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Wijnands, R.A.D.; Degenaar, N.D.; Page, D.

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Testing the deep-crustal heating model using quiescent neutron-star very-faint X-ray transients and the possibility of partially accreted crusts in accreting neutron stars

R. Wijnands,1* N. Degenaar2† and D. Page3

1Astronomical Institute 'Anton Pannekoek', University of Amsterdam, Postbus 94249, 1090 GE Amsterdam, the Netherlands
2Department of Astronomy, University of Michigan, 500 Church Street, Ann Arbor, MI 48109-1042, USA
3Instituto de Astronomía, Universidad Nacional Autónoma de México, Mexico D.F. 04510, Mexico

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ABSTRACT
It is assumed that accreting neutron stars in low-mass X-ray binaries are heated due to the compression of the existing crust by the freshly accreted matter which gives rise to a variety of nuclear reactions in the crust. It has been shown that most of the energy is released deep in the crust by pycnonuclear reactions involving low-Z elements (the deep-crustal heating scenario). In this paper we discuss if neutron stars in the so-called very-faint X-ray transients (VFXTs; those transients have outburst peak 2–10 keV X-ray luminosities $<1 \times 10^{36}$ erg s$^{-1}$) can be used to test this deep-crustal heating model. We demonstrate that such systems would indeed be very interesting objects to test the deep-crustal heating model with, but that the interpretation of the results might be challenging because of the large uncertainties in our estimates of the accretion rate history of those VFXTs, both the short-term (less than a few tens of thousands of years) and the one throughout their lifetime. The latter is particularly important because it can be so low that the neutron stars might not have accreted enough matter to become massive enough that enhanced core cooling processes become active. Therefore, they could be relatively warm compared to other systems for which such enhanced cooling processes have been inferred. However, the amount of matter can also not be too low because then the crust might not have been replaced significantly by accreted matter and thus a hybrid crust of partly accreted and partly original, albeit further compressed matter, might be present. This would inhibit the full range of pycnonuclear reactions to occur and therefore possibly decrease the amount of heat deposited in the crust. More detailed calculations of the heating and cooling properties of such hybrid crusts have to be performed to be conclusive. Furthermore, better understanding is needed about how a hybrid crust affects other properties such as the thermal conductivity. A potentially interesting way to observe the effects of a hybrid crust on the heating and cooling of an accreting neutron star is to observe the crust cooling of such a neutron star after a prolonged (years to decades) accretion episode and compare the results with similar studies performed for neutron stars with a fully accreted crust. We also show that some individual neutron-star low-mass X-ray binaries might have hybrid crusts as well as possibly many of the neutron stars in high-mass X-ray binaries. This has to be taken into account when studying the cooling properties of those systems when they are in quiescence. In addition, we show that the VFXTs are likely not the dominate transients that are associated with the brightest ($\sim 10^{33}$ erg s$^{-1}$) low-luminosity X-ray sources in globular clusters as was previously hypothesized.

Key words: dense matter – binaries: close – stars: neutron – X-rays: binaries.

1 INTRODUCTION
Neutron-star low-mass X-ray binaries (LMXBs) harbour neutron stars which are accreting matter from a close-by low-mass (typically...
<1 M⊙) companion star which transfer mass due to Roche lobe overflow. In most systems the neutron star does not continuously accrete matter. Generally, those systems (called X-ray transients) are in their quiescent state in which they do not accrete at all or only at a very low rate. Only occasionally, they exhibit bright X-ray outbursts during which their X-ray luminosities increase by several orders of magnitude. Such outbursts are most probably caused by a very large increase in the mass accretion rate on to the neutron stars due to instabilities in the accretion disc (see the review by Lasota 2001). The X-ray transients can be divided in sub-groups based on their peak 2–10 keV X-ray luminosities in outburst. In particular, a special sub-group has been called the very-faint X-ray transients (VFXTs) that have peak luminosities between 1 × 10^34 and 1 × 10^36 erg s⁻¹ (see Wijnands et al. 2006), whereas the brighter transients have peak luminosities of 10^{36−39} erg s⁻¹.

In quiescence, neutron-star transients can still be detected using sensitive X-ray satellites, and it has been found that in many systems a soft, most likely thermal, component is present with a typical blackbody temperature of 0.1–0.3 keV (see e.g. van Paradijs et al. 1987; Asai et al. 1996; Campana et al. 1998; Rutledge et al. 1999, and references to those papers). In addition, for many systems an additional spectral component above 2 keV has also been detected (the non-thermal power-law component; see e.g. Asai et al. 1996; Rutledge et al. 1999), which can even dominate the 0.5–10 keV X-ray flux in some systems (e.g. Campana et al. 2002a; Jonker et al. 2004; Wijnands et al. 2005; Degenaar, Patruno & Wijnands 2012b). The origin of the non-thermal component is not clear (see e.g. the discussions in Campana et al. 1998; Degenaar et al. 2012b) but it is generally assumed that the soft component is the thermal emission from the neutron-star surface, either due to very-low-level residual accretion on to the surface or due to the cooling of the neutron star that has been heated by the matter accreted during the outbursts.

During the accretion phases, matter accumulates on the surface of the neutron star. This matter compresses the underlying layers of the neutron-star crust. If the accretion continues long enough the original catalysed crust can be completely replaced by a new crust made of accreted matter (Sato 1979; Haensel & Zdunik 1990b). The original crust is pushed down into the neutron star until it fuses together with the core. The composition of the accreted crust should be quite different from the original, catalysed crust, i.e. richer in low-Z elements (Haensel & Zdunik 1990b). It has been postulated that when the accreted matter sinks into the crust due to the compression induced by freshly accreted material on to the star, a chain of non-equilibrium reactions occur in the crust that generate heat (electron captures, neutron drips and pycnonuclear reactions; Haensel & Zdunik 1990a, 2003, 2008; Gupta et al. 2007). Most of the heat is released deep in the crust (at densities > 10^{12} g cm⁻³) due to pycnonuclear reactions involving low-Z elements. This heat is conducted inwards, heating the core, and outwards, where it is emitted as thermal emission from the surface. This model has been called the ‘deep-crustal heating model’ (Brown, Bildsten & Rutledge 1998). This model has been tested by comparing the observed thermal emission of quiescent neutron stars with predictions based on estimations of their time-averaged accretion rates (see Section 2.1). Another exciting possibility is to study the thermal relaxation of accretion-heated neutron-star crusts after the end of accretion outbursts (see also Section 4.3).

