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Abstract: I would like to attack a certain view: the view that the concept of identity can fail to apply to some things although, for some positive integer *n*, we have *n* of them. The idea of entities without self-identity is seriously entertained in the philosophy of quantum mechanics (QM). It is so pervasive that it has been labelled the Received View (French and Krause 2006. *Identity in Physics: A Historical, Philosophical, and Formal Analysis*. Oxford: Oxford UP: 105). I introduce the Received View in Section 1. In Section 2 I explain what I mean by "entity" (synonymously, by "object" and "thing"), and I argue that supporters of the Received View should agree with my characterization of the corresponding notion of *entity* (*object, thing*). I also explain what I mean by "identity", and I show that supporters of the Received View agree with my characterization of that notion. In Section 3 I argue that the concept of identity, so characterized, is one with the concept of oneness. Thus, it cannot but apply to what belongs to a collection with *n* elements, *n* being a positive integer. In Section 4 I add some considerations on the primitiveness of identity or unity and the status of the Identity of Indiscernibles. In Section 5 I address the problem of how reference to indiscernible objects with identity can be achieved.

Keywords: identity, individuation, philosophy of quantum physics, identity of indiscernibles, metaphysics of physics

> Ταυτότης ἑνότης τίς ἐστιν.
> Identity is a certain unity.
> -Aristotle, *Metaphysics* 1018a5

1 The Received View

Lewis famously claimed: "There is never any problem about what makes something identical to itself: nothing can ever fail to be" (Lewis 1986, 192–3). Whatever a is, "$a = a$" has been taken as a trivial truth if there is any.

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However, the idea of entities without self-identity is taken seriously in the philosophy of quantum mechanics (QM). That quantum particles lack self-identity is not understood here as meaning that they are different from themselves. It means that the concept of identity does not meaningfully apply to them. To see why one may come to have such an idea, let us start with a friendly example.

I give you a sealed wooden box with some little balls in it and I ask you to guess their number. You have no way to look inside. However, by shaking it, listening to the sounds produced by the balls, and carefully considering the vibrations, you guess that the number of balls in there is two. As it happens, you got it right.

It seems to follow that the balls are different: if there are two balls, one must not be the other. However, one may be suspicious of claims of difference *solo numero*. One may think that to ground such claims one needs to individuate[1] the relevant things somehow, so that one can say: "This is one", "That's the other". And you cannot put your finger on either of the two balls, for they are locked in the box.

That the balls are indiscernible for you, one may retort, is a merely epistemic issue, with no relevance for facts concerning identity and difference. That you cannot single out either ball has to do with your contingent epistemic situation. If you could open the box, you would easily discern them. Suppose that the two balls share the same intrinsic properties, a property being intrinsic when something has it independently from the existence of any other thing (Lewis 1983; Langton and Lewis 1998). The balls have the same size, colour, mass, chemical composition, etc. They can still be individuated, in principle, via their spatiotemporal location: one is never where the other is.

Spacetime coordinates are so important for the individuation of ordinary things that Quine (1975) proposed to make of them a criterion of identity for material objects: *a* is the same material object as *b* when they share the same spatiotemporal location. But the Quinean criterion is in trouble when we move to objects of QM.[2] Suppose that, instead of balls in a box, we are dealing with particles in a quantum mechanical system. Particles can be indiscernible in that

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1 That is, according to the *Oxford English Dictionary:* "to distinguish from others of the same kind; to individualize; to single out". So understood, individuation is an epistemic notion: it concerns the cognitive procedures we use to establish whether we have a case of identity. A detailed theory of individuation is proposed in Wiggins (2001).

2 Quine's criterion has been contested also by neo-Aristotelians like Wiggins (2001), who claim that numerically distinct things can occupy the same region of spacetime, provided they fall under different sortal concepts (on this debate, see also Baker (1997), Thomson (1998), Varzi (2000)). Some admit coincident objects of the same sort: see Kit Fine's double letter example in Fine (2000).

they share the same intrinsic properties, like mass, charge, etc. Additionally, no impenetrability assumption holds, as it did in classical mechanics. In the formalism of QM particles do not generally get well-defined trajectories in spacetime (see the classic Reichenbach (1956), and the nice discussion in Ladyman and Ross (2007, 134–5)). Physicists can gather empirical evidence that a system includes *n* particles in situations where individuating the particles is much less feasible than your opening the box for the balls. Cortes (1976) considers the case of a mirror-lined box with two photons, whose trajectories cross in such a way that one cannot determine the history of each photon anymore even though there is, at any time during the observation, a fixed number of them: two – as determined by measuring the total frequency of energy v in the box and getting $2hv$, and by employing Planck's equation. Domenech and Holik (2007) speak of the extraction of two electrons via ionization of an atom of helium.[3]

At this point one may conclude that the problem is not merely an epistemic one – having to do with how we single out things – but an ontological one – a problem of identity. Schrödinger drew such a conclusion. According to him, not only "atoms – our modern atoms, the ultimate particles – must no longer be regarded as identifiable individuals" (Schrödinger 1996, 162), but also we must accept that the particles lack identity:

> It is not a question of our being able to ascertain the identity in some instances and not being able to do so in others. It is beyond doubt that the question of "sameness", of identity, really and truly has no meaning (Schrödinger 1996, 121).

