The use of conditionals in argumentation: a proposal for the analysis and evaluation of argumentatively used conditionals

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Logical approaches to conditionals

2.1 Introduction

In the search for a model to analyse and evaluate argumentatively used conditionals, let us start with the Stoics, who were the first to thoroughly discuss the nature of conditionals. Their interest in conditionals probably was prompted by two issues. First of all, they were not so interested in demonstration – the principal topic of Aristotelian syllogistic logic – but more in every day, dialectical arguments.5 One of the prevailing dialectical arguments was of the form ‘if p then q’ and ‘if p then not-q’, therefore ‘not p’, which seems to make an interest in conditionals inevitable (Kneale & Kneale 1984: 128). Secondly, in their logic the validity of an argument was closely connected to the truth of the corresponding conditional. If the premises are taken together as the antecedent and the conclusion as the consequent, a valid inference schema would yield a conditional that is true under all circumstances. In order to explain the difficult notion of validity, one had to explain the notion of the truth of a conditional (which was perhaps just as difficult) (Sanford 1992: 19).

One of the contributors to this discussion was Philo of Megara, pupil of Diodorus Cronos (end of the 4th century BC). His stance on the nature of conditionals is conveyed by Sextus Empiricus in his Pyrrhoneiae Hypotyposes (ii, 110-112):

[1] Philo says that a sound conditional is one that does not begin with a truth and end with a falsehood, e.g. when it is day and I am conversing, the statement ‘If it is day, I am conversing’.

(Kneale & Kneale 1984: 128)

Philo’s definition can be seen as an early representation of the definition of material implication.6 It is referred to as the truth-functional

5 This is not to say that Aristotle failed to discuss hypotheticals altogether. He discerned two types of hypothetical syllogism, *modus ponendo ponens* and *modus tollendo tollens*. His pupil Theophrastus developed Aristotle’s initial ideas.

6 The term ‘material implication’ was first used by Russell and Whitehead in *Principia Mathematica* (1962: 7).
definition, since for establishing the truth of the conditional one only needs to know the truth values of the atomic propositions. A conditional is not true if the antecedent is true and the consequent is false, in all other situations the conditional is true. Or, to use the truth-table notation developed in the beginning of the 20th century:

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<tr>
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<tr>
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<td>2</td>
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<td>3</td>
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<td>4</td>
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The only situation in which the conditional is false, is the situation where the antecedent is true and the consequent is false (the situation depicted in line 2 of the truth-table). In all other situations the conditional is true.

Ever since the truth-functional definition of conditionals was presented, various authors have argued that this definition is problematic. The problems associated with it can be seen as difficulties that arise when the definition of material implication is applied to ordinary language conditionals. The definition does not lack something needed for dealing with the logical properties of conditionals within the framework of propositional logic. On the contrary, this definition renders commonly accepted argument forms like *modus ponens* and *modus tollens* valid.\(^7\) In that sense, it performs exactly as can be expected of it. It is only when this definition is applied to particular instances of if-then-sentences that it yields undesirable or ‘counterintuitive’ results.\(^8\)

The problems that have been associated with the definition of material implication can be divided into two broad categories: problems associated with the definition itself, and problems associated with logical operations that are valid for material implications but render unaccep-

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\(^7\) If the if-then connective is interpreted according to the definition of material implication, and the conjunction of both premises is taken as the antecedent and the conclusion is taken as the consequent of a conditional sentence, one arrives at a conditional that is true under all circumstances. This means it is impossible for the premises to be true while the conclusion is false – the argument is therefore valid.

\(^8\) For instance, Sanford writes (1992: 58): ‘The success of Frege’s programme for propositional logic is uncontroversial. About the suitability of material conditionals for representing conditionals in ordinary language, on the other hand, controversies have raged.’
table results when applied to (some) ordinary language conditionals. In the first part of this chapter I will describe these problems in detail. The second part will describe solutions developed by various modern logical approaches to conditionals.

2.2

Problems associated with the definition of material implication

To start with problems associated with the definition itself, let us look at the first line of the truth-table. According to the material implication definition of conditionals, a conditional is true when both the antecedent and the consequent are true. If this interpretation of conditionals is correct, the conditional in 5 is true:

5  If 2 times 2 is 4, then Paris is the capital of France.

The capital of France is Paris, 2 times 2 is 4, therefore both the antecedent and the consequent are true. But it is doubtful whether speakers not trained in propositional logic would find this conditional acceptable, since the antecedent and the consequent seem to be unrelated. Apart from being sound in a truth-functional sense, ordinary language conditionals seemingly have to possess another property: between the antecedent and the consequent there must be some sort of connection.

A similar difficulty seems to arise when we look at the 3rd and 4th line of the truth-table. Both 6 and 7 are true according to the truth-functional definition of conditionals, simply because the antecedent is false:

6  If Philo of Megara had died at the age of one, he would have been the first to formulate a truth-functional definition of conditionals.

7  If Philo of Megara had died at the age of one, his contribution to the study of conditionals would have been nil.

In 6 the antecedent is false while the consequent is true (as far as we know), in 7 the antecedent is false and the consequent is false as well. 6 represents line 3 of the truth-table, 7 line 4. Although according to the truth-functional definition both 6 and 7 are true, 6 seems to be unacceptable, while 7 seems to pose no problem.
The definition of material implication does not only render conditionals with a false antecedent true that seem to be false to an ordinary language user. Sometimes it even attaches a truth-value to a conditional in a situation where it is rather awkward to do so. Take the conditional in 8:

8 If Susan takes an aspirin, her headache will disappear.

This conditional is true when Susan takes an aspirin and her headache disappears. It is false when she takes an aspirin and her headache does not disappear. But what if Susan does not take an aspirin at all? In that situation it seems to make more sense to leave the truth of the conditional undecided, whereas according to the definition of material implication it would be true because the antecedent is false.

Example 5, 6 and 8 can be seen as instances of the paradoxes of material implication (Blumberg 1967: 15). From the definition of material implication it follows that a conditional is true when the consequent is true or the antecedent is false. That implies one can infer \((A \rightarrow B)\) from either not-\(A\) or \(B\). In the case of ordinary language conditionals, for a conditional to be sound it does not seem to suffice that either the consequent is true or the antecedent false. Seemingly, there has to be a connection between the antecedent and the consequent as well. If Philo of Megara had died at the age of one, he would not have been the first to come up with a truth-functional account of conditionals, simply because he wouldn’t have lived long enough to do so. Which makes it acceptable to say that as a consequence of his dying young, his contribution to the study of conditionals would have been nil.

Although line 1, 3, and 4 have been discussed frequently, line 2 of the truth-table has escaped such treatment. There seems to be a widespread consensus that a conditional with a true antecedent and a false consequent is false. In fact, this is seen as part of the core meaning of conditionals. Kahane, for instance, writes in the introduction to Logic and Philosophy:

So all that a material conditional affirms not to be the case is what any conditional affirms not to be the case, namely that it is not the case that its antecedent is true and its consequent false (1995:25).