In Section 2, we briefly describe the deep-crustal heating model and compare the model with the available data. We also calculate the time-scale on which the core reacts to changes in the long-term averaged accretion rate. In Section 3 we calculate, in the framework of the deep-crustal heating model, the expected quiescent luminosity of neutron-star VFXTs, in order to use those systems to test the deep-crustal heating model. We argue that it might be possible that during their life those systems might not have accreted enough matter to have fully replaced their original neutron-star crust with an accreted one which could significantly inhibit the pycnonuclear heating reactions. In Section 4 we discuss how the VFXTs can still be used to test the model and also discuss other potential sources which might harbour neutron stars with only partly accreted crusts.

### 2 QUIESCENT NEUTRON STARS AND THE DEEP-CRUSTAL HEATING MODEL

In this heating/cooling model one can calculate the thermal state of the neutron star with a simple energy balance consideration by writing

\[
\frac{dE_{\text{th}}}{dt} = C_V \frac{dT}{dt} = H - L_\nu - L_\nu,
\]

(1)

where \( E_{\text{th}} \) is the thermal energy of the neutron star, \( C_V \) its total specific heat and \( T \) its core temperature. \( H \) is the total heating rate, and the two energy sinks are the star’s thermal photon luminosity, \( L_\nu \), and its neutrino luminosity \( L_\nu \).

The deep-crustal heating scenario \( H \) is taken as a time average

\[
H \rightarrow \langle H \rangle = \frac{\langle M \rangle Q_{\text{nuc}}}{m_u}
\]

(2)

\[
\approx 10^{33} \frac{\langle M \rangle}{10^{-11} M_\odot \text{ yr}^{-1}} \frac{Q_{\text{nuc}}}{1.5 \text{ MeV}} \text{ erg s}^{-1}
\]

where \( \langle M \rangle \) is the long-term time-averaged mass accretion rate on to the neutron star, \( Q_{\text{nuc}} \) the amount of heat, per accreted nucleon, deposited in the crust and \( m_u \) the atomic mass unit. Theoretical predictions (e.g. Haensel & Zdunik 2008) obtain values for \( Q_{\text{nuc}} \) between 1 and 2 MeV.

When \( \langle M \rangle \) has been stable for a long enough time the neutron star is in thermal equilibrium (see Section 2.2 for estimates of the thermal response time-scale). Hence, from equation (1) with \( dT/dt = 0 \), one obtains the expected \( \langle L_\nu \rangle \), or the observable quiescent thermal luminosity \( L_\text{q} \), as

\[
L_\text{q} = \langle H \rangle - \langle L_\nu \rangle.
\]

(3)

Brown et al. (1998) showed that when \( \langle L_\nu \rangle \) is negligible the very simple relation

\[
L_\text{q} = \langle H \rangle = \langle M \rangle Q_{\text{nuc}}/m_u
\]

(4)

agreed with several observations of quiescent neutron-star LMXB transients. However, Colpi et al. (2001) emphasized that many such systems are hot enough that \( \langle L_\nu \rangle \) is not necessarily negligible and could even be in a regime where \( L_\nu \gg L_\gamma \). In the case that fast neutrino emission is possible (as one would expect for a massive neutron star) this would explain the low values of \( L_\text{q} \) observed from several systems that are discrepant with the simplified equation (4) since with a very large \( L_\nu \) one could have \( L_\text{q} \ll \langle M \rangle Q_{\text{nuc}}/m_u \).

Neutrino emission processes can be roughly divided into two categories, either ‘slow’ or ‘fast’ that differ in their temperature dependence and efficiency (Yakovlev & Pethick 2004; Page, Geppert &...
Weber 2006; Page & Reddy 2006). Slow processes include the modified Urca (‘MUrca’) and several bremsstrahlung processes whose emissivities can be roughly written as $\epsilon^\text{slow}_{\nu} = QT^8_{\nu} \text{erg cm}^{-3} \text{s}^{-1}$ where $T_9 = 10^{9}$ K. Values for $Q$ range from $\sim 10^{19}$ for the bremsstrahlung processes to $10^{21}$ for the MUrca. A higher emissivity for the MUrca has been proposed by Voskrenensky & Senatorov (1986), called the Medium-Modified Urca process (‘MMUrca’), resulting from the softening of the pion mode and that is a precursor to the pion condensate. The MUrca has a $Q$ that reaches $10^{23}$ and it smoothly merges into the pion condensate emissivity (see below) with growing density. The fast processes have emissivities $\epsilon^\text{fast}_{\nu} = QT^6_{\nu} \text{erg cm}^{-3} \text{s}^{-1}$. The simplest and most efficient fast process is the direct Urca (‘DUrca’) with nucleons and this process has $Q \approx 10^{22}$. In the presence of hyperons other DUrca processes are possible with slightly reduced efficiencies. Other fast, but less efficient, processes are possible in the presence of a meson condensate (either pion or kaon) that also has a $T^6$ dependence and $Q$ in the range $10^{24}$–$10^{26}$ for pions and $10^{21}$–$10^{23}$ for kaons. Finally, if the neutron star has an inner core with deconfined quark matter, similar DUrca processes are present with efficiencies that can match the nucleon DUrca one.

The resulting neutrino luminosity is then given by

$$L^\text{slow}_{\nu} = \frac{4}{3} \pi R^3 \cdot Q^\text{slow} T^8_{\nu} \equiv N^\text{slow} T^8_{\nu}$$

and

$$L^\text{fast}_{\nu} = \frac{4}{3} \pi R^3_p \cdot Q^\text{fast} T^6_{\nu} \equiv N^\text{fast} T^6_{\nu}$$

where $R$ is the radius of the neutron star and $R_p$ the radius of the inner core (i.e. high density) region where the given fast process is acting. There are theoretical uncertainties on $Q$ of a factor of a few for the slow processes and the DUrca ones, but they are much larger in the case of the meson condensates. Moreover, for the slow processes $R \sim 10$ km while for the fast ones $R_p$ can range from $\sim$ 0 km, i.e. almost no fast neutrino emission, to almost 10 km depending on the equation of state (EOS; in the case it allows such processes) and the mass of the neutron star. We take as a typical value $R_p \sim 5$ km but varying it can change $N^\text{fast}$ by almost three orders of magnitude.

In the presence of pairing (causing superfluidity and/or superconductivity), neutrino emission processes can be strongly altered (see e.g. Page et al. 2006). In low-mass neutron stars where the MUrca processes can be strongly suppressed by pairing, bremsstrahlung processes may become the dominant sources of neutrinos. In the fast cooling scenarios $L^\text{fast}$ can also be significantly reduced by pairing. Moreover, pairing opens a new neutrino emission channel from the constant formation and breaking of Cooper pairs (dubbed the ‘pair breaking and formation’ or ‘PBF’ process). The corresponding emissivity can be roughly approximated by $\epsilon^\text{PBF}_{\nu} \approx QT^6_{\nu} \text{erg cm}^{-3} \text{s}^{-1}$ where $Q$ can reach $10^{22}$ in optimal conditions depending on the type of pairing and its corresponding critical temperature $T_c$ (Page et al. 2009).

