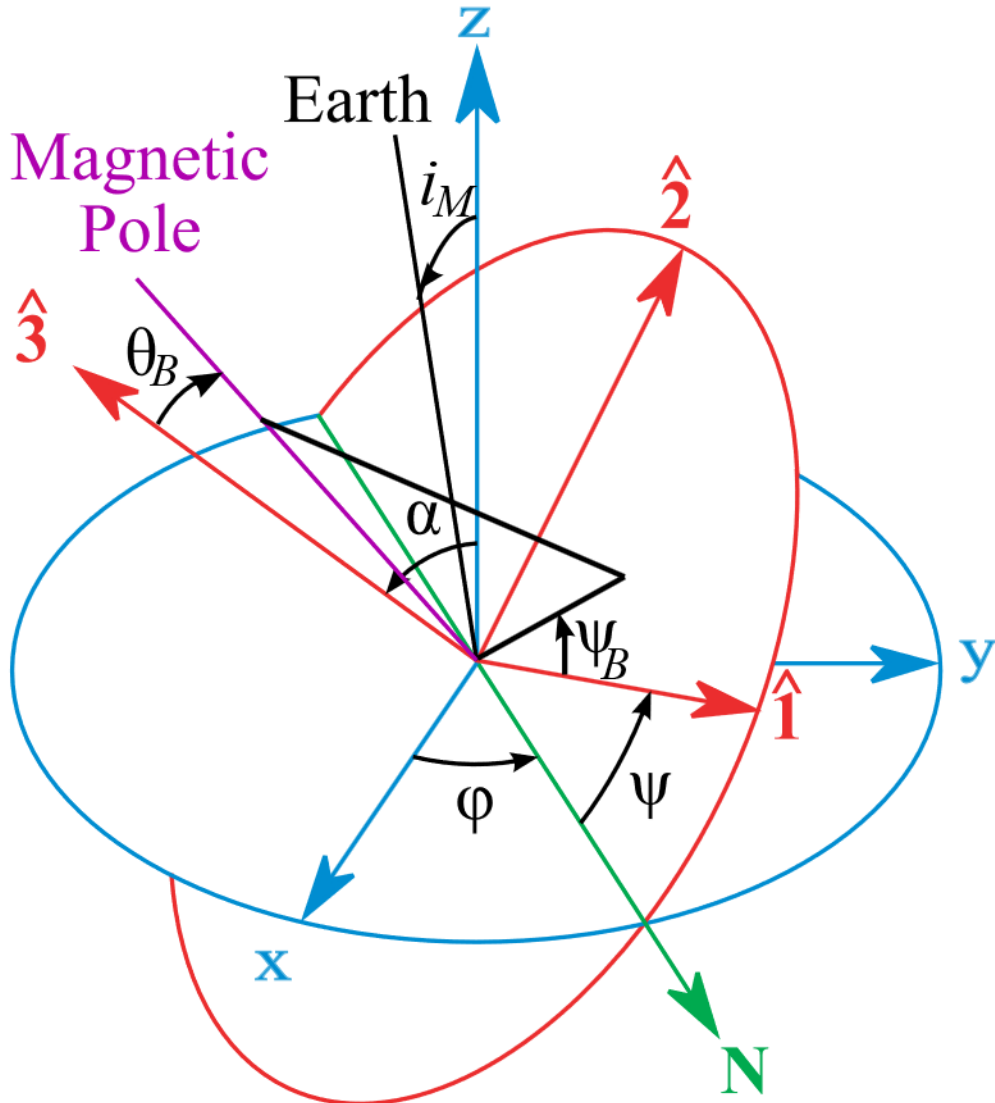
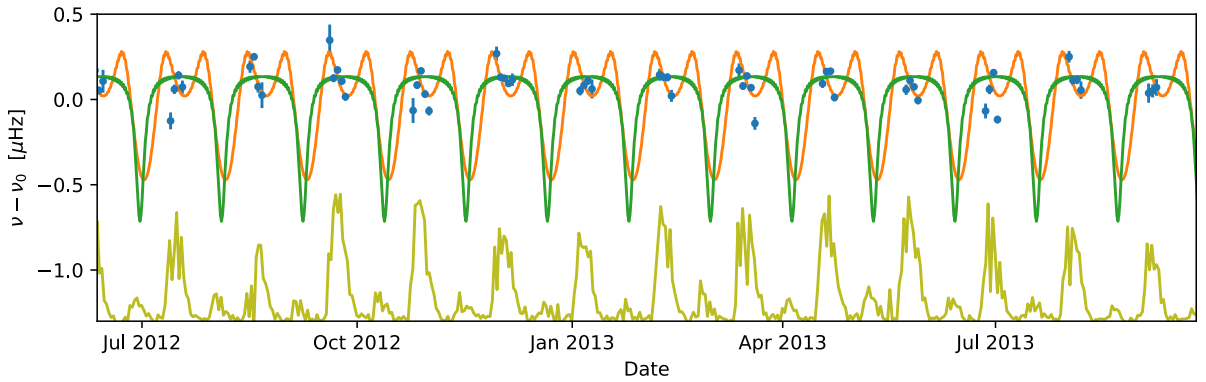

Complex rotational dynamics of the neutron star in Hercules X-1 revealed by X-ray polarization