Nevertheless, even regarding this undisputed line of the truth table, problematic instances can be formulated. Take for instance the following example given by Mackie:
...let us test it [the definition of material implication – jmg] further by considering four cases. All four begin with a father saying to a child ‘If you poke your finger into that monkey’s cage, you will get it nipped off’ and with the child poking its finger into the cage nonetheless. (...) In the fourth case the monkey snarls, snaps at the finger, but just at that moment a large object falls from the roof of the cage and deflects the monkey’s attack (1973: 107).

Is the conditional in this case true or false? The antecedent is true, the child has poked its finger in the cage. The consequent is false, since the finger is not nipped off. So strictly speaking the conditional is false. But to call this conditional false seems rather far-fetched. The conditional expresses a prediction on the basis of a generalized conditional, and to reject this conditional on the basis of one counter-example that can easily be explained away, would be rather premature. 9

To sum up, for line 1, 3 and 4 of the truth-table, examples can be formulated that comply with the truth-functional definition of conditionals but seem to be unacceptable anyway. And for line 2 an example can be thought of where the truth of the antecedent and the falsity of the consequent do not suffice for the falsity of the conditional.

Apart from the problems associated with the definition of material implication, various authors discern problems when conditionals are submitted to logical operations that are valid for material conditionals. The discussion of this subject has been concentrated on ‘contraposition’, ‘transitivity’ and ‘antecedent-strengthening’, but this does not mean that these operations are the only three raising difficulties.

Contraposition is the name given to the logical operation whereby both the antecedent and the consequent are negated and the order is reversed. That is, ‘A \( \rightarrow \) B’ is converted into ‘\( \neg B \rightarrow \neg A \)’. In classical propositional logic ‘A \( \rightarrow \) B’ is equivalent with ‘\( \neg B \rightarrow \neg A \)’ whenever the first is true (or false), the second is true (or false) as well – as a consequence contraposition is valid. From the conditional ‘If Thomas

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9 According to Mackie this conditional is false (1973: 107): ‘As long as we stand firm on attending what was said, we must say the same about the fourth [that it is false-jmg]: there the father would be presumably withdraw to claiming that all he meant was that there was a serious risk of the finger’s being bitten off, and this has been thoroughly confirmed.’ At least this example shows that also with reference to the 2nd line of the truth table a conditional can mean something more (or less?) than is captured in the definition of material implication.
is a vegetarian, he does not eat steaks’ one can infer that ‘If Thomas does(not not) eat steaks, he is not a vegetarian’. However, if one considers some natural language conditionals, this operation doesn’t seem to be acceptable.

First of all, contraposition is problematic for conditionals where the antecedent forms some kind of condition for a speech act expressed in the consequent, as in 9:

9    If you would appreciate it, I could ask the manager for a special discount.

In 9 the antecedent expresses a condition that must be fulfilled in order for the offer in the consequent to become ‘uttered’, which means no one has offered to do anything as long as the condition in the antecedent is not satisfied. If contraposition is applied to 9, one arrives at 9’:

9’   If I could not ask the manager for a special discount, you would not appreciate it.

Although 9’ might be true as well, it cannot be inferred from the conditional in 9. Sometimes conditionals of this type don’t lend themselves to contraposition at all. It is hard to see for instance what contraposition would amount to when applied to ‘If you don’t mind me asking, why did you move out of Amsterdam anyway.’

The second category of conditionals that do not allow contraposition are conditionals like the example in 10 given by Jackson (1987: 48):

10   If it rains, it will not rain heavily.

The antecedent of 10 conveys some sort of concession. If this proposition is put on a scale, it would occupy a less extreme position than the proposition expressed in the consequent. These relative positions on a scale can be externalised by adding ‘at least’ to the consequent. (If it rains, at least it will not rain heavily). Contraposition in this example results in 10’, which is clearly not acceptable:

10’  If it rains heavily, it will not rain.

The third category of conditionals that do not comply with contraposition are conditionals like the one in 11: 10
If the U.S. halts the bombing, then North Vietnam will not agree to negotiate.

In 11 the antecedent of the conditional depicts a certain course of action, while the consequent shows what happens if this course of action is taken. Contraposition of this conditional results in:

If North Vietnam agrees to negotiate, then the U.S. will not have halted the bombing.

The strange thing about 11’ is that the course of action for the U.S. now seems to be the dependent on the willingness to negotiate.

Contraposition is not the only logical operation that yields undesirable results when applied to certain ordinary language conditionals; transitivity poses problems as well. The principle of transitivity is expressed in an argument form called ‘hypothetical syllogism’. In logic, ‘A→B’ and ‘B→C’ together justify the conclusion ‘A→C’. For instance, if one accepts that Theodore did not murder Philip if he was in jail at the time of the murder, and one accepts that he was in jail at the time of the murder if the prison records show this, one can conclude that if the prison records show that Theodore was in jail at the time of the murder, he did not commit the murder.

A well-known example of ordinary language conditionals that do not allow transitivity is the following example given by Cooper (1978: 183):

If Brown wins the election, Smith will retire to private life.
If Smith dies before the election, Brown will win it. So if Smith dies before the election, Smith will retire to private life.

Transitivity is not only problematic when applied to conditionals concerning the future, it can be at times just as problematic in the case of so called subjunctive conditionals, which is demonstrated by the following example given by Jackson (1987:80):

If it had snowed, I would have gone skiing.
If there had been a blizzard, it would have snowed.
So: If there had been a blizzard, I would have gone skiing.

Examples (11) and (11’) are given by Stalnaker (1975: 173-174)
Hypothetical syllogism is closely connected to another argument form that sometimes leads to unacceptable outcomes when applied to ordinary conditionals, namely, the operation called ‘strengthening the antecedent’. An example of strengthening the antecedent is given under 14:

14 If Max is a bat, Max can fly.
So: If Max is a bat and has been born this morning, Max can fly.

Although the premise of this argument is acceptable, the conclusion certainly is not because newly-born bats cannot fly yet. That the reasoning is valid becomes clear when the underlying hypothetical syllogism is made explicit, as in 14’:

14’ If Max is a bat and has been born this morning, then Max is a bat.
If Max is a bat, then Max can fly.
So: If Max is a bat and has been born this morning, then Max can fly.

A and B taken together imply A, moreover, A implies C, therefore A and B taken together imply C. This means that from two acceptable premises, using the valid argument form of hypothetical syllogism, one may arrive at a conclusion that is clearly unacceptable.

In summation: apparently one runs into serious problems when the definition of material implication is taken as the definition of all ordinary language conditionals. This definition gives rise to the paradoxes of material implication: from something false anything can follow, and something true can follow from anything. Moreover, when ordinary language conditionals are applied in logically valid operations like contraposition, hypothetical syllogism and strengthening the antecedent, unacceptable outcomes may occur.

The most common explanation for these problems and paradoxes is that the meaning expressed in the truth-functional definition must not be seen as the ‘whole’ meaning of conditionals. As Kahane puts it:

The connective ‘⊃’ has been defined so as to capture the bare minimum in truth-functional terms that is asserted by a conditional. (...)