The photon luminosity is simply $L_\gamma = 4\pi R^2 \sigma_T (T^\infty)^4$ where $\sigma_T$ is the Stefan–Boltzmann constant and $T^\infty$ the star’s redshifted effective temperature. $T_9$ has to be related to the internal temperature $T$ and, as a simple rule, one can use $T_9 \approx 10^{9} T^{1/2}_{\text{K}}$. This implies that $L_\gamma \approx 7 \times 10^{52} T^9_{\text{K}} \text{erg s}^{-1}$. A detailed study of accreted neutron-star envelopes and the resulting $T \sim T_c$ relationship can be found in Potekhin, Chabrier & Yakovlev (1997) and Yakovlev et al. (2004). The latter work shows that the $T$ dependence of $L_\gamma$ ranges between $T^{4.7}$ and $T^{2.3}$, depending on the actual chemical composition of the accreted envelope.

## 2.1 Comparison of deep-crustal heating with data

Predictions for $L_\gamma$ as a function of $\dot{M}$ for the various neutrino emission scenarios described above are shown in Fig. 1, and compared with data (similar to what has been done by Yakovlev & Pethick 2004; Heinke et al. 2007, 2009a, 2010).

In this figure the band ‘Heating’ shows the average heating rate $\langle H \rangle$ from equation (2) with $Q_{\text{acc}}$ ranging from 1 to 2 MeV. Any star located on this line balances its heating only by its $L_\gamma$ and is thus in the photon cooling regime of equation (4). However, an object located below it has significant neutrino losses; it is in the neutrino cooling regime and the difference between its observed $L_\gamma$ and the corresponding value of $\langle H \rangle$, at the same $\langle M \rangle$, on the ‘Heating’ lines directly gives its $L_\gamma$ from equation (3).

Each curve in Fig. 1, for the various neutrino cooling scenarios, is given by the energy balance of equation (3). The parameter set we use is consistent with the one proposed by Yakovlev, Levenfish & Haensel (2003). The photon luminosity is chosen as $L_\gamma = 7 \times 10^{54} T^9_{\text{K}} \text{erg s}^{-1}$ and the heating rate, or luminosity, is obtained from equation (2) with $Q_{\text{acc}} = 1.5 \text{ MeV}$. The four upper lines, ‘Brems.’, ‘MUrca’, ‘PBF’ and ‘MMUrca’, correspond to the four slow neutrino cooling scenarios. We use, for the corresponding $L_\gamma$ in equation (5), $N^\text{Brems} = 5 \times 10^{34}$, $N^\text{MUrca} = 5 \times 10^{39}$, $N^\text{PBF} = 5 \times 10^{40}$ and $N^\text{MMUrca} = 5 \times 10^{41}$. As mentioned above, the ‘PBF’ scenario can have a very wide range of efficiencies and the case considered here corresponds to the most efficient one as deduced for maximal compatibility of the ‘minimal cooling’ scenario with data from isolated cooling neutron stars (Page et al. 2004, 2009), and with the interpretation of the observed cooling of the neutron star in the supernova remnant Cassiopeia A (Page et al. 2011; Page 2012). The ‘MMUrca’ scenario actually covers a wide range of neutrino emission efficiencies that strongly depend on the neutron-star mass and smoothly merge into the pion cooling scenario.
The three pairs of lines ‘Kaon’, ‘Pion’ and ‘DUrca’ show the prediction for each corresponding scenario when maximum and strongly reduced $L_{\text{fast}}^\text{fast}$ are assumed. We use, for the corresponding $L_\nu$ in equation (6), $\lambda_{\text{Kaon}}$ ranging from $5 \times 10^{40}$ to $5 \times 10^{43}$, $\lambda_{\text{Pion}}$ from $5 \times 10^{41}$ to $5 \times 10^{45}$, and $\lambda_{\text{DUrca}}$ from $5 \times 10^{41}$ to $5 \times 10^{45}$. When $L_\nu \gg L_{\nu}$ one has $L_{\nu}^\text{fast} = \langle H \rangle$ and since $L_{\nu}^\text{fast} \propto T^6$, if the neutrino efficiency is reduced by a factor of $10^3$, $T^2$ must be 10 times higher for $L_{\nu}^\text{fast}$ to keep matching $\langle H \rangle$. Since $L_{\nu} \propto T^2$, the resulting predicted $L_{\nu}$ is an order of magnitude higher. Reduced efficiency of $L_{\nu}^\text{fast}$ can be either due to an emissivity lower than quoted above or due to a smaller $R_\nu$. Thus, to each one of these fast neutrino processes correspond a band of at least one order of magnitude width in predicted $L_{\nu}$.

A comparison of these predictions with the data first shows that only a few objects are on the ‘photon cooling’ line, i.e. are described by equation (4), and that a large number of the observed quiescent neutron-star LMXBs have an inferred $L_\nu$ which requires significant neutrino emission (e.g. Colpi et al. 2001). In most cases one sees that $L_\nu$ dominates over $L_{\nu}$ by one to two orders of magnitude. Enhanced neutrino emission implies a large neutron-star mass that must have been accreted during the lifetime of the X-ray binary, unless most neutron stars can be born this massive. The latter possibility would conflict with the observation of young isolated cooling neutron stars: the cooling of these stars is driven by neutrino emission and their observed luminosity can be reproduced with a $L_\nu$ similar to the ‘PBF’ model of Fig. 1 (see e.g. Page et al. 2009).  

The conclusion is that unless neutron stars in LMXB are born more massive than isolated ones, a large fraction of the neutron stars shown in Fig. 1 must have accreted a significant amount of mass during their lifetime. A way around this conclusion is that the presently deduced $\langle M \rangle$ is not representative of the recent history of these systems (i.e. $\langle M \rangle$ was significantly lower in the past) and for this we examine in the next subsection their thermal inertia. An alternative is that there is something missing in the deep-crustal heating model.