This is a paradigmatic statement of the Received View. Moving to quantum states, the View is motivated on the basis of the way permutations of particles are counted in statistical QM. The number of permutations of m elements of a set with n members is given by: $n!/(n-m)!$ ($n!$ being n factorial). Permutations are taken here as ordered tuples. However, in the quantum statistical framework we divide by $m!$, getting the binomial coefficient: $n!/m!(n-m)!$. We find this way of counting in Planck's early work on QM and in Dirac's unification of the Bose-

3 "For example, we could count how many electrons has an Helium atom imagining the following process […]. Put the atom in a cloud chamber and use radiation to ionize it. Then we would observe the tracks of both, an ion and an electron. It is obvious that the electron track represents a system of particle number equal to one […]. The only thing that cares is that we are sure that the track is due to a single electron state, and for that purpose, the identity of the electron does not matter. If we ionize the atom again, we will see the track of a new ion (of charge 2e), and a new electron track. Which electron is responsible of the second electron track? This query is ill defined, but we still do not care. Now, the counting process has finished, for we cannot extract more electrons. The process finished in two steps, and so we say that an Helium atom has two electrons" (Domenech and Holik 2007, 862).

Einstein statistics for bosons and the Fermi-Dirac statistics for fermions: the key combinatorial formula used in Dirac (1926) "is simply a suitably amended version of the well-known expression for the number of ways in which *m* objects can be selected from a set on *n* objects: that is, the number of combinations of *n* things taken *m* at a time" (French and Krause 2006, 102).

The binomial coefficient numbers *m*-combinations, which are unordered collections. According to the statistics, thus, swapping two otherwise indiscernible particles in a system does not originate an arrangement which is considered, for any purpose, different from the arrangement one started with. We have a determinate number of quanta distributed across states, but there is no fact of the matter as to which particle gets which state. Since the origins of QM "this was taken to imply that the particles had lost their identity and were, in some sense, non-individuals" (French and Krause 2006, xi; see also Ladyman and Ross 2007, 134). Physicists express the idea with lively metaphors:

> The possibility that one of the identical twins Mike and Ike is in the quantum state E_1, and the other in the quantum state E_2, does not include two differentiable cases which are permuted on permuting Mike and Ike: it is impossible for either of these individuals to retain his identity so that one of them will always be able to say "I'm Mike" and the other "I'm Ike". Even in principle one cannot demand an alibi of an electron! (Weyl 1931, 241).

2 Entity and Identity

2.1 "Entity"

I use "entity" synonymously with "thing" or "object". These terms are employed in diverging ways in the debates we are to address. But I adopt a minimal characterization:

> An object is anything that can be the value of a variable, that is, anything we can talk about using pronouns, that is, *anything*. (Van Inwagen 2002, 180)

So understood, terms like "object" or "thing" stand for blanket notions: they provide no restriction to our quantifiers. "Every x is such that, if x is a thing, then x is F" means nothing more and nothing less than "Every x is F". "Some x is such that x is a thing and x is F" means nothing more and nothing less than "Some x is F".

At the very least, entities – objects, things – have features, properties, and can stand in relations. Therefore, certain predicates standing for the relevant properties or relations can be true of them. "Being a property-bearer", though, is a gloss of the concept *thing* or *object*, not a full-fledged definition (it is likely that one cannot do better with such basic notions; we will soon find ourselves having to say the same about *one* and *identical with*). For one can then wonder, what is a property or a relation? It is something things can bear, or stand in. Thus, properties and relations are things in their turn, bearers of (further) properties: *as old as* partitions the set of people into equivalence classes, *divides* is a partial order on N+.

Then quantum particles are things, too: physicists talk about particles in a system, quantify over them, and ascribe properties to them, like having a certain momentum, position, or spin. We recall that Ladyman and Ross' aforementioned 2007 book, articulating their ontic structuralist ontology motivated by our best natural science, and in particular by fundamental physics, is called *Every Thing Must Go*, and that they sometimes "go on to deny that, strictly speaking, there are 'things'" (Ibid: 121). Sometimes they admit, however, that "there are objects in our metaphysics, but they have been purged of their intrinsic natures, identity, and individuality. [...] Real patterns are the objects of genuine existential quantification." (131, 239). What is denied in ontic structuralist metaphysics is a *specific* and widespread conception of things as self-subsistent individual substances, with intrinsic natures independent from the relational structures they are embedded in.[4]

But when *thing* (*entity*, *object*) is understood as above, things will not go. We cannot dispense with them in our thought and theorizing, for we cannot even say how to dispense with them without saying something, that is, without speaking of things. Such notions make for what quantum physicist Sunny Auyang called the "categorical framework", which is there whether we think about the quantum realm or about Austin's moderate-sized specimens of dry goods:

> Consider the quantum postulate that says the state vector representing an isolated physical system contains a complete description of its characteristics. Despite the technical jargon, the logic is plain. Something has certain properties, which are mathematically represented by a state vector. [...] The understanding is achieved [...] by extending to the quantum realm certain logical forms of thoughts, such as the subject-predicate form of propositions. [...] If the framework is destroyed, say, by the demise of the concepts of properties and predication, then the quantum world becomes mystical. When "it is such and so" becomes logically illegitimate, we do not know what to think. (Auyang 1995, 12–13).

