematic theory that predicts and explains under what kind of circumstances what kind of linguistic answers correspond to which notions of answerhood. There, another point of view becomes important, which is that of the third main approach to interrogatives, the imperative-epistemic one.

4.4. The imperative-epistemic approach

The last main approach to the theory of interrogatives that can be discerned in the formal semantics tradition, is the imperative-epistemic one. It should be noted right at the outset that this approach differs from the categorial and the propositional view considerably. It does not just take another perspective, it also has a rather different aim. Whereas all the theories we have discussed so far are descriptive in this sense that they aim at a description and an explanation of how interrogatives function in natural language, the theories within the present approach are directed rather differently. This certainly holds for the original work of Aqvist, whose primary interest is in a logical theory of interrogatives. In developing such a logical theory the relation with natural language is a subject of relatively minor importance. The work of the other main proponent of the imperative-epistemic approach, that of Hintikka, is more explicitly oriented towards natural language. But his analysis does not aim at developing a systematic theory of interrogative expressions in natural language, at least not in the way that the other theories do. Since, however, the relationship with natural language in Hintikka's work has a more prominent place than in Aqvist's, we will draw mainly on the former in our characterization of the aims and methods of the imperative-epistemic approach.

What guides the analysis of interrogatives in this approach
is the way in which they function in ordinary communication. In normal circumstances, the utterance of an interrogative is meant as a means to acquire information. It functions as an exhortation to provide the questioner with certain information, characterized by the content of the interrogative. The semantic content of an interrogative then is identified with such a request. In other words, theories in this approach subscribe to something like the following principle:

\[ \text{(IE) The semantic interpretation of an interrogative is a request for information (knowledge)} \]

Generally, the semantic interpretation of an interrogative contains two elements, an imperative one and an epistemic one. These appear explicitly in the paraphrase that according to principle (IE) can be given of an interrogative. Consider the following two examples:

(30) Does John walk in the garden?
(31) Bring it about that I know whether John walks in the garden
(32) Who walks in the garden?
(33) Bring it about that I know who walks in the garden

These examples illustrate a rather particular feature of this approach. Interrogatives are analyzed by embedding them under a sequence of two logical operators. This means that if we are to understand (31) and (33), for example, as representing the meaning of (30) and (32) respectively, as principle (IE) tells us to do, we should already know what the meaning of the embedded interrogatives is. But the latter are not assigned a meaning independent of their direct counterparts. And, given the equivalence thesis, they could not be. But then it follows, so it seems, that an imperative-epistemic paraphrase does not provide us with a proper semantic interpretation of the interrogative at all. Rather, it must be viewed as a theory of pragmatics of interrogatives, as a theory of pragmatic answer-
hood relations. It is a theory not of what an interrogative means, but of how an interrogative with a certain meaning can be used. So, in fact, it presupposes a semantics rather than providing one.

This interpretation of the contribution of the imperative-epistemic approach to the theory of interrogatives and the relation of answerhood in general, can be further illustrated by considering in slightly more detail how Hintikka goes about analyzing interrogatives like (30) and (32).

As far as the content of interrogatives is concerned, the most important part of the paraphrase consists of the epistemic operator and its argument. Together they form, what Hintikka calls, the desideratum expressed by the interrogative. I.e. they give a description of the epistemic state that the addressee is asked to bring about. The desiderata of (30) and (32) can be written as (34) and (35) respectively:

(34) $K_I(\text{John walks in the garden}) \lor K_I\neg\text{John walks in the garden}

(35) \exists x[K_I(x \text{ walks in the garden})]

A few remarks are in order. First of all, the formulas (34) and (35) are not mere paraphrases, but expressions of an interpreted language, that of Hintikka's epistemic logic. The value of this analysis of interrogatives hence derives from the value Hintikka's epistemic logic has. But that will not concern us here.

The arguments of the epistemic operator $K_I$ are, of course, sentential complements. As (34) shows, knowing whether $\phi$ is analyzed as knowing that $\phi$ or knowing that not-$\phi$, and knowing who has a certain property, is analyzed in (35) as knowing of someone that he or she has that property. Of course, as paraphrases of the entire expressions 'knowing whether' and 'knowing who' this is correct. But, and this is important, these analyses do not assign an independent meaning to the respective $wh$-complements. And this exactly what the independent meaning thesis, and the compositionality constraint require. So, though
the analysis may be useful in other respects, it cannot be viewed as a semantic theory of wh-complements and interrogatives, at least not as one that meets the general constraints we formulated earlier. And it is hard to see how the analysis could be reformulated so as to meet these requirements after all. For it is restricted to extensional cases, essentially. 'Knowing whether' can indeed be analyzed as 'knowing that or knowing that not', but such a paraphrase is impossible for intensional constructions, such as 'wondering whether'. In fact, the existence of both extensional and intensional complement embedding verbs once more emphasizes the need for an independent semantic object that can function as the interpretation of a wh-complement, and of the corresponding interrogatives.

Another remark needs to be made here. As Hintikka recognizes, (35) is not the only desideratum that can be associated with the interrogative (32). It corresponds roughly with the so-called mention-some interpretation of the interrogative. And besides that, there is also the so-called mention-all interpretation, the desideratum of which Hintikka formulates as in (36):\[53\]

\[(36) \forall x [x \text{ walks in the garden } \rightarrow K_1(x \text{ walks in the garden})]\]

Notice that this mention-all interpretation does not imply exhaustiveness as we discussed it in section 3.2. Consequently, it is not accounted for that on its mention-all interpretation, (32) entails (30). An answer to (32) on its reading (36) implies positive answers to such yes/no-interrogatives as (30), but not their negative ones.

This brings us to the last remark, which concerns answerhood. For this we need another notion besides that of the desideratum of an interrogative, that of its matrix. The matrix is the argument of the epistemic operator in the desideratum. So, it is a formula with a free variable. An answer is a (are all) true instance(s). In this sense, the analysis indicates how linguistic answers come about.
An answer is a complete answer if its incorporation into the information (knowledge) of the questioner makes the desideratum true. Hence, it is essentially a pragmatic notion. Whether something constitutes an answer depends on the information already available. Consider (37) as an answer to (32) on the reading on which its desideratum is (35):

(37) Peter walks in the garden.

Incorporating (37) leads to (38):

(38) $K_I(\text{Peter walks in the garden})$

Whether (37) is a complete answer depends on whether the questioner knows who Peter is, i.e., whether (39) holds:

(39) $\exists x K_I(x = \text{Peter})$

For only in combination with (39) does (38) amount to (35), the desideratum of (32).

In a similar manner, complete answers to other readings of interrogatives, and partial answers, can be defined.

These considerations indicate that the major contribution of the imperative-epistemic approach lies in the pragmatics of interrogatives and of question-answering. It emphasizes that question-answering takes place in a pragmatic context, and hence, that pragmatic notions of answerhood are important. What it does not provide, however, is a systematic semantic theory of interrogatives. 'Logical forms' are assigned to natural language expressions on a rather ad hoc basis. No systematic relationship between the syntactic derivation of interrogatives and these forms is provided. Moreover, as we already argued above, the analyses that are given cannot be interpreted as giving the semantic content of interrogatives. This holds not only for the epistemic element in the analysis, but also for the imperative element. This part depends essentially on the use to which the inter-
rogative is put. It may be that the normal use is a request to bring about a certain epistemic state, but interrogatives can be put to other uses as well. An example that readily comes to mind is an exam situation. In that case, Hintikka says, the analysis of (32) is not (33), but (40):

(40) Show me that you know who ...

Rather than making the notion of logical form, i.e. of semantic content, depend on the circumstances of use, one would prefer an analysis that allows one to show how, given some independently provided semantic analysis, different uses in different circumstances come about. And that presupposes that semantic content and pragmatic aspects are distinguished systematically.

So, it seems that the imperative-epistemic approach can most fruitfully be viewed, not as a rival to the categorial and propositional approach, but rather as a companion. Supposing that some fusion of the latter two can be designed to give a systematic account of the semantic content of interrogatives, and of the semantics of linguistic answers, including the characterization of the notion of a standard semantic answer, it seems feasible to supplement it with the insights of the imperative-epistemic approach in order to gain a satisfactory account of the essentially pragmatic nature of the question-answer relationship.

4.5. Conclusion

Our discussion of the problems and prospects of the three major approaches in the theory of interrogatives has been a general one. As such it does not do justice to the many interesting analyses of particular phenomena that the various theories within these approaches provide. For such details the reader is referred to the works cited, and to the discussion of particular proposals in the papers to
follow. Our aim here has been a modest one: to indicate the main lines of thinking each approach embodies. Given a general characterization of such a starting point, it is possible to distinguish, independently of the details of any particular analysis, its strong and its weak sides.

We hope to have shown that the various approaches are in a sense complementary. The categorial approach and the propositional approach both constitute theories about the (syntax and) semantics of interrogatives, but, since they focus on different empirical aspects, it seems that their insights do not contradict each other, but rather can be expected to be fruitfully combined. The categorial approach focusses on the relationship between interrogatives and answers as linguistic expressions. Propositional theories concentrate on the development of a uniform semantic analysis in terms of semantic answerhood. An overall theory should account for both, and it seems that, ideology set aside, such a theory can profit from both approaches. The imperative-epistemic approach, in our view, has to be considered to constitute a theory about the pragmatics of interrogatives and question-answering. Although the viewpoint of information exchange is, of course, essential to a really comprehensive account of question-answering, it has been largely ignored by theories in the first two approaches. In this case too, the results, though not the interpretation that people working in this approach give of them, seem to be incorporable in an overall theory. And they should be, for an adequate theory of interrogatives, answers, and the question-answer-relation that does not account for these pragmatic aspects, is essentially incomplete.

We hope that from the detailed analyses of various kinds of phenomena that are given in the papers to follow, the contours of such a more encompassing theory will emerge. The theory that can be distilled from these papers is like a propositional one in that it defines a uniform semantic object for all interrogatives and wh-complements, avoiding some of the problems with entailment and coordination that other theories run into. It deals with linguistic answers in a way
that is akin in spirit to the categorial approach, accounting for the fact that the interpretation of both sentential and constituent answers depend on the interpretation of the interrogatives they are used to answer. Further, it develops a systematic theory of the question-answer relationship, defining various notions of semantic and pragmatic answerhood in such a way that the relationships between these are reckoned with. This theory is not developed explicitly in what follows, since these papers are primarily analyses of various semantic and pragmatic phenomena pertaining to interrogatives and answers. But we trust that given the overview of the problems and prospects in this paper, the connections between what is said and done in the various separate papers is sufficiently clear.
Notes

* We would like to thank Theo Janssen for his critical remarks on an earlier version, which, we hope, have led to substantial improvements, and Johan van Benthem for encouragement.

1. Implicitly at least, such a position seems to be held by many who feel sympathetic towards the opinion, most clearly and convincingly advocated by Wittgenstein, that there is no internal, logical system underlying all of language, that its various parts are related only indirectly and in diverse ways, and that hence there is no reason whatsoever to suppose that what constitutes an illuminating analysis of one part can be extended to others fruitfully as well. A famous passage of the Philosophische Untersuchungen brings this home forcefully (Wittgenstein, 1953, par. 65):

"Hier stossen wir auf die grosse Frage, die hinter allen diesen Betrachtungen steht. - Denn man könnte mir nun einwenden: "Du machst dir's leicht! Du redest von allen möglichen Sprachspielen, hast aber nirgends gesagt, was denn das Wesentliche des Sprachspiels, und also der Sprache ist. Was allen diesen Vorgängen gemeinsam ist und sie zur Sprache, oder zu Teilen der Sprache macht. [...]"

Und das ist wahr. - Statt etwas anzugeben, was allem, was wir Sprache nennen, gemeinsam ist, sage ich, es ist diesen Erscheinungen gar nicht Eines gemeinsam, weswegen wir für alle das gleiche Wort verwenden, -sondern sie sind mit einander in vielen verschiedenen Weisen verwandt. Und dieser Verwandtschaft, oder dieser Verwandtschaften wegen nennen wir sie alle "Sprachen"."

This expresses an opinion which, we feel, is quite alien to the tradition in which language is studied with formal means and methods. Unless the contrary has been proven (but what would a proof to that effect look like?), it is assumed that 'language' denotes a set of phenomena that do have a common core, be it perhaps one that can be described only rather abstractly. One of the aspects of the enterprise is to find out what this common core is, and this is done by constructing one and scrutinizing it, to find out to what extent it fits the phenomena, and to what extent it gives an insightful, explanatory account of them. It is, of course, one of Wittgenstein's claims that, though abstractly such a common basis for all of our language can be constructed, it is bound to lack any explanatory power (cf. par. 13 and the surrounding sections of the Untersuchungen). Such a claim can be refuted only by actually constructing a common basis, by actually
developing a general theory of language which indeed does connect and elucidate and explain various parts of language. The working hypothesis of the formal tradition that this is possible, that it can give an interesting account of some fundamental principles of language in general, should not be taken for another one, viz. that a formal approach is the way, i.e. the only way, to study language. Other perspectives, other approaches, may contribute each in their own way to our knowledge of and insight in this one of the most fundamental of human capacities. And it may be that it are these various approaches that are related only by means of family resemblances. Perhaps we will never be able to come up with a unique ultimate theory that encompasses all these perspectives. But that is an entirely different matter.


5. See G&S 1981, section 2.2, where this idea is used in a formal statement of Gricean conversational maxims as correctness conditions.

6. The standard work on compositionality is Janssen 1983. See also G&S 1982c for a discussion of compositionality and logical form.

7. Unlike the compositionality principle, which has been studied in depth, and of which the content and the consequences are well-known (see Janssen 1983), this principle lacks a formal theory. One thing that can be noticed is that it is independent of the compositionality principle. A compositional analysis may very well violate this principle. So, it seems to be another constraint on derivations, over and above the requirement of compositionality. How exactly it should be formalized and implemented depends on various aspects of the organization of a grammar. Since it concerns the relationship between syntax and semantics, it is a constraint on both syntactic and semantic rules. An example of a framework that seems to comply with it, is the very restricted framework proposed by Landman & Moerdijk (see Landman & Moerdijk 1983).

8. As an 'explanation' of the human capacity to deal with a potential infinite number of linguistic constructions it is adduced by various people, from Frege to the pre-Fregean Katz (see Frege 1923, Katz 1966).

A field that is often claimed to be outside the scope of compositional semantics is that of lexical semantics (see e.g. Baker & Hacker 1980, which contains various other kinds of criticisms on formal semantics, of a Wittgensteinian nature as well). But recent work of Moortgat (see Moortgat 1984) and others shows remarkable progress in this area as
well. Of course, there are bound to be exceptions to the compositional rule, but that is besides the point. What is important is that taking compositionality as a lead, results in clear and well-organized analyses that cover important areas.

9. For a defense of this view see G&S 1978. A similar position is taken e.g. in Gazdar 1979.

It may be helpful to say something about terminology here. The term semantics is used to refer to that part of an overall theory of meaning that deals with truth-conditional aspects. Another part of such a theory deals with those aspects of meaning that cannot be described in terms of truth-conditions, reference, and so on, but that are of a conversational nature. For this part the term pragmatics is reserved. So, at least most of the time, 'pragmatics' refers to a specific part of the overall study of language use, viz. that part that is concerned with Gricean conversational maxims, with correctness conditions, and especially the informational elements that play a role there.

10. Grice's original purpose was to show that a classical, truth-conditional analysis of the meaning of connectives is basically correct, once it is supplemented with a conversational analysis that explains various other aspects of their meaning (see Grice 1967).

Grice's intentions may explain why his theory has attracted many people working in the formal tradition, even though Grice himself is supposed to be a 'non-formalist'.
19. See e.g. Keenan & Hull (1973), Hintikka (1976), Belnap & Steel (1976), Bennett (1977,1979), Belnap (1982).


21. A good overview of recent developments can be gotten from Soames (1979,1982).

22. A glimpse of the wealth of material available can be gotten by consulting the bibliography compiled by Egli & Schleichert, which appeared as an appendix in Belnap & Steel (1976). And much more has appeared since then.

23. The same classification is used in Kiefer (1983a).

24. Thus Hull, for example, in Hull (1975), starts out with the remark that "an answer ... is linguistically a noun-phrase", and without any further consideration goes on to develop a categorial theory.

Another example is Hausser, who notes that certain structural correlations exist between interrogatives and non-sentential answers, which do not exist between interrogatives and full, sentential answers. The former exhibit a certain categorial match, whereas the latter combine freely. Hausser concludes from this observation that non-sentential answers therefore are primary, and that interrogatives are to be considered syntactically as functions from non-sentential answers to full sentences. This has immediate repercussions for the semantics: the semantic interpretation of an interrogative is a set of denotations of the type corresponding to its 'characteristic' non-sentential answers. See Hausser 1976, 1983, Hausser & Zaefferer 1978.

In these cases, empirical considerations, concerning surface syntactical phenomena, rather than theoretical ones decide upon the way in which the analysis proceeds.

25. For example, Karttunen, in Karttunen (1977), takes a propositional view on single constituent interrogatives, and then argues against assigning multiple constituent interrogatives to a different, more complex category, as was proposed by Wachowicz, in Wachowicz (1974), as follows. He observes that there are hardly any distributional differences between single and multiple constituent interrogatives, and concludes that they ought to be assigned to the same syntactic category, and hence, to the same semantic type. For that keeps the overall grammar simpler. This is a formal, and not a material line of argumentation. No arguments are adduced that multiple constituent interrogatives ought to be analyzed in terms of propositions that relate to the semantics of these expressions themselves directly.

A similar type of argumentation can be found in G&S 1982a. There, a specific type of propositional view, viz. that wh-complements denote propositions, is argued for by the observation that they interact systematically with sentential
complements, and that hence overall simplicity is served by assigning both to the same syntactic category and the same semantic type.

26. Clear examples of this line of reasoning can be found in the works of the two main proponents of the imperative-epistemic view, Aqvist and Hintikka. See for example Aqvist, 1975, section 2. Hintikka, 1974, section 2, expresses it as follows:

"In spite of this somewhat gloomy view of the current scene, I believe that the key to the logic of questions is fairly straightforward. In a way, nothing could be simpler. If there is anything here that virtually all parties agree on, it is the idea that a question is a request for information. The questioner asks his listener to supply a certain item of information, to make him know a certain thing. Thus all that there is to the logic of questions is a combination of the logic of knowledge with the logic of requests (optatives, imperatives)."

And that is about all the theoretical motivation that is given. As for the aim of the analyses of Aqvist and Hintikka, see section 4.3.

27. See Hausser (1976), 1983), Hausser & Zaefferer (1978), Tichy (1978), Scha (1983). Extensive discussion of some of the details of these categorial analyses, especially of those of Tichy and Scha, can be found in G&S 1984b.

28. For example, in Hausser & Zaefferer we find the following:

"This shows that redundant answers are not very interesting from a semantical point of view since their semantic representation is identical to that of ordinary declarative sentences."


It should be noted that once intonation patterns are taken into consideration, and are considered to be an integral part of the 'form' of expressions, it seems that sentential answers and constituent answers do have the same distributational properties.


30. See also G&S 1984b, section 2.2. Notice that if, as was suggested in note 28, we consider intonation to be an aspect of form too, sentential answers depend not just for their interpretation, but also for their form on the context of an interrogative.


32. In fact, Hausser constructs constituent answers as sentential expressions, by introducing a special kind of expression, called a 'context-variable', which ranges over the type of sets of denotations of the type of the constituent. The
interrogative is taken as the value of the context-variable. This hidden sentential character of constituent answers, we take it, is Hausser's way to account for the fact that answers convey information, i.e. express propositions.

The function-argument relation between interrogative and constituent answer is constructed differently in Hausser 1976, and Hausser 1983, relevant factors being, among others, scope phenomena.

33. Such independent motives can be found especially in Tichy's paper. They are discussed in G&S 1984b, section 1.

34. For a further evaluation, see G&S 1984b. In that paper there is extensive discussion of the matter of how to build in exhaustiveness. The paper also contains critical remarks on analyses related to the categorial approach, such as that of Bäuerle 1979.

35. See G&S 1984c, section 3.1 for formal definitions of general rules of entailment, and coordination.

36. In Hausser 1976, we find the following (page 21):

"Furthermore, I fail to see in what intuitive sense (1) [Bill knows who arrived] should have anything to do with a question."

See Belnap 1981, page 7, for some critical remarks.

In Hausser (1983) an analysis of wh-complements is developed, which, however, runs into several problems that we will not go into here.


A more extensive discussion of Karttunen, and of Bennett & Belnap, can be found in G&S 1984c, section 3. G&S 1982a also contains some discussion of Karttunen's analysis.

38. As a matter of historical curiosity, we will go into the relation between Hamblin's and Karttunen's analysis in some detail.

Karttunen phrases the difference between Hamblin's analysis and his own as follows (Karttunen 1977, page 9,10):

"Hamblin's idea was to let every [interrogative] denote a set of propositions, namely the set of propositions expressed by possible answers to it. [...] I choose to make [interrogatives] denote the set of propositions expressed by their true answers instead of the set of propositions expressed by their possible answers."

This formulation suggests a basic difference, but this is mere appearance, caused by a terminological confusion. Hamblin's analysis is carried out in the framework of Montague's 'English as a Formal Language' (EFL), and that of Karttunen in the PTQ-framework. What is called 'denotation' in EFL, is called 'sense' in PTQ. If we use PTQ-terminology to describe both Hamblin and Karttunen, we get the following. For Hamblin, the sense of an interrogative is a set of
propositions, being its possible answers. For Karttunen, the sense of an interrogative is a function, having as its domain the set of indices and as its range the set of all sets of propositions that are possible answers. At an index, this function yields as its value that set which consists of the true answers. That is the denotation of an interrogative. Hamblin's notion of the sense of an interrogative does not give rise to a corresponding notion of denotation in the standard way. It is not a function having the set of indices as its domain. But, of course, if we ask ourselves what the denotation could be in Hamblin's case at a certain index, one can think of nothing else but taking the true answers at that index from the set of possible answers that constitutes the sense. And then we are back at Karttunen. In other words, apart from some differences of detail which are not relevant and which we leave out of consideration here, there is no material difference between the two. The only, but not unimportant difference is that Karttunen's analysis allows for a standard characterization of, and relation between, sense and denotation, whereas Hamblin's approach calls for non-standard notions of sense and denotation.

39. For extensive discussion of such interrogatives, and of Bennett & Belnap's way of accounting for them, see G&S 1984c.

That Karttunen should be interpreted as is done in the text, can be substantiated by the following quotation (Karttunen, 1977, page 10):

"[...] questions denote sets of propositions that jointly constitute a true and complete answer to the questions [...]"

See also Belnap 1982, section 2.2.

40. Karttunen does not speak about the matter at all, and Hamblin only vaguely. Belnap (1982) contains some remarks, but no real theory.

41. The criticism to follow are worked out in formal detail in G&S 1984c, section 3.

42. For a detailed diagnosis, see G&S 1984c, sections 3.2.1 and 3.2.2, and note 26. As is argued there, exhaustiveness plays an important role in these matters.

43. See note 42.

44. See G&S 1984c, section 4.

45. See G&S 1982a, section 1.8.

46. This holds for all the frameworks in which existing propositional theories are formulated. A possible solution might be to use a framework with structured propositions. For our solution of this problem, see G&S 1984b.
47. See Aqvist 1965.

48. See Hintikka 1974, 1976, 1978, 1983. As for the descriptive aims of Hintikka's analyses, they are evident, for example, from the introduction in Hintikka 1976. Hence, one should not be misled by the word 'logic' as it occurs in the quotation given in note 26. We think one can safely read 'logic' there as 'logical semantics'.

49. Analyses that propose a performative paraphrase for interrogatives, such as that of Lewis 1972, are left out of consideration here. The justification for doing so lies partly in the fact that some of the criticisms that are raised against epistemic-imperative paraphrase theories can be raised against such theories too, partly because the entire performative analysis enterprise can be argued to be fundamentally wrongly directed. See e.g. the criticisms made in Gazdar 1979.

50. Similar criticisms are raised throughout the work of Belnap.


52. See Karttunen 1977, section 1.4, and G&S 1982a, section 1.8.

53. Besides these two, Hintikka distinguishes several others (see Hintikka 1983, section 7). The problem is that Hintikka's analysis does not give a general characterization of these different desiderata. They have to be stated separately, and ad-hoc.

For some remarks concerning the status of the mention-all/mention-some contrast, see G&S 1984c, section 5.

54. See note 42.

55. In other words, the analysis does not conform to the compositionality principle. Hintikka, by the way, has his doubts about the possibility of providing a compositional semantics for natural language.

References


Belnap, N. & T. Steel, The logic of questions and answers, Yale University Press, New Haven, 1976

Bennett, M., 'A response to Karttunen on questions', Linguistics and Philosophy, 1, 1977

Idem, Questions in Montague grammar, Indiana University Linguistics Club, Bloomington, 1979


Grewendorf, G., 'What answers can be given?', in: Kiefer (1983a)

Grice, H., Logic and conversation, William James Lectures, ms., Harvard University, 1967

Groenendijk, J. & M. Stokhof, 'Modality and conversational information', Theoretical Linguistics, 2, 1975


Idem, 'Semantic analysis of wh-complements', Linguistics and Philosophy, 5, 1982a
Idem, 'Formal and acquainted objects in the theory of information', ms., Amsterdam, 1982b

Idem, 'Over logische vorm', Tijdschrift voor Tekst- en Taalwetenschap, 2, 1982c


Idem, 'Questions and linguistic answers', ms., Amsterdam, 1984b

Idem, 'Coordinating interrogatives', ms., Amsterdam, 1984c


Hausser, R., 'The logic of questions and answers', ms., München, 1976

Idem, 'The syntax and semantics of English mood', in: Kiefer (1983a)


Hintikka, J., Knowledge and belief, Cornell University Press, Ithaca, 1962


Idem, The semantics of questions and the questions of semantics, Acta Philosophica Fennica, 28 (4), 1976


Idem, 'New foundations for a theory of questions and answers', in: Kiefer (1983a)


Janssen, T., Foundations and applications of Montague grammar, diss., Amsterdam, 1983

Karttunen, L., 'Syntax and semantics of questions', Linguistics and Philosophy, 1, 1977

Karttunen, L. & F. Karttunen, 'Even questions', ms., Austin, 1976


Kiefer, F. (ed.), *Questions and answers*, Reidel, Dordrecht, 1983a

Idem, 'Introduction', in: Kiefer (1983a)

Landman, F., 'Representation of information and information growth. Part I: elimination and the Conway-paradox', ms., Amsterdam, 1984a

Idem, 'Representation of information and information growth. Part II: domain theory', ms., 1984b


Moortgat, M., 'Word formation and Frege's principle', ms., Leiden, 1984

Partee, B. & M. Rooth, 'Generalized conjunction and type ambiguity', ms., Amherst, 1982a

Idem, 'Conjunction, type ambiguity, and wide scope "or"', ms., Amherst, 1982b


Idem, 'How presuppositions are inherited: a solution to the projection problem', *Linguistic Inquiry*, 13, 1982


Wachowicz, K., 'Multiple questions', *Linguistica Silesiana*, 1, 1974

II

SEMANTIC ANALYSIS OF WH-COMPLEMENTS

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This paper presents an analysis of wh-complements in Montague Grammar. We will be concerned primarily with semantics, though some remarks on syntax are made in section 4. Questions and wh-complements in Montague Grammar have been studied in Hamblin (1976), Bennett (1979), Karttunen (1977) and Hausser (1978) among others. These proposals will not be discussed explicitly, but some differences with Karttunen’s analysis will be pointed out along the way.

Apart from being interesting in its own right, it may be hoped that a semantic analysis of wh-complements will shed some light on what a proper analysis of direct questions will look like. One reason for such an indirect approach to direct questions is the general lack of intuitions about the kind of semantic object that is to be associated with them. A survey of the literature reveals that direct questions have been analyzed in terms of propositions, sets of propositions, sets of possible answers, sets of true answers, the true answer, properties, and many other things besides. As far as wh-complements as such are concerned, we do not seem to fare much better, but there is this clear advantage: we do have some intuitions about the semantics of declarative sentences in which they occur embedded under such verbs as know, tell, wonder. What kind of semantic object we may choose to associate with wh-complements is restrained by various facts about the semantics of these sentences.

This paper is organized as follows. In section 1 we discuss a number of semantic facts concerning declarative sentences containing wh-complements, leading to certain conclusions regarding the kind of semantic object that is to be associated with wh-complements. In section 2 we show that Ty2, the
language of two-sorted type theory, gives suitable means to represent the semantics of wh-complements, and that Ty2 can take the place of IL in PTQ as a translation medium. In section 3 we indicate how the analysis proposed can be implemented in a Montague Grammar and how the semantic facts discussed in section 1 are accounted for. In section 4 a possible syntax for wh-complements which suits our semantics is outlined in some detail. Section 5 deals with the coordination of complements, whilst in section 6 we tie up some loose ends and make a speculative remark on the semantics of direct questions.
1. **Semantic properties of wh-complements**

In this section a number of semantic properties of wh-complements will be traced by considering the validity of arguments in which sentences containing them occur. The conclusion of our considerations will be that there are good reasons to assume wh-complements to denote the same kind of semantic object as that-complements: propositions. The differences between the two kinds of complements will be explained in terms of differences in sense.

1.1. **Whether-complements and that-complements**

Consider the following valid argument, of which one of the premises contains a whether-complement and the conclusion a that-complement.

(I) John knows whether Mary walks  
Mary walks  
John knows that Mary walks

The validity of this type of argument reflects an important fact of sentences containing whether-complements and, by implication, of whether-complements themselves. As (I) indicates, there is a relation between the semantic object denoted by **whether Mary walks** and the proposition denoted by **that Mary walks**. Similarly, the validity of (II) is based on a relation between the semantic object denoted by **whether Mary walks** and the proposition denoted by **that Mary doesn't walk**.
(II) John knows whether Mary walks
Mary doesn't walk
John knows that Mary doesn't walk

Together, (I) and (II) indicate that the actual truth value of Mary walks determines whether the relation holds between whether Mary walks and that Mary walks, or between whether Mary walks and that Mary doesn't walk.

The following examples show that the validity of (I) and (II) does not depend on the factivity of the verb know:

(III) John tells whether Mary walks
Mary walks
John tells that Mary walks

(IV) John tells whether Mary walks
Mary doesn't walk
John tells that Mary doesn't walk

Since x tells that ϕ does not imply that ϕ is true, the validity of (III) and (IV) cannot be accounted for in terms of factivity, and neither should the validity of (I) and (II) if, as we do, one assumes that it has to be explained in a similar way.

The overall suggestion made by (I)-(IV) is that there is a relationship between sentences in which a whether-complement occurs embedded under verbs as know or tell and similar sentences containing a that-complement. The most simple account of this relationship would be to claim that whether ϕ and that (not) ϕ denote the same kind of semantic object. Taking that (not) ϕ to denote a proposition, this amounts to claiming that whether ϕ denotes a proposition too.
1.2. **Index dependency**

Although on this account both that- and whether-complements denote propositions, they do this in different ways. The contrast between (I) and (III) on the one hand, and (II) and (IV) on the other, shows that which proposition whether \( \phi \) denotes depends on the actual truth value of \( \phi \). This marks an important difference in meaning between that- and whether-complements. The denotation of that-complements is index independent: at every index that \( \phi \) denotes the same proposition. The denotation of a whether-complement may vary from index to index, it is index dependent. At an index at which \( \phi \) is true it denotes the proposition that \( \phi \); at an index at which \( \phi \) is false it denotes the proposition that not \( \phi \). In other words, whereas the propositional concept which is the sense of a that-complement is a constant function from indices to propositions, the propositional concept which is the sense of a whether-complement (in general) is not. So, although, at a given index, a whether-complement and a that-complement may have the same denotation, their sense will in general be different.

1.3. **Extensional and intensional complement embedding verbs**

The difference in sense between that-complements and whether-complements plays an important role in the explanation of the semantic properties of sentences in which they are embedded. Embedding a complement under a verb semantically corresponds to applying the interpretation of the verb to the sense of the complement, i.e. to a propositional concept. This is the usual procedure for functional application, motivated by the assumption that no context can, a priori, be trusted to be extensional. We speak of an extensional context if a function always operates on the denotation of its arguments, and not on their sense.

As a matter of fact, such verbs as know and tell are extensional in this sense, and moreover, the validity of the
arguments (I)-(IV) is based upon this fact. Verbs such as know and tell operate on the denotations of their complements, i.e. on propositions, and not on their sense, i.e. propositional concepts. The extensionality of these verbs will be accounted for by a meaning postulate which reduces intensional relations between individual concepts and propositional concepts to corresponding extensional relations between individuals and propositions.

However, there are also complement embedding verbs which do create truly intensional contexts. In terms of Karttunen's classification, inquisitive verbs (ask, wonder), verbs of conjecture (guess, estimate), opinion verbs (be certain about), verbs of relevance (matter, care) and verbs of dependency (depend on) count as such. The assumption that no extensional relation corresponds to the intensional one denoted by these verbs explains why arguments such as (I)-(IV) do not hold for them. That some of these verbs (e.g. guess, estimate, matter, care) can be combined with that-complements, while others (ask, wonder, depend on) cannot (at least not without a drastic change in meaning, cf. note 9), is an independent fact that needs to be accounted for as well.

