UNIVERSITY OF AMSTERDAM

UvA-DARE (Digital Academic Repository)

Absolute Contradiction, Dialetheism, and Revenge

Berto, F.

DOI
10.1017/S175502031400001X
Publication date
2014
Document Version
Final published version
Published in
Review of Symbolic Logic

Link to publication

Citation for published version (APA):
Berto, F. (2014). Absolute Contradiction, Dialetheism, and Revenge. *Review of Symbolic Logic*, *7*(2), 193-207. https://doi.org/10.1017/S175502031400001X

General rights
It is not permitted to download or to forward/distribute the text or part of it without the consent of the author(s) and/or copyright holder(s), other than for strictly personal, individual use, unless the work is under an open content license (like Creative Commons).

Disclaimer/Complaints regulations
If you believe that digital publication of certain material infringes any of your rights or (privacy) interests, please let the Library know, stating your reasons. In case of a legitimate complaint, the Library will make the material inaccessible and/or remove it from the website. Please Ask the Library: https://uba.uva.nl/en/contact, or a letter to: Library of the University of Amsterdam, Secretariat, Singel 425, 1012 WP Amsterdam, The Netherlands. You will be contacted as soon as possible.

UvA-DARE is a service provided by the library of the University of Amsterdam (https://dare.uva.nl)

Download date:04 Dec 2021

THE REVIEW OF SYMBOLIC LOGIC
Volume 7, Number 2, June 2014

ABSOLUTE CONTRADICTION, DIALETHEISM, AND REVENGE

FRANCESCO BERTO
Department of Philosophy, University of Amsterdam
and
Northern Institute of Philosophy, University of Aberdeen

Abstract. Is there a notion of *contradiction*—let us call it, for dramatic effect, "absolute"—making *all* contradictions, so understood, unacceptable also for dialetheists? It is argued in this paper that there is, and that spelling it out brings some theoretical benefits. First it gives us a foothold on undisputed ground in the methodologically difficult debate on dialetheism. Second, we can use it to express, without begging questions, the disagreement between dialetheists and their rivals on the nature of truth. Third, dialetheism has an operator allowing it, against the opinion of many critics, to rule things out and manifest disagreement: for unlike other proposed exclusion-expressing-devices (for instance, the entailment of triviality), the operator used to formulate the notion of absolute contradiction appears to be immune both from crippling expressive limitations and from revenge paradoxes—*pending* a rigorous nontriviality proof for a formal dialetheic theory including it.

> Nothing is, and nothing could be, literally both true and false. [...] That may seem dogmatic. And it is: I am affirming the very thesis that [the dialetheists] have called into question and—contrary to the rules of debate—I decline to defend it. Further, I concede that it is indefensible against their challenge. They have called so much into question that I have no foothold on undisputed ground. So much the worse for the demand that philosophers always must be ready to defend their theses under the rules of debate.
>
> – David Lewis, *Logic for Equivocators*

§1. Debating dialetheism. In his 1969 criticism of Hegel and Marx's "dialectical logic", Popper observed that arguing against someone who accepts contradictions is methodologically puzzling. Let $\mathrm{T} = \{A_1, \ldots, A_n\}$ be a theory or belief set. One criticizes the T-theorist, or T-believer, by inferring from premises in $\{A_1, \ldots, A_n\}$, via rules of inference he accepts, some B he rejects. A standard *reductio* move takes $B = \sim A_i$, $1 \leq i \leq n$. But the Marxist, who accepts "dialectical contradictions" in reality, can be unyielding: he can maintain his T without releasing A_i, and take $\sim A_i$ on board too. One who finds contradictions acceptable may not revise beliefs on pain of contradiction. Popper saw in this the death of criticism, freedom and democracy.

Long before the cold war, Aristotle had stated in his *Metaphysics* (1005b 25-6) that, when someone claims "For some A, A and not-A are both true" (the villain was, in that

Received: October 20, 2012.

© Association for Symbolic Logic, 2014
doi:10.1017/S175502031400001X

case, Heraclitus), we should wonder if one actually thinks what one says. Contemporary followers of Aristotle also wonder whether, when making such claims, the *dialetheist*—as we nowadays call one who accepts contradictions: see Berto (2007), Berto & Priest (2013)—plays tricks with the meaning of "not", or with that of "true":

> The fact that a logical system tolerates A and $\sim A$ is only significant if there is reason to think that the tilde means 'not'. Don't we say 'In Australia, the winter is in the summer', 'In Australia, people who stand upright have their heads pointing downwards', 'In Australia, mammals lay eggs', 'In Australia, swans are black'? If 'In Australia' can thus behave like 'not' [...], perhaps the tilde means 'In Australia'? (Smiley, 1993, p. 17)[1]

> [The dialetheist's] 'truth' is meant to be *truth*, his 'falsity' is meant to be *falsity* [and] his 'contradictories' are meant to be *contradictories*. Yet [...] while 'truth' and 'falsity' are only subcontraries in [the dialetheist's] language, that does not show, in any way, that *truth* and *falsity* are only subcontraries. For no change of language can alter the fact, only the mode of expression of them, as we saw before. And one central fact is that *contradictories* cannot be true together—by definition. (Slater, 1995, pp. 452–453)

When philosophers dispute on the content of basic concepts, discussions notoriously face methodological impasses. We cannot inspect such notions as *predication*, *truth, negation*, etc., without resorting to them. It is then hard to decide when some party starts to beg the question, or who carries the burden of proof. It is not easy to tell whether a nonstandard account of an operator carries with it a real disagreement with the mainstream view of *that* operator, or it just characterizes something else using the same name. A typical symptom of the situation is the abundant use of *italics*, as in the quote above, to stress that a wannabe-notion is not the real *notion*.[2] The dialogue on dialetheism easily gets squeezed between change-of-meaning Quinean qualms and reciprocal charges of question-begging. One understands David Lewis' surrender, quoted at the beginning of this paper.[3]

§2. Rejection, arrow-*falsum*, explosion. To improve the debate, we need that "foothold on undisputed ground" Lewis felt he lacked. We can approach the issue via what has been called the *exclusion problem* for the dialetheist. As many critics have noticed (see e.g., Batens, 1990; Parsons, 1990; Littman & Simmons, 2004; Shapiro, 2004; Berto, 2008), the dialetheist's aforesaid ability to swallow contradictions may turn out to be, in fact, a Pyrrhic victory: he may not manage to avoid things he does not want, and/or to express his avoidance. In the dialetheist's mouth, "$\sim A$" may not rule out A, since for him

---

1 Peter van Inwagen once told me that the in-Australia-operator joke is due to R.L. Sturch.

2 Tappenden (1999) and Varzi (2004) talk of the wide use of what they call the Argument from Italics, in debates on nontruth-functional (nonadjunctive, supervaluational) accounts of conjunction and disjunction: "You say 'Either A or B' holds; then *either A or B* (stamp the foot, bang the table!) must hold!"