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4 Schrödinger already protested against the "obstinately lingering doctrine" of substance and accident as one at odds with the nature of quantum reality (Schrödinger 1964, 76). He connected such doctrine with the subject-predicate structure of ordinary languages, and quoted with approval Russell's mention of the metaphysical prejudices coming from it (Ibid.).

2.2 "Identity"

According to authors like Wiggins (2001), the concept of identity is as primitive as the one of predication and co-originary with it: *a* is *F* if and only if *a* is some *F*, that is, *Fa* if and only if $\exists x(x = a \,\&\, Fx)$ (a similar point is made, more recently, by Otávio Bueno (2014), who uses it to argue against the idea that quantum particles can lack identity). To appreciate the claim, one needs to lay out a few features of the concept of identity at issue (a general introduction to the topic is Curtis and Noonan (2014)). The idea of things lacking identity is often explained metaphorically by physicists and philosophers, for instance, via comparisons between particles and virtual money in a bank account (see Hesse (1970)), or in contexts in which the underlying theory of identity is not fully spelt out. I will stick to the presentation of the Received View proposed by Steven French and Décio Krause in their book *Identity in Physics* and in various papers. For there the concept of identity in play is rigorously clarified, and assigned certain features. Such features will be essential for the point I aim to make below. In order not to beg any question, I need to show that the ascription of such features to identity is shared by French and Krause's systematization of the Received View.

I should stress that French and Krause do not subscribe to the Received View rather than to the opposite view that particles are endowed with (self-) identity. Instead, they show that our best current physics is underdetermined between two such opposite metaphysical packages. They then provide a set-theoretic framework, called quasi-set theory (Ch. 7), suitable for the mathematics of QM if one interprets the theory in conformity with the Received View. But they also show (Ch. 4) that the view according to which particles are individuals to which the concept of identity applies is compatible with our current physics.

Firstly, then, identity is not a relation between names of objects, but between the objects themselves (French and Krause 2006, 4–5). Unlike the *Begriffschrift*'s Frege, that is, we do not believe that names "*a*" and "*b*" stand in that relation when they name the same thing. Rather, we "will follow the later Frege and insist that to state that item *a* is identical to item *b*, written symbolically, $a = b$, means that there are not two distinct items in reality, but only one" (Ibid).[5]

5 In a famous passage of the *Metaphysics*, part of which is quoted at the beginning of this paper, Aristotle argues that in order to say that the same thing is identical with itself (αὐτὸ αὐτῳ ταὐτόν) one has to somehow duplicate it and treat it as two things (ὡς δνσί). In his Commentary Aquinas says that if the thought that something is identical with itself implies some reduplication, this cannot mean that something is *simul, sub eodem*, one and not one. It has to be one

Secondly, identity is not sortal-relative (as in Geach (1967), Deutsch (1998)): it cannot happen that a is the same F as b, where “F” stands for some sortal predicate, without this entailing congruence with respect to all properties (in particular, a and b’s being the same G for some different sortal G). Although they speak of the opportunity of applying some sortal theory to quantum particles (ibid: 347–50), French and Krause never question the Indiscernibility of Identicals (see e. g. ibid: 251, 255), the claim that if a is b, then any property of a is a property of b and vice versa. A theory of sortal concepts is anyway in principle compatible with a conception of identity that complies with the Indiscernibility of Identicals: Wiggins (2001) makes for a paradigmatic example.

Thirdly, identity admits no vagueness or degrees (as in van Inwagen (1990, Section 18)): there are no things a and b, such that it is *de re* indeterminate or vague whether $a = b$ or not (the key paper in this area is the one-page Evans (1978)). In French and Krause’s quasi-set-theoretic framework either the objects at issue are such that the concept of identity applies to them, or not. If the concept of identity applies to a and b, either they are the same, or not (and if they are the same, they are members of the same sets, French and Krause (2006, 277)). Unlike other set-theoretic frameworks for quantum theory such as Dalla Chiara and Toraldo di Francia’s (1993) quaset theory, French and Krause’s is no fuzzy set theory (see ibid: 293, 319). If the concept of identity does not apply to a or b, then it does not make sense to claim, within the theory, that it is indeterminate or vague whether a is b: “$a = b$” is not a well-formed formula (see ibid: 276). And no operator (expressing indeterminacy, or of other kind) can yield a well-formed formula when prefixed to a non-well-formed formula.

3 Identity as Unity

While the point that facts about cardinality entail facts about identity has been made in the literature (e. g., besides the aforementioned Bueno (2014), see Jantzen (2011), Dorato and Morganti (2013)), *this* paper aims at phrasing it in terms of a priori conceptual analysis. French and Krause’s (2006) reconstruction shows that our best fundamental physics is underdetermined between two

and not one under different respects. And the different respects are, on the one hand, one object, and on the other hand, a subject who, thinking the object as identical with itself, considers it as two (*utitur eo quod est unum secundum rem, ut duobus:* see *In Metaph*. 912). But then, the only duplication we have is a duplication in ways of representing.

incompatible metaphysical packages: one according to which particles are individuals, that is, identity applies to them; the other one – the Received View – according to which it does not. As French and Krause (2006, 189–93) also show, ordinary considerations concerning theory choice in science do not help either. When "the problem is, it is not always clear what it is that physics teaches us!" (Ibid: 190), some clarification may come from conceptual analysis.

