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Does an Unstable Keynesian Unemployment Equilibrium in a non-Walrasian Dynamic Macroeconomic Model Imply Chaos?

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I. Introduction

Simonovits (1982) introduced a non-Walrasian dynamic macromodel which is based on the disequilibrium model with inventory dynamics due to Honkapohja and Ito (1980). This model describes transactions on two markets (one market for labor and one market for goods) and contains five parameters. Obviously, there is an equilibrium which is the so-called Keynesian unemployment equilibrium point. For suitable choices of the parameters, this equilibrium point is locally stable and, according to Simonovits (1982), the Keynesian unemployment equilibrium point is also globally stable, i.e., the trajectories of all initial values converge to the equilibrium point.

For many parameter values the equilibrium point is unstable; this is in fact the case when the product of the two eigenvalues at the equilibrium point is greater than 1. Simonovits' (1982) conjecture says: "If the Keynesian unemployment equilibrium point is unstable, then there is chaos".

First, we recall some definitions of chaos. We restrict our attention to systems whose trajectories are bounded. A first definition is the following. A system is chaotic if there exist infinitely many periodic points with different period and if there is an uncountable set of aperiodic points. This kind of chaos is nowadays called Li-Yorke chaos; see Li and Yorke (1975) and Diamond (1976). Before turning to a second definition, we need the notion of "sensitive dependence on initial values". A system may be said to have sensitive dependence on initial values if we can find, with probability p (where $0 < p \leq 1$), a point x such that for every open neighborhood U of x there is a point y in U such that the trajectories of x and y will not be close

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to each other forever. Note that the size of the neighborhood U may be arbitrarily small. We may now give the second definition. A system is chaotic if it has sensitive dependence on initial values; see Ruelle (1979) and Guckenheimer (1979).

II. Simonovits' Model

For clarity, we recall the model as presented by Simonovits (1982). We consider two markets, one for goods and one for labor. We write $L^D(t)$ (resp. $L^S(t)$) for the demand for (resp. supply of) the number of units of labor at time t, with integer $t \geq 0$, and we write $L(t)$ for the minimum of these two values. Similarly, we write $Y^D(t)$ (resp. $Y^S(t)$) for the demand for (resp. supply of) the number of units of goods at time t, and we write $Y(t)$ for the minimum of these two values. Note that $L(t)$ and $Y(t)$ represent the actual transactions on the markets at time t.

The inventory $I(t)$ at time t is given by the difference between the supply and the transactions of the goods at time t, i.e., $I(t) = Y^S(t) - Y(t)$. Note that $I(t) \geq 0$ for every integer $t \geq 0$.

Simonovits made the following assumptions:

A-1. The supply of labor is constant: $L^S(t) = d$ for some $d > 0$.

A-2. Labor is the only input for production, and the production function is linearly homogeneous. Hence, the aggregate supply of goods equals the inventory at time $t-1$ plus production: $Y^S(t) = I(t-1) + \delta L(t)$ for some $\delta > 0$.

A-3. The demand for goods is a linear function of labor: $Y^D(t) = a + bL(t)$ for some $a > 0$, $b \geq 0$; and the production is profitable: $\delta > b$.

A-4. The optimal level of inventory $I_{opt}(t)$ at time t is proportional to the expected demand $E[Y^D(t)]$ for goods at time t: $I_{opt}(t) = \beta E[Y^D(t)]$ for some $\beta \geq 0$.

A-5. The firms have naive expectations of the demand for goods: $E[Y^D(t)] = a + bL(t-1)$.

A-6. Production minus demand, both taken at full employment, is positive: $(\delta - b)d - a > 0$.

Under the above assumptions, Simonovits' model is given by:

$$L(t+1) = \begin{cases} 0 & \text{if } I(t) \geq (\beta+1)[a + bL(t)] = I_1(t) \\ [(\beta+1)\{a + bL(t)\} - I(t)]/\delta & \text{if } I_2(t) \leq I(t) \leq I_1(t) \\ d & \text{if } I(t) \leq I_1(t) - \delta d = I_2(t) \end{cases}$$

$$I(t+1)=\begin{cases} I(t)-a & \text{if } L(t+1)=0 \\ \langle \{b(\beta+1)\{(\delta-b)L(t)-a\}+bI(t)+a\beta\delta\}/\delta\rangle_+ & \text{if } 0<L(t+1)<d \\ I(t)+(\delta-b)d-a & \text{if } L(t+1)=d \end{cases}$$

where $\langle x\rangle_+$ denotes the maximum value of $\{0,x\}$.

