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Risk sharing with private and public information*

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Abstract

According to the conventional view on efficient risk sharing (Hirshleifer, 1971), better information on future idiosyncratic income realizations harms risk sharing by evaporating insurance opportunities *ex-ante*. In our model, risk-averse agents receive public and private signals on future income realizations and engage in insurance contracts with limited enforceability. When considered separately, better public and private signals are detrimental to welfare. In contrast to the conventional view, we show that when private information is sufficiently precise, more informative public signals can improve the allocation of risk. First, more informative public signals increase the riskiness of the consumption allocation, deteriorating risk sharing. Second, however, more informative public signals mitigate the welfare costs of private information and improve risk sharing. When private signals are sufficiently precise, the positive effect of better public information dominates the negative effect. The positive effect of public information can be quantitatively important in international risk sharing.

JEL classification: D80, F41, E21, D52
Keywords: Consumption risk sharing; Social value of information, Limited commitment

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1 Introduction

Should benevolent governments strive to provide better forecasts on real GDP to improve households' consumption smoothing against country-specific shocks? The theoretical literature (Hirshleifer, 1971, Schlee, 2001) offers sharp predictions for welfare effects of issuing such public forecasts. Better information on future realizations of idiosyncratic shocks harms risk sharing and welfare by limiting insurance possibilities from an *ex-ante* perspective. Similar to Hirshleifer, we consider an environment in which better public and private information on future idiosyncratic shocks are separately detrimental to welfare. However, as our main theoretical result, we show that if households have private information at their disposal to forecast future shock realizations, releasing better publicly available forecasts can improve welfare. In particular, this happens whenever private information is sufficiently precise.

As in Krueger and Perri (2011), we consider a model with risk-averse agents that seek insurance against idiosyncratic fluctuations of their income. While there is no restriction on the type of insurance contracts that can be traded, the contracts suffer from limited commitment because every period agents have the option to default to autarky. The option to default is in particular tempting for agents whose current income is high and these agents face a tension between the higher current consumption in autarky and the future benefits of the insurance promised in the contracts.

Each period – and this is the new element here – agents receive a private and a public signal on their future income shock realization. Thus, the agents have fore-knowledge on future shocks. By changing the expected value of autarky, the signals directly affect the expected benefit of insurance. Insurance contracts can be made contingent on public signals at no additional costs. To render private information contractible requires providing correct incentives. Agents that receive a low private signal have an incentive to report a high private signal because then their reported outside option is higher which improves their bargaining position. As a consequence, the optimal contract prescribes transfers to these agents and tracking these agents' true willingness to share the income risk becomes costly.

As our main novel theoretical contribution, we formally show that better public information can be beneficial for risk sharing when private information is sufficiently precise and enforcement constraints matter. The positive effect of public information on risk sharing is intriguing but can be understood as a trade-off between costs and benefits from releasing more precise public signals. Better public information has two opposite effects. First, in the spirit of Hirshleifer (1971), more precise public information in advance of trading limits risk-sharing possibilities which increases the riskiness of the consumption allocation. Second – and this is the new effect here – more informative public signals reduce the costs to track households' true willingness to share the income risk which improves risk sharing and social welfare. When private information is sufficiently precise, risk sharing improves with better public information and agents prefer informative public signals.

The importance of providing better public information is a matter of an ongoing discussion on the international policy floor. On the one hand, international organizations such as the IMF repeatedly call on countries to increase their transparency with respect to the timely provision of national statistics data needed for computing reliable GDP forecasts.[1] On the other hand, there are also requests aimed at the IMF to improve the quality of the public data and GDP forecasts they supply.[2] As our second contribution, we therefore evaluate the importance of the insurance-information mechanism in the international risk-sharing environment.

For this application, agents in our model are representative households or governments of small countries that seek insurance against country-specific shocks to real GDP. The public signals capture information on a country's future GDP that is publicly available to other countries, for example, the real GDP growth forecast provided in the IMF World Economic Outlook (WEO). Private signals are exclusively observed by a particular country and can capture how transparent a country is. A high precision of private signals indicates secrecy, i.e., useful information on future real GDP exists but that information is not publicly shared.

Based on the Penn World Tables, we construct a panel that comprises 70 countries. After controlling for observable differences between countries such as the level of development and financial openness, we find a significant U-shaped relationship between the quality of public information and the degree of international risk sharing. Countries with low data quality achieve a degree of risk sharing that is higher than the degree of risk sharing in countries with medium quality but comparable to the amount of risk sharing in countries with high data quality.

To take the model to the data, we calibrate the volatility of country-specific income shocks to match the corresponding estimated volatility of real GDP in the Penn World Tables. For

[1] Christine Lagarde, the head of the IMF, made this point during several public appearances, one example being her speech *Harnessing the Power of Transparency* at the IMF Atlantic Council, Washington, DC, 8 February 2017, `https://www.imf.org/en/News/Articles/2017/02/08/sp02082017-Harnessing-the-Power-of-Transparency`.

[2] See the report issued in 2014 by the Independent Evaluation Office of the IMF, *IMF Forecasts. Process, Quality and Country Perspectives.* `www.ieo-imf.org/ieo/pages/CompletedEvaluation181.aspx`

the quality of public information, we vary the precision of public signals until the implied mean forecast squared-error for GDP growth in the model yields the error as estimated from the WEO data. The unobservable precision of private signals is identified using the model such that the cross-country consumption distribution in equilibrium mimics the degree of risk sharing estimated in the country panel.

The empirical U-shaped relationship suggests that better public information could actually worsen risk sharing. Our quantitative results show that this is not the case because the empirical relationship is primarily the result of unobserved differences in secretiveness across countries. We find that private information is sufficiently precise in the sense of our main theoretical result for all country groups and better public data quality improves risk sharing.

To address the currently debated policies mentioned above, we contrast two scenarios for the release of better public information. In the first scenario, data quality improves because the IMF improves their real GDP growth forecast for a group of countries. The second scenario captures the possibility that countries change their disclosure policies toward transparency and release their private information on the country's future GDP growth to the worldwide public. Quantitatively, the improvements in risk sharing resulting from changes in disclosure policies of countries towards transparency are ten times more important than the improvements achieved by better IMF forecasts. In both scenarios better data quality in a particular group of countries can trigger positive spillover effects to risk sharing and welfare in other countries.

Related Literature Methodologically, the most closely related paper is Broer, Kapicka, and Klein (2017) who analyze the effect of unobservable income shocks on consumption risk sharing of U.S. households when contract enforcement is limited. As in Atkeson and Lucas (1992, 1995), agents can be induced to report their income truthfully, and private information reports become contractible but do not increase the state space. Relative to these papers, we consider an environment with contains publicly observable signals and income as well as unobservable private signals such that the number of state variables increases and the dimensionality expands cubically. Our information structure with signals about future endowments is similar to Hassan and Mertens (2015) who study business cycle implications of private information on future productivity shocks in a closed economy. Our focus is on documenting and rationalizing the role of public data quality in international risk sharing.

With this paper, we seek to build a bridge between the theoretical literature on the social value of information (Hirshleifer, 1971, Schlee, 2001) and the quantitative literature on sovereign risk (Arellano, 2008, Bai and Zhang, 2012).

Hirshleifer (1971) is among the first to point out that perfect information makes risk-averse agents ex-ante worse off if such information leads to evaporation of risks that otherwise could have been shared in a competitive equilibrium with full insurance. Schlee (2001) provides general conditions under which better public information about idiosyncratic risk is undesirable in a competitive equilibrium. There are two main differences to these papers in our work. First, we consider optimal insurance contracts with limited enforcement that hinder agents from full insurance. Second, we allow for two sources of information, private and public. Notably, our main theoretical result on the positive effect of public information only applies in the empirically realistic case of partial insurance and when private information is important.

Arellano (2008) and Bai and Zhang (2012) consider economies in which small countries engage in international risk sharing using non-state contingent bonds. Arellano (2008) focuses on the interaction of sovereign default with output, consumption and interest rates over the business cycle. Bai and Zhang (2012) ask whether financial liberalization helped to improve on the insurance of idiosyncratic country risk. None of these papers focus on the relationship between data quality and risk sharing. Further, with non-state contingent bonds, improving public and private data quality would both have an indistinguishable and monotone positive effect on risk sharing. In our empirical results, we find a non-monotone relationship between risk sharing and the quality of public information.

The remainder of the paper is organized as follows. In the next two sections, we present and analyze the theoretical model. Afterwards, we provide details on cross-country differences in risk sharing with respect to the quality of public information. In Section 5, we take the model to the data and study its normative implications. The last section concludes.

2 Environment

Preferences and endowments Consider a world economy that is populated by a continuum of representative households or benevolent governments of small countries indexed i. As in Krueger and Perri (2011), households can engage in insurance contracts with limited enforceability to hedge their consumption against country-specific income shocks. Further, households receive privately and publicly observed signals on their future income shock realizations.

The time is discrete and indexed by t from zero onward. Households have identical preferences over consumption streams

$$U_0\left(\{c_t\}_{t=0}^{\infty}\right) = (1-\beta)\,\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t), \tag{1}$$

where the instantaneous utility function $u : \mathbb{R}_+ \to \mathbb{R}$ is strictly increasing, strictly concave and satisfies the Inada conditions.

The income stream of household i, $\{y_t^i\}_{t=0}^{\infty}$, is governed by a stochastic process with two possible time-invariant realizations $y_t^i \in Y \equiv \{y_l, y_h\}$, with $0 < y_l < y_h$. Income realizations are independent across households and evolve across time according to a first-order Markov chain with constant transition probabilities $\pi(y'|y)$. The Markov chain induces a unique invariant distribution of income $\pi(y)$ such that average income is $\bar{y} = \sum_y y\pi(y)$. The history of income realizations $(y_0, ...y_t)$ is denoted by $y^t \in Y^{t+1}$. Current-period income is publicly observable and the transition probabilities of the Markov chain are public knowledge.

Information Each period, household i receives a public signal $k_t^i \in Y$ (observed by all representative households) and a private signal $n_t^i \in Y$ (observed only by household i) that inform about her income realization in the next period. As income, both signals have two realizations and are assumed to follow exogenous independent first-order Markov processes. The precision of the public signal is denoted by the conditional probability $\kappa = \pi(y' = y_j | k = y_j)$, while the precision of the private signal is given by $\nu = \pi(y' = y_j | n = y_j)$, $\kappa, \nu \in [1/2, 1]$. Uninformative signals are characterized by precision 1/2 while perfectly informative signals exhibit precision of one. The transition probabilities of the two signals, $\pi(k' = y_j | k = y_i)$ and $\pi(n' = y_j | n = y_i)$ are chosen to yield a consistent joint distribution of income and the two signals. Consistency requires that the joint invariant distribution of income and the two signals $\pi(y, k, n)$ exhibits the invariant income distribution as marginal distribution

$$\pi(y) \doteq \sum_{(k,n)} \pi(y, k, n),$$

and that the conditional income distribution $\pi(y'|y)$ follows from integrating the income distribution that is conditional on income and signals over of any pair of signals

$$\pi(y'|y) \doteq \sum_{(k,n)} \pi(y'|y,k,n)\pi(k,n|y).$$

In the following, we employ an i.i.d. income process such that the Markov matrix $\pi(y'|y)$ is symmetric. In this case, the signal processes that yield a consistent joint distribution of income and signals are characterized by the same transition probabilities as income

$$\pi(k' = y_j|k = y_i) = \pi(n' = y_j|n = y_i) = \pi(y' = y_j|y = y_i).$$

The publicly observable part of the state vector is $s_t = (y_t, k_t)$, $s_t \in S$, where $S = Y \times Y$. The whole state vector is $\theta_t = (s_t, n_t)$, with $\theta_t \in \Theta = S \times Y$. The public history of the state is $s^t = (s_0, ...s_t) \in S^{t+1}$; the history of the public and the private state is denoted by $\theta^t = (\theta_0, ..., \theta_t) \in \Theta^{t+1}$. The conditional distribution of signals and income is a time-invariant Markov chain described by transition matrices P_θ, P_s with the conditional probabilities $\pi(\theta'|\theta)$ and $\pi(s'|s)$ as entries.[3]

Utility allocation and truth-telling Households differ with respect to their initial utility entitlements w_0 and the initial state θ_0. As illustrated in Figure 1, each period after receiving the current income and the two signals, households deliver a report on the current realization of the private signal. Define a reporting strategy $\tilde{z}^t \in Y^{t+1}$ as the sequence $\{\tilde{z}_t(w_0, \theta^t)\}_{t=0}^t$ mapping (w_0, θ^t) into a report of the current private signal realization. For all θ^t, a utility allocation is $h = \{h_t(w_0, s^t, \tilde{z}^t)\}_{t=0}^\infty$ and the consumption allocation c can be obtained as $c = \{C[h_t(w_0, s^t, \tilde{z}^t)]\}_{t=0}^\infty$, where $C : \mathbb{R} \to \mathbb{R}_+$ is the inverse of the instantaneous utility function u. An allocation is compatible with truth-telling if it induces a household not to misreport her current signal realization. We follow Fernandes and Phelan (2000) and consider temporary truth-telling constraints where only one-period deviations are permitted. A truth-telling

[3] The computation of the conditional probabilities can be found in Appendix A.7.

compatible allocation satisfies for all θ^t, all reports $\tilde{z}_t$ and all periods t

$$(1-\beta)u[C(h_t)] + \beta \sum_{\theta_{t+1}} \pi(\theta_{t+1}|\theta^t)\, \mathrm{U}_{t+1}[\{C(h_\tau)\}_{\tau=t+1}^{\infty}] \geq$$

$$(1-\beta)u[C(\tilde{h}_t)] + \beta \sum_{\theta_{t+1}} \pi(\theta_{t+1}|\theta^t)\, \mathrm{U}_{t+1}[\{C(\tilde{h}_\tau)\}_{\tau=t+1}^{\infty}], \quad (2)$$

with $h_t = h_t(w_0, \theta^t)$, $h_\tau = h_\tau(w_0, \theta^\tau)$, $\tilde{h}_t = h_t(w_0, s^t, n^{t-1}, \tilde{z}_t)$, $\tilde{h}_\tau = h_\tau(w_0, s^\tau, \tilde{z}^\tau)$, $\tilde{z}^\tau = (n^{t-1}, \tilde{z}_t, n_{t+1}..., n_\tau)$, and $U_{t+1} = (1-\beta)\, \mathrm{E}_{t+1} \sum_{\tau=t+1}^{\infty} \beta^{\tau-t-1} u\,(c_\tau)$.

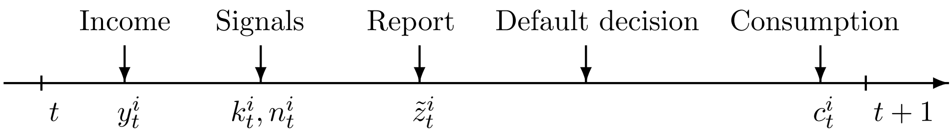

Figure 1: Timing of events with public and private information

Risk sharing arrangements To protect their consumption from undesirable fluctuations, households can engage in insurance contracts that cover the complete state space but have limited enforceability because households have the option to default to autarky. After reporting their current private signal realization, households can decide to participate in international risk-sharing contracts implementing the allocation c or to deviate into autarky forever consuming only their income. Households have no incentive to default if the allocation satisfies enforcement or participation constraints for each history θ^t for all periods t:

$$(1-\beta)u[C(h_t)] + \beta \sum_{\theta_{t+1}} \pi(\theta_{t+1}|\theta^t)\, \mathrm{U}_{t+1}[\{C(h_\tau)\}_{\tau=t+1}^{\infty}] \geq$$

$$(1-\beta)u(y_t) + \beta \sum_{y_{t+1}} \pi(y_{t+1}|\theta_t) U_{t+1}\left(\{y_\tau\}_{\tau=t+1}^{\infty}\right) \equiv U^{Aut}(\theta_t), \quad (3)$$

with as the value of the outside option (autarky).

Efficient allocations Let Φ_0 be a distribution over initial utility promises w_0 and the initial shocks θ_0. In the following definitions, we summarize the notions of constrained feasible and efficient allocations.

Definition 1 *An allocation $h = \{h_t(w_0, \theta^t)\}_{t=0}^{\infty}$ is constrained feasible if* *(i) the allocation delivers the promised utility* w_0

$$w_0 = (1-\beta)\,\mathrm{E}_{\theta_0}\left[\sum_{t=0}^{\infty}\beta^t h_t(w_0,\theta^t)\right] = U(w_0,\theta_0,c); \tag{4}$$

(ii) the allocation satisfies truth-telling (2) *and enforcement constraints* (3) *for each history* θ^t *in each period* t

(iii) and the allocation is resource feasible for each history θ^t *in each period* t

$$\sum_{\theta^t}\int\left[C\left(h_t\right)-y_t\right]\pi(\theta^t|\theta_0)\,\mathrm{d}\,\Phi_0 \leq 0. \tag{5}$$

Atkeson and Lucas (1992, 1995) show by applying a duality argument that efficient allocations can be computed either by directly maximizing households' utility over the distribution Φ_0 or, alternatively, by minimizing resource costs to deliver the promises made in Φ_0.

Definition 2 *An allocation* $\{h_t(w_0,\theta^t)\}_{t=0}^{\infty}$ *is efficient if it is constrained feasible and either*

(i) maximizes households' ex-ante utility over the distribution Φ_0

$$\mathrm{E}\,U(c) = \int U(c)\,\mathrm{d}\,\Phi_0$$

(ii) or, alternatively, there does not exist another constrained feasible allocation $\{\hat{h}_t(w_0,\theta^t)\}_{t=0}^{\infty}$ *with respect to* Φ_0 *that requires fewer resources in at least one period* t

$$\exists t: \quad \sum_{\theta^t}\int\left[C\left(\hat{h}_t\right)-C\left(h_t\right)\right]\pi(\theta^t|\theta_0)\,\mathrm{d}\,\Phi_0 < 0.$$

The second part of the definition is relevant with a continuum of agents and when allocations depend on the state history. Using the first approach, there would be a continuum of promise-keeping constraints (4) the planner would have to respect which makes the approach to optimize directly over ex-ante utility impossible.

