Dynamic models of research and development

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Chapter 3

Cartels and Innovation

3.1 Introduction

There are compelling reasons for rival firms to set up R&D cooperatives. These "organizations, jointly controlled by at least two participating entities, whose primary purpose is to engage in cooperative R&D" (Caloghirou *et al.*, 2003) allow risks to be spread, secure better access to financial markets, and pool resources such that economies of scale and scope in both research and development are better realized. In the words of John Kenneth Galbraith (1952, pp. 86 – 87, emphasis added): "Most of the cheap and simple innovations have, to put it bluntly and unpersuasively, been made. Not only is development now sophisticated and costly but *it must be on a sufficient scale* so that success and failures will in some measure average out." Moreover, R&D cooperatives internalize technological spillovers - the free flow of knowledge from the knowledge creator to its competitors.[1] Sustaining R&D cooperatives is thus perceived to diminish the failure of the market for R&D.[2]

However, as Scherer (1980) observes: "the most egregious price fixing schemes in American history were brought about by R&D cooperatives", an observation that confirms a widely-aired suspicion (see, e.g., Pfeffer and Nowak (1976), Grossman and Shapiro (1986),

[1] Bloom *et al.* (2007) estimate that a 10% increase in a competitor's R&D is associated with up to a 2.4% increase in a firm's own market value. Not surprisingly, internalizing technological spillovers is one of the prime reasons for firms to join an R&D cooperative (Hernan *et al.*, 2003; see also Roeller *et al.*, 2007).

[2] This motivates in particular why independent firms are allowed to cooperate in R&D. See Martin (1997) for an overview of the policy treatment of R&D cooperatives in the E.U., the U.S., and Japan.

and Brodley, 1990).[3] The channels through which cooperation in R&D can facilitate product market collusion have been examined in a number of theoretical studies (see, e.g., Martin (1995), Greenlee and Cassiman (1999), Cabral (2000), Lambertini *et al.* (2002) and Miyagiwa, 2009). As Martin (1995) puts it: "common assets create common interests, and common interests make it more likely that firms will non-cooperatively refrain from rivalrous behavior." (Martin, 1995, p. 740).[4] While price fixing may lead to a reduction of standard surplus measures, in this chapter we challenge the view that extending cooperative behavior to the product market necessarily diminishes consumer surplus and total surplus.

Geroski (1992) argues that it is the feedback from product markets that directs research towards profitable tracks and that, therefore, for an innovation to be commercially successful there must be strong ties between marketing and development of new products. Jacquemin (1988) observes that R&D cooperatives are fragile and unstable. He reasons that when there is no cooperation in the product market, there exists a continuous fear that one partner in the R&D cooperative may be strengthened in such a way that it will become too strong a competitor in the product market. Preventing firms from collaborating in the product market may therefore destabilize R&D cooperatives, or prevent their creation in the first place. Our focus is on the incentives to develop further an initial technology ('ideas'). In general, we find that product market collusion fosters R&D investment incentives because more of the ensuing economic rents can be appropriated by the investing firms. As a result, if firms collude, they will bring more initial technologies to full maturation. And this is unambiguously welfare enhancing.

Static models of R&D predict total surplus to go down if members of an R&D cooperative

[3] Goeree and Helland (2008) find that in the U.S. the probability that firms join an R&D cooperative has gone down due to a revision of antitrust leniency policy in 1993. This revision is perceived as making collusion less attractive. Goeree and Helland (2008) conclude that "Our results are consistent with RJVs [research joint ventures] serving, at least in part, a collusive function." Related evidence is reported by Duso *et al.* (2010). They find that the combined market share declines if partners in an RJV compete on the same product market ("horizontal RJVs"), while it increases if members of the RJV are not direct rivals ("vertical RJVs"). The laboratory experiments of Suetens (2008) show directly that members of an RJV are more likely to collude on price.

[4] In a similar vein, Fisher (1990, p. 194) concludes that "...[firms] cooperating in R&D will tend to talk about other forms of cooperation. Furthermore, in learning how other firms react and adjust in living with each other, each cooperating firm will get better at coordination. Hence, competition in the product market is likely to be harmed."

collude in the product market.[5] But a static view of the world necessarily ignores an important aspect of R&D: time. It takes time for an initial idea to be developed towards a marketable product; continuous process innovations *gradually* reduce production costs (Utterback, 1994). In this chapter, therefore, we develop a dynamic model of R&D to examine the welfare implications of product market collusion by firms of an R&D cooperative. This analysis builds upon our discussion in Chapter 2, where we developed a global framework for an innovating monopolist.

Static models of R&D also predict that the marginal benefit of any R&D investment increases if firms collude in the product market. That is, firms are willing to spend more resources on R&D if the intensity of product market competition is diminished through some collusive agreement.[6] This suggests that any initial idea (that is, any initial level of marginal costs) is more likely to be developed further if firms collude in the product market. Therefore, in a formal analysis, no level of initial marginal costs should be excluded from the analysis, in particular marginal costs that exceed the choke price (that is, the lowest price at which the quantity sold is zero). Moreover, requiring marginal costs to be below the choke price at all times implicitly imposes R&D activity and production to coexist at all times. Surely this assumption is quite unlikely to hold for new technologies at their early stages of development. Research starts long before a prototype sees the light; development begins long before the launch of a new product. To properly assess the welfare implications of product market collusion induced by an R&D cooperative, this development phase should be included in the analysis.

Therefore, a distinguishing feature of our approach is that we provide a *global* analysis. That is, we consider all possible values of initial marginal costs, including those above the

[5] d'Aspremont and Jacquemin (1988) are the first to show that a scenario where firms cooperate in R&D and collude in the ensuing product market yields a lower total surplus than the situation where firms cooperate in R&D only.

[6] Again, d' Aspremont and Jacquemin (1988) are the first to show this formally. This touches upon the debate between *Schumpeter Mark I* ("...new combinations are, as a rule, embodied, as it were, in new firms which generally do not arise out of the old ones but start producing beside them;...in general it is not the owner of stage-coaches who builds railways"; Schumpeter, 1934, p. 66) and *Schumpeter Mark II* ("As soon as we go into the details and inquire into the individual items in which progress was most conspicuous, the trail leads not to the doors of those firms that work under conditions of comparatively free competition but precisely to the doors of the large concerns...and a shocking suspicion dawns upon us that big business may have had more to do with creating that standard of living than with keeping it down"; Schumpeter, 1943, p. 82).

choke price. Hence, we allow research efforts to precede production.[7] Also, we do not limit ourselves to an analysis of equilibrium paths but we consider *all* trajectories that are candidates for an optimal solution. This enables us to determine the location of *critical points* - points at which the optimal investment function qualitatively changes. In particular, we determine the value of marginal costs for which R&D investments are terminated, and for which they are not initiated at all. The size of these critical cost levels is affected by firm conduct. Extending the R&D cooperative agreement to product market collusion can lead to qualitatively different long-run solutions, despite starting from an identical initial technology.

For a global analysis we have to use bifurcation theory. This gives us a bifurcation diagram that indicates for every possible parameter combination the qualitative features of any market equilibrium. Like in the monopoly case discussed in Chapter 2, it yields four distinct possibilities: (i) initial marginal costs are above the choke price and the R&D process is initiated; after some time production starts and marginal costs continue to fall with subsequent R&D investments; (ii) initial marginal costs are above the choke price and the R&D process is not initiated, yielding no activity at all; (iii) initial marginal costs are below the choke price and the R&D process is initiated; production starts immediately, marginal costs continue to fall over time, and the steady-state is reached that is characterized by continuous R&D investments, and (iv) initial marginal costs are below the choke price and the initiated R&D process is progressively scaled down; production starts immediately but the technology (and production) will die out over time; the firms leave the market. To date, the literature has considered possibility (iii) only, and only partially so.

We then compare two different scenarios across these possibilities. In the first scenario, labeled 'competition', firms cooperate in R&D and compete on the concomitant product market. In the second scenario, labeled 'collusion', cooperation in R&D is extended to collusion in the product market. We then compare the qualitative properties of these two scenarios in order to assess the potential set-back of R&D cooperatives in that they can serve

[7] Here we deviate from the related literature that, with no exception, restricts the analysis to initial levels of marginal costs that are below the choke price (cf. Petit and Tolwinski (1999), Cellini and Lambertini (2009), Lambertini and Mantovani (2009), and Kovac *et al.* (2010)). As will become clear below, this restriction excludes a crucial part of the parameter space.

as a platform to coordinate prices.