Simonovits conjectured that for $b(\beta+1)/\delta>1$, there is chaos in the dynamics of the above system.

III. The Simonovits Model with Specified Parameter Values

We now present a specified Simonovits model in which the equilibrium is unstable; this system would exhibit chaos if the conjecture were true. We choose: $a=8$, $b=9$, $d=10$, $\beta=0.3$, and $\delta=10$. These values satisfy the condition in Simonovits' conjecture, because $b(\beta+1)/\delta=1.17>1$.

Proposition. *The set of periodic points of the specified Simonovits system consists of one attracting periodic orbit with period 28, one repelling periodic orbit with period 28, and the unstable equilibrium. Furthermore, the stable periodic orbit will attract the trajectories of almost all initial values.*

Corollary. The specified Simonovits model does not exhibit chaos in the dynamics.

The point (8,24) is the Keynesian unemployment equilibrium of the specified Simonovits system. The equilibrium is unstable; see Section IV for details. The initial part of the orbit of (8,23.5), which consists of 20,000 points, is shown in Figure 1. The figure shows not only that the orbit of (8,23.5) converges to an attracting periodic orbit with period 28, but also that it converges very rapidly.

Remark: The specified model is structurally stable w.r.t. the parameters; the qualitative behavior in the dynamics of this model will not change when the parameter values are varied slightly.

IV. The Analysis

We consider the discrete dynamic system that is associated with Simonovits' model. We write L_t for $L(t)$ and I_t for $I(t)$, and $(L_{t+1}, I_{t+1})=F(L_t, I_t)$, where F maps the set W (in the nonnegative quadrant of the plane) consisting of points (x,y) with $0\le x\le 10$ and $y\ge 0$ into itself. The action of the map F may be described in a geometric way.

We write S_T for the points (x,y) with $y=11.7x+10.4$; S_B for (x,y) with $y=11.7x-89.6$; S_L for $(0,y)$; and S_R for $(10,y)$.

Let V be the region in W bounded by the four lines S_T, S_B, S_L, and S_R, let Z be the region in W above V, and let U be the region in W below V; see Figure 2.