1.4. Constituent complements

Consider the following arguments, of which one of the premisses contains a wh-complement with one or more occurrences of wh-terms such as who, what, which girl.

(V) John knows who walks
     Bill walks
     John knows that Bill walks

(VI) John knows which man walks
     Bill walks
     John knows that Bill walks
(VII) John knows which man which girl loves
    Suzy loves Peter and Mary loves Bill
    ________________________________
    John knows that Suzy loves Peter and that Mary
    loves Bill

Given the usual semantics, these arguments are valid.³
Again, this can be explained in a very direct way if we take
c constituent complements to denote propositions. The validity
of (V)-(VII) no more depends on the factivity of know than
does the validity of (I) and (II). This will be clear if one
substitutes the non-factive tell for know in (V)-(VII). The
validity of all these arguments does depend on the extensionality
of know and tell. As was the case with whether-complements,
which proposition a constituent complement denotes depends
on what is in fact the case. For example, which proposition
is denoted by who walks depends on the actual denotation of
walk. If Bill walks, the proposition denoted by who walks
should entail that Bill walks; if Peter walks, it should
entail that Peter walks. This index dependent character can
more generally be described as follows. At an index i, who
walks denotes that proposition p, which holds true at an
index k iff the denotation of walk at k is the same as its
denotation at i.

1.5. Exhaustiveness

This more general description of the proposition denoted by
who walks not only implies, as is supported by argument (V),
that for John to know who walks he should know - de re - of
everyone who walks that he does, but also implies that of
someone who doesn't walk, he should not erroneously believe
that she does. That this is right appears from the validity
of the following argument:

(VIII) John believes that Bill and Suzy walk
    Only Bill walks
    __________________
    John doesn't know who walks
If only Bill walks and John is to know who walks, he should know that only Bill walks and he should not believe that someone else walks as well. We will call this property of propositions denoted by constituent complements their exhaustiveness.

Another way to make the same point is as follows. For a sentence John knows \( p \), where \( p \) is a wh-complement, to be true, it should hold that if one asks John the direct question corresponding to \( p \), one gets exactly the correct answer. So, if only Bill walks and John knows who walks is to be true, John should answer: 'Bill' when asked the question: 'Who walks?', and not for example: 'Bill and Suzy do'. A similar kind of exhaustiveness is exhibited by whether-complements of the form \( \text{whether } \phi \text{ or } \psi \). Consider the following argument:

\[(\text{IX}) \quad \text{John knows whether Mary walks or Bill sleeps} \]
\[\text{Mary doesn't walk and Bill sleeps} \]
\[\text{John knows that Mary doesn't walk and that Bill sleeps} \]

The validity of this argument illustrates that the proposition denoted by an alternative whether-complement is exhaustive too. At an index \( i \), \( \text{whether } \phi \text{ or } \psi \) denotes that proposition \( p \) that holds at an index \( k \) iff the truthvalues of both \( \phi \) and \( \psi \) at \( k \) are the same as at \( i \).

In fact, one can distinguish different degrees of exhaustiveness of complements. Exhaustiveness to the lowest degree implies that for John to know who walks, he should know of everyone who walks that he/she does (and not merely of someone). This is the interpretation of exhaustiveness Karttunen defends (against Hintikka). Exhaustiveness to a stronger degree is used above. Not only do we require that John knows of everyone who walks that he/she does, but also that of no one who doesn't walk, John erroneously believes that he/she does. Exhaustiveness to at least this degree is required to explain the validity of arguments like (VIII). Since Karttunen only incorporates exhaustiveness to the lowest
degree he is unable to account for the validity of (VIII) and (IX). Whether he does consider these arguments to be valid is unclear to us. His analysis forces him to neglect stronger forms of exhaustiveness for a reason not related to this, which will be discussed in the next section.

We feel that an even stronger notion of exhaustiveness is called for. Suppose that John knows of everyone who walks that he/she does; that of no one who doesn't walk, he believes that he/she does; but that of some individual that actually does not walk, he doubts whether he/she walks or not. In such a situation, John would not say of himself that he knows who walks. We see no reason to override his judgment and to claim that in this situation, John does know who walks. This seems to suggest that for John to know who walks, he should not only know of everyone who walks that he/she does, but also of everyone who doesn't; that he/she doesn't. This would mean that (X) (and its inverse) is a valid argument:

\[(X) \quad \text{John knows who walks} \]
\[\text{John knows who doesn't walk} \]

In view of the plausible arguments for exhaustiveness given above, there seems to be only one type of situation in which knowing who walks may not turn out to be the same as knowing who doesn't, i.e. which gives rise to counterexamples against (X). This is the type of situation in which the subject of the propositional attitude is not fully informed as to which set of individuals constitutes the domain of discourse. More in particular, only if a certain individual which in fact belongs to the domain of discourse and which in fact does not walk, does not belong to what John considers to be the domain of discourse, the situation can arise that John knows the positive extension of the predicate \text{walk} without also knowing its negative extension. Such a situation would be a counterexample against (X). (Of course, similar counterexamples can be constructed against the inverse of (X).)
In our formal analysis, we will not deal with cases like these, and consequently, we will accept the validity of (X), for the following reason. Incorporating into the framework of possible world semantics the type of situation in which individuals are not fully informed about what constitutes the domain of discourse is possible, for example by allowing the domain of discourse to vary with possible worlds, but at a cost. It creates a number of well-known problems, for which no definitive solution is yet available. We refrain from incorporating this aspect because of the problems it raises, and we feel free to do so because it is not inherent to an analysis of wh-complements.\(^5\)

Another observation that somewhat weakens the significance of (X), is the following. That one must know the negative extension of a predicate as well as its positive extension, in order to know who satisfies it, appears less dramatic if one realizes that wh-terms, like all other quantifiers, are usually restricted to some, contextually or otherwise specified, subset of the entire domain of all entities. If someone asks who walks?, then he/she does not, or at least not usually, want a specification of all walkers on this earth, but rather a specification which exhausts the walkers in some restricted domain. Such restrictions are usually left implicit, but are there nonetheless. In fact, a contextual restriction functions as a 'hidden' common noun in the wh-term. In the next section, we will see that arguments similar to (X) which contain wh-terms of the form \(\text{which}\) instead of who, unlike (X) are not always valid. Again, the phenomenon of contextual restriction is not specific for wh-complements, but occurs with every kind of quantification in natural language. We therefore feel free to ignore it in our formal analysis.
1.6. A de dicto/de re ambiguity of constituent complements

Sentences in which constituent complements containing wh-terms of the form which $\delta$ occur exhibit a certain kind of ambiguity, which resembles the familiar de dicto/de re ambiguity, and which will henceforth be referred to as such. For example, whether the following argument is valid or not depends on how the conclusion is read.

\[(XI) \quad \text{John knows who walks} \]
\[\quad \text{John knows which girl walks} \]

That (XI) is valid could be argued for as follows. Since the set of girls is a subset of the set of individuals, and since if one knows of a set which of its elements have a certain property, one also knows this of every subset of that set, it cannot fail to hold that John knows which girl walks if he knows who walks. Here the conclusion is taken de re.

On the other hand, one might point out that (XI) is not valid by presenting the following situation. Suppose that just one individual walks. Suppose further that it is a girl. If John knows of this individual that she is the one that walks, but fails to believe that she is a girl, then the premiss of (XI) is true, but its conclusion is false. In this line of reasoning the conclusion is taken de dicto. It takes for granted that the conclusion should be read in such a way that if John is to know which girl walks, he should believe of every individual which is in fact a girl and walks, not only that she walks, but also that she is a girl. Within the first line of reasoning, this assumption is not made. So, whether (XI) is valid or not depends on how the conclusion is read. If we assign it a de re reading (XI) is valid, under a de dicto reading it is not. The de re reading of the conclusion of (XI) can be paraphrased as Of each girl, John knows whether she walks.
This de dicto/de re ambiguity also plays a role in an argument like (XII), which is analogous to argument (X) discussed in the previous section.

(XII) John knows which man walks

John knows which man doesn't walk

Even if we assume the domain of discourse to be the same for every possible world, i.e. if we exclude the kind of counterexample discussed with respect to (X), this argument, unlike its counterpart (X), is not valid as such. It is valid iff both the premiss and the conclusion are read de re, its inverse is then valid as well. Under all other possible combinations of readings (XII) is not valid. Consider e.g. the de dicto/de re combination. Suppose the premiss is true. This is compatible with there being an individual of which John erroneously believes that it is a man, but rightly believes that it does not walk. However, in such a situation, if the conclusion is read de dicto, it is false. Similar examples can be constructed to show that (XII) is also invalid on the two other combinations of readings. This shows, by the way, that the de dicto and de re readings involved are logically independent.

Once we take into account the type of situation, described in the previous section, in which individuals are not fully informed as to which set of individuals constitutes the domain of discourse, arguments like (XII) are no longer valid, even if premiss and conclusion are read de re. For then, the same kind of counterexample as we outlined against (X) can be constructed. The same holds if we incorporate contextual restrictions on quantification in our semantic framework. Then again, arguments like (X), and (XII) read de re are no longer valid in view of the possibility that the subject of the propositional attitude may be mistaken as to which subset of the domain of discourse is determined by the contextual restriction. As we said above, such a contextual restriction functions as a 'hidden' common noun
in the wh-term, thus allowing for de dicto readings with respect to it. The type of situation in which individuals are not fully informed about what constitutes the domain of discourse can be viewed in this way too (e.g. as misinformation about the denotation of the predicate entity). So, there are striking similarities between the three cases, which is also evident from the fact that the counterexamples that can be constructed in each case, are structurally the same. However, only the de dicto/de re ambiguity of constituent complements is particular to an analysis of wh-complements, the other phenomena being of a more general nature.

The possibility of distinguishing de dicto and de re readings of constituent complements marks an important difference between Karttunen's analysis and ours. Karttunen can account only for de re readings. As a result, arguments like (XI) come out valid in his analysis. Nevertheless, (XII) is not a valid argument in Karttunen's theory. This is caused by the fact that he incorporates exhaustiveness only in its weakest form. He explicitly rejects stronger forms of exhaustiveness because, combined with the fact that his analysis accounts only for de re readings, this would make arguments like (X) and (XII) valid. Rejecting strong exhaustiveness, Karttunen is able to regard (XII) as invalid but for the wrong reason, as can be seen from the fact that (XI) still is valid in his analysis. Worse, he thereby deprives himself of the means to account for the validity of arguments like (VIII) and (IX). We believe that an analysis which can both account for exhaustiveness and for the fact that the validity or invalidity of (XI) and (XII) depends on how the conclusion is read, is to be preferred.

1.7. Implicatures versus presuppositions

From the previous discussion, in particular from sections 1.4 and 1.5, it will be clear that we consider the following arguments to be valid ones:
One might object to the validity of these arguments by pointing out that John knows who walks presupposes that at least/exactly one individual walks, and that John knows whether Peter walks or Mary walks presupposes that at least/exactly one of the alternatives is the case. Therefore, one might continue, the first premiss of these arguments is semantically deviant in some sense, say lacks a truth value, if the second premiss happens to be true.

We adhere to the view, also advocated by Karttunen, that it is better to regard these phenomena as (pragmatic) implicatures and not as presuppositions in the strict semantic sense. More generally, we believe that many of the arguments put forward in Kempson (1975), Wilson (1975) and Gazdar (1979) showing that presupposition is a pragmatic notion should hold for presuppositions of wh-complements as well. (See also the discussion in section 5.)

In Karttunen's analysis, (XIII)–(XVI) are valid as well. The validity of (XIII) and (XV), however, has to be secured by a special clause in a meaning postulate relating know + wh to know that. The need for this special clause explains it–
self by the fact that the validity of (XIII) and (XV) is at odds with not incorporating exhaustiveness. One would expect that in an analysis in which (VIII) and (IX) of section 1.5 are not valid, (XIII) and (XV) would not be valid either.

1.8. Towards a uniform treatment of complements

A distinctive feature of our analysis is that wh-complements are taken to be proposition denoting expressions. This is an important difference between our approach and that of others. To mention only two, in Karttunen's they denote sets of propositions, and in Hausser's they are of all sorts of different categories. From this difference other differences follow, e.g. the possibility of a uniform treatment of complements. For, besides the fact that it provides a simple and direct account of the validity of the various arguments discussed above, the hypothesis that that- and wh-complements denote the same kind of semantic object makes it possible to assign them to the same syntactic category. This seems especially attractive in view of the fact that it is possible to conjoin wh- and that-complements:

(1) John knows that Peter has left for Paris, and also whether Mary has followed him
(2) Alex told Susan that someone was waiting for her, but not who it was

Further, if both kinds of complements can belong to the same syntactic category, we are no longer forced to assume there to be two complement taking verbs know, of different syntactic categories, and of different semantic types: one which takes that- and one which takes wh-complements. We need not acknowledge two different relations of knowing which are only linked indirectly, i.e. by a meaning postulate. This happens for example in Karttunen's analysis. There wh-complements denote sets of propositions, and that-complements denote propositions. Consequently, there are two
relations of knowing. Karttunen reduces the relation to sets of propositions to the relation to propositions by postulating that \( x \) stands in the first relation to a set of propositions iff \( x \) stands in the second relation to all the elements of this set. (Actually, his postulate is slightly more complex, but that is irrelevant here.) Not only is this a rather cumbersome way of accounting for our intuition that there is one verb know, it is also not at all clear whether a strategy like this is applicable in all cases. A case in point are truly intensional verbs which take both wh-complements and that-complements, such as guess and matter. If we categorize wh-complements and that-complements differently, the problem arises how to account for the obvious semantic relation (identity) between the two verbs guess (or matter, etc.) we are then forced to assume. In these cases one cannot reduce the one to the other, for obvious reasons. For example, John guesses who comes to dinner does not mean the same as for all \( x \), if \( x \) comes to dinner, then John guesses that \( x \) comes to dinner. In what other way the interpretation of the two verbs could be related adequately, is quite unclear. In the analysis proposed in this paper, there is no problem at all. Since wh-complements and that-complements are of the same syntactic category, no verbs need to be duplicated in the syntax. The extensionality of verbs such as know and tell can be accounted for by means of a meaning postulate. As for truly intensional verbs such as guess and matter, they express the same relation to a propositional concept, be they combined with a wh-complement or with a that-complement. The semantic differences between the two constructions are accounted for by the different properties of the propositional concepts expressed by wh-complements and that-complements respectively.

Of course, there are also verbs such as wonder, which take only wh-complements, and verbs such as believe, which take only that-complements. The relevant facts can easily be accounted for by means of syntactic subcategorization or, preferably, in lexical semantics, by means of meaning postulates.
2. Ty2 and the semantic analysis of wh-complements

In section 1 we have sketched informally the outlines of a semantics for wh-complements. In particular, we argued that wh-complements denote propositions and do this in an index dependent way. The description of this index dependent character involves comparison of what is the case at different indices. This leads to the choice of a logical language in which reference can be made to indices and in which relations between indices can be expressed directly. The language of two-sorted type theory, Gallin's Ty2, is such a language. In this section we will show that it serves our purpose to express the semantics of wh-complements quite well.

Ty2 is a simple language. Rather than by stating the explicit definitions, we will discuss its syntax and semantics by comparing it with IL, the language of intensional logic of PTQ, thereby indicating how Ty2 can be put to the same use as IL in the PTQ system. We will also make some methodological remarks on the use of Ty2. For a formal exposition and extensive discussion of Ty2, the reader is referred to Gallin (1975).

2.1. Ty2, the language of two-sorted type theory

The basic difference between IL and Ty2 is that s is not introduced only in constructing more complex, intensional types, but that it is a basic type, just like e and t. Complex types can be constructed with s in exactly the same way as with e and t. As is to be expected, the set of possible denotations of type s is the set of indices. Since
it is a type like any other now, we will also employ constants and variables of type s. This means that it is possible to quantify and abstract over indices, making the necessity operator $\Box$ and the cap operator $\wedge$ superfluous.

A model for Ty2 is a triple $<A, I, F>$, $A$ and $I$ are disjoint non-empty sets, $A$ is to be the set of individuals, $I$ the set of indices. $F$ is an interpretation function which assigns to every constant a member of the set of possible denotations of its type. Notice the difference with the interpretation function $F$ of IL-models, which assigns senses and not denotations to constants. The interpretation of a meaningful expression $a$ of Ty2, written as $\langle a \rangle_{M, g}$, is determined with respect to a model $M$ and an assignment $g$ only. (As usual, $g$ assigns to every variable a member of the set of possible denotations of its type.)

The important difference with interpretations in IL, is that the latter also need an index to determine the interpretation of an expression. This role of indices as a parameter in the interpretation is taken over in Ty2 by the assignment functions. The effect of interpreting in IL an expression with respect to an index $i$ is obtained in Ty2 by interpreting expressions with respect to an assignment which assigns to a free index variable occurring in the expression the index $i$. To an index dependent expression of IL (an expression of which the denotation varies from index to index) there corresponds an expression in Ty2 which contains a free index variable. The result is an expression the interpretation of which varies from assignment to assignment. A formula $\phi$ is true with respect to $M$ and $g$ iff $\langle \phi \rangle_{M, g} = 1$; $\phi$ is valid in $M$ iff for all $g$, $\phi$ is true with respect to $M$ and $G$; $\phi$ is valid iff for all $M$, $\phi$ is valid in $M$.

2.2. Translating into Ty2

To illustrate the difference between IL and Ty2, consider first how the English verb walk translates into Ty2. Instead of simply translating it into a constant of type $f(IV)$, it is
translated into the expression \( \text{walk}(v_0, s) \), in which \( \text{walk} \) is a constant of type \( <s, f(IV)> \), and \( v_0, s \) is a variable of type \( s \), so the full translation of the verb is an expression of type \( f(IV) \).

All translations of basic expressions will contain the same free index variable. For this purpose we use \( v_0, s \), the first variable of type \( s \), which from now on we will write as \( a \). Therefore, the translation of a complex expression will be interpreted with respect to the index assigned to \( a \) by the assignment function.

The rules for translating PTQ English into Ty2 can be obtained by using the fact that \( \lambda \alpha \) expresses the same function in Ty2 as \( \alpha \) in IL, \( \sim \alpha \) is the same as \( \alpha(a) \); and \( \alpha \) corresponds to \( \forall a \). Consider the following examples of Ty2 analogues of (parts of) some PTQ translation rules, in which \( \sim \) abbreviates 'translates into'.

\((T:1)\)

(a) If \( \alpha \) is in the domain of \( g \), then \( \alpha \sim g(\alpha)(a) \)

With the usual exceptions, \( g \) associates a basic expression of category \( A \) with a Ty2 constant \( \alpha' \) of type \( <s, f(A)> \), giving its sense. The full translation of \( \alpha, \alpha'(a) \), gives as usual its denotation.

\((T:1)\)

(b) \( \text{be} \sim \lambda P \lambda x P(a)(\lambda a \lambda y[x(a) = y(a)]) \)
(c) \( \text{necessarily} \sim \lambda P \forall a[P(a)] \)
(d) \( \text{John} \sim \lambda P[P(a)(\lambda aj)] \)
(e) \( \text{he} \sim \lambda P[P(a)(x_n)] \)

\((T:2)\)

If \( \delta \in P_{CN} \), and \( \delta \sim \delta' \), then every \( \delta \sim \lambda P \forall x[\delta'(x) \rightarrow P(a)(x)] \)

\((T:4)\)

If \( \alpha \in P_{T} \), \( \delta \in P_{IV} \), \( \alpha \sim \alpha' \), and \( \delta \sim \delta' \), then \( F_4(\alpha, \delta) \sim \alpha'(\lambda a \delta') \)

Of course, the meaning postulates of PTQ can be translated into Ty2 as well. (Notice that the rigid designator view of proper names like \( \text{John} \) is already implemented in its
The translation of a sentence is illustrated in (3):

\[
\begin{align*}
\text{man} & \quad \text{man}(a) \\
\text{every man} & \quad \lambda \forall x [\text{man}(a)(x) \land \text{P}(a)(x)] \\
\text{walk} & \quad \text{walk}(a) \\
\text{every man walks} & \quad \lambda \forall x [\text{man}(a)(x) \land \text{P}(a)(x)] (\lambda a [\text{walk}(a)]) \\
& \Rightarrow \\
& \quad \forall x [\text{man}(a)(x) \land \text{walk}(a)(x)] \\
& \Rightarrow \\
& \quad \forall u \text{ man}_*(a)(u) \land \text{walk}_*(a)(u)
\end{align*}
\]

2.3. That-complements and whether-complements in Ty2

The proposition denoting expression which is to be the translation of a that-complement that \( \phi \) can be constructed from the translation of \( \phi \) by using abstraction over indices. For example, the sentence Mary walks translates into the formula walk\_*(a)(m); from this formula we can form the expression \( \lambda a [\text{walk}_*(a)(m)] \). Its interpretation \((\lambda a [\text{walk}_*(a)(m)])_{M,g}\) is that proposition \(p \in \{0,1\}^I\) such that for every index \(i: p(i) = 1\) iff \((\text{walk}_*(a)(m))_{M,g[i/a]}^\ast\). By \(g[x/y]\) we will understand that assignment \(g'\) which is like \(g\) except for the possible difference that \(g'(y) = x\). So, \(\lambda a [\text{walk}_*(a)(m)]\) denotes the characteristic function of the subset of the set of indices at which it is true that Mary walks.

Notice that \(\lambda a [\text{walk}_*(a)(m)]\) does not contain a free index variable. This makes it the index independent expression it was argued to be in 1.1 and 1.2. Its sense, denoted by the expression \(\lambda a \lambda a [\text{walk}_*(a)(m)]\), is a constant function from indices to propositions.
In section 1.1 we circumscribed the denotation of whether Mary walks as follows: at an index at which it is true that Mary walks it denotes the proposition that Mary walks, and at an index at which it is false that Mary walks it denotes the proposition that Mary doesn't walk. Another way of saying this is that at an index $i$ whether Mary walks denotes that proposition $p$ such that for every index $k$, $p$ holds true at $k$ iff the truth value of Mary walks at $k$ is the same as at $i$. In Ty2 this can be expressed by the index dependent proposition denoting expression (4), the interpretation of which is given in (4').

\[
\lambda i[\text{walk}_*(a)(m) = \text{walk}_*(i)(m)]
\]

\[
\lambda i[\text{walk}_*(a)(m) = \text{walk}_*(i)(m)] \Gamma_{M,g} \text{ is that}
\]

proposition $p \in \{0, 1\}$ such that for every index $k \in I: p(k) = 1$ iff

\[
[[\text{walk}_*(a)(m) = \text{walk}_*(i)(m)]_{M,g[k/i]} = 1
\]

iff

\[
[[\text{walk}_*(a)(m)]_{M,g[k/i]} = [[\text{walk}_*(i)(m)]_{M,g[k/i]}
\]

iff

\[
[[\text{walk}_*(a)(m)]_{M,g} = [[\text{walk}_*(i)(m)]_{M,g[k/i]}
\]

So, at the index $g(a)$, the expression (4) denotes the characteristic function of the set of indices at which the truth value of Mary walks is the same as at the index $g(a)$. The index dependent character of whether-complements discussed in 1.1 and 1.2 is reflected by the fact that a free index variable occurs in their translation. The expression

\[
\lambda a \lambda i[\text{walk}_*(a)(m) = \text{walk}_*(i)(m)]
\]

denoting the propositional concept which is the sense of whether Mary walks, does not denote a constant function. For different indices its value may be a different proposition.

2.4. Constituent complements in Ty2

The kind of expressions which denote propositions in the required index dependent way can be constructed not only from formulas, such as $\text{walk}_*(a)(m)$ in (4), but from expressions of
arbitrary type. Let a/a/ and a/i/ be two expressions such that where the first has free occurrences of a, the second has free occurrences of i, and vice versa. Then the expression (5) denotes a proposition in an index dependent way, as its interpretation given in (5') shows.\(^\text{10}\)

\[(5) \, \lambda i[a/a/ = a/i/]\]

\[(5') \, [\lambda i[a/a/ = a/i/]]_{M,g} \text{ is that proposition } p \in \{0,1\}^I \text{ such that for every } k \in I, p(k) = 1 \text{ iff } [a/a/]_{M,g} = [a/i/]_{M,g}[k/i].\]

Expressions serving as translations of wh-complements will always be of this form. The translation of a whether-complement has been given in (4). There a/a/ is the formula walk*(a)(m). An example of an expression which will serve as the translation of a constituent complement is:

\[(6) \, \lambda i[\lambda u[\text{walk}^*(a)(u)] = \lambda u[\text{walk}^*(i)(u)]].\]

In this case, a/a/ is \(\lambda u[\text{walk}^*(a)(u)]\), an expression of type <e,t>. At an index g(a), (6) denotes that proposition which holds at an index k iff \([\lambda u[\text{walk}^*(a)(u)]]_{M,g}[k/i]\) is the same set as \([\lambda u[\text{walk}^*(i)(u)]]_{M,g}[k/i]\). I.e. at an index g(a), (6) denotes that proposition which holds true at an index k iff the denotation of walk* at that index k is the same as at the index g(a). And this is precisely the index dependent proposition which, in section 1.4, we required to be the denotation of the constituent complement who walks.

2.5. Methodological remarks on the use of Ty2

In this section we will defend our use of Ty2 against some objections that are likely to be raised against it.

A first objection might be that translations in Ty2 are (even) less 'natural' than those in IL. In view of the fact that within a compositional semantic theory the level of translation, be it in Ty2 or in IL, is in principle
dispensable, we do not see that there is empirical motivation for this kind of objection.

A second objection that is often raised against the use of a logical language which allows for reference to and quantification over indices, is that it involves stronger ontological commitments than a language in which the relevant phenomena are dealt with by means of intensional operators. We do not think that this objection holds. It is not the object language in isolation, but the object language together with the meta-language in which its semantics is described that determines ontological commitments. Since the statement of the semantics of intensional operators involves reference to and quantification over indices as well, the commitments are the same. The dispensability of the translation level even strengthens this point.

A more serious reason for preferring an operator approach to a quantificational approach might be that for some purposes one does not need the full expressive power of a quantificational language and therefore prefers a language with operators which has exactly the, restricted, expressive power one needs. In fact, in section 6.2 we will point out that by the introduction of a new intensional operator to IL, one can get a long way in the semantic analysis of wh-complements. However, phenomena remain which escape treatment in this intensional language, an example is discussed in 6.1.

Taking the semantic analysis of tense into consideration as well, we think a lot can be said in favour of a logical language in which reference to and quantification over indices is possible. It appears that analyses set up in the Priorian fashion tend to become stronger and stronger, up to a point where if there is still a difference in expressive power with quantificational logic at all, this advantage is annihilated by the unintuitiveness and complexity of the language used. For an illuminating discussion of these points, see Van Benthem (1978). In fact, we think that Ty2 provides a suitable framework for the incorporation of a semantic analysis of tense in the vein of Needham (1975) into a Montague Grammar as well.
3. Wh-complements in a Montague Grammar

In this section we will outline how the semantic representations of complements in Ty2, given in section 2, can systematically be incorporated in the framework of a Montague Grammar. We will not present the syntactic part of our proposal in detail. In particular, the definitions of the various syntactic functions occurring in the syntactic rules will not be stated until section 4. We will concentrate on the explanation of the semantic facts discussed in section 1.

3.1. Whether-complements and that-complements

Complements are expressions which denote propositions. Therefore, they should translate into expressions of type $<s,t>$. In PTQ there is no syntactic category which is mapped onto this type. Therefore we add the following clauses to the definitions of the set of categories and the function $f$ mapping categories into types;

If $A \in \text{CAT}$, then $\overline{A} \in \text{CAT}; f(\overline{A}) = <s,f(A)>$

So, $\overline{\text{e}}$ will be the category of complements. Complement embedding verbs, such as know, tell, wonder and believe will be of category IV/\overline{\text{e}}. As we remarked in section 1.8, the categories $\overline{\text{e}}$ and IV/\overline{\text{e}} will have to be subcategorized, since not all of these verbs take all kinds of complements. This can be done in an obvious way, with which we will not be concerned here.

In (7) an analysis tree of a sentence containing a that-complement is given together with its translation. Here and
elsewhere, notation conventions and meaning postulates familiar from PTQ are applied whenever possible.

(7) John knows that Mary walks, t
   know(a)(laj, (lala[walk, (a)(m)]))

The syntactic rule deriving a that-complement and the corresponding translation rule are:

(S:THC) If \( \phi \in P \), then that \( \phi \in P\bar{\xi} \)
(T:THC) If \( \phi \sim \phi' \), then that \( \phi \sim \lambda\phi' \)

The rule which embeds the complement under a verb is a simple rule of functional application. The corresponding rule of translation follows the usual pattern:

(S:IV/\bar{\xi}) If \( \delta \in P_{IV}/\bar{\xi} \) and \( \rho \in P_{\bar{\xi}} \), then \( P_{IV}/\bar{\xi}(\delta, \rho) \in P_{IV} \)
(T:IV/\bar{\xi}) If \( \delta \sim \delta' \) and \( \rho \sim \rho' \), then \( P_{IV}/\bar{\xi}(\gamma, \rho) \sim \delta'(\lambda\rho') \)

Sentence (7) expresses that an intensional relation of knowing exists between the individual concept denoted by \( \lambda aj \)
and the propositional concept denoted by \( \lambda a \lambda l[\text{walk}_a(a)(m)] \).

By means of a meaning postulate, to be given below, this intensional relation will be reduced to an extensional one.

In (8) an analysis tree and its translation of a sentence containing a whether-complement are given:

(8) John knows whether Mary walks, \( t \)
\[
\text{know}(a) (\lambda a j, \lambda a l[\text{walk}_a(a)(m) = \text{walk}_a(i)(m)])
\]

The rule which forms a whether-complement from a sentence, and the corresponding translation rule are as follows. (An asterisk indicates that a rule will later be revised.)

\[(S:WHC^*) \text{ If } \phi \in P_T, \text{ then whether } \phi \in P_T^e\]
\[(T:WHC^*) \text{ If } \phi \sim \phi', \text{ then whether } \phi \sim \lambda i[\phi' = \lambda a \phi'](i)\]

Whether-complements can be generated by a more general rule:\n
\[(S:WHC) \text{ If } \phi_1, \ldots, \phi_n \in P_T, \text{ then whether } \phi_1 \text{ or } \ldots \text{ or } \phi_n \in P_T^e\]
\[(T:WHC) \text{ If } \phi_1 \sim \phi'_1, \ldots, \phi_n \sim \phi'_n, \text{ then whether } \phi_1 \text{ or } \ldots \text{ or } \phi_n \sim \lambda i[\phi'_1 = [\lambda a \phi'_1](i) \wedge \ldots \wedge \phi'_n = [\lambda a \phi'_n](i)]\]
Obviously, $(S:\text{WHC}^*)$ and $(T:\text{WHC}^*)$ are special cases of $(S:\text{WHC})$ and $(T:\text{WHC})$.

In general, whether-complements of the form whether $\phi_1$ or ... or $\phi_n$ are ambiguous between an alternative and a yes/no reading. The following two trees and their translations illustrate this ambiguity.

\begin{equation}
\text{(9) whether John walks or Mary walks, } t \\
\lambda i[(\text{walk}_*(a)(j) = \text{walk}_*(i)(j)) \land \\
(\text{walk}_*(a)(m) = \text{walk}_*(i)(m))]
\end{equation}

\begin{align*}
\text{John walks, } t & & \text{Mary walks, } t \\
\text{walk}_*(a)(j) & & \text{walk}_*(a)(m)
\end{align*}

\begin{equation}
\text{(10) whether John walks or Mary walks, } t \\
\lambda i[(\text{walk}_*(a)(j) \lor \text{walk}_*(a)(m)) = \\
(\text{walk}_*(i)(j) \lor \text{walk}_*(i)(m))]
\end{equation}

\begin{align*}
\text{John walks or Mary walks, } t & & \\
\text{walk}_*(a)(j) \lor \text{walk}_*(a)(m)
\end{align*}

3.2. Extensional and intensional complement embedding verbs

In section 1.3 we stated that verbs such as know and tell are extensional. The meaning postulate guaranteeing this reads as follows:

\begin{equation}
(M:\text{IV/t}) \exists Mx\forall vi[\delta(i)(x,r) = M(i)(x(i),r(i))]
\end{equation}

$M$ is a variable of type $<s,<s,t>,<e,t>>$; $x$ of type $<s,e>$; $r$ of type $<s,<s,t>>$; $i$ of type $s$; and $\delta$ is the translation of know, tell, etc.