Our analysis yields three key findings: (i) if firms collude, the range of initial marginal costs that leads to the creation of a new market is larger, (ii) collusion in the product market accelerates the speed with which new technologies enter the product market, and (iii) the set of initial marginal costs that induces firms to abandon the technology in time is larger if firms do not collude in the product market. Related, we show that there are parameter configurations whereby collusion in the product market yields higher total surplus. We thus qualify the conclusion of Petit and Tolwinski (1999, p. 206) that "[collusion] is socially inferior to other forms of industrial structures", a conclusion that is based on a local analysis and which can be seen as representative for the related literature.

Our results are not without policy implications. When designing antitrust policies, it is important to understand that these policies not only affect current markets, but also markets that are not yet visible. Preventing firms from colluding in the product market reduces the number of potential R&D trajectories that successfully lead to new markets. In itself this constitutes a welfare loss. However, because not developing further an initial technology does not surface as a direct surplus loss, this welfare loss remains hidden. It is left for future research to assess empirically the size of this hidden cost.

3.2 The model

Time t is continuous: $t \in [0, \infty)$. There are two *a priori* fully symmetric firms which both produce a homogenous good at constant marginal costs. In every instant, market demand is:

$$p(t) = A - Q(t), \tag{3.1}$$

where $Q(t) = q_1(t) + q_2(t)$, with $q_i(t)$ the quantity produced by firm i at time t, and where $p(t)$ and A are respectively the market price at time t and the choke price.

Each firm can reduce its marginal cost by investing in R&D. In particular, firm i exerts R&D effort $k_i(t)$ and as a consequence of these investments, its marginal cost evolves over time as follows:

$$\frac{dc_i}{dt}(t) \equiv \dot{c}_i(t) = c_i(t)\left(-k_i(t) - \beta k_j(t) + \delta\right), \tag{3.2}$$

where $k_j(t)$ is the R&D effort exerted by its rival and where $\beta \in [0, 1]$ measures the degree of spillover. The parameter $\delta > 0$ is the constant rate of decrease in efficiency due to the ageing of technology and organizational forgetting.[8] Both firms have an identical initial technology $c_i(0) = c_j(0) = c_0$, which is drawn by Nature. The cost of R&D efforts per unit of time $\Gamma_i(k_i(t))$ takes the form

$$\Gamma_i(k_i) = bk_i^2, \tag{3.3}$$

where $b > 0$ is inversely related to the cost-efficiency of the R&D process. Hence, the R&D process exhibits decreasing returns to scale (Schwartzman (1976); see also the discussion in Chapter 2). Both firms discount the future with the same constant rate $\rho > 0$. Either firms' instantaneous profit therefore equals:[9]

$$\pi_i(q_i, Q, k_i, c_i) = (A - Q - c_i)q_i - bk_i^2, \tag{3.4}$$

yielding total discounted profit:

$$\Pi_i(q_i, Q, k_i, c_i) = \int_0^\infty \pi_i(q_i, Q, k_i, c_i)e^{-\rho t}dt. \tag{3.5}$$

The model has five parameters: A, β, b, δ, and ρ. The analysis can be simplified by considering a rescaled version of the model which, as defined in Lemma 9, carries only three parameters: β, ϕ, and $\tilde{\rho}$ (see Appendix 2.A in Chapter 2 for the proof).

Lemma 9. *By choosing the units of* t, q_i, q_j, c_i, c_j, k_i, *and* k_j *appropriately, we can assume*

[8] Assuming an exogenous depreciation rate is common in the literature. See footnote 5 in Chapter 2 for a discussion.

[9] Implicitly we assume here that firms know market demand in advance of production. Relaxing this assumption would make the analysis more complex, while it would not alter any of our conclusions as to the comparison of the competitive and collusive scenario, provided that under both scenarios firms have identical expectations about future demand.

$A = 1$, $b = 1$, *and* $\delta = 1$. *This yields the following rescaled version of the model:*

$$\tilde{\pi}_i(\tilde{q}_i, Q, \tilde{k}_i, \tilde{c}_i) = (1 - \tilde{Q} - \tilde{c}_i)\tilde{q}_i - \tilde{k}_i^2, \tag{3.6}$$

$$\tilde{\Pi}_i(\tilde{q}_i, \tilde{Q}, \tilde{k}_i, \tilde{c}_i) = \int_0^\infty \tilde{\pi}_i(\tilde{q}_i, \tilde{Q}, \tilde{k}_i, \tilde{c}_i) e^{-\tilde{\rho}\tilde{t}} d\tilde{t} \tag{3.7}$$

$$\dot{\tilde{c}}_i = \tilde{c}_i \left(1 - \left(\tilde{k}_i + \beta \tilde{k}_j\right) \phi\right), \quad \tilde{c}_i(0) = \tilde{c}_0, \quad \tilde{c}_i \in [0, \infty) \, \forall \, \tilde{t} \in [0, \infty) \tag{3.8}$$

$$\tilde{q}_i \geq 0, \quad \tilde{k}_i \geq 0 \tag{3.9}$$

$$\tilde{\rho} > 0, \quad \phi > 0 \tag{3.10}$$

with conversion rules: $q_i = A\tilde{q}_i$, $q_j = A\tilde{q}_j$, $k_i = \frac{A}{\sqrt{b}}\tilde{k}_i$, $k_j = \frac{A}{\sqrt{b}}\tilde{k}_j$, $c_i = A\tilde{c}_i$, $c_j = A\tilde{c}_j$, $\pi_i = A^2\tilde{\pi}_i$, $\pi_j = A^2\tilde{\pi}_j$, $\phi = \frac{A}{\delta\sqrt{b}}$, $t = \frac{\tilde{t}}{\delta}$, $\tilde{\rho} = \frac{\rho}{\delta}$.

The rescaled version of the model introduces a new parameter ϕ, which captures the profit potential of a technology in hand: a higher (lower) A implies higher (lower) potential sales revenue, a higher (lower) b implies that each unit of R&D effort costs the firm more (less), whereas a higher (lower) δ implies that each unit of R&D effort reduces the marginal cost by less (more). Therefore, a higher (lower) ϕ corresponds to a higher (lower) profit potential of a technology. For notational convenience, we henceforth omit tildes.

3.3 Competition versus Collusion

This section derives the necessary conditions for optimal production and investment schedules in case firms cooperate in R&D, but compete in the product market (a scenario labelled 'competition'), and in case firms cooperate in R&D and collude in the product market (a scenario labelled 'collusion').

3.3.1 Competition

Both firms operate their own R&D laboratory and production facility, and while they select their output levels non-cooperatively, they adopt a strictly cooperative behavior in determining their R&D efforts so as to maximize joint profits. These assumptions amount to imposing *a priori* the symmetry condition $k_i(t) = k_j(t) = k(t)$.[10] As $c_i(0) = c_j(0) = c_0$, this implies that $c_i(t) = c_j(t) = c(t)$. Equation (3.8) thus reads as:

$$\dot{c} = c(1 - (1 + \beta)\phi k). \tag{3.11}$$

It may seem reasonable to assume that when firms cooperate in R&D, they also fully share information, that is, to assume the level of spillover to be at its maximum ($\beta = 1$; see Kamien *et al.*, 1992). For the sake of generality, we do not *a priori* fix the value of β at its maximal value. There are also intuitive arguments for not doing so as there might still be some *ex post* duplication and/or substitutability in R&D outputs if firms operate separate laboratories (see also the discussion in Hinloopen, 2003).

The instantaneous profit of firm i is

$$\pi_i(q_i, Q, k, c) = (1 - Q - c)q_i - k^2, \tag{3.12}$$

with $Q = q_1 + q_2$, yielding its total discounted profit over time:

$$\Pi_i(q_i, Q, k, c) = \int_0^\infty \pi_i(q_i, Q, k, c)e^{-\rho t}dt. \tag{3.13}$$

As firms cooperatively decide on their R&D efforts, the only independent decisions are those of production levels. However, as quantity variables do not appear in the equation for the state variable (3.11), production feedback strategies of a dynamic game are simply static Cournot-Nash strategies of each corresponding instantaneous game.

Maximising π_i over $q_i \geq 0$ gives us standard Cournot best-response functions for the product market:

$$q_i(q_j) = \begin{cases} \frac{1}{2}(1 - c - q_j) & \text{if} \quad q_j < 1 - c, \\ 0 & \text{if} \quad q_j \geq 1 - c. \end{cases} \tag{3.14}$$

[10] We consider symmetric equilibria only. See Salant and Shaffer (1998) for a specific example of a static model of R&D in which it is optimal for firms in an R&D cooperative to make unequal investments.