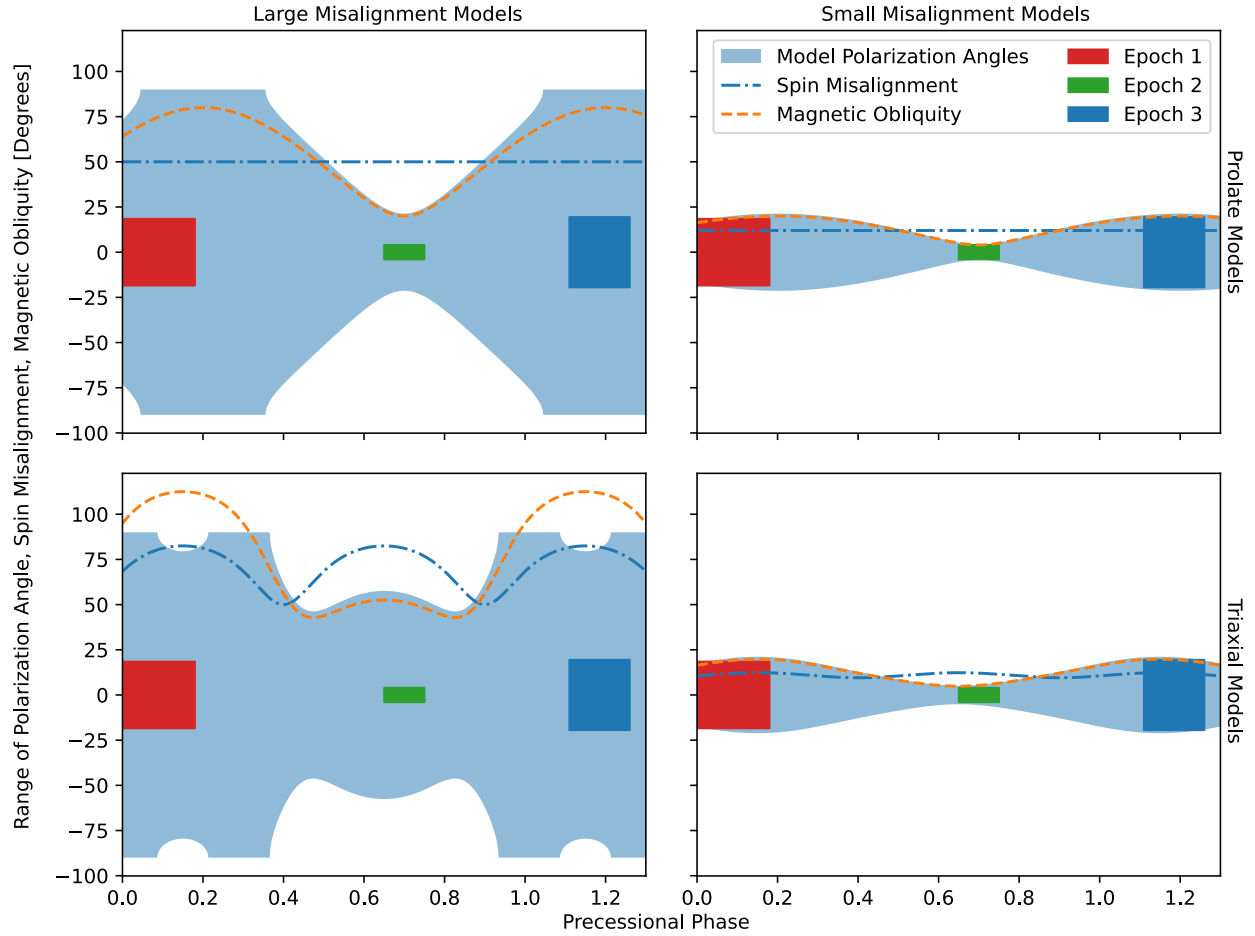
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Supplementary Figure 1: **Euler Angles as defined in the Methods.** The axes $\hat{1}$, $\hat{2}$ and $\hat{3}$ denote the body axes of the neutron star crust, and the axes x , y and z denote a frame of reference with the direction to Earth fixed. The line of sight is indicated by the black line labelled “Earth”, makes an angle i_M with the z -axis and lies in the $x - z$ -plane. The magnetic pole is indicated by the purple line, makes an angle θ_B with the $\hat{3}$ -axis and lies in the $\hat{2} - \hat{3}$ -plane. The vector labeled N denotes the line of nodes about which the rotation by α is performed.



Supplementary Figure 2: **Spin Frequency Deviation.** After the long-term trend of the spin frequency of Hercules X-1 is removed¹⁸ from the Fermi GBM frequency measurements, the variation with superorbital period becomes apparent. The errorbars on the frequency difference are $1-\sigma$. The curves are not a fit to the data, but rather the expectations from the two small-misalignment precession models outlined in the text. The results from the prolate model in Fig. 3 are superimposed in green (single hump), and those from triaxial model are shown in orange (double hump). The X-ray flux as observed by the Swift observatory is depicted by an olive curve along the bottom of the plot.



Supplementary Figure 3: **Four models for the precession.** The extent of the polarization angle swing as a function of precessional phase for the four precession models is depicted by the light-blue bands, and the extent of best-fitting RVM PAs from the three epochs is superimposed in red, green and dark-blue bands from left to right. Top Left: a prolate model¹⁷ with spin misalignment of 50° and magnetic obliquity of 30° . Top Right: a prolate model with spin misalignment of 11° and magnetic obliquity of 8° . Bottom Left: a triaxial model¹⁸ with a magnetic obliquity of 30° . Bottom Right: a triaxial model¹⁸ with a magnetic obliquity of 7.5° . The blue dot-dashed curve traces the spin misalignment angle α , and the orange dashed curve traces θ , the angle between the spin axis and the magnetic axis.