When symbolizing conditionals of ordinary English, it must be remembered that most everyday conditionals are not merely
truth-functional. Thus, when they are symbolized by ‘⊃’, only part of their meaning is captured (namely, the part they share with all conditionals), the other part being lost (1995: 25).

In response, various logics have been developed in order to capture the parts of meaning that have been lost.

2.3

Solutions to the problems associated with material implication

One of the first (modern) reactions to the definition of material implication came from C.I. Lewis. In an article in Mind, ‘Implication and the Algebra of Logic’, he introduces his concerns about this definition in the following manner:

The development of the algebra of logic brings to light two somewhat startling theorems: (1) a false proposition implies any proposition, and (2) a true proposition is implied by any proposition (1912: 522).

The theorems Lewis refers to were first made explicit by Whitehead and Russell in their *Principia Mathematica* and are currently known as the paradoxes of material implication.

It has been suggested that these paradoxes only appear to be paradoxical, due to confusion, which apparently began when Russell and Whitehead used the term ‘implication’ to refer to the conditional construction. In his article ‘External and Internal Relations’, Moore discusses the unusual meaning given to the word ‘implies’:

And these results, it seems to me, appear to be paradoxical, solely because, if we use ‘implies’ in any ordinary sense, they are quite certainly false. Why logicians should have thus chosen to use the word ‘implies’ as a name for a relation, for which it never is used by any one else, I do not know (1922:295).

But the confusion only seems to have started there. According to the *Encyclopaedia of Philosophy* a second shift in meaning has taken place.

Second, the meaning of ‘materially implied by’ is shifted from that of ‘if-then’ to that of ‘follows logically from’ (that is, from the truth-
functional connective ‘→’ to the relation of deducibility or logical consequence). Once this shift is made, the results are indeed ‘paradoxical’: a true sentence follows logically from any sentence, and any sentence follows logically from a false sentence. However, such consequences follow not from the definition of ‘P→Q’ but from a confusion between ‘P→Q’ and ‘P; therefore Q’. The first set of symbols represents a truth-functional compound, the second an (obviously invalid) argument form (...) (Blumberg 1967: 16).

This second shift of meaning could well have been caused by the close connection between the truth of a conditional and the validity of an argument form. The premises and conclusion of a valid argument can be expressed in a conditional sentence which is true under all circumstances. Consider for example the following argument:

15  Either John or Mary is coming to the conference  
    John is not coming so: Mary is coming to the conference

can be expressed in the corresponding conditional ‘If either John or Mary is coming to the conference and John is not coming, then Mary is coming.’ Conditionals such as these can be seen as expressing logical consequence or ‘if..., then’ in the sense of ‘following logically from’.

While some attribute the paradoxical nature of the theorems to confusion, others have contended that these theorems rather are merely pragmatically awkward. Grice for instance argues that ‘the other parts of meaning’ that are lost in the definition of material implication are not parts of the meaning of the conditional, but are in fact conversational implicatures (1989: 83). He discusses two ways in which these implicatures can emerge.  

The first explanation Grice offers is accomplished by an analysis based on the Cooperative principle and the maxims of conversation. If someone asserts ‘if p then q’ – according to Grice – he asserts something that is logically weaker than the negation of p or the assertion of q. This amounts to an infringement of the first maxim of Quantity: if a more informative statement is of interest to the interlocutor, the speaker is expected to assert this statement. Grice considers the more

11 For an explanation of the main concepts of Grice’s theory of conversation, see Grice 1989: 22-40.
informative statement ‘not-\( p \)' or ‘\( q \)' to be of interest, since:

No one would be interested in knowing that a particular relation (...) holds between two propositions without being interested in the truth-value of at least one of the propositions concerned, unless his interest were of an academic or theoretical kind (...). Either because we know (...) that the use of language for practical purposes is more fundamental than (...) its use for theoretical purposes, or because it is simply a well-known fact about human nature that practical interests are commoner than theoretical interests (...), we are justified in assuming, in the absence of any special contextual information, that an interest is practical more than theoretical (1989: 61).

The interlocutor, assuming that the speaker observes the Cooperative Principle, can explain this infringement of the maxim of Quantity by supposing that asserting the more informative ‘not-\( p \)' or ‘\( q \)' would amount to an infringement of the second maxim of Quality (which states that you must have adequate evidence for what you say). Therefore, whenever ‘if \( p \), then \( q \)' is asserted under normal circumstances, the interlocutor concludes that the speaker does not put forward this assertion because he knows / has evidence that not-\( p \), or that \( q \) (that is, he does not assert ‘if \( p \) then \( q \)' on the basis of one of the paradoxes of material implication). To be sure, that is exactly what makes the paradoxes of material implication pragmatically odd.

The second way an implicature might come about is closely connected to the role conditionals fulfil in discourse. Grice argues that conditionals might be fitted for the special role of presenting cases:

...in which a passage of thought, or inferential passage, is envisaged from antecedent to consequent, and possibly to a further consequent with respect to which the first consequent occupies the position of antecedent (1989: 77).

If such a function can be ascribed to conditionals, a speaker might in asserting a conditional ‘implicate’ that he is using the conditional in this way. Furthermore, if it is a precondition for this use that the speaker’s knowledge of this conditional is not founded on his knowing that not-\( p \) or that \( q \) (but for instance rather on a strong connection between the antecedent and the consequent), then this information might be implicated as well.
Whether they ascribe the paradoxical nature of \( q \to (p \to q) \) and \( \neg p \to (p \to q) \) to confusion or to pragmatic factors, the above mentioned authors agree that the definition of material implication does reflect the meaning of ‘if..., then’. Many other authors however have disputed just that. The paradoxes of material implication and the strange or counterintuitive results of applying contraposition, hypothetical syllogism and antecedent strengthening to ordinary conditionals, have led them to abandon the definition of material implication and to formulate an alternative definition that precludes such conundrums.

C.I. Lewis for instance found the abovementioned theorems not only somewhat startling, but even went on to dispute the fact that the definition of material implication reflected the true meaning of ‘to imply’. This definition states that a proposition materially implies another proposition if it is not the case that the antecedent is true and the consequent is false. Suppose someone says: ‘If it is sunny, then John is in the garden’. According to the definition, this sentence would be true if it is in fact sunny and John is in fact in the garden, no matter where John would be the next sunny day.