This expresses the fact that the constraint $q_i \geq 0$ is binding when $q_j \geq 1 - c$. Solving for Cournot-Nash production levels, we obtain

$$q^N = \begin{cases} \frac{1}{3}(1-c) & \text{if} \quad c < 1, \\ 0 & \text{if} \quad c \geq 1. \end{cases} \tag{3.15}$$

Consequently, the instantaneous profit of each firm is

$$\pi(c,k) = \begin{cases} \frac{1}{9}(1-c)^2 - k^2 & \text{if} \quad c < 1, \\ -k^2 & \text{if} \quad c \geq 1. \end{cases} \tag{3.16}$$

The dynamic optimization problem of the R&D cooperative reduces to finding an R&D effort schedule k^* for each firm that maximizes the total discounted joint profit of the two firms, taking into account the state equation (3.11), the initial condition $c(0) = 0$, and the boundary condition $k(t) \geq 0$ which must hold at all times. Note that according to (3.11), if $c_0 > 0$, then $c(t) > 0$ for all t. The state space of this problem is the interval $[0, \infty)$ of marginal cost levels.

To solve this problem, we introduce the current-value Pontryagin function (also called pre-Hamilton or un-maximized Hamilton function)[11]

$$P(c,k,\lambda) = \begin{cases} \frac{1}{9}(1-c)^2 - k^2 + \lambda c(1-(1+\beta)\phi k) & \text{if} \quad c < 1, \\ -k^2 + \lambda c(1-(1+\beta)\phi k) & \text{if} \quad c \geq 1, \end{cases} \tag{3.17}$$

where λ is the current-value co-state variable of a firm in the R&D cooperative. The co-state, or shadow value, measures the marginal worth of the increment in the state c for each firm in the cartel at time t when moving along the optimal path. As marginal cost is a "bad", we expect $\lambda(t) \leq 0$ along optimal trajectories.

We use Pontryagin's maximum principle to obtain the solution to our optimization problem.

[11]We omit a factor of 2 for combined profits to obtain the solution expressed in per-firm values. Due to symmetry, maximizing the per-firm total profit corresponds to maximizing the two firms' combined total profit.

Maximising over the control $k \geq 0$ yields

$$k = -\frac{1}{2}\lambda c(1+\beta)\phi, \tag{3.18}$$

whenever the value on the right hand side is nonnegative, and $k = 0$ otherwise. The maximum principle states further that the optimizing trajectory necessarily corresponds to the trajectory of the state-costate system

$$\dot{c} = \frac{\partial P}{\partial \lambda}, \quad \dot{\lambda} = \rho\lambda - \frac{\partial P}{\partial c},$$

where k is replaced by its maximizing value. For $\lambda \leq 0$, relation (3.18) gives a one-to-one correspondence between the co-state λ and the control k. We use this relation to transform the state-costate system into a state-control system which an optimizing trajectory has to satisfy necessarily as well. This system consists of two regimes (following the two part composition of the Pontryagin function). The first one corresponds to $c < 1$ and positive production ($q = (1-c)/3$). The second one corresponds to $c \geq 1$ and zero production.[12] The state-control system with positive production consists of the following two differential equations:[13]

$$\begin{cases} \dot{k} = \rho k - \frac{(1+\beta)\phi}{9} c(1-c), \\ \dot{c} = c\left(1 - (1+\beta)\phi k\right). \end{cases} \tag{3.19}$$

[12]Recall from Lemma 9 that $A = 1$ in the rescaled model. In the non-rescaled model, the analogous conditions for positive and zero production are $c(t) < A$ and $c(t) \geq A$, respectively.

[13]Our closed-loop solution differs from that of Cellini and Lambertini (2009), who consider the case when marginal cost is always lower than the choke price. This is so because their proof that the open-loop and closed-loop solutions coincide is flawed by the fact that in their derivation of the closed-loop solution, players' output choices are not properly treated as functions of the state variable. The derivations of the authors implicitly assume that if marginal cost within the R&D cooperative changes, the opponent's quantity does not change, which is the violation of the feedback principle underlying the closed-loop solution. It is also counterintuitive as firms in the R&D cooperative are supposed to jointly decide on their R&D efforts taking into account that marginal cost in any period affects the ensuing Nash-equilibrium profits. Our calculations also show that the solution of Cellini and Lambertini (2009) yields situations in which per-firm profits in the competitive scenario exceed those of the collusive scenarios, which obviously contradicts the notion that in case of collusion firms maximize joint profits.

The state-control system with zero production is given by

$$\begin{cases} \dot{k} = \rho k, \\ \dot{c} = c\,(1 - (1+\beta)\phi k)\,. \end{cases} \tag{3.20}$$

3.3.2 Collusion

If firms collude, they adopt a strictly cooperative behavior in determining both their R&D efforts and output levels. These assumptions amount to imposing *a priori* the symmetry conditions $k_i(t) = k_j(t) = k(t)$ and $q_i(t) = q_j(t) = q(t)$. Equation (3.8) reads therefore as:

$$\dot{c} = c(1 - (1+\beta)\phi k). \tag{3.21}$$

The profit of each firm in every instant is:

$$\pi(q, k, c) = (1 - 2q - c)q - k^2, \tag{3.22}$$

yielding its total discounted profit over time:

$$\Pi(q, k, c) = \int_0^\infty \pi(q, k, c)e^{-\rho t} dt. \tag{3.23}$$

The optimal control problem of the two firms is to find controls q^* and k^* that maximize the profit functional Π subject to the state equation (3.21), the initial condition $c(0) = c_0$, and two boundary conditions which must hold at all times: $q \geq 0$ and $k \geq 0$.[14] Notice again that according to (3.21), if $c_0 > 0$, then $c(t) > 0$ for all t.

The current-value Pontryagin function now reads as:

$$P(c, q, k, \lambda) = (1 - 2q - c)\,q - k^2 + \lambda c\,(1 - (1+\beta)\phi k)\,, \tag{3.24}$$

where λ is the current-value co-state variable. It now measures the marginal worth at time t

[14] Again, due to symmetry, maximizing per-firm total profit is the same as maximizing the two firms' combined total profit.

of an increment in the state c for a colluding firm when moving along the optimal path.

The necessary conditions for the solution to the dynamic optimization problem consist again of a state-control system which has two regimes. As in the competitive case, the first regime corresponds to $c < 1$ and positive production ($q = (1 - c)/4$), while the second corresponds to $c \geq 1$ and zero production.

The state-control system in the region with positive production reads as

$$\begin{cases} \dot{k} = \rho k - \frac{(1+\beta)\phi}{8} c(1-c), \\ \dot{c} = c\left(1 - (1+\beta)\phi k\right), \end{cases} \tag{3.25}$$

whereas the state-control system with zero production is

$$\begin{cases} \dot{k} = \rho k, \\ \dot{c} = c\left(1 - (1+\beta)\phi k\right). \end{cases} \tag{3.26}$$

3.4 Analysis

The systems (3.19) – (3.20) and (3.25) – (3.26) have the same structure as the state-control system for the monopoly problem considered in Chapter 2. Indeed, consider the system

$$\dot{c} = c\left(1 - (1+\beta)\phi k\right), \tag{3.27}$$

$$\dot{k} = \rho k - \alpha(1+\beta)\phi c(1-c)\chi_{[0,1]}(c), \tag{3.28}$$

where $\chi_A(c) = 1$ if $c \in A$ and $\chi_A(c) = 0$ if $c \notin A$. The monopoly system in Chapter 2 as well as systems (3.19) – (3.20) and (3.25) – (3.26) are instances of the system (3.27) – (3.28), with $\alpha = 1/9$ for the competitive scenario, $\alpha = 1/8$ for the collusion scenario, and $\alpha = 1/4$ for the monopoly studied in Chapter 2.

Therefore, the analysis of system (3.27) – (3.28) corresponds to the analysis of the system in Chapter 2. Accordingly, we confine ourselves to stating the principal results of system (3.27) – (3.28), including its bifurcation diagrams. All proofs are in the appendices of Chapter 2.

The first result gives the properties of the steady states of the state-control system.

Proposition 2. *Let*

$$D = \frac{1}{4} - \frac{\rho}{\alpha(1+\beta)^2\phi^2}.$$

Depending on the value of D, there are three different situations.