3 Analysis

In this section, we deliver our main theoretical result that better public information can be either beneficial or detrimental to social welfare depending on how important the private information friction is. Further, we show that better private information unambiguously worsens welfare.

To derive the analytical results in this section, we consider the two income states y_h, y_l as equally likely and the realizations as independent across time and agents. Correspondingly, signals are i.i.d. as well and can indicate either a high income ("good" or "high" signals) or a low income ("bad" or "low" signals) in the future.

3.1 A positive value of public information in risk sharing

Efficient allocations depend on the history of income shocks and signal realizations. Thereby, the length of the history is endogenous and can comprise the infinite history of the state which limits possibilities for deriving analytical results to special cases. To analyze how better public and private information affect risk sharing, we follow Coate and Ravallion (1993) and restrict attention to memoryless allocations that do not depend on past events but just on current realizations of the state.

Definition 3 *An allocation $\{h_t(w_0, \theta^t)\}_{t=0}^{\infty}$ is a memoryless allocation (denoted h_{ML}) if:*

$$\forall \theta^t \quad \left\{h_t(w_0, \theta^t)\right\}_{t=0}^{\infty} = \{h_t(\theta_t)\}_{t=0}^{\infty} \equiv h_{ML}.$$

A utility allocation in this simplified setting is $h_{ML} = \left\{u\left(c_{ik}^j\right)\right\}$, with $c_{ik}^j = C[h(y = y_j, k = y_i, n = y_k)]$.

As a first step, consider the incentives of high-income agents with a good public signal to truthfully report a bad private signal

$$\begin{aligned}(1-\beta)u(c_{hl}^h) + \frac{\beta(1-\beta)\left[\kappa(1-\nu)V_{rs}^h + (1-\kappa)\nu V_{rs}^h\right]}{\kappa(1-\nu)+(1-\kappa)\nu} + \beta^2 V_{rs} \\ \geq (1-\beta)u(c_{hh}^h) + \frac{\beta(1-\beta)\left[\kappa(1-\nu)V_{rs}^h + (1-\kappa)\nu V_{rs}^h\right]}{\kappa(1-\nu)+(1-\kappa)\nu} + \beta^2 V_{rs}, \quad (6)\end{aligned}$$

High-income agents with a good public signal will truthfully report a good private signal when

$$\begin{aligned}(1-\beta)u(c_{hh}^h) + \frac{\beta(1-\beta)\left[\kappa\nu V_{rs}^h + (1-\kappa)(1-\nu)V_{rs}^h\right]}{\kappa\nu+(1-\kappa)(1-\nu)} + \beta^2 V_{rs} \\ \geq (1-\beta)u(c_{hl}^h) + \frac{\beta(1-\beta)\left[\kappa\nu V_{rs}^h + (1-\kappa)(1-\nu)V_{rs}^h\right]}{\kappa\nu+(1-\kappa)(1-\nu)} + \beta^2 V_{rs}, \quad (7)\end{aligned}$$

with the continuation values defined as

$$V_{rs}^h = \frac{1}{4}\sum_{i\in\{l,h\}}\sum_{k\in\{l,h\}} u(c_{ik}^h), \quad V_{rs}^l = \frac{1}{4}\sum_{i\in\{l,h\}}\sum_{k\in\{l,h\}} u(c_{ik}^l), \quad V_{rs} = \frac{V_{rs}^h + V_{rs}^l}{2}.$$

With memoryless allocations, truth-telling incentives can only be trivially satisfied which is summarized in the following lemma.

Lemma 1 (Memoryless allocations and truth-telling) *Consider a memoryless allocations that satisfies truth-telling. Then private information is non-contractible and the consumption allocation is independent from private-signal reports, i.e.*

$$c_{ik}^j = c_i^j \quad \forall i, k, j.$$

Proof. With memoryless allocations, future consumption depends only on future shock realizations such that from (6), it follows that $c_{hl}^h \geq c_{hh}^h$. From (7), one gets $c_{hh}^h \geq c_{hl}^h$ such that $c_{hl}^h = c_{hh}^h = c_h^h$. Using similar arguments, $c_{ih}^j = c_{il}^j = c_i^j$, for all i, k, j. ■

Thus, a utility allocation can be summarized as $h_{ML} = \left\{u\left(c_i^j\right)\right\}$, with $c_i^j = C[h(y = y_j, k = y_i)]$ and social welfare is

$$\mathrm{E}\, U(c) = (1-\beta)\frac{1}{4}\sum_{t=0}^{\infty}\sum_{j\in\{l,h\}}\sum_{i\in\{l,h\}} \beta^t u\left(c_i^j\right). \tag{8}$$

Resource feasibility requires in each period t

$$\frac{1}{4}\sum_{j\in\{l,h\}}\sum_{i\in\{l,h\}} c_i^j = \frac{1}{2}\sum_{j\in\{l,h\}} y_j. \tag{9}$$

Further, constrained feasibility requires that allocations are consistent with rational incentives to participate. For example, the enforcement constraints of high-income households with high public and high private signals are

$$\begin{aligned}(1-\beta)u(c_h^h) + \frac{\beta(1-\beta)\left[\kappa\nu V_{rs}^h + (1-\kappa)(1-\nu)V_{rs}^l\right]}{\kappa\nu + (1-\kappa)(1-\nu)} + \beta^2 V_{rs} \geq \\ (1-\beta)u(y_h) + \frac{\beta(1-\beta)\left[\kappa\nu u(y_h) + (1-\kappa)(1-\nu)u(y_l)\right]}{\kappa\nu + (1-\kappa)(1-\nu)} + \beta^2 V_{out}, \end{aligned} \tag{10}$$

while the constraints for high-income agents with low public and high private signals read

$$(1-\beta)u(c_l^h) + \frac{\beta(1-\beta)\left[(1-\kappa)\nu V_{rs}^h + \kappa(1-\nu)V_{rs}^l\right]}{(1-\kappa)\nu + \kappa(1-\nu)} + \beta^2 V_{rs} \geq$$
$$(1-\beta)u(y_h) + \frac{\beta(1-\beta)\left[(1-\kappa)\nu u(y_h) + \kappa(1-\nu)u(y_l)\right]}{(1-\kappa)\nu + \kappa(1-\nu)} + \beta^2 V_{out}, \tag{11}$$

with

$$V_{rs}^h = \frac{1}{2}\left[u(c_h^h) + u(c_l^h)\right], \quad V_{rs}^l = \frac{1}{2}\left[u(c_h^l) + u(c_l^l]\right)$$

and

$$V_{rs} = \frac{V_{rs}^h + V_{rs}^l}{2}, \quad V_{out} = \frac{u(y_h) + u(y_l}{2}.$$

As a next step, we provide the definition of an optimal memoryless allocation.

Definition 4 *An optimal memoryless arrangement (ML arrangement) is a consumption allocation $\{c_i^j\}$ that maximizes households' utility (8) subject to resource feasibility (9) and enforcement constraints in all periods $t \geq 0$.*

The optimal memoryless arrangement exists and is unique. The arrangement and social welfare are continuous functions in the precision of the public and private signal. The proof follows from the maximum theorem under convexity and is omitted here. Further, one can show that in memoryless arrangements only participation constraints of high-income agents can be binding.[4] Allocations depend only on public information but not on private information. Thus, for each public state allocations must be consistent with the participation incentives of households with high private signals as the highest outside option value. For this reason, only constraints (10) and (11) can be binding in the optimal memoryless arrangement.

We begin our analysis with the effect of private information on risk sharing. In the following proposition, we show that the relationship between risk sharing and private information is monotone.

Proposition 1 (Risk Sharing and Private Information) *Let participation constraints of high-income agents with a good private signal (10) and (11) be binding. Assume that autarky is not the only constrained feasible memoryless arrangement and consider $\kappa \in [0.5, 1)$. Then risk sharing and social welfare are decreasing in the precision of the private signal for $\nu \in [0.5, 1)$.*

[4] A proof for this result can be found for example in Lepetyuk and Stoltenberg (2013). In optimal history-dependent arrangements also participation constraints of low-income agents are occasionally binding.

The proof is provided in Appendix A.1.

The welfare effect of increases in private signal precision is negative independent of the precision of public signals. The intuition for this result is as follows. An increase in private signal precision increases the value of the outside option of all high-income agents because only the high private signal is relevant for the optimal arrangement. As a consequence, consumption of high income agents increases, and risk sharing and welfare decrease. Despite its positive implications, Proposition 1 has also normative implications that we study in Section 5. In particular, we can compute how a move toward transparency affects risk sharing by reducing private signal precision and increasing public signal precision.

In the following Theorem, we provide our main theoretical result that improvements in public information can either benefit or worsen risk sharing and welfare depending on the importance of the private information fiction.

Theorem 1 (Positive Value of Public Information) *Consider the case when the participation constraints (10) and (11) of high-income agents are binding. Assume that autarky is not the only constrained feasible memoryless arrangement. Then there exists a precision of the private signal* $\bar{\nu}$*, such that for* $\nu \geq \bar{\nu}$ *and* $\nu \in [0.5, 1)$*, risk sharing and social welfare are increasing in the precision of the public signal* κ *over* $\kappa \in [0.5, 1)$*.*

The proof is provided in Appendix A.2. The logic of the proof is as follows. First, we show that for an uninformative private signal the social value of information is negative, while for a perfectly informative signal the effect is positive. Continuity then implies that there exists a level of private information for which the welfare effect of better public information is positive.

There is a negative and a positive effect on risk sharing as a result of releasing better public information when private signals are informative. First – and in the spirit of the traditional Hirshleifer (1971) effect – more precise public information in advance of trading limits risk-sharing possibilities. Second – and this is the new effect here – more informative public signals facilitate a better tracking of households' true willingness to share the income risk which improves risk sharing and social welfare.

The main theorem is illustrated in Figure 2 that depicts social welfare as a function of public information precision for different precisions of private information. When private information is uninformative or not precise (see the upper two functions), the negative effect dominates and social welfare is decreasing in public signal precision. For $\nu = 0.70$, the negative and the

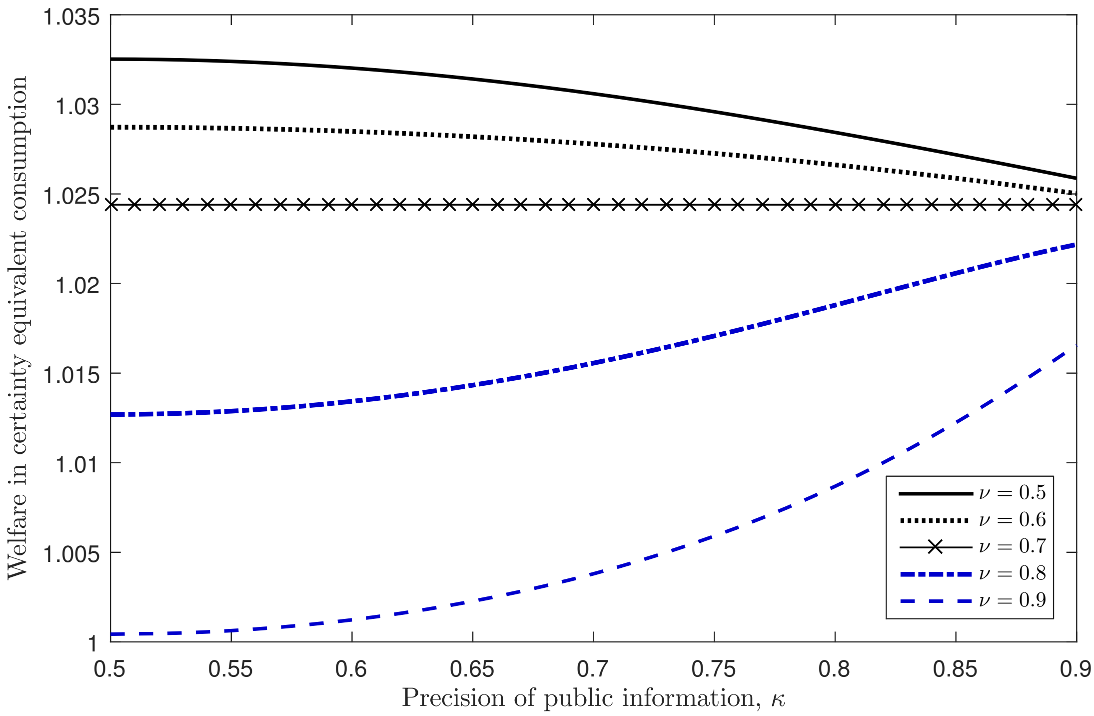

Figure 2: Welfare effects of public information for different precisions of private information.

positive effect neutralize each other. However, when private signals are sufficiently precise the latter positive effect dominates the negative effect, and social welfare increases in public signal precision (see the lower two functions).

To gain intuition, consider an increase in the precision of the public signal. By (10) and (11), this results in an increase in the value of the outside option for high-income agents with a good public signal and a decrease for agents with a bad public signal as illustrated in Figure 3.

Agents with a bad signal are more willing while the agents with a good signal are less willing to share their current high income. When the private signal is uninformative (see the lower part of Figure 3 with $\nu = 0.50$), the changes in the value of the outside option of high-income agents with a good signal ($V^h_{h,out}$) and with a bad signal ($V^h_{l,out}$) are symmetric.

The high-income agents with a good public signal have a lower marginal utility of consumption and thus require more additional resources than the high-income agents with a bad public signal are willing to give up. In sum, the average consumption of high-income agents increases which by resource feasibility reduces the risk-sharing possibilities for low-income agents. As a consequence, the allocation becomes riskier ex-ante and social welfare decreases.

When the private signal is sufficiently informative and public-signal precision increases (see the upper part of Figure 3 for $\nu = 0.90$), the value of the outside option of high-income agents with good public signals increases less than the outside option of high-income agents with bad

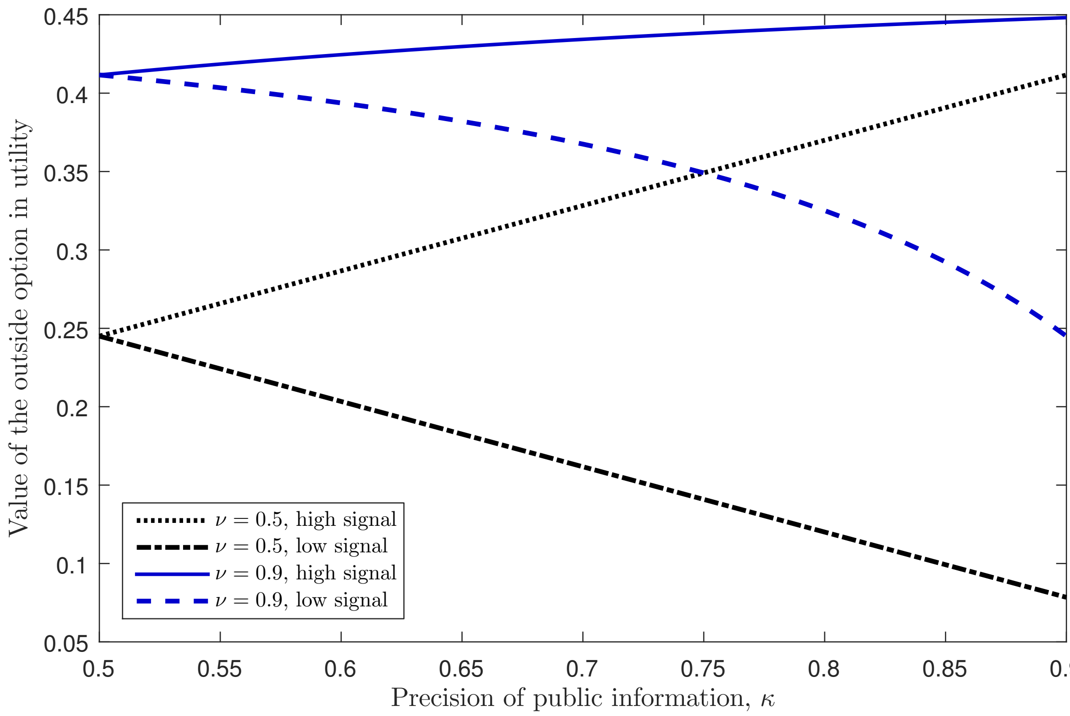

Figure 3: Outside option values of high-income agents as functions of public information precision for different precisions of private information.

public signals decreases (in absolute terms). The asymmetric change in the outside option creates room for redistribution from high to low-income agents stemming from high-income agents with low public signals. For a sufficiently informative private signal, better public information facilitates additional risk-sharing transfers between high-and low-income agents and social welfare increases.

In the following section, we discuss whether the positive value of public information as summarized in Theorem 1 also arises when we allow allocations to depend on the history of shocks.

3.2 Discussion: history-dependence and truth telling

The main theorem is derived for memoryless allocations that are in general not efficient. In this section, we study whether the positive effect of public information also applies to allocations that are not memoryless. We subsequently allow for allocations to depend on the history of the public but not on the truth-full reports of the private shock realizations (non-contractible private information), and eventually consider allocations that can depend on agents' truth-full reports of their private signal realizations (contractible private information).

We analyze a two-period economy. For the second period, we assume that agents respect the commitments made in the first period. Otherwise, if voluntary participation were allowed in both periods, there would be no risk sharing because agents would always choose to consume their endowments. Social welfare is given by the expected utility of consumption before any risk has been resolved

$$\mathrm{E}\left[u(c_1) + u(c_2)\right], \tag{12}$$

with a strictly increasing and strictly concave instantaneous utility function $u(c)$ and a normalized discount factor of one.