3 For another quotation: "To conduct a debate, one needs common ground; and in this case [scil. the debate on the Law of NonContradiction], the principles *not* in dispute are so very much less certain than noncontradiction itself that it matters little whether or not a successful defence of noncontradiction could be based on them" (Lewis, 2004, p. 176).

both A and $\sim A$ may obtain.[4] Nor are "A is false" or "A is not true" going to help: they may not rule out A's being true either.

Priest has proposed a pragmatic approach: according to him, the dialetheist can rule things out by *rejecting* them. We can take rejection as a mental state a cognitive agent, k, can entertain towards a sentence—or rather, the expressed proposition. Rejection is the contrary of acceptance or belief (or, belief to a degree above a given threshold):[5] k's rejecting something is to be seen as k's positively refusing to believe it. The linguistic acts manifesting acceptance and rejection are, respectively, assertion and denial. The mental and linguistic pair can diverge in important respects, but we can safely run them together for our purposes. Take the pragmatic operators, "$\vdash_k$" and "$\dashv_k$" ("k accepts/asserts (that)", "k rejects/denies (that)"). Now rejection/denial is often taken to boil down to the acceptance/assertion of negation via the so-called Frege–Geach reduction:

$$\text{(FG)} \dashv_k A =_{\text{df}} \vdash_k \sim A.$$

FG expresses the mainstream position on the connection between acceptance/assertion, rejection/denial, and negation:

> To deny a statement is to affirm another statement, known as the negation or the contradictory of the first. (Quine, 1951, p. 1)

> After all, disbelief is just belief in the negation of a proposition (Sorensen, 2003, p. 153)

But suppose A is a dialetheia. It is a basic principle of rationality that we ought to accept something when we have good evidence of its truth. Then we ought to accept $\sim A$ without thereby rejecting A: if there are dialetheias, FG has to go. Thus, Priest (2006, p. 104) has claimed that rejection is to be taken as a primitive, *sui generis* act directly pointing at (the proposition expressed by) A.

FG can be disputed, in fact, also independently from dialetheism. For dually, if there are truth-value gaps and A is one, we should reject it without thereby accepting $\sim A$ (see e.g., Parsons, 1984; Field, 2008, pp. 73–74). Nor does one need to be either a gapper or "paracompletist", or a glutter or dialetheist, to dispute FG.[6] The reduction may be problematic even for a philosopher who sticks to classical logic: for unlike the pair consisting of a sentence and its (classical) negation, acceptance and rejection need not be exhaustive. Whereas an *ideal* cognitive agent might be expected to be able to make up its mind on any assertable content, real agents are, for many A, not in the position to either accept or reject A rationally, due to their lacking information, or to their bounded cognitive capacities.

So the dialetheist can rule things out by rejecting them, with no need to pass through a negation not strong enough to support exclusion. Besides, no dialetheically intractable revenge Liar paradox formulated via rejection/denial is expected, for "being a force operator [$\dashv_k$] has no interaction with the content of what is uttered" (Priest 1987, p. 108): we are dealing with a pragmatic device, not with a logical connective.

---

[4] "Paraconsistent negation [...] does not rule out the sentence that is negated and is intended not to rule this out. This is not an objection against paraconsistent negation, just as is no objection to a violin that it is useless to hammer nails in the wall. But if we want to express the rejection of some sentence, we cannot recur to paraconsistent negation" (Batens, 1990, p. 223).

[5] Dialetheic treatments of degrees of belief in a proposition and its negation in terms of probability theory have been proposed by Priest (1987, chap. 7) and Beall (2009, chap. 5).

[6] An anonymous referee suggested the following point.

But as pointed out by Shapiro (2004, pp. 339–340), Field (2008, pp. 387–388), and others, precisely this makes $\dashv_k$ not fully satisfactory, due to the well-known expressive limitations of pragmatic operators. We can straightforwardly reject a whole claim, and perhaps also reject it conditionally on something else: we may reject A conditionally on assumption B when we have a conditional degree of belief $P(A|B)$ below a certain threshold. However, we cannot make our rejection work as the antecedent of a conditional, nor embed it in more deeply structured claims. This makes it difficult to conduct and express our disputes on what should and should not be ruled out: rejection/denial does not allow us to say everything we need to say.

Crippling expressive limitations aside, notice for later purposes how the Priestian proposal crucially needs acceptance and rejection to *rule out* each other. If rejection failed to preclude acceptance, that is to say, if agent k could both accept and reject the same claim, *simul, sub eodem*, then we would be back where we started:

> Someone who rejects A cannot simultaneously accept it any more than a person can simultaneously catch a bus and miss it, or win a game of chess and lose it. If a person is asked whether or not A, he can of course say ‘Yes and no’. However this does not show that he both accepts and rejects A. It means that he accepts both A and its negation. Moreover a person can alternate between accepting and rejecting a claim. He can also be undecided as to which to do. But do both he can not. (Priest, 1989, p. 618)

Another tool sometimes proposed as an exclusion-expressing device is arrow-*falsum*, “$\rightarrow \bot$”, where $\rightarrow$ is a detachable conditional and $\bot$ is or entails something unacceptable also for a dialetheist (see Priest, 1987, p. 291; 2006, pp. 105–106). Let “*Tr*” be a predicate expressing transparent truth for the relevant language, that is, such that for any A, $Tr\langle A\rangle$ and A (with $\langle A\rangle$ the appropriate name of A) are interchangeable in all (nonopaque) contexts. Not all dialetheic accounts of truth are transparent: the one in Priest (1987) isn’t, whereas Beall’s (2009) theory (called BXTT = transparent truth theory based on the weak relevant logic B, plus Excluded Middle) is; sticking to transparent *Tr* simplifies the exposition while prejudging nothing substantive for our purposes. In the semantics of the basic paraconsistent-dialetheic logic LP, augmented with a detachable $\rightarrow$, we can take *Tr* as governed by introduction/elimination rules:

$$\text{(T-In)}\ \ A \models Tr\langle A\rangle$$

$$\text{(T-Out)}\ \ Tr\langle A\rangle \models A.$$

Now let $\bot = \forall x Tr(x)$. Then the dialetheist may try to rule out A by uttering: “$A \rightarrow \bot$”; for the trivial claim that everything is true is too much even for him (though it may not be too much for *everyone*; trivialism has actually, and interestingly, been defended in the literature: see Kabay, 2010).