1. *If $D > 0$, the state-control system with positive production* (3.25) *has three steady states:*

   i. $(c^C, k^C) = (0, 0)$ *is an unstable node,*

   ii. $(c^C, k^C) = \left(\frac{1}{2} + \sqrt{D}, \frac{1}{(1+\beta)\phi}\right)$ *is either an unstable node or an unstable focus, and*

   iii. $(c^C, k^C) = \left(\frac{1}{2} - \sqrt{D}, \frac{1}{(1+\beta)\phi}\right)$ *is a saddle-point steady state.*

2. *At $D = 0$, there are two steady states:*

   i. $(c^C, k^C) = (0, 0)$*, which is an unstable node, and*

   ii. $(c^C, k^C) = \left(\frac{1}{2}, \frac{1}{(1+\beta)\phi}\right)$*, which is a semi-stable steady state.*

3. *If $D < 0$, the origin $(c^C, k^C) = (0, 0)$ is the unique steady state of the state-control system with positive production, which is unstable.*

The system consequently exhibits a saddle-node bifurcation at $D = 0$.

The stable manifold of the saddle-point steady state is one of the candidates for an optimal solution. As neither the Mangasarian nor the Arrow concavity conditions are satisfied, the stable manifold is not necessarily optimal. Note that Proposition 2 already implies that there should be other candidates for optimality as there is a parameter region for which there is no saddle point, and hence no stable manifold to it. We have the following result.

Corollary 2. *The set of candidates for an optimal solution consist of the stable path of the saddle-point steady state and the trajectory through the point $(c, k) = (1, 0)$.*

The thick black lines L_1 and L_2 in Figure 3.1 indicate these two candidates. In this figure, the dotted vertical line $c = 1$ separates the region with zero production from the region of

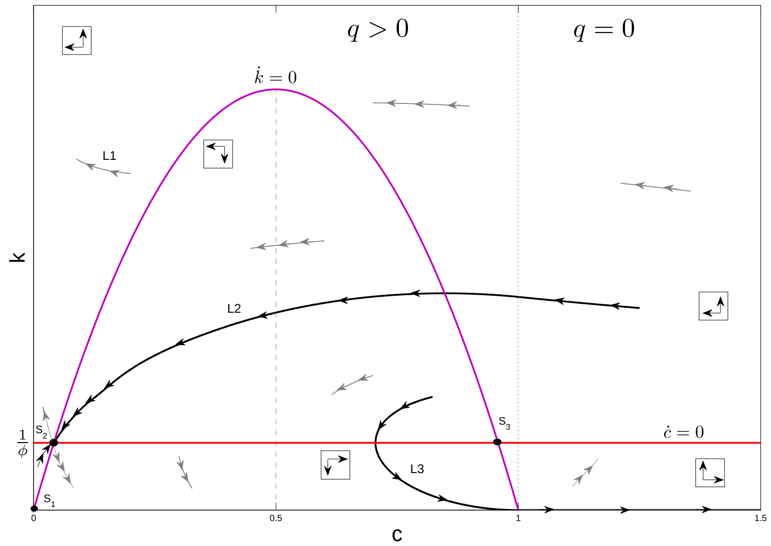


Figure 3.1: *Candidate trajectories in the state-control space.*

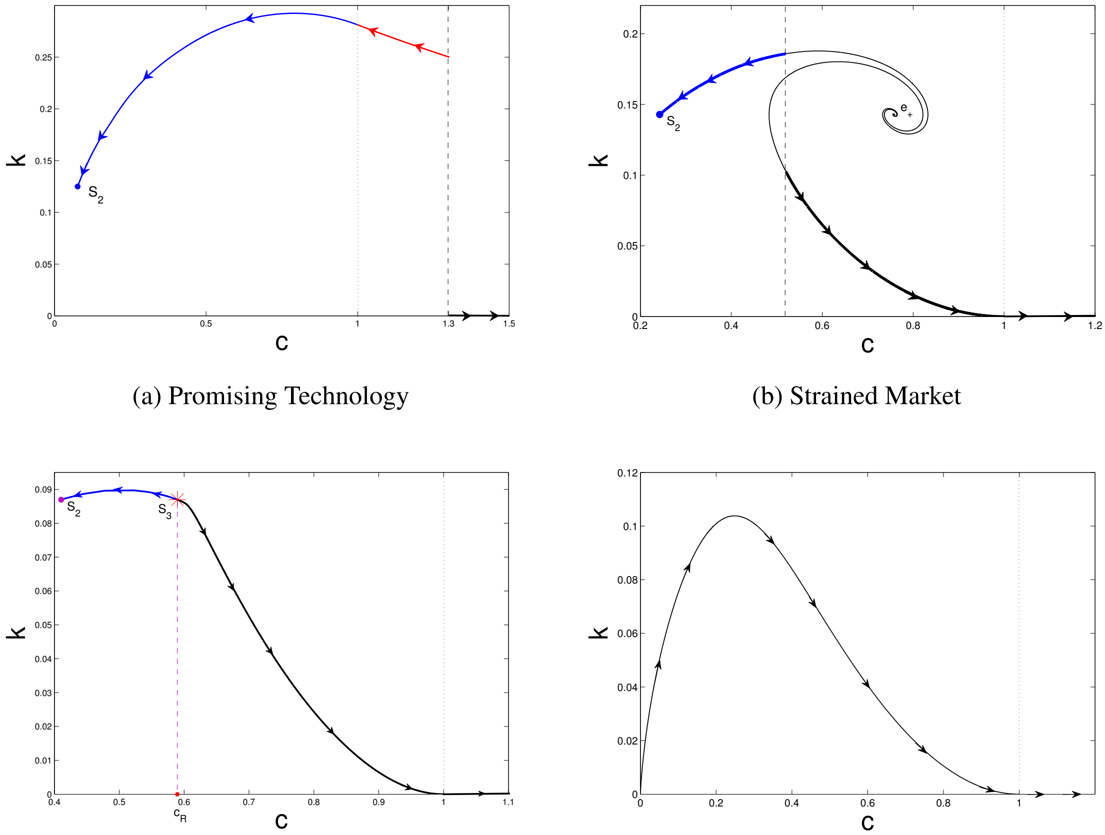
positive production. We label the trajectory L_2 the "exit trajectory", as both firms eventually leave the region $0 < c < 1$ of positive production when following this trajectory.

To assess the dependence of the solution structure on the model parameters, we carry out a bifurcation analysis. This consists in identifying the parameter values at which the qualitative structure of the optimal dynamics changes. These 'bifurcating' values bound open parameter regions such that the optimal dynamics are qualitatively equal for all parameter values in the region (see Wagener (2003), Kiseleva & Wagener, 2010, 2011). As for any point in the region, a sufficiently small change in the parameter value does not lead to a qualitative change of the dynamics, these regions are said to characterize *stable* types of dynamics.

In Chapter 2, we identified four distinct stable types. Figure 3.2 illustrates these types; Figure 3.3 shows the corresponding bifurcation diagram.

The first type is one of a "Promising Technology", where there is an *indifference threshold*[15] in the region of no production. In an optimal control problem, an indifference threshold is a point in state space where the decision maker is indifferent between two optimal trajectories that have distinct long-term limit behavior. In case of a Promising Technology, there is a point $\hat{c} > 1$, such that for $0 < c_0 \leq \hat{c}$, it is optimal to start developing the initial technology,

[15] Also known as Skiba, Dechert-Nishimura-Skiba or DNSS point; see Grass *et al.* (2008).



(a) Promising Technology (b) Strained Market

(c) Uncertain future (d) Obsolete technology

Figure 3.2: The four stable types of dynamics.

ending up in the saddle-point steady state in the region of positive production. In particular, this happens for initial values of c that are in the no-production region. If $c_0 \geq \hat{c}$, it is optimal not to initiate R&D efforts as in this case potential future profits do not suffice to compensate for losses that would be incurred in the initial periods during which firms would invest in R&D but would not produce yet. Note that for $c_0 = \hat{c}$, there are two entirely different R&D investment policies, which are, nevertheless, both optimal.

The second type corresponds to a “Strained Market”, where there is an indifference threshold in the region of positive production: $0 < \hat{c} < 1$. The new feature here is that for $\hat{c} \leq c_0 < 1$, the firm eventually leaves the market. It is optimal however to invest in R&D to slow down the technological decay.