Requiring this formula to hold in all models guarantees that to certain intensional relations between individual concepts
and propositional concepts, extensional relations between
individuals and propositions correspond. We extend the sub-
star notation convention of PTQ as follows:

\[(SNC) \delta_\ast = \lambda a \lambda p \lambda u (\delta(a)(\lambda a p)(\lambda a u))\]

\(p\) is a variable of type \(<s,t>\), \(u\) of type \(e\)

Combining (MP:IV/\xi) with (SNC) we can prove that (11) is
valid: 13

\[(11) \ \forall i [\delta(i)(x, r) = \delta_\ast(i)(x(i), r(i))]\]

If we apply (11) to the translations of (7) John knows that
Mary walks and (8) John knows whether Mary walks, we get the
following results:

\[(7') \ know_\ast (j, \lambda a [walk_\ast(a)(m)])\]
\[(8') \ know_\ast (j, \lambda i [walk_\ast(a)(m) = walk_\ast(i)(m)])\]

Formula (7') expresses that the individual John knows the
proposition that Mary walks. In (8') it is expressed that
John knows the proposition denoted by
\(\lambda i [walk_\ast(a)(m) = walk_\ast(i)(m)]\). As has been indicated in
section 2.2, which proposition is denoted by this expression
at \(g(a)\) depends on the truth value of \(walk_\ast(a)(m)\) at \(g(a)\). More
generally, we can prove that the following holds: 14

\[(12) \ [\lambda i [\phi/a/ = \phi/i/]]_{M, g} = \begin{cases} [\lambda i [\phi/i/]]_{M, g} & \text{if} \\
[\phi/a/]_{M, g} & \text{if} \\
[\lambda i [\neg \phi/i/]]_{M, g} = 0 & \text{if} \end{cases}\]

Given (12), it is obvious that the arguments (I) and (II) of
section 1.1 are valid. Their translations are:

\[(I') \ know_\ast(a)(j, \lambda i [walk_\ast(a)(m) = walk_\ast(i)(m)])\]
\[walk_\ast(a)(m) \]
\[know_\ast(a)(j, \lambda a [walk_\ast(a)(m)])\]
(II') \[
\begin{align*}
\text{know}_*(a) \{ j, \lambda i[\text{walk}_*(a)(m)] = \text{walk}_*(i)(m) \} \\
\lambda \text{walk}_*(a)(m) \\
\text{know}_*(a) \{ j, \lambda a[\neg \text{walk}_*(a)(m)] \}
\end{align*}
\]

Since (MP:IV/\neg) also holds for \textit{tell}, the arguments (III) and (IV) are rendered valid in exactly the same way. And precisely because (MP:IV/\neg) does not hold for intensional verbs, arguments like (I)-(IV) cannot be constructed for them. The relations expressed by these verbs are not extensional in object position, their second argument is irreducibly a propositional concept.

Argument (IX), concerning the exhaustiveness of alternative whether-complements, is discussed in section 3.4. The arguments (XV) and (XVI) of section 1.7 are left to the reader.

3.3. Single constituent complements with who

First we consider constituent complements which contain just one occurrence of the wh-term who. An example of an analysis tree of a sentence containing such a complement, together with its translation is:

(13) John knows who walks, t
\[
\begin{align*}
\text{know}_*(a) \{ j, \lambda i[\text{walk}_*(a)(u)] = \lambda u[\text{walk}_*(i)(u)] \} \\
\lambda P[P(a)(\lambda a)] \\
\text{know}_*(a) \{ \lambda i[\lambda u[\text{walk}_*(a)(u)] = \lambda u[\text{walk}_*(i)(u)] \} \\
\text{who walks}, \bar{t} \\
\text{know}(a) \\
\lambda i[\lambda u[\text{walk}_*(a)(u)] = \lambda u[\text{walk}_*(i)(u)] \\
\text{who walks}, tAAAA \\
\lambda x_0[\text{walk}(a)(x_0)] \\
\text{he}_0 \text{walks}, t \\
\text{walk}(a)(x_0)
\end{align*}
\]
Constituent complements are formed from sentences containing a syntactic variable, but in an indirect way. First a so-called abstract is formed, an expression of category \( t//e \). The wh-term \( \text{who(m)} \) is placed at the front of the sentence, certain occurrences of the variable are deleted, others are replaced by suitable pro-forms. For details see section 4. In fact, our use of the phrase 'wh-term' is rather misleading. Unlike the wh-terms in Karttunen's analysis for example, they do not belong to a fixed syntactic category. In this they are like their logical language counterpart, the \( \lambda \)-abstraction sign. Why this is necessary is explained in section 3.8. This rule of abstract formation and its translation are:

\[
(S:AB1) \text{ If } \phi \in P_t, \text{ then } F^A_{B1,n}(\phi) \in P_{t//e} \\
(T:AB1) \text{ If } \phi \sim \phi', \text{ then } F^A_{B1,n}(\phi) \sim \lambda x_n(\phi')
\]

The translation of an abstract is a predicate denoting expression. From these abstracts constituent complements are formed. The syntactic rule that does this is a category changing rule. The corresponding translation rule turns predicate denoting expressions into proposition denoting expressions in the way indicated in (5) in section 2.4.

\[
(S:CCF*) \text{ If } \chi \in P_{t//e'}, \text{ then } F^C_{CF}(\chi) \in P_P \\
(T:CCF*) \text{ If } \chi \sim \chi', \text{ then } F^C_{CF}(\chi) \sim \lambda i[\chi' = [\lambda x'](i)]
\]

The intermediate level of abstracts is not strictly needed for single constituent complements, but, as shall be argued in section 3.8, it is essential for a correct analysis of constituent complements that contain more than one occurrence of a wh-term. (Moreover, an attractive feature of our analysis is that another kind of wh-construction, relative clauses, can both syntactically and semantically be treated as abstracts as well, see section 4.5.)

We are now able to show that argument (V) of section 1.4 is valid. Its translation is:
(V') \text{know}_x(a)(j, \lambda i[\text{walk}_x(a)(u)] = \lambda u[\text{walk}_x(i)(u)])

\text{walk}_x(a)(b)

\text{know}_x(a)(j, \lambda a[\text{walk}_x(a)(b)])

From \[\text{[\text{walk}_x(a)(b)]}_{M,g} = 1,\] it follows that
\[\text{[\lambda u[\text{walk}_x(a)(u)]]}_{M,g} = 1.\] So, at every index k such that \[\text{[\lambda i[\lambda u \text{walk}_x(a)(u)] = \lambda u[\text{walk}_x(i)(u)]]}\] \[\text{[\text{[\lambda u[\text{walk}_x(i)(u)]]}]}_{M,g} = 1,\] it also holds that \[\text{[\lambda u[\text{walk}_x(i)(u)]]}_{M,g} = 1.\]

I.e. at every such index k: \[\text{[\lambda a[\text{walk}_x(a)(b)]]}_{M,g} = 1.

Under the assumption that to know a proposition is to know its entailments, this means that (V') is valid. The assumption in question can be laid down in a meaning postulate in a straightforward way.

3.4. Exhaustiveness

It is easy to see that argument (VIII) of section 1.5, illustrating the exhaustiveness of the proposition denoted by a constituent complement is valid too. Its translation is:

(VIII') \text{believe}_x(a)(j, \lambda a[\text{walk}_x(a)(b) \land \text{walk}_x(a)(s)])

\text{vu}[b = u \leftrightarrow \text{walk}_x(a)(u)]

\text{[know}_x(a)(j, \lambda i[\text{walk}_x(a)(u)] = \lambda u[\text{walk}_x(i)(u)]])

Suppose the conclusion is false and the second premiss is true. Then \[\text{[\lambda u \text{walk}_x(a)(u)]}_{M,g} = 1.\] From this it follows that \[\text{[\lambda u[\text{walk}_x(a)(u)]]}_{M,g} = 1.\ Under the assumption that knowing implies believing, also to be laid down in a meaning postulate, it follows that the first premiss is false. So, (VIII') is valid. We leave it to the reader to verify that the similar arguments (XIII) and (XIV) of section 1.7 are valid too.

Argument (IX), showing the exhaustiveness of whether-complements, translates as follows:
From the truth of the second premiss it follows that for every index $k$ such that $\text{eval}(\lambda i[\text{walk}_*(a)(m) = \text{walk}_*(i)(m)] \land (\text{sleep}_*(a)(b) = \text{sleep}_*(i)(b)])_M, g[k/\alpha] = 1$ it holds that $\text{eval}(\neg\text{walk}_*(a)(m) \land \text{sleep}_*(a)(b))_M, g[k/\alpha] = 1$ and thus that for every such index $k$ it holds that $\text{eval}(\lambda u[\neg\text{walk}_*(a)(m) \land \text{sleep}_*(a)(b)])_M, g[k/\alpha] = 1$.

As we already indicated in our discussion of exhaustiveness in section 1.5, argument (X), which translates as (X'), comes out valid in our formal analysis.

As we argued in section 1.5, the fact that (X') is valid is not due to the incorporation of exhaustiveness, but is a consequence of the fact that the only type of situation which can give rise to counterexamples to (X'), the situations in which the subject of the propositional attitude is not fully informed as to what constitutes the domain of discourse, is not dealt with in the semantic framework used here.

Situations of misinformation about what subset of the domain is determined by a contextual restriction on the range of who, can be regarded as a subtype of this kind of situation. Once either one of these two aspects, which being of a general nature need to be built into the semantic framework anyway, is incorporated, counterexamples to (X') can be constructed, which are structurally the same as those discussed in the next section with regard to argument (XII).
3.5. **Single constituent complements with which**

The analysis of constituent complements in which one occurrence of a wh-term of the form which δ occurs is illustrated in the following example:

(14) John knows which man walks, t

\[
\text{know}_*(a)\{j, \lambda i[\lambda u \text{man}_*(a)(u) \land \text{walk}_*(a)(u)]
\]

\[
= \lambda u[\text{man}_*(i)(u) \land \text{walk}_*(i)(u)]
\]

Again, the complement is formed in two steps. First, from a sentence containing a syntactic variable, and a common noun phrase an abstract is formed. The syntactic function which does this is quite similar to the one forming abstracts with who. The syntactic rule and the translation rule are:

\[
(S:AB2) \text{ If } \phi \in P_t \text{ and } \delta \in P_{CN}, \text{ then } F_{AB2,n}(\delta, \phi) \in P_t//e
\]

\[
(T:AB2) \text{ If } \phi \sim \phi' \text{ and } \delta \sim \delta', \text{ then } F_{AB2,n}(\delta, \phi) \sim \lambda x_n(\delta'(x_n) \land \phi')
\]
The translation is a complex predicate denoting expression. It denotes the conjunction of the predicate denoted by the common noun phrase and the predicate that can be formed from the sentence.

The second step is to apply the category changing rule (S:CCF*) which turns abstracts into complements. This way of constructing complements like which man walks gives rise to the de dicto reading discussed in section 1.6. The proposition 

\[ \lambda u[\text{man}(a)(u) \land \text{walk}(a)(u)] = \lambda u[\text{man}(i)(u) \land \text{walk}(i)(u)] \]

holds at an index \( k \) iff the intersection of the set of men and the set of walkers at \( k \) is the same as at \( g(a) \). If John knows this proposition, it is implied that if a certain individual is a walking man, John knows both that it is a man and that it walks. In view of this, (XII'), the translation of (XII) with both the premiss and the conclusion in the de dicto reading is not valid:

\[
(\text{XII'}) \quad \text{know}(a)(j, \lambda u[\text{man}(a)(u) \land \text{walk}(a)(u)]
= \lambda u[\text{man}(i)(u) \land \neg \text{walk}(i)(u)]
\]

A counterexample can be constructed as follows. Suppose that for some assignment \( g \) and for some individual \( d \) it holds that:

\[ \llbracket \text{walk}(a) \rrbracket_M^g(d) = \llbracket \text{man}(i) \rrbracket_M^g(d) = \llbracket \text{walk}(i) \rrbracket_M^g(d) = 0, \]

and \( \llbracket \text{man}(a) \rrbracket_M^g(d) = 1. \) Then we can construct a model in which the proposition which is the argument in the premiss holds at \( g(i) \), whereas the proposition which is the argument in the conclusion does not. So, the proposition in the premiss does not entail the proposition in the conclusion, which, given the usual semantics of know would be the only way in which the premiss could imply the conclusion. By a similar argument it can be shown that the inverse of (XII') is not valid either.
3.6. De re readings of constituent complements

In section 1.6 we argued that (XII) is valid iff both its premiss and its conclusion are read de re (excluding situations in which individuals may not be fully informed about the domain of discourse). This means that a second way to derive sentences containing constituent complements should be added to the syntax. In this derivation process common noun phrases are quantified into sentences containing a common noun variable \( \text{one}_0, \text{one}_1, \ldots \), which translate into \( o_0, o_1, \ldots \) of type \( \langle \langle s, e \rangle, t \rangle \). The rule of common noun quantification and the corresponding translation rule are as follows:

\[
\begin{align*}
(S:CNQ) & \text{ If } \phi \in P_t \text{ and } \delta \in P_{CN'} \text{ then } F_{CNQ,n}(\delta, \phi) \in P_t \\
(T:CNQ) & \text{ If } \phi \sim \phi' \text{ and } \delta \sim \delta', \text{ then } \lambda \alpha F_{CNQ,n}(\delta, \phi) \sim \lambda \alpha F_{CNQ,n}(\delta', \phi')
\end{align*}
\]

The sentence \textit{John knows which man walks} can now also be derived as follows:
(15) John knows which man walks

\[
\text{know}_*(a) \left( j, \lambda i[\lambda u[\text{man}_*(a)(u) \land \text{walk}_*(a)(u)]
\right.
\]

\[
= \lambda u[\text{man}_*(a)(u) \land \text{walk}_*(i)(u)])
\]

\[
\text{man}
\]

The proposition denoted by the complement in the premiss at \(g(a)\) is the same as the one denoted by the complement of the conclusion at \(g(a)\). The first proposition holds true at an index \(k\) iff the intersection of the set of men at \(g(a)\) and the set of walkers at \(g(a)\) is the same as the intersection
of the set of men at g(a) and the set of walkers at k. Clearly, this is the case iff the intersection of the set of men at g(a) and the set of non-walkers at g(a) is the same as the intersection of the set of men at g(a) and the set of non-walkers at k, i.e. iff the second proposition holds true at k. So, both (XII") and its inverse are valid arguments.

We leave it to the reader to satisfy her/himself that (XI) with its conclusion read de dicto is not valid, whereas with the conclusion read de re it is.

3.7. Multiple constituent complements

In this section we will outline our treatment of constituent complements in which more than one wh-term occurs. The construction of multiple constituent complements starts out with a sentence containing more than one syntactic variable. By using one of the abstract formation rules given above, an abstract is obtained from such a sentence. From this abstract, a 'higher level' abstract is formed. This process can be repeated as long as there are variables left, each time resulting in an abstract of one level higher. This means that there is not just one category of abstracts, but a whole set of abstract categories. The definition of this set and of the corresponding set of abstract types are as follows:

(a) AB is the smallest subset of CAT such that
   (i) t///e £ AB
   (ii) if A £ AB, then A/e £ AB
(b) AB' is the smallest subset of TYPE such that
    if A £ AB, then f(A) £ AB'

To the two rules which formed abstracts from sentences, one for who and one for which, there correspond two rules, or better rule schemata, which from an abstract form an abstract of one level higher:

(S:AB3) If x £ P_A, A £ AB, then F_{AB3} n(x) £ P_A/e
(S:AB4) If $\chi \in P_A$, $A \in AB$, and $\delta \in P_{CN}$, then $F_{AB4,n}(\delta, \chi) \in P_{A/e}$

The two syntactic functions of this pair of rules differ from those of the former pair. In particular, the wh-term is not placed in front of the abstract, but is substituted for a certain occurrence of the syntactic variable. As a matter of fact, this is the main reason for distinguishing the two pairs of rules; the new translation rules follow the same pattern as the old ones. This is most obvious in the case of who:

(T:AB3) If $\chi \sim \chi'$, then $F_{AB3,n}(\chi) \sim \lambda x_n \chi'$

Like the syntactic rule, the translation rule is a rule schema, making use of the fact that the syntactic rule of the logical language forming $\lambda$-abstracts is a rule schema as well: abstracts $\lambda x a$ can be formed from a variable $x$ and an expression $a$ of arbitrary type.

For which $\delta$ the situation is slightly more complicated. The old translation:

$$\lambda x_n[\delta'(x_n) \land \phi']$$

cannot be used as such in case $\phi$ is not a sentence, but an abstract. The conjunction sign $\land$ does not have the variable character that the $\lambda$-abstractor has.

We therefore extend our logical language with a new kind of expressions which do have this flexible character. These expressions are called restricted $\lambda$-abstracts and are of the form $\lambda x[\alpha]_\beta$. The abstraction is restricted to those entities which satisfy the predicate denoted by $\alpha$. We will use these new expressions in the translation rule (T:AB4) as follows:

(T:AB4) If $\delta \sim \delta'$ and $\chi \sim \chi'$,
then $F_{AB4,n}(\delta, \chi) \sim \lambda x_n[\delta']_\chi'$

So, the translation is a restricted $\lambda$-abstract, where the abstraction is restricted to the individual concepts which
satisfy the translation of the common noun phrase \( \delta \) in which \( \delta \).

The new clause in the definition of the logical language and its interpretation are as follows:

\[
(R\lambda) \quad \text{If } x \in \text{VAR}_a, \ a \in \text{ME}_{<a,t>} \text{ and } \beta \in \text{ME}_{b}, \ b \in \text{AB}',
\]

then \( \lambda x[\alpha]\beta \in \text{ME}_{<a,b>}
\]

\[
[[\lambda x[\alpha]\beta]]_{M,g} \text{ is that function } h \in D_{M,<a,b>}
\]

such that for all \( d \in D_{M,a} \)

\[
h(d) = [[\beta]]_{M,g[x/d]} \text{ if } [[\alpha]]_{M,g}(d) = 1,
\]

\[
= \text{zero}_{b} \quad \text{if } [[\alpha]]_{M,g}(d) = 0,
\]

where \( \text{zero}_{c} = 0 \); \( \text{zero}_{<a,b>} \) is the constant function from \( D_{M,a} \) to \( \text{zero}_{b} \)

The expressions \( \beta \) are restricted to expressions of abstract types, i.e. they are \( n \)-place predicate expressions \((n \geq 1)\).

A more general definition of restricted \( \lambda \)-abstraction for arbitrary types is possible, if we are prepared to have zero elements of type \( e \) and type \( s \) as well. The expression \( \lambda x[\alpha]\beta \) is an abstract of one level higher than \( \beta \), i.e. an \( n + 1 \) place predicate expression. When applied to an argument \( d \) of which the one-place predicate denoted by \( \alpha \) is true,

\[
[[\lambda x[\alpha]\beta]]_{M,g}(d) \text{ denotes the same } n \text{-place predicate as the unrestricted abstract } [[\lambda x\beta]]_{M,g} \text{ applied to } d. \text{ When } \alpha \text{ is false of } d, \text{ }[[\lambda x[\alpha]\beta]]_{M,g}(d) \text{ denotes a zero } n \text{-place predicate: a predicate which invariably gives the value } 0, \text{ no matter to which arguments it is applied.}
\]

The category changing rule \((S:CCF^*)\) which formed constituent complements from expressions of abstract category \( t///e \), can now be generalized to a constituent complement formation rule scheme \((S:CCF)\) which applies to expressions of arbitrary abstract category. The corresponding translation rule \((T:CCF)\) remains essentially the same as the old one:

\[
(S:CCF) \text{ If } x \in P_A, \ A \in \text{AB}, \text{ then } F_{CCF}(x) \in P_{e}
\]

\[
(T:CCF) \text{ If } x \sim x', \text{ then } F_{CCF}(x) \sim \lambda i[x'] = [\lambda x'](i)
\]
The following analysis trees are examples of the derivation of sentences containing multiple constituent complements with who and which:

(16) who loves whom, $\bar{t}$
\[
\lambda i[\lambda u \lambda v[\text{love}_x(a)(u,v)] = \lambda u \lambda v[\text{love}_x(i)(u,v)]
\]

who loves whom, $(t//e) / e$
\[
\lambda x_1 \lambda x_0[\text{love}(a)(x_0,x_1)]
\]

who loves him, $t//e$
\[
\lambda x_0[\text{love}(a)(x_0,x_1)]
\]

he$_0$ loves him$_1$, $t$
love$(a)(x_0,x_1)$

(17) which man which girl loves, $\bar{t}$
\[
\lambda i[\lambda u[\text{girl}_x(a)] \lambda v[\text{man}_x(a)(v) \land \text{love}_x(a)(u,v)]
= \lambda u[\text{girl}_x(i)] \lambda v[\text{man}_x(i)(v) \land \text{love}_x(i)(u,v)]
\]

which man which girl loves, $(t//e) / e$
\[
\lambda x_0[\text{girl}(a)] \lambda x_1[\text{man}(a)(x_1) \land \text{love}(a)(x_0,x_1)]
\]

girl, $CN$ which man he$_0$ loves, $t//e$
\[
\text{girl}(a) \lambda x_1[\text{man}(a)(x_1) \land \text{love}(a)(x_0,x_1)]
\]

man, $CN$ he$_0$ loves him$_1$, $t$
\[
\text{man}(a) \land \text{love}(a)(x_0,x_1)
\]

It can in general be proved that if $\beta$ is an $n$-place predicate expression, taking arguments of type $a_1, \ldots, a_n$, and $x_1, \ldots, x_n$ are variables of type $a_1, \ldots, a_n$ respectively, then $\lambda x[a]\beta$ is equivalent to $\lambda x_1[\ldots, \lambda x_n[a(x) \land \beta(x_1, \ldots, x_n)]].$ This means that the translation of the second line of (17) is
equivalent to: $\lambda x_0 \lambda x_1 [\text{girl}(a)(x_0) \land \text{man}(a)(x_1) \land \text{love}(a)(x_0, x_1)]$.

So the top line of (17) is equivalent to:

$$(17') \lambda i [\lambda u \lambda v [\text{girl}_*(a)(u) \land \text{man}_*(a)(v) \land \text{love}_*(a)(u, v)]]$$

This means that it is possible to reformulate (T:AB2) in terms of restricted $\lambda$-abstraction. (The same holds for (T:AB1) and (T:AB3) if that turns out to be necessary, cf. the remarks on argument ($X$) in sections 3.4 and 1.5.) We leave it to the reader to verify that the arguments (VI) and (VII) of section 1.4 are valid. The proof of their validity runs parallel to that of (V'), given in section 3.3.

The analysis of constituent complements presented here can easily be extended to cover complements with expressions like why, where, when, etc. as well. What is needed are syntactic variables that range over the proper kinds of entities. Further the set of abstract categories has to be extended, to cover abstraction over these variables. The syntactic and the corresponding translation rules have the same form as the rules discussed above.

3.8. Why abstracts are necessary

As we already stated in section 3.3, the level of abstracts is not strictly needed for the analysis of single constituent complements, they could be formed directly from sentences. However, abstracts (or some similar distinct level of analysis) seem to be essential for a correct analysis of multiple constituent complements. The reasons behind this can be outlined as follows.

Without the intermediary level of abstracts, one would need a syntactic rule which forms (multiple) constituent complements by introducing a (new) wh-term into a complement. On the semantic level such a rule would have to transform an expression of the form (a) into one of the form (b):
(a) \( \lambda i[\alpha/a/ = \alpha/i/] \)
(b) \( \lambda i[\lambda x[(...\alpha...)/a/] = \lambda x[(...\alpha...)/i/] ] \)

The problem is to make this transition in a compositional way. A possibility that might suggest itself is to treat wh-terms not as a kind of abstractors, but as a kind of terms that can only be introduced by means of a quantification rule. We might translate who as in (c), and formulate a quantification rule which, when applied to a wh-term \( \beta \) and a complement \( \rho \), translates as (d):

(c) \( \lambda P \forall x[P(a)(x)] \)
(d) \( \lambda j[\beta(\lambda a \lambda x_n(\rho(j)))] \), where \( \beta \) translates a wh-term and \( \rho \) a complement and \( x_n \) is the variable quantified over

If we apply (d) to the term (c) and a complement of the form (a), the result is (e), which is equivalent to (f). The expression (f) is of the form (b), so in this case we have succeeded in making a transition from an expression of the form (a) to an expression of the form (b) in a compositional way.

(e) \( \lambda j \forall x[\lambda x_n[\alpha/a/ = \alpha/j/](x)] \)
(f) \( \lambda i[\lambda x_n[\alpha/a/ = \lambda x_n a/i/] \)

However, this approach is only possible as long as we do not take wh-terms of the form which \( \delta \) into consideration. A term of the form which \( \delta \) would translate as (g). Applying (d) to a term of the form (g) and a complement of the form (a) results in (h):

(g) \( \lambda P \forall x[\delta(x) \cdot P(a)(x)] \)
(h) \( \lambda j[\forall x[\delta(x) \cdot (\lambda x_n[\alpha/a/ = \alpha/j/)](x)] \]

The expression (h) is equivalent to (i):

(i) \( \lambda i[\lambda x_n[\delta(x_n) \land \alpha/a/] = \lambda x_n[\delta(x_n) \land \alpha/i/] \)
But, since both occurrences of $\delta$ in (i) contain a free occurrence of $a$, this results only in de re readings of complements, not in de dicto ones. Result (i) is not of the required form (b). The de dicto reading would be expressed by (j):

$$\lambda i[\forall x[\delta(x) \land (\lambda x \, a)(x)]/a/ = [\delta(x) \land (\lambda x \, a)(x)]/i/]$$

This formula (j) is equivalent to one of the form (b), but it seems impossible to obtain (j) from (a) and (g) in a compositional way. Although we lack a formal proof, we are convinced that there is no way to proceed from (a) and (g) to an expression which gives de dicto readings. Consequently, we feel that the level of abstracts is indeed necessary, it is necessary to account for de dicto readings of multiple constituent complements. 15

In a nutshell, this is the reason why Karttunen's approach, being a quantificational one, can only account for the de re readings. The fact that Karttunen uses existential rather than universal quantification is not essential. It has to do with the fact that in his analysis complements denote sets of propositions instead of single propositions and with the fact that he does not take into account the exhaustiveness of wh-complements.

This is also the reason why it is impossible to treat wh-terms as terms, i.e. as expressions of (a subcategory of) the category T. In a quantificational approach like Karttunen's, wh-terms can be treated as 'normal' terms. From a syntactic point of view, this may be an advantage. However, as we hope to have shown, the quantificational approach has important semantic shortcomings. And it seems that semantic considerations lead us to the abstractor view of wh-terms. This means that wh-terms have to be treated as syncategorematic expressions (or, alternatively, as expressions belonging to the whole range of categories $(t///e)/t$, $((t///e)/e)/(t///e)$, etc.).
4. Details of a possible syntax for wh-complements

4.1. Background assumptions

In section 3 we explained how the semantic analysis of wh-complements proposed in this paper can be incorporated systematically in the framework of Montague grammar. There we did not bother about the syntactic details. In this section we will try to be a little bit more explicit. We will sketch one possible syntax of wh-constructions which is suitable for our semantics. The syntax presented here is in the line of the modifications of Montague's original syntax as proposed by Partee (see Partee, 1976, 1979a and 1979b) and others. Some of its aspects will remind the reader of work done in transformational grammar. Of course, we do not claim that the analysis of wh-complements presented here is new. Moreover, we do not attempt to solve all of the notoriously difficult syntactic problems in this area. We merely wish to show in this section that our semantic analysis of wh-complements can be combined with a feasible syntactic analysis.

In what follows the following assumptions concerning the syntax are made. The syntax produces not plain strings, but labelled bracketings (or, equivalently, phrase structure trees). The labeled bracketings account for the intuitions about the constituent structure of expressions and contain all the information which is needed for syntactic purposes. The constituent structure of an expression is, in general, not enough to determine its semantic interpretation. The semantic interpretation of an expression is determined by its derivation, which is encoded in its analysis tree.
Further it is assumed that the facts concerning pronominalization, reflexivization and 'wh-movement' are to be accounted for in terms of structural properties, i.e. properties of labelled bracketings, such as Reinhart's notion of c-command (see Reinhart, 1976). For an analysis of pronominalization and reflexivization in terms of structural properties in the Montague framework the reader is referred to Landman and Moerdijk (1981). Their paper also contains an analysis of some wh-constructions which, like the one presented here, uses structural properties, but differs from our analysis in several other respects.

4.2. 'Wh-preposing' and 'preposable occurrences'

We will concentrate on the rules which build abstracts. There are four of them, two 'preposing' rules, (S:AB1) and (S:AB2), and two 'substitution' rules (S:AB3) and (S:AB4). We start with (S:AB1), the rule which produces abstracts with preposed who(m). We want this rule to produce structures such as (18b)-(21b) from structures such as (18a)-(21a):

\[(S:AB1)\) operates on sentential structures containing one or more occurrences of a syntactic variable \(he_0\). It creates a structure labelled AB by 'preposing' the wh-term who(m),
substituting a trace (i.e. empty node) for some, 'preposable', occurrences of \( h_n \) and anaphorizing the others. The occurrences of \( h_n \) which are replaced by a trace share certain structural properties. They are called the \textit{wh-p-antecedent occurrences of} \( h_n \). One of these occurrences is replaced by a \textit{WHT-trace}, the others by \textit{T-traces}. Traces are left because in order for pronominalization, reflexivization and abstract formation to work properly, the structural properties of certain expressions in the original structure have to be recoverable. In effect, leaving traces is nothing but building into the structure those aspects of derivational history which continue to have syntactic relevance.

We add two general remarks. First, notice that labels like \( AB \) and \( WHT \) are not category labels. \( AB \) acts as a variable over category labels, \( WHT \) labels expressions which are introduced syncategorematically. The use of such labels does not present semantic problems since it is the derivational history, and not the structure, of an expression that determines its meaning. Second, as structures (21) show, the output of a category changing rule no longer contains the original category label: the complement of \textit{know} is of the form \( t_{WHT}[\text{who}] \ldots \) and not of the form \( t_{AB-WHT}^{T} \ldots \). This is based on the assumption that information about the old category is no longer syntactically relevant. Nothing in our analysis, however, depends on this assumption.

The notion of \textit{wh-p-antecedent occurrence} is not only needed to distinguish those occurrences of \( h_n \) which are to be replaced by a trace, it will also be used to determine whether a given structure is a proper input for \( (S;AB1) \). Before giving a definition, let us point out what will be understood by an occurrence. Formally, an occurrence of an expression \( a \) in a structure \( \beta \) is an ordered pair \(<n,X[a{-}]>\), where \( n \) defines a position in \( \beta \), \( X \) is the label of \( a \) and \( (-) \) is the set of features that determines the morphological form. In what follows we will not use the term 'occurrence' so strictly. For example we will write \( t[h_{im0}] \).
instead of \( \text{he}_0(\text{acc}) \), etc. The notion of wh-p-antecedent occurrence is defined as follows:

(WH-P) The wh-p-antecedent occurrences of \( \text{he}_n \) in \( \phi \) are those occurrences \( a \) of \( \text{he}_n \) in \( \phi \) such that:

(i) \( a \) is not c-commanded by another occurrence of \( \text{he}_0 \) in \( \phi \);

(ii) \( a \) is not dominated by a node \( t \) such that that node is directly dominated by a node \( A: A \neq t \);

(iii) if \( a \) occurs in a coordinate structure in \( \phi \) then for every coordinate \( \psi \) there is a wh-p-antecedent occurrence of \( \text{he}_n \) in \( \psi \).

We will give a few examples to illustrate this. In these examples only the relevant aspects of the structures are represented. First consider (22):

(22) \( \text{he}_0 \) loves \( \text{him}_0 \)self

\( a \quad \beta \)

\( a \) is a wh-p-antecedent occurrence of \( \text{he}_0 \), but \( \beta \) isn't, since \( \beta \) is c-commanded by \( a \). So, (22) will give rise to (22a) but not to (22b):

(22) (a) \( \text{AB}[\text{who}_t[\text{WHT}[] \text{loves himself}]\]

(22) (b) \( *_{\text{AB}}[\text{who}_t[\text{WHT}[] \text{loves}_w]]\]

Next consider (23):

(23) \( \text{he}_0 \) says\( _t[\text{that}_t[\text{Mary loves him}_0]]\]

\( a \quad \beta \)

Again \( a \) is a wh-p-antecedent occurrence, and \( \beta \) is not. Not only because \( \beta \) is c-commanded by \( a \), but also because \( \beta \) is dominated by a \( t \) which is directly dominated by a \( t \). So, (23) will lead to (23a), but not to (23b):
Another example illustrating condition (ii) is (24):

(24) John says $\bar{\varepsilon}[\text{that}_t[\text{he}_0 \text{ loves } Mary]]$

$\alpha$ is not a wh-p-antecedent occurrence, because it is dominated by a $t$ which is directly dominated by $\bar{t}$. Thus (24a) will not be derivable from (24):

(24) (a) $*_AB[\text{who}_t[\text{John says-}\text{that}_t[\text{he}_0 \text{ loves } Mary]]]$.

Notice that condition (ii) excludes any occurrence of a syntactic variable in an embedded clause. As (25a) indicates, this is too strong:

(25) (a) $AB[\text{whom}_t[\text{John says}\text{-}\text{that}_t[\text{he}_0 \text{ loves } Mary]]]$.