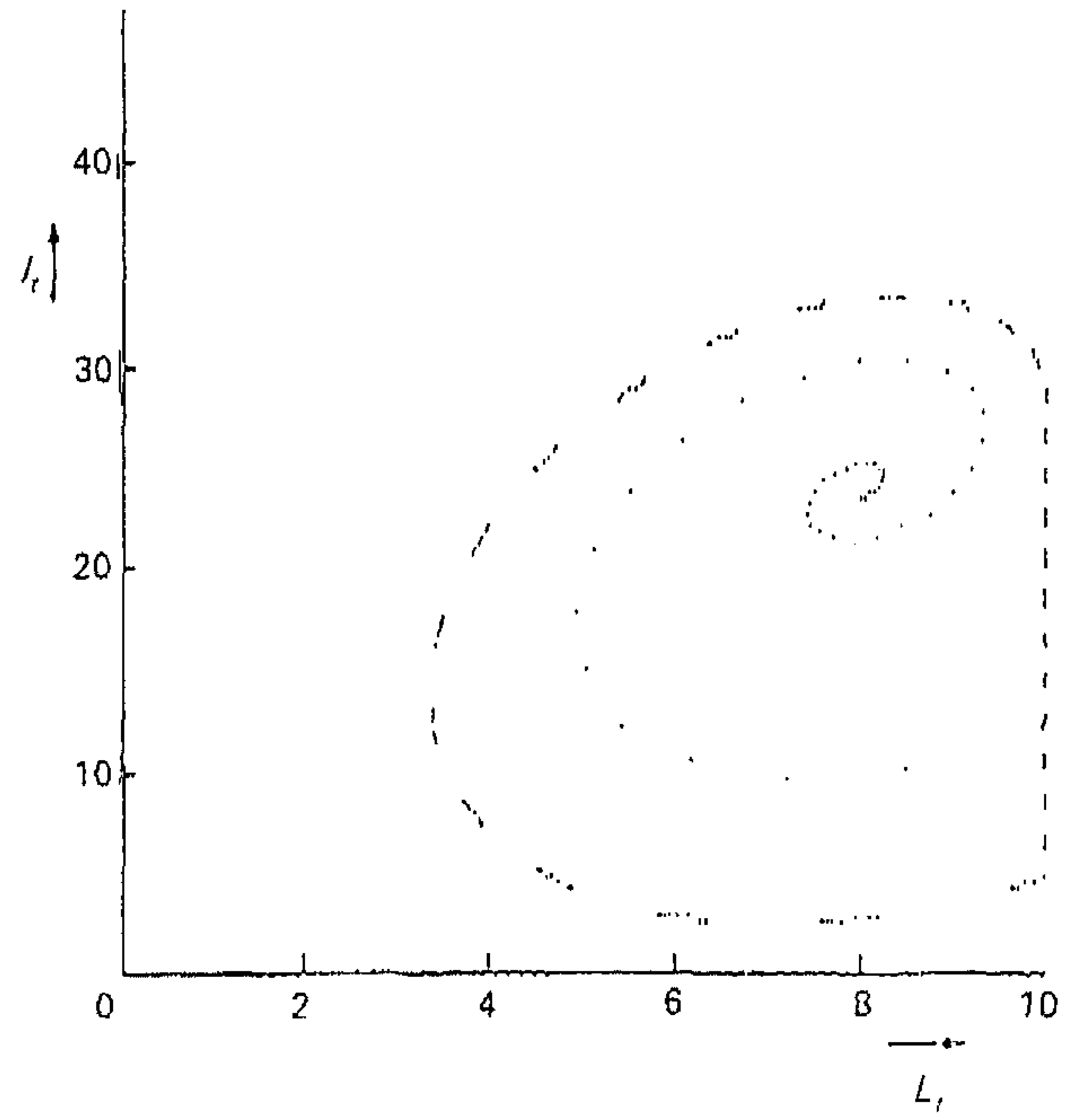

Fig. 1.