From Lewis’ point of view, a proposition only truly implies another proposition if it is not just ‘not the case’ that the antecedent is true and the consequent false, but ‘impossible’ for the consequent to be false while the antecedent is true. In order to develop a logic of conditionals that reflects this quality, he introduced a symbol for what he called ‘strict implication’ and developed logical systems in which the paradoxes of material implication cannot be proven. Since these logical systems make use of modal terms like ‘necessary’ and ‘possibly’, they are commonly known as modal logics.\(^{12}\)

Unfortunately, logical theories based on strict implication do have paradoxes of their own. If the consequent of a conditional is formed by a proposition that is necessarily true, this conditional will be true, irrespective of the truth or falsity of the antecedent. Something similar applies to a conditional of which the antecedent is necessarily false: such a conditional will always be true, whether the consequent is true or not. Lewis was not overly concerned by these results having nevertheless held onto his definition of strict implication. Other logical theories like possible world semantics and probability theory have elaborated on this basic notion of strict implication since then. But relevance

\(^{12}\) For an overview of the different systems of modal logic and the discussions associated with them, see Bull & Segerberg 2001.
logicians see the occurrence of the paradoxes of strict implication as a sign that strict implication doesn’t capture the true meaning of ‘to imply’ and have set themselves the task of improving this definition.

Relevance logicians are concerned with the formal analysis of the notion of ‘logical implication’.13 They contest the view that material implication has anything to do with implication, since ‘implication’ must be seen as:

...‘entailment,’ or ‘the converse of deducibility’(...), expressed in such logical locutions as ‘if... then –,’ ‘implies,’ ‘entails,’ etc., and answering to such conclusion-signalling logical phrases as ‘therefore,’ ‘it follows that,’ ‘hence,’ ‘consequently,’ and the like (Anderson & Belnap 1975: 5).14

Therefore, material implication is not a kind of implication, implication is not yet formally analysed, while relevance logic aims at doing just that.

As a starting point for their analysis, relevance logicians use natural deduction. Relevance logicians wish to interpret $A \rightarrow B$ as ‘$A$ entails $B$’ or ‘$B$ is deducible from $A$’. In a system of natural deduction this means that – by the rules of derivation of the system – $B$ can somehow be deduced from the hypothesis $A$. However, in most systems of natural deduction the paradoxes of material implication are deducible as well. That is why relevance logicians eventually reject this (official) natural deduction concept of proof from hypotheses as a basis for proper implication and propose two adjustments: the addition of the notion of necessity and of relevance.

13 Dunn and Restall (2002) claim that relevance logicians are not exclusively concerned with the notion of logical implication. However, given that their article is not quite clear as to what relevance logicians are exactly concerned with, and since Anderson’s and Belnap’s books Entailment: the Logic of Relevance and Necessity (volume I, 1975; volume II, 1992) are still seen as the standard work on the subject, this discussion of relevance logic will mainly be based on these earlier works.

14 Judging from this exposition, relevance logicians seem to be subjected to the confusion mentioned before: ‘$p$ materially implies $q$’ is taken to mean ‘$p$, therefore $q$’. However, they vehemently disagree with the view that the paradoxes of material implication are based on this confusion and consider ‘If $p$, then if $q$, then $p$’ to be just as objectionable as ‘$p$ implies ($q$ implies $p$)’ (Anderson & Belnap 1975: 13).
That necessity must be added to arrive at a definition of implication is motivated by the basic assumption in logic that the validity of an inference depends on formal considerations alone. According to Anderson and Belnap, this:

...amounts to saying that the validity of a valid inference is no accident of nature, but rather a property a valid inference has necessarily. Still more accurately: an entailment, if true at all, is necessarily true (1975: 14).

Anderson and Belnap argue that this means that truths deduced from necessary truths are necessary truths as well. The paradoxes of material implication do not fulfil this condition, since an example can be developed whereby a statement that is not necessarily true can be deduced from a necessarily true statement. For instance, if it is true that John is in the garden, then one can deduce that ‘If a bachelor is an unmarried man, John is in the garden’. In this implication, a necessarily true statement implies a statement that is not necessarily true, which is in direct conflict with the condition mentioned above.

Yet the logical system Anderson and Belnap devise to include the notion of necessity, suffers from the same trouble as C.I. Lewis’s modal logic: the paradoxes of strict implication. An arbitrary proposition \( p \) implies \( q \) when \( q \) is necessarily true. This means that the implication \( A \rightarrow (B \rightarrow B) \) is valid: since it is impossible for \( (B \rightarrow B) \) to be false, it is impossible that the antecedent is true while the consequent is false. Anderson and Belnap conclude from this that a logical system that includes the notion of necessity still cannot capture the true meaning of ‘to imply’.

According to Belnap and Anderson, the deduction of the necessary proposition \( (B \rightarrow B) \) from an ordinary proposition \( A \) is unacceptable because \( A \) is irrelevant to \( (B \rightarrow B) \). Anyone putting forward this deduction would be guilty of a fallacy of relevance. Although Anderson and Belnap do not provide a definition of relevance, they claim that for a proposition \( A \) to be relevant in the deduction of another proposition \( B \), proposition \( A \) must actually be used in deducing \( B \). To keep track of the propositions used in a deduction, they propose a subscripting device and develop logical systems that comply with this notion of relevance. Only if the conclusion of a deduction contains the same subscript as an hypothesis earlier introduced, can this hypothesis be relevant for the conclusion of the deduction. And only if a proposition \( B \) follows necessarily from a proposition \( A \) with the same subscript, can one conclude that \( A \) implies \( B \).
Both strict implication and its combinations with the notion of relevance are advanced as a way to circumvent the paradoxes of material implication. Yet the paradoxes of material implication are not the only problems connected to a truth-functional definition of ‘if-then’. Some logical operations that are valid in propositional logic produce unacceptable outcomes when applied to ordinary conditionals. Conditional logics, like possible world semantics and probability theory, are aimed at solving this problem: they try to explain why contraposition, hypothetical syllogism and strengthening the antecedent yield such undesirable results.

Possible world semantics was aimed originally at providing a semantics for logical systems based on the notion of strict implication. Since truth tables cannot be used for determining the truth of sentences that contain words like ‘possible’ or ‘necessary’, these logical systems necessitate a different way of evaluating the truth of a sentence. Possible world semantics provides such a semantics by introducing the notion of an accessible possible world.

Stalnaker introduces the main concepts of possible world semantics by examining how one proceeds in evaluating a certain conditional sentence. He presents the hypothetical situation of a true/false political opinion survey in which one is confronted with the statement ‘If the Chinese enter the Vietnam conflict, the United States will use nuclear weapons’. How does one go about determining whether one believes this statement or not? According to Stalnaker, the evaluation moves along the following lines:

First, add the antecedent (hypothetically) to your stock of beliefs; second, make whatever adjustments are required to maintain consistency (without modifying your belief in the antecedent); finally, consider whether or not the consequent is then true (1975: 169).

This procedure is now known as the Ramsey test.

From these belief conditions, the transition to truth conditions is made by introducing the concept of a possible world: a world in which the antecedent of the conditional is true and which has been adjusted so that no contradictions arise. ‘Necessity’ is defined as ‘true in all accessible possible worlds’, while ‘possibility’ is defined as ‘true in some accessible possible world’. Worlds are accessible when they comply with a restriction associated with the type of necessity or possibility under consideration. For instance, in the case of physical necessity /
possibility, the worlds accessible are those in which the same laws of nature hold (D. Lewis 1973: 5).