But this won’t work either, for various reasons. First, a dialetheist from Hartford may want to disagree on “Hartford is in Rhode Island” on the basis of plain empirical evidence. It seems strange that he can only express this by claiming, “If Hartford is in Rhode Island, then the absurd *falsum* obtains”. Can’t we have slightly gentler forms of disagreement? The purely accidental falsity of A does not seem to warrant $A \rightarrow \bot$, especially since a detachable arrow for the dialetheist cannot be a mere material conditional.

More decisively, arrow-*falsum* suffers a Curry problem, as highlighted by Field (2008, pp. 388–389). Take the standard Curry sentence:

$$K = Tr\langle K\rangle \rightarrow \bot$$

(informally: "Anything follows from my truth").[7] This cannot be a dialetheia, for its having a designated value would give us trivialism. So we must rule out K. However, we cannot express this by uttering "$K \rightarrow \bot$", for this just is K, and will give us $\bot$ by *modus ponens*, which the detachable arrow licenses. What does the trick is the weaker:

$$(A \rightarrow \bot) \vee (A \rightarrow (A \rightarrow \bot)).$$

However, this triggers its revenge, too, via another Curry-like sentence:

$$K_1 = (Tr\langle K_1 \rangle \rightarrow \bot) \vee (Tr\langle K_1 \rangle \rightarrow (Tr\langle K_1 \rangle \rightarrow \bot)).$$

To rule this out, we now need a still weaker:

$$(A \rightarrow \bot) \vee (A \rightarrow (A \rightarrow \bot)) \vee (A \rightarrow (A \rightarrow (A \rightarrow \bot))).$$

This triggers

$$K_2 = (Tr\langle K_2 \rangle \rightarrow \bot) \vee (Tr\langle K_2 \rangle \rightarrow (Tr\langle K_2 \rangle \rightarrow \bot)) \vee (Tr\langle K_2 \rangle \rightarrow (Tr\langle K_2 \rangle \rightarrow (Tr\langle K_2 \rangle \rightarrow \bot)))$$

... And we are off and running (the construction is the perfect dual, for paraconsistent dialetheists, of the hierarchy of stronger and stronger "determinately-true" operators for paracompletists, described in Sections 15.2 and 15.3 of Field's book).

Arrow-*falsum* at most *approximates* exclusion. As the index on K increases, the corresponding veto formulated via arrow-*falsum* gets endlessly strictly weaker. It does not matter what system of ordinal notation one uses for the indexing: following the arrow-*falsum* route, no catch-all exclusion-expressing device is expected.[8] In his critical review of Field's book, Priest (2010, p. 136) grants the point to Field, and claims to "have no inclination to go down this path" (or, one may say, up this ladder).

The problem generalizes: *no* sentential operator, \$, that applied to A outputs a \$$A$ which has a designated value only if A doesn't, can be a dialetheic exclusion-expressing device. For then we have the explosive logical consequence:

---

[7] For a friendly introduction to the Curry paradox, see Beall (2008). The literature on the topic makes for a burgeoning field: the paradox is often taken to require a revision of the classical operational, and perhaps structural, principles governing the conditional (and maybe logical *entailment* itself—see the following footnote). It has been therefore studied especially in the areas of relevant and substructural logics: see for example, Meyer *et al.* (1979); Priest (1987, chaps. 6 and 19); Slaney (1989); Restall (2000, chap. 2); Zardini (2011); Shapiro (2011); Beall & Murzi (forthcoming).

[8] Beall & Murzi (forthcoming) highlight that the traditional conditional-Curry paradox has a largely isomorphic entailment-Curry counterpart, formulated by expressing the logical entailment relation in the relevant language. Field (2008), Beall (2009), and others, have proposed to deal with the conditional-Curry paradox by dismissing Conditional Proof or (half of) the Deduction Theorem for the conditional, but the corresponding solution for the entailment-Curry paradox is less plausible. Beall and Murzi suggest (without mandating) that rejecting structural Contraction, thus admitting that entailment is noncontractive, may be a way to address the entailment-Curry paradox: the analogy with the approach to the conditional-Curry paradox, which rejects contraction for the arrow, suggests this route. I mention this recent development in the debate on Curry only to set it aside, for it brings no interesting news for the dialetheic quest for an exclusion-expressing device. If one tries to rule out A via the arrow-*falsum* trick, one faces the (conditional-)Curry problem described above via "Anything follows from my truth". Should one try to rule out A via an entailment-*falsum* trick, one would face an entailment-Curry revenge in the shape described by Beall and Murzi ("The argument from me to triviality is valid"). This would be no progress for the dialetheist.

$$\{A, \$A\} \models \perp.$$

We can then always build the relevant revenge sentence, which gives triviality, using \$. The same happens if, as per a view entertained (but not endorsed) by Beall, we introduce a positive operator, J, such that JA aims at expressing that A is just true, true and not false: if "[A]'s being just true should 'rule out' its being false" (Beall, 2009, p. 63), we still get $\{JA, \sim A\} \models \perp$. Now the revenge is:

$$L = J{\sim}Tr\langle L\rangle^9.$$

§3. Primitive exclusion. The upshot of this situation, it seems to me, is that exclusion had better not be characterized as a *logical* concept at all. Specifically, it had better not be defined as the logical entailment of an explosive notion. As Priest (2006, p. 107) acknowledges, for instance, $A \rightarrow \perp$ is still *logically* compatible with A, given that LP has its so-called "trivial model": if all atomic sentences of the relevant language are both true and false (or both true and false at the base world where truth is evaluated, if we have a worlds semantics for LP plus arrow), then all sentences are true and false.