Start with the concepts of *countability* and *cardinal*. A countable set is one for which there exists an onto mapping from the positive integers to it (or, a one-to-one mapping from it to the positive integers), so that the members of the set can be arranged in a list. Such a mapping cannot be available, however, for aggregates of particles lacking identity. If it were, the particles could be ordered, which is what cannot be done according to the Received View. The framework of quaset theory developed by authors like Toraldo di Francia and Dalla Chiara (1993), as well as French and Krause's quasi-set theory, aim at disentangling quantities from ordering. In Toraldo di Francia's words:

> Can we distinguish *this* and the *other* in a system of two electrons? As is well known, this cannot be done [...]. Here, cardinal numbers seem to take over the role we had previously attributed to ordinal numbers. A system of identical particles has a *cardinality*; but we cannot tell which is the first, the second, and so on. (Toraldo di Francia 1978, 65).

French and Krause claim that "*countable* is ambiguous, between cardinality and ordinality", and that "it is only the latter which requires determinate distinctness" (French and Krause 2006, 15). They also say that "quantum particles cannot be ordered or counted, but only aggregated in certain amounts." (Ibid: 276–7). Their quasi-set theory aims at representing collections of quantum particles that have a cardinal but no ordinal. But then, there is such a thing as the number of particles in one such collection or (quasi-)set. It might be that "there is no way of counting [in the "ordinal sense"] or of distinguishing them", but "there is a sense in saying, say, that there are k electrons in a certain level of a certain atom" (Ibid: 289). Here "k" is a placeholder for a positive integer.

French and Krause speak of "quasi-cardinals" for, unlike what happens standardly when cardinals are defined as particular ordinals, the cardinals of quasi-set theory are taken as primitive (Ibid: 276). This does not change much: "given the concept of quasi-cardinal, there is a sense in saying that there may exist a certain quantity of m-atoms obeying certain conditions, although they cannot be named or labelled" (Ibid: 277; I will come to the issue of naming in our final Section). Aggregates of quanta can be called "uncountable", but only in a peculiar sense: like the set of the reals, they are uncountable in that there is no injection from them to the positive integers, or no surjection from the positive integers to them. However, one can label one such aggregate with a single positive integer: the number of particles in the aggregate. It may be a small number, like *two*, as in the case of Cortes' two photons in a mirror-lined box, or in Domenech and Holik's experiment. One may insist that in aggregates of quanta "there is no difference in principle about which one has which properties"; still we can "heap them up in different quantities with a total measure of one, two, or three, and so on" (Teller 1995, 12). Aggregates of quantum objects can have positive integers making for the number of objects in the collection.

In the framework of quasi-sets proposed to mathematically ground the Received View, then, neither vagueness nor sortal-relativity prevent the application of a notion of identity which is all of a piece with the notion of *being one*. If particles *a* and *b* were such that it is *de re* vague or indeterminate whether *a* is the same as *b*, it might be indeterminate whether we have to count one or two of them. If identity were relative to a sortal, so that *a* can be the same *F* as *b* but not the same *G* because *a* is *G* while *b* is not, then there would be no single answer to the question, "How many things are there?". But quasi-set theory does not claim that, whereas the concept of absolute identity does not apply to quantum particles, a relativized one does, whereby one can say that *a* and *b* are the same *F* for the relevant sortal *F* without this entailing congruence with respect to all properties. The theory has an absolute notion of identity which applies to ordinary objects (French and Krause 2006, 277). And the *urelemente* in the theory, which represent quantum particles, are no vague objects with indeterminate identities. If this were the case, there would be no single (quasi-)cardinal associated to aggregates of such particles – but there is:

> As in quantum physics, we may reason as if a certain element does or does not belong to the quasi-set: the law of the Escluded Middle $x \in y \vee x \notin y$ remains valid, even if we cannot verify which case holds. The idea fits with what happens with the electrons in an atom; in general we know how many electrons there are, and we can say that some of them *are* in that atom, but we cannot tell *which* particular electrons are in the atom. (Ibid: 293)

Suppose the question, "How many particles of such-and-such kind are in this system?", gets a single, determinate, non-sortal-relative answer via the mentioning of some positive integer. Suppose that the answer, as in the examples of Cortes' mirror-lined box with photons and of Domenech and Holik's electrons of a helium atom, is *two*. It seems to make sense, then, to claim that one of the particles is not the other, that is, they are different (Bueno 2014, Section 3, makes a similar point). Once *identity* has been characterized as per the three features above (objectual, not vague, not sortal-relative), this is what claims of difference and identity mean. That a sentence of the form "$a = b$" is true, under this reading of "$=$", means that we need to count one thing: the thing named "*a*", which happens to be the thing named "*b*" (flag again the *naming* issue, to be addressed below). That we, instead, count two things, means that that sentence is false. But then its negation, "$\neg(a = b)$", is true. So a and b are different. And if the concept of difference meaningfully applies to a and b, the one of identity does as well. "$a = b$" is meaningful together with its negation: adding or removing a negation in front of such a meaningful sentence cannot turn it into a meaningless one. The concept of identity cannot but apply to whatever the concept of difference applies to: if – to use Ryle's jargon – we have no category mistake in the latter case, we have no such mistake in the former.