2.2 Thermal response time-scale

Since human X-ray observations of neutron-star LMXBs only span, in the best case, a few decades, the estimated $\langle M \rangle$ are highly uncertain in many cases. Of particular importance is an estimate of the thermal response time-scale $\tau_{\text{th}}$ of a neutron star after a significant change of $\langle M \rangle$. This can be easily obtained from equation (1) after specifying $C_\nu$. Most of a neutron-star specific heat is provided by its core and, for degenerate Fermions, $C_\nu \propto T$ so we simply write $C_\nu = C \cdot T$ erg K$^{-1}$ (Yakovlev & Pethick 2004; Page et al. 2006). Then equation (1) gives

\[ \frac{dT}{dt} = \frac{2}{C} (\overline{H} - \overline{T}), \]

where $\overline{H}$ and $\overline{T}$ are short-term time averages of the heating term $H$ and the cooling term $L_\nu = L_{\nu} + L_{\nu}$. By ‘short-term’ time average we mean averaged over many accretion outbursts but during a time-span still much shorter than $\tau_{\text{th}}$. We assume that at time $t = 0$ there is an abrupt change of $\overline{H}$ from $\overline{H}_0$ to $\overline{H}_1$, and that at times $t < 0$ the star was in a stationary state at a temperature $T_0$. Then at $t < 0$ one had $\overline{T}_0 = \overline{H}_0$. At $t > 0$ the star begins to react and the resulting evolution of $T^2$ is illustrated by the thick curves in Fig. 2.

\[ \tau_{\text{th}} \approx \frac{C T_{\text{high}}^2}{2 \langle H \rangle_{\text{high}}}. \]

where we also used that $\overline{T}_0 = \overline{H}_0$. If $\overline{H}_1 \ll \overline{H}_0$ or $\overline{H}_1 \gg \overline{H}_0$ this expression for $\tau_{\text{th}}$ will be dominated by the high $\overline{H}$, high $T$, term and we can write

\[ \tau_{\text{th}} \approx 2 \times 10^4 f_{\text{SF}} \frac{L_{\text{eq}}}{10^{52} \text{erg s}^{-1}} \frac{10^{-11} M_\odot}{\langle M \rangle} \text{yr}. \]

Models show that $C \approx 10^{10}$ when $T$ is measured in kelvin, with only a small dependence on the mass of the neutron star. However, it can be reduced by up to a factor of 10 in the presence of pairing (Page et al. 2006), i.e. $C \rightarrow f_{\text{SF}} C$ with $f_{\text{SF}}$ ranging from 1 (no pairing) down to 0.1 (maximum extent of superfluidity/superconductivity). $\langle H \rangle$ is obtained from $\langle M \rangle$ with equation (2). Notice that since $L_{\nu} \propto T^2$ one has the fortuitous, but useful, result

\[ \tau_{\text{th}} \approx 2 \times 10^4 f_{\text{SF}} \frac{L_{\text{eq}}}{10^{52} \text{erg s}^{-1}} \frac{10^{-11} M_\odot}{\langle M \rangle} \text{yr}. \]

Several tracks (the dotted lines) for various values of $\tau_{\text{th}}$ are also shown in Fig. 1.

Notice that the obtained $\tau_{\text{th}}$ is just the Kelvin–Helmoltz timescale $\tau_{\text{KH}} = E_\odot/L_\odot$ since $E_\odot = \int C \cdot dT = \frac{1}{2} C T^2$. However, when a star transits between a high and a low $\langle M \rangle$ states we have two $\tau_{\text{KH}}$, and our analysis tells us that the relevant one is the high state one. If the energy loss is due to neutrinos the high state $\tau_{\text{KH}}$ is the shortest one since $L_\nu \propto T^4$ or $T^6$ and thus $\tau_{\text{KH}} \propto T^{-4}$ or $T^{-6}$. This is intuitively clear from Fig. 2: if the star starts from a low $\langle M \rangle$ its heating is driven by the new high $\langle M \rangle$ while if the star starts from a high $\langle M \rangle$ its cooling is driven by the remaining previous high $L_\nu$.

However, if the cooling is driven by the photon luminosity $L_{\nu} \propto T^2$ then $\tau_{\text{th}}$ is essentially independent of $\langle M \rangle$; equations (2) and (9)
give $\tau_{\text{th}} \approx 10^3 f_{\text{fs}}$ yr, as is confirmed by the $\tau_{\text{bar}}$ line in Fig. 1. In both cases, it is simply the shortest $\tau_{\text{th}}$ that determines the time-scale.

Returning to the issue raised at the end of Section 2.1 about the mass of these neutron stars, we see from Fig. 1 that the ones requiring fast neutrino emission, and hence a high mass, have $\tau_{\text{th}} \sim 10^3 - 10^4$ yr. Postulating that they have a low $L_q$ because $\langle M \rangle$ was much lower in the past and suddenly increased recently would require that this increase occurred in the last $10^3 - 10^4$ for all of them. Such a claim is hardly sustainable and the most natural interpretation of the large spread in observed $L_q$, for a given $\langle M \rangle$, is the result of a large range of neutron-star masses: more massive stars should have a larger neutrino emission efficiency and hence appear fainter.

2.3 Evolution of neutron stars with varying $\langle M \rangle$

When $\langle M \rangle$ changes, the $L_q$ of the neutron star will slowly evolve on a time-scale $\tau_{\text{th}}$. It will follow a trajectory, in the $L_q$ versus $\langle M \rangle$ plane, given by equation (3) which is just one of the many curves plotted in Fig. 1, depending on its neutrino emission efficiency (i.e. and therefore very likely its mass).

If neutrino cooling dominates the energy balance, i.e. $L_q = \langle H \rangle$, we have for the star’s core temperature $T$, $T^2 \propto \langle H \rangle^{1/3}$ for fast ($T^2$) neutrinos and $T^2 \propto \langle H \rangle^{1/4}$ for slow ($T^2$) neutrinos. Since $L_q \propto T^2$ we obtain for fast neutrino emission

$$L_q \propto \langle M \rangle^{1/3}$$

and

$$L_q \propto \langle M \rangle^{1/4}$$

for slow neutrino emission. For low enough $\langle M \rangle$ these neutrino trajectories curve down and merge into the photon cooling trajectory $L_q = (Q_{\text{acc}}/m_a)\langle M \rangle$. These simple power-law behaviours are clearly seen in Fig. 1. However, for a very rapid change in $\langle M \rangle$ the neutron star may be found ‘off-trajectory’ for a time $\sim \tau_{\text{th}}$ that can be as large as $10^4$ yr for very-faint systems.

Notice that from the theoretical point of view the photon trajectory is the cleanest one: it only depends on $Q_{\text{acc}}$, equation (4), which is known within a factor of 2, 1–2 MeV, within the deep-crustal heating model. The neutrino trajectories (which depend on the core temperature $T$) are plagued with the uncertainty on how to relate $T_y$ with the core $T$. This $T_y$–$T$ relationship depends on the outer layer chemical composition and the star’s surface gravity: for a given neutron-star model (encapsulated in our parameters $N$ and $C$ for the neutrino luminosity and the specific heat) the predicted $L_q$ can vary by almost one order of magnitude because of the $T_y$–$T$ relationship (Potekhin et al. 1997; Yakovlev et al. 2004).