Non-contractible private information One property of memoryless allocations is that private information is non-contractible (see Lemma 1). As an intermediate step, we consider allocations in which private information is non-contractible per assumption but can depend on the history of public signals and income. Let $c^j_{i,1}$ be first-period consumption of agents with public signal k_i and income y_j and $c^{jk}_{i,2}$ second-period consumption of agents with public signal k_i and income y_j in the first period and income y_k in the second period with $i, j, k \in \{l, h\}$. The planner chooses $\{c^j_{i,1}\}, \{c^{jk}_{i,2}\}$ to maximize (12) subject to resource feasibility in the first and second period

$$\frac{1}{4}\left(c^h_{h,1} + c^l_{h,1} + c^h_{l,1} + c^l_{l,1}\right) = \frac{1}{2}\sum_{j\in\{l,h\}} y_{j,1}$$

$$\frac{1}{8}\Big[(1 + z_1 - z_2)\left(c^{hh}_{h,2} + c^{lh}_{h,2} + c^{hl}_{l,2} + c^{ll}_{l,2}\right)$$
$$+(1 + z_2 - z_1)\left(c^{hl}_{h,2} + c^{hh}_{l,2} + c^{ll}_{h,2} + c^{lh}_{l,2}\right)\Big] = \frac{1}{2}\sum_{j\in\{l,h\}} y_{j,2},$$

and enforcement constraints with $0.5 \leq z_1, z_2 \leq 1$ defined as

$$z_1 = \frac{\kappa\nu}{\kappa\nu + (1-\kappa)(1-\nu)}$$

and

$$z_2 = \frac{(1-\kappa)\nu}{(1-\kappa)\nu + \kappa(1-\nu)}.$$

Thereby, $(1+z_1-z_2)/2$ is the measure of agents for which public signals and income realizations in the second period coincide, $(1 + z_2 - z_1)/2$ the measure of agents where this is not the case. For $\kappa \to 1$, the first measure approaches one and the second one vanishes; for $\kappa \to 0.5$, the two measures boil down to 0.5. As one example, the enforcement constraints for high-income agents in the first period with a good public and good private signal are

$$u(c_{h,1}^{h}) + z_1 u(c_{h,2}^{hh}) + (1 - z_1)u(c_{h,2}^{hl}) \geq u(y_{h,1}) + z_1 u(y_{h,2}) + (1 - z_1)u(y_{l,2}).$$

With allocations that depend on the history of the public state, the planner can now offer better inter-temporal smoothing to high-income agents which facilitates more transfers to low-income agents and reduces consumption dispersion compared to memoryless allocations. Figure 4 summarizes social welfare as a function of public signal precision for different degrees of precisions of private signals. When private signals are not very precise (as for $\nu = 0.50, 0.60$), social welfare is decreasing in public signal precision; when private information is sufficiently precise, social welfare improves with better public information (see the graphs for $\nu = 0.80, 0.90$).

Figure 4 is qualitatively similar to Figure 2 and the mechanism at work is the same as with memoryless allocations. In particular, better public information affects the outside options of high and low-income agents asymmetrically similar to Figure 3. When private signals are sufficiently precise, high-income agents with bad public signals are willing to transfer more resources than high-income agents with good private signals require to be indifferent with autarky. The better public information decreases the average consumption of high-income agents and increases the average consumption of low-income agents, resulting in a less risky allocation ex-ante. For $\nu = 0.80$ and $\nu = 0.90$ and low degrees of κ, the optimal allocation features initially no risk sharing. As public information improves further, optimal allocations provide risk sharing and social welfare improves with more precise public signals.

Contractible private information The planner can also encourage agents to truthfully report their private signals by rendering them at least as well off as when lying about the private signal and private information becomes contractible. In this case, the planner chooses allocations that are contingent on public signals and income and on the truthfully reported private signal realizations in the first period. Let $c_{im,1}^{j}$ be first-period consumption of agents with public signal k_i, private signal n_m and income y_j and $c_{im,2}^{jk}$ is second-period consumption defined accordingly. The planner chooses $\{c_{im,1}^{j}\}, \{c_{im,2}^{jk}\}$ to maximize (12) subject to resource feasibility, enforcement constraints, and truth-telling constraints. Thus, the allocations computed here are efficient. With the inclusion of truth-telling constraints, the dimensionality increases. Instead of 12 elements with non-contractible private information, the allocation now contains 24 elements.

For example, truth-telling constraints of agents with a high income, a high public, and a

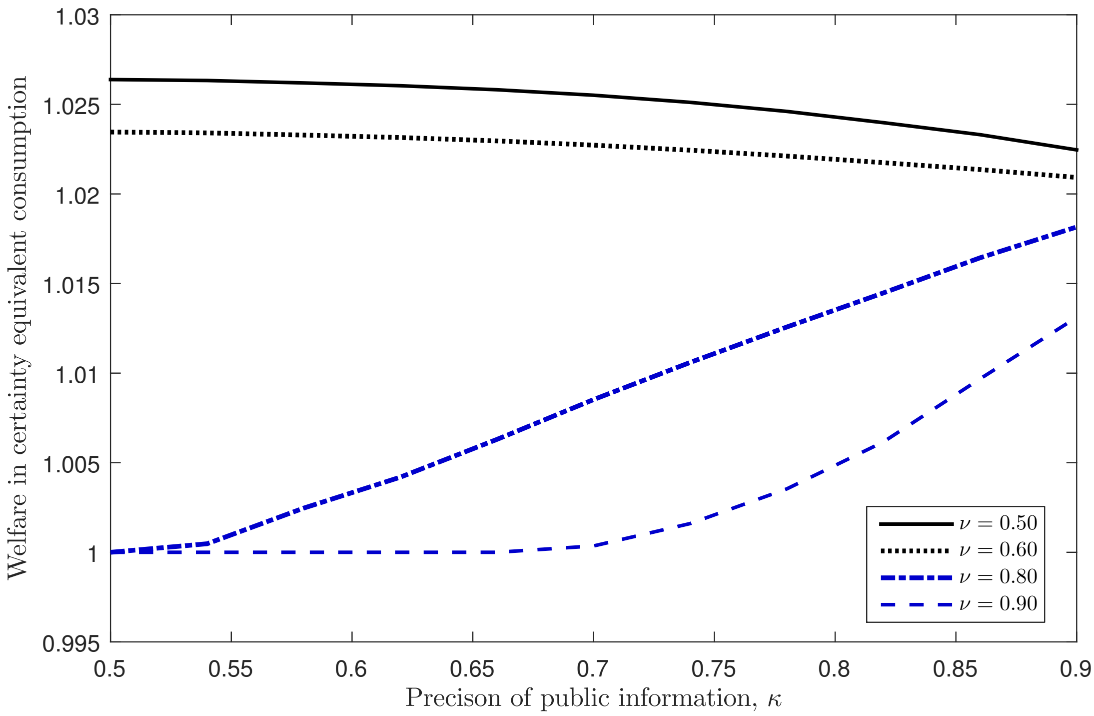

Figure 4: Non-contractible private information: welfare effects of public information for different precisions of private information.

low private signal are given by

$$u(c^h_{hl,1}) + (1 - z_2)u(c^{hh}_{hl,2}) + z_2 u(c^{hl}_{hl,2}) \geq u(c^h_{hh,1}) + (1 - z_2)u(c^{hh}_{hh,2}) + z_2 u(c^{hl}_{hh,2}).$$

For informative private signals, the optimal allocation is characterized by higher transfers from high-income to low-income agents in the first period than with non-contractible private information. The additional transfers are stemming from agents with a low private signal as the agents with binding truth-telling constraints. Agents with a low private signal are willing to transfer more in the first period to be insured in the low-income state in the second period because this state is likely to realize for them. To discourage these agents to lie, the corresponding consumption for a high private signal in the low-income state in the future must be lower, i.e., $c^{hl}_{hh,2} < c^{hl}_{hl,2}$. In optimum, agents with a low private signal are indifferent between lying and telling the truth. The truth-telling constraints of agents with good private signals, however, do not bind because these agents have no incentive to misreport.

Figure 5 depicts social welfare as a function of the precision of public information for various precisions of the private signals. Qualitatively, Figure 5 resembles the main message from Theorem 1: the better public information improves welfare when private signals are sufficiently precise.

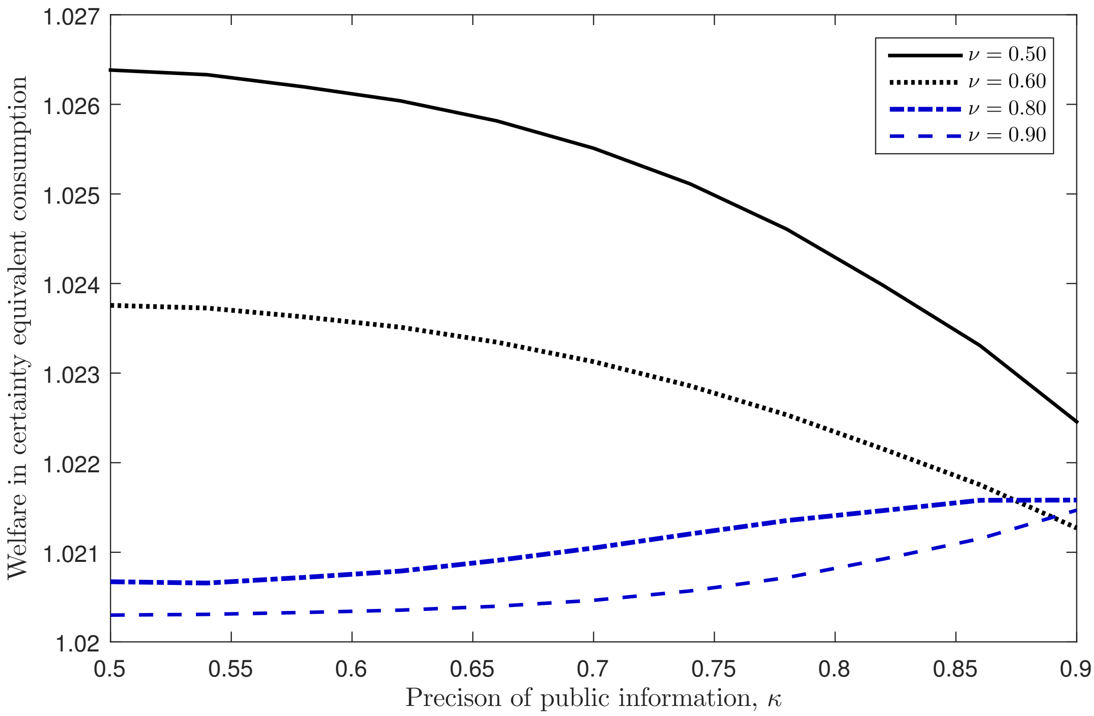

Figure 5: Contractible private information: welfare effects of public information for different precisions of private information.

Even with truth-telling, private information is costly such that social welfare with only private signals is lower than welfare with only public signals. In particular, the average consumption of high-income agents is higher with private than with public information. With private signals precise enough, the average consumption of high-income households in the second period decreases with better public information, thereby reducing consumption dispersion ex-ante. For low precision of private information, the opposite applies. Comparing Figures 5 and 4 for $\nu = 0.8, 0.9$, efficient allocations can provide risk sharing when non-contractible private information can only deliver autarky.

For the same pair of private and public signal precision, the welfare difference between allocation with non-contractible and contractible private signal reports can be quantitatively important (compare Figures 4 and 5). Intuitively, the planner can with contractible private information always choose allocations that do not depend on private signal reports. This does not necessarily imply that the welfare effects of better public signals are quantitatively significantly different when risk sharing is calibrated to the same target in both types of allocations. When we evaluate the quantitative importance of better public information in international risk sharing, we calibrate the model to match risk sharing degrees and public information precision estimated from the data. This leads to a different degree of private information in allocations with contractible and non-contractible private information. As an example, we calibrate the

Table 1: Welfare effects of public information with private information.

	Non-contractible, $\Delta\kappa$	Contractible, $\Delta\kappa$
$\sigma_y = 0.25$	0.15	0.16
$\sigma_y = 0.20$	0.08	0.04

Notes: $\Delta\kappa$ captures the relative change in welfare for an increase in public signal precision measured in certainty equivalent consumption expressed in percent.

precision of private information such that each period 40 percent of the variation in logged income is insured for $\kappa = 0.50$. The welfare effect of public information $\Delta\kappa$, is computed by comparing welfare with uninformative public signals to welfare with public signals that are as precise as the calibrated precision of private signals. As displayed in Table 1, the social value of public information is positive in both environments and very similar. While for a higher variability of income, public information has a slightly larger marginal gain with contractible private information (see first the row), the reverse applies for the lower variability of income (see the second row).

3.3 Optimal stationary allocations

As the main result in the previous section, we find that the positive effect of public information as summarized in Theorem 1 prevails when allocations depend on the history of public shocks or allocations that are additionally contingent on truthfully reported private signal realizations. In the following, we study the quantitative importance of better public information in international risk sharing in an infinite horizon economy. Even in an environment with two periods, the increase in dimensionality resulting from history-dependence and truth-telling constraints is significant. The cubic increase in dimensionality compared to existing studies as Broer et al. (2017) is amplified with an infinite horizon because then a potential infinite history of each shock must be tracked. In light of the robustness results from the previous section and for reasons of tractability, we, therefore, restrict attention to optimal stationary allocations with non-contractible private information in the following.

More specifically, we consider history-dependent allocations that respect enforcement constraints and resource feasibility for all possible histories of public and private information but allocations that only depend on the history of public signals and income as the publicly observable part of the state vector. As in Krueger and Perri (2011), we restrict attention to stationary allocations in which the distribution of current utility and utility promises is constant across time. As originally shown by Atkeson and Lucas (1992), a stationary allocation is optimal if it is a solution to a standard dynamic programming problem and satisfies resource feasibility. This dynamic programming problem adapted to our environment is described next.

A financial intermediary is responsible for allocating resources to a particular household. There are many intermediaries acting under perfect competition that can inter-temporally trade resources with each other at the given shadow price $1/R$ with $R \in (1, 1/\beta]$. The equilibrium interest rate is the interest rate that guarantees resource feasibility. Given a utility promise w, a public state $s = (y, k)$, a constant R, the planner chooses a portfolio of current utility $h\left(w\left(s\right), s\right)$ and future promises $w'(w\left(s\right), s; s')$ for each future income realization y' and signal k'. In doing so, the intermediary takes into account that households differ with respect to their private signal realization in every period which affects their evaluation of the portfolio $(h\left(w\left(s\right), s\right), \{w'(w\left(s\right), s; s')\})$. This portfolio is required to minimize the discounted resources costs with the dependence of controls on $\left(w\left(s\right), s\right)$ scrapped for notational convenience:

$$V(w, s) = \min_{h, \{w'(s')\}} \left[\left(1 - \frac{1}{R}\right) C(h) + \frac{1}{R} \sum_{s'} \pi(s'|s) V\left(w'(s'), s'\right) \right] \tag{13}$$

to deliver the promised value $w(s)$ and to satisfy the participation constraints:

$$w = \sum_{n} \pi\left(n|s\right) \left[\left(1 - \beta\right) h + \beta \sum_{s'} \pi\left(s'|s, n\right) w'\left(s'\right) \right] \tag{14}$$

$$\left(1 - \beta\right) h + \beta \sum_{s'} \pi\left(s'|s, n\right) w\left(s'\right) \geq U^{Aut}\left(s, n\right), \forall n, \tag{15}$$

where $\pi\left(n|s\right)$ is the probability of a given private signal realization conditional on the realization of the public state in the current period.[5]

In the recursive formulation, the enforcement constraints (15) are imposed for the current period. This is different from Krueger and Perri (2011) who impose the enforcement constraint on continuation values, i.e., $w'(s') \geq U^{Aut}(s', n')$. If households can sign contracts that are contractible on the whole state vector, both recursive formulations are equivalent. When private information is not contractible, imposing the enforcement constraint on continuation values is not equivalent to imposing the constraints in the current period. In Appendix A.6, we show that the recursive formulation (13)-(15) implies looser constraints than imposing the enforcements

[5] The resulting recursive problem and the properties of its solution bear resemblance to those in Krueger and Perri (2011) for autocorrelated income processes.

constraints on promises. Unlike in Krueger and Perri (2011), the value function depends on y, k even if income is i.i.d. because the financial intermediary needs to know the current realization of s in the enforcement constraints (15).

A stationary allocation $\{h_t(w_0, s^t)\}_{t=0}^{\infty}$ is an optimal allocation if it is induced by an optimal policy from the functional equation above with $R > 1$ and satisfies the resource constraint (5) with equality. In Appendix A.4, we provide a condition for the existence of risk sharing that is less restrictive than the corresponding condition with memoryless allocations.

In the following section, we present an empirical application of our theoretical model. There, we investigate the relationship between risk sharing and the quality of public information in a panel of countries.

4 Information and international risk sharing in the data

In this section, we will look at the correlation between international risk sharing and quality of public information through the lens of our model. To empirically assess how risk sharing varies across countries that differ with respect to public data quality, we employ data on real consumption and real GDP per capita from the Penn World Tables, version 8.1. We generate measures of risk sharing by regressing the idiosyncratic component of changes in consumption per capita in country i on the idiosyncratic component of changes in GDP per capita.

We use two different measures to asses the country's quality of public information on real GDP. In the model, the higher the quality of information, the smaller the forecast error of the next period realisation of income. Thus, our first measure of public information quality is based on data from the IMF World Economic Outlook. We compute mean squared forecast errors for real GDP growth for all countries in our sample and normalise them by the variance of country GDP growth.