When the number of things (in a system) is given by positive integer n, these things cannot lack self-identity. So understood, identity amounts to unity: to be self-identical is to count as one. This is how Aristotle characterizes identity in the quote opening our paper: ταυτότης ἑνότης τίς ἐστιν. So says Heidegger:

> To every being as such there belongs identity, the unity with itself. (Heidegger 1957, 26)

Auyang similarly claims:

> Identity does not say anything beyond *one thing*; rather, it discloses the meaning of *being an entity*, and the disclosure signifies our primordial understanding. [...] "*A* is *A*" is tautological because identity is constitutive of the general concept of the individual that *A* is. (Auyang 1995, 125)

4 Primitive Identity and Indiscernibility

Once identity is so strictly coupled with unity, identity ought to share the *primitiveness* of unity. In this Section, I firstly embrace the view that identity is primitive. Secondly, I argue that the primitiveness of identity begs no question in the debate on whether quantum particles have identity, in relation to the status of Leibniz's Identity of Indiscernibles. Thirdly, I grant that my asserting the primitiveness of identity commits me to *haecceitates*, "thisnesses", but I argue that one can understand *haecceitas* as oneness, in a metaphysically innocent way, in the framework of standard set theory. Fourthly, I draw some consequences of the availability of haecceitates, so understood, for the distinction between epistemic issues of individuation and metaphysical issues of identity.

4.1 The Primitiveness of Identity

That some concepts are primitive seems uncontroversial: if each notion were definable in terms of others, we would have a vicious regress or a large *circulus in definiendo* (Williamson 2007, 50–1). Definitions must come to an end.

There being primitive concepts, it is likely that there is no no fool-proof algorithm for them. Many take the concept of set as primitive. We explain to students that *sets* are *aggregates* or *collections* of objects, and we elucidate by giving examples, but these are no definitions. Kripke famously claimed of the notion of *reference* that "philosophical analyses of some concepts like reference, in completely different terms which make no mention of reference, are very apt to fail" (Kripke 1972, 94). And in *The Question of Ontology*, Kit Fine treated the notion of *reality* as primitive: "we seem to have a good intuitive grasp of the concept"; but he does "not see any way to define the concept of reality in essentially different terms" (Fine 2009, 175).

If *is a set, refers to, is real* are plausible primitives given their fundamental role in our understanding of mathematics, language, and the world – one may say: if they are part of what Auyang called the "categorical framework" – so is *is one*. Philosophers ask difficult questions about the conditions under which things x_1, ..., x_n are one in the sense of composing a unity, that is, of having a mereological sum with exactly x_1, ..., x_n as its parts (the Special Composition Question: van Inwagen (1990)). This does not change that, in Graham Priest's words:

> The notion of being one thing is, perhaps, our must fundamental notion. One cannot say anything, think anything, cognize anything, without presupposing it'. (Priest 2014, xv)

If being self-identical is one with being one, and being one is primitive because of such fundamentality, identity ought to be primitive as well.

4.2 The Identity of Indiscernibles

Does my committing to the primitiveness of identity beg any question against the Received View? I think not. That identity is primitive is consistent with *both* of the metaphysical views between which physics is underdetermined according to French and Krause (that particles have identity; that they do not). It is generally agreed by both parties that there are no identity criteria for particles, such that their identity can be reduced to them. And no non-trivial version of the Identity of Indiscernibles,

(IdIn) $\forall F(Fa \leftrightarrow Fb) \rightarrow a = b$

applies to particles either way – were a "non-trivial" version of (IdIn) is one which does not already embed identity within the properties we quantify over in the antecedent of the conditional.[6] If the Received View is right and the concept of

6 See Forrest (2010) for a general introduction to the topic. Saunders (2003) offers a subtle analysis of different ways of understanding (IdIn). We obtain formulations of different strength

identity does not apply to particles, then (IdIn) is simply moot with respect to them, rather than finding in them a counterexample, as rightly pointed out, e. g., by Ladyman and Ross (2007, 135). On the other hand, if particles are endowed with identity, then any non-trivial formulation of (IdIn) should be violated by some particles: both by fermions and by bosons (French and Krause (2006, 153–6)); see also Ladyman and Ross (2007, 135–6)); and if not by fermions (usually with the help of Pauli's Exclusion Principle: see Massimi 2001 on this), at least by bosons (van Fraassen 1989; Saunders 2003, 2006). Thus, to claim that *being self-identical* is primitive because it is one with *being one*, and that because the latter is primitive, the former is, does not pre-judge issues in the comparison between the Received View and its denial.

