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Essays in nonlinear dynamics in economics and econometrics with applications to monetary policy and banking

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Chapter 2

Monetary Policy, Banking and Heterogeneous Agents

2.1 Introduction

The need for a framework which would incorporate financial frictions in DSGE models was stressed long before the 2007-2009 financial crisis (Bernanke and Gertler, 1989; Kiyotaki and Moore, 1997). The body of literature in this topic has grown substantially thereafter, bringing significant changes to monetary policy conduct (Rotemberg and Woodford, 1997; Woodford, 2003). It is surprising, as argued by Goodfriend and McCallum (2007) and Casares, Miguel and Poutineau (2010), that the role of the banking sector was left unexplored in the monetary policy analysis until recently.

The framework used in this study clarifies this oversight. Firstly, by introducing profit-maximizing bankers at the micro level, one may explicitly study the impact of their individual behavior on the macro aggregates. Secondly, the differentiation of the capital market allows to investigate the relationship between various types of interest rates (Goodfriend, 2005). Thirdly, by having government bonds which serve for collateral purposes, one observes the direct influence of public policy on the monetary aggregates.

Most noticeably, however, a banking sector *per se* is an important, if not the most important part of each economy (Levine, 1997). Since it is a general source of liquidity, its problems may easily spread over the other sectors, bringing them down eventually. Especially, the recent history proves that banking sector disturbances might result in sovereign crises, as recently took place in the euro zone (Grammatikos and Vermeulen, 2012). Therefore, a detailed study of the banking sector's role in the monetary framework is required in order to (i) understand its transmission mechanism and (ii) endow the monetary authorities with the sufficient preventive tools.

The goal of this chapter is twofold. Firstly, we assess the determinacy properties of different monetary policies in the DSGE model with a banking sector of Goodfriend and McCallum (2007). The model is built within the standard new Keynesian framework where the aggregate dynamics is a direct consequence of individual utility maximizing behavior of forward-looking agents. Secondly, we relax the assumption of agents' homogeneity and investigate how the presence of the backward-looking (or boundedly rational after Hommes (2013)) agents influences the determinacy of the equilibrium. We introduce agents' heterogeneity at the micro level, which means that each agent is solving the individual optimization problem simultaneously. It is an important distinction from a variety of models which neglect this aspect and allow for agents' heterogeneity at the macro level only. Clearly, such a concept violates the Subjective Expected Utility (SEU) theory and in our view is inappropriate. Instead, we follow the classical approach where the macro behavior is a direct consequence of agents' micro optimal plans.

The latter part of this study is motivated by a growing body of research which shows explicitly that agents differ in forming expectations. This phenomenon was confirmed by both survey data analysis (Carroll, 2003; Mankiw et al., 2003; Branch, 2004) as well as laboratory experiments with human subjects (Hommes et al., 2005; Assenza et al., 2011; Hommes, 2011; Pfajfar and Zakelj, 2011). The heterogeneity among agents was proved to have important implications on the determinacy properties in the new Keynesian models (Branch and McGough, 2009; Massaro, 2013). We follow this approach and assess its implication within the framework

with a banking sector.

This chapter is organized as follows. Section 2.2 describes the workhorse model and discusses the implications of the banking sector on monetary policy conduct. In Section 2.3 we relax the assumption of a representative agent structure and introduce boundedly rational backward-looking agents. Section 2.4 presents the numerical results and Section 2.5 concludes.

2.2 The model

In this section we develop the workhorse version of the model. Since the complete derivation, with the first order conditions and aggregation, is described in detail in the original paper of Goodfriend and McCallum (2007), we skip it in the main part of this text. However, for the reader's convenience, the complete derivation is given in Appendix 2.A.

The model space consists of a continuum of farmers who provide labor supply to the production and banking sectors at the same time t (n_t and m_t , respectively). Additionally, each farmer manufactures a differentiated product and sells it in the monopolistically competitive environment. As in the standard new Keynesian framework, it is assumed that only a fraction $(1 - \omega)$ of all farmers can adjust their prices fully flexibly. The remaining part takes the prices from the previous period (Calvo, 1983). Given these conditions, the goal of each farmer is to maximize her expected utility, which is a linear combination of consumption and leisure, over the infinite horizon.

In the utility maximization problem, each farmer has to take into account three constraints: (i) the budget constraint, (ii) the production constraint and (iii) the banking constraint. The first of these is the standard intertemporal budget constraint which ensures that the net income and bond/money holdings in one period are being transmitted to the next period. The second constraint is a direct consequence of the production technology, which in this case is of the Cobb-Douglas type. Assuming market clearing, the production (Y_t) in each period is the consequence of the amount of capital (K_t) and labor (n_t^d) involved, corrected for their output

elasticities: η and $(1 - \eta)$, respectively. The banking constraint assumes that the level of consumption (C_t) has to be rigidly related to the level of deposits held at a bank. One may view this as if all the transactions were being facilitated through the banking sector and each agent may consume a part V of her wealth only. A bank is then allowed to use $(1 - rr)$ fraction of the deposits to produce loans using the Cobb-Douglas production function with collateral (col_t) and labor (m_t^d) as production factors and α and $(1 - \alpha)$ being the output elasticities. The collateral consists of two parts, i.e. the discounted level of real bond holdings $B_{t+1}/(P_t^A(1 + r_t^B))$, with P_t^A being the aggregate price level and r_t^B the interest rate on bonds, and real level of capital $q_t K_{t+1}$, corrected for the inferiority of capital to bonds for collateral purposes, v . The last term results from the fact that bonds, contrary to capital goods, do not require substantial monitoring effort in order to verify their market value (Goodfriend and McCallum, 2007).

Such a banking sector setting captures several important aspects of financial intermediation. Firstly, it enters the consumer utility maximization problem at the micro level. Secondly, it builds a clear link between households and a production sector. Thirdly, because of its dependence on governmental securities, it comprises the monetary policy transmission mechanism (through the repo market).

There are two main simplifications of the original model. Firstly, we abstract from the capital shocks in the loan production function. We assume that the capital level is at its steady state level and the productivity shocks are transmitted through the labor channels only. This simplification does not affect the final results as in the determinacy analysis the stochastic terms do not play a role (Blanchard and Kahn, 1980). Secondly, we assume a zero tax rate. Eventually, the role of government is narrowed to issuing bonds in each period at some exogenously given level, and paying the interest.

Given the specification above, we may now turn to derivation of three model equations: the Investment-Savings (IS) curve, the Phillips curve and the banking curve. The first two of these build the standard new Keynesian model. The last one is the direct consequence of the presence of the banking sector and describes its role in the aggregate dynamics explicitly.