This would have to be derived from the structure (25):

(25) John says $\bar{\varepsilon}[\text{that}_t[\text{he}_0 \text{ loves } Mary]]$

If we weaken condition (ii) by adding:

... unless the case of $\alpha \neq$ nominative and
$A = \bar{\varepsilon}$-that

then $\alpha$ in (25) counts as a wh-p-antecedent of $\text{he}_0$. Notice that $\beta$ in (23) is still excluded by condition (i). By $\bar{\varepsilon}$-that, of course, we mean to label the subcategory of that-complements. That the above weakening should be restricted to that-complements is made clear by (26):
Another example illustrating condition (ii) involves a subordinate clause:

(27)  the fact \[the fact_0[he is ill]\] bothers him_0
      \alpha  \beta

\(\alpha\) is not a wh-p-antecedent occurrence, \(\beta\) is. So, from (27) we can obtain (27a), but not (27b):

(27)  (a)  \(A_B[whom_0[the fact_0[he is ill]] bothers_0[he is ill]]\)
(27)  (b)  *\(A_B[whom_0[the fact_0[he is ill]] bothers_0[he is ill]]\)

As a last example, consider (28):

(28)  t_[Mary loves him_0] t/t if t_[Suzy hates him_0]
      \alpha  \beta

\(\alpha\) is a wh-p-antecedent occurrence, \(\beta\) is not, which predicts that (28a) can result from (28), but not (28b):

(28)  (a)  \(A_B[whom_0[t[Mary loves_0]]] t/t[Suzy hates_0]]\)
(28)  (b)  *\(A_B[whom_0[t[Mary loves_0]]] t/t[Suzy hates_0]]\)

The coordinate structure constraint (iii) prevents the derivation of (29a) from (29):

(29)  t_[he_0 walks] and t_[Peter talks]
(29)  (a)  *\(A_B[who_0[t[he_0 walks]] and t[Peter talks]]\)
Notice that in case we weaken condition (ii) as indicated above, there is a wh-p-antecedent occurrence of \( \text{he}_0 \) in (30), but not in (31) according to (iii):

\[
\begin{align*}
(30) \text{John says } & \, \exists[\text{that}_t[\text{t}[\text{Peter loves him}_0] \, \text{and} \, \text{t}[\text{Mary kisses him}_0]]] \\
(31) \text{John says } & \, \exists[\text{that}_t[\text{t}[\text{Peter loves him}_0] \, \text{and} \, \text{t}[\text{Mary kisses Bill}]]]
\end{align*}
\]

Notice further that (32) does not contain a wh-p-antecedent occurrence of \( \text{he}_0 \) since, although \( \alpha \) and \( \beta \) are dominated by a node \( t \) which is directly dominated by another node \( t \), they also occur in a \( t \) (i.e. the entire coordinate structure) which is directly dominated by \( t \):

\[
(32) \text{John says } \exists[\text{that}_t[\text{t}[\text{he}_0 \text{ walks} ] \, \text{and} \, \text{t}[\text{he}_0 \text{ talks}]]]
\]

All those occurrences of \( \text{he}_n \) in \( \phi \) which are not wh-p-antecedent occurrences according to (WH-P) we call wh-p-anaphor occurrences of \( \text{he}_n \) in \( \phi \). The formulation of the syntactic rule \( \text{(S:AB1)} \) now runs as follows:

\[
\text{(S:AB1) If } \phi \in P_t, \text{ then } F_{AB1,n}(\phi) \in P_{t//e}
\]

Condition: \( \phi \) contains one or more wh-p-antecedent occurrences of \( \text{he}_n \), all of which have the same case \( c \).

\[
F_{AB1,n}(\phi) = AB[\text{WHT}[\text{who}(c)],_t[\phi^{'}}], \text{ where } \phi^{'} \text{ comes from } \phi \text{ by performing the following operations:}
\]

(i) if \( c = \) nominative then replace the first, else replace the last, wh-p-antecedent occurrence of \( \text{he}_n \) in \( \phi \) by \( \text{WHT}[ ] \);

(ii) delete all other wh-p-antecedent occurrences of \( \text{he}_n \) in \( \phi \), i.e. replace them by \( _t[ ] \);

(iii) anaphorize all wh-p-anaphor occurrences of \( \text{he}_n \) in \( \phi \).
The examples (18)-(32) illustrate the working of this rule. The condition which restricts the application of (S:AB1) deals with the familiar cases of case-conflict. It would become superfluous once a theory of features, e.g. in the line of Landman and Moerdijk (1981), is incorporated. Clause (i) is stated in terms of case, we do not want to exclude the possibility to formulate it in terms of structural properties. The anaphorization operation in (iii) here comes to simply removing indices.

The second 'wh-preposing' rule, which preposes wh-terms of the form \textit{which} \( \delta \), is a minor variation of the one just given. It reads as follows:

\[(S:AB2) \text{ If } \phi \in P_t \text{ and } \delta \in P_{CN}, \text{ then} \]
\[ F_{AB2,n}(\delta, \phi) \in P_{t//e} \]
Condition: as in (S:AB1).
\[ F_{AB2,n}(\delta, \phi) = AB[WH][\text{which } \delta(\varepsilon)[\phi']], \text{ where } \phi' \]
comes from \( \phi \) by performing the following operations:
(i) and (ii) as in (S:AB1)
(iii) as in (S:AB1), taking into account the (number and) gender of \( \delta \).

Examples similar to the ones already given for (S:AB1) can easily be constructed.

4.3. Wh-reconstruction

Interesting cases of application of (S:AB2) are those in which the common noun \( \delta \) is not lexical, but itself complex and contains an occurrence of a syntactic variable, e.g.:

(33) \[ AB[\text{which } \text{poem of him}_0 \text{ he}_0 \text{ likes best } \text{WH}] \]
\[ \alpha \quad \beta \]

(34) \[ AB[\text{which man who loves him}_0 \text{ he}_0 \text{ likes best } \text{WH}] \]
\[ \alpha \quad \beta \]
Notice that in both structures \( \alpha \) and \( \beta \) do not c-command each other. If it were the case that \( \beta \) c-commanded \( \alpha \), then this could be used to explain why (35a) and (36a) are acceptable, whereas (35b) and (36b) are not (on coreferential readings, of course):

\[
\begin{align*}
(35)(a) & \quad [\text{which poem of him } t[\text{every poet likes best } \text{WHT}]] \\
(35)(b) & \quad *[\text{which poem of every poet } t[\text{he likes best } \text{WHT}]] \\
(36)(a) & \quad [\text{which man who loves her } t[\text{every girl likes best } \text{WHT}]] \\
(36)(b) & \quad *[\text{which man who loves every girl } t[\text{she likes best } \text{WHT}]]
\end{align*}
\]

A natural condition (see Reinhart, 1976, 1979) on antecedent-anaphor relations is that an anaphor does not c-command its antecedent. Notice that although \( \beta \) does not c-command \( \alpha \), it does c-command the trace of the wh-term in which \( \alpha \) occurs. It seems that in the process of deriving (35a) from (33) structural relations such as c-command are not determined on (33) as such, but on what is called the wh-reconstruction of (33). 17, 18 This notion is defined as follows:

\[(\text{WH-R}) \quad \text{The wh-reconstruction of a structure } \phi \text{ is that structure } \phi' \text{ which is the result of replacing, bottom up, each substructure of the form } [\text{WHT}[\gamma] t[\psi]] \text{ by } [t[\psi]], \text{ which is the result of substituting the wh-term } \gamma \text{ for its trace in } \psi\]

Notice that the existence of a unique trace for each occurrence of a wh-term is guaranteed by the direction of the reconstruction process (bottom up) and the nature of the proposing rules (S:AB1) and (S:AB2).

For every structural property \( P \) we define a corresponding structural property \( P' \) as follows:
(RSP) \( a \) has the structural property \( P' \) in the structure \( \phi \) iff \( a \) has the structural property \( P \) in the wh-reconstruction of \( \phi \).

From now on we will refer to structural properties \( P' \) as \( P \), e.g. from now on c-command stands for c-command'.

At this point a remark on the nature of WHT-traces is in order. In fact a WHT-trace is nothing but a T-trace in a special structural position. So, WHT-traces are marked T-traces. However, whether or not a T-trace is in this special structural position, can always be determined, so the special marking is not essential.

We could do without WHT-traces and only use T-traces. The wh-reconstruction is then defined as follows:

\[(WH-R')\] The wh-reconstruction of a structure \( \phi \) is that structure \( \phi' \) which is the result of replacing, bottom up, each substructure of the form \([\text{WHT}][\gamma][\psi]t[\gamma']\) by \([\text{t}][\psi']\), which is the result of substituting \( \gamma \) for the first T-trace in \( \psi \) if \( \gamma \) has nominative case, and for the last T-trace in \( \psi \) otherwise.

Of course, if one extends the present analysis to the more difficult cases involving pied-piping etc., the definition of wh-reconstruction might become more complicated. However, we feel that a reconstruction in terms of structural positions of T-traces will always be possible. In fact it has to be since this seems to be the only explanation for the fact that language users are able to interpret wh-constructions at all. A language user is capable of recognizing a hole in a structure (i.e. a trace), he will be capable of determining its category and its structural properties, but it seems unlikely that he is able to distinguish between subcategories of holes, if the subcategory information in question represents structural information which is not also present in the structure itself.
4.4. Wh-substitution and substitutable occurrences

Other cases where we need wh-reconstruction than the ones discussed above, involve the other two abstract formation rules, the wh-substitution rules. These rules form abstracts from abstracts by substituting who(m), which δ, for an occurrence of a syntactic variable. They are highly parallel to the previous two. However, they operate on a type of occurrences of syntactic variables which is a bit less constrained than wh-p-antecedent occurrences. The difference is that the substitution rules are allowed to operate on occurrences which are inside a complement. Consider three examples:

\[(37)\] (a) \[AB[\text{who} \ 
(37)\] (b) \[AB[\text{who} \ 
\]

(38) (a) \[AB[\text{who} \ 
(38)\] (b) \[AB[\text{who} \ 
\]

(39) (a) \[AB[\text{who} \ 
(39)\] (b) \[AB[\text{who} \ 
\]

The multiple constituent complement in the (b)-sentences can be constructed from the single constituent complements in the (a)-sentences. To see that the substitution rules are more liberal than the preposing rules, compare (38) with (26) and (39) with (24). This leads to the following notion of wh-s-antecedent occurrence:

\[(\text{WH-S})\] The wh-s-antecedent occurrences of \(\text{he}_n\) in \(\phi\) are those occurrences \(\alpha\) of \(\text{he}_n\) in \(\phi\) such that:

(i) \(\alpha\) is not c-commanded by another occurrence of \(\text{he}_n\) in \(\phi\);

(ii) \(\alpha\) is not dominated by a node \(t\) such that that node is directly dominated by a node \(A: A \neq t, \tilde{t}\);

(iii) if \(\alpha\) occurs in a coordinate structure in
\[
\phi \text{ then for every coordinate } \psi \text{ there is a wh-s-antecedent occurrence of } \text{he}_n \text{ in } \psi
\]

(WH-S) only differs from (WH-P) in that in clause (ii) A may be either t or t. So occurrences within subordinate clauses other than complements are still out of bounds. As an example consider (40):

\[
(40) \quad \text{AB[which man}_{\text{RC}}[\text{who}_{\text{t}}[\text{WHT[loves him}_{0}[\text{WHT]}[\text{walks}]]_{\text{t}}[\text{WHT}]]_{\text{t}}[\text{WHT}]]_{\text{t}}[\text{WHT}]]_{\text{t}}[\text{WHT}]
\]

According to (WH-S) α is not a wh-s-antecedent occurrence of he₀, since RC ≠ t, t. (In section 4.5 we will identify RC as a subcategory of t////e.) The wh-s-anaphor occurrences of heᵣ in φ are those which are not wh-s-antecedent occurrences of heᵣ in φ. The two wh-substitution rules can now be formulated as follows:

\[
(\text{S:AB3}) \quad \text{If } \chi \in P_A, A \in \text{AB}, \text{ then } F_{\text{AB},n}(\chi) \in P_A/e \\
\text{Condition: } \chi \text{ contains one or more wh-s-antecedent occurrences of } \text{he}_n, \text{ all of which have the same case } c. \\
F_{\text{AB},n}(\chi) = \chi', \text{ where } \chi' \text{ comes from } \chi \text{ by performing the following operations:} \\
(\text{i}) \quad \text{if } c = \text{nominative then replace the first,} \\
\quad \text{else the last, wh-s-antecedent occurrence} \\
\quad \text{of } \text{he}_n \text{ in } \chi \text{ by } \text{WHT[who(c)]}; \\
(\text{ii}) \quad \text{delete all other wh-s-antecedent} \\
\quad \text{occurrences of } \text{he}_n \text{ in } \chi, \text{ i.e. replace them} \\
\quad \text{by } \text{t[ ];} \\
(\text{iii}) \quad \text{anaphorize all wh-s-anaphor occurrences of} \\
\quad \text{he}_n \text{ in } \chi
\]

\[
(\text{S:AB4}) \quad \text{If } \chi \in P_A, A \in \text{AB}, \text{ and } \delta \in P_{\text{CN}}, \text{ then} \\
F_{\text{AB},n}(\delta, \chi) \in P_A/e \\
\text{Condition: as in (S:AB3).} \\
F_{\text{AB},n}(\delta, \chi) = \chi', \text{ where } \chi' \text{ comes from } \chi \text{ by}
\]
performing the following operations:

(i) if c = nominative, then replace the first, 
else replace the last, wh-s-antecedent 
occurrence of \( h_{\text{en}} \) in \( \chi \) by \( \text{wHT}[\text{which } \delta(c)] \);

(ii) as in (S:AB3);

(iii) as in (S:AB3), taking into account the 
(number and) gender of \( \delta \)

Given these rules (37b)-(39b) can be derived from the 
corresponding (a)-structures. Two other examples are:

\[
\begin{align*}
(41)(a) & \quad AB[\text{who}_t[t[\text{wHT}[ ]] \text{loves } h_{0}] and \\
& \quad t[T[ ] \text{kisses } h_{0}])])
\end{align*}
\]

\[
\begin{align*}
(41)(b) & \quad AB[\text{who}_t[t[\text{wHT}[ ]] \text{loves}_T[ ]] and \\
& \quad t[T[ ] \text{kisses whom}])])
\end{align*}
\]

\[
\begin{align*}
(42)(a) & \quad AB[\text{which girl}_t[t[he_0 \text{ loves}_T[ ]] \text{and} \\
& \quad t[he_0 \text{ kisses}_{\text{wHT}[ ]}])])
\end{align*}
\]

\[
\begin{align*}
(42)(b) & \quad AB[\text{which girl}_t[t[\text{which man loves}_T[ ]] \text{and} \\
& \quad t[T[ ] \text{kisses}_{\text{wHT}[ ]}])])
\end{align*}
\]

The notion of wh-reconstruction plays an essential role in 
determining the wh-s-antecedent occurrences of a syntactic 
variable and thereby in the way in which (S:AB3) and (S:AB4) 
function. Consider again (33):

\[
\begin{align*}
(33) & \quad AB[\text{which poem of } h_{0} t[he_0 \text{ likes best}_{\text{wHT}[ ]}]]
\end{align*}
\]

If the structural notions like c-command were not redefined 
as in (RSP), then both \( \alpha \) and \( \beta \) would count as wh-s-antecedent 
ocurrences. Together with the 'same-case'-condition this 
means that we could not derive (43):

\[
\begin{align*}
(43) & \quad AB[\text{which poem of } h_{0} t[\text{which poet} \\
& \quad \text{likes best}_{\text{wHT}[ ]}]]
\end{align*}
\]
However, given the fact that the c-command notion used in (WH-S) is redefined as in (RSP), in fact only β counts as a wh-s-antedecedent occurrence in (33), since β c-commands (in the old sense) α in the wh-reconstruction of (33). This means that (43) can be derived from (33).

4.5. Relative clauses

We will end section 4 by indicating how another type of wh-constructions, that of relative clauses, can be treated in this framework. Observe that the kind of expressions formed by (S:AB1) can not only be used to form complements from, but can also be used as relative clauses. Relative clauses are constructed in exactly the same way and are subject to exactly the same constraints (in English at least). So all the relevant examples given above apply here too.

Semantically we can regard relative clauses as abstracts, i.e. predicate denoting expressions, too. So, relative clauses are taken to be constructed from sentences containing a wh-p-antecedent occurrence of a syntactic variable by the first abstract formation rule (S:AB1). This means that the category t///e, the category of expressions produced by the two proposing abstract formation rules (S:AB1) and (S:AB2), has to be split into two subcategories, (t///e)1, which contains the results of (S:AB1), and (t///e)2, which contains the results of (S:AB2). Expressions of the first subcategory can then be used as input in two rules which combine them with a common noun or a term. These rules can be formulated as follows:

\[
\begin{align*}
(S:RRC) & \quad \text{If } \delta \in P_{CN}, \chi \in P_{(t///e)1}, \\
& \quad \text{then } F_{RRC}(\delta, \chi) \in P_{CN}, \text{ where } F_{RRC}(\delta, \chi) = \delta \chi \\
(T:RRC) & \quad \text{If } \delta \sim \delta', \chi \sim \chi', \\
& \quad \text{then } F_{RRC}(\delta, \chi) \sim \lambda x[\delta'(x) \land \chi'(x)] \\
(S:NRC) & \quad \text{If } \alpha \in P_T, \chi \in P_{(t///e)1}, \text{ then } F_{NRC}(\alpha, \chi) \in P_T, \\
& \quad \text{where } F_{NRC}(\alpha, \chi) = \alpha \chi
\end{align*}
\]
(T:NRC) If \( a \sim a', \chi \sim \chi', \) then
\[
F_{\text{NRC}}(a, \chi) \sim \lambda P[a'(\lambda a \lambda x[P(a)(x) \land \chi'(x))])
\]

Rule (S:RRC) produces restrictive relative clause constructions, (S:NRC) non-restrictive relative clause constructions. Both rules do not, as they stand, account for the necessary agreement in number and gender. This could be handled either by a theory of features as proposed by Landman and Moerdijk (1981) or by a mechanism of subcategorization as proposed by Janssen (1980b).

The two translation rules are straightforward. In fact, the analysis of restrictive relative clause constructions can be regarded as an analysis of the CN-S type, with this difference that (S:RRC) does not take a sentence as such, but an abstract formed from a sentence (see Janssen, 1981, for extensive discussion of the various types of analyses of restrictive relative clause constructions). The semantic part of the analysis of non-restrictive relative clause constructions is in essence the one given by Rodman (1976).

The fact that both types of wh-constructions, viz. relative clause constructions and constituent complements, at a certain level of analysis can be regarded as constructions of the same category, in our opinion supports the existence of the level of abstracts as a separate level of analysis.
5. Coordination of complements

5.1. The need for complement-level terms

In section 1.8 we argued that the fact that wh-complements and that-complements can be coordinated is an argument in favour of treating them as belonging to the same syntactic category. We have not yet shown how the coordination of complements is to be carried out. The reason for this is that a proper account involves complications which might have obscured the basic principles of our analysis of the semantics of wh-complements. In order to give a proper account of the coordination of complements, one needs to analyze them as a kind of terms, as expressions denoting not propositions as such, but sets of properties of propositional concepts. This 'higher level' analysis is needed to ensure that the following three types of complements come out as they should:

- (a) whether (ψ and ψ) 'conjunctive complement'
- (b) whether ψ and whether ψ 'conjunction of complements'
- (c) whether ψ or ψ 'alternative complement'

The relation between alternative complements and disjunctive complements, i.e. complements of type whether (ψ or ψ), has already been discussed in section 3.1, examples (9) and (10). A fifth type of complement is disjunction of complements, i.e. complements of type whether ψ or whether ψ. They will not be discussed since they are analogous to conjunctions of complements.

The difference between conjunctive complements and
conjunctions of complements is clear from the difference in meaning between sentences of the form (44) and (45):

(44) Bill wonders whether (φ and ψ)
(45) Bill wonders whether φ and whether ψ

Whereas (45) implies that Bill wonders whether φ, (44) does not. In other words, (45), but not (44), is equivalent to (46):

(46) Bill wonders whether φ and Bill wonders whether ψ

This means that conjunctions of complements should be analyzed in such a way that complement taking verbs distribute over the complements which are their conjuncts.

The difference between conjunctions of complements and alternative complements may be a little harder to grasp. At first they may seem equivalent, but we will argue that they are not. Consider the following sentence forms:

(47) Bill wants to know whether φ or ψ
(48) Bill wants to know whether φ and whether ψ
(49) Bill knows whether φ

Obviously, (48) is false if (49) is true. It may seem that this holds for (47) too. However, in our opinion this is not the case without further qualification. The truth of (49) as such does not imply the falsity of (47). That it seems to do so is caused by the implicature carried by alternative complements that (according to the subject) exactly one of the alternatives holds. If (Bill assumes that) either φ or ψ is true, but not both, then it would indeed follow from (49) that (47) is false. As we already argued in section 1.7, however, we are dealing here with an implicature, and not with an implication. That it is an implicature is also clear from the fact that it can be cancelled, as is illustrated in the following example:
(50) Bill wanted to know whether Mary, or John, or Peter, or Harry or \{all four of them\} witnessed the murder.

Sentence (50) contains an alternative complement of the form whether \(\phi_1\), or \(\phi_2\), or \(\phi_3\), or \(\phi_4\), or \(\phi_5\). It is not a contradiction, which means that the implicature that exactly one of the alternatives is true, is cancelled in (50). This means that the truth of (51):

(51) Bill knew that Mary witnessed the murder

is compatible with the truth of (50), as is shown by (52), which is not contradictory:

(52) Already having concluded that Mary witnessed the murder, Bill wanted to know whether Mary, or John, or Peter, or Harry, or all four of them, witnessed the murder.

Sentence (52) is not necessarily false. But, to be sure, uttering it one would strictly speaking violate the Gricean maxims. On the other hand, (53) is a contradiction:

(53) Already having concluded that Mary witnessed the murder, Bill wanted to know whether Mary and whether John and whether Peter and whether Harry witnessed the murder.

This means that alternative complements and conjunctions of complements, despite their seeming similarity, may denote different propositions. The similarity is explained by the fact that if the implicature is not cancelled, then on the assumption of its truth, (49) implies that (47) is false.

An indirect argument which leads to the same conclusion, involves the relation between constituent complements and alternative complements. Semantically, constituent
complements are equivalent to alternative complements. In case one deals with a finite (sub)domain and \( d_1, \ldots, d_n \) name all the elements, the alternative complement corresponding to a constituent complement can be written down, as the following pair of sentences illustrates:

(54) Bill investigated who did it
(55) Bill investigated whether \( d_1 \) did it, or \( \ldots \), or \( d_n \) did it

Clearly, (54) and (55) are equivalent. Now, again, (56) is not a contradiction:

(56) Already having established that Peter didn't do it,
    Bill investigated who did it

Given the equivalences of (54) and (55), this means that (57) isn't a contradiction either:

(57) Already having established that Peter didn't do it,
    Bill investigated whether Mary did it, \( \ldots \), or Peter did it, or \( \ldots \)

Like (52), (57), though not necessarily false, may violate the Gricean maxims. Notice that (56) is much less likely to be in conflict with these maxims than (57). On the other hand, (58) is contradictory:

(58) Already having established that Peter didn't do it,
    Bill investigated whether Mary did it and whether Harry did it \( \ldots \) and whether Peter did it \( \ldots \)

And this leads to the same conclusion as above: despite their seeming similarity, which can be explained in terms of implicatures, alternative complements and conjunctions of complements express different propositional concepts.
5.2. Analyzing complements as complement-level terms

The facts discussed in section 5.1, in particular the fact that complement taking verbs distribute over the complements which make up a conjunction of complements, point towards a 'higher level' analysis of complements. For different reasons, such a higher level analysis of that-complements is proposed in Delacruz (1976). He argues that that-complements are to be analyzed in terms of sets of properties of propositions. In our analysis this comes to considering complements to be expressions which denote sets of properties of propositional concepts. It should be noted that kicking complements upstairs in this way does not change anything fundamental in our semantic analysis. The rule which transforms complements 'old style' into complement terms, i.e. expressions of category \( t/(t/\bar{t}) = CT \), is as follows:

\[
\begin{align*}
(S;CTF) & \text{ If } \rho \in P, \text{ then } F_{CTF}(\rho) \in P_{CT} \\
(T;CTF) & \text{ If } \rho \sim \rho', \text{ then } F_{CTF}(\rho) \sim \lambda R[R(a)(\lambda a \rho')]
\end{align*}
\]

where \( R \) is a variable of type \(<s,<<s,<s,t>,t>>\>

The reason to keep the intermediary stage of expressions of category \( \bar{t} \), is that they are needed as input for a rule which quantifies terms into complements (see section 4.3).

The syntactic rule is a category changing rule. The translation rule shows that the complement term formed from a complement \( \rho \) denotes the set of properties of the propositional concept expressed by \( \rho \). Complement-embedding verbs are now of a higher level too, of course. They are expressions of category IV/CT. The complement-embedding rule remains a simple rule of functional application. Sentence (8) of section 3.1 is now analyzed as follows:
(59) John knows whether Mary walks, t
\[
\text{know}(a)(\lambda a \lambda R[R(a)(\lambda a \lambda i[\text{walk}_*(a)(m) = \text{walk}_*(i)(m)]])
\]

John, T know whether Mary walks, IV
λP[\text{P}(a)(\lambda aj)] \text{know}(a)(\lambda a \lambda R[R(a)(\lambda a \lambda i[\text{walk}_*(a)(m) = \text{walk}_*(i)(m)])]

know, IV/CT whether Mary walks, CT
\text{know}(a) \lambda R[R(a)(\lambda a \lambda i[\text{walk}_*(a)(m) = \text{walk}_*(i)(m)]])

whether Mary walks, t
\lambda i[\text{walk}_*(a)(m) = \text{walk}_*(i)(m)]

(59) expresses that an intensional relation of knowing holds between an individual concept and the intension of a set of properties of a propositional concept. The following meaning postulate reduces this high-level intensional relation into a low-level extensional one, i.e. to a relation between an individual and a proposition.

\[\forall x \forall y \forall i[\delta(i)(x,R) = (R(i)(\lambda i \lambda R[M(i)(x(i),r(i))])],\]

M is a variable of type \(\langle s,<s,t>,<e,t>\rangle\);
x of type \(\langle s,e\rangle\); R of type \(\langle s,\langle s,\langle s,<s,t>,t>,t\rangle,\rangle\); i type s; r of type \(\langle s,<s,t\rangle\rangle\) and \(\delta\) is the translation of know, tell, etc.

The substar notation convention is now extended as follows:

\[\delta_* = \lambda i \lambda p \lambda u[\delta(i)(\lambda iu,\lambda iR[R(i)(\lambda ip)]])],\]

p is a variable of type \(\langle s,t\rangle\); u of type e; R of type \(\langle s,\langle s,\langle s,<s,t>,t\rangle,t\rangle\rangle\); p of type \(\langle s,t\rangle\)

Combining (MP:IV/CT-E) with (SNC) one can prove that (60) is valid:
Applying (60), we get the following reduced translation of (59):

\[(59') \text{know}_\ast(a)(j, \lambda i[\text{walk}_\ast(a)(m) = \text{walk}_\ast(i)(m)]\]

This is exactly the same result as we obtained in our low-level analysis. For those verbs, such as wonder, which are extensional in subject position, but intensional in object position, we propose the following meaning postulate which reduces the high-level intensional relation expressed by these verbs to a low-level intensional one.

\[(\text{MP:IV/CT-I}) \exists \text{NVxVRVi}[\delta(i)(x,R) = R(i)(\lambda i l r[N(i)(x(i),r)])],\]
\[N \text{ is a variable of type } \langle s, \langle s, s, t \rangle, e, t \rangle\]

Further, we introduce the following notation convention:

\[(\text{CNC}) \delta_\ast = \lambda i l r l u[\delta(i)(\lambda i u, \lambda i l r[R(i)(r)])]\]

Combining (MP:IV/CT-I) with (CNC) one can prove that (61) is valid:

\[(61) \forall i[\delta(i)(x,R) = R(i)(\lambda i l r[\delta_\ast(i)(x(i),r)])]\]

Given (61) the following is the reduced translation of Bill wonders whether Mary walks:

\[(62) \text{wonder}_\ast(a)(b, \lambda a l i[\text{walk}_\ast(a)(m) = \text{walk}_\ast(i)(m)])\]

5.3. Complement coordination

Let us now turn to complement coordination, which necessitates this move to the complement term level (we
restrict ourselves to conjunction, the rule for disjunction is completely analogous:

\[(S:\text{CTCO}) \text{ If } \Sigma, \Theta \in \mathcal{P}_\text{CT}, \text{ then } \Sigma \text{ and } \Theta \in \mathcal{P}_\text{CT}\]

\[(T:\text{CTCO}) \text{ If } \Sigma, \Theta \sim \Sigma', \Theta', \text{ then } \Sigma \text{ and } \Theta \sim \lambda R[\Sigma'(R) \land \Theta'(R)]\]

These rules can be illustrated by considering the derivation of the three types of complements (a), (b) and (c):

(a') whether \((\phi \land \psi), \text{ CT}\)
\[
\lambda R[R(a)(\lambda a \lambda i[(\phi/a \land \psi/a) = (\phi/i \land \psi/i)])]
\]
\[
\text{whether } (\phi \land \psi), \bar{t}
\lambda i[(\phi/a \land \psi/a) = (\phi/i \land \psi/i)]
\]

(b') whether \(\phi\) and whether \(\psi\), CT
\[
\lambda R[R(a)(\lambda a \lambda i[(\phi/a = \phi/i)]) \land R(a)(\lambda a \lambda i[(\psi/a = \psi/i)])]
\]
\[
\text{whether } \phi, \bar{t} \quad \text{whether } \psi, \bar{t}
\lambda i[(\phi/a = \phi/i)] \quad \lambda i[(\psi/a = \psi/i)]
\]

(c') whether \(\phi \lor \psi\), CT
\[
\lambda R[R(a)(\lambda a \lambda i[(\phi/a = \phi/i) \land (\psi/a = \psi/i)])]
\]
\[
\text{whether } \phi \lor \psi, \bar{t}
\lambda i[(\phi/a = \phi/i) \land (\psi/a = \psi/i)]
\]

It can be proved that the complement terms (a'), (b') and (c') denote different sets of properties of propositional
concepts. Sentences of the form (44) and (45) are now translated as follows:

\[(44') \text{ Bill wonders whether } (\phi \text{ and } \psi), t\]
\[\text{wonder}(a)(\lambda b, \lambda a \lambda R[R(a)](\lambda a \lambda i[\phi/a] \land \psi/a/) = (\phi/i/ \land \psi/i/))\]

\[(45') \text{ Bill wonders whether } \phi \text{ and whether } \psi, t\]
\[\text{wonder}(a)(\lambda b, \lambda a \lambda R[R(a)](\lambda a \lambda i[\phi/a] = \phi/i/) \land R(a)(\lambda a \lambda i[\psi/a] = \psi/i/))\]

If we apply (MP:IV/CT-I) to these translations, we get the following results:

\[(44'') \text{ wonder}(a)(b, \lambda a \lambda i[\phi/a] \land \psi/a/) = (\phi/i/ \land \psi/i/))\]

\[(45'') \text{ wonder}(a)(b, \lambda a \lambda i[\phi/a] = \psi/i/) \land \text{ wonder}(a)(b, \lambda a \lambda i[\psi/a] = \psi/i/))\]

Of course, \(45''\) is also the translation of (46):

\[(46) \text{ Bill wonders whether } \phi \text{ and Bill wonders whether } \psi\]

This illustrates that complement-embedding verbs distribute over a conjunction of complements, but the fact that (44'') does not imply (45'') shows that they do not distribute over a conjunctive complement.

The difference between (44'') and (45'') can also be illustrated using the following meaning postulate:

\[\text{(MP:INQ)} \forall x \forall y [\delta(i)(x, r) \rightarrow \neg \text{know}_+(i)(x, r(i))]\]
where \(\delta\) is wonder\(+\), investigate\(+\), ask\(+\), etc.

Given (MP:INQ), which captures a central part of the meaning of inquisitive verbs, (44'') and (45'') imply (63) and (64) respectively:

\[(63) \neg \text{know}_+(a)(b, \lambda i[\phi/a] \land \psi/a/) = (\phi/i/ \land \psi/i/))\]
\[(64) \neg \text{know}_+(a)(b, \lambda i[\phi/a] = \phi/i/) \land \neg \text{know}_+(a)(b, \lambda i[\psi/a] = \psi/i/))\]
Using the same meaning postulate we can also illustrate the difference between (47) and (48). Using (MP:INQ), (47) implies (65), whereas (48) implies (64):

\[(65) \forall \text{know}_x(a)(b,\lambda i((\phi/a = \phi/i) \land (\psi/a = \psi/i)))\]

One might think that not just (65), but also the stronger (64) follows from (47). This is, however, again a matter involving implicatures. Although (64) is not an implication of (47), it is an implication of (48). And, as we have seen above, (48) in its turn follows from (47) on the assumption of the truth of the implicature that exactly one of the alternatives holds. But that means that (64) follows from (47) too, if this implicature is true.