We label the third type, which occurs only in a small part of the parameter space, the “Uncertain Future”. Instead of an indifference point, here a repelling steady state divides the

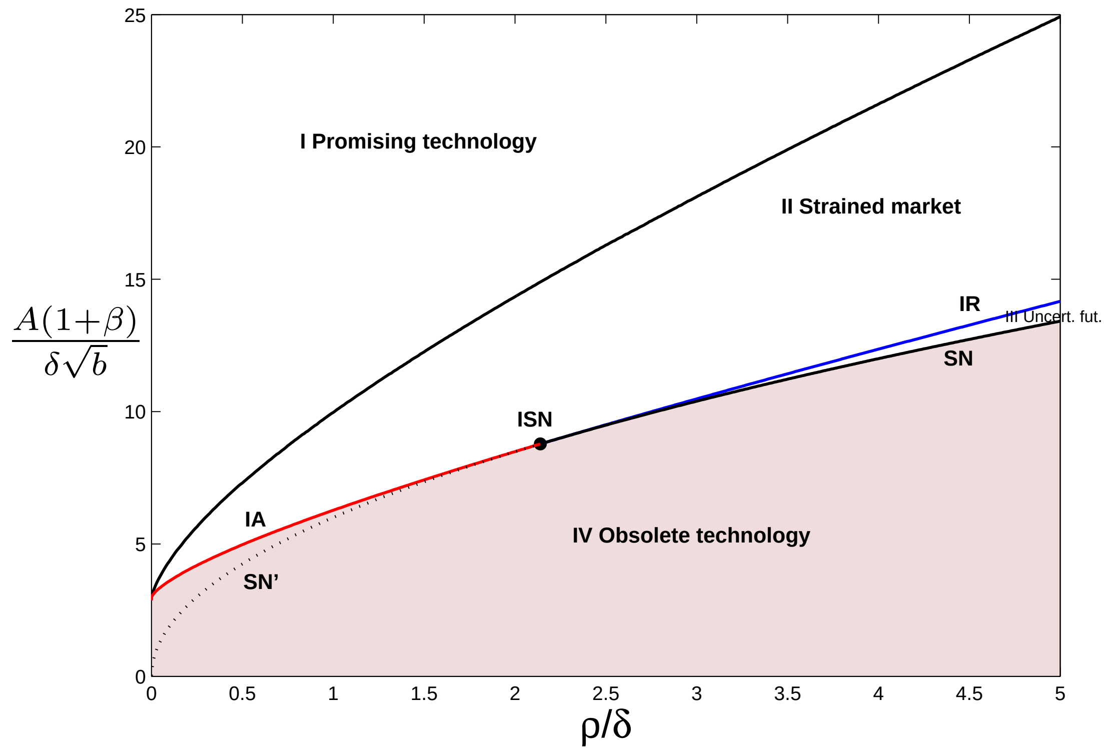


Figure 3.3: *Bifurcation diagram for* $\beta = 1$.

initial states that optimally converge to the steady state with positive production and those with no production region. If the system starts exactly at the repelling point, it stays there indefinitely; when it starts close to it, it stays there for a long period of time, after which it converges to one of the steady states.

The fourth type typifies the dynamics of an "Obsolete Technology". Whatever the initial state, the firms let the technology decay and eventually leave the market. In the region of positive production, the decay is again slowed down by R&D investments.

In the bifurcation diagram, the uppermost curve represents parameter values for which the indifference point is exactly at $c = 1$. At the saddle-node curve (SN), an optimal repeller and an optimal attractor collide and disappear. The curve SN' corresponds to saddle-node bifurcations in the state-control system that do not correspond to optimal dynamics. At the indifference-attractor bifurcations (IA),[16] an indifference point collides with an optimal attractor and both disappear. Finally, at an indifference-repeller bifurcation, an indifference

[16] For the terminology, see Kiseleva & Wagener (2010, 2011).

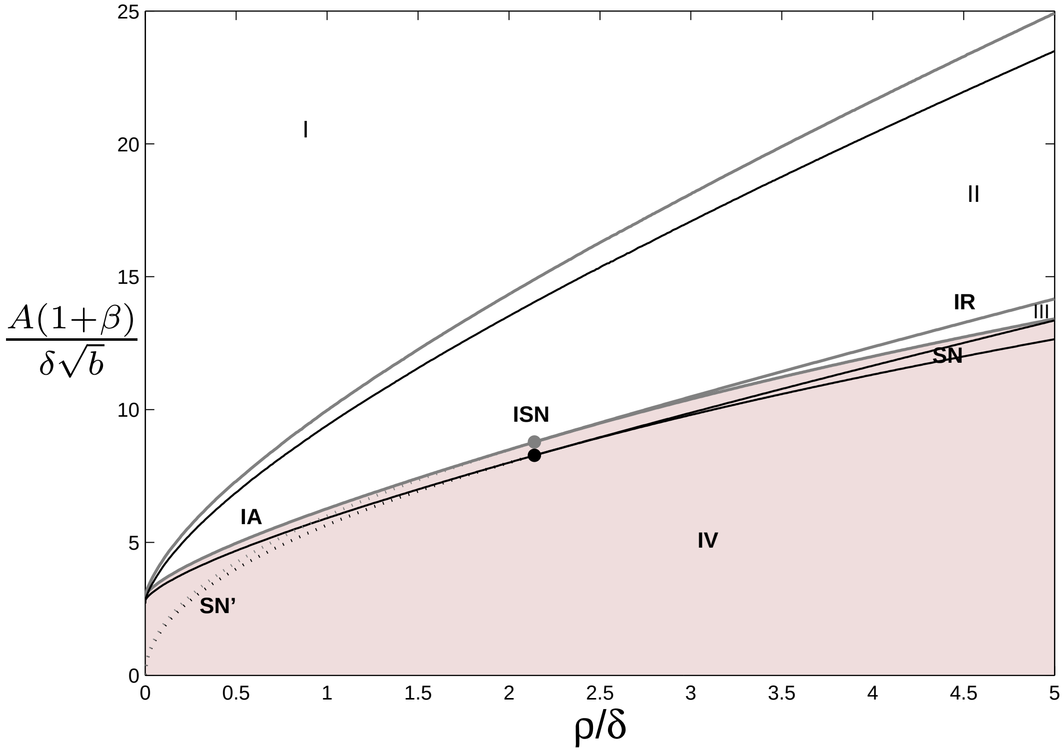


Figure 3.4: *Bifurcation diagram.* The plot depicts the bifurcation curves for the competitive scenario (grey) together with those of the collusive scenario (black). Throughout, the collusive curves pass through lower values of $A(1+\beta)/(\delta\sqrt{b})$ (non-rescaled variables) when compared at equal values of ρ/δ (non-rescaled).

point turns into an optimal repeller. The central indifference-saddle-node (ISN) bifurcation point at $(\tilde{\rho}, \phi) \approx (2.14, 8.78)$ organizes the bifurcation diagram. The curve representing indifference points at $c = 1$ obtains a value of $\phi \approx 2.998$ for $\tilde{\rho} = 1 \times 10^{-5}$.

3.5 Collusion and the incentives to innovate

Having characterized the global optimum of both the competitive and the collusive scenario, we can compare their respective bifurcation diagrams. These are superimposed in Figure 3.4. Qualitatively, there is no difference between the diagrams. There are, however, important quantitative differences which the following observation summarizes:

Numerical observation 1. *Over the entire parameter space we observe that if firms collude, the bifurcation curves lie below the concomitant curves in case firms compete.*

This observation has two corollaries. First, the “Promising Technology” region is larger if firms collude. Put differently, if firms collude, the situation where firms first invest in R&D, and only after some initial development period start producing, is more likely to occur. Second, if firms collude, the “Obsolete Market” region is smaller. That is, due to collusion, it is less likely that firms either do not develop further an initial technology, or that they invest in R&D only to abandon the technology in time.

Numerical observation 2. *Over the entire parameter space we observe that whenever a threshold value of initial marginal costs exists in both scenarios (be it an indifference point or a repeller), it is larger if firms collude.*