2.2.1 The IS curve

The model implies the presence of two Lagrange multipliers: λ_t for the budget constraint and ξ_t for the production constraint. They represent the shadow values, or the utility gains, of unit values of consumption and production respectively (Casares, Miguel and Poutineau, 2010). In particular, from the banking labor demand optimality condition we know that

$$\lambda_t^i = \frac{\xi_t^i}{\varphi_t^i} = \frac{\frac{\phi}{C_t^i}}{1 + \left(\frac{1-rr}{V}\right) \chi_t^i}, \quad (2.1)$$

where φ_t^i is the individual marginal production cost, ϕ is the utility weight on consumption and we explored the fact that the χ_t^i might be viewed as the individual marginal loan management cost, or simply the marginal banking cost (Goodfriend and McCallum, 2007; Casares, Miguel and Poutineau, 2010)¹. To put it more formally, imagine the cost minimization problem of a representative bank in a situation without collateral cost. The total cost function may be rewritten as $TC_t = m_t^d w_t$, where w_t is the real wage. The minimization problem includes the loan production constraint with a Lagrangian multiplier (here perceived as a marginal cost (Walsh, 2010)), denoted by χ_t . The first order condition implies that $\chi_t = V w_t m_t^d / ((1 - rr)(1 - \alpha)C_t)$. In fact, χ_t^i is parallel to the individual marginal production cost that is being often referred to in the standard new Keynesian framework (Walsh, 2010). One may view that as a general variable describing the situation in the banking sector, i.e. the higher it is the less effective the loan management is. As it is shown later, this variable is of crucial importance as it becomes a link between a standard new Keynesian model and the banking system.

Eq. (2.1) gives the first overview of the model behavior. Firstly, the shadow value of production equals the shadow value of consumption corrected for the marginal production cost. In other words, additional consumption has to turn up in either increased production or decreased production costs. Secondly, λ_t is the marginal utility of consumption corrected for the marginal banking cost. Put differently, each additional unit of consumption requires more deposits, which

¹We include superscript i to underline the individual level of the relationship which is explored in detail later. In the representative agent structure it may be omitted as every agent behaves the same.

may be raised at the cost χ_t . It is straightforward to notice that the lower the marginal banking cost, the relatively cheaper the additional consumption. On the other hand, a highly inefficient banking sector limits the incentives to increase consumption.

Substituting Eq. (2.1) into the bond optimality condition, we finally arrive at the familiar Euler equation

$$\beta E_t^i \left(\frac{\frac{\phi}{C_{t+1}^i}}{1 + \left(\frac{1-rr}{V}\right) \chi_{t+1}^i} \right) = \frac{\frac{\phi}{C_t^i}}{1 + \left(\frac{1-rr}{V}\right) \chi_t^i} (1 + E_t^i \pi_{t+1}) \left(\frac{1 - \frac{1-rr}{V} \chi_t^i \Omega_t^i}{1 + r_t^B} \right), \quad (2.2)$$

where $(1 + E_t^i \pi_{t+1}) = P_{t+1}^A / P_t^A$ is the inflation rate and $\Omega_t^i = \alpha C_t^i / col_t^i$.

Following Goodfriend (2005), let us introduce a one-period default-free security with the nominal rate denoted by r_t^T . Since we additionally assume that it cannot serve for collateral purposes, r_t^T represents a pure intertemporal rate of interest and serves as a benchmark for other interest rates. From the agent optimization problem, we know that $1 + r_t^{i,T} = E_t^i \lambda_t^i P_{t+1}^i / (\beta \lambda_{t+1}^i P_t^i)$ so that it includes the discounted difference between expected changes in shadow prices and actual prices. An important distinction is that the pricing of this fictitious security is done at the individual level which is not strange given its completely artificial and agent-dependent nature. Eventually, the last term of Eq. (2.2) might be rewritten as the reciprocal of $(1 + r_t^{i,T})$.

At the same time, let us assume that each bank can obtain funds from the interbank market at the common rate r_t^{IB} . It can then loan them to agents at the rate $r_t^{i,T}$. The profit maximization of a bank implies that the marginal costs of obtaining funds has to be equal their marginal profit so that

$$(1 + r_t^{IB})(1 + \chi_t^i) = (1 + r_t^{i,T}). \quad (2.3)$$

Inserting Eq. (2.3) into Eq. (2.2) and taking the log approximation around the steady state we have

$$\hat{Y}_t^i = E_t^i \hat{Y}_{t+1}^i + \left(\frac{1-rr}{V} \right) E_t^i \tilde{\chi}_{t+1}^i - \left(\frac{1-rr}{V} + 1 \right) \tilde{\chi}_t^i - (\hat{r}_t^{IB} - E_t^i \pi_{t+1}), \quad (2.4)$$

where tildes and hats denote deviations and percentage deviations from the steady state, respectively, and we explored the market clearing condition².

As in the standard new Keynesian framework, we define the potential output as the output under completely flexible prices and wages (Walsh, 2010). We additionally assume that in such a situation there is a fixed proportion between employment in the production and banking sector, $n_t^d \propto m_t^d$. Following Walsh (2010), price flexibility implies that all agents can adjust their prices immediately, which gives that the marginal cost of production φ_t is equal $(\theta - 1)/\theta$ across all individuals, where θ is the elasticity of substitution between consumption goods. The labor optimality condition implies that the real wage has to be equal the marginal rate of substitution between leisure and consumption, corrected for the presence of the banking sector. Combining the above-mentioned points with Eq. (2.1) and the production constraint, we finally get that under flexible prices and wages, the supply of labor of each individual is fixed so that if the capital stock is in the steady state (as we assume throughout the model) the log deviations of the potential product depend only on exogenous disturbances, $\hat{Y}_t^f = (1 - \eta)(A1_t - \bar{A}1)$. Subtracting them from both sides of Eq. (2.3) and omitting the i superscript, we finally arrive at the aggregate IS curve corrected for the presence of a banking sector

$$x_t = E_t x_{t+1} + \left(\frac{1 - rr}{V} \right) E_t \tilde{\chi}_{t+1} - \left(\frac{1 - rr}{V} + 1 \right) \tilde{\chi}_t - [\hat{r}_t^{IB} - E_t \pi_{t+1}] + u_t, \quad (2.5)$$

where $x_t = \hat{Y}_t - \hat{Y}_t^f$ is the output gap measure and u_t is the disturbance term that depends only on exogenous productivity shocks.

It is straightforward to notice that when skipping the banking sector variables from Eq. (2.5) we obtain the standard new Keynesian IS curve. What is important, is that the aggregate dynamics is affected not only by the current, but also expected future values of the banking variables. In other words, the way the agents form their expectations about future banking sector conditions seems to play a role in determining current production. The impact of the banking sector is limited by (i) the reserve requirement, rr , and (ii) the proportion of consumption that has to

²Following literature, we take the zero inflation steady state.

be covered by deposits, V . Clearly, the lower the minimum reserve requirement, the larger the loan production so that the importance of the banking sector increases, *ceteris paribus*. At the same time, if the consumption-to-deposits coverage ratio is large, relative size of the banking sector is smaller so that its impact decreases.