To sum up, treating complement coordination like we do enables us to account for the difference in meaning between (a), (b) and (c). The facts discussed above show that (45) implies (47) which in its turn implies (44). An interesting fact to note is that in this respect too there is a difference between intensional and extensional complement embedding verbs. Consider (66)-(68):

\[(66) \text{Bill wonders whether John walks and Mary walks} \]
\[(67) \text{Bill knows whether John walks and whether Mary walks} \]
\[(68) \text{Bill knows whether John walks or Mary walks} \]

It turns out that (67) and (68) are equivalent and that both imply (66). The equivalence of (67) and (68) may at first sight seem counterintuitive since there are clearly differences between them. However, as we argued above, in section 1.7, these differences do not concern truth conditional aspects of meaning, but are of a pragmatic nature.
6. Two loose ends and one speculative remark

6.1. A scope ambiguity in wh-complements

In this section we will show how a certain type of scope ambiguity can be accounted for in our analysis. A prime example is the ambiguity of sentence (69), extensively discussed in Karttunen and Peters (1980):

(69) Bill wonders which professor recommends each candidate

In order to facilitate the exposition we will discuss a simpler sentence, (70), and return to (69) at the end of this section:

(70) Bill wonders whom everyone loves

Following Karttunen and Peters we claim that (70) has three different readings. Two of them, (70a) and (70b), can be obtained in a straightforward way with the rules already available:

(70a) \( \text{wonder}_+(a)(b, \lambda \alpha i[\lambda v[\forall u[\text{love}_+(a)(u,v)]]] = \lambda v[\forall u[\text{love}_+(i)(u,v)]]] \)

'Bill wonders who is loves by everyone'

(70b) \( \forall u[\text{wonder}_+(a)(b, \lambda \alpha i[\lambda v[\text{love}_+(a)(u,v)]] = \lambda v[\text{love}_+(i)(u,v)]]] \)

'For each person Bill wonders who is loved by that person'
(70a) can be obtained by direct construction, (70b) by quantifying everyone into the sentence Bill wonder whom he loves. Given (MP:INQ), (70b) implies that for each person Bill does not know who is loved by that person. This predicts that the following is a contradiction:

\[(71)\] Bill knows that Suzy loves only John, but he still wonders whom everyone loves

Following Karttunen and Peters we assume that (71) is not necessarily false. This means that (70) also has a reading which has a weaker implication than (70b), viz. that Bill doesn't know for each person who is loved by that person. The obvious way to try to obtain readings like this is to quantify terms not only into sentences but also into complements. For this purpose we add the following rule:

\[
(S:QC) \text{ If } a \in P_T, \rho \in P_T, \text{ then } F_{ON,n}(a,\rho) \in P_T
\]

\[
(T:QC) \text{ If } a, \rho \sim a', \rho', \text{ then } F_{QC,n}(a,\rho) \sim \lambda i[a'(\lambda \lambda x_n[\rho'(i)])]
\]

Given these rules a third reading of (70) can be obtained as follows:

\[(70c)\] Bill wonders whom everyone loves, t

\[\begin{align*}
\text{wonder}_+(a)(b, \lambda \lambda i[\nu u[\lambda v[\text{love}_+(a)(u,v)]]
\end{align*}\]

\[\lambda i[\nu u[\lambda v[\text{love}_+(a)(u,v)]] = \lambda v[\text{love}_+(i)(u,v)]]\]

\[\begin{align*}
\text{whom everyone loves, } \bar{t}
\end{align*}\]

\[\lambda i[\nu u[\lambda v[\text{love}_+(a)(u,v)]] = \lambda v[\text{love}_+(i)(u,v)]]\]

\[\begin{align*}
\text{everyone, } T \quad \text{whom he_0 loves, } \bar{t}
\end{align*}\]

\[\lambda P[\forall x P(a)(x)] \quad \lambda i[\lambda y[\text{love}(a)(x_0,y)]] = \lambda y[\text{love}(i)(x_0,y)]\]
Universal quantification semantically amounts to a (possibly infinite) conjunction. Suppose we are dealing with finite cases so that we can write these conjunctions down. (This is of course not an essential restriction.) Then (70) (a)(b)(c) are equivalent to the conjunctions (70) (a')(b')(c') (in which \(d_1, \ldots, d_n\) name all the individuals):

\[
(70a') \text{ wonder}_+(a)(b, \lambda \alpha \lambda \iota [\lambda \nu [\text{love}_x(a)(d_1,\nu)]
\mathrm{^\wedge} \ldots \mathrm{^\wedge} \text{love}_x(a)(d_n,\nu)]
= \lambda \nu [\text{love}_x(i)(d_1,\nu)] \wedge \ldots \wedge \text{love}_x(i)(d_n,\nu)])
\]

\[
(70b') \text{ wonder}_+(a)(b, \lambda \alpha \lambda \iota [\lambda \nu [\text{love}_x(a)(d_1,\nu)]
\mathrm{^\wedge} \ldots \wedge \text{wonder}_+(a)(b, \lambda \alpha \lambda \iota [\lambda \nu [\text{love}_x(a)(d_1,\nu)]
= \lambda \nu [\text{love}_x(i)(d_1,\nu)]])
\]

\[
(70c') \text{ wonder}_+(a)(b, \lambda \alpha \lambda \iota [\lambda \nu [\text{love}_x(a)(d_1,\nu)]
\mathrm{^\wedge} \ldots \mathrm{^\wedge} (\lambda \nu [\text{lo}v\_e_x(i)(d_n,\nu)])
= \lambda \nu [\text{love}_x(i)(d_n,\nu)])
\]

It can be proved that (70a'), (70b') and (70c') express different propositions. In connection with this, it may be useful to point at the correspondence between (70a') and conjunctive complements, between (70b') and conjunction of complements, and between (70c') and alternative complements.

The implications resulting from application of (MP:INQ) to (70) (a)(b)(c) reflect the intuitions about the differences between the three readings of (70):

\[
(70a") \lnot \text{know}_x(a)(b, \lambda \iota [\lambda \nu [\text{vu}[\text{love}_x(a)(u,\nu)]]
= \lambda \nu [\text{vu}[\text{love}_x(i)(u,\nu)])
\]

\[
(70b") \text{vu}[\lnot \text{know}_x(a)(b, \lambda \iota [\lambda \nu [\text{love}_x(a)(u,\nu)]]
= \lambda \nu [\text{love}_x(i)(u,\nu)])
\]

\[
(70c") \lnot \text{know}_x(a)(b, \lambda \iota [\text{vu}[\lambda \nu [\text{love}_x(a)(u,\nu)]]
= \lambda \nu [\text{love}_x(i)(u,\nu)])
\]

It is interesting to note that, like in section 5.3 and of course for the same reasons, there is a difference between
extensional and intensional complement embedding verbs. If the matrix verb is extensional the (c)-reading collapses into the (b)-reading. This result is in accordance with the fact that (71), in contrast with sentence (70) has only two readings:

(71) Bill knows whom everyone loves

The results of quantifying into the sentence and the complement respectively are:

\[
(71b) \forall u[\text{know}_x(a)(b, \lambda i[\lambda v[\text{love}_x(a)(u,v)] = \lambda v[\text{love}_x(i)(u,v)]]])
\]

\[
(71c) \text{know}_x(a)(b, \lambda i[\forall u[\lambda v[\text{love}_x(a)(u,v)] = \lambda v[\text{love}_x(i)(u,v)]]])
\]

We leave it to the reader to verify that (71b) and (71c) are indeed equivalent, stressing the fact that this equivalence is essentially due to the fact that (71b) and (71c) concern relations between individuals and propositions, and not, as (70b) and (70c) do, relations between individuals and propositional concepts.

This difference between extensional and intensional complement embedding verbs also accounts for the fact that (72) is equivalent with (73) and with (74) on the reading where everyone has widest scope (but see the remarks in sections 1.5 and 3.4), whereas (75) is not equivalent with (76) (nor with (77) on the reading with everyone having widest scope):

(72) Bill knows who walks
(73) Of everyone, Bill knows whether he/she walks
(74) Bill knows whether everyone walks
(75) Bill wonders who walks
(76) Of everyone, Bill wonders whether he/she walks
(77) Bill wonders whether everyone walks
Notice that despite the equivalence of (72) and (73), (78) and (79) need not be equivalent:

(78) Bill knows which man walks
(79) Of every man, Bill knows whether he walks

(78) and (79) are equivalent only if (78) is read de re. Analogously, (70), on its reading (70c), is equivalent to (80), but (82) is equivalent to (81), on its third reading, only if (82) is read de re:

(70) Bill wonders whom everyone loves
(80) Bill wonders whom who loves
(81) Bill wonders whom every man loves
(82) Bill wonders whom which man loves

This means that quantifying a term into a complement always results in a de re reading of the common noun contained in the term (if any). So our approach predicts that (69) is equivalent to one reading of (83), viz. the one in which which candidate is read de re:

(69) Bill wonders which professor recommends each candidate
(83) Bill wonders which professor recommends which candidate

Whether this is a completely satisfactory result is, to be honest, beyond the scope of our intuitions.

6.2. Wh-complements in an extension of IL

In section 2.5 we said that one can get a long way in the analysis of complements by adding a new intensional operator to IL. As a matter of fact, one could come quite as far as the end of section 5, since the phenomena that resist an
adequate treatment in such an intensional language are phenomena like those discussed in the previous section 6.1.

The new operator, called $\Delta$, can be introduced in IL as follows:

(i) If $\alpha \in ME_L$, then $\Delta \alpha \in ME_{<s,t>}$

$[[\Delta \alpha]]_{M,k,g}$ is that $p \in \{0,1\}^I$ such that for every $i \in I$: $p(i) = 1$ iff $[[\alpha]]_{M,k,g} = [[\alpha]]_{M,i,g}$

With the aid of $\Delta$, the translations of the complement formation rules discussed in section 3 can be formulated as follows:

(T:THC') If $\phi \sim \phi'$, then that $\phi \sim \Delta \phi'$

(T:WHC') If $\phi \sim \phi'$, then whether $\phi \sim \Delta \phi'$

(T:WHC') If $\phi_1, \ldots, \phi_n \sim \phi'_1, \ldots, \phi'_n$, then whether $\phi_1, \text{ or } \ldots, \text{ or } \phi_n \sim \Delta \phi[p \wedge [p = \phi'_1 \vee \ldots \vee p = \phi'_n]]$

(T:CCF') If $\chi \sim \chi'$, then $F_{CCF}(\chi) \sim \Delta \chi'$

The phenomena that cause this approach to fail have in common that their treatment requires the possibility to quantify terms into complements. An example of such a phenomenon is the 'third reading' of sentence (20), mentioned in section 6.1. Another example is the reading of (84):

(84) John will tell whether every president walks

in which the term every president has narrow scope with respect to the tense, but wide scope with respect to the complement. On this reading (84) is true if at some time in the future John tells of every individual which at that time is a president whether he or she walks or not.

In order to obtain these readings, we need to be able to quantify terms into complements. This rule of quantification (S:QC) and its translation rule (T:QC) were stated in
section 6.1:

(R:QC) If $a \in P_T$ and $\rho \in P_{\mathcal{E}}$, then $F_{QC, n}(a, \rho) \in P_{\mathcal{E}}$

(T:QC) If $\alpha, \rho \sim \alpha', \rho'$, then $F_{QC, n}(\alpha, \rho) \sim \lambda i[\alpha'(\lambda a \lambda x_n[\rho'(i)])]$

The difficulty in formulating a translation rule in IL + $\Delta$ is that we cannot express the equivalent of $\rho'(i)$. We can only express the equivalent of $\rho'(a)$, namely $\forall \rho'$. (Notice that $\forall a$ expresses the proposition that is true at every index.) In IL + $\Delta$ we could only arrive at the translation rule:

(T:QC') If $\alpha, \rho \sim \alpha', \rho'$, then $F_{QC, n}(\alpha, \rho) \sim \lambda i[\alpha'(\lambda a \lambda x_n[\forall \rho'])]$

If $\psi'$ is of the form $\forall a$, the resulting expression denotes a proposition that holds true at every index, instead of denoting a proposition in the required index dependent way.

6.3. Remark on the semantics of direct questions

At the beginning of this paper, we expressed the hope that an adequate semantics of wh-complements might give a clue to the semantics of direct questions as well. At first sight, it seems that little or nothing speaks against simply associating direct questions with the same semantic objects we associated wh-complements with. An objection that might come to mind is this. Suppose $\phi$ is true. Then the direct questions *Does John know whether $\phi$?* and *Does John know that $\phi$?* denote the same proposition. Wouldn't this mean that asking the first question comes to the same thing as asking the second one? No, no more than that asserting a declarative sentence $\phi$ comes to the same thing as asserting a declarative sentence $\psi$ in case $\phi$ and $\psi$ happen to have the same truth value. Although the denotations of the two questions are the same, their senses still are different.
Another interesting issue is to what extent we could consider the proposition denoted by a question to be the proposition expressed by an answer to it. At first sight, it seems to make a good deal of sense to say that the proposition denoted by a question at a given index, is the proposition expressed by a true answer to that question at that index, and that hence the sense of a question could be described as a function from indices to true answers. However, things are more complicated. Compare the following sentences:

(85) Who won the Tour de France in 1980?
(86) Joop Zoetemelk won the Tour de France in 1980
(87) The one who ended second in 1979 won the Tour de France in 1980

Of course, (86) is a true answer to (85). However, in many cases (87) counts as a true answer as well. But it cannot be the case that both (86) and (87) express the proposition denoted by (85), since (86) and (87) clearly express different propositions. In our analysis, (86) expresses the proposition denoted by (85). In order to grant (87) the status of answerhood as well, one would need some property, in between 'denoting the same truth value' and 'expressing the same proposition', which (86) and (87) share. Such a property requires something in between truth values and possible worlds. It could very well be that the notion of possible fact, in the sense of Veltman (1981), is what is needed. One might then take a declarative sentence to be an answer to a question iff the possible fact expressed by the sentence is in some way related to the proposition denoted by the question. Then (86) and (87) would both qualify as answers to (85), since although they do not express the same proposition they do presumably express the same possible fact. It should be noted that this would not involve a change in the semantics of questions, it would be a refinement of the semantics needed for a satisfactory account of the
property of answerhood (and probably of many other things besides).

So, we conclude that it is misleading to interpret the proposition denoted by a question as the unique true answer to it. Both (86) and (87) should count as answers to (85). In fact, we believe that (86) should not even be granted a special status, even though it expresses the same proposition as (85) actually denotes. For there are situations in which (87) is a better answer to (85), for example by being more informative, than (86) is. In our opinion, this holds quite generally. Within the semantic limits set by the denotation of a question, what counts as a good answer is determined by pragmatic factors. These concern, among other things, the information available to the hearer, the information of the speaker about the information of the hearer, etc.

Pragmatic considerations again are all important in the following example:

(88) Where can one buy Italian newspapers?
(89) At the Centraal Station (one can buy Italian newspapers)
(90) At the Atheneum Newscentre (one can buy Italian newspapers)

Clearly, there are situations in which each of (89) and (90) on its own constitutes a proper answer to (88). But the propositions expressed by (89) and (90) are only part of (entailments of) the proposition denoted by (88). Some have taken this to show that questions are ambiguous between an existential (examplificatory) and a universal (exhaustive) reading. This runs counter to the exhaustiveness, even to the lowest degree, which we ascribe to wh-complements. Like Karttunen, we feel that again this is a pragmatic rather than a semantic phenomenon. Whether a question asks for a complete answer or for an incomplete one, depends on the needs of the one asking it. For example,
(88) when asked by an Italian tourist is properly answered, at least in most cases, by indicating one place where Italian newspapers are sold: what the tourist wants is a newspaper. (This does not mean that (89) and (90) in every such situation are equally good; other pragmatic factors, such as the acquaintance of the questioner with the various locations, etc. may be involved.) But when (88) is asked by someone who is interested in setting up a distribution network in Amsterdam for foreign newspapers, clearly an exhaustive answer to (88) is called for. So again, what counts as an answer is determined by pragmatic factors within the limits set by the semantics of the question.

Of course, these are just a few, rather speculative remarks, and a lot more has been (and still should be) said on these matters. But they seem to lead us to the conclusion that no semantic theory on its own can be expected to provide a satisfactory account of question-answer relations. Evidently, a pragmatic theory is called for. However, such a theory should be based on an adequate semantic theory. It is our hope that the semantic theory of wh-complements developed in this paper contributes to the survey of the semantic space within which pragmatic factors determine the question-answer relationship.
Notes

* Part of the material presented in this paper appeared as G & S 1981. We would like to thank Renate Bartsch, Elisabet Engdahl, Roland Hausser, Fred Landman, Alice ter Meulen, Ieke Moerdijk, Zeno Swijtink, Henk Verkuyl, and in particular Johan van Benthem, Theo Janssen, Lauri Karttunen and the anonymous referees of Linguistics and Philosophy for their comments and criticisms on earlier versions, which have led to many improvements.

1. We are told by one of the referees that David Lewis has developed a similar idea concerning whether-complements in an unpublished paper. We have not seen the paper, therefore we are unable to draw a comparison.

   [Added in proof: In the meantime we have obtained a copy of a recent version of Lewis' 1974 note, which under the title 'Whether' report is to appear in a Festschrift of which the publication data are not known to us. In this paper, Lewis discusses the index dependent character of whether-complements and proposes an analysis in terms of double indexing. We cannot argue for it here, but we feel that Lewis' analysis, in which whether-complements are taken to be expressions of sentence type, is less natural and less general than ours, in which they are considered to denote propositions. In particular, by taking the sense of complements to be propositional concepts, our analysis solves the problems with intensional (see section 1.3) complement embedding verbs which Lewis' proposal runs into.]

2. In order to avoid terminological confusion, let us point out that the way we use the terms 'extensional' and 'intensional' here, is a generalization of the terminology used in PTQ which does not fully conform to the traditional use. So, know is extensional in our sense of the term since it operates on the denotation of the complement that is its argument. But it is intensional in the traditional sense since the denotation of a complement is an intensional entity, viz. a proposition.

3. If their conclusions are read de re, these arguments are valid. If their conclusions are read de dicto, however, they are not. It turns out that the combination of treating proper names as rigid designators and verbs such as know as relations between individuals and propositions does not make it possible to distinguish a de dicto reading of the conclusions of these arguments. This is not correct, it
should be possible to distinguish a de dicto reading of these sentences, while maintaining a rigid designator view of the proper names at the same time.

4. Complements of this form are ambiguous between an alternative and a yes/no reading. The latter might be indicated as whether \( \phi \) or \( \psi \). In section 3.1 we show how this ambiguity is accounted for. In (IX) the alternative reading is meant.

5. That this is so, can be seen from the fact that the same phenomenon can be observed with other types of sentences. For example, it is not unreasonable to distinguish between a de dicto and a de re reading of the sentence John believes that everyone walks. Its de re reading would be true iff John believes of every individual that is in the domain of discourse that he/she walks, whereas its de dicto reading would be true iff John believes of every individual that according to him is in the domain of discourse that he/she walks. Yet within a possible world semantics, this distinction can be made only if one allows for varying domains in some sense. Since we are dealing here with a general problem of the semantics of propositional attitudes within an intensional framework, and not with a problem that is specific to finding a correct semantics for wh-complements, and since this paper is about the latter and not about the former, we will not try to solve it here.

6. Karttunen discusses argument (X). His reasons for not accepting (X) as valid accord with our remarks in the previous section on the type of situations that can give rise to counterexamples against (X). However, unlike Karttunen, we do not interpret the possibility of counterexamples as an argument against strong exhaustiveness.

7. For a proposal which makes it possible to consider infinitival complements to be proposition denoting expressions as well, see G & S 1979.

8. There still remains the verb know which takes NP's as in John knows Mary. An argument in favour of regarding this verb to be different from the one taking complements might be that in such languages as German and Dutch the difference is lexicalized. On the other hand, in a sentence like John knows Mary's phone number, the verb know seems to be quite like the complement taking know in many respects. (See also note 10.)

9. As a matter of fact, Karttunen argues against Hintikka's analysis (in Hintikka, 1976) by pointing out that John wonders who came cannot be paraphrased, as Hintikka would have it, as Any person is such that if he came then John wonders that he came. Unlike such verbs as guess and matter, wonder seems to be a truly ambiguous lexical item (in other languages, e.g. in Dutch, the difference in meaning is lexicalized). What arguments like the one used in the text and the one used by Karttunen in our opinion really show is that there is an essential difference between extensional and intensional complement embedding verbs, and that
10. The possibility of constructing these proposition denoting expressions from expressions $a$ of arbitrary type is quite interesting also in view of sentences like John knows Mary's phone number, mentioned in note 8. If we simply apply procedure (5) with the translation of the term Mary's phone number substituted for $0/a/$, we seem to obtain exactly the proposition John needs to know if he is to know Mary's phone number. The point was brought to our attention by Barbara Partee.

11. Notice that in PTQ complements are in fact taken to be of category $t$. When embedded under complement taking verbs, we semantically apply the interpretation of the verb to the sense of the complement. This makes that proposition denoting expressions do occur in PTQ translations. Because of this, one might think that the new category $t$ is superfluous. But it is not, since we want complements to denote propositions and to have propositional concepts as their sense.

12. For those who find it unbearable, c.q. unnatural, that the translation of whether $\phi$ or $\psi$ does not contain a disjunction, we present the following equivalent alternative:

$$\lambda i[\lambda p[p(a) \land [p = \lambda a \phi' \lor \ldots \lor p = \lambda a \phi']]]$$

13. For those complement embedding verbs for which (MP:IV/t) is not defined (i.e. the intensional ones), (11) holds trivially in case they are combined with a that-complement, since the sense of a that-complement is a constant propositional concept.

14. As (12) shows, whether-complements resemble if then else statements of certain programming languages. In Janssen (1980a) the latter are used as counterexamples to the validity of cap-cup elimination in IL. It seems that wh-complements are natural language counterexamples. If $\rho$ translates a wh-complement, then $\lambda a(\rho(a)) \neq \rho$, i.e. $\forall \rho \neq \rho$.

15. Engdahl in Engdahl (1980) presents a modification of Karttunen's framework in which a kind of de dicto readings can be obtained by means of a special storage mechanism. However, it turns out that, in order to obtain correct results, restrictions on the order of quantification of ordinary terms and wh-terms are necessary. But this means that in her framework too, a special level of analysis in between sentences and complements has to be distinguished.

16. Notice that condition (ii) allows the derivation of (i)(a) from (i), though it blocks (i)(b):
(i) The manRC[whoWHT[loves him]]kisses himWHT[ ]

(i) (a) AB[whomRC[whoWHT[loves him]]kissesWHT[ ]]

(i) (b) AB[whomRC[the manRC[whoWHT[loves him]]kissesWHT[ ]]]

Structures like (i)(a) are not generally considered to be well formed. These are problematic cases having to do with cross-over phenomena, which are not dealt with here and which, to our knowledge, present a problem to any account of Wh-constructions.

17. Of course, there is more to the antecedent-anaphor relation than c-command (see Landman and Moerdijk (1981) for an extensive discussion within the Montague framework). In the case discussed here, a consequence of using c-command and Wh-reconstruction is that (i):

(i) Which picture that John saw, he likes best

cannot be obtained with coreferentiality of John and he. How these and related problems are to be solved, is quite unclear.

18. It is sometimes claimed, e.g. in Engdahl (1980), that a structure like (35a) has to be ambiguous, since the related direct question allows for two different kinds of answers: functional ones like his last, and pair-list ones like: Gorter, 'Mei'; Kouwenaar, 'Elba'; Gerhardt, 'In tekenen'. For a long time we have thought, following Benett (1979), that functional readings could be regarded as a kind of shorthand for pair-list ones, and that only the latter would have to be accounted for in the semantics. However, in view of Engdahl's arguments and in view of such expressions as (i) and (ii):

(i) which woman no man loves

(ii) which woman few men love

which do not have a pair-list reading, but only a functional one (beside the direct reading), we are convinced now that functional readings are independent of pair-list ones. Moreover, they do not only occur with structures like (35a), but as (i) and (ii) show, are a quite general phenomenon. In G & S 1983 we propose to analyze functional readings by means of Skolem-functions. Abstract (35a) for example is then translated as (35a') and (i) as (i'):

(35) (a') λf[Vu[poem-of*(a)(u,f(u))]
∧ Vu[poet*(a) (u) -> like-best* (a) (u,f (u)) ]]

(i') λf[Vu[woman*(a)(f(u))]
∧ Vu[man*(a) (u) -> H love* (a) (u,f (u) ) ]]]

In these formulas f is a variable ranging over functions from individuals to individuals. Complements are formed from these expressions in the usual way.
19. Our notion of wh-reconstruction thus serves syntactical purposes only. In this respect it seems to differ from related notions, e.g. the one proposed in Van Riemsdijk and Williams (1980), where it plays a role in establishing the logical form of wh-constructions.

20. Actually clause (i) in (S:AB3,4) may be a bit too strict, since who loves whom and kisses him is well-formed, but cannot be derived here.

21. Belnap calls this 'the unique answer fallacy' (see Belnap, 1982). We agree with him that it is a mistake to think that every question has in every situation a unique true answer. But we have a different diagnosis as to how and where this has to be accounted for. We cannot do justice here to the many interesting arguments Belnap puts forward, but as will become clear from what follows, we feel that there is far more pragmatics between questions and answers than is accounted for in Belnap's theory.

22. A framework in which this kind of information of language users can be formally represented can be found in G & S (1980) and Van Emde Boas et al. (1981).
References

Belnap, N., 'Questions and Answers in Montague Grammar', in S. Peters and E. Saarinen (Eds.), Processes, Beliefs and Questions (Reidel, Dordrecht, 1982).


Karttunen, L., 'Syntax and Semantics of Questions', in Linguistics and Philosophy 1, 1977), 3-44.


Riemsdijk, H. van, and E. Williams, NP-Structure (unpublished paper, 1980).


III

INTERROGATIVE QUANTIFIERS
AND SKOLEM-FUNCTIONS

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0. Introduction

This paper discusses a particular problem in the analysis of questions: the proper account of what we will call the 'functional' reading of questions. The analysis we will propose is a further refinement of an analysis of questions in the framework of Montague Grammar which we have presented elsewhere (see G&S 1981b, 1982). Although we will make use of that analysis at some points, the contents of this paper will pretty much stand on their own.

Our interest in the problem of functional readings of questions was raised by Elisabet Engdahl's discussion of it in her dissertation (Engdahl 1980). To our knowledge, she was the first to discuss this phenomenon in any detail.

The notion of connectedness, though not treated explicitly, comes in at several points. The connectedness of questions and answers is used as a heuristic means in the analysis of questions. This in its turn may eventually contribute to an account of the question-answer relationship itself, which can be regarded as one of the fundamental types of connected discourse. Furthermore, some of the constructions which we will discuss exhibit an interesting kind of binding pattern, being a form of connectedness at sentence level. Lastly, the phenomenon of functional readings is, we will argue, also to be observed with certain kinds of indicative sentences, as appears from the various ways in which such sentences can be continued in a larger discourse. Here connectedness at discourse level comes in again.

The particular problem we want to discuss in this paper concerns questions like (1) and (2) in connection with answers of type (a), (b) and (c):

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(1) Which woman does every man love?
   (a) Mary (individual answer)
   (b) John loves Mary, Bill loves Suzy, ...
       (pair-list answer)
   (c) His mother (functional answer)

(2) Which of his relatives does every man love?
   (a) Mary
   (b) John loves (his wife) Mary, Bill loves (his sister) Suzy, ...
   (c) His mother

With respect to these examples, two facts call our attention. First of all, a question like (1) allows for three different types of answers. The first type is an answer like (a), which specifies a particular individual that is the woman that is universally loved by the men. This we call an individual answer. The second type of answer is exemplified by (b): it gives a list of all pairs of men and women such that the man loves the woman. This we call a pair-list answer. Answers of the third type (c), finally, specify a function, in this case one which for every man x, when applied to x gives the woman x loves as value. Answers such as (c) are the ones we are interested in here. We will refer to them as functional answers. The main points to be discussed are whether functional answers are a separate type of answers, and if so how this can be accounted for in the analysis of questions.

The second fact concerning the examples given above that we want to point out is that a question like (2) allows for only two types of answers: pair-list answers such as (b) and functional ones such as (c). An individual answer like (a) is excluded. Question (2) differs from (1) in that the wh-term which of his relatives contains a pronoun, his, that seems to be bound by the term every man. Not in all cases, however, this binding relation is of the usual sort, as we shall see below.

Before turning to the main topic of this paper, an account of functional answers, we will first say a few words
about the difference between individual answers and pair-list answers.

1. Scope-ambiguities in questions

An obvious way to deal with the difference between individual answers and pair-list answers is to relate them to different readings of a question like (1). These readings can be accounted for in terms of a scope-ambiguity. The reading corresponding to the individual answer is the one in which the wh-term which woman has wide scope with respect to the quantified term every man. The reading corresponding to the pair-list answer is the one where every man has wide scope over which woman. These two readings of (1) can be paraphrased as (1a) and (1b) respectively:

(1a) Which woman is such that every man loves her?
(1b) For every man, which woman does he love?

If an account along these lines is to work, two conditions have to be fulfilled. First, wh-terms have to be treated as scope-bearing elements, just as normal quantified terms. Second, questions have to be derivable in (at least) two different ways.

In the analysis developed in G & S 1981b, 1982, these two conditions are fulfilled as far as wh-complements, i.e. indirect questions, are concerned. In the present paper we will assume that at least as far as the problems we want to discuss here are concerned, the semantics of indirect and direct questions is the same. Therefore, we feel free to analyse direct questions via their indirect counterparts. Our analysis is carried out within the framework of a modified Montague grammar. Syntactically the grammar is enriched with an account of constituent structure, more or less along the lines pointed out by Partee (see Partee 1973, 1979). As for the semantics, the usual logical language of intensional type theory is replaced by a language of two-
sorted type theory. In this language explicit reference to and quantification over indices is allowed. What necessitates this change of translation medium is explained in G & S 1982, section 6.2.

The main features of our syntactic analysis of constituent questions are the following. We start with a sentence with one of more free term variables PRO\textsubscript{n}, PRO\textsubscript{k}, ... Choosing one of these variables, say PRO\textsubscript{n}, the sentence is transformed into a so-called abstract by 'preposing' a wh-term and replacing certain occurrences of PRO\textsubscript{n} by a trace, and others, if any, by suitable anaphoric pronouns. What happens with an occurrence of PRO\textsubscript{n} depends on its structural position in the original sentence. Next other wh-terms may be introduced, choosing other variables, by a similar process. After that, the abstract is transformed into a wh-complement by a category changing rule.

Semantically, we regard questions as proposition denoting expressions. Of particular importance is the index dependent character we ascribe to the denotation of questions. Which proposition a question denotes at an index depends on what is the case at that index. Loosely speaking, the proposition denoted by a question at some index is the true exhaustive answer to that question at that index.

Let us illustrate these general remarks by considering a concrete analysis tree plus translation of (the wh-complement corresponding to) question (1):\textsuperscript{4}
The abstract which woman every man loves is constructed from the common noun woman and the sentential structure every man loves \( \text{PRO}_1 \). In this process the wh-term which woman is formed and 'preposed'. The occurrence of \( \text{PRO}_1 \) is replaced by a wh-trace, i.e. an empty node labelled WHT. What semantically corresponds to this process of abstract formation is \( \lambda \)-abstraction over the free variable which occurs in the translation of the syntactic variable \( \text{PRO}_1 \). This makes
wh-terms scope-bearing elements. In the structure given above, the scope of *which woman* includes the universal quantifier in the translation of *every man*. The translation of the entire abstract denotes at an index \( i \) the set of women \( x \) such that for every man \( y \) at \( i \), \( y \) loves \( x \) at \( i \). The abstract is transformed into a proposition denoting complement. The distinction between abstracts and complements is not needed for syntactic purposes, but is semantically motivated. Since the distinction is not essential to the problems discussed in this paper, we will not motivate it here, but refer the reader to G & S 1982. The complement *which woman every man loves* denotes at an index \( a \) the proposition which holds at precisely those indices \( i \) in which the set of women who are loved by every man is the same as at \( a \). If at an index \( a \) Mary is the only woman whom is universally loved by the men, then the complement denotes at \( a \) the proposition that Mary, and only Mary, is loved by every man. In that situation, the answer *Mary* would be the 'true, complete answer' to question (1). On this reading the question can be answered by what we have called an individual answer. We therefore call this reading of question (1) its *individual reading*.

So, the first condition for questions to exhibit a scope ambiguity, i.e. that wh-terms have scope, is fulfilled. The second condition was that there be two ways to construct questions, that there be two derivations for them. This requirement is an immediate consequence of the central methodological principle of Montague grammar (and logical grammar in general): the principle of semantic compositionality. This principle says that the meaning of an expression is a function of the meanings of its parts and the way in which these parts are put together. In other words, the meaning of an expression is a function of the meaning of its parts and the way in which it is derived. Save for cases of lexical ambiguity, the principle of semantic compositionality therefore requires: different meanings, different derivations. If an expression is ambiguous between \( n \) readings, there have to be (at least) \( n \) different ways to derive it.
As we have indicated above, the derivation of question (1) given in (3) is the one which gives the reading that corresponds to individual type answers. It is the reading we paraphrased as (1a):

(1a) Which woman is such that every man loves her?

The proposition denoted by (1) on this derivation specifies women who are universally loved by the men. It remains to be shown that we can create another way to derive questions which gives the type of reading that corresponds to the pair-list type answers. As we have already remarked above, the obvious way to do this is to allow wh-terms and other terms to have different scope with respect to one another.

The usual way to create a scope ambiguity in Montague grammar is illustrated by the two derivations plus translations of the sentence every man loves a woman given in (4) and (5): 5

\[
\text{∀x}[\text{man(a)}(x) \rightarrow \exists y[\text{woman(a)}(y) \land \text{love(a)}(x,y)]]
\]

\[
\lambda P \forall x[\text{man(a)}(x) \rightarrow P(a)(x)]
\]

\[
\text{love(a)}[\lambda a \lambda P \forall y[\text{woman(a)}(y) \land P(a)(y)]]
\]

\[
\text{love(a)}[\lambda P \forall y[\text{woman(a)}(y) \land P(a)(y)]]
\]
The derivation in (4) results in the so-called 'direct' reading, in which every man has wide scope over a woman. The 'indirect' reading, in which a woman has widest scope, is obtained by quantifying in the term a woman into the sentence every man loves PRO\(_1\). This derivation is given in (5). Notice by the way that both derivations assign one and the same constituent structure to the sentence in question. Derivational ambiguities do not necessarily result in structural ambiguities, i.e. in different constituent structures.