4.3 Primitive Thisness Is Innocent

If one claims that, their identity being primitive, (at least some) QM particles defy any non-trivial form of (IdIn), and one specifies that they also cannot have their identity grounded in their space-time trajectories as is commonly admitted in the philosophy of QM, is one not committed to some haecceity, or "primitive thisness", or "transcendental individuality?" In his seminal paper, Adams characterized this as:

> the property of being identical with a certain individual – not the property that we all share, of being identical with some individual or other, but my property of being identical with me, your property of being identical with you, etc. (Adams 1979, 6)

depending on whether we include spatiotemporal properties, only non-spatiotemporal ones (so the debate on (IdIn) quickly involves questions about the nature of spacetime), or only monadic properties as opposed to relational ones. A famous armchair criticism of (IdIn) based on thought experiment is due to Black (1952), where the relevant scenario involves two indiscernible iron spheres. Hacking (1975) criticizes Black by reinterpreting the scenario as involving one globe in Riemannian spacetime, and French (1995) criticizes Hacking for misunderstanding the relevant physics. Whether we take this or that non-trivial version of (IdIn) as a metaphysical law, or rather as contingently true (Casullo 1984), or as contingenly false (French and Krause 2006), all that matters for us is that any trivial formulation will not allow a reduction of identity to identity-independent concepts: "Once predicables involving ' = ' or its congeners and its derivatives are included within the range of the variable, the formula is neither an analytical explication nor even a serviceable elucidation of identity. For the formula manifestly presupposes identity."(Wiggins 2001, 63). One of the most sophisticated discussions of the topic of (IdIn) in the context of QM is the recent Muller (2015). Muller argues that various putative counterexamples to (IdIn) miss the kind of objects that actually do the discerning: these, called *relationals,* can only be discerned via relations, not via properties. Se also Muller and Saunders (2008).

I admit being identical with a as a property of each a. But various metaphysicians are suspicious of the notion of haecceity or primitive thisness, which they find obscure and objectionable (see e. g. Wiggins 2001, 125–6). I answer that the haecceitistic properties I need to appeal to, in a set-theoretic context where properties are represented by their extension, are innocent. And QM is based on standard mathematics, that is, on standard set theory. Although I take unity as a primitive notion, I allow the addition of a set-theoretic gloss to it: for a to be one, I claim, is for there to be $\{a\}$: to be one thing is to have one's singleton. Each thing, a, has the property of being identical with a. This property, extensionally, is nothing but $\{a\}$, which manifests the thing's unity. And the existence of its singleton – its haecceity, in this sense – is guaranteed to each thing in standard set theory. Primitive thisness or haecceity, so understood, is kosher to the extent that standard mathematics, that is, standard set theory, is.

4.4 Indiscernibility as Structure-Relative

Singletons also help us understand what goes on when some mathematical structures, like those employed in QM, represent certain individuals as indiscernible. Because each thing is self-identical (that is, one; that is, such that it has a singleton), any indiscernibility between things must be relative to a structure and internal to its viewpoint. A non-discriminating structure lacks some properties, which are, however, always there, as can be seen from the outside – ultimately, from the viewpoint of the complete set-theoretic structure (say, the standard well-founded universe of ZF: the cumulative hierarchy of sets).

Let me clarify a bit such metaphorical talk of "viewpoints". Within a set-theoretic structure (a group, a ring of functions, a poset, a lattice of sets, or else), one can call elements a and b of the structure indiscernible when there is an automorphism f on the domain making for the support of the structure, that is, an isomorphism (a structure-preserving bijection) from the structure to itself, such that $f(a) = b$. Automorphisms are permutations. When a structure is well-ordered, i. e., each non-empty subset of the domain has a least element with respect to the relevant total ordering, its only automorphism is the identity function. In such a structure there are no distinct but indiscernible elements (French and Krause 2006, 265 call such structures "rigid").

Many structures admit non-trivial automorphisms. For instance $<Z, +>$, the additive group of integers, has the automorphism f: $Z \rightarrow Z$, $f(x) = -x$. From the viewpoint of the structure we cannot tell any number from its additive inverse, for instance 2 and −2. To make them discernible we can add order-imposing properties like *being greater than zero,* unavailable within the structure.

However, all of these are set-theoretic structures. Thus, for any a which belongs to any such structure we have ZF guaranteeing its $\{a\}$. From the broader viewpoint of the standard cumulative set-theoretic hierarchy, each thing will have its unity, that is, its identity. Any structure can be rigidified, if trivially, by adding to each element of the structure its singleton.

The distinction between an internal or structure-relative viewpoint and an external one can be mapped to a distinction between the epistemological viewpoint of an *observer* who cannot, even in principle, discern some things from within a structure, and the metaphysical viewpoint of things as they *are* in themselves. This comes with a sharp separation between issues of identity, which pertain to metaphysics, and issues of individuation, which pertain to epistemology.[7] From this perspective, to claim, e. g., that "we are unable to identify individual electrons, hence it is meaningless to speak of the self-identity of electrons" (Hesse 1970, 50), looks like a slide from epistemology to metaphysics. We may be unable to individuate some electrons in a system. But we should not extend our theoretical limitations to reality and conclude that the electrons themselves lack identity, when a conceptual investigation of the notion of identity-as-unity reassures us that, insofar as we have a positive integer giving the number of them, they do not.