2.2.2 The Phillips curve

The model allows us also to derive the explicit formula for the Phillips (or Aggregate Supply) curve. We know that all the farmers share the same production technology and face the same constant demand elasticities. We know from the Calvo lottery that a fraction ω of agents cannot adjust their prices in a given period t . Profits of some future date $t + k$ are affected only if an agent did not receive a chance to adjust prices between t and $t + k$. Therefore, the probability of having lower expected profits in period k is ω^k . Having pointed that out, the price optimality condition has to be corrected for the nominal price rigidities in the long run and by iterating forward it might be viewed as

$$E_t^i \sum_{k=0}^{\infty} \beta^k \omega^k \left[(1 - \theta) \left(\frac{P_t^i}{P_{t+k}^A} \right) + \theta \left(\frac{\xi_{t+k}^i}{\lambda_{t+k}^i} \right) \right] \left(\frac{1}{P_t^i} \right) \left(\frac{P_t^i}{P_{t+k}^A} \right) C_{t+k}^A = 0. \quad (2.6)$$

Solving for optimal price setting, we arrive at

$$\frac{P_t^i}{P_t^A} = \frac{E_t^i \sum_{k=0}^{\infty} \beta^k \omega^k C_{t+k}^A \varphi_{t+k}^i \left(\frac{P_{t+k}^A}{P_t^A} \right)^\theta}{E_t^i \sum_{k=0}^{\infty} \beta^k \omega^k C_{t+k}^A \left(\frac{P_{t+k}^A}{P_t^A} \right)^{\theta-1}}, \quad (2.7)$$

where $\varphi_t^i = \xi_t^i / \lambda_t^i$ is the individual marginal production cost (Goodfriend and McCallum, 2007).

Skipping the i superscript and taking a log approximation, after some algebra we obtain³

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \hat{\varphi}_t, \quad (2.8)$$

³For a detailed derivation see the appendix of Chapter 8 from Walsh (2010).

where $\kappa = \frac{(1-\omega)(1-\beta\omega)}{\omega}$. We further explore the fact that given the Cobb-Douglas production function, the steady state log deviations of the marginal production cost might be viewed as an output gap measure (Goodfriend and McCallum, 2007). Finally, we arrive at the standard new Keynesian Phillips curve

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t. \quad (2.9)$$

What is important is that the situation in the banking sector does not affect the inflation level directly but only through the consumption channel. The absence of the banking variables in Eq. (2.9) is a consequence of the banking sector specification. The level of consumption is rigidly related to the amount of deposits in the banking sector. Therefore, changes in the banking sector would result in a different deposit level, which would shake the consumption eventually. However, there is no direct link to the inflation in the meantime.

2.2.3 The banking sector curve

Since the presence of the banking sector affects the aggregate evolution of the IS and (indirectly) Phillips curves, it is also necessary to describe its dynamics. Observing that $\varphi_t = q_t K_t / (\eta C_t)$, the capital optimality condition implies

$$1 - \frac{v(1-rr)}{V} \Omega_t^i \chi_t^i = \beta(2-\delta) E_t^i \left[\frac{\left(1 + \left(\frac{1-rr}{V}\right) \chi_t^i\right) \varphi_{t+1}^i}{\left(1 + \left(\frac{1-rr}{V}\right) \chi_{t+1}^i\right) \varphi_t^i} \right]. \quad (2.10)$$

Observe that the LHS of Eq. (2.10) is almost identical with the numerator of the last term in Eq. (2.2). The only difference comes from the inferiority of capital to bonds for collateral purposes, v . Applying the same interest rate reasoning to the log approximation of the LHS of Eq. (2.10), we see that $-v(1-rr)\Omega_t^i \chi_t^i / V = -v(r_t^{IB} - r_t^B + \chi_t^i)$. Since the interbank rate r_t^{IB} and the government bond rate r_t^B are both short-term rates, they should be close to each other around the equilibrium (Goodfriend and McCallum, 2007). Additionally, given the fact that v is relatively small, we neglect the influence of $v(r_t^{IB} - r_t^B)$. Eventually, after taking the deviations

from the steady state of Eq. (2.10), iterating forward and skipping the i superscript, we get

$$\left(v + \frac{1 - rr}{V}\right) \tilde{\chi}_t = \frac{1 - rr}{V} E_t \tilde{\chi}_{t+1} - (E_t x_{t+1} - x_t). \quad (2.11)$$

Given Eq. (2.11) it is clear that the marginal cost of banking depends on (i) expectations about the banking situation in the future and (ii) the current and expected future production. In particular, the expectations about higher next period marginal banking costs work as a self-fulfilling prophecy, increasing also today's cost. This positive feedback structure reflects, to at least some degree, financial market sentiment and herding behavior. When investors see that the banking sector is going to face difficulties the next day, they will adjust their today's positions accordingly. On the other hand, given the link between the banking sector and consumption, high expectations about next period output gap decrease today's marginal banking cost (negative feedback). Imagine that people expect that there will be a decrease in production in the next period. Since the banking sector is a source of funding, there will be gradually less effort involved in the loan production, bringing today's marginal cost down.

The effects on the current banking situation are proportional to the size of the banking sector, expressed by $(1 - rr)/V$, being more prominent for smaller banking sectors. Smaller banking sectors are more vulnerable to changes in the production sector as the relatively higher part of the banking capital is involved. On the other side, a bigger banking sector might be viewed as being more stable in the sense that the production sector affects it to the lower extent. It should be kept in mind, however, that the model does not say that big banks are ultimately stable as a high drop in today's production can cause the marginal banking cost to skyrocket. Eq. (2.11) predicts only that this effect will be more prominent in the environment with a smaller banking sector.

At the same time, the inferiority of capital to bonds for collateral purposes, v , also plays a role in determining the current marginal banking cost. In particular, let us consider the extreme case when capital cannot serve as a collateral, i.e. $v = 0$. Banks do not have access to capital then so that the only link between them and the production sector is through loans. If there

is a production shock, it affects the bond holdings and labor in the banking sector, making it more severe. In this sense, using capital as collateral serves as a hedge against production sector disturbances. When banks can access capital, in the presence of a production shock, its magnitude is being partially absorbed by the capital part.

2.3 The influence of heterogeneity

So far, we assumed that all the agents are the same and each of them faces the same optimization problem. Before turning to the numerical results, let us first consider what happens in the environment with heterogeneous agents. Contrary to the standard representative agent framework, we allow a part $(1-\gamma)$ of agents to be boundedly rational in forming their expectations⁴. In other words, we assume that a constant proportion of agents is uniformed or unable to form rational expectations. This implies that we may divide our continuum of farmers into two groups: those with rational expectations (E^{RE}) producing good $j \in [0, \gamma]$ and those with boundedly rational expectations (E^{BRE}) producing good $j \in [\gamma, 1]$. By rational agents we mean forward-looking fundamentalists who try to analyze the economy and form their expectations accordingly. Both groups of agents behave as if everybody in the economy was of their type.

To be able to aggregate the results over both groups, we follow the methodology proposed by Branch and McGough (2009) and we impose similar seven axioms on expectation operators:

1. expectations operators fix observables,
2. if z is a forecasted variable and has a steady state, then $E^{RE} \bar{z} = E^{BRE} \bar{z} = \bar{z}$,
3. expectations operators are linear,
4. if for all $k \geq 0$, z_{t+k} and $\sum_{k=0}^{\infty} \beta^{t+k} z_{t+k}$ are forecasted variables then

$$E_t^\tau \left(\sum_{k=0}^{\infty} \beta^{t+k} z_{t+k} \right) = \sum_{k=0}^{\infty} \beta^{t+k} E_t^\tau z_{t+k} \text{ for } \tau \in \{RE, BRE\},$$

⁴Throughout this chapter we use the term ‘rational’ to refer to forward-looking whereas ‘boundedly rational’ to express backward-looking expectations.