The same kind of procedure can be followed in the case of questions. In (6) a second way to derive question (1) is given, in which the term every man is quantified into the complement which woman PRO\(_1\) loves:

\[ s_{\text{WH}}[\text{which woman}] s_{\text{T}}[\text{every man}] s_{\text{TV}}[\text{loves}] s_{\text{WH}}[ ] \]

\[ \lambda_i[y[\text{man}(a) (y) \land \lambda x[\text{woman}(a) (x) \land \text{love}(a) (y, x)]] = \lambda x[\text{woman}(i) (x) \land \text{love}(i) (y, x)]] \]

\[ \lambda P[y[\text{man}(a) (y) \land P(a) (y)]] \lambda i[\lambda x[\text{woman}(a) (x) \land \text{love}(a) (x, y, x)]] = \lambda x[\text{woman}(i) (x) \land \text{love}(i) (x, y, x)] \]
As is evident from the corresponding translation, the derivation process exemplified in (6) results in a reading of question (1) in which the term every man has wide scope over the wh-term which woman. The proposition denoted at an index a by the complement thus constructed, is the set of indices i such that for every man y at a it holds that the set of women that y loves at i is the same as the set of women y loves at a. Clearly, on this derivation, question (1) receives the reading paraphrased as (1b) above:

(1b) For every man, which woman does he love?

Such a question is answered by specifying for every man the woman (or women) he loves, i.e. by giving a list of pairs of men and women such that the man loves the woman. So, on this second reading question (1) is answered by what we have called a pair-list answer, hence this reading is called the pair-list reading.

Summing up our results, we conclude that individual answers and pair-list answers correspond to different readings of questions. These different readings stem from a scope ambiguity: wh-terms and normal quantified terms may stand in different scope relations to one another. Within the framework of Montague grammar it is possible to account for this ambiguity since wh-terms can be treated semantically as scope-bearing elements and since the usual 'quantifying in' device for handling scope ambiguities can be extended to questions.

Finally let us point out that the account just given of the ambiguity of questions between an individual and a pair-list reading enables one to explain why there is no individual reading for question (2):

(2) Which of his relatives does every man love?

This question cannot be answered by specifying an individual, as in the individual answer Mary, thus (2) lacks what we have
called the individual reading. The reason for this is the following. In Montague grammar the standard way to deal with anaphoric pronouns is also by means of quantification rules. Sentence (7), for example, is derived by quantifying in the term every man in the sentence PRO₁ loves PRO₁'s mother:

(7) Every man loves his mother

In the quantification process one of the occurrences of the syntactic variable which is quantified is replaced by the term which is quantified in, while any other occurrences become suitable anaphoric pronouns. Semantically, they turn up as bound variables. If the grammar is enriched with an account of constituent structure, various structural conditions may be formulated which govern this process (for a theory along these lines, see Landman & Moerdijk 1981, 1983).

As for question (2), it seems that in order to get an anaphoric pronoun his in the wh-term which of his relatives, the term every man should have wide scope. I.e. it has to be quantified into the question which of PRO₁'s relatives PRO₁ loves. But, as we have seen with regard to question (1), this would result in a pair-list reading. So, there is no way to derive (2) with his bound by every man which assigns it an individual reading. And this accounts for the impossibility of individual answers such as Mary to questions such as (2).

2. Functional readings of questions

We now turn to the third type of answers to questions which we distinguished: functional answers. With many others, we believed for a long time that answers like his mother to questions like (1) and (2) are just a kind of abbreviation, a more economic way of expressing pair-list answers. For suppose that things are as in the situation depicted in figure 1:
The arrow represents the love-relation. In this situation, the question *Which woman does every man love?* or *Which of his relatives does every man love?* can be answered by means of a pair-list answer as well as by means of a functional answer. The pair-list answer would be (8), the functional answer would be (9):

(8) John loves Mary, Bill loves Suzy and Peter loves Jane
(9) Every man loves his mother

Both answers cover the situation in question. This is not surprising, of course, for extensionally a function is just a list of pairs. So, if one answers the question by (9) instead of by (8), this seems to be merely for reasons of convenience. If the list of pairs gets longer, abbreviating the list by means of a function becomes more attractive. But that would be a fact of language use, not one of semantics. Both a pair-list answer and a functional answer would express the same complete true answer. And as far as the semantics of questions is concerned, there would be no reason to distinguish between the two.

But can functional answers and pair-list ones really always be equated? There seem to be several reasons to doubt this.

First of all, someone may know the answer *His mother* to the question *Which woman does every man love?* without being
able to present the corresponding pair-list answer. This may happen simply because he does not know of every man which woman is his mother. And vice versa, someone might be able to present a complete list of pairs of men and women such that the first loves the second, without knowing that in each case the woman is the mother of the man. So, it may be true that John knows which woman every man loves in the functional sense (he knows that every man loves his mother), without him knowing this in the pair-list sense. And vice versa, he may know it in the pair-list sense (he can give an exhaustive list of pairs of men and women such that the man loves the woman), without knowing it in the functional sense. This means that in a given situation, the sentence John knows which woman every man loves may be true "in a certain sense", but false "in another". One way to account for this possibility is to ascribe two senses, i.e. two readings, to this sentence. And it seems plausible that if the sentence in question is ambiguous in this way, this ambiguity stems from the complement. For the same ambiguity can be observed in case of the corresponding direct question Which woman does every man love?.

A second argument for the non-equivalence of functional and pair-list answers is the following. Suppose we change the situation of figure 1 into that of figure 2:

\begin{itemize}
  \item Son mother
  \item John Mary
  \item Bill Suzy
  \item Peter Jane
\end{itemize}

(fig. 2)
In this new situation, the complete pair-list answer to the question *Which woman does every man love?* has to be extended with the pair <Bill, Mary>:

(10) John loves Mary, Bill loves Mary and Suzy, and Peter loves Jane

Since Mary is not Bill's mother (Suzy is), the extension of the function his mother is no longer identical with the list of pairs that constitutes a complete pair-list answer. Still it seems that if someone asks the question *Which woman does every man love?*, the functional answer his mother, in this situation too, may constitute a fully satisfactory and complete answer. If this is true (as we think it is) it means that the question can be understood in different ways. Sometimes we use it to ask for a functional answer, and sometimes it serves to elicit a pair-list answer. If we use it in the first way in the situation described by figure 2, the functional answer his mother is the true complete answer. If we use it in the second way, the pair-list answer (10) is the true complete answer. Since the two are not equivalent, it follows that the question should have two non-equivalent readings corresponding to these two different kinds of answers. The functional answer cannot be regarded systematically as a mere abbreviation of the pair-list answer. If a question at an index a denotes the proposition to be expressed by what at a is a complete and true answer to it, and if there are two non-equivalent but equally satisfactory complete and true answers, then the conclusion must be that the question is ambiguous.

Perhaps the strongest arguments for distinguishing a separate functional reading of questions stem from examples such as (11)-(16):

(11) Which woman does no man love?
   (a) Mary
   (b) *John loves Mary, Bill loves Suzy, ...*
   (c) His mother
(12) Which of his relatives does no man love?
   (a) *Mary
   (b) *John loves Mary, Bill loves Suzy, ...
   (c) His mother

(13) Which woman do few men love?
   (a) Mary
   (b) *John loves Mary, Bill loves Suzy, ...
   (c) Their mother

(14) Which woman do many men love?
   (a) Mary
   (b) *John loves Mary, Bill loves Suzy, ...
   (c) Their mother

(15) Which of their relatives do few men love?
   (a) *Mary
   (b) *John loves Mary, Bill loves Suzy, ...
   (c) Their mother

(16) Which of their relatives do many men love?
   (a) *Mary
   (b) *John loves Mary, Bill loves Suzy, ...
   (c) Their mother

These questions differ from questions (1) and (2) in that they do not allow pair-list answers, where (1) and (2) do. Pair-list answers to these questions simply do not make sense. This does not only hold for terms with the determiners no, few or many as in the examples above, it holds for many others besides. They are listed in the second column in figure 3.
If functional answers would be just alternative, more concise ways of expressing pair-list answers, it would be hard to explain why questions such as (11)-(16) can be answered in a functional way, but do not permit a pair-list answer. To prevent pair-list answers to them, we have to exclude their pair-list reading. But then, no reading is available to which the functional answers would correspond if the two were identified. This shows that we need to distinguish functional from pair-list answers, and hence to postulate a separate functional reading for questions.

Why is it impossible to answer these questions by giving a list? Intuitively, the reason seems to be the following. If we are to be able to give a list, the term in question has to be associated with a definite set, otherwise we would not know what to make a list of. If we are asked to give a list of pairs of men and women such that the man loves the woman, we are only able to do this if we can pick the men from a definite set. With a question like Which woman does every man love? it is clear what we should do, the definite set is the set of every man. And the same holds for e.g. Which woman
do the two men love? In this case the set consists of the
two men, identified or specified either by the non-linguistic
context or by previous discourse. Things are completely
different with a question like Which woman do few men love?
There isn't any definite set of few men from which we can
construct our list. And hence it is impossible to answer such
a question by means of a pair-list answer.

In our analysis, the fact that questions with
non-universal subject terms do not have a pair-list reading
is mirrored by the fact that quantification of non-universal
terms into questions is ruled out. In order to derive
questions with pair-list readings we need to quantify terms
into questions. If we would apply this procedure in case of
non-universal terms, we would wind up with completely wrong
results. For example, quantifying in no man into which woman
PRO, loves would result in the following translation, which
does not represent a meaning of the question which woman no
man loves:

\[
\lambda i(\forall y[\text{man}(a)(y) \rightarrow \exists i[ \exists x[\text{woman}(a)(x) \land \text{love}(a)(y,x)] = \\
\lambda x[\text{woman}(i)(x) \land \text{love}(i)(y,x))]])
\]

At an index a this expression denotes the set indices i such
that for no man x at a the set of women whom he loves at i
is the same as the set of women he loves at a. For no man
this proposition entails the proposition which identifies
the woman (or women) he loves.

The explanation given above of why pair-list answers are
not possible with questions like (11)-(16) seems reasonable
enough. Since functional answers are possible, however, this
constitutes a conclusive argument against the equation of
functional answers with pair-list answers.

Where does all this leave us? We seem to be forced to
distinguish, quite generally, three different readings for
questions. In some cases some readings are excluded, for
reasons which we have indicated. The individual reading of
questions, i.e. the reading which gives rise to the
individual type answers, corresponds to direct construction,
exemplified in (3) above. The pair-list reading is the result of quantifying in. This construction is exemplified in (6). It is restricted to universal terms. At first sight the functional reading appeared to be a simple variant of the pair-list reading, but as we have argued above, it is not. This means that the functional reading cannot be derived by the quantifying-in process. On the other hand, though akin to it in some respect, the functional reading obviously is not equivalent to the individual reading either. Following the methodological principle of compositionality, we postulate a third way to derive questions.

At this point an interesting phenomenon can be observed. As we said, the functional reading cannot be obtained by quantifying in since the wh-term has to have wide scope over the subject term, so, semantically the subject term cannot bind anything inside the wh-term. Syntactically, however, in such questions as (2), (12), (15) and (16), the subject term, in some way or other, has to bind the pronoun in the wh-term. Here semantic and syntactic binding are not parallel in the way they usually are, a fact that hitherto seems to have escaped attention.

3. **Functional readings and Skolem-functions**

In this section we will sketch our solution to the problem of functional readings of questions. In section 4 we will indicate some further uses of the apparatus in similar problematic cases.

Questions like (2) and (12) are discussed extensively by Elisabet Engdahl (Engdahl 1980). She does not discuss functional readings of questions such as (1), (11), (13)-(16). Her proposal for the analysis of the functional readings of (2) and (12) is not fully satisfactory, and moreover is not general enough to deal with the other cases.\(^{14}\)

As for our own solution, since our framework is one in which we want to give an explicit model-theoretic semantics
for natural language, there are two things which we will have to do. First of all, we will have to indicate what the interpretation of questions on their functional reading is. Secondly, if we have succeeded in this, we will have to provide explicit syntactic and semantic rules which, building up the interpretation of the whole from the interpretation of the parts, give us the required results.

Our proposal is to use so-called Skolem-functions in the analysis of functional readings of questions. Let us consider the simple question (18) in connection with the functional answer (c):

(18) Whom does every man love?

(c) His mother

The answer His mother specifies a function from individuals to individuals. When applied to an individual, say John, it gives the mother of that individual, say Mary, as its value. What answer (c) expresses is that this function, call it f, is such that for every man x when f is applied to x it gives as value an individual that x loves. So, on its functional reading question (18) asks which function f is such that for every man x, x loves f(x).

This suggests the following translation (19) for (18) on its functional reading. For comparison we add the translation (20) of the individual reading of (18):

(19) $\lambda f[\forall x[\text{man}(a)(x) \land \text{love}(a)(x,f(x))]]$

(20) $\lambda y[\forall x[\text{man}(a)(x) \land \text{love}(a)(x,y)]]$

Functions from individuals to individuals like f used above, are called Skolem-functions. They can be used to change the order of quantifiers in a formula like $\forall x \exists y \phi(x,y)$ in order to obtain an equivalent formula $\exists f \forall x \phi(x,f(x))$. In order to illustrate this, look at the picture in figure 4:
In the situation depicted in figure 4 it holds that \( \forall x \exists y \ x \rightarrow y \) and also that \( \exists f \forall x \ x \rightarrow f(x) \), viz. the following function:

\[
(21) \quad g(1) = 2, \ g(2) = 3, \ g(3) = 4, \ g(4) = 1
\]

Of course, there may be more such functions as in the situation depicted in figure 5:

In this situation there are two functions that make \( \exists f \forall x \ x \rightarrow f(x) \) true, viz. \( g \) and \( h \):

\[
(22) \quad h(1) = 2, \ h(2) = 3, \ h(3) = 4, \ h(4) = 2
\]

Question (1) on its functional reading asks not for any function such that for every man \( x \), \( x \) loves \( f(x) \), but for a function which always yields a woman as its value:

(1) Which woman does every man love?

(c) His mother

(c') *His father
Whereas question (18) can be answered functionally with His father, this answer is not possible for question (1), since the father-function is not a function into the set of women. So, a question like (1) restricts the set of possible functions that may constitute an answer to it on its functional reading. In the case of (1) this restriction on admissible functions f can be formulated as: \( \forall x \text{ woman}(a)(f(x)) \). As a whole, (1) may be translated into (23). For comparison we give again the translation of (1) on its individual reading as (24).

(23) \( \lambda f[\forall x \text{ woman}(a)(f(x)) \land \forall x[\text{ man}(a)(x) \rightarrow \text{ love}(a)(x,f(x))] ] \)

(24) \( \lambda y[\text{ woman}(a)(y) \land \forall x[\text{ man}(a)(x) \rightarrow \text{ love}(a)(x,y)]] \)

The most interesting case is a question like (2):

(2) Which of his relatives does every man love?
   (c) His mother
   (c') His first grade teacher

This question too formulates a restriction on the functions that can be specified as answers to it. Here the restriction can be formulated as: \( \forall x \text{ relative-of}(a)(f(x),x) \). The functional reading of (2) can then be represented as (25):

(25) \( \lambda f[\forall x \text{ relative-of}(a)(f(x),x) \land \forall x[\text{ man}(a)(x) \rightarrow \text{ love}(a)(x,f(x))] ] \)

It is clear that thus interpreted (c) constitutes an acceptable answer to (2), but (c') does not. Notice that the variable \( x \) in \( \text{ relative-of}(a)(f(x),x) \), which corresponds to the pronoun his in the wh-term which of his relatives is not bound by the universal quantifier in the translation of every man. Rather, it is bound by the universal quantifier in the restriction on the function. Still, the effect is as if it is bound by every man since for every choice of a man \( x, f(x) \) is a relative of \( x \). This is the result of restricting \( f \) in such a way that when applied to an individual it gives a
relative of that individual as its value. So, although we can say that the pronoun his in the wh-term is 'bound' in a certain sense by the term every man, it is not connected with it in the usual direct way of being translated as a variable which is bound by the quantifier in the translation of the term. Rather, the pronoun depends on the term indirectly, via the dependency of the Skolem-function and the way in which it is restricted. In constructions like these, the pronoun is neither a variable bound by a term, nor is it a pronoun of laziness or a discourse anaphor. Rather it signals a separate kind of dependency, a functional dependency. This is a rather unusual kind of semantic binding which allows us to account for a semantic relation between two terms which, in a sense, is the reverse of their syntactic relation.

As a last example, consider question (12), a question with a non-universal subject term. Such questions do not allow pair-list answers but they do have a functional reading. In (26) the functional reading of (12) is represented:

(12) Which of his relatives does no man love?
(26) \( \lambda f[\forall x \text{ relative-of}(a)(f(x),x) \land \\
\forall x[\text{man}(a)(x) \rightarrow \forall \text{love}(a)(x,f(x))] ] \)

The expression in (26) denotes the set of functions \( f \) such that for every \( x \), \( f(x) \) is a relative of \( x \), and for no man \( x \) it holds that \( x \) loves \( f(x) \). Answering (12) on this reading by a functional answer like His mother is specifying one of those functions, and expresses that no man loves his mother. For other questions with non-universal subject terms, the functional reading can be represented in a similar fashion.

What we have ended up with now are formulas that correctly represent the interpretations of questions on their functional readings. But as we said earlier, this constitutes only half of the job. Writing down a formula that represents the meaning of a sentence is one thing, finding a compositional translation procedure which results in this formula, or in one that is equivalent to it, is quite another. (For example, it is no problem to write down
formulas which represent the meaning of Bach-Peters sentences or donkey-sentences. What is difficult is to construct a compositional procedure that produces them.

We cannot deal here with the syntax of wh-constructions in detail. For our analysis the reader is referred to G & S 1982, section 4. We will restrict ourselves to giving an informal indication of the contents of the relevant syntactic rules, by discussing some examples. What is important is that to these syntactic rules compositional translation rules correspond, thus providing a compositional semantics for the expressions produced.

Consider to begin with the derivation tree (27), which gives the functional reading of question (1), and compare it with (3), the derivation tree which resulted in the individual reading of (1):

(27) \[ AB[WHT[\text{which woman}]_{S,T[every man]}_{TV[loves]_{WHT[ ]}}]] \]

\[ CN[\text{woman}] \quad S[T[every man]_{TV[loves]_{T[PRO'S_1]}}] \]

\[ T[every man] \quad S[T[PRO_2]_{TV[loves]_{T[PRO'S_{1,2}]}}] \]

\[ T[PRO_2] \quad TV[loves]_{T[PRO'S_{1,2}]} \]
In order to obtain the functional reading, a new kind of syntactic variable of category T is introduced. It is a double-indexed variable of the form $\text{PRO}^{S}_{m,n}$. The two indices $m$ and $n$ of these syntactic variables correspond to the indices of the two free variables $f_m$ and $x_n$ in their translation, which is given in (28):

\begin{equation}
\text{PRO}^{S}_{m,n} \sim \lambda P[P(a)(f_m(x_n))]
\end{equation}

Here '$\sim$' is to be read as 'translates into'. $P$ is a variable of type $<s,<e,t>$, $w$ of type $s$, $f_m$ of type $<e,e>$ and $x_n$ of type $e$. The translation $\lambda P[P(a)(f_m(x_n))]]$ denotes at $a$ the set of properties $P$ which the individual $f_m(x_n)$, the value of $f_m$ for $x_n$, has at $a$.

The new syntactic variables behave like all other expressions of category T. So we can form the sentence (29):

\begin{equation}
s_T[\text{PRO}_2]_{IV}[\text{TV}[\text{loves}]]_T[\text{PRO}^{S}_{1,2}]]
\end{equation}

in the usual way. Into this sentence we can quantify every man for variables carrying index 2. The existing quantification rule has to be adapted slightly in view of the possible occurrences of this new kind of syntactic variable. What is important is that features for number and
gender of the term that is quantified in are taken over by all those occurrences of variables with the relevant index that are not replaced by the term itself. Thus, quantifying in every man into (29) for PRO₂ results in (30):

\[(30) \ s_{T}^{\text{every man}}_{IV}^{T_{V}}[\text{loves}]_{T}^{[\text{PRO}^{'S_{1}}]}]\]

in which PRO'S₁ carries the features male, singular, third person, because it is bound by the male, singular, third person term every man. The translation rule corresponding to the modified quantification rule remains unaltered. Syntactically, quantifying in removes the second index on a variable PRO'S₂,ₙ, semantically it binds the variable xₙ, ranging over individuals, by the translation of the term which is quantified in.

From sentence (30) and the common noun woman an abstract is formed. If we compare this stage of the derivation of the functional reading with the corresponding stage of the derivation of the individual reading, we notice that syntactically the difference is minimal. Where the former has an occurrence of a syntactic variable PRO'S^ in its input sentence, the latter has an occurrence of PRO. The resulting abstracts are in both derivations the same:

\[(31) \ AB^{\text{which woman}}_{WHT}[\text{every man}]_{S}^{T_{IV}[\text{loves}]_{WHT}[\text{]]}]\]

They are formed by the same syntactic process. Informally, the relevant syntactic rules read as follows.

On the individual reading the abstract is derived by means of (S:AB2):

\[(S:AB2) \ \text{If } \delta \text{ is a CN and } \phi \text{ is an S containing one or more occurrences of } \text{PRO}_n \text{ which satisfy certain structural constraints, then } F_{AB,2}^{\text{which } \delta}^{\phi'} \text{ is an AB of the form } AB^{\text{which } \delta}^{\phi'}, \text{ where } \phi' \text{ comes from } \phi \text{ by replacing certain of the occurrences of } \text{PRO}_n \text{ by traces and all the others}\]
by anaphoric pronouns which take over the features for gender and number from the CN $\delta$

The translation rule corresponding to $\text{(S:AB2)}$ is $\text{(T:AB2)}$:

$$\text{(T:AB2)} \quad \text{If } \delta \sim \delta' \text{ and } \phi \sim \phi', \text{ then}$$
$$F_{\text{AB2},n}(\delta,\phi) \sim \lambda x_n[\delta'(x_n) \land \phi']$$

On the functional reading the abstract is derived by means of a quite similar syntactic rule $\text{(S:AB2/f)}$:

$$\text{(S:AB2/f)} \quad \text{If } \delta \text{ is a CN and } \phi \text{ is an S containing one or more occurrences of PRO'S which satisfy certain structural constraints, then}$$
$$F_{\text{AB2/f},n}(\delta,\phi) \text{ is an AB of the form }$$
$$\text{AB}_{\text{WHT}}[\text{which } \delta] \phi', \text{ where } \phi' \text{ comes from } \phi \text{ by replacing certain of the occurrences of PRO'S which take over the features for gender and number from the CN } \delta$$

The corresponding translation rule is $\text{(T:AB2/f)}$:

$$\text{(T:AB2/f)} \quad \text{If } \delta \sim \delta' \text{ and } \phi \sim \phi', \text{ then}$$
$$F_{\text{AB2/f},n}(\delta,\phi) \sim \lambda f_n[\forall x \delta(f_n(x)) \land \phi']$$

On its individual reading the abstract underlying which woman every man loves denotes the set of individuals $y$ such that $y$ is a woman and for every man $x$ it holds that $x$ loves $y$. On its functional reading the abstract denotes the set of functions $f$ from individuals to individuals such that $f$ is a function into the set of women and for every man $x$ it holds that $x$ loves $f(x)$. So, on the individual reading the common noun woman in the wh-term which woman functions as a restriction on individuals, on the functional reading it acts as a restriction on Skolem-functions.

As a second example, consider the derivation tree plus
translation of the functional reading of question (2), which of his relatives does every man love?:

(32) 
[which relative of him][S[T[every man]IV[loves]WHT]]

\[
\begin{array}{c}
CN[CN/T[relative of]T[PRO_3]] \\
CN/T[relative of]T[PRO_3] \\
S[T[every man]IV[loves]T[PRO'S_1]] \\
T[every man]S[T[PRO_2]IV[loves]T[PRO'S_{1,2}]] \\
T[PRO_2]IV[loves]T[PRO'S_{1,2}] \\
IV[loves]T[PRO'S_{1,2}] \\
TV[love]T[PRO'S_{1,2}]
\end{array}
\]
The new element in this derivation is that in forming the abstract from the sentence a common noun is used which itself contains a free syntactic variable which gets bound in the process of abstract formation. In deriving the abstract which relative of him every man loves from the common noun relative of PRO₁ and the sentence every man loves PRO'S, two variables get bound: the functional variable in PRO'S in the S and the individual variable in PRO₁ in the CN. The syntactic rule which does this can informally be stated as follows:

(S:AB5) If δ is a CN with one or more occurrences of PROₙ and φ is an S with one or more occurrences of PRO'S which satisfy certain structural constraints, then F_{AB5,n,m}(δ,φ) is an AB of the form \( \text{AB}_{\text{WHIT}}[\text{which } δ'][φ'] \), where δ' comes from δ by replacing the occurrences of PROₙ by
anaphoric pronouns which take over the
features for gender and number from PRO'S,m,
and where $\phi'$ comes from $\phi$ as in (S:AB2/f)

The syntactic process codified in this rule is quite like
that described in the previous two rules of abstract
formation (S:AB2) and (S:AB2/f). The only difference lies
in the fact that in addition the syntactic variable PRO,n in
the CN is bound and takes over the features for number and
gender from the variable PRO'S,m in the S, and thereby
indirectly from the term by which the latter variable in its
turn is partly bound. This syntactic binding process is not
paralleled by the normal semantic binding process. Although
syntactically _every man_ binds _him_ in which relative of _him_,
semantically the variable in the translation of _him_ is not
inside the scope of the quantifier in the translation of
_every man_. Rather it is bound in the translation of the
restriction which the _wh_-term places on the functions. This
is expressed in the translation rule corresponding to
(S:AB5):

\[(T:AB5) \quad \text{If } \delta = \delta' \text{ and } \phi = \phi', \text{ then} \]
\[F_{AB5,n,m}(\delta,\phi) = \lambda f_m[\forall x_n \delta'(f_m(x_n)) \land \phi']\]

Of course this description of the derivation process of
functional readings of questions gives a mere indication of
what a detailed syntactic analysis would look like. This is
true in particular for the remarks on how morphological
features function in this process. However, we are confident
that such a detailed analysis can be carried out, on the
basis of the syntax of _wh_-constructions defined in _G & S 1982_
and a theory of morphology as proposed in _Landman & Moerdijk
1981, 1984_.

More important in the context of the present paper is
that our remarks have shown (and not merely indicated) that
it is indeed possible to give a compositional semantics for
questions which accounts for individual, pair-list and
functional readings. This is shown by the compositional
translation rules defined above. In fact it is the methodological principle of semantic compositionality that more or less directly leads to an analysis like the one just outlined. If one accepts compositionality as a requirement on one's grammar, one is bound to associate a derivational ambiguity with every non-lexical semantic ambiguity.

At this point it may be usefull to stress again the difference between derivation and constituent structure. Constituent structure is what we have intuitions about, intuitions which may take the form of well-formedness judgements and which can be elicited by means of various kinds of tests. Constituent structure embodies our intuitions about what the parts of an expression are, how they combine into larger parts, how they depend on one and another, etc. But as to how these constituent structures are derived, we do not have any intuitions at all. The derivational process is not directly linked with syntactic intuitions. The analysis of questions given in this paper illustrates this. The various types of derivations which we distinguished, for example the three derivations (3), (6) and (27) of question (1), are of course primarily semantically motivated. This is also evident from the fact that all of them assign the same constituent structure to the question. Quite generally, one may say that within the framework of Montague grammar the theory of syntactic structure is embodied, not in the derivations, but in the constituent structures which the grammar assigns to the expressions it produces.

One may perhaps object against the semantically motivated level of derivations in the syntax, feeling that syntax should deal with syntactic properties of expressions only. But then one has to give up the compositionality requirement. For given the fact that constituent structure as such does not determine semantic interpretation, any grammar that is set up to give a compositional semantics for the expressions it produces, will have to contain some level of analysis which is primarily semantically motivated, a level which contains in addition to the information which the constituent structure of an expression provides all other
aspects which are needed to fix its semantic interpretation. One may very well argue about the precise contents of the level of analysis and its exact place in the grammar. One may prefer storage mechanisms (cf. footnote 17) or interpretation strategies over derivations, but given the common goal of logical grammar, a compositional semantics for natural language, a level of analysis like that of derivations has to be incorporated in the grammar, some way, somewhere.20

4. Functional readings of other constructions

In this section we will point out briefly other types of constructions than questions where functional readings seem to play a role.

Consider sentence (33):

(33) Every man loves a woman

A sentence such as (33) can be continued in a larger discourse in (at least) three different ways. These continuations are remarkably like the three ways in which the question Which woman does every man love? can be understood:

(33) (a) Mary
    (b) John loves Mary, Bill loves Suzy, ...
    (c) His mother

We call them the individual continuation, the pair-list continuation and the functional continuation accordingly. Sentence (33) is generally assumed to have two readings. The individual continuation would match the reading of (33) which is the result of constructing it indirectly, i.e. by quantifying in a woman (see (5)), which consequently gets wide scope:

(34) ∃y[woman(a)(y) ∧ ∀x[man(a)(x) → love(a)(x,y)]]
So, the individual continuation (33)(a), *Mary*, is to be regarded as a specification of an individual that is loved by every man, that is said to exist by (33) on its reading (34). The other reading of (33) is of course the one which results from the direct construction (see(4)):

(35) ∀x[man(a)(x) → ∃y[woman(a)(y) ∧ love(a)(x,y)]]

At first sight nothing speaks against taking both the pair-list continuation (33)(b) and the functional continuation (33)(c) as matching this reading of (33). In (35) it is expressed that for all men there is a woman whom he loves. This fact may well be specified either by giving a list of pairs, as in (33)(b), or by giving a function, as in (33)(c). On this view the functional continuation would be a convenient abbreviation of a pair-list continuation.

But now consider sentence (36):

(36) There is a woman whom every man loves

This sentence can be continued in two ways only, individually and functionally:

(36)(a) Mary

(b) *John loves Mary, Bill loves Suzy, ...

(c) His mother

A pair-list continuation does not result in a well-formed, interpretable discourse. Two facts call our attention. First of all, with respect to (33) the suggestion was to take the functional continuation as a mere abbreviation of a pair-list continuation. This strategy will not work, however, in case of (36), since in this case the pair-list continuation is not possible while the functional continuation is. Secondly, a sentence such as (36) is often regarded (and offered) as a disambiguation of a sentence like (33). (36) is considered to have only one reading, being the indirect reading (34) of
(33), in which a woman has wide scope over every man. This is in accordance with the fact that an individual continuation is possible for (36). But it conflicts with the previously mentioned suggestion that the functional continuation of (33) corresponds to its direct reading (35). For this leaves us at a loss as to how to account for the functional continuation of (36).

A possible solution is to assign to (36) a second, 'functional' reading of which (36)(c) is the functional continuation. This reading may be represented as follows:

\[
(37) \exists f [ \forall x \, \text{woman}(a)(f(x)) \land \forall x [\text{man}(a)(x) \rightarrow \text{love}(a)(x,f(x))] ]
\]

So, (36) can also be read as asserting that there is a function f into the set of women such that for every man x it holds that x loves f(x). The functional continuation (36)(c) specifies this function as the mother-function, much in the same way as the individual continuation (36)(a) specifies the woman that is universally loved among the men, that is asserted to exist by (36) on its reading (34), as the individual Mary.

But here a problem presents itself, for (37) is equivalent to (35). And (35) intuitively does not represent a reading of (36), an intuition which is supported by the fact that it is (35) that makes the pair-list continuation possible for (33), a type of continuation which does not exist in connection with (36). So, postulating reading (37) for (36) in order to account for the possible functional continuation (36)(c), seems to allow the impossible pair-list continuation (36)(b) as well.

A formally correct and intuitively appealing solution to this problem is to restrict the domain of the quantifier \( \exists f \) in (37) to some subset of the totality of all Skolem-functions. If we do this, (37) is no longer equivalent to (35) and we have a representation of (36) which accounts for the functional continuation without allowing the pair-list one. This seems a quite reasonable move to make, for if one asks for the specification of a function (with a question on its
functional reading), or asserts the existence of a function and gives a specification of it, one obviously is not satisfied with any old specification of any old weird functional relationship between individuals. If someone asserts that there is some function f such that for all x, x loves f(x), and on our demand to specify this function, starts listing all pairs <x,y> such that x loves y, this simply will not do. Somehow quantification over functions is restricted. It would seem that functions that are allowed, must be either conventional in some sense (such as the mother-function, the wife-function, etc.) and thus in some sense computable, or they must be made computable by the context. Compositions of such acceptable functions will in most cases result in acceptable functions. The exact principle, or principles, underlying this restriction are not entirely clear to us, but that something like this is going on seems quite likely.