Although providing a semantics for logical systems based on the notion of strict implication triggered the research in the field of possible world semantics, some logicians working in this field abandoned the notion of strict implication as a way to treat ordinary conditionals. After all, ordinary conditionals often cannot be submitted to the logical operations of contraposition, transitivity and strengthening the antecedent, while strict conditionals even in very weak modal logics allow for these logical operations. As a result conditionals are not treated as strict conditionals, although possible world semantics is still viewed as a valuable tool in evaluating ordinary conditionals. As Nute and Cross put it:

The basic intuition, that a conditional is true just in case its consequent is true at every member of some set of worlds at which its antecedent is true, may be yet salvageable. We can avoid Transitivity, etc. if we allow that the set of worlds involved in the truth conditions for different conditionals may be different (2001: 9).

Possible world semantics explains the failure of contraposition, transitivity and strengthening the antecedent by referring to two crucial concepts in this theory: the selection of worlds and the relative closeness of worlds. In possible world semantics, a selection function is defined in order to decide on the set of worlds significant for the evaluation of the conditional. The relative closeness relates to the position of one world in relation to other worlds and the actual world. If one takes the actual world as a starting point, all possible worlds can be positioned as either close to or further away from the actual world, depending on the similarity / dissimilarity that exists between the actual and the possible world. When a possible world is very similar to the actual world, this world is close to it, when there are many differences, this world is ‘far’ away from it.

In order to evaluate a conditional, one has to check whether the consequent of this conditional is true in the possible worlds in which the antecedent is true and that are closest to the actual world. If the consequent is true in those worlds, the conditional is true, if not, the conditional is false. Failures of contraposition, transitivity and strengthening the antecedent arise when the conditional(s) that form the premise of the argument are true according to these truth conditions, whereas the conditional that forms the conclusion is false.
A situation where contraposition fails is given in the following: suppose a father with two children says ‘If we had more than two children, we would not have had more than ten children’.¹⁶ This conditional is true when in the possible worlds closest to the actual world in which the father has more than two children, he does not have more than ten children. Since worlds close to the actual world can be thought of where the father has three, four or in any case less than eleven children, this conditional is true. However, contraposition yields a conditional that is false. ‘If we would have had more than ten children, we would not have had more than two children’ is false since in the worlds closest to the actual world in which the statement ‘we would have had more than ten children’ is true, the statement ‘we would not have had more than two children’ is false.

Transitivity fails in situations where the antecedent of the first premise is more far-fetched than the antecedent of the second premise. As a result, the worlds closest to the actual world where the antecedent of the first premise holds, differ from the worlds closest to the actual worlds where the antecedent of the second premise holds. That is why the consequent of the second premise can be false in the nearest world where the antecedent of the first premise holds.

Both Stalnaker (1975: 173) and D. Lewis (1973: 33) use the following example to make this point clear:

If J. Edgar Hoover had been born a Russian, then he would have been a communist.
If he had been a communist, he would have been a traitor.
Therefore: If he had been born a Russian, he would have been a traitor.

Both Stalnaker and Lewis consider the world in which Edgar J. Hoover is born a Russian to be less like the actual world than the world in which he is a communist. In the world closest to the actual world where he is born a Russian, J. Edgar Hoover would not be an American citizen (whereas he would be an American citizen in the closest world in which he is a communist). As a result, the consequent ‘he would have been a traitor’ is not true in this world.

¹⁵ One of the most difficult questions possible world semantics has to answer is ‘how does one decide which worlds are closest to the actual world?’ For an overview of the different viewpoints on this issue see Nute & Cross 2001: 9-28.

¹⁶ This is an example from Sanford (1992: 109).
The failure of strengthening the antecedent is closely related to that of transitivity. Suppose someone says ‘If the cherries would have been cheap, Martha would have bought them’. This conditional would be true when in the possible worlds with cheap cherries closest to the actual world Martha would indeed have bought the cherries. Along the same line of reasoning, the conditional ‘If the cherries would have been rotten, they would have been cheap’ would be true. However, if these two conditionals are used as premises in a hypothetical syllogism, the conclusion ‘If the cherries would have been rotten, Martha would have bought then’ would be false. Similarly, the antecedent cannot be strengthened to ‘If the cherries would have been cheap and would have been rotten’, since in the world closest to the actual world where this antecedent would be true (which is different from the closest world with cheap cherries), the consequent ‘Martha would have bought them’ would be false. In cases like these, antecedent strengthening fails.

When one starts from the truth conditions for conditionals as described by possible world semantics, situations can be thought of where contraposition, transitivity and strengthening the antecedent lead to an argument with true premises and a false conclusion. Therefore, these argument forms are invalid and the formal systems of possible world semantics are designed in such a way that contraposition, transitivity and strengthening cannot be proven within these logical systems.

Just as with possible world semantics, probability theory provides an explanation for unacceptable inferences that may obtain when contraposition, transitivity and strengthening the antecedent are applied to everyday conditionals, albeit this is achieved from a different starting point. According to probability theorists, the truth-functional soundness criterion does not always suffice for establishing whether a certain deduction is sound. Adams gives the following example of the failure of orthodox truth-functional logic:

Imagine a man about to eat some very good and non-poisonous mushrooms who is informed ‘if you eat these mushrooms, you will be poisoned’, which leads the man not to eat the mushrooms while making the statement ‘true’ (i.e. materially true) at the same time. Obviously the man would have been better off not to have arrived at this allegedly ‘true’ conclusion, and this type of example should make it questionable that reasoners should want to be guided in their reasoning by the principles of orthodox logic, if those
are designed to lead them to conclusions which are ‘true’ in this unwanted sense (1975:x).

Adams claims that orthodox truth-functional logic fails because it is designed for reasoning from certainties to other certainties. However, in everyday reasoning one often doesn’t start from absolute certainties (or rather, at least the truth of the premises often cannot be established at the time the reasoning takes place) and therefore the conclusion is no certainty either.

Probability theory is designed to provide a soundness criterion that can be applied in such circumstances. The key to probability theories is that they do not start from the truth of propositions but from the probability of propositions. Reasoning should not just be evaluated by a truth conditional soundness criterion, but also by a ‘probabilistic soundness criterion’, according to which reasoning is sound if it is impossible for the premises of an inference to be probable while its conclusion is improbable. Or to be more precise: in a probabilistically valid argument the uncertainty of the conclusion cannot exceed the sum of the uncertainties of the premises (Adams 1975:2). Contraposition, hypothetical syllogism and strengthening the antecedent in some instances fail to meet the probabilistic soundness criterion, and are therefore not endorsed in probability theory.