5. expectation operators satisfy the law of iterative expectations,
6. if z is a forecasted variable at time t and time $t+k$ then $E_t^\tau E_{t+k}^{\tau'} z_{t+k} = E_t^\tau z_{t+k}$ for $\tau \neq \tau'$,
7. all agents have common expectations on expected differences in limiting wealth and marginal banking cost.

Our contribution to the original methodology comprises axiom 7, which describes the limiting behavior of the expectation operators. Since we add the banking sector to the model, we have to include it also in the expectation formation. Branch and McGough (2009) assume that both types of agents have common expectation on their limiting wealth. It allows to represent the aggregate expectations operator as a weighted average of group expectations. Otherwise, there is an extra term on the limiting behavior of expectations that complicates the dynamics (see Eq. (2.41) from Appendix 2.B). A similar pattern might be observed when aggregating the banking sector (Eq. (2.49) from Appendix 2.B). The aggregate dynamics of the system is therefore influenced by how agents predict the banking sector behaves over the infinite horizon.

Axiom 7 might be viewed as an agreement among all agents that in the far future their banking sectors will be equivalent or will at least generate the same marginal costs. From the macroeconomic perspective, one may think of it as if both groups of agents were trying to reach the banking sector technological frontier. Since there is a common technology, both types of agents should be heading towards the same frontier eventually, satisfying axiom 7.

Proposition 2.3.1. *In the presence of fraction $(1 - \gamma)$ of boundedly rational agents, if agents' expectations satisfy axioms 1-7 then the model from Eq. (2.5), (2.9) and (2.11) can be rewritten as*

$$x_t = \bar{E}_t x_{t+1} + \left(\frac{1 - rr}{V} \right) \bar{E}_t \tilde{\chi}_{t+1} - \left(\frac{1 - rr}{V} + 1 \right) \tilde{\chi}_t - [\hat{r}_t^{IB} - \bar{E}_t \pi_{t+1}] + u_t, \quad (2.12)$$

$$\pi_t = \beta \bar{E}_t \pi_{t+1} + \kappa x_t, \quad (2.13)$$

$$\left(v + \frac{1 - rr}{V} \right) \tilde{\chi}_t = \frac{1 - rr}{V} \bar{E}_t \tilde{\chi}_{t+1} - (\bar{E}_t x_{t+1} - x_t), \quad (2.14)$$

where $\bar{E}_t = \gamma E_t^{RE} + (1 - \gamma) E_t^{BRE}$.

The proof of Proposition 2.3.1 can be found in Appendix 2.B.

2.4 Numerical analysis

As opposed to the standard framework, the central bank policy instrument is the interbank interest rate, \hat{r}_t^{IB} (not the bond rate). In fact, this is the monetary policy tool used in practice (Goodfriend and McCallum, 2007). As argued by Bernanke and Woodford (1997), to close the model we use the forward-looking Taylor rule of the form

$$\hat{r}_t^{IB} = \rho_x E_t^{RE} x_{t+1} + \rho_\pi E_t^{RE} \pi_{t+1}, \quad (2.15)$$

where ρ_x and ρ_π are constant weights on output and inflation variability, respectively. We follow a common approach and assume that the central bank does not target the situation in the banking sector directly. Including a banking sector variable in the monetary rule would extend the monetary policy analysis to a three-dimensional problem so that the interpretation of the results would not be straightforward anymore. Instead, the purpose of this study is to observe how the standard monetary policy rule behaves in the environment with a present banking sector.

2.4.1 Formation of expectations

Throughout the model, we assume that the economy consists of two types of agents that are homogeneous within each group. The first type of agents, $i = RE$, are those who form rational expectations. We abstract here from the standard understanding of rationality, where agents have full knowledge and capacities to perfectly predict the future. Instead, we rather view them as being forward-looking fundamentalists, who collect information and form their expectations accordingly. They are not aware of the presence of the other type of agents so that they form their expectations as if everybody in the economy was rational in forming the expectations

(Branch and McGough, 2009).

The second type of agents is not able to form rational expectations and use simple backward-looking heuristics instead to predict the future. Following Evans and Honkapohja (2001) we assume them to have adaptive expectations of the form

$$E_t^{BRE} z_{t+1} = \mu^2 z_{t-1}, \quad (2.16)$$

where z is either x , π or $\tilde{\chi}$. Parameter $\mu > 0$ describes the magnitude and the direction of the expectations. If $\mu > 1$, the influence of the past is being extrapolated to the future so that we would call those expectations extrapolative. On the other hand, when $\mu < 1$, this influence disappears over time and we would call those expectations adaptive⁵. When $\mu = 1$, the boundedly rational agents form naive expectations (Evans and Honkapohja, 2001).

Given the expectation operators for both groups of agents, we may rewrite the aggregate expectations as

$$\bar{E}_t z_{t+1} = \gamma E_t^{RE} z_{t+1} + (1 - \gamma) \mu^2 z_{t-1}, \quad (2.17)$$

with z being either x , π or $\tilde{\chi}$.

2.4.2 Calibration and numerical results

DSGE models often exhibit indeterminacy, i.e. there is no unique path guiding the equilibrium. In such a situation, the quantities and prices might not be even locally determinate, making the monetary policy conduct more unstable (Woodford, 1994). Therefore, it is important to make sure that the monetary tools provide a determinate structure of the economy.

⁵In the literature, adaptive expectations are being recognized as the whole group of operators of the form similar to Eq. (2.16). However, for clarity purposes, we distinguish here between extrapolative and adaptive expectations when $\mu > 1$ and $\mu < 1$, respectively.

Table 2.1: Calibration values for the model parameters.

Parameter	V	rr	ν	κ	β
Value	0.31	0.005	0.2	0.05	0.99

Let us write the complete model in the matrix form

$$\begin{pmatrix} B & 0 \\ 0 & I_3 \end{pmatrix} \begin{pmatrix} y_{t+1} \\ y_t \end{pmatrix} = \begin{pmatrix} F & -C \\ I_3 & 0 \end{pmatrix} \begin{pmatrix} y_t \\ y_{t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_t \\ 0 \end{pmatrix}, \quad (2.18)$$

where $y = (x, \pi, \tilde{\chi})'$, $\varepsilon = (u, 0, 0)$ is a vector of exogenous shocks and B , F and C are the coefficient matrices described in detail in Appendix 2.C.