Our second measure capturing the quality of public information on the real GDP are the data quality grades from the Penn World Tables. The data-quality grades rank from the highest data-quality A to the lowest quality D.[6] This measure, however, does not correspond exactly to the meaning of public information precision in the model. As we demonstrate it later on, the two measures are strongly correlated.

[6] Thereby, a low data quality grade can result for example because information about real GDP is not reported and thus incomplete or when real GDP figures are sensitive with respect to the particular method used in aggregation.

Except for the sake of robustness, the second measure of public information provides a rationale to bin countries into groups. Sorting countries into bins will be useful for the quantitative evaluation in the next section. In particular, the sorting allows us to explicitly consider trade and risk sharing between and within the groups of countries that differ in terms of the precision of private and public information. Without sorting countries into bins but considering all 70 countries separately, would prohibitively increase the dimensionality of the numerical problem we have to solve. As a consequence, we could only study trade and risk sharing within a group of agents with the same precisions of both types of information which is not realistic.

As our main result in this section, we find that the correlation between public information and risk sharing in the cross section follows a U-shape. For low quality of public information, improvements in public information are correlated with decreases in risk sharing, while further improving data quality however ameliorates the degree of international risk sharing. The differences in risk sharing captured in the U-shaped relationship survive controlling for observable country-differences such as the stage of economic development and several financial openness measures. Further, we argue that the differences in risk sharing are also not merely the result of measurement error in GDP data.

To facilitate comparisons with related studies, we build our sample starting with the data of Kose, Prasad, and Terrones (2009), a study that utilised Penn World Tables data to investigate the evolution of the risk sharing measures in time. However, Kose et al. (2009)'s sample contains only one D-quality grade country (Togo). Thus, we expand the sample by adding D-quality grade countries with population no smaller than 1 million inhabitants. Eventually, our data set comprises observations on 70 countries for the years 1990-2004 after dropping outliers. In Appendix A.9, we provide further details about the final sample.

Results As a first step, for each country in our data we compute the mean squared error of one year ahead IMF GDP growth rate forecasts relative to actual realized real GDP growth over the sample period. This measure exactly corresponds to the treatment of information in our model presented in the next section, however, because this measure is scale dependent, we normalize it by country's realized GDP growth rate variances. In the first row of Table 2, we report the averages of normalized mean squared errors for each of the public data quality country groups. Evidently, both data quality measures are correlated and the resulting monotone pattern is aligned well with the data quality grade assignment from the Penn World Tables.

We proceed with documenting the relationship between data quality grade and international risk sharing. Following Obstfeld (1995), for each country i in our sample we separately run a first stage risk sharing regression:

$$\Delta \ln (c_{it}) - \Delta \ln (C_t) = \beta_{0i} + \beta_{1i} [\Delta \ln (y_{it}) - \Delta \ln (Y_t)] + \varepsilon_{it}, \tag{16}$$

where $\Delta c_{it}/y_{it}$ stands for country's i consumption/GDP per capita in year t. Capital letter variables are sample aggregates that capture uninsurable aggregate risk component. The standard measure of risk sharing is $\beta_{\Delta y,i} = 1 - \beta_{1i}$ which attains 0 (no insurance) if changes in country's i consumption growth react one-to-one to changes in country's i GDP growth rate and 1 if consumption growth does not react to changes in GDP growth at all (full insurance).

When we bin the countries according to the PWT data quality grades we find that the A-countries average insurance measure equals 0.46, for B-countries it is 0.13 and for C-countries it's 0.26. Thus, unconditionally, the insurance relationship with public data quality is U-shaped. For D countries, we face data limitations: except for one country, there is no consumption data available in the Penn World Tables. For this reason, we exclude these countries from the risk-sharing regressions. The IMF data on GDP growth rate forecasts and realizations includes D countries and we employ these countries as a reference point for calibrating public information precision.[7]

A significant fraction of between-country dispersion in international risk sharing measures can be attributed to the level of economic development. In particular, richer countries tend to have better quality institutions, the rule of law and investor protection.[8] Thus, we remove the effect of the level of development on between country dispersion in insurance they obtain against idiosyncratic GDP fluctuations. To do that we regress the country specific risk sharing coefficients $1 - \beta_{1i}$ on the log of the average GDP per capita $\ln (\overline{y_i})$.

$$1 - \beta_{1i} = \gamma_0 + \gamma_{GDP} \ln (\overline{y_i}) + \zeta_i.$$

We then introduce our insurance measure $\beta_{\Delta y,i}$ by centering the residuals ζ_i from this regression around the sample mean for $1-\beta_{1i}$. The estimates of the second stage regression are displayed in

[7] Note that while the model postulates a causal relationship between risk sharing and information, the data from the PWT allow only to estimate a correlation between the two variables.

[8] Kose, Prasad, and Terrones (2009) employ a discrete measure, splitting countries into OECD and non-OECD economies.

Table 2: Data quality, forecast accuracy and risk sharing

	Data Quality Grade			
	A	B	C	D
MSE_j, normalized	1.02	1.24	1.30	1.49
Risk sharing, $\beta_{\Delta y,i}$	0.36	0.09	0.33	–
Means test p-values (B group as reference)	0.03	–	0.02	–

Notes: $MSFE_j$ is the annual mean-squared forecast error in country group $j \in \{A, B, C\}$ normalized by the variance of GDP growth. IMF World Economic Outlook Forecast for GDP growth in year t is given by the Fall forecast in $t-1$, the GDP growth realization is given by the value reported in Fall $t+1$. Data: Penn World Tables, IMF World Economic Outlook, 1990–2004.

Table 9 in Appendix A.9 together with the robustness checks that we discuss after summarising the results of our main exercise.

In the second row of Table 2, we report the conditional risk sharing measure for the data-quality groups A, B and C. This exercise demonstrates that the inferior unconditional risk sharing of countries in groups B and C compared to the highest grade A countries is indeed partly driven by differences in level of economic development. For example, grade C countries tend to have lower average GDP per capita levels than countries in A and B group. In fact, all of the advantage of group A over group C can be attributed to the level of economic development; B countries' decrease in risk sharing from 0.13 to 0.09 is due to their relatively high level of GDP per capita as compared to C countries.

The main message is that improvements in public data quality on country's idiosyncratic GDP risk do not exhibit a monotone correlation with risk sharing. Instead, the relationship follows a U-shape, both for the unconditional and conditional risk sharing measures. Increasing data quality from grade C to B worsens risk sharing but further improvements ameliorate risk sharing.

To test the significance of the conditional risk sharing measure, we regress the averages on A- and C-group dummies. We find the positive differences with regard to the B-group to be significant at p-values reported in the bottom row of Table 2 that are below 3%. The same picture emerges from alternatively employing the normalised mean squared forecast error as public information measure. Figure 7 is a scatter plot of the non-linear correlation between risk sharing and public data quality in this case. As with the data-quality grades, the non-monotone relationship is also significant on the 5% level. Further details of this test are displayed in Table 10 in Appendix A.9 together with the robustness checks that we discuss next.

Robustness We test the robustness of our main result, the U-shaped relationship between public data quality and risk sharing in two ways.

First, we add additional regressors in the second-stage regression. We regress country-specific risk sharing measure $\beta_{\Delta y,i}$ on the level of average economic development and measures of financial integration. We try three measures of financial integration. The first one is the Chinn-Ito index (Chinn and Ito, 2006) which is derived from a set of underlying measures of financial openness of a country. The other two measures are the (logs of) ratio of total assets and liabilities to GDP taken from the External Wealth of Nations Database. Unlike the Chinn-Ito index which is a continuous *de jure* measure of financial integration, the other two measures are *de facto* measures. Results of the baseline regression and robustness checks are displayed in table 10 in Appendix A.9.

Second, we partition the countries into high, medium and low public data quality groups splitting the sample into terciles based on the average normalised IMF forecast mean squared error. We again find a U-shaped relationship that is significant with p-values below 5% (Table 11, Appendix A.9) even when the effects of economic development and financial integration are controlled for.

Measurement Error Theoretically, the U-shaped pattern that we find in the data could be driven by a measurement error in the explanatory variable that is potentially the largest for C countries. It is well known that a classical measurement error in the explanatory variable can introduce an attenuation bias, thus lowering the estimated β_{1i} coefficients in regression (16). Formally, if the true data Δy_{it}^* is measured with noise, $\Delta y_{it} = \Delta y_{it}^* + \varepsilon_{m.err.}$ and the true underlying first stage coefficient is β_{1i}^* then the estimated value $\hat{\beta}_{1i}$ in the limit reads:

$$\hat{\beta}_{1i} = \frac{\sigma^2_{\Delta \ln\left(y_{it}^*\right) - \Delta \ln(Y_t^*)}}{\sigma^2_{\Delta \ln\left(y_{it}^*\right) - \Delta \ln(Y_t^*)} + \sigma^2_{m.err.}} \beta_{1i}^* \equiv \tilde{\lambda} \beta_{1i}^* \tag{17}$$

The attenuation factor $\tilde{\lambda}$ depends solely on the variances of the measurement error and the true data Δy_{it}^*. Getting rid of this error could bring the C-group risk sharing measure below the one of the B-countries, breaking the U-shaped pattern as a result. It seems unlikely, however, that the measurement error could be the sole driver of the U-shaped pattern.

In theory, there are two ways of breaking the U-shaped pattern, either making it a decreasing or an increasing one (moving from low to high data quality). The first case requires the insurance for the A countries to be mechanically over-estimated because of the attenuation bias. The second case calls for the same effect for C countries to take place. The monotonicity of country data quality ranking implies that in order to introduce attenuation bias in A countries we would need to introduce it in greater magnitudes also for B and C countries. Thus, it seems impossible to reconcile the attenuation bias argument with the claim that the true relationship between public data quality and risk sharing is in fact decreasing.

It is still possible, however, that the true relationship between data quality and risk sharing is increasing and our results are driven by the measurement error. To this effect we mechanically introduce attenuation bias in the first stage regression estimates and redo our empirical exercise, including the second stage regression and means difference test.

First, we introduce the attenuation bias only in the C-countries data putting $\tilde{\lambda}_C < 1$ with $\tilde{\lambda}_B = \tilde{\lambda}_A = 1$. We find the minimum size of attenuation bias that destroys the significance of the U-shaped pattern on the 10 percent significance level to be equal to $\tilde{\lambda}_C = 0.86$. In other words, approximately the sixth part of observed fluctuations in GDP of C-countries would have to come from mismeasurement.

However, this treatment postulates that A and B countries have not only identical data qualities but more importantly there is no measurement error. If we realistically allow for the measurement error in B countries data, exogenously specifying its relative magnitude to be a third of this in C countries data, we arrive at $\tilde{\lambda}_C = 0.69$ and $\tilde{\lambda}_B = 0.85$ to break the significance of the U-shaped relationship. Now, one third of all observed fluctuations in GDP of C-countries must stem from mismeasurement which does not seem plausible. The implied magnitude of the measurement error in C-countries is a conservative estimate as it needs to be even larger if one also allows for mismeasurements in A-countries data. We conclude that although the measurement error can to some extent affect the size of between country groups differences in risk sharing, it is unlikely to break the U-shaped pattern. In the following section, we take the theoretical model to the data to explore the normative implications of the empirical relationship established in this section.

5 Quantitative evaluation

In this section, we take the model presented in Section 2 to the data and we explore its positive and normative implications. Quantitatively, we find that the model can capture the differences in risk sharing that result from the differences in the quality of public information as observed in the data. While the U-shaped relationship in the data seems to suggest better public data quality may deteriorate risk sharing, the theoretical model clarifies that better public information nevertheless improves risk sharing under different risk-sharing regimes and model specifications.

5.1 Calibration

Preferences, endowments and outside options We start with standard values for the specification of preferences. The instantaneous utility function features log preferences and we choose an annual discount factor of $\beta = 0.89$ as for example recently employed in Bai and Zhang (2012). The logarithm of income of country i is modeled as an auto-regressive process

$$\log(y_{it}) = \rho \log(y_{it-1}) + \epsilon_{it},$$

where $\rho \in [0, 1)$ and ϵ_{it} is normally distributed with mean zero and variance σ_ϵ^2. To capture the volatility of log income, we compute the variance of the idiosyncratic GDP component $\ln y_{it} - \ln Y_t$ after removal of country-specific growth trends to eventually arrive at an idiosyncratic income risk standard deviation of 0.082.

We normalize the mean of income to one, and employ the method proposed by Tauchen and Hussey (1991) to approximate the income process by a Markov process with two states and time-invariant transition probabilities to yield the unconditional variance observed in our data set. The joint distribution of income and signals is therefore approximated by 8 states for each of the three countries.[9] For our baseline, we employ an i.i.d. income process, i.e., $\rho = 0$.

For the quantitative results, we allow countries to engage in self insurance in the outside option. In case of defaulting to the outside option, countries loose all their consumption claims. Further, access to financial markets is restricted. While countries can save unlimited amounts in a non-state contingent bond with real gross return $R^{Aut} > 0$, they cannot borrow. Thus, the value of the outside option is a solution to an optimal savings problem that can be written in recursive form as follows

$$v(\theta, a) = \max_{0 \le a' \le y + aR^{Aut}} \left[(1 - \beta) u(aR^{Aut} + y - a') + \beta \sum_{\theta'} \pi(\theta'|\theta) v'(\theta', a') \right].$$

[9] The cardinality of the set of all θ_t is N^3 which makes computing optimal allocations a challenging task.

Table 3: Baseline parameters and data moments

	Parameter	Value
σ	Risk aversion	1
β	Discount factor	0.89
R^{Aut}	Real return savings in autarky	1.02
κ_A	Precision public information A countries	0.78
κ_B	Precision public information B countries	0.70
κ_C	Precision public information C countries	0.68
ρ	Auto-correlation	0
σ_ϵ	SD of i.i.d. term	0.082
Θ	Income-signals states	3×8
var_y	Variance log income	0.01
$\beta_{\Delta y,A}$	Risk-sharing coefficient A countries	0.36
$\beta_{\Delta y,B}$	Risk-sharing coefficient B countries	0.09
$\beta_{\Delta y,C}$	Risk-sharing coefficient C countries	0.33
$MSFE_A$	Scaled mean-squared forecast error A countries	1.02
$MSFE_B$	Scaled mean-squared forecast error B countries	1.24
$MSFE_C$	Scaled mean-squared forecast error C countries	1.30
$MSFE_D$	Scaled mean-squared forecast error D countries	1.49

Thus, outside option values are given by the following vector

$$U^{Aut} = v(\theta, 0).$$

Again, we follow Bai and Zhang (2012) and we set $R^{Aut} = 1.02$.

Information, forecast errors and risk sharing measures To calibrate the precision of public information κ_j for the countries with data quality $j \in \{A, B, C\}$, we compute the percentage reduction of income risk $\tilde{\kappa}$ as measured by the reduction in the mean-squared forecast error resulting from conditioning expectations on informative public signals in our model

$$\tilde{\kappa}(\kappa) = \frac{\text{MSFE}_y - \text{MSFE}_s}{\text{MSFE}_y}, \tag{18}$$

with

$$\text{MSFE}_y = \sum_y \pi(y) \sum_{y'} \pi(y'|y) \left[y' - \text{E}(y'|y)\right]^2$$

$$\text{MSFE}_s = \sum_s \pi(s) \sum_{y'} \pi(y'|s) \left[y' - \text{E}(y'|s)\right]^2,$$

and $\pi(s)$ as the joint invariant distribution of income and public signals. If signals are uninformative, $\tilde{\kappa}$ is equal to zero and if signals are perfectly informative, $\tilde{\kappa}$ equals one. We choose D countries as reference point and express the scaled mean-squared forecast error relative to these countries. To be more precise, we assume that $\kappa_D = 1/N$ such that $\text{MSFE}_D = \text{MSFE}_y$. For example, A countries' mean-squared forecast error is 32 percent lower than the corresponding forecast error in D countries. For the given income process, we adjust κ such that $\tilde{\kappa}_A = 0.32$ which results in a calibrated precision of public information of $\kappa_A = 0.78$. For the other country groups we receive $\tilde{\kappa}_B = 0.17$ and $\tilde{\kappa}_C = 0.13$ leading to $\kappa_B = 0.70$ and $\kappa_C = 0.68$, respectively. In Table 3, we summarize the mean-squared forecast errors and the resulting parameter values for precision of public information in the three country groups.

To identify the unobserved private information in the different country groups, we employ the theoretical model. As shown in Proposition 1, risk sharing in the model decreases monotonically in the precision of private information for given public signal precision. Thus, given the public information precision κ_j pinned down from the IMF forecast, we seek to identify the precision of private information ν_j by varying it until risk sharing in the model mimics the degrees of risk sharing $\beta_{\Delta y,j}$ for $j \in \{A, B, C\}$ as listed in Table 3. Eventually, this results in a reduction in the mean-squared forecast error resulting from conditioning expectations on informative public and private signals which is defined as follows

$$\tilde{z}(\kappa, \nu) = \frac{\text{MSFE}_y - \text{MSFE}_\theta}{\text{MSFE}_y}, \tag{19}$$

with

$$\text{MSFE}_\theta = \sum_\theta \pi(\theta) \sum_{y'} \pi(y'|\theta) \left[y' - \text{E}(y'|\theta)\right]^2,$$

and $\pi(\theta)$ as the joint invariant distribution of income, public and private signals.

5.2 Risk-sharing environments and public data improvements scenarios

Risk-sharing environments Agents in our model correspond to representative households across country groups A, B, C. The world economy comprises three groups of countries indexed by A, B, C, each group comprises a continuum of households with measure 1/3. The country groups differ only with respect to the importance of private and public information and the degree of risk sharing, preferences, endowments and the real interest rate for savings in autarky are identical.