Assuming that quantification over Skolem-functions is indeed restricted, we can not only explain that (36) has a functional reading but not a pair-list reading, it also becomes reasonable to consider (33) to be 3-ways ambiguous. The third reading of (33) will be the same as the second, functional reading of (36), reformulated as (37'):

\[
(37') \exists f[R(f) \land \forall x \text{woman}(a)(f(x)) \land \forall x[\text{man}(a)(x) \rightarrow \text{love}(a)(f(x))]]
\]

Here R is to be filled by some predicate over Skolem-functions which expresses the restriction to 'conventional', 'computable' functions.

Another sentence that illustrates the usefulness of distinguishing functional readings from pair-list readings is (38):

\[
(38) \text{There is a woman whom no man loves}
\]

Like (36) this sentence has a functional continuation, but no pair-list continuation. The functional reading of (38) is represented by (39):
Finally, it may be noted that the special binding properties we found in questions like:

(2) Which of his relatives does every man love?

occur also in certain indicative sentences. An example is (40):

(40) Every man loves one of his relatives

This sentence does not have a reading in which the term one of his relatives is quantified in, for then the pronoun his could not be bound by every man. This appears also from the fact that (40) does not allow an individual continuation, it cannot be continued by specifying an individual. The sentence has a pair-list continuation which corresponds to the reading which results from quantifying in every man in PRO's relatives. It also allows a functional continuation which matches the reading which results from quantifying in one of PRO's relatives in the sentence every man loves PRO's relatives by means of a process which is completely analogous to that by means of which the functional reading of a question like (2) is derived and which was described above in rule (S:AB5). In this case too, the syntactic binding of his in one of his relatives by every man is not paralleled by the usual semantic binding: the variable in the translation of his is not bound by the quantifier in the translation of every man. This is shown by the following representation of the functional reading of (40):

(41) \exists f[R(f) \land \forall x \text{ relative-of}(a)(f(x),x) \land \\
\forall x[\text{man}(a)(x) \rightarrow \text{love}(a)(x,f(x))]]

The pronoun his gets bound semantically in the restriction on the range of the Skolem-function f. The effect is the
same as in the case of the corresponding question: for every man \( x \), \( f(x) \) denotes one of \( x \)'s relatives. Notice that since (41) expresses restricted quantification over Skolem-functions, it is not equivalent to (42), which represents the pair-list reading of (40):

\[
(42) \forall x [\text{man}(a)(x) \rightarrow \exists y [\text{relative-of}(a)(y,x) \land \text{love}(a)(x,y)]]
\]

So, we assign to (40) two distinct readings, the functional one and the pair-list one.

Formula (41) also represents the only reading of sentence (43):

\[
(43) \text{There is one of his relatives that every man loves}
\]

This sentence allows neither an individual continuation nor a pair-list one. It can only be continued with a specification of a function. In this case the need to distinguish functional readings is quite evident, the functional reading being the only one (43) has.

The reason why (43), (38) and (36) do not have a pair-list reading is that in order to obtain this reading the term every man, c.q. no man would have to be quantified into a relative clause, which is not allowed: the scope of any term inside a relative clause is restricted to that relative clause. The reason why (43), unlike (38) and (36), also does not have an individual reading is the same as why this reading does not occur with (40): it would leave the pronoun his in one of his relatives unbound.

5. Conclusion

What we have tried to show in this paper were two things: first of all, that questions have functional readings and that these readings are independent from other readings, and secondly, that an account of functional readings can be given within the framework of Montague grammar.
As for the first objective, we think that the arguments given in this paper are convincing. The phenomenon of functional readings is a real one, which even extends to other types of constructions, as we have indicated in the previous section.

Concerning the account of functional readings which we sketched above, we are less satisfied. We do believe that the rules which we have proposed give a compositional analysis of functional readings. However, we cannot reason away some doubts as to the plausibility (let alone elegance) of the syntactic part of our analysis. We would prefer one which would involve less complications in the syntax. Such an analysis would require a major modification of the framework of Montague grammar. And of the available alternatives, none strikes us as definitely superior in this respect. And it may be relevant to stress again that whatever kind of analysis one may come up with, functional readings should be represented as distinct readings of questions (and other constructions), and thus require some level of representation on which these constructions are disambiguated.
Notes

* We would like to thank Elisabet Engdahl for some stimulating discussions and Renate Bartsch and Johan van Benthem for their comments on some preparatory notes.

1. An individual answer may, of course, specify more individuals. So, if both Mary and Suzy are loved by every man, (a') is an individual answer too:

(a') Mary and Suzy

Something similar holds for pair-list answers and functional answers: (b') is also a pair-list answer to question (1), and (c') a functional answer:

(b') John loves Mary, John loves Suzy, Bill loves Suzy, ...
(c') His mother and his grandmother

For simplicity's sake, we stick in what follows to the most simple case.

2. There are situations in which it does seem to be possible to give an individual answer to a question like (2). Suppose we quantify over the set of men in our family. These men have the same (blood-)relatives. Then the following is possible:

(2') Which of his (blood-)relatives does every man (in our family) love?
   (a) Aunt Mary

However, it is quite clear that in this situation the answer (2')(a) is to be regarded as a special case of a functional answer. It specifies a constant function, in this case a function which for every argument gives aunt Mary as value.

Individual answers to (2) are also possible if the pronoun his is a free (deictic) pronoun:

(3") Which of his (= John's) relatives does every man love?
   (a) (John's) aunt Mary

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Unlike \((2')\)(a), which looks like an individual answer, but is a functional one, \((3'')\)(a) is an individual answer. 

A last remark concerns what apparently are mixed answers:

\[(1)\] Which woman does every man love?  
\[\text{(d) Mary and his mother}\]

This answer \((d)\) seems to be a combination of an individual and a functional answer, but is, we think, better regarded as a functional answer. The answer gives (the composition of) two functions, the constant function to Mary and the mother-function.

3. 'Loosely speaking', for, as we argued in G&S 1982, section 6.3, the link between the semantic interpretation of questions and the question-answer relationship is not as direct as the formulation in the text suggests. More in particular, pragmatic factors seem to play a predominant role when it comes to characterizing what constitutes a correct answer to a question in a given situation. But for our present purposes, these aspects may be ignored.

4. Throughout we will not bother about certain details, such as mentioning rule numbers, distinguishing between verbs and their extensional counterparts by means of substars, etc. The formulas in the translation trees will be the reduced forms at each step.

5. From now on, we will leave out irrelevant syntactic and semantic information in the analysis trees and translation trees.

6. We will not give the actual rule, it can be found in G&S 1982, section 6.1, where a more extensive motivation for the existence of this rule can be found.

7. See also footnote 2.

8. See e.g. Bennett (1977), who says that a pair-list answer: "might be given in a very compressed way" in the form of a functional answer, and adds that: "Obviously, for epistemic reasons, someone is more likely to give an answer like the second one than like the first."

9. We disregard for the moment the individual reading which the indirect question, and consequently the sentence as a whole, also has.

10. This is not to deny that sometimes a list of pairs may, for the sake of convenience or for some other reason, be abbreviated by a function. The point is that this is not always the case, that functional answers do have a status of their own and that hence questions have a functional reading.
11. Notice that the following list of pairs:

(b') John doesn't love Mary, Bill doesn't love Suzy, ...

does not constitute an answer to a question like (11).

12. The distinction between universal and non-universal terms originates from a discussion of the specific/non-specific contrast in the use of terms, where it proved to be useful too (see G & S 1981). Using some terminology from recent studies on generalized quantifiers (see e.g. Barwise & Cooper 1981, Zwarts 1981) we can define a universal term as one for which it holds that the set on which it lives is a subset of every set in the set of sets denoted by it. Formally:

A term D(A) is universal iff \( \forall X: X \in [[D(A)]] \Rightarrow A \subseteq X \)

The distinction between universal and non-universal terms also seems to play a role when it comes to determining when quantifying in is allowed, though there things are not as straightforward as one might wish. However, the following seems to hold at least: a non-universal term may not be quantified over another non-universal term.

13. This restriction on quantification into questions was not stated in G & S 1982.

14. We cannot discuss the relevant arguments here, since that would take us too far afield, they are given in G & S 1981c. Recently, Engdahl has come up with another proposal for the analysis of functional readings which in some respects is quite like the analysis proposed in the present paper.

15. Notice that (19) is an abstract, not a complement. From now on, we can restrict our attention to the level of abstracts since nothing changes in the way abstracts are turned into complements, i.e. proposition denoting expressions. So, the proposition denoted by a question can be 'read of' the translation of the abstract underlying it. E.g. the abstract (19) is turned into the following complement:

\[
\lambda i[\lambda f[\forall x[\text{man}(a)(x) \rightarrow \text{love}(a)(x,f(x))]] = \\
\lambda f[\forall x[\text{man}(i)(x) \rightarrow \text{love}(i)(x,f(x))]]]
\]

16. Skolem-functions first made their appearance on the linguistic and philosophical stage in a play called 'What is a branching quantifier and why?', which ran for a short but stormy period in the seventies. For some reviews, see Hintikka (1974), Günthner & Hoepelman (1975) and Barwise (1979).

17. We extend the PTQ-mechanism of quantification rules and syntactic variables to account for scope ambiguities and binding phenomena. It is fairly easy to transpose our
entire analysis into a framework which uses Cooper-stores as an alternative (see e.g. Cooper (1975), Engdahl (1980)). However, the use of storage mechanisms is not without problems. E.g. it is not quite clear that the use that is made of Cooper-stores in the literature always obeys the compositionality requirement. See Landman & Moerdijk (1983) for a thorough analysis of Partee & Bach's (1981) extension of the storage approach.

18. Instead of analyzing (2) we take (2'):

(2') Which relative of him does every man love?

which is simpler in that we do not have to take into account the analysis of possessive constructions. Of course, for the problems under discussion in this paper it makes no essential difference.

19. On the pair-list reading of this abstract, syntactic and semantic binding are parallel in the usual way. There every man has which relative of him syntactically as well as semantically inside its scope. For this we need the notion of wh-reconstruction defined in G & S 1982, section 4.3.

20. From this, by the way, one may conclude that the controversy between those who require their grammar to give an explicit compositional semantics and those who restrict semantics in the grammar to those aspects determined by pure, autonomous syntax, is not an empirical dispute, but a methodological one.

21. Notice that in this case having recourse to the mechanism of functional readings is essential. Of course, the functional reading of (38), which (39) represents can also be expressed without quantification over Skolem-functions:

(39') ∀x[man(a)(x) → ∃y[woman(a)(y) ∧ love(a)(x,y)]]

But it is impossible to obtain (39') in a compositional way, using the straightforward translation of no man as λP∀x[man(a)(x) → ¬P(a)(x)].

22. For an extensive discussion, see Rodman (1976). The constraint in question is incorporated in the syntax of relative clauses given in G & S 1982, section 4.5.
References


Bennett, M., 'Questions in Montague Grammar', IULC, 1979

Cooper, R., Montague's semantic theory and transformational syntax, dissertation, Amherst, 1975

Engdahl, E., The syntax and semantics of questions in Swedish, dissertation, Amherst, 1980


Idem, 'Interrogative quantifiers and Skolem-functions. Preparatory notes', manuscript, Amsterdam, 1981c


Idem, 'Compositionality and the analysis of anaphora', Linguistics and Philosophy 6, 1983

Idem, 'Compositional semantics and morphological features', Theoretical Linguistics, 10, 1983
Partee, B., 'Some transformational extensions of Montague Grammar', *Journal of Philosophical Logic* 2, 1973


Rodman, R., 'Scope phenomena, "movement transformations" and relative clauses', in: B. Partee (ed.), *Montague Grammar*, New York, 1976

Zwarts, F., 'Negatief polaire uitdrukkingen I', *GLOT* 4, 1981
ON THE SEMANTICS OF QUESTIONS
AND THE PRAGMATICS OF ANSWERS

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0. Introduction

There is a vast, and rapidly growing, literature on questions and question-answering. The subject has had the longstanding and almost continuous attention in many areas of study, including linguistics, logic, philosophy of language, computer science, and certainly others besides. Many proposals for the analysis of questions and answers at different levels and in different fields and frameworks exist. The aim of this paper is no other than to add another proposal to this long list. We will not discuss the work of others, or point at the relative merits of our own. This is an ill-practice which we hope to make good for at some time in the future.

The analysis of questions and answers we will propose, is a fairly simple and straightforward one. Our most basic assumption, which perhaps strikes the uninitiated as rather trivial, is that there is no hope for an adequate theory of question-answering that does not take absolutely seriously the fact that a correct question signalizes a gap in the information of the questioner, and that a correct answer is an attempt to fill in this gap as well as one can by providing new information. So, information should be a crucial notion in any acceptable theory of question-answering. Whether a piece of information, a proposition, provides an answer to a question of a certain questioner, depends on the information it conveys and on the information the questioner already has. This makes the notion of answerhood essentially a pragmatic one. But no pragmatics without semantics. It is not information as such, but only information together with the semantics of a question, that determines whether a proposition counts as a suitable answer.
Although it can be read quite independently of it, this paper is a follow-up of our paper on the semantic analysis of indirect questions (G & S 1982). In the final section of that paper, we expressed the hope that our analysis of indirect questions would shed some light on what a proper analysis of direct questions looks like. We share the opinion that a fully adequate theory of questions should deal with direct and indirect questions in a uniform way. The semantics of direct and indirect questions should be intimately related. The aim of this paper is to argue that our semantics for indirect questions, which enabled us to explain a number of semantic facts about sentences in which questions occur embedded under such verbs as know and wonder, can also be made to work in an analysis of the question-answer relation, thus satisfying a requirement Belnap has formulated for semantical theories of (indirect) questions (see Belnap 1981).

In this paper we explore one possible account of the question-answer relation. This analysis stays within the possible worlds framework, within which we also developed our analysis of indirect questions. This framework has its inherent shortcomings and the analysis developed here is bound to inherit them. But it seems clear to us that our analysis, when suitably rephrased, can be incorporated in a different, more sophisticated, epistemic pragmatic theory.

Although this paper is clearly related to our earlier work on indirect questions, it differs from it in perspective to a considerable extent. Whereas our former paper primarily dealt with the syntax and semantics of certain linguistic constructions, this paper hardly refers to language or linguistics at all. When we talk about questions or (propositions giving) answers here, we do not mean interrogative or indicative sentences, i.e. linguistic objects, but the objects that serve as their interpretation, i.e. semantic, modeltheoretic objects.

Still, in the end, it is language that matters. We would not be satisfied if the semantic objects we discuss could
not be linked in a systematic way to linguistic expressions. However, we are confident that, in principle, this will constitute no major problem. We feel that our confidence is justified by the fact that there is a well-defined syntactic relationship between direct and indirect questions. Since we have already given a compositional syntax and semantics for indirect questions and since the semantics of indirect and direct questions is the same, we feel that a compositional analysis of direct questions will be possible.

We share the basic view of questions and answers expressed here with many others. One of them, whom we should mention, is Hintikka. To our knowledge, he was the first to develop a theory of questions and answers (see Hintikka 1974, 1976, 1978) in which the notion of an answer "does not depend only on the logical and semantical status of the question and its putative answer, [...] but also on the state of knowledge of the questioner at the time he asks the question" (Hintikka 1978, p. 290).

1. Questions as partitions

In G & S (1982) questions were analyzed as proposition denoting expressions. At an index, a question denotes a proposition, which we will call the true semantic answer at that index. So, the sense (meaning) of a question is a propositional concept, a function from indices to propositions, which at every index yields as its value the proposition that is the true semantic answer to that question at that index.

Let us immediately remark two things about this notion of semantic answerhood. Calling these answers 'semantic' indicates first of all that the resulting notion of answerhood is a limited one, indeed a limiting case of the true notion of an answer, which, in our opinion, is essentially a pragmatic notion. Secondly, it signalizes that when we are talking about questions and answers in this paper, we do not
talk about linguistic entities, but refer to semantic objects. (But for reasons of readability, we italicize expressions referring to these objects.)

In this paper we will view questions as partitions of the set of indices, a perspective which is different from, though equivalent with, the propositional concepts view taken in G & S (1982). A partition of a set $A$ is a set of non-empty subsets of $A$ such that the union of those subsets equals $A$ and no two of these subsets overlap. Formally:

\[ (1) \quad A \text{ is a partition of } A \iff \forall X \in A: X \neq \emptyset, \bigcup_{X \in A} X = A, \forall X, Y \in A: X \cap Y = \emptyset \lor X = Y \]

If we view a question as a partition of the set of indices $I$, each element of that partition, a set of indices, represents a proposition, a possible semantic answer to that question. Consider the question \textit{whether $\phi$}. This question has two possible semantic answers: \textit{that $\phi$}, and \textit{that not $\phi$}. The two sets of indices corresponding to these two propositions divide the total set of indices in two non-overlapping parts. So, a single \textit{whether}-question (a yes/no question) makes a bipartition on the set of indices (except for the tautological question, see section 3). Figure 1 below gives a pictorial representation.

Constituent questions can be viewed as partitions as well. The possible semantic answers to the question \textit{who $G$'s}, are propositions that express that the objects $a_1, \ldots, a_n$ are the ones that $G$. Such propositions exhaustively and rigidly specify which objects have the property $G$ at an index. The sets of indices that represent the possible semantic answers form a partition of $I$. They do not overlap (the various propositions each exhaustively specify a certain set of individuals), and their union equals $I$ (the property $G$ is a total function). Partitions made by constituent questions can also be represented pictorially (in finite cases, at least), see figure 2.
whether φ \hspace{2cm} who G's
\begin{align*}
\text{that } φ & \quad \text{nobody G's} \\
\text{that not } φ & \quad \text{a}_1 \text{ is the one that G's} \\
& \quad \text{a}_2 \text{ is the one that G's} \\
& \quad \text{a}_1 \text{ and } a_2 \text{ are the ones that G} \\
& \quad \text{everybody G's}
\end{align*}

\begin{figure}
\begin{align*}
\text{figure 1}
\end{align*}
\end{figure}

\begin{figure}
\begin{align*}
\text{figure 2}
\end{align*}
\end{figure}

So, generally, a constituent question can be regarded as an \text{n-fold partition of I}, where \text{n} is the number of possible denotations of the (complex or simple) predicate involved in the question.

That the propositional concept view of questions and the partition view are equivalent is easy to see. In G & S (1982) questions were represented by expressions of the following form:

\begin{align*}
(2) \ & \lambda j[a/i/ = a/j/]
\end{align*}

Here \text{i} and \text{j} are variables of type \text{s}, ranging over indices, and \text{a}/\text{i}/ and \text{a}/\text{j}/ are two expressions which differ only in that where the one has free occurrences of \text{i} the other has free occurrences of \text{j}. The sense of a question, 
\[
[\lambda i \lambda j[a/i/ = a/j/]]_{M,g}
\]

is a semantic object of type \text{<s, s, t>}, i.e. a relation between indices. This relation holds between two indices if and only if the denotation of \text{a} is the same at both. It is easy to check that this relation is reflexive, symmetric and transitive, i.e. that it is an equivalence relation. To every equivalence relation \text{R} on
on a set $A$ corresponds a partition of $A$, the elements being
the equivalence classes of $A$ under $R$. So, the semantic
object expressed by a question $Q$ can be regarded as a
partition of the set of indices $I$:

\[(3) \quad I/Q = \text{def} \{ [i]_Q \mid i \in I \}\]

where $[i]_Q$, the set $\{ j \in I \mid Q(i)(j) \}$, is the answer to $Q$
at $i$. This means that the partition $I/Q$ is the set of
possible semantic answers to $Q$.

2. Questions, answers and information

Above we have characterized the proposition denoted by a
question at a certain index as the true, semantic answer to
that question at that index. As we noted in G & S (1982),
this semantic notion of answerhood can hardly do as a
satisfactory explication of the intuitive notion of
answerhood. E.g. the proposition that is a semantic answer
to the question who $G$'s, gives a rigid specification of the
objects that have the property $G$. If the objects are
individuals, such a specification might be given using the
individual's proper names, assuming the latter to be rigid
designators. There are many problems with the consequent
rigid notion of answerhood. For one thing, in an actual
speech situation, it may very well be the case that, for one
reason or other, no such names are available to the speech
participants. Further, there are situations in which
identification of objects by means of descriptions could
serve just as well, and sometimes even better. However, a
proposition in which an object that has a certain property
is identified by means of a proper name, is not equivalent
to, and in general even logically independent of, a propo-
sition in which this identification is carried out by means
of a description. Yet, in many cases, the latter
provide excellent answers to questions. There is no purely
semantic way to relate these answers 'by description' to the semantic answers 'by naming'. And, of course, this is not to be expected. The relationship between questions and answers cannot be isolated from the purpose of posing questions and of answering them: to fill in a gap in the information of the questioner. And consequently, whether two semantically unrelated propositions can serve equally well as an answer to a question, cannot be decided without taking this information into account. So, the question-answer relation is essentially of a pragmatic nature.

Within the limits of possible world semantics, the information of a speech participant can simple-mindedly be represented as a non-empty subset of the set of indices. Each index in such an information set represents a state of affairs that is compatible with the information in question. Evidently, the amount of information is inversely proportional to the extension of the corresponding set. Information is maximal if the information set is a singleton, and minimal if it equals I.

Considerations like those presented above, lead us to a relativization of questions and answers to information sets. Notice that although from a semantic point of view, i.e. if we take the full set of indices into account, a description will, in general, not be a rigid specification of an object, it may very well be that it is such a rigid specification if we limit ourselves to a subset of I. In fact, if a speech participant has the information to which object a description refers, such a description will function pragmatically as a rigid designation of that object. So, although descriptions and proper names in general will not be semantically equivalent, they may very well happen to be pragmatically equivalent.
3. Some formal properties of questions

The cardinality of a question \( I/Q \) equals the number of possible semantic answers to it. The lowest possible cardinality of \( I/Q \) is 1 (since we do not allow \( I = \emptyset \), in that case it would hold for all \( Q: I/Q = \emptyset \)). In this case \( I/Q = \{I\} \). We call this the tautological question in \( I \). Its only answer is the tautology. E.g. if \( \phi \) is a tautology or contradiction, then the single whether question whether \( \phi \) is the tautological question. The questions whether \( (\phi \lor \neg \phi) \) and whether \( (\phi \land \neg \phi) \) have the equivalent answers yes, \( \phi \lor \neg \phi \), and no, \( \neg (\phi \land \neg \phi) \), respectively. Tautological constituent questions are e.g. who G's or does not G, and, which F is not an F. One could very well say that the tautological question does never arise. A question that has only one possible answer is not a proper question at all.

Some operations on questions (partitions) result in new questions (partitions), as do the 1-place operations that take the union of two elements of partition:

\[
\begin{array}{c|c|c}
I/Q & A_1, A_2 I/Q \\
\hline
A_1 & A_1 \lor A_2 \\
A_2 & A_3 \\
A_3 & A_4 \\
A_4 & \\
\end{array}
\]

\[(figure 3)\]
This operation can be defined as follows:

\[ \bigcup_{X,Y \in I/Q} \{Z \mid Z = X \cup Y \vee \{Z \neq X \& Z \neq Y \& Z \in I/Q\}\} \]

(The 1-place operation that takes the complements of all the elements of a partition does not in general result in a partition again. It does so only when it operates on a bipartition, in which case it maps it onto itself, which reflects the equivalence of the questions whether \( \phi \) and whether not-\( \phi \).)

A two-place operation on partitions that results in a new partition, is the one that takes the non-empty intersections of all the elements of the two partitions on which it operates:

\[
\begin{array}{ccc}
I/Q & I/R & I/Q \cap I/R \\
A_1 & B_1 & A_1 \cap B_1 \\
A_2 & B_2 & A_2 \cap B_2 \\
\end{array}
\]

(figure 4)

This intersection operation can be defined as follows:

\[ I/Q \cap I/R = \{X \cap Y \mid X \in I/Q \& Y \in I/R \& X \cap Y \neq \emptyset\} \]

In the pictorial representation of the intersection of two partitions, the dividing lines of each of the two partitions return.

An alternative whether question whether \( \phi \) or \( \psi \) can be constructed as the intersection of the two bipartitions whether \( \phi \), and whether \( \psi \). In general, an alternative whether-
question with n terms can be constructed stepwise from n bipartitions, i.e. from n single whether-questions. In fact, any non-tautological question can be constructed by intersection from a number of bipartitions. E.g. the constituent question who G's can be constructed in this way from the questions whether \( a_1 \) G's, whether \( a_2 \) G's, etc.

The union operation on two partitions is defined as follows:

\[
I/Q \cup I/R = \{ Z \mid Z \neq \emptyset \land \exists X \subseteq I/Q, \exists Y \subseteq I/R : \\
Z = \bigcup_{x \in X} \bigcap_{y \in Y} \exists Z' \mid Z' \neq \emptyset \land \exists X \subseteq I/Q, \\
\exists Y \subseteq I/R : X \cap Y = Z \}
\]

In a pictorial representation of the union of two partitions, only those dividing lines are retained that the two have in common, as is illustrated in figure 5.

\[
\begin{align*}
\text{I/Q} & \quad \text{I/R} \\
A_1 & \quad B_1 \\
A_2 & \quad B_2 \\
A_3 & \quad B_3 \\
\text{I} & \quad \text{I} \\
\text{I/Q} \cup \text{I/R} & = A_1 \cup B_1 \cup A_2 \\
& = A_3 \cup B_2 \cup B_3
\end{align*}
\]

(figures 5)

The union operation will play no role in the remainder of this paper. It has no straightforward linguistic analogue.

More important in the present context is the following inclusion relation between partitions.

\[
I/Q \subseteq I/R \iff \forall X \in I/Q \exists Y \in I/R: X \subseteq Y
\]

The inclusion relation holds between two questions I/Q and I/R if every semantic answer to Q implies a (unique) semantic
answer to R. It is a kind of implication relation between questions. I/Q \subseteq I/R means that I/Q is a refinement of I/R, i.e. that every dividing line in I/R is a dividing line in I/Q as well. See the example in figure 6.

The following facts can be seen to hold:

\begin{enumerate}
\item[(8)] For all I/Q: I/Q \subseteq \{1\}
\item[(9)] For all I/Q: \{\{i\} \mid i \in I\} \subseteq I/Q
\item[(10)] I/Q \cap I/R \subseteq I/Q
\item[(11)] I/Q \subseteq I/R iff I/Q \cap I/R = I/Q
\item[(12)] I/Q \subseteq I/Q \cup I/R
\item[(13)] I/Q \subseteq I/R iff I/Q \cup I/R = I/R
\end{enumerate}

It can easily be checked that \subseteq is a partial order on the set of all partitions of I. \subseteq is a reflexive, antisymmetric and transitive relation. The operations \cap and \cup satisfy idempotency, commutativity, associativity and absorption.

The set of all questions in I, i.e. the set of all partitions of I, forms a complete lattice under \subseteq. The tautological question \{I\} is its maximal element (8). It is the least demanding question. Its counterpart \{\{i\} \mid i \in I\} is the most demanding one. It asks everything that can be asked. It might be phrased as 'What is the world like?'. It is the minimal element of the lattice (9). The bipartitions
(single whether-questions) are the dual atoms. $\sqcap$ and $\sqcup$ are the meet and join.

We have seen in section 2 that in order to obtain a pragmatic notion of answerhood, we are interested in relativizing questions and answers to information sets, i.e. to non-empty subsets of $I$. Doing so, we get pictures such as the following:

\[
\begin{array}{c}
I/Q \\
A_1 \\
J \\
A_2 \\
A_3 \\
\end{array}
\]

(figures 7)

In the situation depicted in figure 7, $A_1$ and $A_2 \in I/Q$ are the semantic answers to $Q$ that are compatible with $J$. $A_3$ is not compatible with $J$, since $A_3 \cap J = \emptyset$. The set of semantic answers compatible with $J$, $I/Q^J$, can be defined as follows:

\[
(14) \quad I/Q^J = \{X \mid X \in I/Q \land X \cap J \neq \emptyset\}
\]

Of course it will always hold that $I/Q^J \subseteq I/Q$.

A second notion that suggests itself is the partition that a question $Q$ restricted to $J$ makes on $J$. We will write this as $J/Q$, and will simply speak of the partition that $Q$ makes on $J$. This notion can be defined as follows:

\[
(15) \quad J/Q = \{X \cap J \mid X \in I/Q \land X \cap J \neq \emptyset\}
\]

The notions $I/Q^J$ and $J/Q$ are related as follows:
The inclusion relation between partitions can now be generalized as follows:

\[(17) \ J/Q \subseteq K/R \iff \forall X \in J/Q \ \exists Y \in K/R: X \subseteq Y\]

The following fact can be observed:

\[(18) \ J/Q \subseteq K/R \iff J \subseteq K \& J/Q \subseteq J/R\]

Notice that (18) implies (19):

\[(19) \ J/Q \subseteq I/Q\]

This expresses that the partition that Q makes on I is preserved when Q is restricted to J, in the sense that it may be compatible with less semantic answers, but that every answer in (element of) J/Q will be a subset of a semantic answer.

The limiting case is where J/Q contains just one element (provided that J is non-empty), i.e. where J/Q = \{J\}. In this case, Q could be called the tautological question in J. But we will preserve the notion of the tautological question as a purely semantic one, and will not use it when talking about information sets. Instead we define:

\[(20) \ J \text{ offers an answer for Q iff } J/Q = \{J\}\]

If an information set offers an answer to a question, the question can be said to be decided by that information, the information provides a (unique) answer.

Fact (18) guarantees that when one's information increases then one remains at least as close to an answer to a question.
4. To have a (true) answer and to know an answer

An information set represents information of an individual \( x \) at an index \( i \). We will add an individual parameter and an index parameter to information sets. We can distinguish two kinds of information sets, doxastic sets and epistemic sets. We will call both kinds of sets information sets. A doxastic set \( D_{x,i} \) is a non-empty set of indices, representing the consistent beliefs of \( x \) in \( i \). An epistemic set \( E_{x,i} \) represents the knowledge of \( x \) in \( i \). Since what one knows should be true, \( i \) should be an element of \( E_{x,i} \). The epistemic and the doxastic set of \( x \) in \( i \) are related, since what one knows, one also believes. So, we can formulate the following general constraints:

\[
E_{x,i} \subseteq I, \quad i \in E_{x,i} \\
D_{x,i} \subseteq E_{x,i}, \quad D_{x,i} \neq \emptyset
\]

Since we have \( D_{x,i} \subseteq E_{x,i} \subseteq I \), we also have for any question \( Q \):

\[
D_{x,i} \cap Q \subseteq E_{x,i} \cap Q \subseteq I \cap Q
\]

The notion of an information set offering an answer, defined in (20), applies to doxastic and epistemic sets. And (22) assures us that if \( E_{x,i} \) offers an answer to \( Q \), then \( D_{x,i} \) offers an answer to \( Q \) as well.

We are also interested in the notion of an information set offering a true answer to a question. If an information set \( J_{x,i} \) offers a true answer, this need not be a true answer. In the situation in figure 8(b), \( J_{x,i} \) offers an answer, but not a true one, whereas in 8(c) and 8(d), \( J_{x,i} \) offers a true answer. (In 8(a) \( J_{x,i} \) does not offer an answer at all, regardless of where \( i \) is situated.) But notice that since \( i \) has to be an element of \( E_{x,i} \), the situations depicted in 8(b) and 8(c) cannot occur if \( J_{x,i} \) is to be an epistemic set, but only if it is a doxastic set. A doxastic set need
not contain only true information about $i$. But still, as 8(c) illustrates, it may offer a true answer.

![Diagram](image)

(figure 8)

We can define the notions of an information set offering an answer or a true answer to a question as follows:

(23) $J_{X,i}$ offers an answer to a question $Q$ iff $J_{X,i}/Q = \{J_{X,i}\}$

$J_{X,i}$ offers a true answer to $Q$ iff $J_{X,i} \cup \{i\}$ offers an answer to $Q$

Since $E_{X,i} \cup \{i\} = E_{X,i}$, $E_{X,i}$ offers a true answer to $Q$ iff $E_{X,i}$ offers an answer to $Q$. This does not hold for $D_{X,i}$. What does hold is that if $D_{X,i}$ offers a true answer to $Q$, then it offers an answer, but not necessarily the other way around.

So, (23) gives rise to the following three possibilities:

(24) $x$ has an answer to $Q$ in $i$ iff $D_{x,i}$ offers an answer to $Q$.

$x$ has a true answer to $Q$ in $i$ iff $D_{x,i}$ offers a true answer to $Q$.

$x$ knows an answer to $Q$ in $i$ iff $E_{x,i}$ offers an answer to $Q$. 


To know an answer implies to have a true answer, but not the other way around, since $D_{x,i} \cup \{i\}$ may be a proper subset of $E_{x,i}$. And to have a true answer implies to have an answer.

5. Pragmatic answers

We are now almost in the position to define the wider, pragmatic notion of answerhood that we are after, i.e. the notion of a proposition giving an answer with respect to an information set. A proposition gives an answer to a question in an information set, if the information set to which that proposition is added offers an answer. So, in order to calculate whether a proposition $P$ gives an answer to a question $Q$ in an information set $J_{x,i}'$, we first update $J_{x,i}$ with $P$, which results in a new information set $J_{x,i}'$, and then check whether $J_{x,i}'$ offers an answer to $Q$.