To study the determinacy properties, we apply the methodology developed by Blanchard and Kahn (1980). Since it does not depend on the exogenous disturbances, we omit ε in our further analysis. The determinacy is a result of the properties of the solution matrix M , where

$$M = \begin{pmatrix} B^{-1}F & -B^{-1}C \\ I_3 & 0 \end{pmatrix}. \quad (2.19)$$

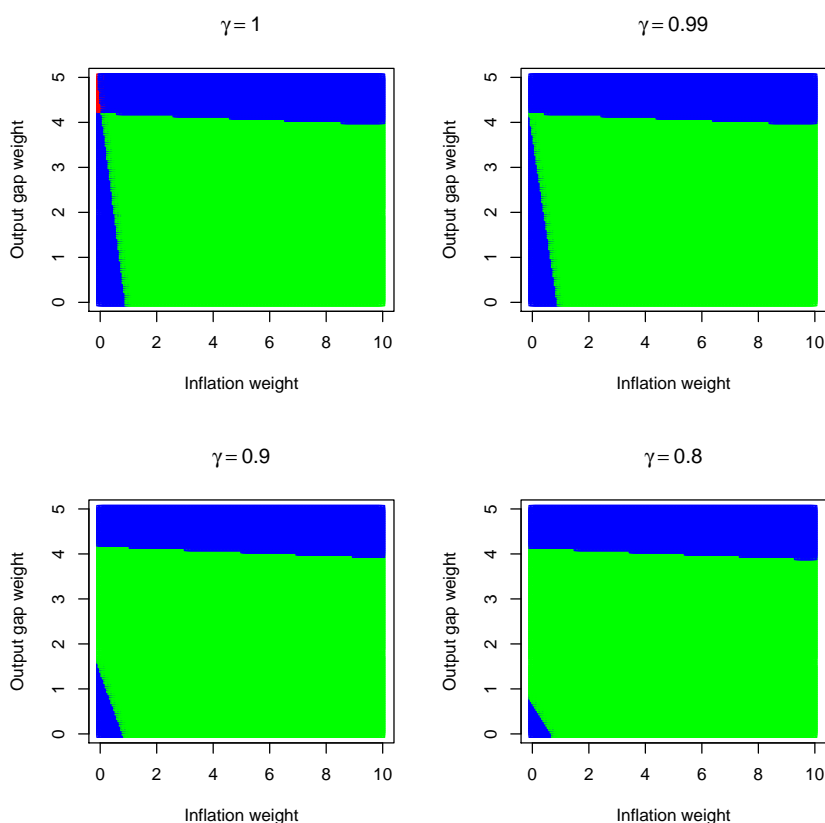
The equilibrium of the system is determinate only if the number of eigenvalues that are outside the unit circle is equal to the number of non-predetermined variables (or the forward-looking variables (Walsh, 2010)), which is 3 in this case. Having more eigenvalues outside the unit circle implies explosiveness and fewer of them implies indeterminacy. The degree of indeterminacy is equal to the number of non-predetermined variables less the number of eigenvalues outside the unit circle (Evans and McGough, 2005).

We calibrate our model accordingly to Goodfriend and McCallum (2007). The detailed values are presented in Table 2.1.

The determinacy properties are studied for extrapolative and adaptive expectations separately. For the former, the μ parameter is set to 1.1 and for the latter to 0.9 (Branch and McGough, 2009). The ranges for policy parameters ρ_x and ρ_π are set from 0 to 5 and 10,

respectively, in order to show the complete behavior of the system. The results are presented in Figs 2.1 and 2.2.

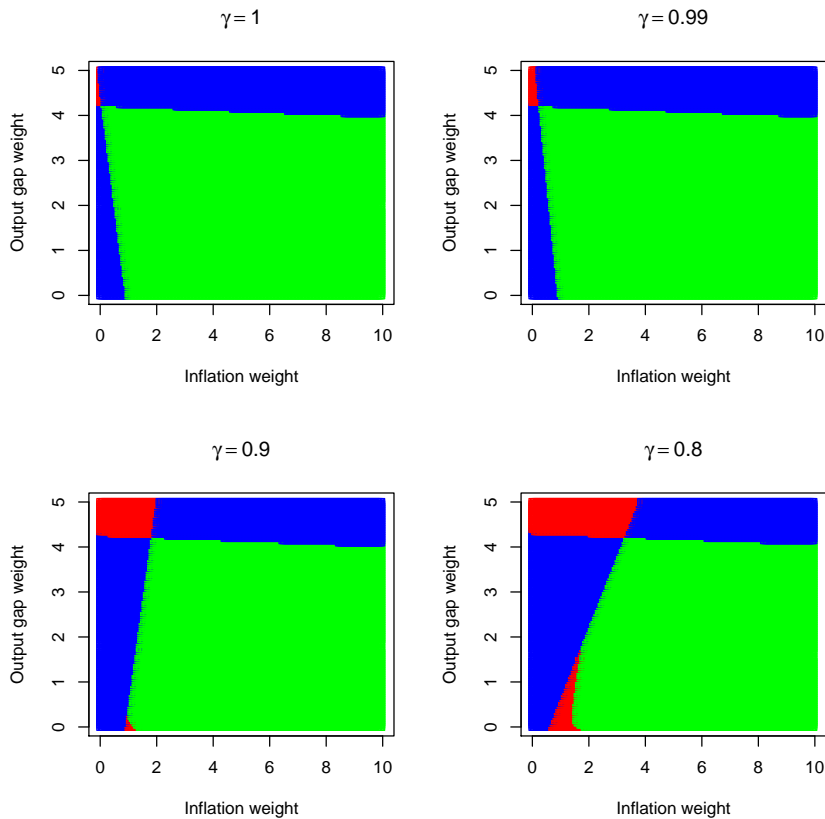
Figure 2.1: Determinacy properties ($\mu = 0.9$). Green color describes determinacy, blue order 1 indeterminacy and red order 2 indeterminacy.



Firstly, the results confirm the 'rotating' behavior of the system from Branch and McGough (2009). With adaptive expectations the system rotates counterclockwise so that the determinacy area increases. With extrapolative expectations the system rotates clockwise decreasing the determinacy area.

Secondly, the location of the indeterminacy of order one and two is in line with the figures presented in Branch and McGough (2009). In fact, the only difference lies in the size of those areas, comparing with the original paper. This, however, is the consequence of the banking calibration parameters and the different specification of the utility function. In fact, if we allow for extra parameter describing the intertemporal substitution elasticity of consumption in the

Figure 2.2: Determinacy properties ($\mu = 1.1$). Green color describes determinacy, blue order 1 indeterminacy and red order 2 indeterminacy.



utility function, σ , the determinacy area is narrowed from the top, being more similar to the results from Branch and McGough (2009) and Bullard and Mitra (2002).

Thirdly, the presence of the banking sector has one important impact on determinacy properties. When agents form extrapolative expectations ($\mu = 1.1$), a new region of indeterminacy of order 2 arises for too lenient inflation targeting. In the case with adaptive expectations ($\mu = 0.9$) there is no similar effect.

2.5 Conclusions and discussion

The goal of this chapter was twofold. Firstly, we derived a workhorse model for monetary policy analysis with the present banking sector. Secondly, we relaxed the assumption of the

representative agent structure and investigated the effects of the presence of boundedly rational agents.

The results suggest that the presence of a banking sector changes the determinacy structure of the equilibrium. Given that agents form adaptive expectations, the determinacy structure rotates counterclockwise, so that more lenient output gap and inflation targeting still guarantees determinacy.

The problem arises when backward-looking agents extrapolate the past performance over their future forecasts. The presence of the banking sector brings additional indeterminacy area for lower inflation targeting parameter. In other words, in the environment with a fraction of extrapolative agents, if the monetary policy does not fight inflation sufficiently well, it may not reach the equilibrium in the long run.