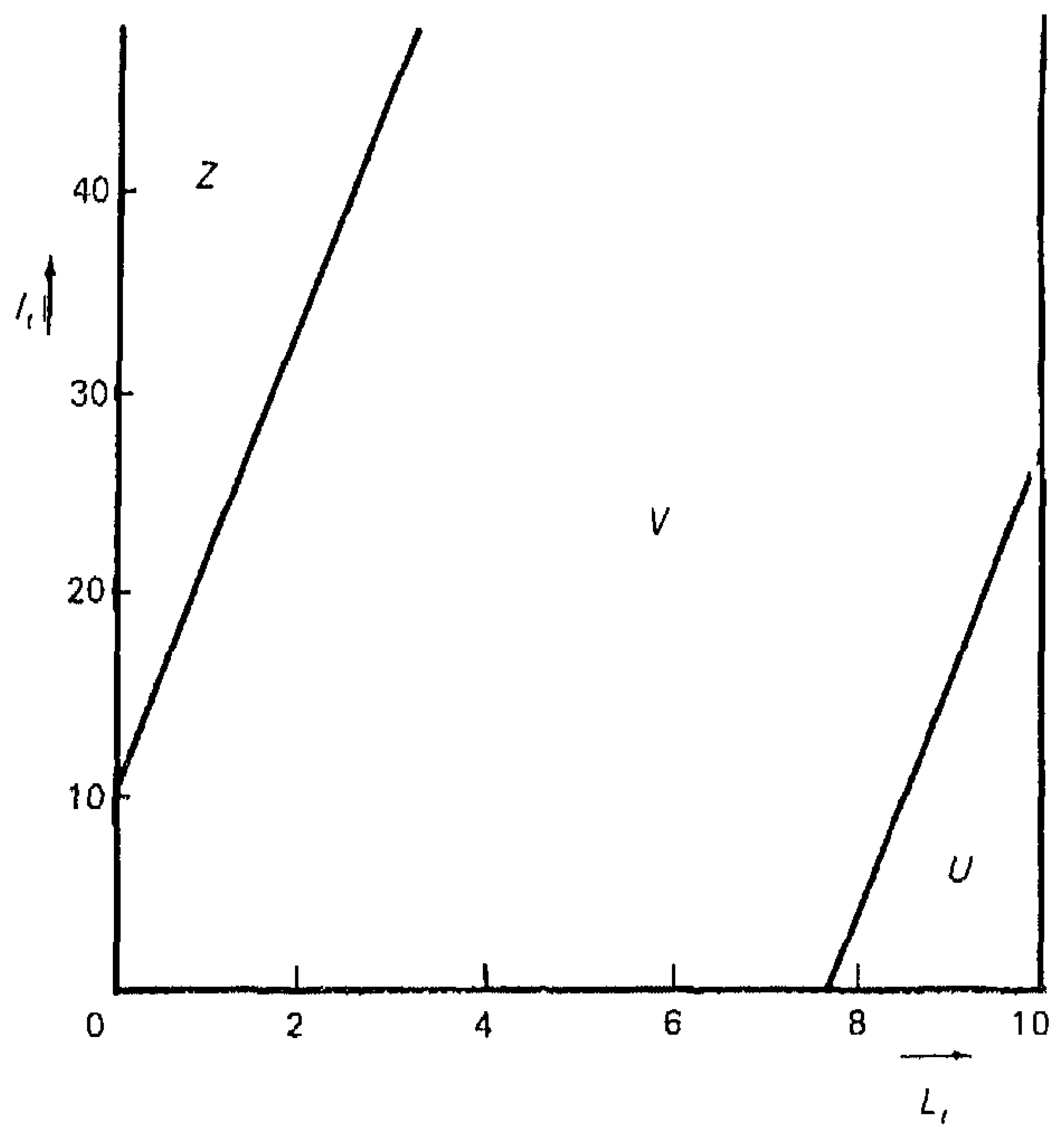

Fig. 2.

The map F is defined as follows: $F(L_t, I_t) = (0, I_t - 8)$ for (L_t, I_t) in Z; $F(L_t, I_t) = (10, I_t + 2)$ for (L_t, I_t) in U; and for (L_t, I_t) in V $F(L_t, I_t) = (1.17L_t - 0.1I_t + 1.04, \langle 1.17L_t + 0.9I_t - 6.96\rangle_+)$.

The map F is piecewise linear and continuous. The eigenvalues of the derivative of F in the equilibrium point (8,24) are $1.035 \pm i\sqrt{0.3142849}$. Consequently, the equilibrium is unstable.

Now we consider the line segment S_P, where S_P consists of the points (x, y) with $x = 10$ and $27.4 \leq y < 29.4$. Straightforward computation shows that for every initial point (x_0, y_0) in W, there is a smallest positive integer N so that the N'th iterate (x_N, y_N) is in S_P. Therefore, it is sufficient to consider only those orbits which start in S_P.

In order to analyze the specified Simonovits model, we consider the return map g on S_P which is defined as follows. Let $(10, I_0)$ be in S_P. Then $g(10, I_0) = (10, I_m)$, where $(10, I_m)$ is the image of $(10, I_0)$ under the m'th iterate of F such that $(10, I_m)$ is in S_P, and $(10, I_k)$ is not in S_P for each integer k with $0 < k < m$. The map g is in fact a one-dimensional map from the interval $[27.4, 29.4)$ into itself; see Figure 3. We would like to emphasize that if $(10, I'_0)$ is in S_P, $g(10, I'_0) = (10, I'_n)$, and $I_0 \neq I'_0$, then n and m might be different. In particular, if $27.4 \leq I_0 < I^*$ then $m = 29$ and if