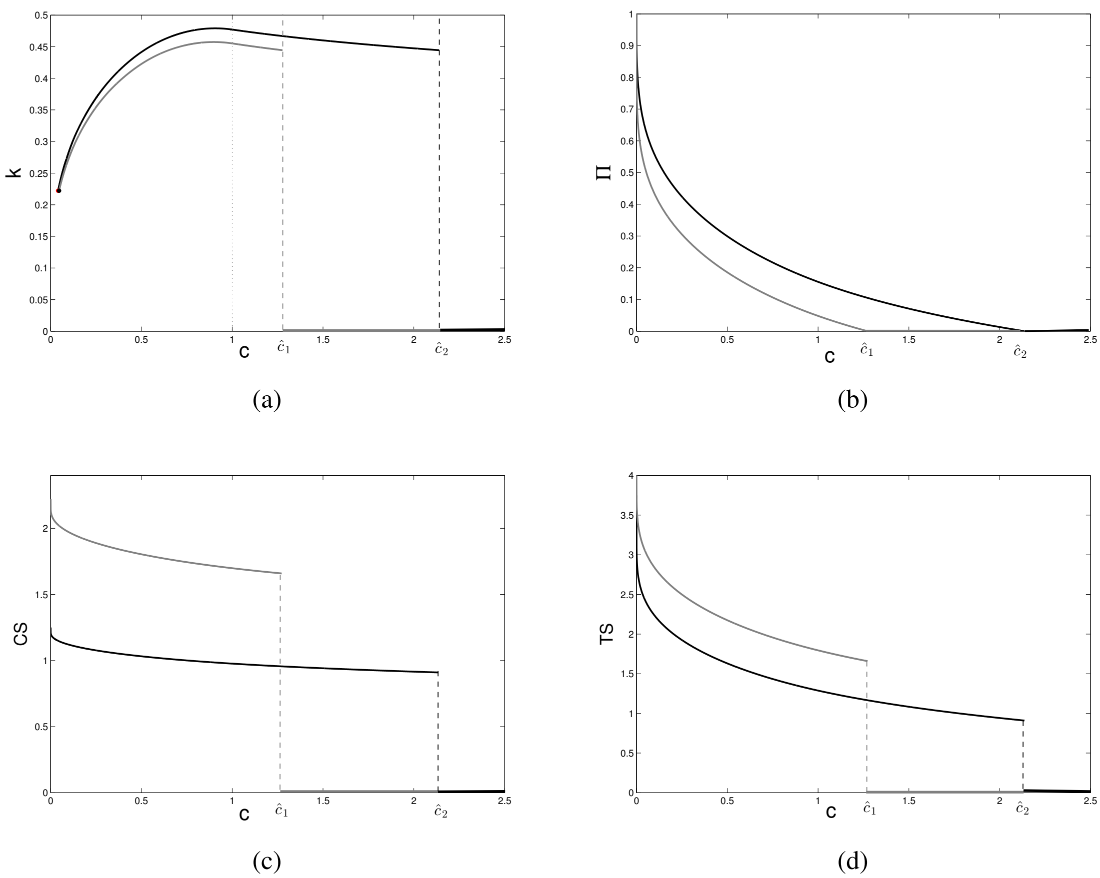


Figure 3.5: *State-control space (a), total discounted profit (b), consumer surplus (c), and total surplus (d), when the indifference point is in the region with zero production.* Parameters: $(\beta, \tilde{\rho}, \phi) = (1, 0.1, 2.25)$. Grey curves correspond to competition, whereas the black ones correspond to colllusion. For all $c_0 \in (\hat{c}_1, \hat{c}_2)$, the collusive scenario brings about higher consumer surplus and total surplus than the competitive scenario.

The implications of this observation are twofold. First, if firms collude, the set of initial technologies that are developed further and that lead to the saddle-point steady state is larger. Figure 3.5 illustrates this implication. If the initial technology draw c_0 falls in the non-empty interval $(\hat{c}_1, \hat{c}_2)$, the firms will develop the technology and this will eventually give rise to a new market, but only if firms collude. If they compete, neither firm will develop the technology.

Note that a higher value of initial marginal cost implies larger early-stage losses because there is no profitable production yet. Obviously, these losses are more quickly off-set by future profits if firms collude, due to higher mark-ups. Therefore, under collusion, firms can afford to invest more in R&D prior to production, and thereby to bring down over time a higher initial level of marginal cost.

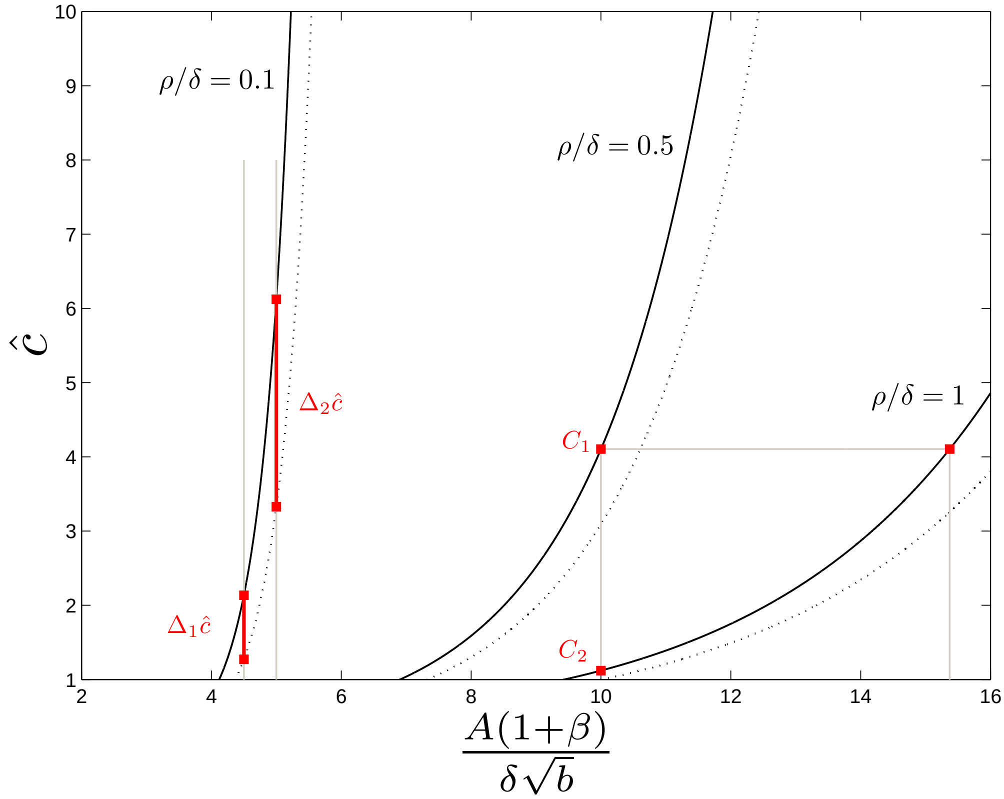


Figure 3.6: *Dependence of the indifference point* $\hat{c}$ *on model parameters.* Curves are drawn for three fixed values of $\tilde{\rho}$. Curves for competition (dotted) lie below the curves for collusion (full).

For this situation, Figure 3.6 illustrates some comparative statics of the indifference points. Obviously, these points are positively related to market size and R&D efficiency. Note however that also the difference $\Delta\hat{c}$ between $\hat{c}_{\text{competitive}}$ and $\hat{c}_{\text{collusive}}$ increases if the R&D

process becomes more efficient and/or if the market size becomes larger, the more so the lower the discount rate is. This inelasticity corresponds in Figure 3.6 to a larger slope of the convex curves. Because future mark-ups are positively related to both market size and R&D efficiency, an increase in either one of these has a larger (positive) effect on future profits if firms collude. And these future benefits feature more prominently in total discounted profits if the discount rate is lower. Put differently, indifference points occur at smaller values if the discount rate goes up, all else equal (cf. the relative location of C_1 and C_2 in Figure 3.6).

A particular situation arises when the indifference point under collusion is above the choke price, while it is below the choke price if firms compete. This is the case for all points in Figure 3.4 in between the two bifurcation curves that separate the Promising Technology region from the Strained Market region. In any such a situation, only colluding firms may develop further a technology which requires investments in advance of production. Competing firms never develop it further as for them the exit trajectory is optimal for all initial costs above the choke price. Obviously, the latter scenario yields a lower total surplus.[17]

Second, if firms collude, the set of initial technologies that triggers no investment in R&D at all or that induces firms to select the exit trajectory is smaller. Figure 3.7 illustrates this for the Strained Market region. The strained investment circumstances, in the sense of a high depreciation rate in comparison to the market size and R&D efficiency, induce competing firms to exit the market in due time for all $c_0 > \hat{c}_1$. In contrast to this, colluding firms exit the market only for $c_0 > \hat{c}_2$, which is again due to larger mark-ups in the product market. Initial technologies c_0 in the interval $(\hat{c}_1, \hat{c}_2)$ are therefore brought to full maturation only by colluding firms, which leads to a direct welfare gain of collusion.

So far we can conclude that due to collusion (i) it is more likely that we have a Promising Technology, and if so, that it is more likely to be developed further, (ii) it is less likely that we have an Obsolete Technology, and if so, it is more likely that firms invest in R&D, albeit temporarily, and (iii) if the technology causes a Strained Market or if it induces an Uncertain Future, it is less likely that it will be taken of the market in due time. In sum, due to collusion it is more likely that firms invest in R&D, and that these investments eventually lead to a

[17] In the next section, we discuss what our analysis implies for competition policies.