The literature on international financial markets points to significant variation in the degree of integration across countries and over time (see Bekaert et al. (2007) for a discussion). Thus, below we consider two polar different risk-sharing environments to illustrate the working of public and private information. First, we consider only trade that occurs within each group of countries, i.e., between agents with the same precision of private and public signals. Thus, the risk sharing takes place in segmented markets, the environment is as described in Section 2 and we refer to this trading arrangement as *Segmented-markets risk sharing.*

The *World risk sharing* environment considers risk sharing between agents with the same but also with agents with different precision of signals. The solution concept and equilibrium features extend naturally from the within-group environment described in Section 2. Let $h^j = \left\{h_t(w_0^j, s^t)\right\}_{t=0}^{\infty}$ denote the utility allocation, $c^j = \left\{C\left(h_t^j(w_0^j, s^t)\right)\right\}_{t=0}^{\infty}$ the consumption allocation and Φ_0^j the invariant distribution over initial promises w_0^j and initial shocks s_0 in country group j. Efficient allocations c^A, c^B, c^C, maximize world social welfare

$$\sum_j \mathrm{E}\, U(c^j) = \sum_j \int U(c^j)\, \mathrm{d}\, \Phi_0^j, \quad j = A, B, C$$

subject to promise keeping and enforcement constraints for households in the country groups A, B and C and resource feasibility for each history θ^t in each period t

$$\sum_j \sum_{\theta^t} \int \left[C\left(h_t^j\right) - y_t\right] \pi^j(s^t|s_0)\, \mathrm{d}\, \Phi_0^j \leq 0, \quad j = A, B, C. \tag{20}$$

Better public data quality scenarios As a normative question, we ask how better public data quality affects risk sharing and welfare. We consider two scenarios for the release of better public information.

In the first scenario, public data quality improves while private information precision remains unaffected. The idea of the scenario is that the IMF improves her real GDP growth forecast for a particular group of countries while these countries do not change their disclosure policies. Such a change is not information neutral because it reduces the mean-squared forecast error from conditioning on signals such that $\tilde{\kappa}$ and $\tilde{z}$ increase. We refer to this scenario as the ***IMF Scenario***.

The second scenario captures the possibility that countries change their disclosure policies toward transparency and release their private information on future country risk. More formally,

Table 4: IMF Scenario: welfare and risk sharing effects of better public information

	Risk-sharing effect			Welfare effect, %		
	A	B	C	A	B	C
All countries, $\kappa = 0.5 \to \kappa_j$	0.110	0.090	0.040	0.027	0.041	0.010
C countries, $\kappa_C \to \kappa_B$	–	–	0.010	–	–	0.002
B countries, $\kappa_B \to \kappa_A$	–	0.085	–	–	0.032	–

Notes: Segmented-markets risk sharing. The table lists changes in risk sharing and welfare resulting from changes in the quality of public information with constant private information precision. Risk sharing effects are expressed as absolute changes, welfare effects as percentage changes in certainty equivalent consumption. $\kappa_C = 0.68$, $\kappa_B = 0.70$ and $\kappa_A = 0.78$.

we compute $\hat{\kappa}_j$ such that

$$\tilde{\kappa}(\hat{\kappa}_j) \doteq \tilde{z}(\kappa_j, \nu_j)$$

and compare the properties of the resulting allocation in the absence of private information to the original allocation with informative signals of precisions κ_j, ν_j. While such a policy improves the quality of public information, it is information neutral because it does not reduce the mean-squared forecast error for future income. We refer to this scenario as the ***Transparency Scenario*** of better public data quality.

In the following subsection, we start with analyzing the effects of better public information on risk sharing in segmented markets. In the next step, we also allow for risk sharing between country groups.

5.3 Segmented-markets risk sharing

Quantitatively, we find the theoretical model can explain the differences in risk sharing for groups A, B and C. Thereby, the private information friction plays the most important role in countries of group B. To capture the low degree of risk sharing of $\beta_{\Delta y,B} = 0.09$, the private information friction is with $\nu_B = 0.74$ more severe than in country groups A with $\nu_A = 0.63$, and in C countries, $\nu_C = 0.62$, respectively. For these values of ν, the model matches the risk sharing in A, $\beta_{\Delta y,A} = 0.36$ and in C, $\beta_{\Delta y,C} = 0.33$.

IMF Scenario As our main result in this section, we find that – in contrast to what the empirical relationship seemingly suggests – better public data quality improves risk sharing and social welfare for all country groups. The reason for this result is that risk sharing in the model – unlike in the estimation – not only depends on the quality of public information but also on the unobserved precision of private information. If the precision of private signals necessary to

Table 5: Transparency Scenario: welfare and risk sharing effects of better public information

	Risk-sharing effect			Welfare effect, %		
	A	B	C	A	B	C
C countries, $\kappa_C \to 0.70, \nu_C \to 0.5$	–	–	0.268	–	–	0.07
B countries, $\kappa_B \to 0.76, \nu_B \to 0.5$	–	0.505	–	–	0.156	–
A countries, $\kappa_A \to 0.78, \nu_A \to 0.5$	0.234	–	–	0.059	–	–

Notes: Segmented-markets risk sharing. The table lists changes in risk sharing and welfare resulting from changes in the quality of public information. Risk sharing effects are expressed as absolute changes, welfare effects as percentage changes in certainty equivalent consumption. Risk sharing effect expressed as difference. Starting from $\kappa_C = 0.68$, $\kappa_B = 0.70$ and $\kappa_A = 0.78$, public signal precision increases are information neutral.

capture the degrees of risk sharing in the data is sufficiently high in the sense of Theorem 1, better public information improves risk sharing.

As an initial step, we compute the following counterfactual. Suppose the IMF was to provide uninformative GDP growth forecasts. As displayed in Table 4, for B countries, reducing the quality of public information from $\kappa_B = 0.70$ to $\kappa = 0.5$ (the public information quality in D countries) would lead to a monotone decrease in risk sharing from 0.09 to an autarkic allocation with zero risk sharing. The resulting welfare losses are roughly 0.04% measured in certainty equivalent consumption.

A relevant question is by much risk sharing and welfare would improve when the IMF improves the quality of information for a particular group of countries to reach the next quality category. The results for this exercise are displayed in Table 4. When C countries improve their quality of public information to achieve the same quality of information as in B countries, risk sharing improves from 0.36 to 0.37 and welfare increases by 0.002%. For B countries, the improvements in risk sharing and welfare are larger. Risk sharing increases from 0.09 to 0.18 and welfare improves by 0.032%.

Transparency Scenario In this paragraph, we consider the second possibility of improving public data quality. Here, countries in group A, B and C reveal their private information on future shocks to real GDP by releasing better public information. To be more precise, private information is rendered uninformative and public signal precision is increased until the resulting mean-squared forecast error equals the forecast error in the original situation with informative public and private signals. For example, for C countries the mean-squared forecast error reduction that agents achieve with private signal precision $\nu_C = 0.62$ and public signal precision $\kappa_C = 0.68$ – relative to D countries – is 0.16. With uninformative private signals,

Table 6: Segmented-markets versus world risk sharing

	Segmented markets			*World*		
	A	B	C	A	B	C
Risk sharing, $\beta_{\Delta y,j}$	0.311	0.185	0.304	0.36	0.09	0.33
Signal precision, ν_j	0.63	0.74	0.62	0.60	0.85	0.61

public signals with precision $\kappa_C = 0.70$ yield the same reduction in the mean-squared forecast error. Thus, the improvement in public data quality is information neutral. It is a quantitative question whether the welfare effects of the transparency scenario are positive. On the one hand, reducing private signal precision improves risk sharing and welfare. However, simultaneously increasing the precision of public signals increases the riskiness of the consumption allocation when private information is uninformative (see Theorem 1). In Table 5, we display the resulting effects on risk sharing and welfare in case of the transparency scenario.

Improving public data quality by releasing private information has positive effects on risk sharing and welfare, and the effects are quantitatively more important than in case of the IMF scenario. For example, in B countries risk sharing improves by 0.51 and welfare improves by 0.16% each year, permanently.

As an intermediate summary, we find that improving public data quality yields positive welfare effects in both scenarios. However, improving public data quality by encouraging countries to release their private information on future country risk is quantitatively more important for risk sharing than improvements in the IMF's GDP growth forecasts.

5.4 World risk sharing

In reality, A, B and C countries engage in risk sharing contracts with each other. This case is analyzed in the *World risk sharing* environment. Despite of this change in the risk sharing environment, the model can generate the U-shaped relationship between public data quality and risk sharing. Further, we find that better public information in one country group now also has positive spillover effects to countries in other groups, both in case of the IMF and even more so in the Transparency scenario of better public data quality.

Segmented-markets versus world risk sharing To gain intuition on the difference between within-and world risk sharing, we start with the calibrated values of signal precision from the segmented-markets risk sharing calibration, i.e., $\nu_A = 0.63$, $\nu_B = 0.74$ and $\nu_C = 0.62$. As displayed in the first column of Table 6, after opening up trade between the country groups the degree of risk sharing for group B countries increases while A and B countries realize lower degrees of risk sharing.

The changes are a consequence of the possibility to smooth consumption not only across income states within each country group, but also to smooth it across income states *across* country groups. To compensate for the low degree of risk sharing in B countries, the optimal allocation features additional transfers from A and C countries to B countries as the country groups with the lowest degree of risk sharing.

The possibility of engaging in risk-sharing arrangements not only within but also between country groups imposes a challenge for the model to match the risk sharing estimates from the data. Nevertheless, we find that the model can again capture risk sharing across country groups. Matching the target risk sharing ratios, requires now more precise private signals in B countries with $\nu_B = 0.85$ and a less severe private information friction in A. Countries of group C are characterized by a similar degree of private information as in the segmented-markets risk-sharing environment (see the second column of Table 6).

We proceed with our quantitative evaluation and explore the normative implications of better public information with world risk sharing.

IMF Scenario Similar to the segmented markets setting, even though the model captures the U-shaped relationship from the data, better public information in one country group improves risk sharing and welfare for these countries.

As displayed in the first row of Table 7, better public information in C countries (to a small extent) improves risk sharing and welfare in these countries but hardly affects the other country groups. Again, there is a positive effect of better information in C countries because private information is sufficiently precise in these countries in the sense of Theorem 1. However, private information in C countries is relatively small such that the positive effect of better public information in these countries is quantitatively small.

Improving public information in B countries affects almost exclusively risk sharing and welfare in B countries, the spillover effects to other countries are negligible. Here the positive effect of better public information facilitates better risk sharing within the B countries as a direct effect. Since risk sharing in B countries is lower than in A and C countries, the optimal allocation predominantly prescribes more transfers from high to low-income agents within the

Table 7: IMF Scenario: welfare and risk sharing effects of better public information

	Risk-sharing effect			Welfare effect, %			
	A	*B*	*C*	*A*	*B*	*C*	*World*
C countries, $\kappa_C \to \kappa_B$	0.002	0.003	0.009	0.001	0.001	0.001	0.001
B countries, $\kappa_B \to \kappa_A$	−0.001	0.038	−0.000	−0.000	0.014	−0.000	0.004

Notes: World risk sharing. The table lists changes risk sharing and welfare resulting from changes in the quality of public information. Risk sharing effects are expressed as absolute changes, welfare effects as percentage changes in certainty equivalent consumption. $\kappa_C = 0.68$, $\kappa_B = 0.70$ and $\kappa_A = 0.78$.

Table 8: Transparency Scenario: welfare and risk sharing effects of better public information

	Risk-sharing effect			Welfare effect, %			
	A	*B*	*C*	*A*	*B*	*C*	*World*
C countries, $\kappa_C \to 0.70, \nu_C \to 0.5$	0.051	0.075	0.160	0.020	0.044	0.025	0.030
B countries, $\kappa_B \to 0.84, \nu_B \to 0.5$	0.073	0.404	0.082	0.033	0.103	0.037	0.058
A countries, $\kappa_A \to 0.78, \nu_A \to 0.5$	0.112	0.056	0.046	0.018	0.032	0.017	0.022

Notes: World risk sharing. The table lists changes in risk sharing and welfare resulting from changes in the quality of public information. Risk sharing effects are expressed as absolute changes, welfare effects as percentage changes in certainty equivalent consumption. Risk sharing effect expressed as difference. Starting from $\kappa_C = 0.68$, $\kappa_B = 0.70$ and $\kappa_A = 0.78$, public signal precision increases are information neutral.

B countries.

Transparency Scenario In Table 8, we summarize the quantitative results for the Transparency scenario. Confirming the results from the within-risk sharing, better public data quality has positive welfare effects that are quantitatively important. Compared to the IMF scenario, the direct and spillover welfare effects of better data quality are over ten times larger. For example, for B countries improving data quality by releasing private information improves risk sharing by 0.4, thereby improving welfare in all three country groups and facilitating up to 0.10% higher annual consumption, permanently.

Remarkably, the transparency scenario yields also both positive direct and indirect welfare effects when country groups A and C improve their data quality. In all cases, B countries with the lowest initial degree of risk sharing benefit the most from the improvements in data quality, directly (see the second row) and indirectly (see the fifth column). This result follows from optimal insurance that ideally seeks to equalize risk sharing across country groups. However, full equalization is prevented by countries' heterogenous incentives to participate in international risk sharing resulting from different precisions of public and private information across country groups.

6 Conclusions

In this paper, we have returned to the old question of effects of public information on risk sharing. We extended the previous theoretical work allowing for private and public information released in advance of trading constrained by limited commitment. We find that improvements in public information can improve welfare. Better public information has two opposite effects. First, it has a detrimental effect on risk sharing by limiting risk-sharing possibilities in the spirit of Hirshleifer (1971). Second, it mitigates the private information friction which improves risk sharing. The second effect dominates if private information is sufficiently precise.

Then, we used the model to address the currently debated calls for provision of better public information in the international risk sharing setting. Based on the Penn World Tables, we found a U-shaped correlation between risk sharing and data quality, one that would not be possible to explain with existing models of information and risk sharing. The data suggest that better public information quality may deteriorate risk sharing for countries of low public data quality. However, we have found that this is not the case. When interpreted through the lenses of our model, the U-shaped pattern stems from the underlying heterogeneity in private information that countries have (how secretive they are). Furthermore, the private information friction is severe enough for the public information improvements to have positive welfare effects.

The improvements in risk sharing can either result from improvements in the quality of public information provided by the IMF or – and quantitatively more important – when countries move its disclosure policies toward transparency. In both cases, the improvements in public data quality can induce positive spillover effects to consumption smoothing in other countries.

The quality of public information is not only relevant for international risk sharing but can also be a relevant factor in other environments where also private information is present. One example includes commercial banks facing idiosyncratic liquidity shocks against which they seek insurance. Banks have private information about their future liquidity position, and there is also a public source of information on liquidity risk of individual banks provided by the Federal Reserve System (Fed).

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A Appendix

A.1 Proof of Proposition 1

In the proof, we employ an envelope theorem to show that social welfare decreases in better private information. Take the derivative of social welfare with respect to ν

$$\frac{\partial V_{rs}}{\partial \nu} = \frac{1}{4}\left[u'(c_h^h)\frac{\partial c_h^h}{\partial \nu} + u'(c_l^h)\frac{\partial c_l^h}{\partial \nu} - u'(c^l)\left(\frac{\partial c_h^h}{\partial \nu} + \frac{\partial c_l^h}{\partial \nu}\right)\right] \leq 0, \tag{21}$$

where we used that in the optimal memoryless arrangement consumption of low-income agents is the same for a good and a bad public signal, $c_h^l = c_l^l = c^l$ and that u is Inada. The derivatives of consumption with respect to the signal precision follow from the implicit function theorem. Further, in optimum, the following conditions holds

$$\frac{u'(c_h^h) - u'(c^l)}{F_{c_h^h} + G_{c_h^h}} = \frac{u'(c_l^h) - u'(c^l)}{F_{c_l^h} + G_{c_l^h}},$$

with both denominators being strictly positive. Using this and dividing by $u'(c_l^h) - u'(c^l) < 0$ leads to a negative derivative of social welfare with respect to ν when

$$\begin{aligned}\left(F_{c_h^h} + G_{c_h^h}\right)\frac{\alpha_1 G_{c_l^h} - \alpha_2 F_{c_l^h}}{F_{c_h^h}G_{c_l^h} - F_{c_l^h}G_{c_h^h}} + \left(F_{c_l^h} + G_{c_l^h}\right)\frac{\alpha_2 F_{c_h^h} - \alpha_1 G_{c_h^h}}{F_{c_h^h}G_{c_l^h} - F_{c_l^h}G_{c_h^h}} &\geq 0 \\ \Leftrightarrow (\alpha_1 + \alpha_2)\frac{F_{c_h^h}G_{c_l^h} - F_{c_l^h}G_{c_h^h}}{F_{c_h^h}G_{c_l^h} - F_{c_l^h}G_{c_h^h}} &\geq 0 \\ \Leftrightarrow \alpha_1 + \alpha_2 \geq 0,&\end{aligned}$$

with the coefficients α_1, α_2 defined as

$$\alpha_1 = -\frac{\partial F}{\partial \nu} = \beta(1-\beta)(u(y_h) - V_{rs}^h - u(y_l) + V_{rs}^l)\frac{\kappa(1-\kappa)}{(\kappa\nu + (1-\kappa)(1-\nu))^2} \geq 0$$

$$\alpha_2 = -\frac{\partial G}{\partial \nu} = \beta(1-\beta)(u(y_h) - V_{rs}^h - u(y_l) + V_{rs}^l)\frac{\kappa(1-\kappa)}{((1-\kappa)\nu + \kappa(1-\nu))^2} \geq 0,$$

where strict inequalities apply and social welfare is strictly decreasing in ν for $\kappa \in [0.5, 1)$.