2.4 Testing the deep-crustal heating model

So far, this deep-crustal heating model has mostly been tested using sources within a relatively small $\langle M \rangle$ range, between $5 \times 10^{-12}$ and $5 \times 10^{-10}$ $M_\odot$ yr$^{-1}$, as can be seen from Fig. 1. For higher $\langle M \rangle$ the model will be difficult to test because it straddles in the range of the persistently bright sources and quiescent measurements are impossible for such sources. However, at the low $\langle M \rangle$, between $1 \times 10^{-13}$ erg s$^{-1}$ and $5 \times 10^{-12}$ erg s$^{-1}$, it is likely that sources can be added; in particular the neutron-star VFXTs can have $\langle M \rangle$ in this range (see Section 3.1). We now discuss the prospect of using those systems to test the deep-crustal heating model (see also the brief discussion in Wijnands 2008).

3 THE QUIESCENT LUMINOSITY OF NEUTRON-STAR VFXTs

From the results of the previous section, illustrated in Fig. 1, we can estimate the expected quiescent luminosities $L_q$ of VFXTs from estimates of their long-term average accretion rates. However, it is notoriously difficult to estimate $\langle M \rangle$ for any X-ray transient and even more so for VFXTs because they are difficult (despite the sensitivity of X-ray instruments in orbit; Wijnands et al. 2006) to detect and therefore most of their outbursts are likely missed. For the deep-crustal heating model, the $\langle M \rangle$ which is needed is the $M$ averaged over a time $\tau_{\text{th}}$ that (as can be seen from Fig. 1) ranges between $10^4$ and $10^5$ yr at $M \sim 10^{-12} M_\odot$ yr$^{-1}$ and $L_q \sim 10^{36}$ erg s$^{-1}$.

3.1 Estimation of $\langle M \rangle$ in VFXTs

An estimation of an upper limit on $\langle M \rangle$ for the VFXTs can be obtained using some simple assumptions. As already stated in the Introduction, VFXTs have peak X-ray luminosities (2–10 keV) $< 1 \times 10^{36}$ erg s$^{-1}$. This limit was chosen in the classification of Wijnands et al. (2006) because it roughly corresponds to the sensitivity limit of the past and present X-ray all-sky monitors in orbit (e.g. BeppoSAX/NFC, RXTE/ASM, Swift/BAT, Integral, MAXI) for sources at 8 kpc. Typically, those instruments are sensitive to X-ray outbursts which have a peak flux (2–10 keV) above 10 mCrab,$^3$ which corresponds to a 2–10 keV flux of $\sim 2 \times 10^{-10}$ erg s$^{-1}$ cm$^{-2}$ and a limiting 2–10 keV luminosity sensitivity of $1 \times 10^{36}$ erg s$^{-1}$ (for 8 kpc; which was assumed the typical distance towards VFXTs since most have been found in the Galactic bulge).

Typically the bolometric luminosity is then a factor of 2–3 higher (see in ’t Zand, Jonker & Markwardt 2007) and thus the upper limit on the bolometric peak luminosity would be $< 3 \times 10^{36}$ erg s$^{-1}$. Assuming perfect efficiency of the accretion process, this accretion luminosity is related to the accretion rate in outburst $\dot{M}$ through $L = GM\dot{M}/R$, with $G$ the gravitational constant, $M$ the mass of the neutron star and $R$ the radius of the star. Here we use a ‘canonical’ neutron star with a mass of 1.4 solar masses and a radius of 10 km. The luminosity upper limit then results in an upper limit on the peak mass accretion rate during outburst of $< 3 \times 10^{-10} M_\odot$ yr$^{-1}$, assuming that the duty cycle (DC) (with DC = $t_o/(t_o + t_q)$, $t_o$ being the outburst duration time and $t_q$ the quiescent duration time) of VFXTs is between 1 and 10 per cent (as is typically observed for the recurrent bright transients), then $\langle M \rangle \lesssim 10^{-12} - 10^{-11} M_\odot$ yr$^{-1}$. This is a rather conservative upper limit because it assumes a ‘step-function’ outburst in which the sources are always accreting just below the limits set by the all-sky instruments when they are in outburst. However, many systems will have peak luminosities well below this level (e.g. Degenaar & Wijnands 2010) and during the outbursts the peak luminosities are only reached during a small fraction of the outburst, similar to the outburst profiles of brighter transients (e.g. see Chen, Shrader & Livio 1997). This will significantly lower this upper limit for the VFXTs. However, this limit already shows that indeed VFXTs have very low $\langle M \rangle$; i.e. in the range which has hardly been used to test the deep-crustal heating model.

Moreover, for several VFXTs more stringent constraints can be obtained on their $\langle M \rangle$. Degenaar & Wijnands (2010) have estimated

$^3$ Some instruments or programs are more sensitive (e.g. the RXTE/PCA bulge scan project reached about 1 mCrab; Swank & Markwardt 2001). However, for the purpose of this paper, we assume a conservative limit of 10 mCrab.
the \( \langle M \rangle \) of several VFXTs near the Galactic centre using 4 years of Swift X-ray Telescope monitoring data on Sgr A*. The majority of the sources considered in that paper had \( \langle M \rangle < 2 \times 10^{-12} \, M_{\odot} \, \text{yr}^{-1} \) and were typically in the range \( 10^{-13} \) to \( 10^{-12} \, M_{\odot} \, \text{yr}^{-1} \). This demonstrates that at least some VFXTs have extremely low \( \langle M \rangle \) and it might be possible that a large fraction of the VFXTs have similar \( \langle M \rangle \).

3.2 Estimation of \( L_q \) in VFXTs

A naive estimation of the possible range of \( L_q \)'s for neutron-star VFXTs would simply consider the various trajectories of Fig. 1: we can consider the sample of objects depicted in Fig. 1 and move them along their respective cooling trajectories into the range of the (very low) \( \langle M \rangle \) above estimated for the VFXTs. There are, however, two immediate issues with this extrapolation:

(1) How representative are the above estimated \( \langle M \rangle \) of VFXTs with their real long-term \( \langle M \rangle \)?

(2) Since the spread in the \( L_q \) observed for the brighter transients exhibited in Fig. 1 is likely due to a large spread in neutron-star masses (as argued above) how representative is the mass distribution of the bright transients for the mass range of the VFXTs?