The factoring out of permutations in the statistics of QM described in Section 2 above, then, has in this sense no metaphysical significance: the ruling out of some permutations from the overall counting needs to be understood as a restriction on observables, that is, on what we can discern. This is the epistemic

7 The distinction is often discussed in debates on identity criteria (for an introduction, see Chapter 3, Section 2 of Berto and Plebani 2015). Talk of criteria of identity is traced back to Frege's *Grundlagen der Arithmetik* claim that "if we are to use symbol a to signify an object, we must have a criterion for deciding in all cases whether b is the same as a, even if it is not always in our power to apply this criterion" (Frege 1884, § 52). Some have detected a mixture of metaphysics and epistemology in the Fregean phrasing (see Williamson 1990, 148–9). A criterion of identity is not a criterion of individuation, such as the checking of fingerprints by the police in order to tell whether suspect a is public enemy b. A criterion of individuation concerns the ways in which we can come to know whether we have a case of identity. A criterion of identity, instead, is supposed to specify the conditions under which a and b are the same, independently of how we can decide whether they are the same or not. In Kit Fine's words, "the problem of [the criterion of] identity is not the epistemological question of saying how we can identify the object"; rather, it is the "metaphysical question of what, in the real world, explains the identity of the object" (Fine 1982, 102).

reading of the Indistinguishability Postulate by Greenberg and Messiah (1964), 250: when a permutation is applied to a system, we cannot distinguish the result from the original non-permuted state. Nothing follows concerning the metaphysical issue of the identity of the relevant objects.

> From this perspective, states formed by a particle permutation are not counted not because they do not exist, but because they are simply not available to the particles of the relevant symmetry type. With the reduction in statistical weight now explained by the inaccessibility of certain states, rather than by the non-classical metaphysical nature of the particles as non-individuals, one can continue to regard them as individuals for which certain states are now inaccessible. (French and Krause 2006, 148)

5 Reference to Indiscernible Objects

I have been postponing a problem. I have so far presupposed that we can refer, by pinning names on them or by making of them values of variables, to subatomic particles which may be indiscernible but which, if we are right, have identity. It is not obvious that we can do so. Ladyman and Ross (2007, 136) claim that the problem is especially acute for those, like me, who see particles as endowed with identity. Indeed the quaset and quasi-set frameworks were developed having this issue in mind.[8] In order for "$a = b$" to be truth-evaluable we need "a" and "b" to refer.[9] Given the understanding of identity as oneness, if "a" and "b" refer to one thing, that statement is true. Otherwise, it is false.

But how can we pin a singular term on exactly one of two indiscernible photons? Talk of causal connectedness to one of two entangled particles, for instance, does not make much sense. In van Fraassen's words:

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8 Even David Wiggins' forceful remark reported above, namely that identity is co-originary with predication because Fa if and only if $\exists x(x = a \,\&\, Fx)$, now does not look so innocent. As French and Krause (2006, 320) point out, we cannot introduce an individual constant "a" for a subatomic particle so straightforwardly in quasi-set theory. For one would normally do it only after proving some existence condition of the form "$\exists x(x = a)$", and you need identity to be applicable to the thing for this to be feasible.

9 This may not perforce be the case for "$a = a$". There are free logics (see Bencivenga (2002) for an introduction) with supervaluational-style semantics, where "$a = a$" gets a truth value even when "a" is denotationless. The idea is that for any way of extending a partial evaluation into a total one, that is, whatever denotation we could assign to that term, "$a = a$" would end up being true (or, "super-true"). This is not so for "$a = b$" anyway; for there will be ways of extending making it true, and ways of extending making it false.

> However obscure the notion of causal chain may be, this view of reference would seem to preclude differential naming of two photons in the same state – since entering into distinct causal chains would surely distinguish them in a way that quantum mechanics does not recognize. (van Fraassen 1998, 89)

Besides, one cannot have uniquely referring definite descriptions. My view that subatomic particles have self-identity, that is, unity, does not help. Although each particle, a, uniquely instantiates the property of being a, that is, extensionally, uniquely belongs to $\{a\}$, one cannot pick out a starting from its haecceity. For one cannot uniquely refer to the property of being a, or to $\{a\}$, until one has managed to uniquely refer to a. So that property cannot help to fix the reference of "a".

Switching from singular terms to the quantifier-variable idiom of general sentences may appear to improve the situation. One need not share the persuasion, of Fregean ascendency, that quantification over objects in a domain is possible only on the presupposition that singular reference to the elements of the domain via singular terms is available. It seems intuitive that we can quantify over, and talk in general of, things we have no way to achieve singular reference to. Maybe the particles of QM correspond to what Timothy Williamson called "elusive objects" (Williamson 2007, 16–7): things we can think and speak of collectively – as elusive objects, for instance – although we cannot single them out and individually refer to them for we lack the causal interactions or the descriptions which would be required. Given that the number of photons in a mirror-lined box is two, we can quantify over these things, it seems, and claim, truthfully, that there's at least two of them: $\exists x \exists y \neg (x = y)$. Diversity, and thus identity, still are meaningfully applied to them although we cannot label them individually.

However, in the semantics of elementary languages we normally give the recursive truth conditions of quantified formulas like "$\exists x \exists y \neg (x = y)$" by first assigning values to the variables in the open formula "$\neg (x = y)$". We then express the truth conditions for the whole formula by quantifying in the metalanguage over reassignments of values to variables. A formula of the form "$\exists x A[x]$" is true (in an interpretation), with respect to assignment s, if and only if "$A[x]$" is true for some reassignment, s_1, that differs from s only with respect to the object assigned to "x". This presupposes that we assign values to variables to begin with. And if one finds pinning singular terms, "a", "b", on things one cannot single out and individuate problematic, one should find assigning such things as values of variables "x", "y", equally problematic.