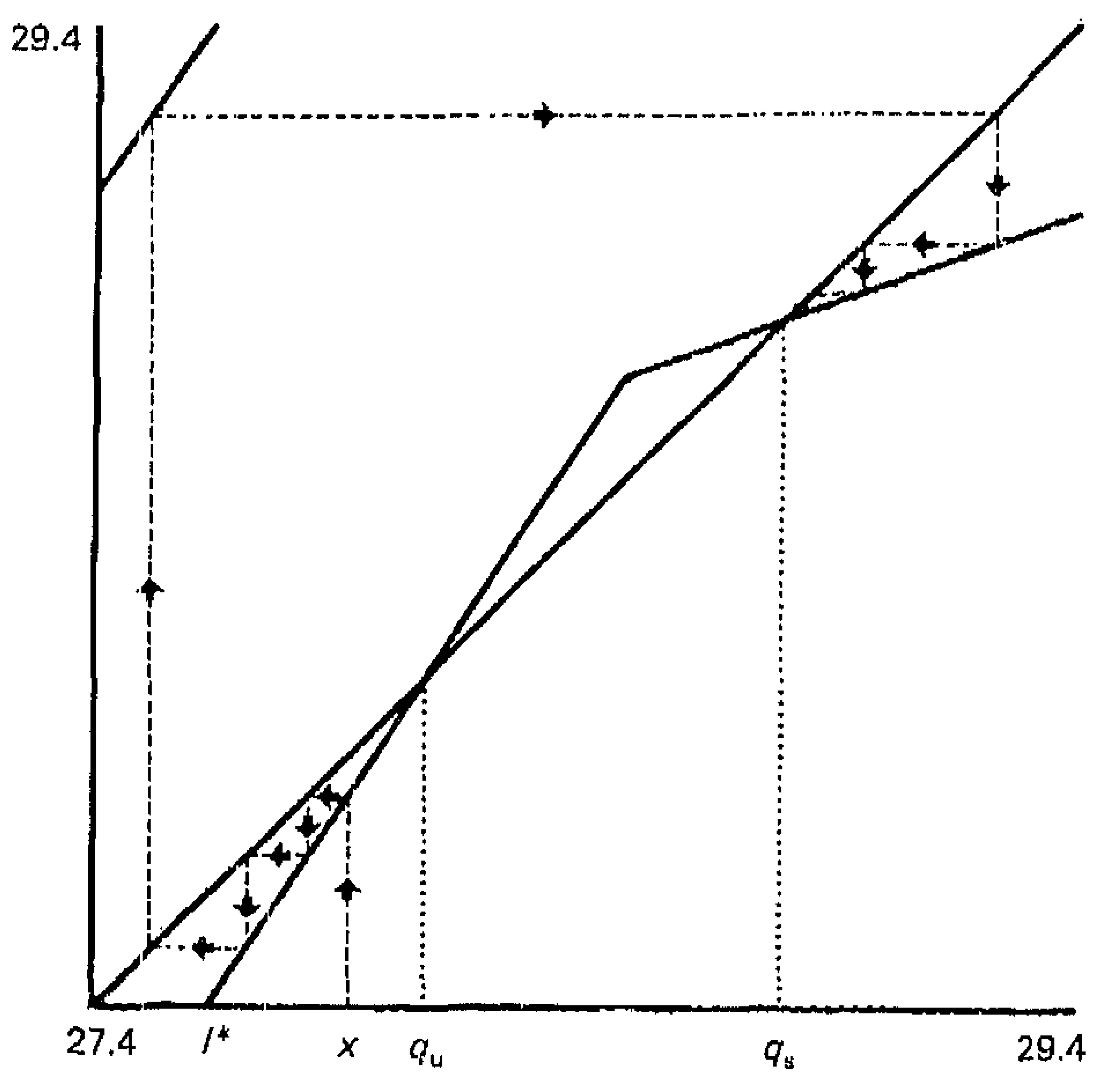

Fig. 3.

$I^* \leq I_0 < 29.4$ then $m = 28$, where I^* is the solution of $F^{28}(10, I^*) = (10, 27.4)$ and this is a point of discontinuity of the return map g.

Let J denote the interval $[27.4, 29.4)$. Write q_s (resp. q_u) for the stable (resp. unstable) fixed point of g in J. Obviously, if $x = q_u$, then the n'th iterate of x under g is equal to q_u for every integer $n \geq 0$.

We now show that if x is in J and x is different from q_u, then the n'th iterate of x under g is going to q_s as n goes to infinity. This is clearly true if $x = q_s$, because the n'th iterate of x under g equals q_s for each n. From now on we assume that x is not a fixed point of g.

Case 1. $q_u < x < q_s$. In this case the sequence $\{g^n(x)\}$ is strictly increasing and bounded, and it converges to q_s as n goes to infinity.

Case 2. $q_s < x < 29.4$. The sequence $\{g^n(x)\}$ is strictly decreasing and bounded, and it converges to q_s as n goes to infinity.

Case 3. $27.4 \leq x < q_u$. There exists a positive integer N such that the N'th iterate of x under g is in the interval $(q_s, 29.4)$, because if $27.4 \leq x < I^*$, then $q_s < g(x) < 29.4$ and, if $I^* \leq x < q_u$, then there is a positive integer K such that the sequence $\{g^n(x)\}$ is strictly decreasing for $0 \leq n \leq K$ and $27.4 \leq g^K(x) < I^*$. Therefore the sequence $\{g^n(x)\}$ converges to q_s as n goes to infinity. This completes the proof of the proposition.

Note that the sequence $F^k(10, q_s)$ with $1 \leq k \leq 28$ is the stable periodic orbit of the specified Simonovits model.

Conclusion: In the dynamics of the system, there is neither Li-Yorke chaos nor sensitive dependence on initial values.

V. Concluding Remarks

Independently of the two definitions of chaos given above, the dynamics of the specified Simonovits model does not show chaos. In fact the dynamic behavior is very regular, and this regularity will persist when the parameter values are varied slightly.

For one specified Simonovits model with an unstable Keynesian unemployment equilibrium, we have shown that there is no chaos. Several other very interesting phenomena might occur, e.g. coexistence of two stable cycles and coexistence of three chaotic attractors. Some important questions are: (i) How many simple attractors can coexist? (ii) What is the number of chaotic attractors that can occur simultaneously? (iii) Does the existence of a simple attractor imply that there is no sensitive dependence on initial values?

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