Now a dialetheist can reject the trivial model, as Beall (2009, p. 34) does, on the ground of its theoretical uselessness.[10] But such a rejection is not strictly *logically* motivated, in the following sense. It is an assumption packed in the semantics of classical logic that truth and falsity (*in* a model, which in our context are close enough to truth and falsity to make no significant difference anyway) do not overlap. So no model in which some (atomic) formula or other gets assigned both values is a classically *admissible* interpretation. Such interpretations, among which is the trivial model, are ruled out for reasons that don't go beyond classical logic: they are simply disallowed by the logical theory (or, admittedly, its semantics). But the trivial model *is* LP-admissible in this sense:[11] it is not excluded purely on logical grounds in a typical dialetheic framework.[12]

---

[9] Further reflections on the "just-true" problem can be found in Beall (2009, chap. 3). I will stop the discussion here to give room to the paper's positive proposal, but I mention a hierarchy of partial-limited "just-true" operators $J_0, J_1, \ldots$, described on pages 58 and 59 of that chapter: as n increases, each J_n applies to a larger fragment of the relevant language including J_{n-1}, with J applying to the ground, truth-predicate-free fragment. The obvious limit of such a hierarchy seems to be, again, the lack of a single cover-all notion of *being just true*, that is, true and ruling out falsity. The hierarchy looks like an asymptotic approximation to the concept.

[10] As pointed out by a referee.

[11] See for example, the presentation of LP and FDE (the logic of First Degree Entailment, due to Belnap and Dunn) in Priest's *Introduction to NonClassical Logic*, chapters 7 and 8: one gets "classical logic" from FDE, which is itself a sub-logic of LP, by ruling out, that is, by declaring inadmissible, certain interpretations: "If an interpretation satisfies both *Exclusion* [scil., No atomic formula is both true and false] and *Exhaustion* [No atomic formula is neither] [. . .], we have what is, in effect, an interpretation for classical logic." (Priest 2001, p. 149)

[12] This much seems suggested by the following Priestian quotation: "I am frequently asked for a criterion as to when contradictions are acceptable and when they are not. It would be nice if there were a substantial answer to this question—or even if one could give a partial answer, in the form of some algorithm to demonstrate that an area of discourse is contradiction free. But I doubt that this is possible. Nor is this a matter for surprise. Few would now seriously suppose that one can give an algorithm—or any other informative criterion—to determine when it is rational to accept something. There is no reason why the fact that something has a certain syntactic form—be it $p \wedge \neg p$ or anything else—should change this. One can determine the acceptability of any given contradiction, as of anything else, only on its individual merits." (Priest, 1998, p. 423).

Indeed, I submit that exclusion had better not be defined *at all*. Exclusion should be taken as a primitive concept with a general metaphysical import. There are reasons for so taking it. First, that there must be primitive notions is uncontroversial: were all notions definable in terms of others, we would face either a bad infinite regress, or a (large) *circulus in definiendo* (on this, see Williamson, 2007, pp. 50–51). Definitions have to come to an end.

There being primitive concepts, no fool-proof decision procedure for them is likely to be available. Many take the concept of *set*, for instance, as a candidate primitive. We say that a set is an aggregate or collection of objects, but that is no definition To elucidate the concept, we give examples and hope for the best. Kripke notoriously claimed of the notion of *reference* that "philosophical analyses of some concepts like reference, in completely different terms which make no mention of reference, are very apt to fail" (Kripke, 1972, p. 94). In *The Question of Ontology*, Kit Fine ascribed the status of primitive to the notion of *reality*: "we seem to have a good *intuitive* grasp of the concept", but he does "not see any way to define the concept of reality in essentially different terms" (Fine, 2009, p. 175).[13]

If *is a set*, *refers to*, and *is real* are candidate primitives given their basic role in our understanding of mathematics, language, and actuality, then *excludes* is so basic to our experience of the world, that it can be one as well. It is likely to show up in the most rudimentary thing new-borns learn to do: distinguishing objects, recognizing a border between something and something else, or acknowledging that this thing's being *here* rules out its simultaneously being *there*. We know that if an ordinary material object is uniformly green, it cannot simultaneously be uniformly red; that if it's shorter than one inch then it cannot be longer than a mile. The notion is shared by the dialetheist, of course – we had examples of exclusion from his mouth in the passage flagged above, for instance, x's catching the bus and x's *simul, sub eodem* missing the bus. Besides, that passage showed that some incompatible situations, such as agent k's accepting a claim and agent k's rejecting that very claim, are *needed* for Priest's aforementioned pragmatic strategy to make sense (a point stressed in Berto, 2008 already).

There is something in exclusion being characterized as a primitive completely general and, in this sense, metaphysical (contrast logical) feature of our experience the world. For this makes plausible the view that the holding of worldly exclusion relations is (only) ascertainable fallibly and *a posteriori*. We have seen that, in a sense, there is no purely logically warranted exclusion for the dialetheist: given any A, in LP there is a model—if not others, the trivial one—both for A and for any other sentence. So "there is no *logical* guarantee against a person being a trivialist" (Priest, 2006, p. 107). But when we believe that some exclusion relation holds, and we find out that we were wrong, we never play the game of inferring *falsum* via an explosive logical entailment. We may have had defeasible evidence supporting the view that a given property P, say, *being a mammal*, is incompatible with a given property Q, say, *being en egg-layer*. When we went down under and found a counterexample, the least plausible move we could make was to logically infer $\perp$. An exclusionary hypothesis simply is *always withdrawn* when refuted.

[13] To be sure, none of these is uncontroversially taken as a primitive notion by everyone (for any candidate primitive one could find, I guess, some philosopher who has tried to give a definition, reducing it to something else). As one referee appropriately pointed out, although sethood is normally taken as primitive, precisely some dialetheic and para-classical theories of sets take the notion as definable. Similarly, against the Kripkean suggestion, some have tried to provide reductive accounts of reference, for example, famously, Fodor (1975).

Now we can exploit the primitive, shared concept of exclusion by attempting to define a dialetheic-friendly exclusion-expressing device, not yielding dialetheically intractable revenges, and via which we get our Lewisian foothold on undisputed ground: a notion of *contradiction* making any contradiction unacceptable by dialetheists as well as their rivals. As we will see, such a notion may bring many theoretical benefits.