This account of the semantics of the quantifiers is often called the "objectual reading". Switching to the main contender, the substitutional reading (Haack 1978, 42), is not going to help: that "$\exists x A[x]$" is true if and only if some substitution instance "$A[x/a]$" is true presupposes that "a" gets its denotation to begin with.

The first thing to say on this issue is that, although one motivation for a quaset or quasi-set theory is to account for objects that, as French and Krause say, cannot be labelled from the very start, it is not clear to me what help will come from switching to the formal apparatus of quaset or quasi-set theory. Whatever problem is caused to the semantics of variables and quantifiers in ordinary logic by the indiscernibility of particles is inherited by the variables and quantifiers of quasi-set theory. Supporters of the Received View, including quaset and quasi-set theorists, say a lot of things about subatomic particles allegedly lacking identity, and do it by employing the quantifier-variable idiom (see e. g. French and Krause 2006, 277–81). Declaring some objects identity-less is not going to help with quantification over *them*. The quasi-set theory presented in the book and in various papers has variables that range over things allegedly lacking identity, variables which can be bound by quantifiers. To pick one example at random, one who claims:

> either the non-individual y belongs to the quasi-set A or not, as in the case of an atom, where an electron either belongs or does not belong to it, although we cannot name it unambiguously. Here, y does not act as a name for an individual (French and Krause 2006, 319)

… while denying that we can name arbitrary particles, is using variables to refer to them and say things about them.

Even if this is right, it is better to look for a positive account of how we *can* assign one of several indiscernible objects as the value of a variable or the referent of a term. The solution to this issue, in my opinion, consists simply in claiming that we can fix the reference of an individual constant or variable arbitrarily. When we do so, we just refer to one of a bunch of objects, and we do not and cannot know which.

There is nothing special with quantum objects in this respect. Arbitrary selection of referents is in place also with ordinary objects of everyday experience. One can think of a man who crossed the Tower Bridge on January 1st, 1980, and name him "Pete". There are many such men, of course, and even if one has all the men who crossed the Tower Bridge on that day before him, one will still not know which one "Pete" refers to. An account that fits the view is given in Breckenridge and Magidor (2012), Kearns and Magidor (2012). They convincingly defend the view that this goes on also when we reason, in mathematics as well as elsewhere, via stipulations of the form "Let a be an arbitrary F" ("Let f be an arbitrary homomorphism on a group", "Let Pete be an arbitrary

Londoner"), typically in Universal Generalization and Existential Instantiation (indeed, Bueno (2014) uses Universal Generalization and arbitrary reference to argue that identity is presupposed by quantification).

In Existential Instantiation, one establishes that something is such and so, $\exists x A[x]$, and assumes: "Now let it be a", $A[x/a]$. Then one derives from the assumption some conclusion, B, not involving a, and discharges the assumption. The argument works precisely because a is an arbitrary thing satisfying $A[x]$; there is no point in wondering which one.[10] One can tell a dual story for Universal Generalisation. Arbitrary reference to objects may be a special kind of reference, but is not *per se* reference to objects of a special kind, e. g., objects lacking identity. Pete is arbitrarily referred to, and is an arbitrary Londoner. It would be gratuitous to say that Pete is thereby a special kind of thing, or of Londoner.

Arbitrary reference so understood, as Breckenridge and Magidor (2012, 398) highlight, is especially suitable to explain what goes on when we refer to indiscernible objects. Two different numbers n are such that $n^2 + 1 = 0$. Call them i and $-i$. There is no non-haecceitistic feature we can resort to in order to discern them, but we can refer to either arbitrarily. If we buy this view of reference to arbitrary objects, the upshot for the particles of QM is pretty much the same as for i and $-i$. As French and Krause also claim:

> Consider the superposition formed by a pair of electrons and the spin states "up" and "down" [...]. In this case, we can assert that "there is one electron which has spin up", but not "*this* individual has spin up"; there does not appear to be any way of "picking out" the individual which has one name, rather than the other [...]. In these cases we simply have to accept that the labels refer, but we cannot say *to which* individuals they refer (French and Krause 2006, 214 and 230)

We can achieve singular reference to things we are not able to single out, such as subatomic particles in entangled states. One such thing can be made the value of a variable, or the referent of a term. That we do not and cannot know which particle in a two-particle system is the one named "a" (or made the value of "x") and which is the one named named "b" (or made the value of "y"), does not change that each of the two singular terms (respectively, variables) has received its own denotation (or, value). "$a = b$" is perfectly meaningful and truth-evaluable in the context. And because the number of particles in the

10 The term "$\varepsilon x A$" had just this function in Hilbert's epsilon calculus, famously in connection to the Axiom of Choice. We are not discussing issues connected to the AC in detail here, since we have been dealing only with finite collections of particles.

system is two, it is false. Then its negation is true. Via (twofold) Existential Generalization, "$\exists x \exists y(x = y)$" is true as well.

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