In fact this pattern might have significant consequences for the actual monetary policy conduct. Pfajfar and Zakelj (2011) suggest that the fraction of extrapolative agents might be as high as 30%, even larger than in our analysis. Given the fact that the estimated Taylor rule parameters vary usually in the region of $(0,1)$ for the output gap weight and of $(1,2)$ for the inflation weight (Taylor, 1999; Woodford, 2003), this may suggest that the system is very close to indeterminacy, if not indeterminate already, which arises as a consequence of the banking sector. Therefore, it seems vital for the monetary policy to address the issue of agents' heterogeneity and investigate in detail how they form their forecasts. There could be many solutions to the problem raised above, however, it is beyond the scope of this chapter to discuss them in detail. Assuming that the inflation and output weights are set to satisfy the goals of the monetary policy, there seem to be still ways out of the problem. For instance, one may think of increasing the clarity and flexibility of capital, somehow reducing its inferiority for collateral purposes. This would make current marginal banking cost more robust with respect to the future disturbances and thereof could decrease the influence of destabilizing extrapolative expectations. Another solution would be smaller minimum capital requirement, however, this could translate into higher banking sector leverage and eventually may cause more problems than it originally aimed to

solve.

It is clear that households' expectations play an important role in determining the monetary policy, especially when a banking sector is present. However, this research shows just the top of an iceberg and more study is required in order to fully understand the phenomenon of banking in the modern economy. In particular, a straightforward extension of this study is to endogenize the fraction of rational agents, making it dependent on other systemic variables.

Appendix 2.A Baseline derivation

The utility of a farmer is defined as a weighted average of her consumption and leisure and takes the form

$$U^i(C_t^i, n_t^i, m_t^i) = \phi \log(C_t^i) + (1 - \phi) \log(1 - n_t^i - m_t^i), \quad (2.20)$$

where ϕ is the relative preference weight on consumption and t is the time subscript. C_t^i represents a composite consumption good and is of the standard Constant Elasticity of Substitution (CES) form, as in Dixit and Stiglitz (1977)

$$C_t^i = \left(\int_0^1 c_{jt}^{\frac{\theta-1}{\theta}} dj \right)^{\frac{\theta}{\theta-1}}, \quad (2.21)$$

with θ being the elasticity of substitution.

The farmer's decision problem is to maximize her discounted expected utility subject to the budget and technology constraints. Assuming a cashless limit (Woodford, 2003; Branch and McGough, 2009), we may define the former in real terms as

$$w_t(n_t^i + m_t^i) + q_t(1 - \delta)K_t^i + \frac{Y_t^i P_t^i}{P_t^A} + \frac{B_t^i}{P_t^A} = w_t(n_t^{i,d} + m_t^{i,d}) + C_t^i + q_t K_{t+1}^i + \frac{B_{t+1}}{P_t^A(1 + r_t^B)}, \quad (2.22)$$

where K_t^i is capital level with q_t being its real price and δ the depreciation rate, w_t is the real wage and B_t^i are the nominal bond holdings with the nominal interest equal r_t^B . Y_t^i is the production level, P^i is the price of the individual good and P_t^A is the aggregate price level, as in the Dixit-Stiglitz setup. Superscript d denotes the amount of labor demanded by a given farmer. Superscript i and subscript t relate to the agent and time dimensions, respectively.

Contrary to the standard new Keynesian framework, there is a capital market in the model. Its role is twofold. Firstly, capital serves as a production factor in the farmers' technology. Secondly, it is used as a collateral in the banking sector to produce loans. For simplicity, it is assumed that the aggregate capital stock is on a steady state growth path (Goodfriend and McCallum, 2007). What is important is that farmers are allowed to trade it so that its market

price q_t may fluctuate.

The production constraint requires that

$$Y_t^i = K_t^{i\eta} \left(e^{A1_t} n_t^{i,d} \right)^{1-\eta}, \quad (2.23)$$

where $A1_t$ is an aggregate productivity disturbance and η is the capital elasticity measure.

A novelty in the model is the presence of the banking sector. Its main role is to facilitate transactions between production and consumption sides of the economy. Since the medium of exchange is the crucial role of the monetary policy analysis, the model does not distinguish between transaction balances and time deposits at the banks. In this simple form, it implies that the farmer's consumption in each period has to be rigidly related to the deposits held at a bank (Goodfriend and McCallum, 2007). In other words, in each period, the level of consumption (C_t^i) has to be covered by some constant fraction of the real deposits (VD_t^i/P_t^A). Since each bank has to hold a given level of reserves at the central bank (rr), the nominal amount of loans it may produce from deposits held by farmer i is constrained by $L_t^i = (1 - rr)D_t^i$. At the same time, the real loan production depends on the collateral and loan monitoring, and is assumed to be of a Cobb-Douglas form

$$\frac{L_t^i}{P_t^A} = F \left(\frac{B_{t+1}^i}{P_t^A(1 + r_t^B)} + vq_t K_{t+1}^i \right)^\alpha \left(e^{A2_t} m_t^{i,d} \right)^{1-\alpha}. \quad (2.24)$$

The loan monitoring is assumed to be proportional to the labor supplied to the banking sector by farmer i and $A2_t$ is the productivity disturbance similar to the one in the production sector. Since capital stock require a substantial monitoring effort to confirm its physical condition, its inferiority to bonds for collateral purposes is expressed by v (Goodfriend and McCallum, 2007).

The complete intertemporal farmers's maximization problem (with a presence of the banking sector) may be written as

$$\max_{n_t^i, m_t^i, n_t^{i,d}, m_t^{i,d}, P_t^i, K_{t+1}^i, B_{t+1}^i} E_t^i \sum_{k=0}^{\infty} \beta^k \left[\phi \log(C_{t+k}^i) + (1 - \phi) \log(1 - n_{t+k}^i - m_{t+k}^i) \right]. \quad (2.25)$$

subject to the budget constraint (Eq. 2.22) and production constraint (Eq. 2.23).

Before solving the optimization problem, from Eq. (2.24) we know that

$$C_t^i = \frac{VF}{1-rr} (b_{t+1}^i + vq_t K_{t+1}^i)^\alpha \left(e^{A2t} m_t^{i,d} \right)^{1-\alpha}, \quad (2.26)$$

where $b_{t+1}^i = B_{t+1}^i / (P_t^A (1 + r_t^B))$. Additionally, by imposing market clearing we know that the good produced by farmer i is equal to its demand

$$Y_t^i = \left(\frac{P_t^i}{P_t^A} \right)^{-\theta} C_t^A, \quad (2.27)$$

where C_t^A is the aggregate consumption level that each individual takes as given.