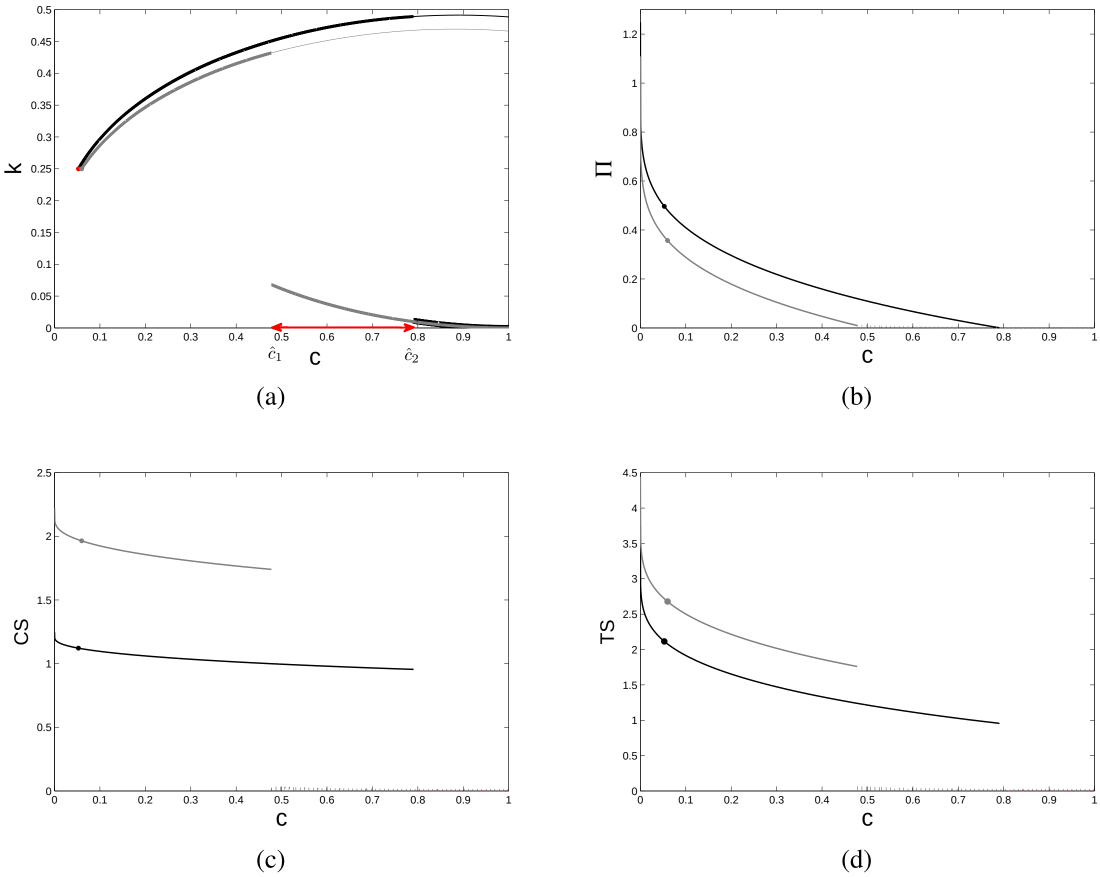


Figure 3.7: *State-control space (a), total discounted profit (b), consumer surplus (c), and total surplus (d), when the indifference point is within the region with positive production.* Parameters: $(\beta, \tilde{\rho}, \phi) = (1, 0.1, 2)$. Grey curves correspond to competition, whereas the black ones correspond to collusion. Full curves correspond to the stable path, whereas the dotted ones to the exit trajectory. Dots indicate the saddle-point steady state. For all $c_0 \in (\hat{c}_1, \hat{c}_2)$, the collusive scenario brings about higher consumer surplus and total surplus than the competitive scenario.

steady state with positive production.

The next observation is about the intensity of the R&D process as such.

Numerical observation 3. *Over the entire parameter space we observe that whenever both scenarios trigger either the exit trajectory or the stable path towards the saddle-point steady state, the trajectory of the collusive scenario lies above that of the competitive scenario.*

This observation implies the following. First, whenever both scenarios lead to the saddle-point steady state, marginal costs in the collusive scenario are lower than in case of competition, because colluding firms have invested more in cost-reducing R&D to arrive at the long-run equilibrium. Put differently, collusion yields a higher production efficiency. Second, if the initial technology leads to production after some initial development period only, colluding firms will enter this production phase more quickly. That is, at every instant of the pre-production phase, colluding firms invest more in R&D in order to bring some initial level of marginal costs below the choke price. As a result, less favorable initial technologies will be brought to the market if firms collude. Third, colluding firms abandon obsolete technologies at a lower pace. This implication, that a monopolist holds on longer to a technology that is bound to leave the market, has a similar vein as the argument of Arrow (1962), that a monopolist has less incentive to invest in R&D than an otherwise identical but perfectly competitive market, because by doing so the monopolist replaces current monopoly profits by future (higher) monopoly profits. Here, of course, the alternative for the colluding firms is to exit the market more quickly (rather than staying in the market as a monopolist, as in Arrow, 1962), an alternative that for them is not optimal (see Figure 3.8).

3.6 Competition policies

Summarizing the results of the previous section, we have found that the collusive scenario is more R&D intensive: R&D investment levels are higher and the set of initial technologies that is developed further is larger. The price to be paid for this increased innovation intensity is the higher mark-up in the product market. It should therefore not come as a surprise that the welfare comparison between the two scenarios yields a mixed picture.

First, as alluded to in the previous section, if the firms develop an initial technology in such a way that this leads to a positive production steady state, then this always yields a higher total surplus over the alternative of no R&D investment at all. Indeed, in Figure 3.5, for all $c_0 \in (\hat{c}_1, \hat{c}_2)$, the collusive scenario is the better alternative.

Numerical observation 4. *Over the entire parameter space we observe that whenever both scenarios have an indifference point above the choke price, the collusive scenario yields higher consumer surplus and total surplus than the competitive scenario for all initial technologies in between the two indifference points.*

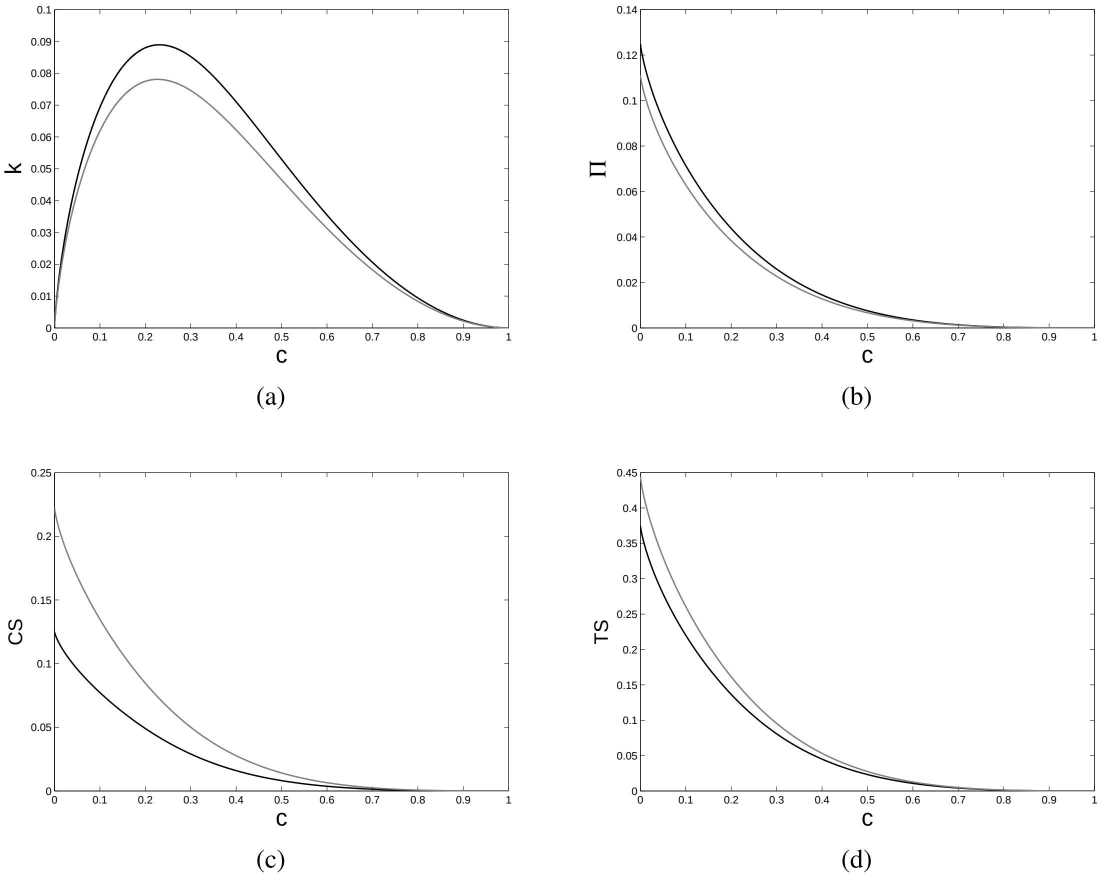
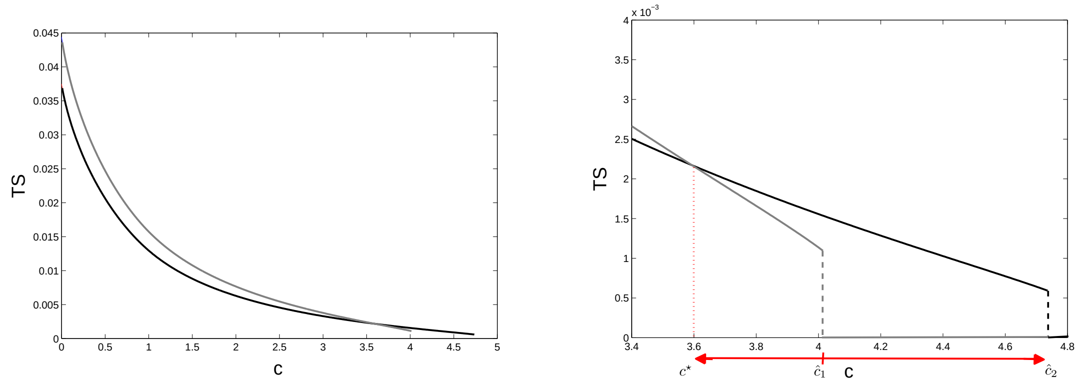