3.2.1 The \( \langle M \rangle \) versus \( \tau_{\text{th}} \) issue

The above limits on the \( \langle M \rangle \) assumes that the current observed behaviour of VFXTs is representative of their general behaviour. However, this does not need to be true, and it might be that some fraction of the VFXTs are actually sources which usually accrete at much higher rates and we only observe them during a small period in which their \( \langle M \rangle \) is much less (although see the discussions in King & Wijnands 2006; Wijnands et al. 2006, demonstrating that this is likely not true for the majority of the VFXTs).

The issue is how long has this period of very low \( \langle M \rangle \) lasted compared to the neutron-star relaxation time \( \tau_{\text{th}} \). As discussed in Section 2.3, if the time-scale of the evolution of \( \langle M \rangle \) is larger than the corresponding \( \tau_{\text{th}} \) the neutron star will evolve along one of the trajectories exhibited in Fig. 1: this is the Case B discussed below. However, in the opposite case we have:

Case A. The VFXTs in this class are normally bright transients or even bright persistent sources and we only observe them during a short-lived VFXT episode, shorter than their \( \tau_{\text{th}} \). This is not unfeasible, because several bright systems have exhibited faint to very-faint outbursts as well (e.g. Linares, Wijnands & van der Klis 2007; Degenaar & Wijnands 2009; Fridriksson et al. 2011; Degenaar et al. 2012a). In other words, the estimated \( \langle M \rangle \) is rather an \( \bar{M} \) in the notations of Section 2.2 and the star has not yet adjusted to the new low \( \bar{M} \): the star's present \( L_q \) is still determined by its previous higher \( \langle M \rangle \) and is 'off-trajectory'. In the case the neutron star is not too massive and was previously on a slow neutrino cooling track its present \( L_q \) can be in the range of \( 10^{31} \) to \( 10^{34} \) erg \( \text{s}^{-1} \); such high \( L_q \) can be sustained for more than \( 10^5 \) yr, the corresponding \( \tau_{\text{th}} \) of bright transients on the slow cooling trajectories (\( \tau_{\text{th}} \) being determined by the value in the high \( \langle M \rangle \) regime). With such an \( L_q \) and an \( \langle M \rangle \) in the VFXT regime it would be located in the diagram of Fig. 1 above the photon cooling line. This is a region that is unaccessible for a star in thermal equilibrium.

Case B. The observed behaviour of the VFXTs in this class is representative for their behaviour on a long enough time that they are in a steady state, i.e. they can be correctly located on one of the trajectories of Fig. 1. However, for the Case B systems, one has to consider the second issue, which we will address in the next section.

3.2.2 The mass distribution issue

Extrapolating the observed distribution of \( L_q \) of bright transients to the low \( \langle M \rangle \) regime could allow us to predict the expected distribution of \( L_q \) for the VFXTs. However, isolated young neutron stars, that likely have a mass distribution between 1.2 and 1.6 \( M_{\odot} \), with an observed thermal luminosity, are well described by slow neutrino emission processes (Page et al. 2004, 2009). This is in sharp contrast to what is observed for the bright neutron-star transients. Most of them have an \( L_q \) that requires fast neutrino processes, implying a mass distribution strongly skewed towards higher masses due to the long-term accretion. As a consequence, the \( L_q \) distribution of the VFXTs that have been in the VFXT phase for long enough that they are in a steady state (Case B) should show a significant imprint of their very long term past accretion history and Case B can be divided into two distinct sub-cases:

Case B1. Some systems might have been a VFXT for a period longer than \( \tau_{\text{th}} \) but before that they were similar to the brighter transients. Although originally the temperature of the core should be relatively high, the time spent in the VFXT phase is long enough that the neutron star has adjusted its core temperature (i.e. it has become colder) to the very low accretion rate, but during the time spent before this phase (when the accretion rate was much higher) the source could have accreted enough material for it to cross the mass threshold beyond which fast core cooling could occur. Therefore, those systems should be on one of the 'fast neutrino cooling' trajectories of Fig. 1 and be significantly fainter in quiescence than predicted using the standard deep-crustal heating model, equation (4).

Case B2. Some systems might have been VFXTs throughout their life and the estimated \( \langle M \rangle \) is representative of the \( \langle M \rangle \) throughout their life (we call them primordial VFXTs; see also King & Wijnands 2006, who talked about the possible existence of such systems). Therefore, since it is expected that LMXBs live for \( 10^8 \) to \( 10^9 \) yr, only about \( 10^{-5} \) to \( 10^{-2} \) solar masses (assuming \( \langle M \rangle = 10^{-13} \) to \( 10^{-11} \, M_{\odot} \, \text{yr}^{-1} \); Section 3.1) have been accreted by the neutron stars. Therefore, it is quite likely that most, if not all, of these sources have not been accreting enough matter during their life to increase their masses (assuming they are all born with a mass of \( \sim 1.4 \, M_{\odot} \) above the threshold so that enhanced core cooling would become active. If true, these systems should be on the slow neutrino cooling tracks of Fig. 1 that, in the estimated \( \langle M \rangle \) range of VFXTs, merge with the photon cooling track. Therefore, they should agglutinate on the narrow 'photon cooling' region and indeed be detectable at \( 10^{31} \) to \( 10^{33} \) erg \( \text{s}^{-1} \). However, this assumes that the standard deep-crustal heating is active in those sources, but in the next section we argue that this might not be the case.

3.3 Non-standard heating in neutron-star VFXTs

In the deep-crustal heating scenario, it is explicitly assumed that the original, catalysed crust is fully replaced by an accreted one (Haensel & Zdunik 1990a, 2003, 2008). The low-Z elements present (at high densities) in an accreted crust facilitate a significant amount of energy release due to the pyconuclear reactions. Those pyconuclear reactions occur in the density range from \( 10^{12} \) to \( \sim 5 \times 10^{13} \) g cm\(^{-3}\) (Haensel & Zdunik 1990a, 2003, 2008). It is quite possible
that among the primordial VFXTs (the above Case B2 sources) a group of systems exist in which the crust is not replaced to this depth and a partly accreted and a partly original (albeit further compressed) crust (a hybrid crust) might be present.