A.2 Proof of Theorem 1

Take the derivative of social welfare with respect to κ

$$\frac{\partial V_{rs}}{\partial \kappa} = \frac{1}{4}\left[u'(c_h^h)\frac{\partial c_h^h}{\partial \kappa} + u'(c_l^h)\frac{\partial c_l^h}{\partial \kappa} - u'(c^l)\left(\frac{\partial c_h^h}{\partial \kappa} + \frac{\partial c_l^h}{\partial \kappa}\right)\right] \equiv I(\nu,\kappa), \quad (22)$$

where we used that in the optimal memoryless arrangement consumption of low-income agents is the same for a good and a bad public signal, $c_h^l = c_l^l = c^l$ and that u is Inada. The derivatives of consumption with respect to the signal precision follow from the implicit function theorem. For the latter, re-write the two binding participation constraints

$$F(c_h^h, c_l^h) \equiv (1-\beta)u(c_h^h) + \beta(1-\beta)z_1 V_{rs}^h + \beta(1-\beta)(1-z_1)V_{rs}^l + \beta^2 V_{rs} \\ - (1-\beta)u(y_h) - \beta(1-\beta)\left(z_1 u(y_h) + (1-z_1)u(y_l)\right) - \beta^2 V_{out} = 0,$$

$$G(c_h^h, c_l^h) \equiv (1-\beta)u(c_l^h) + \beta(1-\beta)z_2 V_{rs}^h + \beta(1-\beta)(1-z_2)V_{rs}^l + \beta^2 V_{rs} \\ - u(y_h) - \beta(1-\beta)\left(z_2 u(y_h) + (1-z_2)u(y_l)\right) - \beta^2 V_{out} = 0,$$

with $z_1 \geq z_2$ defined as before as

$$z_1 = \frac{\kappa\nu}{\kappa\nu + (1-\kappa)(1-\nu)}$$

and

$$z_2 = \frac{(1-\kappa)\nu}{(1-\kappa)\nu + \kappa(1-\nu)}$$

In the following, it is useful to employ the sum of derivatives of high-income agents' consumption with respect to κ which is given by

$$\frac{\partial c_h^h}{\partial \kappa} + \frac{\partial c_l^h}{\partial \kappa} = \frac{x_2 F_{c_l^h} + x_1 G_{c_l^h}}{F_{c_h^h} G_{c_l^h} - F_{c_l^h} G_{c_h^h}} - \frac{x_2 F_{c_h^h} + x_1 G_{c_h^h}}{F_{c_h^h} G_{c_l^h} - F_{c_l^h} G_{c_h^h}}$$

where

$$x_1 = -\frac{\partial F}{\partial \kappa} = \beta(1-\beta)(u(y_h) - V_{rs}^h - u(y_l) + V_{rs}^l)\frac{\nu(1-\nu)}{(\kappa\nu + (1-\kappa)(1-\nu))^2} \geq 0$$

$$x_2 = \frac{\partial G}{\partial \kappa} = \beta(1-\beta)(u(y_h) - V_{rs}^h - u(y_l) + V_{rs}^l)\frac{\nu(1-\nu)}{((1-\kappa)\nu + \kappa(1-\nu))^2} \geq 0.$$

The partial derivatives are

$$\begin{aligned}
F_{c_h^h} &= (1-\beta)\left[u'(c_h^h) + \beta\frac{z_1}{2}u'(c_h^h) - \beta\frac{1-z_1}{2}u'(c^l)\right] + \frac{\beta^2}{4}\left[u'(c_h^h) - u'(c^l)\right] \\
F_{c_l^h} &= (1-\beta)\left[\beta\frac{z_1}{2}u'(c_l^h) - \beta\frac{1-z_1}{2}u'(c^l)\right] + \frac{\beta^2}{4}\left[u'(c_l^h) - u'(c^l)\right] \\
G_{c_l^h} &= (1-\beta)\left[u'(c_l^h) + \beta\frac{z_2}{2}u'(c_l^h) - \beta\frac{1-z_2}{2}u'(c^l)\right] + \frac{\beta^2}{4}\left[u'(c_l^h) - u'(c^l)\right] \\
G_{c_h^h} &= (1-\beta)\left[\beta\frac{z_2}{2}u'(c_h^h) - \beta\frac{1-z_2}{2}u'(c^l)\right] + \frac{\beta^2}{4}\left[u'(c_h^h) - u'(c^l)\right].
\end{aligned}$$

Autarky is not the only constrained feasible allocation. This implies that at the optimal memoryless allocation the following holds

$$F_{c_h^h}G_{c_l^h} - F_{c_l^h}G_{c_h^h} > 0. \tag{23}$$

Together with the properties of the utility function $u(\cdot)$, this establishes the unique existence of the continuous differentiable functions $c_h^h(\kappa), c_l^h(\kappa)$, while at the autarky allocation, (23) is negative (see Proposition 3 in Appendix A.3), i.e.,

$$\begin{aligned}
F_{c_h^h}G_{c_l^h} - F_{c_l^h}G_{c_h^h}\Big|_{\{c_i^j\}=\{y_j\}} &= u'(y_h) + \beta\frac{z_1+z_2}{2}[u'(y_h) + u'(y_l)] \\
&\quad - \beta u'(y_l) - \frac{\beta^2}{1-\beta}\frac{1}{2}[u'(y_l) - u'(y_h)] < 0.
\end{aligned}$$

For $\nu = 0.5$, $x_1 = x_2 = x > 0$, $c_h^h \geq c_l^h$ and $\partial c_h^h/\partial\kappa \geq 0$[10], the derivative with respect to κ satisfies

$$I(0.5,\kappa) \leq \frac{1}{4}[u'(c_l^h) - u'(c^l)]\left(\frac{\partial c_h^h}{\partial \kappa} + \frac{\partial c_l^h}{\partial \kappa}\right).$$

which is negative whenever the sum of derivatives with respect to κ on the right-hand side is

[10] At the optimal memoryless arrangement with binding participation constraints, $F_{c_l^h} + G_{c_l^h} > 0$ follows from the first order conditions. For $\nu = 0.5$, $x_1 = x_2 \geq 0$ which implies the positive sign of the derivative when autarky is not the only constrained feasible memoryless arrangement.

positive. Evaluated at $\nu = 0.5$, the sum of derivatives reads (with a strict inequality for $\kappa > 0.5$)

$$\frac{\partial c_h^h}{\partial \kappa} + \frac{\partial c_l^h}{\partial \kappa} = x \frac{F_{c_l^h} + G_{c_l^h} - F_{c_h^h} - G_{c_h^h}}{F_{c_h^h} G_{c_l^h} - F_{c_l^h} G_{c_h^h}}$$
$$= x \frac{(1-\beta)(1+\beta \frac{z_1+z_2}{2} + \frac{\beta^2}{(1-\beta)} \frac{1}{2})(u'(c_l^h) - u'(c_h^h))}{F_{c_h^h} G_{c_l^h} - F_{c_l^h} G_{c_h^h}} \geq 0.$$

For $\nu \to 1$, we get $c_h^h \to c_l^h = c^h$, the derivative of social welfare with respect to κ is

$$I(1, \kappa) = \frac{1}{4}[u'(c^h) - u'(c^l)] \left(\frac{\partial c_h^h}{\partial \kappa} + \frac{\partial c_l^h}{\partial \kappa} \right),$$

and the sum of derivatives is negative

$$\frac{\partial c_h^h}{\partial \kappa} + \frac{\partial c_l^h}{\partial \kappa} \leq 0$$
$$\Leftrightarrow \frac{-\frac{x_2}{x_1}(F_{c_h^h} - F_{c_l^h}) + G_{c_l^h} - G_{c_h^h}}{F_{c_h^h} G_{c_l^h} - F_{c_l^h} G_{c_h^h}} = (1-\beta) \frac{\left(1 - \frac{\kappa^2}{(1-\kappa)^2}\right) u'(c^h)}{F_{c_h^h} G_{c_l^h} - F_{c_l^h} G_{c_h^h}} \leq 0,$$

where we used that for $\nu \to 1$, $F_{c_l^h} \to G_{c_h^h}$, $F_{c_h^h} \to G_{c_l^h}$ and $x_1/x_2 = \kappa^2/(1-\kappa)^2$; for $\kappa > 0.5$, the strict inequality applies. Existence of $\bar{\nu}$ follows from continuity of $I(\nu, \kappa)$ for $\nu \in [0.5, 1)$.

A.3 Information, perfect risk sharing and autarky

Efficient allocations can be characterized by perfect risk sharing. In the following proposition, we summarize how public and private information affect the condition that render perfect risk sharing constrained feasible and thus efficient.

Proposition 2 (Perfect Risk Sharing) *Consider efficient allocations.*

1. *There exists a unique cutoff point, $0 < \bar{\beta}(\kappa, \nu) < 1$, such that for any discount factor $1 > \beta \geq \bar{\beta}(\kappa, \nu)$ the optimal allocation for any signal precision is perfect risk sharing.*

2. *The cutoff point $\bar{\beta}(\kappa, \nu)$ is increasing in the precision of the public and private signal.*

Proof. Let $\bar{V}_{rs} = u(\bar{y})$ be social welfare under perfect risk sharing. First, perfect risk sharing provides the highest ex-ante utility among the consumption-feasible allocations. The existence of $\bar{\beta}(\kappa, \nu)$ follows from monotonicity of participation constraints in β and $\bar{V}_{rs} > V_{out}$. A higher β increases the future value of perfect risk sharing relative to the allocation in the equilibrium without transfers, leaving the current incentives to deviate unaffected. Therefore, if the participation constraints are not binding for $\bar{\beta}(\kappa, \nu)$, they are not binding for any $\beta \geq \bar{\beta}(\kappa, \nu)$. The cutoff point is characterized by the tightest participation constraint (10), i.e., by the participation constraint with the highest value of the outside option. Solving this constraint yields a unique solution for $\bar{\beta}(\kappa, \nu)$ in $(0, 1)$ because $u(\bar{y}) < u(y_h)$. Second, the tightest constraint at the first best allocation is

$$u(\bar{y}) \geq (1-\beta)u(y_h) + \beta(1-\beta)z_1 u(y_h) + \beta(1-\beta)(1-z_1)u(y_l) + \beta^2 V_{out},$$

$z_1 = \kappa\nu/[\kappa\nu + (1-\kappa)(1-\nu)]$. Differentiating the constraint fulfilled with equality with respect to κ using the implicit function theorem gives

$$\frac{\partial \bar{\beta}(\kappa, \nu)}{\partial \kappa} = \frac{\beta(1-\beta)[u(y_h) - u(y_l)]\frac{\partial z_1}{\partial \kappa}}{D} > 0,$$

results in a positive sign for $y_h > y_l$,

$$\frac{\partial z_1}{\partial \kappa} = \frac{\nu(1-\nu)}{[\kappa\nu + (1-\kappa)(1-\nu)]^2} > 0$$

and

$$D = (1-\beta)[u(y_h) - z_1 u(y_h) - (1-z_1)u(y_l)] + \beta[z_1 u(y_h) + (1-z_1)u(y_l) - u(y_l)] > 0.$$

For $\nu \in [0.5, 1)$, the cutoff point increases strictly in precision of the public signal. Similarly, taking the derivative with respect to the ν results in a positive sign as well (a strictly positive sign for $\kappa \in [0.5, 1)$). ■

The cutoff point for perfect risk sharing is determined by the tightest participation constraints which are the ones of high income agents with good public and private signals. The long-term gains from risk sharing can only outweigh the desire to leave the arrangement if agents are sufficiently patient. Furthermore, the value of the outside option at the tightest participation constraint is increasing in the precision of both signals.

Risk sharing in memoryless allocation requires a higher degree of patience than with efficient allocation. In the next proposition, we provide conditions for this case.

Proposition 3 (Autarky with memoryless allocations) *Consider the case when the participation constraints of high-income agents (10) and (11) are binding. Let* $z_1 = \kappa\nu/[\kappa\nu + (1-$

$\kappa)(1-\nu)]$ and $z_2 = (1-\kappa)\nu/[(1-\kappa)\nu + \kappa(1-\nu)]$. If and only if

$$u'(y_h) + \beta\frac{z_1+z_2}{2}[u'(y_h)+u'(y_l)] - \beta u'(y_l) - \frac{\beta^2}{1-\beta}\frac{1}{2}[u'(y_l)-u'(y_h)] \geq 0 \tag{24}$$

autarky is the optimal memoryless arrangement.

Proof. The optimal memoryless arrangement can be analyzed as a fixed-point problem expressed in terms of the period value of the arrangement.

The fixed-point problem is constructed as follows. Let $W = \mathrm{E}\,U(c)$ be the unconditional expected value of an arrangement before any signal has realized. We restrict attention to $W \in [V_{out}, \bar{V}_{rs})$ because per assumption participation constraints for high-income households are binding. The binding participation constraints are given by the following

$$\begin{aligned} u(c_h^h) &+ \frac{\beta\left\{\kappa\nu[u(c_h^h)+u(c_l^h)]/2 + (1-\kappa)(1-\nu)u(2\bar{y}-(c_h^h+c_l^h)/2)\right\}}{\kappa\nu+(1-\kappa)(1-\nu)} \\ &= u(y_h) + \frac{\beta\left[\kappa\nu u(y_h)+(1-\kappa)(1-\nu)u(y_l)\right]}{\kappa\nu+(1-\kappa)(1-\nu)} + \frac{\beta^2}{1-\beta}(V_{out}-W), \end{aligned} \tag{25}$$

$$\begin{aligned} u(c_l^h) &+ \frac{\beta\left\{(1-\kappa)\nu[u(c_h^h)+u(c_l^h)]/2 + \kappa(1-\nu)u(2\bar{y}-(c_h^h+c_l^h)/2)\right\}}{(1-\kappa)\nu+\kappa(1-\nu)} \\ &= u(y_h) + \frac{\beta\left[(1-\kappa)\nu u(y_h)+\kappa(1-\nu)u(y_l)\right]}{(1-\kappa)\nu+\kappa(1-\nu)} + \frac{\beta^2}{1-\beta}(V_{out}-W), \end{aligned} \tag{26}$$

and resource feasibility is used. The objective function of the problem to compute the optimal memoryless arrangement is given by the following expression

$$V_{rs}(W) \equiv \frac{1}{4}\left[u(c_h^h(W)) + u(c_l^h(W)) + 2u(2\bar{y} - (c_h^h(W)+c_l^h(W))/2)\right].$$

The optimal memoryless arrangement should necessary solve the fixed-point problem $W = V_{rs}(W)$. We will show that $V_{rs}(W)$ is strictly increasing. $V(W)$ is also strictly concave, therefore there exist at most two solutions to the fixed-point problem.

From the participation constraints (25) and (26), the derivative of $V(W)$ is given by

$$V_{rs}'(W) = \frac{1}{4}\left[(u'(c_h^h) - u'(c_l))\frac{\partial c_h^h}{\partial W} + (u'(c_l^h) - u'(c_l))\frac{\partial c_l^h}{\partial W}\right]$$

which is strictly increasing in W because perfect risk sharing is not constrained feasible which implies that $\partial c_h^h/\partial W$ and $\partial c_l^h/\partial W$ are negative and $c_h^h, c_l^h \neq \bar{y}$.

By construction, one solution to the fixed-point problem is V_{out}. The concavity of $V_{rs}(W)$ implies that the derivative of $V_{rs}(W)$ at V_{out} is higher than at any partial risk-sharing allocation. Therefore, autarky is the optimal memoryless arrangement if the derivative of $V'_{rs}(w)$ at V_{out} must be smaller than or equal to 1 which implies

$$V'_{rs}(W) = \frac{1}{4}\left[(u'(y_h) - u'(y_l)\right]\left(\frac{\partial c_h^h}{\partial W} + \frac{\partial c_l^h}{\partial W}\right)\Bigg|_{\{c_i^j\}=\{y_j\}} \leq 1$$

The two derivatives are

$$\frac{\partial c_h^h}{\partial W} = -\frac{\begin{vmatrix} \beta^2 & P_{c_l^h} \\ \beta^2 & Q_{c_l^h} \end{vmatrix}}{\begin{vmatrix} P_{c_h^h} & P_{c_l^h} \\ Q_{c_h^h} & Q_{c_l^h} \end{vmatrix}}, \quad \frac{\partial c_l^h}{\partial W} = -\frac{\begin{vmatrix} P_{c_h^h} & \beta^2 \\ Q_{c_h^h} & \beta^2 \end{vmatrix}}{\begin{vmatrix} P_{c_h^h} & P_{c_l^h} \\ Q_{c_h^h} & Q_{c_l^h} \end{vmatrix}},$$

with the auxiliary derivatives P, Q as the partial derivatives of the binding participation constraints (25) and (26) evaluated at the autarky allocation given by

$$\begin{aligned}
P_{c_h^h} &= (1-\beta)[u'(y_h) + \frac{\beta}{2}z_1 u'(y_h) - \frac{\beta}{2}(1-z_1)u'(y_l)] \\
P_{c_l^h} &= P_{c_h^h} - (1-\beta)u'(y_h) \\
Q_{c_l^h} &= (1-\beta)[u'(y_h) + \frac{\beta}{2}z_2 u'(y_h) - \frac{\beta}{2}(1-z_2)u'(y_l)] \\
Q_{c_h^h} &= Q_{c_l^h} - (1-\beta)u'(y_h).
\end{aligned}$$

Using these expression, the sum of the partial derivatives with respect to W evaluated at $\{c_i^j\} = \{y_j\}$ is given by

$$\begin{aligned}
\left(\frac{\partial c_h^h}{\partial W} + \frac{\partial c_l^h}{\partial W}\right) &= -\frac{2\beta^2(1-\beta)u'(y_h)}{(1-\beta)u'(y_h)[P_{c_h^h} + Q_{c_l^h} - (1-\beta)u'(y_h)]} \\
&= \frac{-4\beta}{(1-\beta)\left[u'(y_h)\left(z_1 + \frac{2}{\beta} + z_2\right) - (2 - z_1 - z_2)u'(y_l)\right]}.
\end{aligned}$$

Using this expression in $V'_{rs}(W)$ and collecting terms eventually results in

$$u'(y_h) + \beta\frac{z_1+z_2}{2}[u'(y_h)+u'(y_l)] - \beta u'(y_l) - \frac{\beta^2}{1-\beta}\frac{1}{2}[u'(y_l)-u'(y_h)] \geq 0.$$

Under this condition, the optimal memoryless arrangement is the outside option. If the condition is strictly negative then there exists an alternative constrained feasible allocation that is preferable to the outside option. This result is used in the main theorem.