§4. Absolute contradiction. We resort, in fact, to the idea of *minimal* incompatibility with or exclusion of something—crudely: what follows from anything ruling out that thing. The idea has been explored as a way to characterize logical negation, for instance, in the quantum logic framework of Birkoff & von Neumann (1936) and Goldblatt (1974). Goldblatt's semantics for quantum logic consists of frames whose points are seen as outcomes of possible quantum physics measurements, and the relation between pairs of outcomes precluding one another, usually called "perp", is used to phrase the semantics of negation. Mike Dunn has proposed to "define negation in terms of one primitive relation of incompatibility [...] in a metaphysical framework" (Dunn, 1996, p. 9), and Restall (1999) has developed Dunn's framework to provide the controversial De Morgan negation of relevant logics with a plausible intuitive reading.

For reasons that should be clear by now, though, *we* are not after a logical, sentential operator. That would take us back to the dilemma explored above: either the operator will not be strong enough to support exclusion, or it will suffer from revenge problems. Rather, we resort to an idea from JC Beall's *Spandrels of Truth* and use, albeit for different purposes from his, a predicative underlining functor, "_" (see Beall, 2009, p. 108—Beall uses an overline rather than an underline notation).

Let $\bullet$ stand for our primitive exclusion relation, which can here be taken as holding between properties: "$P \bullet Q$" is to be read as "Properties P and Q are incompatible", or as "The having of P excludes the having of Q", or as "Being P rules out being Q". Talk of properties should not be taken as metaphysically too committing (we could in fact rephrase the view in a strictly nominalistic but more cumbersome fashion). We mean by "property" what Field has called *conceptual* properties, and have as our background a naïve property theory: "there is a conceptual property corresponding to every intelligible predicate" (Field, 2008, p. 3) in our language.

Taking a property P as input, the underline operator outputs its minimal incompatible $\underline{P}$, the having of which is the having of a feature ruling P out. A given property, we may safely suppose, could have various incompatible mates: it may rule out a whole bunch of features. *Being (uniformly) blue*, for instance, excludes *being red*, *being yellow*, *being white*, etc. Then one can then talk of an incompatibility set for P say $\mathrm{I}_P = \{Q|Q \bullet P\}$.

Now one difference between Beall and us is that Beall sees (the counterpart, in his account, of) the underline mate $\underline{P}$ of P as atomic: he talks of it as P's "atomic *contrary*" (Ibid). While the underline mate has indeed much in common with what is traditionally included under the notion of contrariety, we instead take it as *defined* via the primitive $\bullet$. That $\underline{P}$ is the having of this or that feature incompatible with P may be captured by seeing $\underline{P}$ as picking out the join of I_P, $\bigvee\{Q|Q \bullet P\}$. When I_P is finite, it amounts to an ordinary disjunction $Q_1 \vee \ldots \vee Q_n$ with $Q_1, \ldots, Q_n \in \mathrm{I}_P$. But when I_P is infinite (which may happen: think again about the spectrum of colours, and incompatibilities between them), we may want to avoid infinitary disjunctions. Which we can, if we endorse the more general characterization:

$$(\mathrm{Df\_})\ \underline{P}x =_{\mathrm{df}} \exists Q(Qx \,\&\, P \bullet Q).$$

Being $\underline{P}$ means having some feature or other excluding P (when we move from a specific Q incompatible with P to $\underline{P}$, we have an understandable loss of information: this is only to be expected, talking of a *minimal* incompatible). If $|Px|$ stands for the extension of Px, then $|Px| \cap |\underline{P}x| = \varnothing$.[14] And given the standard LP three-valued negation with dialetheias as fixed points (if A is true, $\sim A$ is false; if A is false, $\sim A$ is true; and if A is a dialetheia, $\sim A$ is, too), such a negation is then entailed by the corresponding exclusion or minimal incompatible:

$$\text{(Ent1) } \underline{P}x \models \sim Px.$$

However, we do not stipulate that $|Px| \cup |\underline{P}x| =$ the whole domain of objects. In general, we don't have the converse to Ent1,

$$\text{(Ent2) } \sim Px \models_{?} \underline{P}x.$$

This tracks the insight that a given object x may fail to be P, without thereby having any feature positively ruling out P. Which seems to make intuitive sense. Let $Px = x$ *is translucent*, so say that $\underline{P}x = \exists Q(Qx \;\&\; P \bullet Q) = x$ *is opaque* (x has some feature positively preventing translucency). Let $h =$ the concept *horse*. Then $\sim Ph,$ it's not the case that the concept *horse* is translucent – as one may expect from an abstract object. But that doesn't entail $\underline{P}h$, the concept is opaque – for the very same reason. Or let $Px = x$ *is happy*, so say that $\underline{P}x = \exists Q(Qx \;\&\; P \bullet Q) = x$ *is unhappy* (x has some feature ruling out happiness). Let $h =$ High Street. Then $\sim Ph,$ it's not the case that High Street is happy. It does not follow that $\underline{P}h$: consciousless streets cannot experience sadness. Notice that failures of $Px \vee \underline{P}x$, besides not being failures of $Px \vee \sim Px$ which is logically valid in LP, need not be due to vagueness of any kind: High Street need not be borderline sad in order for it to fail to be happy.

Now we have (what we call, for dramatic effect) our absolute contradiction, which simply is:

$$\text{(AC) } Px \;\&\; \underline{P}x.$$

Unlike what happens with $Px \;\& \sim Px$, also for the dialetheist AC should hold *for no P and no x* Once *ruling out* has been understood as the basic, primitive notion of exclusion described above, nothing can have both a property and anything ruling out that property.

[14] This is phrased in the set-theoretic metatheory of the target theory—which, as one referee pointed out, may raise the issue whether the metatheory itself ought to be dialetheic. The reply depends on the dialetheist one is dialectically engaged with. Some more moderate "merely semantic" dialetheists phrase the semantics of their truth theories using classical set theory (e.g., Mares, 2004; Beall, 2009). They admit "semantic" dialetheias like the Liar, but deny there being inconsistent sets, as per the typical paraconsistent set theories. Since Beall's (2009) transparent theory of truth is my background (mainly for the sake of simplicity, as explained), I stick to this option. But more thoroughgoing dialetheists like Priest want the metatheory to be itself dialetheic. How this relates to the exclusion-expressing problem depends on how the dialetheic set-theoretic framework is developed. As far as I can see, also Priest usually provides (e.g., in *In Contradiction*, chap. 9) ordinary model theories, done within standard set theory, for his dialetheic semantics. A verdict on this issue should wait for precise applications of the (currently being developed: see Weber, 2010, 2012) inconsistent theories of sets to dialetheic theories of truth. I should also add that an intermediate position is plausible: accept that the right theory of sets has an unrestricted Comprehension Principle for sets and underlying paraconsistent logic, but stress that the modicum fragment of set theory needed to phrase the semantics of such theories as LPTT plus arrow, BXTT, etc., is consistent, and full classical logic applies in this restricted area.