Let the Lagrange multipliers be λ_t and ξ_t for the budget and production constraints respectively. By including Eq. (2.26) and Eq. (2.27) into the maximization problem and assuming market symmetry (Goodfriend and McCallum, 2007), the first order conditions provide

$$\frac{-(1-\phi)}{1-n_t^i-m_t^i} + \lambda_t^i w_t = 0, \quad (2.28)$$

$$-\lambda_t^i w_t + \xi_t^i e^{A1t} (1-\eta) \left(\frac{K_t^i}{e^{A1t} n_t^i} \right)^\eta = 0, \quad (2.29)$$

$$\left(\frac{\phi}{C_t^i} - \lambda_t^i \right) \frac{C_t^i (1-\alpha)}{m_t^i} - \lambda_t^i w_t = 0, \quad (2.30)$$

$$C_t^A \left(\frac{P_t^i}{P_t^A} \right)^{-\theta} \left(\frac{(1-\theta)\lambda_t^i}{P_t^A} + \frac{\theta\xi_t^i}{P_t^i} \right) = 0, \quad (2.31)$$

$$\left(\frac{\phi}{C_t^i \lambda_t^i} - 1 \right) \Omega_t^i vq_t - q_t + \beta(1-\delta) E_t^i \left(\frac{\lambda_{t+1}^i}{\lambda_t^i} q_{t+1} \right) + \beta \eta E_t^i \left(\frac{\xi_{t+1}^i}{\lambda_t^i} \left(\frac{e^{A1_{t+1}} n_{t+1}^i}{K_{t+1}^i} \right)^{1-\eta} \right) = 0, \quad (2.32)$$

$$\left(\frac{\phi}{C_t^i \lambda_t^i} - 1 \right) \Omega_t^i - 1 + \beta E_t^i \left(\frac{\lambda_{t+1}^i}{\lambda_t^i} \frac{P_t^A}{P_{t+1}^A} (1+r_t^B) \right) = 0, \quad (2.33)$$

where Ω_t^i is the partial derivative of the deposit constraint $C_t^i = \frac{VL_t^i}{(1-rr)P_t^A}$ with respect to

collateral

$$\Omega_t^i = \frac{\alpha C_t^i}{b_{t+1}^i + \nu q_t K_{t+1}^i}. \quad (2.34)$$

Appendix 2.B The influence of heterogeneous agents

Throughout the following derivation, we assume that each agent belongs to one of the two groups, i.e. $i = \tau \in \{RE, BRE\}$. By superscript A we will refer to the aggregate values.

Appendix 2.B.1 The heterogeneous IS curve

Let us first introduce a benevolent financial institution that helps farmers in hedging the risk associated with the Calvo lottery (Shi, 1999; Mankiw and Reis, 2007). In each period it collects all the income from the market and then redistribute it evenly across farmers. Given this property and assuming cashless limit, the agents' budget constraint becomes

$$w_t(n_t^i + m_t^i) + q_t(1 - \delta)K_t^i + \frac{Y_t^i P_t^i}{P_t^A} + \frac{B_t^i}{P_t^A} + I_{r,t}^i = w_t(n_t^{i,d} + m_t^{i,d}) + C_t^i + q_t K_{t+1}^i + \frac{B_{t+1}}{P_t^A(1 + r_t^B)} + I_{p,t}^i, \quad (2.35)$$

where $I_{r,t}^i$ and $I_{p,t}^i$ are the real receipts from and payments to the insurance agency. Each agent maximizes her expected utility over an infinite horizon, subject to Eq. (2.35) instead of (Eq. 2.22).

We know that the average real income (denoted by Ψ_t^τ) and the average marginal banking cost χ_t^τ obtained by rational and boundedly rational agents are

$$\Psi_t^{RE} = \frac{1}{\gamma P_t^A} \int_0^\gamma P_t^i Y_t^i di \quad \text{and} \quad \Psi_t^{BRE} = \frac{1}{(1 - \gamma) P_t^A} \int_\gamma^1 P_t^i Y_t^i di, \quad (2.36)$$

$$\chi_t^{RE} = \frac{1}{\gamma} \int_0^\gamma \chi_t^i di \quad \text{and} \quad \chi_t^{BRE} = \frac{1}{1 - \gamma} \int_\gamma^1 \chi_t^i di. \quad (2.37)$$

From the above equations it is clear that we may view the aggregate production and aggregate real marginal banking cost as a weighted average of their components, i.e. $Y_t^A = \gamma Y_t^{RE} + (1 -$

$\gamma)Y_t^{BRE}$ and $\chi_t^A = \gamma\chi_t^{RE} + (1 - \gamma)\chi_t^{BRE}$.

Following Branch and McGough (2009), if an agent is of type τ , then her real receipts from and payments to the insurance agency are $I_{r,t}^i = \Psi_t^\tau$ and $I_{p,t}^i = Y_t^i P_t^i / P_t^A$. By market clearing and axiom A2 the steady states of consumption and production are equal at individual and group levels. By imposing market symmetry, the budget constraint (Eq. 2.35) yields

$$\hat{C}_t^\tau = \hat{\Psi}_t^\tau + \frac{B_t^\tau / P_t^A}{\bar{Y}_t^A} - \frac{B_{t+1}^\tau / (P_t^A (1 + \bar{r}_t^B))}{\bar{Y}_t^A} + \frac{q_t (1 - \delta) K^\tau}{\bar{Y}_t^A} - \frac{q_t K_{t+1}^\tau}{\bar{Y}_t^A}, \quad (2.38)$$

where the bars indicate the steady state levels. Bond and capital market clearing require that $\alpha B_t^{RE} = -(1 - \alpha)B_t^{BRE}$ and $\alpha K_t^{RE} = -(1 - \alpha)K_t^{BRE}$. After multiplying Eq. (2.38) by γ for rational and by $(1 - \gamma)$ for boundedly rational agents and summing up, we arrive at

$$\hat{Y}_t^A = \gamma \hat{\Psi}_t^{RE} + (1 - \gamma) \hat{\Psi}_t^{BRE}. \quad (2.39)$$

From Eq. (2.4), (2.37) and (2.38) we have

$$\hat{\Psi}_t^\tau = E_t^\tau \hat{\Psi}_{t+1}^\tau + \left(\frac{1 - rr}{V} \right) E_t^\tau \tilde{\chi}_{t+1}^\tau - \left(\frac{1 - rr}{V} + 1 \right) \tilde{\chi}_t^\tau - (\hat{r}_t^{IB} - E_t^\tau \pi_{t+1}). \quad (2.40)$$

Iterating this equation forward and substituting into Eq. (2.39) we finally get

$$\begin{aligned} \hat{Y}_t^A = & \bar{E}_t \hat{Y}_{t+1}^A + \left(\frac{1 - rr}{V} \right) \bar{E}_t \tilde{\chi}_{t+1}^A - \left(\frac{1 - rr}{V} + 1 \right) \tilde{\chi}_t^A - (\hat{r}_t^{IB} - \bar{E}_t \pi_{t+1}) + \\ & + \left(\gamma \hat{\Psi}_\infty^{RE} + (1 - \gamma) \hat{\Psi}_\infty^{BRE} \right) - \bar{E}_t \left(\gamma \hat{\Psi}_\infty^{RE} + (1 - \gamma) \hat{\Psi}_\infty^{BRE} \right), \end{aligned} \quad (2.41)$$

with $\bar{E}_t = \gamma E_t^{RE} + (1 - \gamma) E_t^{BRE}$ and $\hat{\Psi}_\infty^\tau = \lim_{k \rightarrow \infty} E_t^\tau \hat{\Psi}_{t+k}^\tau$. In fact, Eq. (2.41) is of exactly the same form as in Branch and McGough (2009) but with a banking sector present. Axiom 7 indicates that agents predict their limiting wealth identically, which makes

$$\left(\gamma \hat{\Psi}_\infty^{RE} + (1 - \gamma) \hat{\Psi}_\infty^{BRE} \right) = \bar{E}_t \left(\gamma \hat{\Psi}_\infty^{RE} + (1 - \gamma) \hat{\Psi}_\infty^{BRE} \right). \quad (2.42)$$

Subtracting the log deviations of the potential product from both sides, we finally arrive at the heterogeneous IS curve with a present banking sector

$$x_t = \bar{E}_t x_{t+1} + \left(\frac{1-rr}{V} \right) \bar{E}_t \tilde{\chi}_{t+1}^A - \left(\frac{1-rr}{V} + 1 \right) \tilde{\chi}_t^A - [\hat{r}_t^{IB} - \bar{E}_t \pi_{t+1}] + u_t, \quad (2.43)$$

where $x_t = \hat{Y}_t^A - \hat{Y}_t^{f,A}$ is the output gap measure, the expectation operator is the weighted average of the group expectations $\bar{E}_t = \gamma E_t^{RE} + (1-\gamma) E_t^{BRE}$ and u_t is a disturbance term that depends only on exogenous productivity shocks.