This condition can be related to the corresponding condition in case of uninformative signals ($\kappa = \nu = 0.5$). In this case, $z_1 = z_2 = 0.5$, and the condition reads

$$\begin{aligned}
&u'(y_h)\left[(1-\beta)+\beta\frac{1-\beta}{2}+\frac{\beta^2}{2}\right] - u'(y_l)\beta\left[1-\beta-\frac{1-\beta}{2}+\frac{\beta}{2}\right] \geq 0 \\
&\Leftrightarrow u'(y_h)\left(1-\frac{\beta}{2}\right) - u'(y_l)\frac{\beta}{2} \geq 0 \\
&u'(y_h) \geq \frac{\beta}{2-\beta}u'(y_l),
\end{aligned}$$

which corresponds to the condition derived in Krueger and Perri (2011) and in Lepetyuk and Stoltenberg (2013). From the other end, suppose that autarky is the optimal memoryless arrangement and that participation constraints (10) and (11) are binding. Then, the value of this arrangement $W_{out} = V_{out}$ must be a solution to the fixed-point problem. This requires that the slope of $V_{rs}(W)$ at $\{c_i^j\} = \{y_j\}$ must be necessarily smaller than or equal to unity. Otherwise, due to the concavity of $V_{rs}(W)$, there exists another solution to the fixed-point problem with an allocation that Pareto dominates the autarky allocation. ■

The more patient agents are the more restrictive is the condition stated in the proposition. The effect of information is source dependent. Thereby, the terms $1-z_1$ and $1-z_2$ as the weights to the low-income state in the next period capture the relevance of insurance for high-income agents. An increase in precision of the private signal decreases the importance of insurance by increasing z_1 and z_2, making it more likely that autarky is the only constrained feasible arrangement. The effect of an increase in public information precision leads to an increase in the importance of insurance as the sum of z_1 and z_2 decreases whenever private information is informative. The increase in importance of insurance makes it less likely that autarky is the optimal memoryless arrangement.

A.4 Existence of risk sharing with non-contractible private information

Optimal allocations may feature either perfect risk sharing (all agents consume $\bar{y}$ in all states), no insurance against income risk (autarky, all agents consume their income in all states) or partial risk sharing. The condition for existence of perfect risk sharing is analyzed in Proposition 2 in Appendix A.3. The conditions for the latter two cases are analyzed in the following.

In the standard case without signals, Krueger and Perri (2011) provide conditions for the existence of allocations that are different from autarky. In particular, they show that for

$$\beta \geq \frac{u'(y_h)}{u'(y_l)}$$

allocations with risk sharing exist. In the following proposition, we generalize the standard case by accounting for non-contractible private information.

Proposition 4 *Consider uninformative public signals ($\kappa = 1/2$) and informative private signals ($\nu \geq 1/2$). An optimal allocation with risk sharing exists for*

$$\beta \geq \frac{u'(y_h)}{u'(y_l)} A(\beta, \nu),$$

with

$$A(\beta, \nu) \equiv \frac{2(1-\beta)(1+\nu\beta)+\beta^2}{(1-\beta)[2\beta\nu+4(1-\nu)]+\beta^2} \geq 1,$$

and

$$\frac{\partial A(\beta, \nu)}{\partial \nu} > 0.$$

The proof in provided in Appendix A.5.

As summarized in the following corollary, when there is only public information the condition for the existence of risk sharing resembles the standard condition in the absence of signals.

Corollary 1 *Consider informative public signals ($\kappa \geq 1/2$) and uninformative private signals ($\nu = 1/2$). An efficient allocation with risk sharing exists for $\beta \geq u'(y_h)/u'(y_l)$.*

The corollary follows as a special case of Proposition 5 in Appendix A.6.

The main implications of Proposition 4, Proposition 2 and Corollary 1 are illustrated in Figure 6. Compared to public signals, non-contractible private signals require a higher degree of patience to support partial risk sharing: in region $\mathcal{R}_2$, there exists risk sharing with public

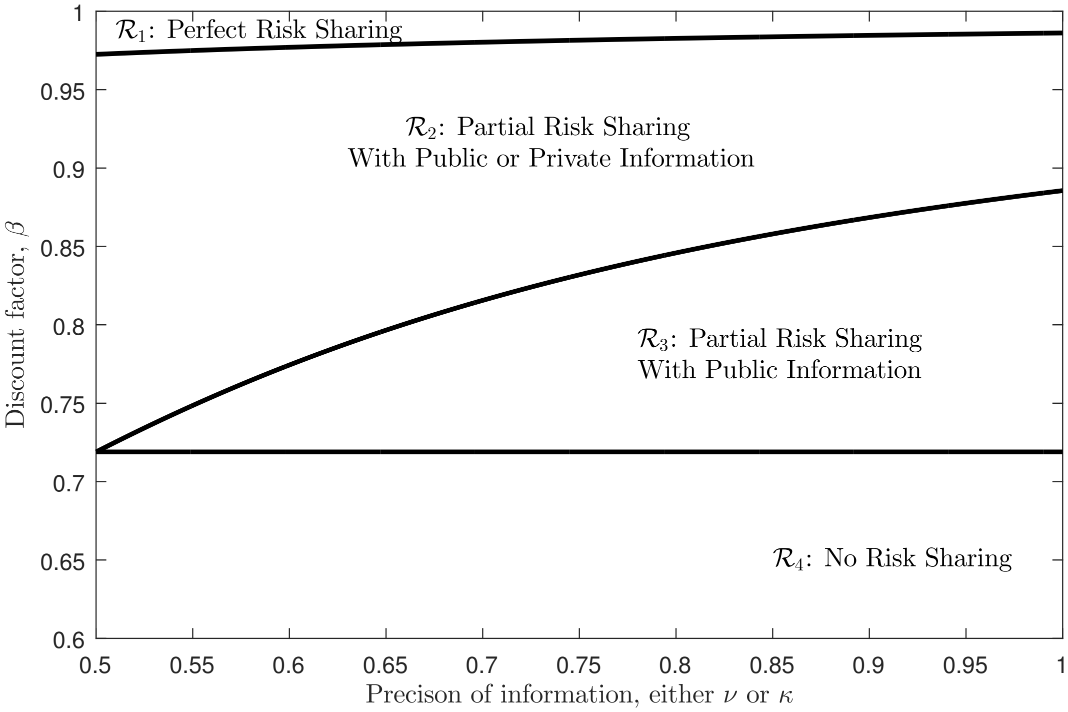

Figure 6: Existence of risk sharing with public and private information.

or private information, while in region $\mathcal{R}_3$ with lower β values, risk sharing exists only with public signals. Further, the term $A(\beta, \nu)$ increases in the precision of private information such that more precise private signals require a higher discount factor to allow for risk sharing. For public signals, the risk sharing region $\mathcal{R}_3$ is independent from signal precision and corresponds to the region in the absence of information. Full risk sharing in region $\mathcal{R}_1$ depends only on the participation incentives of the households with the most valuable outside option – agents with a high income and a high signal – which is identical for public and private information. The outside option value of these agents is increasing in signal precision and a higher degree of patience is required to make perfect risk sharing consistent with the enforcement constraints of these agents. In the following, we characterize how public and private information affect risk sharing.

A.5 Proof of Proposition 4

Let $\pi(y_h) = \pi(n_h) = 1/2$. We construct an alternative constrained-feasible distribution $\hat{\Phi}$ with a one-period income history that dominates the stationary distribution $\Phi^{Aut}(\{U^{Aut}(y), y\}) = q(y)$. We consider a transfer $\delta > 0$ from all agents with high income of measure $1/2$ to agents that had a high income in the previous period but a low income in the current period. Resource feasibility requires that these agents of measure $1/4$ receive 2δ. Agents that receive y_l for two or more times in a row consume y_l and are of measure 1/4. The continuation values for the alternative distribution are defined as follows

$$\{\tilde{w}(y_h, n_h), \tilde{w}(y_h, n_l), \tilde{w}((y_h, y_l), n_h), \tilde{w}((y_h, y_l), n_l), \tilde{w}((y_l, y_l), n_h), \tilde{w}((y_l, y_l), n_l)\}$$

with the following convention: $\tilde{w}(y_h, n_j)$ is the expected utility of an agent whose current income is high, with signal n_j, $\tilde{w}((y_h, y_l), n_i)$ is the continuation value of an agent with current income low and previous income high. The transfer scheme leads to the following individual continuation values:

$$\begin{aligned}
\tilde{w}(y_h, n_h) &= (1-\beta) u(y_h - \delta) + \beta [\nu \tilde{w}(y_h) + (1-\nu) \tilde{w}(y_h, y_l)] \\
\tilde{w}(y_h, n_l) &= (1-\beta) u(y_h - \delta) + \beta [(1-\nu) \tilde{w}(y_h) + \nu \tilde{w}(y_h, y_l)] \\
\tilde{w}((y_h, y_l), n_h) &= (1-\beta) u(y_l + 2\delta) + \beta [\nu \tilde{w}(y_h) + (1-\nu) \tilde{w}(y_l, y_l)] \\
\tilde{w}((y_h, y_l), n_l) &= (1-\beta) u(y_l + 2\delta) + \beta [(1-\nu) \tilde{w}(y_h) + \nu \tilde{w}(y_l, y_l)] \\
\tilde{w}((y_l, y_l), n_h) &= (1-\beta) u(y_l) + \beta [\nu \tilde{w}(y_h) + (1-\nu) \tilde{w}(y_l, y_l)] \\
\tilde{w}((y_l, y_l), n_l) &= (1-\beta) u(y_l) + \beta [(1-\nu) \tilde{w}(y_h) + \nu \tilde{w}(y_l, y_l)],
\end{aligned}$$

with averages defined as

$$\begin{aligned}
\tilde{w}(y_h) &= \frac{1}{2} (\tilde{w}(y_h, n_h) + \tilde{w}(y_h, n_l)) \\
\tilde{w}(y_h, y_l) &= \frac{1}{2} (\tilde{w}((y_h, y_l), n_h) + \tilde{w}((y_h, y_l), n_l)) \\
\tilde{w}(y_l, y_l) &= \frac{1}{2} (\tilde{w}((y_l, y_l), n_h) + \tilde{w}((y_l, y_l), n_l))
\end{aligned}$$

and the average outside options are

$$\begin{aligned}
U^{Aut}(y_h) &= \frac{1}{2} \left(U^{Aut}(y_h, n_h) + U^{Aut}(y_h, n_l) \right) \\
U^{Aut}(y_l) &= \frac{1}{2} \left(U^{Aut}(y_l, n_h) + U^{Aut}(y_l, n_l) \right).
\end{aligned}$$

Private information is not contractible, thus, not both participation constraints can be binding for high-income agents. This implies that high-income agents gain on average compared to autarky.

$$\tilde{w}(y_h) - U^{Aut}(y_h) = \varepsilon \geq 0.$$

For agents with at least two low income realizations in a row, it follows

$$\tilde{w}\left(y_l, y_l\right) - U^{Aut}\left(y_l\right) = \frac{\beta\varepsilon}{2-\beta}.$$

For $\tilde{w}\left(y_h, y_l\right) - U^{Aut}\left(y_l\right)$, we get

$$\tilde{w}\left(y_h, y_l\right) - U^{Aut}\left(y_l\right) = \left(1-\beta\right)\left(u\left(y_l + 2\delta\right) - u\left(y_l\right)\right) + \frac{\beta}{2}\left(\tilde{w}\left(y_h\right) - U^{Aut}\left(y_h\right) + \tilde{w}\left(y_l, y_l\right) - U^{Aut}\left(y_l\right)\right)$$

which implies

$$\tilde{w}\left(y_h, y_l\right) - U^{Aut}\left(y_l\right) = 2\left(1-\beta\right)u'\left(y_l\right)\delta + \frac{\beta}{2}\left(\varepsilon + \frac{\beta\varepsilon}{2-\beta}\right) = 2\left(1-\beta\right)u'\left(y_l\right)\delta + \frac{\beta\varepsilon}{2-\beta}.$$

Thus, we are looking for δ that solves the following equation:

$$\tilde{w}\left(y_h\right) - U^{Aut}\left(y_h\right) = \varepsilon = -\left(1-\beta\right)u'\left(y_h\right)\delta + \frac{\beta}{2}\left[\varepsilon + 2\left(1-\beta\right)u'\left(y_l\right)\delta + \frac{\beta\varepsilon}{2-\beta}\right]$$

Such a transfer is given by:

$$\delta = \frac{\varepsilon - \frac{\beta\varepsilon}{2}\left(1 + \frac{\beta}{2-\beta}\right)}{\left(1-\beta\right)\left(\beta u'\left(y_l\right) - u'\left(y_h\right)\right)} = \frac{2\varepsilon}{\left(2-\beta\right)\left(\beta u'\left(y_l\right) - u'\left(y_h\right)\right)} \geq 0$$

Consistency with participation incentives of high-income agents with a high private signal requires

$$\tilde{w}\left(y_h, n_h\right) - U^{Aut}\left(y_h, n_h\right) = -\left(1-\beta\right)u'\left(y_h\right)\delta + \beta\left(\nu\varepsilon + \left(1-\nu\right)\left[2\left(1-\beta\right)u'\left(y_l\right)\delta + \frac{\beta\varepsilon}{2-\beta}\right]\right) \geq 0$$

which after substituting for δ results in

$$\frac{2\beta\nu(1-\beta) + \beta^2}{2(1-\beta)} \geq \frac{u'\left(y_h\right) - 2\beta\left(1-\nu\right)u'\left(y_l\right)}{\beta u'\left(y_l\right) - u'\left(y_h\right)}$$

Together with the necessary condition $\beta > \frac{u'(y_h)}{u'(y_l)}$ defines the cut-off β

$$\beta \geq \frac{u'(y_h)}{u'(y_l)}A(\beta, \nu)$$

with

$$A(\beta,\nu) \equiv \frac{2(1-\beta)(1+\nu\beta)+\beta^2}{(1-\beta)(2\beta\nu+4(1-\nu))+\beta^2}.$$

This expression is larger than one and increases in ν. For $\beta \geq u'(y_h)A(\beta,\nu)/u'(y_l)$, participation constraints of high-income agents with a low private signals are slack.

A.6 Enforcement constraints on future promises

The recursive problem that we describe in Section 2 imposes the enforcement constraints in the current period rather than in the future as in Krueger and Perri (2011). In this section, we argue that while the difference in the timing of participation constraints is irrelevant in the absence of non-contractible private information it matters with informative private signals. In particular, imposing the enforcement constraints on future promises requires a higher discount factor β to support risk sharing than the patience implicitly summarized in Proposition 4.

With enforcement constraints imposed on future promises, the modified recursive problem can be summarized as follows

$$V(w,s) = \min_{h,\{w'(s')\}} \left[\left(1-\frac{1}{R}\right) C(h) + \frac{1}{R}\sum_{s'} \pi(s'|s) V\left(w'(s'),s\right) \right] \tag{27}$$

subject to promise keeping and enforcement constraints

$$w = (1-\beta)h + \beta \sum_{s'} \pi(s'|s) w'(s') \tag{28}$$

$$w'\left(s'\right) \geq U^{Aut}\left(s',n'\right), \forall s',n'. \tag{29}$$

We refer to the stationary allocations induced by this recursive as modified history-dependent allocations. First, the promise keeping constraint in the modified recursive problem resembles the promise keeping constraint (14). The modified recursive however imposes tighter participation constraints than the recursive problem that we specify in the main text. To see this, shift the promise keeping constraint (14) and the participation constraint (15) forward one period, combining them produces the following constraint on the continuation values that ensures the next period participation constraints can be satisfied:

$$w'\left(s'\right) \geq \sum_{n'} \pi\left(n'\right) U^{Aut}\left(s',n'\right).$$

The enforcement constraint in the modified problem (29) implies that

$$w'\left(s'\right) \geq \max_{n'} U^{Aut}\left(s', n'\right)$$

because private information is non-contractible. Unless private information is absent, the modified recursive problem is more constrained. The condition for the existence of risk sharing is summarized in the following proposition (for i.i.d. income).

Proposition 5 (Modified history dependent arrangements) *A modified history dependent arrangement with risk sharing exists if*

$$\beta \geq \left[\frac{u'(y_h)}{u'(y_l)}\frac{1}{2-z_1-z_2}\right].$$

The proof follows similar steps as in Krueger and Perri (2011). The main implication from the proposition is that the modified recursive problem requires always a higher degree of patience to support risk sharing when private information is informative than in case of optimal allocations as summarized in Proposition 4 in Section A.4.