Contradictions in the old negation-involving sense can be true for the dialetheist, a relevant case being provided by the various Liars (and their negations); but no absolute contradiction can. We have, in this sense, some unquestionable ground in the debate on dialetheism: a notion of *contradiction*, AC, unacceptable by any involved party for any x and P

Next, by means of "_" we can express in a nonquestion-begging fashion exactly what the divergence between dialetheists and their rivals on the concept of truth consists in—thus making implausible the view that foes and friends of consistency are normally talking past each other, or that either party is just victim of a conceptual confusion,[15] on this issue. For in general the disagreement between dialetheists and supporters of consistency has to do with the *extension* of a notion (whose intension) they both grasp and share: the notion of exclusion.

Those who deny that anything could be both true and false (or untrue) take truth and falsity (ditto) as reciprocally exclusive features of truth-bearers, in our primitive sense of exclusion. The dialetheist claims to have counterexamples, like the Liars. Let $L = {\sim}Tr\langle L\rangle$, the so-called strengthened Liar.[16] For the dialetheist L is both true and untrue,

$$Tr\langle L\rangle \& \sim Tr\langle L\rangle.$$

However, precisely because of this, *L is a relevant counterexample to* Ent2:

$${\sim}Tr\langle L\rangle \not\models \underline{Tr}\langle L\rangle.$$

For the dialetheist, being untrue is having no feature incompatible with truth. We have our old negation-involving contradiction; but we don't have the corresponding absolute contradiction or instance of the unacceptable AC, that is, $Tr\langle L\rangle \ \& \ \underline{Tr}\langle L\rangle$. For the dialetheist, in general,

$$Tr\langle{\sim}A\rangle \not\models \underline{Tr}\langle A\rangle$$
$${\sim}Tr\langle A\rangle \not\models \underline{Tr}\langle A\rangle.$$

That is to say: falsity and untruth fail to belong in $\{P|\ P \bullet \text{Truth}\}$.

§5. Revenge? Now for the final (and biggest) problem. Once the dialetheist has (or, cannot but accept something like) "_", can't we use it to obtain our revenge on him? The minimal incompatible with something, that is, $\underline{\text{something}}$, is what rules out in our primitive sense, the thing at issue. Can't we get a self-referential sentence, "I am $\underline{\text{true}}$", for a dialetheically intractable revenge?

Recall the dilemma explored above: pragmatic rejection/denial, $\dashv_k$, is free from revenge due to its being a hardly embeddable force operator, but for this very reason is expressively limited. The underline device is not a pragmatic or force operator for a linguistic or mental act: it is an operator on *contents*, outputting the minimal incompatible of its input; it is thus fully embeddable. On the other hand, nonpragmatic and fully embeddable candidate exclusion-expressing tools like arrow-*falsum*, $A \to \bot$, or just-true/just-false operators like JA, cannot perform their job properly, in Beall's terminology, due to their trivializing

[15] As claimed by Slater (2007).

[16] Field (2008, p. 23), has recently questioned the terminology. Given the transparency of truth, which is (innocuously) assumed above, plus the standard view that falsity just is the truth of negation, the strengthened Liar and the standard one turn out to be equivalent anyway.

"spandrels": the inescapable by-products of their introduction into a language.[17] It is natural to ask, therefore, whether the underline operator can do better.

Prima facie, it can: "_" is not subject to revenge problems in the way its predecessors were. The relevant spandrel in this case is:

$$\text{(UL) } \underline{L} = \underline{Tr}\langle\underline{L}\rangle$$

(the "Underlined Liar"—though calling it a Liar may be inappropriate, as we are about to see). Given (Df_), this claims: $\exists Q(Q\langle\underline{L}\rangle \ \& \ Tr \bullet Q)$, that is, "I have some feature ruling out truth", or "I am incompatible with truth". Now if $\underline{L}$ were true, or a dialetheia, we would be in trouble, for the above T-In entailment would deliver $Tr\langle\underline{L}\rangle \ \& \ \underline{Tr}\langle\underline{L}\rangle$, against the unacceptability of AC.

However, the dialetheist can take $\underline{L}$ as simply *false*: $Tr\langle\sim\underline{L}\rangle$; from which follows, because "*Tr*" is transparent, that is, via T-In and T-Out, that it should also be taken as not true, $\sim Tr\langle\underline{L}\rangle$ The dialetheist does not have to take $\underline{L}$ as a dialetheia or, in general, as having a designated value, just as he does not have to (and had better not) take the Curry sentence as a dialetheia or, in general, as having a designated value (and standardly goes for a contraction-free conditional to deal with Curry: see again Priest, 1987, chaps. 6 and 19, Beall, 2009, chap. 2). As for why $\underline{L}$ need not be taken as designated: $\underline{L}$ claims of itself to have a truth-excluding feature; but for the dialetheist, recall, truth and falsity (or untruth) are *compatible*—denying which would beg the question against him, in the dialectical context, and take us back to the aforementioned methodological impasse. Some truth-bearers live in the intersection of truth and falsity, notably the Liar L But that intersection of truth and falsity is not $\underline{L}$'s home too (nor is it the Curry sentence's home) $\underline{L}$ just falsely claims to have a truth-excluding feature, and its plain falsity does not entail its having a truth-excluding feature. As in general $Tr\langle\sim A\rangle \not\models \underline{Tr}\langle A\rangle$ and $\sim Tr\langle A\rangle \not\models \underline{Tr}\langle A\rangle$, the plain falsity or untruth of $\underline{L}$ need not entail an absolute contradiction.

It is instructive to see how a couple of attempted revenge proofs of $Tr\langle\underline{L}\rangle \ \& \ \underline{Tr}\langle\underline{L}\rangle$ fail. One may start thus (Lemmon-style):

1	(1)	$Tr\langle\underline{L}\rangle$	Supposition
1	(2)	$\underline{L}$	1, T-Out
1	(3)	$\underline{Tr}\langle\underline{L}\rangle$	2,UL
1	(4)	$Tr\langle\underline{L}\rangle \ \& \ \underline{Tr}\langle\underline{L}\rangle$	1, 3, &-In

This does not prove an absolute contradiction yet: (4) is not deduced as a theorem, but depends on supposing (1). This shows only that, if we assume that $\underline{L}$ is true, then we get one absolute contradiction.