To make this last assertion more quantitative, we show in Fig. 3 estimates of the amount of matter that needs to be accreted in order to replace the original catalysed matter. The needed accreted mass is simply

$$\Delta M_{\text{acc}} = 4\pi R^2 y,$$

where $y$ is the accreted column density. This column density determines the pressure reached by the accreted matter since, from hydrostatic equilibrium, $P = gy$ where $g = e^{-\pi} GM/R^2$ is the gravity acceleration in the crust and $e^\pi = (1 - 2GM/Rc^2)^{-1/2}$ the redshift ($G$ being the gravitational constant and $c$ the speed of light). For a given pressure we can obtain the corresponding density $\rho$ using a crust EOS. The EOS of a hybrid crust remains to be calculated but we can bracket it between the EOS of a catalysed crust and a wholly accreted one. For a catalysed crust we use the EOSs of Haensel, Zdunik & Dobaczewski (1989) for the outer crust and of Negele & Vautherin (1973) for the inner crust. For an accreted crust we use the model of Haensel & Zdunik (2008). The results of Fig. 3 show that, fortunately, the needed $\Delta M_{\text{acc}}$ does not depend strongly on the assumed EOS. Pycnonuclear reactions provide about 60–70 per cent of the total $Q_{\text{acc}}$ and one sees that, for a low mass ($1.2 M_\odot$) extended (14 km radius) neutron star, about $2 \times 10^{-2} M_\odot$ of matter needs to be accreted so that $Q_{\text{acc}}$ reaches its optimal value while about $2 \times 10^{-3} M_\odot$ is needed for the accreted matter to reach the threshold of the first possible reaction. For a heavy ($1.8 M_\odot$) compact (10 km radius) neutron star the corresponding numbers are approximately 1 order of magnitude smaller but such a heavy star will likely undergo fast neutrino cooling and has such a low $L_\nu$ in the $\langle M \rangle$ regime of primordial VFXTs that it will be practically unobservable. We note that if such heavy neutron stars could indeed be present in some primordial VFXTs that those neutron stars have to have been born this massive.

The effect on the heating is not clear. It depends on to what depth the crust has been replaced since pycnonuclear reactions occur down to a density of $\sim 5 \times 10^{13}$ g cm$^{-3}$ but can start already at significantly lower density (see e.g. Horowitz, Dussan & Berry 2008). In particular, most of the heat due to pycnonuclear reactions is released in the density range of $10^{12}$ to $10^{13}$ g cm$^{-3}$ (e.g. Haensel & Zdunik 2008) and significantly less mass has to be accreted to replace the crust to those densities (Fig. 3). However, in the most extreme case in which the total amount of accreted matter is only $10^{-3} M_\odot$, only the outer crust, with densities below the neutron drip, has been fully replaced, significantly inhibiting pycnonuclear reactions to occur in the inner crust. Therefore, there might be a sub-group among the primordial VFXTs for which the heating is significantly reduced which would make them even fainter than already inferred from their low $\langle M \rangle$. However, other than fusion reactions, substantial heating may occur just below the neutron drip via cascades of electron capture and neutron emissions (Gupta, Kawano & Möller 2008). Therefore, detailed calculations are required to fully grasp the effect of a partially accreted crust on the thermal state of transiently accreting neutron stars.

4 DISCUSSION

We have estimated the quiescent thermal luminosity of neutron-star VFXTs in order to determine if they can be used to test the deep-crustal heating model in a hardly explored $\langle M \rangle$ regime. Unfortunately, a conclusive answer cannot be given due to the large uncertainties in our knowledge of the accretion rate history of VFXTs. The $\langle M \rangle$ of the source during the last several thousands to tens of thousand years determines how much heat has been deposited in the neutron star over that period and therefore the thermal state of the star. However, the long-term history over the lifetime of the binary determines the amount of matter accreted and therefore if enough matter has been accreted to trigger enhanced neutrino emission processes in the core and if enough matter is accreted to allow the activation of all pycnonuclear heating reactions in the inner crust.

This last point arises because it is quite possible that the amount of matter which primordial VFXTs have accreted during their lifetime is not enough to fully replace the original crust, leaving a crust which is partly replaced by accreted matter and partly still contains the original, albeit further compressed material. It is unclear how such a hybrid crust would react to the accretion of matter and how this would affect the thermal state of the neutron star. Likely less heat is produced because not all pycnonuclear reactions can occur, but it is not clear if other properties of a hybrid crust are also significantly different compared to a fully accreted crusts, such as the thermal conductivity. Beside obtaining more observational data to constrain the models, detailed theoretical calculations have to be performed to investigate the heating and cooling in neutron stars which have a hybrid crust. In particular, it is important to investigate different crustal compositions with a variety of amount of matter accreted (e.g. an update study of the one performed by Sato 1979). This problem might be interesting not only for VFXTs, but also for other types of neutron stars because VFXTs might not be the only sources which harbour neutron stars with hybrid crusts (see Section 4.1). Furthermore, VFXTs might be important to understand low-luminosity X-ray sources in globular clusters (Section 4.2). In addition, they might form an interesting group of sources to try to
study cooling of the neutron-star crust (Section 4.3) after it has been heated during outbursts.

### 4.1 Additional potential sources without fully accreted crusts

Despite that it is generally accepted that most neutron-star LMXBs are rather old systems with ages of $10^8-9$ yr; there are individual sources which are likely much younger. One relatively young system might be the recently discovered transiently accreting 11 Hz X-ray pulsar IGR J17480-2446 in the globular cluster Terzan 5 (also called Terzan 5 X-2; Strohmayer & Markwardt 2010; Papitto et al. 2011). This system is an unusual LMXB because it was expected that the neutron stars in LMXBs should have spin periods $<10$ ms because they are spun up by the accretion of matter (see review by Bhattacharya & van den Heuvel 1991). The slow spin period of IGR J17480-2446 is enigmatic and it has been hypothesized that this is due to the fact that the system has so far only spend a relatively brief time in the Roche lobe overflow phase ($10^7$ to $10^8$ yr; Patruno et al. 2012). If true, this system might be an example of systems which do not have a fully replaced neutron-star crust.

The mass accretion rate of this source during outburst has been estimated to be $3 \times 10^{-9} \frac{M_\odot}{yr}$ (Degenaar & Wijnands 2011a). The DC of this system is poorly constrained but if we assume again values of 1–10 per cent we obtain a time-averaged accretion rate of $3 \times 10^{-11}$ to $3 \times 10^{-10} \frac{M_\odot}{yr}$. Combined with the expected age of the accretion phase this results in a mass accreted on the neutron star of $3 \times 10^{-4}$ to $3 \times 10^{-3} \frac{M_\odot}{yr}$. Although the maximum amount of matter accreted would indicate that the full crust is replaced, it is also quite possible that the neutron star in this system has a hybrid crust as well. Degenaar & Wijnands (2011a) found the quiescent counterpart for this source to be rather cold, significantly colder than expected using standard heating and cooling theory. They suggested that in its neutron-star-enhanced core cooling processes might be active although, as also shown above, probably not enough matter has accreted on the star for the star to have become massive enough to allow such processes to occur in the core. Alternatively, they suggested that the DC might be extremely low of the order of 0.1 per cent.