Proof. We construct an alternative distribution $\hat{\Phi}$ that dominates the stationary distribution $\Phi^{Aut}(\{U^{Aut}(y), y\}) = q(y)$ and show that the distribution is resource-feasible if the condition of the proposition is satisfied. Let $\hat{\Phi}$ be the following distribution over utility promises, income and public signals with a one-period history

$$\begin{aligned}
&\hat{\Phi}(\{U^{Aut}(y_h, k_h, n_h), y_h\}) = \pi_h \pi(k_h), \quad \hat{\Phi}(\{U^{Aut}(y_h, k_l, n_h), y_h\}) = \pi_h \pi(k_l), \\
&\hat{\Phi}(\{U^{Aut}(y_l, k_h, n_h), y_l\}) = (1-\pi_h)(1-\pi_h)\pi(k_h), \\
&\hat{\Phi}(\{U^{Aut}(y_l, k_l, n_h), y_l\}) = (1-\pi_h)(1-\pi_h)\pi(k_l), \\
&\hat{\Phi}(\{\tilde{\omega}_{k_h}^{k_h}, y_l\}) = (1-\pi_h)\pi_h\pi(k_h)\pi(k_h), \quad \hat{\Phi}(\{\tilde{\omega}_{k_l}^{k_h}, y_l\}) = (1-\pi_h)\pi_h\pi(k_h)\pi(k_l), \\
&\hat{\Phi}(\{\tilde{\omega}_{k_h}^{k_l}, y_l\}) = (1-\pi_h)\pi_h\pi(k_l)\pi(k_h), \quad \hat{\Phi}(\{\tilde{\omega}_{k_l}^{k_l}, y_l\}) = (1-\pi_h)\pi_h\pi(k_l)\pi(k_l),
\end{aligned}$$

where $\tilde{\omega}_{k_i}^{k_j} = U^{Aut}(y_l, k_i, n_h) + \varepsilon_{k_i}^{k_j}$ for small $\varepsilon_{k_i}^{k_j}$ and and the upper (lower) index indicates the previous (current) period signal. In the following, we consider two equally likely income state such that $\pi_h = \pi(k) = 0.5$. Let $\{\delta_{ij}^m\}$, $i, j, m \in \{l, h\}$ be transfers in terms of utility, with the first lower index as current income, the second index as the current public signal and the upper index as previous period public signal. The transfers are implicitly defined for the low-income agents by

$$\tilde{\omega}_{k_h}^{k_h} = (1-\beta)(u(y_l) + \delta_{lh}^h) + \beta\left[(1-z_1)U^{Aut}(y_l) + z_1 U^{Aut}(y_h)\right]$$
$$\tilde{\omega}_{k_h}^{k_l} = (1-\beta)(u(y_l) + \delta_{lh}^l) + \beta\left[(1-z_1)U^{Aut}(y_l) + z_1 U^{Aut}(y_h)\right]$$
$$\tilde{\omega}_{k_l}^{k_h} = (1-\beta)(u(y_l) + \delta_{ll}^h) + \beta\left[(1-z_2)U^{Aut}(y_l) + z_2 U^{Aut}(y_h)\right]$$
$$\tilde{\omega}_{k_l}^{k_l} = (1-\beta)(u(y_l) + \delta_{ll}^l) + \beta\left[(1-z_2)U^{Aut}(y_l) + z_2 U^{Aut}(y_h)\right].$$

Utility of high-income are equal to their outside options which do not depend on previous period signals which leads to $\delta_{hh}^h = \delta_{hh}^l = \delta_{hh}$ and $\delta_{hl}^h = \delta_{hl}^l = \delta_{hl}$. Transfers of high-income agents are implicitly defined by

$$U^{Aut}(y_h, k_h, n_h) = (1-\beta)(u(y_h) - \delta_{hh}) + \beta\left[(1-z_1)\frac{\tilde{\omega}_{k_l}^{k_h} + \tilde{\omega}_{k_h}^{k_h}}{2} + z_1 U^{Aut}(y_h)\right]$$
$$U^{Aut}(y_h, k_l, n_h) = (1-\beta)(u(y_h) - \delta_{hl}) + \beta\left[(1-z_2)\frac{\tilde{\omega}_{k_l}^{k_l} + \tilde{\omega}_{k_h}^{k_l}}{2} + z_2 U^{Aut}(y_h)\right],$$

where $U^{Aut}(y_h) = 0.5[U^{Aut}(y_h, k_h, n_h) + U^{Aut}(y_h, k_l, n_h)]$, and $U^{Aut}(y_l)$ defined, accordingly. The marginal utility of low-income agents before the transfer is identical for each combination of past and current public signal and low-income agents receive the same transfer, $\delta_{li}^j = \epsilon/[4(1-\beta)]$ for all i, j. Transfers of high-income agents can be then directly derived from binding participation constraints and the scheme can be summarized by

$$\delta_l \equiv \sum_{i,j} \delta_{li}^j = \frac{\epsilon}{1-\beta} \qquad \delta_{hh} = \beta\epsilon\frac{(1-z_1)}{4(1-\beta)} \qquad \delta_{hl} = \beta\epsilon\frac{(1-z_2)}{4(1-\beta)}.$$

The distribution $\hat{\Phi}$ requires the following increase in resources

$$\begin{aligned}\Upsilon &= \pi_h(1-\pi_h)c'(u(y_l))\delta_l/4 - \frac{\pi_h}{2}c'(u(y_h))(\delta_{hh} + \delta_{hl}) \\ &= \frac{\pi_h(1-\pi_h)\epsilon}{4(1-\beta)}\left[\frac{1}{u'(y_l)} - \frac{\beta(2-z_1-z_2)}{2(1-\pi_h)u'(y_h)}\right].\end{aligned}$$

Solving for β, if

$$\beta \geq \left[\frac{(1-\pi_h)u'(y_h)}{u'(y_l)}\frac{2}{2-z_1-z_2}\right] = \left[\frac{u'(y_h)}{u'(y_l)}\frac{1}{2-z_1-z_2}\right],$$

the constructed allocation $\hat{\Phi}$ uses less resources and dominates Φ^{Aut} by making the low-income agents strictly better off. ■

A.7 Transition laws

There are four transition laws of interest, namely:

- $\pi(s'|s) = \pi(y', k'|y, k)$,
- $\pi(s'|s, n) = \pi(y', k'|y, k, n)$,
- $\pi(y'|s, n) = \pi(y'|y, k, n)$
- $\pi(\theta'|\theta) = \pi(y', k', n'|y, k, n)$.

The conditional probability for income is

$$\pi\left(y' = y_j | k = y_m, n = y_l, y = y_i\right) = \frac{\pi\left(y' = y_j, k = y_m, n = y_l, y = y_i\right)}{\pi\left(k = y_m, n = y_l, y = y_i\right)}.$$

The denominator can be derived by using the following identity

$$\sum_{z=1}^{K} \pi\left(y' = y_z | k = y_m, n = y_l, y = y_i\right) = 1$$

which implies

$$\pi\left(k = y_m, n = y_l, y = y_i\right) = \sum_{z=1}^{K} \pi\left(y' = y_z, k = y_m, n = y_l, y = y_i\right).$$

The elements of the sum on the right hand side are products that depend on z, m, l, i and the precisions of signals (we treat the current income simply as a yet another signal). It follows

$$\pi\left(y' = y_z, k = y_m, n = y_l, y = y_i\right) = p_{iz}\kappa^{\mathbf{1}_{m=z}} \left(\frac{1-\kappa}{K-1}\right)^{1-\mathbf{1}_{m=z}} \nu^{\mathbf{1}_{l=z}} \left(\frac{1-\nu}{K-1}\right)^{1-\mathbf{1}_{l=z}},$$

where p_{iz} is the Markov transition probability for moving from income i to income z. When the signal is wrong, $m \neq Z$, we assume that all income states are equally likely. The general formula for the conditional expectations reads:

$$\pi\left(y' = y_j | k = y_m, n = y_l, y = y_i\right) = \frac{p_{ij}\kappa^{\mathbf{1}_{m=j}} \left(\frac{1-\kappa}{K-1}\right)^{1-\mathbf{1}_{m=j}} \nu^{\mathbf{1}_{l=j}} \left(\frac{1-\nu}{K-1}\right)^{1-\mathbf{1}_{l=j}}}{\sum_{z=1}^{K} p_{iz}\kappa^{\mathbf{1}_{m=z}} \left(\frac{1-\kappa}{K-1}\right)^{1-\mathbf{1}_{m=z}} \nu^{\mathbf{1}_{l=z}} \left(\frac{1-\nu}{K-1}\right)^{1-\mathbf{1}_{l=z}}}. \quad (30)$$

With both signals following independent exogenous processes, the conditional probability for the full state is for all k', n'

$$\pi\left(y' = y_j, k', n' | k = y_m, n = y_l, y = y_i\right) = \pi(k'|k = y_m)\pi(n'|n = y_l) \frac{p_{ij}\kappa^{\mathbf{1}_{m=j}} \left(\frac{1-\kappa}{K-1}\right)^{1-\mathbf{1}_{m=j}} \nu^{\mathbf{1}_{l=j}} \left(\frac{1-\nu}{K-1}\right)^{1-\mathbf{1}_{l=j}}}{\sum_{z=1}^{K} p_{iz}\kappa^{\mathbf{1}_{m=z}} \left(\frac{1-\kappa}{K-1}\right)^{1-\mathbf{1}_{m=z}} \nu^{\mathbf{1}_{l=z}} \left(\frac{1-\nu}{K-1}\right)^{1-\mathbf{1}_{l=z}}}. \quad (31)$$

The transition probabilities for the signals follow the corresponding transition probabilities for income, i.e., $\pi(k'|k = y_m) = \pi(y'|y_m)$ and vise versa for $\pi(n'|n = y_l)$. Finally, the formula for $\pi(y', k'|k, y)$ naturally follows and is given for all k' by

$$\pi\left(y' = y_j, k' | k = y_m, n = y_l, y = y_i\right) = \pi(y'|y = y_m) \frac{p_{ij}\kappa^{\mathbf{1}_{m=j}} \left(\frac{1-\kappa}{K-1}\right)^{1-\mathbf{1}_{m=j}} \nu^{\mathbf{1}_{l=j}} \left(\frac{1-\nu}{K-1}\right)^{1-\mathbf{1}_{l=j}}}{\sum_{z=1}^{K} p_{iz}\kappa^{\mathbf{1}_{m=z}} \left(\frac{1-\kappa}{K-1}\right)^{1-\mathbf{1}_{m=z}} \nu^{\mathbf{1}_{l=z}} \left(\frac{1-\nu}{K-1}\right)^{1-\mathbf{1}_{l=z}}}. \quad (32)$$

For example, with two equally likely i.i.d. income states, the conditional probability of receiving a low income in the future conditional on a low private high, a high public signal and a low income today is given according to (30)

$$\pi\left(y' = y_l | k = y_h, n = y_l, y = y_l\right) = \pi\left(y' = y_l | k = y_h, n = y_l\right) = \frac{(1-\kappa)\nu}{(1-\kappa)\nu + (1-\nu)\kappa} \quad .$$

A.8 Computing the outside options

Here we provide detailed derivations of how the values of the outside options $U^{Aut}(y, k, n)$ were computed. Let P be the transition matrix for income with elements $\pi(y'|y)$. Scrapping the time index we have:

$$U^{Aut}(y, k, n) = (1-\beta)u(y) + \beta(1-\beta)\sum_{y'} \pi(y'|k, n, y)u(y') + \beta^2(1-\beta)\sum_{y''} \pi(y''|y, k, n)u(y'') + \beta^3(1-\beta)\sum_{y'''} \pi(y'''|y, k, n)u(y''') + ...$$

Now, we need to distinguish two cases depending on income autocorrelation. If income is i.i.d, we have that $\pi(y_n|y, k, n) = \pi(y), n \geq 2$. For an autocorrelated income process we have that

$\pi(y_n|y,k,n) = \pi(y_{n-1}|y,k,n)^T P, n \geq 2$. Let $U^{Aut,i}$ be the outside option under iid income and $U^{Aut,p}$ be the outside option with persistent income. By the virtue of infinite sum for matrices formula we then have the following

$$U^{Aut,i}(y,k,n) = (1-\beta)\,u(y) + \beta(1-\beta)\,\pi(y'|k,n,y)^T u(y') + \beta^2\pi\left(y''\right)^T u\left(y''\right)$$
$$U^{Aut,p}(y,k,n) = (1-\beta)\,u(y) + \beta(1-\beta)\pi(y'|k,n,y)^T\,(I-\beta P)^{-1}\,u\left(y'\right).$$

A.9 Data

We list out countries in the sample partitioned accordingly to the data quality grade. We start with the sample of Kose et al. (2009) and add D-grade quality countries. Then, we drop countries with no data on IMF GDP growth rate forecasts and with population lower than 1 million of inhabitants, we also drop outliers, countries that were found out to misreport their statistical data (e.g. Greece) and countries with less than 3 observations.

1. *Grade A*: Austria, Australia, Belgium, Canada, Finland, France, Ireland, Italy, Japan, the Netherlands, Norway, Sweden, the United Kingdom, the United States

2. *Grade B*: Argentina, Chile, Germany, Israel, Korea, New Zealand, Portugal, Spain, Uruguay

3. *Grade C*: Bolivia, Brasil, China, Ivory Coast, Cameroon, Costa Rica, Ecuador, Egypt, Gabon, Ghana, Guatemala, India, Indonesia, Iran, Jamaica, Jordan, Sri Lanka, Mexico, Mauritius, Pakistan, Peru, Philippines, Paraguay, Senegal, Trinidad Tobago, Tunisia, Turkey, Venezuela, South Africa

4. *Grade D*: Angola, Belarus, Central African Republic, Guinea Bissau, Cambodia, Laos, Lesotho, Mongolia, Mozambique, Namibia, Niger, Sudan, Chad, Togo, Uganda, Uzbekistan, Yemen.

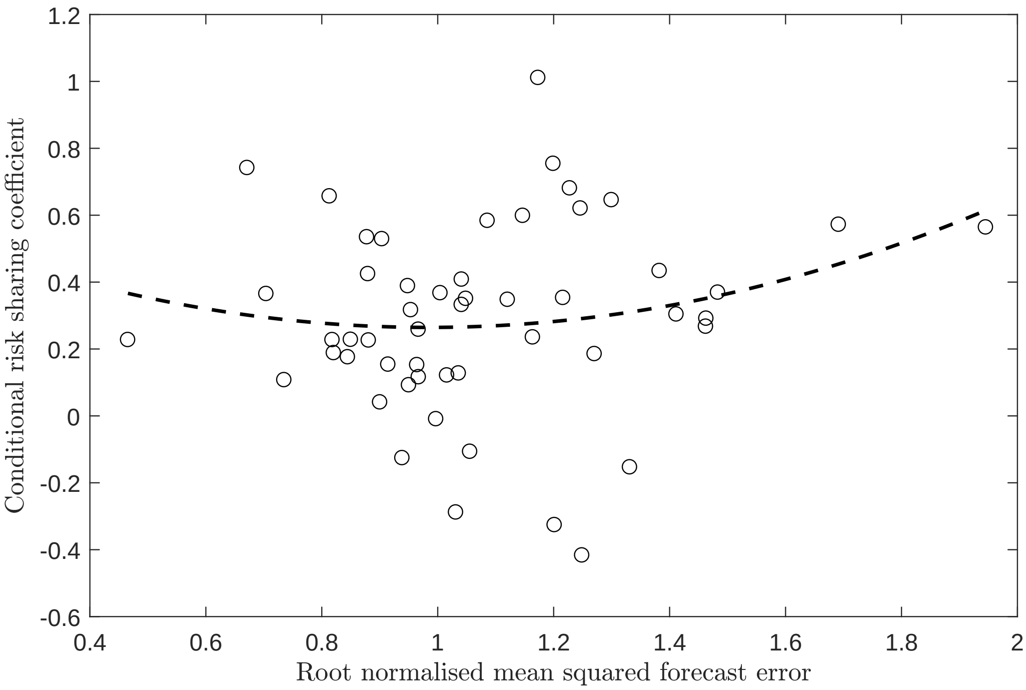

Figure 7: Relationship between public data quality and risk sharing measures.

Table 9: Coefficients of the second-stage regression of the risk sharing coefficient on macroeconomic controls, constant not displayed. Column (4) is the case reported in the main text.

	(1)	(2)	(3)	(4)
average GDP p.c.	0.0818	0.120*	0.103**	0.0947**
	(0.158)	(0.051)	(0.023)	(0.024)
Chinn-Ito index	0.0141			
	(0.746)			
average assets to GDP		-0.0396		
		(0.571)		
average liab. to GDP			-0.0387	
			(0.598)	
Observations	52	52	52	52
R^2	0.100	0.104	0.103	0.098
Adjusted R^2	0.063	0.068	0.067	0.080
F	2.727	2.849	2.825	5.446

p-values in parentheses
$*p < 0.10, **p < 0.05, ***p < 0.010$

Table 10: Test of significance of between-group differences, group-average risk sharing measures regressed on dummies corresponding to low and high public data quality (difference relative to medium quality of data countries). Split of countries according to the Penn World Tables quality grades. Column numbering corresponds to that in Table 9. Column (4) is the case reported in the main text.

	(1)	(2)	(3)	(4)
high quality of public data	0.265**	0.284**	0.284**	0.271**
	(0.028)	(0.018)	(0.018)	(0.025)
low quality of public data	0.244**	0.243**	0.246**	0.244**
	(0.024)	(0.023)	(0.022)	(0.024)
Observations	52	52	52	52
R^2	0.114	0.122	0.124	0.116
Adjusted R^2	0.077	0.087	0.088	0.080
F	3.140	3.416	3.453	3.215

p-values in parentheses
$*p < 0.10, **p < 0.05, ***p < 0.010$

Table 11: Test of significance of between-group differences, group-average risk sharing measures regressed on dummies corresponding to low and high public data quality (difference relative to medium quality of data countries). Split of countries according to terciles of normalised IMF mean squared forecast errors. Column numbering corresponds to that in Table 9.

	(1)	(2)	(3)	(4)
high quality of public data	0.201**	0.211**	0.214**	0.204**
	(0.035)	(0.027)	(0.025)	(0.033)
low quality of public data	0.230**	0.232**	0.232**	0.229**
	(0.014)	(0.013)	(0.013)	(0.014)
Observations	52	52	52	52
R^2	0.135	0.142	0.143	0.135
Adjusted R^2	0.100	0.107	0.108	0.099
F	3.818	4.051	4.089	3.816

p-values in parentheses
$*p < 0.10, **p < 0.05, ***p < 0.010$