There are several important facts to note about the update operation. The first is that it should turn an information set of a certain kind into an information set of the same kind. It should turn a doxastic set into a doxastic set and an epistemic set into an epistemic set. Since $E_{x,i}$ and $D_{x,i}$ are related, they should be updated simultaneously. Secondly, when information sets are updated, they, in general, change. $J_{x,i}'$ need not equal $J_{x,i}$. If a model is determined by the totality of doxastic and epistemic sets of each individual at each index, updating takes us from one model into another. We will not bother to state this in detailed definitions, but it is important to bear these things in mind.

Intuitively, there are two ways to update an information set $J_{x,i}$ with a proposition $P$, that seem to make sense. The first is to check whether $P$ is consistent with $J_{x,i}$, and if so, to add it to it. The second is to check whether $P$ is true (and consistent with $J_{x,i}$) and if so, to add it to it. In fact, if we apply the first method of updating to a
doxastic set $D_{x,i}$, and, at the same time, the second to the corresponding set $E_{x,i}$, with an extra proviso that keeps $D_{x,i}$ and $E_{x,i}$ related in the proper way, the resulting sets $D'_{x,i}$ and $E'_{x,i}$ will be proper information sets again.

We can define the update operation on information sets as follows:

\[(25) \text{update } <P, D_{x,i}, E_{x,i} > = <D'_{x,i}, E'_{x,i} >\]

where $D'_{x,i} = D_{x,i} \cap P$, if $D_{x,i} \cap P \neq \emptyset$

$= D_{x,i}$ otherwise

$E'_{x,i} = E_{x,i} \cap P$, if $i \in P$ and $D_{x,i} \cap P \neq \emptyset$

$= E_{x,i}$ otherwise

The reader can verify that $D'_{x,i}$ and $E'_{x,i}$ satisfy the constraints laid down in (21). We will say that update $<P, D_{x,i}, E_{x,i} > = D'_{x,i}$, and update $<P, E_{x,i}, E_{x,i} > = E'_{x,i}$ iff update $<P, <D_{x,i}, E_{x,i} > = <D'_{x,i}, E'_{x,i} >$.

It may be illuminating to notice that if we start with no information at all, i.e. with $E_{x,i} = D_{x,i} = I$, and continuously update these sets with propositions in accordance with (25), the pair of information sets that results, is, at each step, a pair consisting of a doxastic and an epistemic set, i.e. a pair of sets satisfying (21).

In order to be able to give a definition of a notion of pragmatic answerhood, we need one more auxiliary notion that introduces nothing but a new piece of terminology.

\[(26) \text{Q is a question in } J_{x,i} \text{ iff } J_{x,i} \text{ does not offer an answer to } Q\]

\[Q \text{ is a question in } J_{x,i} \text{ iff there is more than one answer to } Q \text{ that is compatible with } J.\]

We can now give the definition of a proposition giving a (true) answer to a question in an information set as follows (assuming $J_{x,i}$ to be an information set of a certain kind, and update to be the corresponding update operation):
(27) Let \( Q \) be a question in \( J_{x,i} \), then (a proposition) \( P \) gives a (true) answer to \( Q \) in \( J_{x,i} \) iff update \( \langle P, J_{x,i} \rangle \) offers a (true) answer to \( Q \).

What this definition expresses is simply that a proposition answers a question in an information set iff when the information set is updated with the proposition, the question is no longer a question, but is (dis)solved.

Definition (23) of an information set offering a (true) answer, together with definition (25) of the update operation, guarantee that the following facts hold:

(28) \( P \) gives a true answer to \( Q \) in \( E_{x,i} \) iff \( P \) gives an answer to \( Q \) in \( E_{x,i} \).
If \( P \) gives an answer to \( Q \) in \( E_{x,i} \), then \( P \) gives a true answer to \( Q \) in \( D_{x,i} \).
If \( P \) gives a true answer to \( Q \) in \( D_{x,i} \), then \( P \) gives an answer to \( Q \) in \( D_{x,i} \).

In view of (28), we can say, analogously to (24):

(29) \( P \) gives \( x \) an answer to \( Q \) in \( i \) iff \( P \) gives an answer to \( Q \) in \( D_{x,i} \).
\( P \) gives \( x \) a true answer to \( Q \) in \( i \) iff \( P \) gives a true answer to \( Q \) in \( D_{x,i} \).
\( P \) does let \( x \) know an answer to \( Q \) in \( i \) iff \( P \) gives an answer to \( Q \) in \( E_{x,i} \).

The following examples may serve to illustrate the notions of pragmatic answerhood. Consider the situation in figure 9(a):
The vertical division of I is the partition I/whether \( \phi \), the horizontal one is I/whether \( \psi \). Since \( i \in \text{that} \ \phi \) and \( i \in \text{that} \ \psi \), that \( \phi \) and that \( \psi \) are true in \( i \). \( D_{x,i} \) and \( E_{x,i} \) contain the information that if \( \psi \), then \( \phi \). Neither the question whether \( \phi \) nor the question whether \( \psi \) is answered in \( D_{x,i} \) or in \( E_{x,i} \). In this situation, the true proposition that \( \psi \) gives a true answer to the question whether \( \phi \) in \( D_{x,i} \), the answer that \( \phi \). And it also gives that answer to that question in \( E_{x,i} \). Figure 9(b) represents the situation that results after updating \( D_{x,i} \) and \( E_{x,i} \) with that \( \psi \). Update \( \langle \text{that} \ \psi, D_{x,i} \rangle = D'_{x,i} = D_{x,i} \cap \text{that} \ \psi \). And update \( \langle \text{that} \ \psi, E_{x,i} \rangle = E'_{x,i} = E_{x,i} \cap \text{that} \ \psi \). Notice that the pragmatic answer that \( \psi \) is logically independent of the semantic answer that \( \phi \).

As a second example, consider the following situation:
That \( \phi \) is now false in \( i \), but that \( \psi \) is still true. \( D_{x,i} \) still contains the (now false) information that if \( \psi \), then \( \phi \). Since it is false, \( E_{x,i} \) cannot contain this piece of information anymore. In this situation, the true proposition that \( \psi \) still gives \( x \) an answer to the question whether \( \phi \) in \( i \), but no longer a true answer. Then, of course, it cannot let \( x \) know an answer either. A true proposition, even if it gives an answer, need not give a true answer.

Next, consider the following situation:
Both that $\phi$ and that $\psi$ are now false in $i$. As in the first example, both $D_{x,i}$ and $E_{x,i}$ contain the information that if $\psi$, then $\phi$. Since $D_{x,i}$ is compatible with that $\psi$, update $<\text{that } \psi, D_{x,i}> = D'_{x,i} = D_{x,i} \cap \text{that } \psi$. But since $\neg \phi \neq \text{that } \psi$, update $<\text{that } \psi, E_{x,i}> = E'_{x,i} = E_{x,i}$. The false proposition that $\psi$ gives $x$ the false answer that $\phi$ to the question whether $\phi$, and does not let $x$ know an answer.

As a last one in this series of examples, consider the following situation:

![Diagram](figure 12)

That $\phi$ is now true in $i$, but that $\psi$ is still false. The updates of $D_{x,i}$ and $E_{x,i}$ are similar to those in the previous situation. But this time the proposition that $\psi$ does not only give $x$ an answer, it even gives $x$ the true answer that $\phi$. But it cannot let $x$ know an answer, since that $\psi$ is false in $i$. So, a false proposition can give one a true answer, but it can never let one know an answer.

Whereas in the previous series of examples we concerned ourselves with single whether-questions, in the next example we consider a constituent question.
In this situation, the domain of individuals \( D = \{a_1, a_2\} \). \( F \) is a property that is true of exactly one individual. The vertical division of \( I \) is the partition \( I/\)who is the \( F \), the horizontal one is \( I/\)who \( G \)'s. \( D_{x,i} \) contains the (false) information that \( a_1 \) is the \( F \), and the (true) information, also contained in \( E_{x,i} \), that exactly one individual \( G \)'s. The question who \( G \)'s is not answered in \( D_{x,i} \) and \( E_{x,i} \). Both the proposition that \( a_1 \) is the one who \( G \)'s (the shaded area in figure 13 (b)) and the proposition that the \( F \) is the one who \( G \)'s (the dotted area) give an answer to the question who \( G \)'s in \( D_{x,i} \). Notice that the former is a semantic answer, whereas the latter is a pragmatic answer, and that the two are logically independent in \( I \), but pragmatically equivalent in \( D_{x,i} \). Both propositions in fact give a true answer in \( D_{x,i} \). But only the proposition that \( a_1 \) is the one who \( G \)'s does let \( x \) know an answer in \( i \). Notice that even a much
weaker proposition like that if anyone $G$'s then the $F$ does, would already give $x$ a true answer in $i$. And propositions like that nobody $G$'s or that everybody $G$'s, would not give an answer, since they are incompatible with $x$'s information.

6. Partial answers

Although the notion of a pragmatic answer is an essential step towards a satisfactory notion of answerhood, it still calls for further refinements. Pragmatic answers as defined in (27) are always complete answers. If a proposition gives an answer in an information set $J_{x,i}$, the question is always completely solved in that information set. However, in many cases the questioner will already be very happy if her question can be partially solved, i.e. if the set of answers compatible with her information is narrowed down. What we need is a notion of partial pragmatic answerhood.

If a proposition $P$ narrows down an information set $J_{x,i}$ to a proper subset $J'_{x,i}$ such that the answers to $Q$ compatible with $J'_{x,i}$ form a proper subset of the answers compatible with $J_{x,i}$, we will say that $P$ gives a partial answer to $Q$ in $J_{x,i}$. This is exemplified in figure 14(a):

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure14}
\caption{Figure 14}
\end{figure}
As figure 14(b) illustrates, a proposition may be informative with respect to $J_{x,i}$, without giving a partial answer to a question $Q$ in $J_{x,i}$.

We will say that $J'_{x,i}$ in figure 14(a) is closer to an answer to $Q$ than $J_{x,i}$ (whereas in 14(b) $J''_{x,i}$ and $J_{x,i}$ are equally close to an answer to $Q$). The notion of being closer to an answer can be defined as follows:

\[(30) \text{Let } J_{x,i} \text{ be a subset of } K_{x,i}, \text{ then } J_{x,i} \text{ is closer to an answer to } Q \text{ than } K_{x,i} \]
\[\text{iff } I/Q_{x,i} \subseteq I/Q_{x,i}\]

If a proposition is to give a true partial answer in an information set $J_{x,i}$ to a question $Q$, the set of answers to $Q$ compatible with $J_{x,i}$ updated with that proposition should be narrowed down in such a way that the true answer to $Q$ remains accessible. The notion of an information set giving access to a true answer can be defined as follows:

\[(31) J_{x,i} \text{ gives access to a true answer to } Q \text{ iff } [i]_Q \in I/Q_{x,i}\]

A doxastic set need not give access to a true answer, but an epistemic set always will. The notion of an information set being closer to a true answer can now be defined as follows:

\[(32) J_{x,i} \text{ is closer to a true answer to } Q \text{ than } K_{x,i} \text{ iff } J_{x,i} \text{ is closer to an answer to } Q \text{ than } K_{x,i} \]
\[\text{and } J_{x,i} \text{ gives access to a true answer to } Q\]

For epistemic sets, the notions of being closer to an answer and being closer to a true answer coincide, but they do not for doxastic sets. Whereas a doxastic set will always be as least as close to an answer as an epistemic set, it need not be as least as close to a true answer.

We can now define the notion of a proposition giving a (true) partial answer in an information set as follows:
(33) Let Q be a question in $J_{x,i}$, then P gives a (true) partial answer to Q in $J_{x,i}$ iff update $\langle P, J_{x,i} \rangle$ is closer to a (true) answer to Q than $J_{x,i}$.

Of course, (true) pragmatic answers as defined in (27), which we might call complete pragmatic answers, form a subset of the set of (true) partial answers. The facts stated in (28) for complete pragmatic answers, hold for partial answers as well. And the three different notions of pragmatic answerhood that were distinguished in (29) apply also to partial answers.

An important fact to be noticed is that if $J_{x,i}$ is a bipartition (i.e. if Q is, or comes down to, a single whether question in $J_{x,i}$), and P gives a partial answer to Q in $J_{x,i}$, then P gives a complete answer to Q in $J_{x,i}$. This fact is not very satisfactory. We will come back to it in the next section.

We will end this section by giving some examples of propositions giving partial answers in a doxastic set (the difference between a proposition giving a true answer and letting one know an answer, discussed in the previous section, applies to partial answers in much the same way, but will be left out of consideration here). Consider the situation depicted in figure 15.

\[ G = \emptyset \]
\[ G = \{a_1\} \]
\[ G = \{a_2\} \]
\[ G = \{a_3\} \]
\[ G = \{a_1, a_2\} \]
\[ G = \{a_1, a_3\} \]
\[ G = \{a_2, a_3\} \]
\[ G = D \]

The domain $D = \{a_1, a_2, a_3\}$ throughout $I$:
\[ M = \{a_1, a_2\} \]
\[ F = \{a_3\} \]

(figure 15)
The proposition \textit{that if }a_1 \textit{G's then }a_2 \textit{G's}, gives a true partial answer in \(D_{x,i}\). Updating \(D_{x,i}\) with that proposition results in an information set \(D'_{x,i}\) in which the areas 2 and 6 in \(D_{x,i}\) have been cut out. So, the set of semantic answers compatible with \(D_{x,i}\) is smaller than the set of semantic answers compatible with \(D'_{x,i}\), and the true semantic answer that \(a_3\) \textit{is the one who G's is} is still accessible in \(D'_{x,i}\).

As a second example, consider the proposition \textit{that the one who G's is} \(M\). This proposition gives a partial answer in \(D_{x,i}\) as well, but this time not a true one. Updating \(D_{x,i}\) with the proposition \textit{that the one who G's is} \(M\) brings \(D_{x,i}\) down to the areas 2 and 3. The true answer that \(a_3\) \textit{is the one who G's is} no longer accessible from this information set. Notice that the proposition \textit{that the one who G's is} \(M\) would give a complete answer (but again not a true one) in \(D'_{x,i}\), which resulted after updating \(D_{x,i}\) with the proposition \textit{that if }a_1 \textit{G's then }a_2 \textit{G's}.

The answer that \textit{the one who G's is} \(M\) might be called an exhaustive indefinite answer. It exhaustively lists the individuals that (are supposed to) walk, in this case only one, and characterizes them by means of an indefinite description. A non-exhaustive indefinite answer would then be the proposition \textit{that (at least) an }M\textit{ G's. It gives one individual that G's and specifies it in an indefinite way, but leaves open that there are other individuals that G as well. This proposition gives a partial (false) answer in }D_{x,i}\textit{ as well. It cuts the areas 1 and 3 out of }D'_{x,i}\textit{.}

Often, indefinite answers are partial ones, but they can very well be complete, the exhaustive indefinite answer that \textit{the one who G's is} \(F\) gives a complete true answer in \(D_{x,i}\). And notice that an exhaustive definite answer like \textit{that the one who G's is the }F\textit{, need not give a complete answer. It does so in the situation in figure 15, but it would not in an information set in which the question }who\textit{ is the }F\textit{ is not decided.}
7. Indirect answers

We return now to the unsatisfactory fact noticed above, that questions which are bipartitions in an information set can be answered only completely. This implies e.g. that simple whether-questions cannot be answered partially. But it seems that in a sense, they can. Suppose that whether $\phi$ is a question in $J_{x,i}$. The proposition that if $\psi$, then $\phi$, can be a good answer, even in case $\psi$ is not contained in $J_{x,i}$. But it does not give a partial answer according to definition (33). Consider figure 16:

![Diagram](figure 16)

What is going on here is the following. The situation in 16(b) is the one discussed above with respect to figure 9. There we saw that in this situation, that $\psi$ will give an answer to the question whether $\phi$ in $J_{x,i}$. And notice that in the situation depicted in 16(a), that $\psi$ does not yet give an
answer to whether $ \phi$ in $J_{x,i}$. So it seems that, in a sense, $x$ is getting closer to an answer. What the proposition that if $\psi$, then $\phi$ does to $J_{x,i}$ is that it provides a new way of getting an answer to the question whether $\phi$. For $x$ can turn to someone and ask whether $\psi$, and if he is lucky, he gets the answer that $\psi$, which solves his original question whether $\phi$ at the same time. His question whether $\phi$ is related to the question whether $\psi$. This may be very important, e.g. the question whether $\psi$ may be easier to get answered. And not only informants who happen to have the information whether $\phi$, but also informants who do not happen to have that information, but do happen to have the information that $\psi$ can help him out. Notice that whether $\phi$ and whether $\psi$ are not equivalent in the new information set: that not-$\phi$ does not give $x$ an answer to whether $\psi$. In the new information set the proposition that if $\phi$, then $\psi$ also provides usefull information, without qualifying as a (partial) answer. If $x$ updates with this proposition then his original question whether $\phi$ gets even more intimately related to whether $\psi$: it now becomes equivalent to it, for now also that not-$\psi$ tells $x$ something about whether $\phi$, viz. that not-$\phi$.

Similar situations can occur with constituent questions. Suppose that who is the one who $G$'s is a question in $J_{x,i}$. Suppose further, that $x$ has no idea which individual has the property $G$, it may be any individual in the domain. If $x$ also has no idea as to which individual is the $F$, the proposition that the $F$ is the one who $G$'s, will not give a partial answer to her question in $J_{x,i}$. Still, she may be quite satisfied with this answer, because now there is the possibility to turn to another informant and ask the question who is the $F$. A (partial) answer to that question will be a (partial) answer to her original question as well. And her informant may have an answer to the new question without having one to the old one.

In view of these examples, one would like to widen the notion of answerhood, so as to include this indirect kind of
answers. But doing so is a delicate matter. Informally, these indirect answers can be characterized as follows:

(34) Let $Q$ be a question in $J_{x,i}'$, then $P$ gives an indirect answer to $Q$ in $J_{x,i}'$ iff there is some question $R$ in update $<P,J_{x,i}>$ such that $Q$ depends more on $R$ in update $<P,J_{x,i}'>$ than in $J_{x,i}'$ and $R$ is not conversationally equivalent to $Q$ in update $<P,J_{x,i}>$

Dependence is a relation between questions. Intuitively, a question $Q$ depends on a question $R$ if an answer to $R$ tells us something about an answer to $Q$. Relativizing dependence to information sets, we give the following definition:

(35) $Q$ depends on $R$ in $J_{x,i}$ iff

$$\exists X \in I/R \exists Y \in I/Q \exists x, x \in J_{x,i}' : X \cap Y \neq \emptyset$$

According to (35) $Q$ depends on $R$ iff some answer to $R$ compatible with $J_{x,i}'$ gives a partial answer to $Q$ in $J_{x,i}'$. The comparative notion is then defined as follows:

(36) Let $Q$ be a question in $J_{x,i} \subset K_{x,i}'$, then $Q$ depends on $R$ in $J_{x,i}$ more than in $K_{x,i}'$ iff

$$\{X | X \in I/R \times \emptyset \} \cup \exists Y \in I/Q \times \emptyset : X \cap K_{x,i} \cap Y \neq \emptyset \} < \{X | X \in I/R \times \emptyset \} \cup \exists Y \in I/Q \times \emptyset : X \cap J_{x,i} \cap Y \neq \emptyset \}$$

According to (36) $Q$ depends more on $R$ in update $<P,J_{x,i}>$ than in $J_{x,i}'$ iff there are more answers to $R$ that are partial answers to $Q$ in update $<P,J_{x,i}>$ than there are in $J_{x,i}'$. Thus, in update $<P,J_{x,i}>$ the chances of getting an answer to $Q$ through an answer to $R$ are greater than in $J$. As the reader can easily verify, the situations discussed above are covered by this definition.

The notion of conversational equivalence is harder to get a grip on. Elusive though it may be, it is an essential
element in the definition of an indirect partial answer, since it prevents the notion from being totally void. For, without it any proposition that is informative with respect to \( J_{x,i} \) would give an indirect answer to any question \( Q \) in \( J_{x,i} \). This can be shown as follows. Consider a situation in which there are two fully independent (in any sense of the word) atomic propositions \( \text{that } \phi \) and \( \text{that } \psi \). In such a situation, it is out of the question that the proposition \( \text{that } \psi \) would be of any help at all for the question whether \( \phi \). So, \( \text{that } \psi \) should not come out as an indirect partial answer. However, if we add \( \text{that } \psi \) to \( J_{x,i} \), the question whether \( \phi \) can easily be seen to depend more on the question whether if \( \psi \), then \( \phi \), than in the original \( J_{x,i} \). So, all conditions of (34) are fulfilled, except for the last one.

The following informal reasoning may show how cases like these are cancelled by the requirement of conversational non-equivalence. Remember that the whole point of getting a question on which the original one depends more is that it provides the questioner with the opportunity to find an informant who is not able to answer the original question, but is able to answer the one on which it depends more, with a better chance that such an answer indirectly provides an answer to the original question. This is successful only if the two questions are not conversationally equivalent. Two questions are conversationally equivalent if the questioner has to assume that an informant will be able to answer the one question truthfully iff she is able to answer the other truthfully as well. So, if a proposition gives rise to a new question which is conversationally equivalent to the original one, the entire point of providing an indirect answer vanishes.

This can be captured in the following, more precise definition:

\[
(37) \text{Q is conversationally equivalent to R for } x \text{ in } i \text{ iff } \forall y (x \text{ believes to know } y \text{ to know a (partial) answer to } Q \text{ iff } x \text{ believes to know } y \text{ to know a (partial) answer to } R)
\]
What remains to be shown is that in the kind of counterexamples discussed above, the new question is indeed conversationally equivalent to the original one. I.e. if we have to show that under the assumption that that φ and that ψ are totally unrelated, the question whether if ψ, then ψ, to which adding that ψ to Jx,i gives rise, is conversationally equivalent to the question whether φ. This can be done as follows.

Suppose our questioner x asks an informant y whether if ψ, then φ. Suppose y replies that, indeed, if ψ, then φ. The propositions that ψ and that φ are known to be totally unrelated. Thus, x cannot interpret the conditional as expressing some kind of internal relation between φ and ψ, for such an interpretation would be incompatible with his information. Consequently, the only interpretation available for x is that of a straightforward material implication. This means that x has to assume that either y believes that ψ is false, or that φ is true. If x is to incorporate the material implication in his information, he has to make sure that the latter is the case. For, given that his information contains that ψ that is the only situation in which x can assume that y knows the answer to whether if ψ, then φ. But, obviously, this means that in the given circumstances this question is conversationally equivalent to the original question whether φ.

As will be clear from this informal discussion, a formalization of the notion of conversational equivalence involves information of speech participants about each other's information in an essential way. This requires a richer framework, and a more restricted notion of an information set, than we are using here. But, informally at least, the matter seems clear, so, assuming a formalization can be given, (34) indeed defines the notion of indirect partial answerhood.
8. Answers compared

Not all propositions give equally good answers to a question in an information set. In what follows, we will formulate some conditions which can be used in comparing propositions in this respect. These conditions will be seen to be related to the notion of a correct answer to a question in a Gricean, conversational, sense of the word.

First of all, there is a condition pertaining to relevance. When relevance is defined as in (38), a condition of relation can be stated as in (39):

\[(38) \text{Let } Q \text{ be a question in } J_{x,i}, \text{ then } P \text{ is relevant to } Q \text{ in } J_{x,i} \text{ iff } P \text{ gives a (partial) answer to } Q \text{ in } J_{x,i} \]

\[(39) \text{If } P \text{ is a good answer to } Q \text{ in } J_{x,i}, \text{ then } P \text{ is relevant to } Q \text{ in } J_{x,i} \]

Notice that indirect answers are excluded. Of course, this is not correct, but we prefer to leave them out of consideration until they are properly formalized.

Second, there is a condition of quality, i.e. a condition pertaining to truth:

\[(40) \text{Let } Q \text{ be a question in } J_{x,i}, \text{ then } P \text{ is a good answer to } Q \text{ in } J_{x,i} \text{ iff } P \text{ gives a true (partial) answer to } Q \text{ in } J_{x,i} \]

Two things can be noticed. First, since giving a true (partial) answer implies giving a (partial) answer, relevance is subsumed under quality. Second, the condition of quality allows for a weaker and a stronger reading. The
stronger reading results if $J_{x,i}$ is required to be an epistemic set. (In that case relevance would collapse into quality.)

Besides these absolute conditions of relation and quality, there is a relative condition pertaining to the amount of information a proposition gives with respect to a question. Before giving this condition of quantity, we first define some auxiliary notions. Throughout, we assume that $Q$ is a question in $J_{x,i}$ and that $P_1$, $P_2$ give (partial) answers to $Q$ in $J_{x,i}$.

(41) $P_1$ is more informative to $Q$ in $J_{x,i}$ than $P_2$ iff $P_1 \cap J_{x,i}$ is closer to an answer to $Q$ than $P_2 \cap J_{x,i}$.

(42) $P_1$ is less overinformative to $Q$ in $J_{x,i}$ than $P_2$ iff

(i) $P_2$ is not more informative to $Q$ in $J_{x,i}$ than $P_1$; and

(ii) $P_1$ is weaker in $J_{x,i}$ than $P_2$, i.e.

$$P_2 \cap J \subseteq (P_1 \cap J)$$

In terms of (41) and (42) we can define the notion of a more standard answer as follows:

(43) $P_1$ is a more standard answer to $Q$ than $P_2$ iff either

(i) $P_1$ is more informative to $Q$ in $I$ than $P_2$; or

(ii) $P_1$ is less overinformative to $Q$ in $I$ than $P_2$

From (43) it follows that:

(44) If $P_1 \subseteq P_2$, then either

(i) $P_1$ is more informative to $Q$ in $J_{x,i}$ than $P_2$; or

(ii) $P_2$ is less overinformative to $Q$ in $J_{x,i}$ than $P_1$; or

(iii) $P_1$ and $P_2$ are equivalent in $J_{x,i}$, and $P_1$ is a more standard answer to $Q$ than $P_2$, or $P_2$ is a more standard answer to $Q$ than $P_1$
We are now ready to state the following condition of quantity:

(45) \( P_1 \) is a better answer to \( Q \) in \( J_{x,1} \) than \( P_2 \) iff either

(i) \( P_1 \) is more informative to \( Q \) in \( J_{x,1} \) than \( P_2 \); or

(ii) \( P_1 \) is less overinformative to \( Q \) in \( J_{x,1} \) than \( P_2 \); or

(iii) \( P_1 \) and \( P_2 \) are equivalent in \( J_{x,1} \) and \( P_1 \) is a more standard answer to \( Q \) than \( P_2 \).

Clause (45)(i) correctly predicts that a proposition that gives a complete answer is a better answer than one that gives a properly partial one, if it is any good at all, i.e. if it gives a true answer. Complete answers are the most informative ones. ³

Clause (45)(ii) requires a proposition not to give more information than the question asks. For example, suppose that \( J_{x,1} \) contains no information about \( \phi \), or about \( \psi \). Let the question be whether \( \phi \). Then (45) predicts that the proposition that \( \phi \) is a better answer than the proposition that \( (\phi \) and \( \psi ) \). Both are complete answers, and therefore, that \( \phi \) is not more informative than that \( (\phi \) and \( \psi ) \). But the former is weaker in \( J_{x,1} \) than the latter, and therefore less overinformative. (Notice that that \( \phi \) would be a better answer than the possible indirect answer that \( (\phi \) or \( \psi ) \), since it is more informative in this situation.)

However, if the proposition that \( \phi \) is already contained in \( J_{x,1} \), then that \( \phi \) is no longer weaker, but equivalent with that \( (\phi \) and \( \psi ) \) in \( J_{x,1} \). But clause (45)(ii) decides between the two, even in this situation. Both propositions are complete answers to whether \( \phi \) in \( I \), but that \( \phi \) is weaker in \( I \) than that \( (\phi \) and \( \psi ) \), and hence a more standard answer, and therefore a better answer.

To give another example, suppose \( J_{x,1} \) contains the information that not-\( \psi \). Then, the proposition that \( \phi \) and the
proposition that \((\phi \lor \psi)\) are equivalent in \(J_{x,i}\), but that \(\phi\) is a more standard answer to whether \(\phi\), since it is more informative in \(I\) to whether \(\phi\), and therefore a better answer to this question. Of course, this does not mean that the proposition that \((\phi \lor \psi)\) could never be a good answer in this situation. It would be for example, if the one who answers the question is simply not able to express the proposition that \(\phi\) sincerely. The proposition that \(\phi\) may simply not be available as a good answer.

A natural question that arises, is whether in a given set of available good answers, there always is a best one. It can be proved that in a sense this is the case. But only if we make two assumptions. The first is that if two propositions \(P_1\) and \(P_2\) are available, their conjunction \(P_1 \land P_2\) and their disjunction \(P_1 \lor P_2\) are available as well.

The second assumption is that \(J_{x,i}\) is an epistemic set. Then we can prove the following:

\[
(46) \text{Let } Q \text{ be a question in } J_{x,i}, J_{x,i} \text{ an epistemic set, and } P_1, P_2 \text{ different (partial) answers to } Q \text{ in } J_{x,i}. \text{ Then either (i) } P_1 \text{ is a better answer to } Q \text{ in } J_{x,i} \text{ than } P_2; \text{ or (ii) } P_2 \text{ is a better answer to } Q \text{ in } J_{x,i} \text{ than } P_1; \text{ or (iii) } P_1 \land P_2 \text{ is a good answer to } Q \text{ in } J_{x,i} \text{ and a better answer to } Q \text{ in } J_{x,i} \text{ than both } P_1 \text{ and } P_2; \text{ or (iv) } P_1 \lor P_2 \text{ is a good answer to } Q \text{ in } J_{x,i} \text{ and a better answer to } Q \text{ in } J_{x,i} \text{ than both } P_1 \text{ and } P_2.
\]
9. Correctness of question-answering

We have called the conditions given above conditions of relation, quality and quantity. This should remind one of the corresponding Gricean maxims. Conditions like these may be expected to form the core of an explication of the notion of a correct answer, of an answer in accordance with the Gricean maxims. Such a notion of correctness can be formulated informally as follows:

(47) If x has a question Q, then y gives a correct answer to Q for x in expressing P iff y believes that P gives a good answer to Q for x and that there is no P' available such that P' gives a better answer to Q for x than P.

Clearly, the notions of a good, and of a better answer, figure essentially in this definition. But it reflects the subjective, speaker-oriented, nature of the Gricean maxims. Therefore, it relates the notions of a good and of a better answer, which themselves are pragmatic in that they pertain to the information of the questioner, to the information of the one who is answering the question. Thus, a formalization of (47) essentially involves a representation of information about information. We will not attempt such an analysis of information here, the elaborations this would involve go beyond the scope of the present paper. But it may be noted that the subjective correctness notion is based upon the notion of a proposition giving a good answer to a question in an information set, and upon that of one proposition giving a better answer than another. And these notions are defined...
by the conditions stated above.

A closer look at (47) should reveal further that it refers to expressible and available propositions, i.e. that it refers to language. Throughout this paper we have been talking about questions and answers not as linguistic, but as semantic, modeltheoretic objects. But if we come to consider effective question-answering in speech situations, language becomes all important again. A certain proposition may be a good answer, it may even be the best one there is, but this is of little use if we are not able to express it adequately. In determining what the best answer to a question is, we are always dealing with a certain subset of the totality of all true partial pragmatic answers. Roughly, this set contains those propositions which the one who answers the question is able to express linguistically in such a way that the questioner's interpretation of this linguistic expression is a proposition that gives her a true partial pragmatic answer.

The restriction to adequately expressible propositions is highly relevant. The notion of giving a better answer strongly favours semantic answers. This is due to condition (45)(iii). In fact, if we consider all true partial answers to a question, the true semantic answer will obviously be the best one. (And if it is too strong to be given vis à vis the quality maxim, disjunctions of semantic answers will come into play.) But if semantic answers are to be expressed, we need, among other things, semantically rigid designators. And as we noted quite at the outset in section 2, such rigid designators may not be available in the language. And even when they are, they may not be available to the speech participants in the sense that they may not be, or may not be expected to be, rigid in the information of questioner or questionnee. A semantically rigid designator may fail to pick out a unique denotation with respect to a certain information set, whereas at the same time a semantically non-rigid expression may do so, by being pragmatically rigid with respect to that set. Obviously, in such a situation the
latter kind of expression gives better means to express a pragmatic answer.

The restriction to adequately expressible propositions, which (47) makes, is very realistic in predicting that semantic answers are not always the best ones available. So, the theory of pragmatic answers developed in this paper loses none of whatever usefulness it may have, by the fact that ideally semantic answers tend to be the best ones. In fact, that under completely ideal circumstances, which include having a complete, perfect language, being a perfect language user, a perfect logician, and a walking encyclopedia, semantic answers are the best ones, may be viewed as a merit of the present theory. For it correctly links the existence and function of pragmatic answers to their proper source: the human condition.
Notes

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1. An analysis of the relation between linguistic answers and constituent interrogatives makes use of the property or relation in which the latter are based. See G & S (1984) for details. There the theory developed in this paper is applied to linguistic interrogatives-answer pairs.

2. These constraints are familiar from epistemic logic. More constraints would have to be added once we want to deal with information of one individual about information of another, and with consciousness of one's own information state.

3. Notice that we will need the maxim of Manner to help decide between equivalent sentences, since in this framework they express the same proposition.

4. For a proof see G & S (1984), appendix 2.

5. This presupposes that accessibility relations play a role in defining rigid designation. In a model without them, semantic rigidity would imply pragmatic rigidity.
References