An important question probability theory has to answer is: ‘what counts as the probability of a proposition?’ In the case of assertions like ‘it rains’, the answer to this question is simple: the probability of this proposition equals the probability of its truth. The probability of conditional sentences is less easy to establish. Just as in possible world semantics, the Ramsey test generally is taken as a starting point for the evaluation of if-then-sentences. In evaluating the sentence ‘If it rains, the birthday party will be cancelled’, one adds ‘It rains’ to one’s stock of beliefs, adjusts those beliefs so that this assumption is accommodated and then assesses how probable it is that ‘the birthday party will be cancelled’. The probability of a conditional is therefore

17 No matter how convincing this definition of the probability of a conditional might seem at first glance, closer inspection reveals the complexity at hand. David Lewis showed that the definition leads to the so-called ‘triviality result’ when taken in combination with the axioms of probability logic (Lewis 1976: 131-134). One way out of this is to abandon the idea that indicative conditionals have truth-values. For more on this subject, see Bennett 2003 (especially chapters 4 and 5).
a conditional probability: the probability that the consequent holds given the antecedent.\textsuperscript{17} If in 10 out of 20 situations when it rains, the birthday is cancelled, the probability of ‘If it rains, the birthday party will be cancelled’ is $\frac{10}{20} = 0.5$ (50\%).\textsuperscript{18}

Given this definition of the probability of conditionals and the probabilistic soundness criterion, one can show that contraposition, hypothetical syllogism and strengthening the antecedent are unsound. An example of contraposition failing to meet the probabilistic soundness criterion is given by Sanford (1989: 93-94). Take the two propositions $S$ and $T$:

A roll of dice comes up 6.
At least one of the dice comes up 3.

There are 36 equipossible ways for two dice to fall. In 11 of those 36 cases, ($T$) is true: at least one of the two dice comes up 3. In the remaining 25 cases ($T$) is false. ($S$) is true in 5 of the 36 cases, in 31 cases it is false. The distribution of truth and falsity is represented in the following table (black indicates false, white indicates true)\textsuperscript{19}:

<table>
<thead>
<tr>
<th></th>
<th>$T$</th>
<th>$\neg T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S$</td>
<td>$T \wedge \neg S$</td>
<td>$T \wedge T \wedge S$</td>
</tr>
</tbody>
</table>

From this table, it can easily be seen that ‘If $S$, then $T$’ and ‘If not-$T$, then not-$S$’ are not equally probable. The probability of ‘If $S$, then $T$’ is $1/5$: out of the five situations in which $S$ is true, there is one situation in which $T$ is true as well. The probability of ‘If not-$T$, then not-$S$’ is much higher, out of 25 situations in which not-$T$ is true, there are 21 in which not-$S$ is true as well (probability is $21/25$). This makes the

\textsuperscript{17} From this definition it becomes apparent that probability theory can only say something about indicative conditionals, not about counterfactuals. If the antecedent of a conditional is known-to-be false, there would be no situations in which the proposition expressed in the antecedent is true, and therefore the conditional probability would be $n/0$.

\textsuperscript{18} This table is taken from Sanford as well.
argument ‘If not-T, then not-S; therefore, if S, then T’ probabilistically unsound, since the probability of the conclusion is much lower than that of the premise.

That this hypothetical syllogism is probabilistically unsound is illustrated by the following example of Bennett (2003: 145). Imagine a farmer who believes that the gate to the turnip field is closed and that his cows are not in the turnip field. This farmer would accept the following conditional:

17 If the cows are in the turnip field, the gate has been left open.

He would accept it because if he adds the antecedent to his stock of beliefs and adjusts his belief system, his belief in ‘the gate is closed’ would drop (making the proposition ‘the gate has been left open’ more probable). Furthermore, the farmer would accept the following conditional:

18 If the gate to the turnip field has been left open, the cows have not noticed the gate’s condition.

The farmer accepts this consequent, given the antecedent, because he thinks the cows are not in the turnip field. Combining the two conditionals in a hypothetical syllogism, we are left with ‘If the cows are in the turnip field, they have not noticed the gate’s condition’, and this is clearly absurd.

That this instance of hypothetical syllogism is probabilistically unsound can be illustrated with the following table:

<table>
<thead>
<tr>
<th>cows in field (a)</th>
<th>cows noticed condition (b)</th>
<th>gate left open (c)</th>
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</table>

The conditional ‘If the cows are in the turnip field, the gate to the turnip field has been left open’ receives the probability 1: the a-segment lies completely within the c-segment. The probability of the second conditional is reasonably high as well: out of 14 situations in which the gate is open, the cows have not noticed it in 9. The conclusion of the hypothetical syllogism however, has the probability of 0: there is no
situation in which the cows are in the turnip field, but have not noticed the gate’s condition. Here again, just as in the case of the role of dice, a situation can be thought of where the probabilities of the premises are high, whereas the probability of the conclusion is low. Therefore hypothetical syllogisms are not always probabilistically sound.

Since the logical operations of hypothetical syllogism and antecedent strengthening are closely related, the table used to show that hypothetical syllogism can be probabilistically unsound can be also used for proving the probabilistically unsoundness of strengthening the antecedent. Take for instance the following argument:

19 If the gate to the turnip field has been left open, the cows have not noticed it. Therefore, if the gate to the turnip field has been left open, and the cows are in the turnip field, the cows have not noticed that the gate is open.

The probability of the premise is high (9/14), while the probability of the conclusion is zero, which makes the argument probabilistically invalid.

The soundness criterion provided by probability theory renders contraposition, hypothetical syllogism and antecedent strengthening invalid. Therefore, just as in the case of possible world semantics, probability theory is devised in such a way that these three logical operations are excluded.

A final group of logics that have been initiated by the counterintuitive results brought forth by the logical operation of strengthening the antecedent cannot remain unmentioned in a chapter about logical approaches to conditionals. These are the non-monotonic logics or logics for defeasible argumentation. The concept of defeasibility was first introduced by Hart, who worked in the field of legal philosophy (Prakken&Vreeswijk 2001: 229). He noted that in law, a case is not closed if the plaintiff can show that all conditions mentioned in the antecedent of the rule of law are met. The other party can acknowledge all the facts that the plaintiff came up with, and still argue that the conclusion drawn by the plaintiff must be rejected. In a similar manner, the conclusion drawn by a judge can be reversed by a higher court. Logical conclusions drawn in a judicial context are not definite but only tentative.

Conclusions are drawn tentatively not only in judicial contexts but in everyday contexts as well. For instance, a person about to meet his
friends and confronted with the fact that their car is not on the drive-
way, might conclude from this that his friends are out and drive on. But if he comes to learn that the car is being serviced, he will retract his conclusion and ring their doorbell after all.20

Standard logic cannot deal with this kind of reasoning. In standard propositional logic a conclusion once reached on the basis of particular premises remains true, irrespective of other premises that might be added. This makes standard propositional logic monotonic: it’s (deductively valid) inferences never can be undone by new information. This characteristic of logical reasoning is reflected in the logical operation called ‘strengthening the antecedent’: from ‘A→C’ one can deduce ‘(A∧B) → C’.