A natural attempt at a proof by cases would now exploit the fact that (each instance of) $A \vee \sim A$ is logically valid in LP, or in BXTT. So the attempt would go on as:

5	(5)	$\sim Tr\langle\underline{L}\rangle$	Supposition

[17] Beall (2009) defines "spandrels of x" as the "inevitable, and frequently unintended, by-products of introducing x into some environment" (p. 5). He then calls the Curry sentence a spandrel: "Spandrels such as 'If this sentence is [true], then every sentence is true' pose a problem if the given conditional detaches..." (p. 27). As we know, whereas the Liar sentence gets a designated value in the dialetheic treatment (it's a dialetheia), the Curry sentence does not. So apparently some spandrels have dialetheically designated values, some others don't.

But from this no absolute contradiction follows: as (Ent2) fails, something can fail to be P without there being any feature incompatible with its being P. In the specific case: $\underline{L}$'s being false (or untrue) fails to entail its incompatibility with truth, and therefore its truth.

Or, one could try as follows. Given (Ent1), from (4) one can get:

1	(5)	$Tr\langle\underline{L}\rangle$ & $\sim Tr\langle\underline{L}\rangle$	4, Ent1
1	(6)	$\sim Tr\langle\underline{L}\rangle$	5, &-Out

But again, it stops here. One cannot infer from (6) that $\underline{L}$ is incompatible with truth (and thus true), for the usual reason: $\underline{L}$'s failing to be true does not entail its having any feature incompatible with its being true.

The Underlined Liar being undesignated, no dialetheically problematic revenge is straightforwardly expected from it for the dialetheist who uses underlining to express exclusion. But that the underline operator does not face a revenge problem produced by the *prima facie* most obvious candidate does not mean that it is guaranteed to be *robustly* revenge-free (to adapt terminology used by Restall, 1993 in his discussion on Curry-revenge). To ensure that one has avoided a triviality-inducing revenge produced by "_", one needs (a) to introduce it in the context of a well-defined formal dialetheic truth theory T, and (b) to produce a *nontriviality* proof for T: a proof that the introduction of "_" does not make the theory trivial.[18]

The background for the introduction of "_" in this paper has been Beall's (2009) transparent truth theory BXTT, but as claimed above, this was merely due to the (in my view) superior simplicity of a transparent dialetheic theory over a nontransparent one: the setting could be adapted to Priest's (1987) nontransparent theory. Now Beall (chap. 2, Appendix) *does* provide a nontriviality proof for BXTT. This relies on techniques invented by Brady (1983, 1989) to prove the nontriviality of dialetheic set theory, and originally applied to dialetheic truth theories by Priest (1991, 2002, Sections 8.1 and 8.2). Now whereas the nontriviality proof extends to Beall's overline operator, he claims that "if further constraints beyond merely being a contrary-forming connective are imposed on overline mates, then [the] argument for nontriviality may well fail" (p. 110). And unlike Beall's overline operator, "_" is not primitive but defined, using our primitive exclusion relation (and, in the "infinitary" case, by dangerous second-order-ish quantification over predicate variables!). This may or may not make an attempted nontriviality proof break down. But if the operator is characterizable within an appropriate extension of Beall's BXTT theory, and we have a nontriviality result for the theory so expanded, this will be enough to ensure that it is revenge-free.

§6. Acknowledgments. This paper was prepared within the 2013-15 AHRC project *The Metaphysical basis of Logic: the Law of Non-Contradiction as Basic Knowledge* (grant ref. AH/K001698/1), based at the Northern Institute of Philosophy, University of Aberdeen UK. Its main idea, though, first came up in 2011 during a one-year research fellowship at the Institute for Advanced Study, University of Notre Dame: thanks to Vittorio Hösle, Don Stelluto, Carolyn Sherman, and Jo Ann Norris, for making my stay in the US enjoyable,

[18] I am very grateful to a referee for pointing at this direction of further inquiry.

and to the other fellows of the Institute, for many conversations on dialetheism and contradictions.

BIBLIOGRAPHY

Batens, D. (1990). Against global paraconsistency. *Studies in Soviet Thought*, **39**, 209–229.

Beall, J. C. (2008). Curry's paradox. In Zalta, E. N., editor. *The Stanford Encyclopedia of Philosophy*. Stanford, CA: CSLI. Available from: htto://plato.stanford.edu

Beall, J. C. (2009). *Spandrels of Truth*. Oxford, UK: Oxford University Press.

Beall, J. C., & Murzi, J. (forthcoming). Two flavors of Curry paradox. *Journal of Philosophy.*

Berto, F. (2007). *How to Sell a Contradiction. The Logic and Metaphysics of Inconsistency*, Studies in Logic 6, London: College Publications.

Berto, F. (2008). Adynaton and material exclusion. *Australasian Journal of Philosophy*, **86**, 165–190.

Berto, F., & Priest, G. (2013). Dialetheism. In Zalta, E. N., editor. *The Stanford Encyclopedia of Philosophy*. Stanford, CA: CSLI. Available from: htto://plato.stanford.edu

Birkoff, G., & von Neumann, J. (1936). The logic of quantum mechanics. *Annals of Mathematics*, **37**, 823–843.

Brady, R. (1983). The simple consistency of set theory based on the logic CSQ. *Notre Dame Journal of Formal Logic*, **24**, 431–439.

Brady, R. (1989). The non-triviality of dialectical set theory. In Priest, G., Routley, R., and Norman, J., editors. *Paraconsistent Logic. Essays on the Inconsistent*, München, Germany: Philosophia Verlag, pp. 437–471.

Dunn, J. M. (1996). Generalized ortho negation. In Wansing, H., editor. *Negation. A Notion in Focus*, Berlin-New York: De Gruyter, pp. 3–26.

Field, H. (2008). *Saving Truth from Paradox*. Oxford, UK: Oxford University Press.

Fine, K. (2009). The question of ontology. In Chalmers, D., Manley, D., and Wasserman, R., editors. *Metametaphysics*, Oxford, UK: Clarendon Press, pp. 157–77.

Fodor, J. A. (1975). *The Language of Thought*. Cambridge, MA: Harvard University Press.