Figure 3.8: *State-control space (a), total discounted profit (b), consumer surplus (c), and total surplus (d), when the exit trajectory is an optimal solution.* Parameters: $(\beta, \tilde{\rho}, \phi) = (1, 1, 2)$. Grey curves correspond to competition, whereas the black ones correspond to collusion. The competitive scenario brings about higher consumer and producer surplus than the collusive scenario for all c_0.

This observation qualifies the argument that R&D cooperatives make it easier for firms to collude in the concomitant product market and that this is necessarily welfare reducing. Obviously, this fails to be the case for all c_0 in the interval $(\hat{c}_1, \hat{c}_2)$. It is also not necessarily valid in situations where collusion induces firms to select the stable path while competition induces them to exit the market (recall Figure 3.7).

For competition authorities, a particularly difficult situation arises when the initial draw c_0 out of $(\hat{c}_1, \hat{c}_2)$ is above the choke price ($c_0 > 1$). Then the welfare costs of prohibiting firms to collude in the product market do not surface because no production is affected by this prohibition. There is no production yet, and because collusion is prohibited, there will be no production in the future. Yet, in this case, prohibiting firms of an R&D cooperative to collude



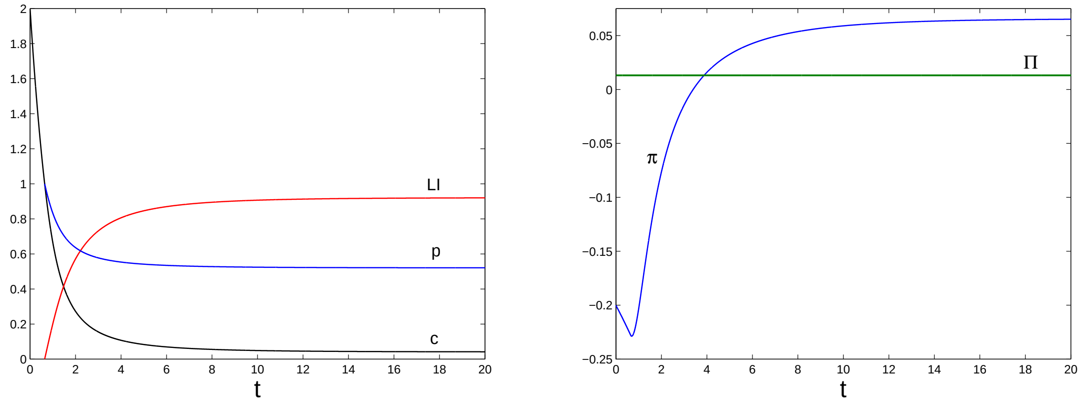
(a) (b)

Figure 3.9: *Total surplus when the indifference point is in the region with zero production.* Parameters: $(\beta, \tilde{\rho}, \phi) = (1, 10, 50)$. Grey curves correspond to competition, whereas the black ones correspond to collusion. $c^\star \approx 3.6$, $\hat{c}_1 \approx 4.01$, $\hat{c}_2 \approx 4.74$. For all $c_0 \in (c^\star, \hat{c}_2)$, total surplus is higher if firms collude in the product market.

in the product market is welfare reducing. To the extent that competition policies are designed to enhance total surplus, a general prohibition of product market collusion is not first-best *per se*. At the same time, and more in line with traditional views, Figures 3.5 and 3.7 suggest that if both scenarios induce firms to select the stable path towards the saddle-point steady state, the competitive scenario yields a higher total surplus (Figure 3.8 contains a similar suggestion in case both scenarios induce firms to select the exit trajectory).[18] However, this is not necessarily the case, as Figure 3.9 illustrates. Although both scenarios would induce firms to select the trajectory towards the saddle-point steady state, for all $c_0 \in (c^\star, \hat{c}_2)$, total surplus is higher if firms collude in the product market. In this example, the discount rate is high: $\tilde{\rho} = 10$, which corresponds, for instance, to $\delta = 0.01$ and $\rho = 0.1$. Also, the initial marginal costs have to be 'high' for the collusive scenario to outperform the competitive scenario in terms of consumer surplus and total surplus. In such an environment, the higher R&D investments and the reduced importance that is attached to future surplus are favorable for the collusive scenario: if firms collude, they reach the production stage more quickly, a benefit that more than off-sets the concomitant welfare loss of increased mark-ups in the future.[19] To

[18] As noted above, over the entire trajectory, collusion yields more R&D investments. Insofar higher investment levels as such are desirable, the case for prohibiting collusion in the product market is weakened.

[19] More precisely, a higher discount rate $\tilde{\rho} = \rho/\delta$ implies either a higher discount rate ρ or a lower δ. With a lower δ, any cost reduction takes longer, such that whenever future benefits are discounted, the time difference in reaching the production stage between the scenarios becomes more pronounced.



(a) (b)

Figure 3.10: *Marginal cost, price, and Lerner Index (a); total discounted profit and instantaneous profit (b), for collusion.* Parameters: $(\beta, \tilde{\rho}, \phi) = (1, 0.1, 2.25)$ and starting point $c_0 = 2$.

illustrate further what difficulties competition authorities face, consider Figure 3.10. Among others, it shows the development of the Lerner index over time towards its long-run level of 0.92 for the parameter configuration of Figure 3.5, where the initial draw $c_0 = 2$ is from the interval $(\hat{c}_1, \hat{c}_2)$. This case illustrates what has been alluded to by Lindenberg and Ross (1981, p. 28): "[The Lerner index] does not recognize that some deviation of P from MC comes from ... the need to cover fixed costs and does not contribute to market value in excess of replacement cost."[20] The high value of the Lerner index is due to collusion, which, in this case, is welfare enhancing. Indeed, this example suggests that the court was right in its ruling of US vs. Eastman Kodak (1995) when it concluded that "Kodak's film business is subject to enormous expenses that are not reflected in its short-run marginal costs." More generally, it illustrates the difficulty in designing optimal competition policies for high-tech industries. This is illustrated further if one considers instantaneous profits and total discounted profits, as in Panel (b) of Figure 3.10. Clearly, after a while, the former are much larger than the latter. But the high instantaneous mark-ups should not be considered as a signal of potential welfare losses, because if it had not been for these mark-ups, in the long run there would have been no market at all.

[20]See Elzinga and Mills (2011) for a critical assessment of the use of the Lerner index; see also Armentano (1999).

3.7 Concluding remarks

We present an analysis of R&D cooperatives whereby the phase prior to production is taken into account, because it is well known that collusion triggers the incentives to invest in R&D. Our global analysis shows that if firms collude in the product market, the set of initial technologies that is developed further increases, and that, in particular, more initial technologies are brought to full maturation. This is a direct welfare gain of product market collusion. Also, the probability that an initial technology induces firms to leave the market altogether is reduced, which again is welfare enhancing.

Our analysis presents a problem for competition policy because it shows that prohibiting collusion in the product market per se is not univocally welfare enhancing. It also shows that the associated welfare costs might not surface because a prohibition of product market collusion affects R&D investment decisions prior to the production phase. Any decision not to develop further some initial technology does not materialize as a welfare cost because no production is affected (yet).