Strengthening the antecedent seems to be invalid in the ‘car-not-on-the-driveway’-example. The original conditional ‘If the car is not on the driveway (A), my friends are out (C)’ cannot be expanded to ‘If the car is not on the driveway (A) and the car is being serviced (B), my friends are out (C)’; the new information makes it necessary to retract the original conclusion C. Inferences corresponding to conditionals like these are non-monotonic and various non-monotonic logics are devised to handle them.21

One way to formalize inferences with tentative conclusions is to reinterpret the conditional used in this type of reasoning. Whereas in monotonic logic the truth of the antecedent of a conditional guarantees the truth of the consequent, in non-monotonic logics no such guarantee is given. The conditional is qualified: if the antecedent holds, normally the consequent holds as well. Such a defeasible conditional is called a default: when there are no exceptional circumstances, the consequent can be derived. On the other hand, if at a later stage one can show that exceptional circumstances obtained, the consequent may have to be retracted.

The default character of conditionals can be formalized in different ways. One possibility is to add an extra ‘normality’ condition to the antecedent of the conditional. Instead of ‘If their car is not on the driveway, my friends are out’ the conditional reads ‘If their car is not on the driveway and there is no abnormal situation with respect to the car, my friends are out’. The car being serviced would count as

21 For a clear and insightful overview of different systems for defeasible reason-
ing see Prakken & Vreeswijk 2001.
an abnormal situation, and therefore in that case the consequent ‘my friends are out’ cannot be derived.\textsuperscript{22} Another possibility would be to re-interpret the meaning of the conditional operator. According to these accounts – that resemble possible world semantics – default conditionals are taken to mean something like ‘In all most normal worlds in which p holds, q holds as well.’ The world in which the car is serviced would not count as a ‘most normal world’ and therefore the truth of ‘my friends are out’ is not guaranteed.

But one does not have to reinterpret the conditionals in first order logic as defeasible conditionals in order to handle non-monotonic inferences. In some systems of non-monotonic reasoning the logical language is expanded in such a way that the expression of ‘defeasible generalizations’ or rules of thumb is made possible. The conditional connective is only used for non-defeasible inferences, hence it need not be adjusted or reinterpreted. The sentence ‘If the car is being serviced, then the car is not on the driveway’ can therefore be expressed as an ‘ordinary’ non-defeasible conditional (since no exceptions to this rule are possible), whereas the sentence ‘If the car is not on the driveway, my friends are out’ is expressed in a new vocabulary.\textsuperscript{23}

A fourth and final way to deal with non-monotonic reasoning is to comprehend defeasibility as a characteristic not of conditionals but of the arguments in which these conditionals occur. In order for defeasibility to emerge there must be a conflict between two arguments. ‘My friends are out, because their car is not on the driveway’ is for instance in conflict with the argument ‘My friends are in, because I just saw John through the window’, since the conclusions cannot both be true. In situations like this, an argument is defeated when one argument is preferred over the other (i.e. when direct proof ‘I saw him’ is preferred over indirect proof ‘their car is not on the driveway’. In a similar fashion, the statement ‘their car is being serviced’ defeats the argument in support of the conclusion that the friends are out. In the original argument, one infers from the effect (the car is not on the driveway) the occurrence of the cause (my friends are out). ‘Their car is being serviced’ successfully undercuts this inference by supplying an alter-

\textsuperscript{22} Systems that make use of an additional condition in the antecedent of the conditional are in fact somewhat more complex: apart from the extra condition, an assumption is added that the situation is normal when there is no evidence to the contrary.

\textsuperscript{23} An example of such a theory is Reiter’s ‘Default logic’ (1980).
The defeasibility of arguments is captured in various logics for defeasible argumentation. Such logical systems consist not only of a logical language, but also define what an argument is, what it means to have a conflict between arguments and under what circumstances an argument can be said to have defeated another. Furthermore, such logical systems give a definition of the status of arguments, for only when arguments are placed in a hierarchy, can one decide which of the two competing arguments defeats the other.

An important difference between logics for defeasible argumentation and propositional logic or possible world semantics is that logics for defeasible argumentation are not concerned with the truth or falsity of propositions. Instead, they are aimed at specifying under what circumstances it is justified to accept a proposition as true. Conditionals that form part of an argument are therefore not defined with reference to the circumstances under which they are true but are subjected to the same criteria as other, non-conditional propositions: they are justified when not undercut or defeated.

2.4 Conclusion

One way of evaluating conditionals is to see whether a conditional is true according to the definition of material implication: when the antecedent is true and the consequent is false, the conditional is false; in all other situations it is true. Many authors have pointed out that this definition creates several problems when applied to ordinary conditionals. This chapter contains a discussion of these problems, as well as a discussion of logical theories that offer ways to solve them.

In short, one can discern two ways of handling this matter. Either one retains the definition of material implication and tries to explain away
the difficulties that arise, or one rejects this definition and replaces it by one that does not give rise to these difficulties. An author following the first strategy is Nieuwint. He argues that the failure of the hypothetical syllogism in 12 is not caused by a flaw in the definition of material implication, but rather results from neglecting the ‘ceteris paribus’-condition:

12 If Brown wins the election, Smith will retire to private life. If Smith dies before the election, Brown will win it. So if Smith dies before the election, Smith will retire to private life.

According to Nieuwint, the Brown that won in the first sentence took part in elections in which Smith took part as well, while the Brown in the second sentence didn’t. As a result, the the proposition ‘Brown wins the election’ that is expressed in the antecedent of the first conditional sentence is not the same as the one expressed in the consequent of the second conditional (1992: 195). That contraposition fails for a sentence like ‘If you are thirsty, there is beer in the fridge’ Nieuwint explains by pointing out that the consequent ‘there is beer in the fridge’ is not the ‘real consequent’. The sentence is elliptical and should read ‘you will be interested to hear that there is beer in the fridge’. Contraposition would then yield ‘If you are not interested to hear that there is beer in the fridge, you are not thirsty’, which is unproblematic.

Although Nieuwint manages to give explanations for common counter-examples to the definition of material implication, these are ad hoc and not always fortunate. Adding a ceteris paribus-condition to the antecedent of conditionals means for instance that modus tollendo tollens becomes more complicated. From a mere denial of the consequent one cannot deduce denial of the antecedent, since it could just as well be that the ceteris paribus-condition is not met. 27 Nieuwint’s explanation for the failure of contraposition makes sense as it stands, but it is not clear why in this example the consequent is elliptical and in others it isn’t.

Where Nieuwint explains the difference between theory and practice in a rather unsystematic way, others take a more systematic approach. Just like Nieuwint, Grice sees the definition of material implication as

26 Not all logical theories try to explain conditional sentences: relevance logic is for instance not so much occupied with the if-then connective as with formalizing what it means ‘to imply’.

40
the correct description of the meaning of conditionals. Extra features added to ordinary conditionals, like the connection between the antecedent and consequent, are not a part of the meaning of conditionals, but are conversational implicatures. They are a result of the way in which conditionals are commonly used. Grice uses his theory of conversation to explain how such conversational implicatures can come about. The failure of contraposition, strengthening the antecedent and hypothetical syllogism for ordinary conditionals might be explainable in a similar way.

By far the largest part of this chapter is dedicated to authors that follow not the first but the second path: they dispose of the definition of material implication altogether and replace it with a new one. All these theories are based on the assumption that the evaluation criteria offered by the definition of material implication are inadequate. New evaluation criteria are proposed and it is shown that the problematic cases are questionable because they fail to meet these new criteria.