Appendix 2.B.2 The heterogeneous Phillips curve

It is important to note that when farmers may hedge against the Calvo risk their production level would be 0 in equilibrium as a result of the free-riding problem. Therefore, following Branch and McGough (2009), we assume that farmers make their pricing decisions as if there was no insuring agency.

Let us take the log approximation of Eq. (2.7)

$$\log P_t^\tau - \log P_t^A = (1-\omega\beta)\hat{\varphi}_t^\tau + \omega\beta E_t^\tau \pi_{t+1} + \omega\beta E_t^\tau \log P_{t+1}^\tau / P_{t+1}^A. \quad (2.44)$$

Branch and McGough (2009) show that the Calvo lottery implies aggregate inflation to follow

$$\pi_t = \frac{1-\omega}{\omega} (\gamma \log P_t^{RE} / P_t^A + (1-\gamma) \log P_t^{BRE} / P_t^A). \quad (2.45)$$

As long as the pricing decisions are homogeneous within each group τ , by multiplying Eq. (2.44) by γ for rational and by $(1-\gamma)$ for boundedly rational agents and adding up, after some algebra we arrive at the final aggregate heterogeneous Phillips curve

$$\pi_t = \beta \bar{E}_t \pi_{t+1} + \kappa \varphi_t^A, \quad (2.46)$$

where $\kappa = \frac{(1-\omega)(1-\beta\omega)}{\omega}$.

Finally, noting that the aggregate marginal production cost is the aggregate output gap measure, the heterogeneous new Keynesian Phillips curve amended for the banking sector may be viewed as

$$\pi_t = \beta \bar{E}_t \pi_{t+1} + \kappa x_t, \quad (2.47)$$

where $\bar{E}_t = \gamma E_t^{RE} + (1 - \gamma) E_t^{BRE}$.

Appendix 2.B.3 The heterogeneous banking sector curve

Taking the steady state log deviations of Eq. (2.10) and iterating forward we get for each group of agents

$$\tilde{\chi}_t^\tau = \left(\frac{1 - rr}{V} \right)^{-1} \left[-v E_t^\tau \sum_{j=0}^{\infty} \tilde{\chi}_{t+j}^\tau + \left(\frac{1 - rr}{V} \right) \tilde{\chi}_\infty^\tau - (x_\infty^\tau - x_t^\tau) \right], \quad (2.48)$$

where $\hat{\chi}_\infty^\tau = \lim_{k \rightarrow \infty} E_t^\tau \hat{\chi}_{t+k}^\tau$ and $x_\infty^\tau = \lim_{k \rightarrow \infty} E_t^\tau x_{t+k}^\tau$.

Given Eq. (2.37) and (2.48), we get

$$\begin{aligned} \tilde{\chi}_t^A &= \gamma \tilde{\chi}_t^{RE} + (1 - \gamma) \tilde{\chi}_t^{BRE} \\ &= \left(\frac{1 - rr}{V} \right)^{-1} \left[-v \left(\gamma E_t^{RE} \sum_{k=0}^{\infty} \tilde{\chi}_{t+k}^{RE} + (1 - \gamma) E_t^{BRE} \sum_{k=0}^{\infty} \tilde{\chi}_{t+k}^{BRE} \right) + x_t \right. \\ &\quad \left. + \left(\frac{1 - rr}{V} \right) (\gamma \tilde{\chi}_\infty^{RE} + (1 - \gamma) \tilde{\chi}_\infty^{BRE}) - (\gamma x_\infty^{RE} + (1 - \gamma) x_\infty^{BRE}) \right] \\ &= \bar{E}_t \tilde{\chi}_{t+1}^A - \frac{vV}{1 - rr} \tilde{\chi}_t^A - \left(\frac{1 - rr}{V} \right)^{-1} (\bar{E}_t x_{t+1} - x_t) \\ &\quad + [(\gamma \tilde{\chi}_\infty^{RE} + (1 - \gamma) \tilde{\chi}_\infty^{BRE}) - \bar{E}_t (\gamma \tilde{\chi}_\infty^{RE} + (1 - \gamma) \tilde{\chi}_\infty^{BRE})] \\ &\quad - \left(\frac{1 - rr}{V} \right)^{-1} [(\gamma x_\infty^{RE} + (1 - \gamma) x_\infty^{BRE}) - \bar{E}_t (\gamma x_\infty^{RE} + (1 - \gamma) x_\infty^{BRE})]. \end{aligned} \quad (2.49)$$

The last two lines disappear due to Axiom 7, which gives

$$(\gamma \tilde{\chi}_\infty^{RE} + (1 - \gamma) \tilde{\chi}_\infty^{BRE}) = \bar{E}_t (\gamma \tilde{\chi}_\infty^{RE} + (1 - \gamma) \tilde{\chi}_\infty^{BRE}) \quad (2.50)$$

$$(\gamma x_\infty^{RE} + (1 - \gamma)x_\infty^{BRE}) = \bar{E}_t (\gamma x_\infty^{RE} + (1 - \gamma)x_\infty^{BRE}) \quad (2.51)$$

so that the final banking curve equation may be written as Eq. (2.14).

Appendix 2.C Model dynamics

The condensed model can be viewed as

$$\begin{pmatrix} B & 0 \\ 0 & I_3 \end{pmatrix} \begin{pmatrix} y_{t+1} \\ y_t \end{pmatrix} = \begin{pmatrix} F & -C \\ I_3 & 0 \end{pmatrix} \begin{pmatrix} y_t \\ y_{t-1} \end{pmatrix}, \quad (2.52)$$

where $y = (x, \pi, \tilde{\chi})'$ and

$$B = \begin{pmatrix} \gamma - \rho_x & \gamma - \rho_\pi & \frac{\gamma(1-rr)}{V} \\ 0 & \beta\gamma & 0 \\ -\gamma & 0 & \frac{\gamma(1-rr)}{V} \end{pmatrix}, \quad (2.53)$$

$$F = \begin{pmatrix} 1 & 0 & \frac{(1-rr)}{V} + 1 \\ -\kappa & 1 & 0 \\ -1 & 0 & v + \frac{(1-rr)}{V} \end{pmatrix}, \quad (2.54)$$

$$C = \begin{pmatrix} (1 - \gamma)\mu^2 & (1 - \gamma)\mu^2 & \frac{(1-\gamma)\mu^2(1-rr)}{V} \\ 0 & \beta(1 - \gamma)\mu^2 & 0 \\ -(1 - \gamma)\mu^2 & 0 & \frac{(1-\gamma)\mu^2(1-rr)}{V} \end{pmatrix}. \quad (2.55)$$