Goldblatt, R. I. (1974). Semantic analysis of orthologic. *Journal of Philosophical Logic*, **3**, 19–35.

Kabay, P. (2010). *On the Plenitude of Truth: A Defense of Trivialism*. Saarbrücken, Germany: Lambert Academic Publishing.

Kripke, S. (1972). Naming and necessity. In Davidson, D., and Harman, G., editors. *Semantics of Natural Language*, Dordrecht: Reidel, pp. 253–355 and 762–799, second edition, expanded ed., Oxford, UK: Blackwell, 1980.

Lewis, D. (1982). Logic for equivocators. *Noûs*, **16**, 431–441.

Lewis, D. (2004). Letters to Beall and Priest. In Priest, G., Beall, J. C., and Armour-Garb, B., editors. *The Law of Non-Contradiction. New Philosophical Essays*, Oxford, UK: Clarendon Press, pp. 176–177.

Littman, G., & Simmons, K. (2004). A critique of dialetheism. In Priest, G., Beall, J. C., and Armour-Garb, B., editors. *The Law of Non-Contradiction. New Philosophical Essays*, Oxford, UK: Clarendon Press, pp. 314–335.

Mares, E. (2004). Semantic dialetheism. In Priest, G., Beall, J. C., and Armour-Garb, B., editors. *The Law of Non-Contradiction. New Philosophical Essays*, Oxford, UK: Clarendon Press, pp. 264–275.

Meyer, R. K., Routley, R., & Dunn, J. M. (1979). Curry's paradox. *Analysis*, **39**, 124–128.

Parsons, T. (1984). Assertion, denial and the Liar paradox. *Journal of Philosophical Logic*, **13**, 137–152.

Parsons, T. (1990). True contradictions. *Canadian Journal of Philosophy*, **20**, 335–354.

Priest, G. (1987). *In Contradiction: A Study of the Transconsistent*. The Hague: Martinus Nijhoff, second edition, revised ed. Oxford, UK: Oxford University Press, 2006.

Priest, G. (1989). Reductio ad Absurdum et Modus Tollendo Ponens. In Priest, G., Routley, R., and Norman, J., editors. *Paraconsistent Logic. Essays on the Inconsistent*, München, Germany: Philosophia Verlag, pp. 613–626.

Priest, G. (1991). Intensional paradoxes. *Notre Dame Journal of Formal Logic*, **32**, 193–211.

Priest, G. (1998). What is so bad about contradictions? *Journal of Philosophy*, **8**, 410–426.

Priest, G. (2001). *An Introduction to Non-Classical Logic. From If to Is*. Cambridge: Cambridge University Press, second edition, Expanded edition, 2008.

Priest, G. (2002). Paraconsistent logic. In Gabbay, D. M., and Guenthner, F., editors. *Handbook of Philosophical Logic*, second edition, Vol. 6, Dordrecht, Netherlands: Kluwer, pp. 287–393.

Priest, G. (2006). *Doubt Truth to Be a Liar*. Oxford, UK: Oxford University Press.

Priest, G. (2010). Hopes fade for saving truth. *Philosophy*, **85**, 109–140. Critical notice of Field (2008).

Quine, W. V. O. (1951). *Mathematical Logic*. New York, NY: Harper & Row.

Restall, G. (1993). How to be really contraction-free. *Studia Logica*, **52**, 381–391.

Restall, G. (1999). Negation in relevant logics. In Gabbay, D. M., and Wansing, H., editors. *What Is Negation?*, Dordrecht, Netherlands: Kluwer, pp. 53–76.

Restall, G. (2000). *An Introduction to Substructural Logics*, New York, NY: Routledge.

Shapiro, S. (2004). Simple truth, contradiction, and consistency. In Priest, G., Beall, J. C., and Armour-Garb, B., editors. *The Law of Non-Contradiction. New Philosophical Essays*, Oxford, UK: Clarendon Press, pp. 336–354.

Shapiro, L. (2011). Deflating logical consequence. *The Philosophical Quarterly*, **61**, 320–342.

Slaney, J. K. (1989). RWX is not Curry-paraconsistent. In Priest, G., Routley, R., and Norman, J., editors. *Paraconsistent Logic. Essays on the Inconsistent*, München, Germany: Philosophia Verlag, pp. 472–481.

Slater, B. H. (1995). Paraconsistent logics? *Journal of Philosophical Logic*, **24**, 451–454.

Slater, B. H. (2007). Dialetheias are mental confusions. In Béziau, J.-Y., Carnielli, W., and Gabbay, D., editors. *Handbook of Paraconsistency*, London: College Publications, pp. 457–466.

Smiley, T. (1993). Can contradictions be true? I. *Proceedings of the Aristotelian Society*, **67**, 17–34.

Sorensen, R. (2003). *A Brief History of the Paradox. Philosophy and the Labyrinths of the Mind*. Oxford, UK: Oxford University Press.

Tappenden, J. (1999). Negation, denial and language change in philosophical logic. In Gabbay, D. M., and Wansing, H., editors. *What is Negation?*, Dordrecht, Netherlands: Kluwer, pp. 261–298.

Varzi, A. (2004). Conjunction and contradiction. In Beall, J. C., Priest, G., and Armour-Garb, B., editors. *The Law of Non-Contradiction*, Oxford, UK: Clarendon Press, 2004, pp. 93–110.

Weber, Z. (2010). Extensionality and restriction in naïve set theory. *Studia Logica*, **94**, 87–104.

Weber, Z. (2012). Notes on inconsistent set theory. In Tanaka, K., Berto, F., Mares, E., and Paoli, F., editors. *Paraconsistency: Logic and Applications*, Dordrecht, Netherlands: Springer, pp. 313–325.

Williamson, T. (2007). *The Philosophy of Philosophy*, Oxford, UK: Blackwell.
Zardini, E. (2011). Truth without contra(di)ction. *Review of Symbolic Logic*, **4**, 498–535.

DEPARTMENT OF PHILOSOPHY, UNIVERSITY OF AMSTERDAM
OUDE TURFMARKT 141-147, 1012 GC AMSTERDAM, NETHERLANDS
E-mail: F.Berto@uva.nl.

NORTHERN INSTITUTE OF PHILOSOPHY, UNIVERSITY OF ABERDEEN
KING'S COLLEGE, HIGH STREET, ABERDEEN AB24 3UB, UK
E-mail: f.berto@abdn.ac.uk.