Unfortunately, the solutions offered suffer from problems of their own. In replacing material implication with strict implication, C.L. Lewis found a solution for the paradoxes of material implication, but his solution itself gave rise to the paradoxes of strict implication. Furthermore, modal logic does not provide an explanation for the strange outcomes that may occur when the logical operations of contraposition, hypothetical syllogism and strengthening the antecedent are applied to ordinary conditionals. Relevance logic circumvents the paradoxes of strict implication by adding an extra ‘relevance-criterion’, but in the system devised to accommodate this, the inference ‘If A or B and not-A, so B’, which up to this point remained unproblematic, has to be rejected as well (Anderson & Belnap 1975: 165).

Conditional logics like probability theory and possible world semantics for counterfactuals are designed in such a way that contraposition, hypothetical syllogism and strengthening the antecedent can not be proved within the logical system. However, the explanations provided by these theories give rise to new difficulties. Possible world semantics has to address the difficult question of the relative closeness of worlds, which has not been satisfactorily answered. Probability theory solves the problems at the very high cost of abandoning the idea that

27 If one does deduce the denial of the antecedent, one is guilty of the fallacy of treating a non cause as a cause (Hamblin 1970: 78).
conditionals can be either true or false. As a result, probability theory works for simple conditional constructions but falls short when applied to compounds with conditionals (Gibbard 1980: 213). Furthermore, probability theory cannot be used to evaluate counterfactuals.

Finally, the various non-monotonic logics have their own difficulties. For instance, if the default character of conditionals is formalized by adding an extra ‘normality’ condition to the antecedent of the conditional, one runs into the same problem as Nieuwint: such conditionals cannot be used in simple applications of *modus tollendo tollens*, since one may not be able decide whether it is the condition expressed in the antecedent or the normality condition that is not met. When defeasibility is conceived as a characteristic of not the conditionals but of the arguments in which these conditionals occur, it becomes unclear as to how the possible strange outcomes in the case of the logical operation of strengthening the antecedent can be explained. To be sure, from the conflict between ‘My friends are out, because their car is not on the driveway’ and ‘My friends are in, because I just saw John through the window’ it follows that one should decide which argument is to be preferred. But this does not explain why ‘If their car is not on the driveway, then my friends are out’ cannot be strengthened to ‘If their car is not on the driveway and I just saw John through the window, then my friends are out’.

One could conclude that the difficulties associated with these different approaches to the evaluation of conditionals are only minor, and will be solved through further investigation. However, this way of perceiving the problems of the truth functional definition has a distinct disadvantage. By disposing of the definition of material implication on grounds of inadequacy, one might run the risk of disposing of something valuable. As was mentioned in the introduction, the definition of material implication forms a connection between the logical truth of a conditional sentence and the validity of an argument. Although the validity of arguments could be established in a different way, perhaps one should not forfeit this approach.

Furthermore, does one want to be rid of the logical operations of contraposition, hypothetical syllogism and strengthening the antecedent? While these logical operations do fail when applied to some ordinary conditionals, they are appropriate when applied to others. Take sentence 20:

20 *If the bottle of olive oil is leaking, you have not closed it properly.*
Contraposition yields ‘If you have closed the bottle of olive oil properly, it is not leaking’, a statement which makes perfect sense. If sentence 20 is combined with ‘If the bag is greasy, the bottle of olive oil is leaking’, the conclusion of this hypothetical syllogism would be ‘If the bag is greasy, you have not closed the bottle of olive oil properly’, which is unproblematic as well. Finally, one could devise examples of antecedent strengthening that pose no problem, such as ‘If the bottle of olive oil is leaking, and you are the one that put it in the bag, you have not closed it properly’.²⁸

Not only are these logical operations acceptable when applied to ordinary conditionals, they also retain their value when they are discerned mainly as patterns of reasoning. If A implies B, and B implies C, then the conclusion that A implies C seems inevitable; hypothetical syllogism indeed makes sense. The same conclusion can be drawn for contraposition: when B holds if A holds (if ‘If A, then B’ is true), then not-B indeed must mean that not-A; exactly what we are informed by contraposition. Finally, a similar plea could be made for strengthening the antecedent. If one really thinks that B holds if A holds, one might be committed to the statement that it does so under all circumstances, i.e. whatever circumstance added to the antecedent, it will not affect the consequent.

If the reasons given above are convincing regarding the value of the definition of material implication and the logical operations related to it, perhaps we should not dispose of it too hastily. That leaves us in

²⁸ One could say that in this example strengthening the antecedent is unproblematic because the proposition added to the antecedent can be interpreted as having influence on the truth / falsity of the consequent. Examples where this is not the case are possible as well, as can be witnessed from the following dialogue:

A: You haven’t closed the bottle of olive oil properly.
B: What do you mean?
A: It is leaking
B: You put it in the bag
A: If the bottle of olive oil is leaking, you have not closed it properly, whether I put it in the bag or not.

This last sentence can be rewritten as ‘If the bottle of olive oil is leaking and I put it in the bag, you have not closed it properly and if the bottle of olive oil is leaking and I didn’t put it in the bag, you have not closed it properly’. 
the following situation: on the one hand we have a group of conditional sentences to which the application of the definition of material implication together with the logical operations yields undesirable outcomes. At this point we have no satisfactory approach which would remove these undesirable outcomes. On the other hand, we have a definition that we don’t want to give up. We appear to be in deadlock.

Fortunately, there is a way out. As we have seen, the definition of material implication is not problematic for all ordinary conditionals. Why not restrict the truth functional definition to the unproblematic cases? We could then maintain this definition and have one group of conditionals for which evaluation criteria are formulated. Furthermore, we will no longer be bothered by problematic cases, since ‘problematic conditionals’ are not subjected to the evaluation criteria provided by the definition of material implication.

The solution appears to be rather simple, but difficulties remain that must be overcome. First of all, dividing conditional sentences into two categories – ones to which the definition of material implication is applicable and ones to which it is not – means we have to give up the idea of a unified theory for all conditional sentences. Secondly, we shall have to find a means to discriminate between these two kinds of conditionals. Only then can we decide whether a particular conditional sentence belongs either to the first or the second category. Finally, the decision that the definition of material implication is not applicable to conditionals belonging to the second category means that, for now, this category is deprived of evaluation criteria. If we want to evaluate all argumentatively used conditionals, we shall have to find out what evaluation criteria could be used for conditionals in this second category.

Giving up the idea of a unified theory of conditionals is something that has been already proposed by some logicians. D. Lewis (1973) for instance has argued that indicative and subjunctive conditionals need different accounts. Also Gibbard argues that one should differentiate between two types of conditionals: epistemic and nearness conditionals. He argues that possible world semantics is concerned with the former while probability theory refers to the latter (1980: 211). The step to differentiate between various types of conditionals is easily made. But a fundamental question remains: how can we differentiate between conditionals in an easy and univocal way?