Although not impossible, this DC seems very low (and possibly improbable in the disc instability model) and therefore we suggest an another possible reason why the source is so faint in quiescence: due to the presence of a hybrid crust, not all the heating reactions can occur in the crust and therefore less heat has been deposited in the neutron star to heat it up to the expected temperature as inferred from its $M(M)$. This conclusion still holds when also taking into account that before the Roche-lob overflow phase a wind-accretion phase occurred. Patruno et al. (2012) estimated that the mass accretion rate in that phase would at most be $10^{-13}$ to $10^{-12} \frac{M_\odot}{yr}$. This phase could have lasted $10^7$–$8$ year and thus at most $10^{-6}$ to $10^{-4} \frac{M_\odot}{yr}$ could have been accreted.

If this is the correct explanation for why the neutron star in IGR J17480-2446 is colder than expected, one has to wonder if a similar argument might also hold for other systems which have been founds to be too cold. For example, the neutron star in SAX J1808.4–3658 seems to be extremely cold (Campana et al. 2002a; Heinke et al. 2007, 2009a). Its $(M)$ has been estimated to be $\sim 10^{-11} \frac{M_\odot}{yr}$ (Heinke et al. 2007) and if it has lived shorter than $10^8$ yr the neutron star should have a hybrid crust. However, this system is an accreting millisecond pulsar with a spin period of 401 Hz (Wijnands & van der Klis 1998). This means that a significant amount of matter has to have been accreted by the neutron star to spin it up to this spin frequency. Typically, the calculations show that up to 0.1 $M_\odot$ (van den Heuvel 1987) is needed to accomplish this (see review by Bhattacharya & van den Heuvel 1991). Therefore, in SAX J1808.4–3658 the neutron-star crust will have been fully replaced, which strongly indicates that in the past the accretion rate of this system was considerably larger than its current inferred $(M)$. Another source which might be relatively young is Circinus X-1. The age of this system is not known, but it has been suggested to be rather young (of the order of $<10^{5}$ yr; see the discussion in Clarkson, Charles & Onyett 2004). Despite that it can accrete on occasions at very high accretion rates (up to $>10^{-8} \frac{M_\odot}{yr}$); this age (if confirmed) is sufficiently low that very likely not the complete crust has been replaced. If the source would go fully quiescence, it would be very interesting to determine the quiescent luminosity of the neutron star in this system.

#### 4.1.1 Neutron stars in high-mass X-ray binaries

In high-mass X-ray binaries (HMXBs) the neutron star accretes either from the strong stellar wind of the companion (e.g. a supergiant star) or from the decretion disc of a B-type star, which is typically observed to be of type B0-B2 in Be/X-ray transients (see review by Reig 2011). Such early-type B stars only live between 10 and 30 million years. Typically in Be/X-ray transients the sources have outbursts with X-ray luminosities of $10^{36-37} \text{erg s}^{-1}$ (corresponding to an outburst accretion rate of $10^{-10}$ to $10^{-9} \frac{M_\odot}{yr}$) when the neutron star moves through the decretion disc at periastron passage. For some sources this occurs once every orbital period (resulting in periodic outbursts; called type-I outbursts) but other sources are only occasionally in outburst. Therefore, it is unclear what fraction of the time the neutron star is actually accreting, but when assuming again a DC of 1–10 per cent, this would result in a $(M)$ of $10^{-12}$ to $10^{-10} \frac{M_\odot}{yr}$ and a total amount of mass accretion throughout the lifetime of the system (assuming the system was a Be/X-ray transient for the whole life of the B star which might be a significant overestimation of the duration of this phase) of $10^{-5}$ to $3 \times 10^{-5} \frac{M_\odot}{yr}$. Thus, it is quite possible that also the neutron stars in some Be/X-ray transients have a hybrid crust. We note that some systems also exhibited the so-called type-II outbursts which are much brighter (peak luminosities of $10^{38} \text{erg s}^{-1}$) which can last for weeks to months but they are very infrequent and not all systems exhibit them. Therefore, we do not expect that those type of outbursts will affect our main conclusion significantly.

Another class of HMXBs transients are the supergiant fast X-ray transients (SFXTs; see e.g. Sidoli 2011) in which the neutron star transiently accretes from the variable dense wind of a supergiant star. However, also very likely in those systems the neutron star has only a partly replaced crust because the supergiants only live very short and despite that the outbursts of those systems can be very bright $(10^{38} \text{erg s}^{-1})$, they are very brief, very infrequent, and most of the time the neutron star is only accreting at much lower rates.
or not at all. Although if before the supergiant phase the neutron star was also already accreting significantly from the companion star (e.g. during an earlier Be phase; Liu, Chaty & Yan 2011), then more of the original crust is replaced.

For the neutron stars in HMXB transients ($\dot{M}$) is typically higher than inferred for the VFXTs (they are typically more in the range observed for the ordinary LMXB transients.). Therefore, it is expected that if standard heating and cooling occur in those systems (as suggested by Brown et al. 1998), their thermal emission should be readily detectable in quiescent. Enhanced core cooling is not expected because they should be relatively light neutron stars since little matter has been accreted (although it might be possible that some systems are born with massive neutron stars; see e.g. Barziv et al. 2001). In contrast, the heating might be affected by what type of crust is present (i.e. fully accreted or hybrid crust) and HXMB transients might be very good candidates to investigate the effect of hybrid crust on the thermal properties of the neutron star. However, the situation for those sources might be complicated by the much stronger magnetic field in those systems ($10^{12} - 13$ G) compared to those of the neutron stars in LMXBs ($10^8 - 9$ G). It is unclear how strong the effects of these stronger magnetic fields are on the heating and cooling of the neutron stars and other related properties (e.g. the thermal conductivity which is severally affects by super strong magnetic fields of $> 10^{13}$ G and therefore likely also by slightly lower fields; Potekhin et al. 1999; Aguilera, Pons & Miralles 2008). More detailed theoretical calculations have to be performed to determine the effect of the magnetic field, in combination with the exact composition and structure of the (possible hybrid) crust.

Observing HMXB transients in their quiescent state could be very useful in this aspect. However, the number of neutron-star Be/X-ray transients so far studied in quiescence is rather limited (for a source list, see Rutledge et al. 2007; Tomskick et al. 2011). So far, the obtained picture is complex. Some systems (like e.g. EXO 2030+375) always remain rather bright in-between outbursts ($> 10^{35}$ erg s$^{-1}$; basically they never transit to quiescence). However, the majority of systems have quiescent luminosities between $10^{32}$ and $10^{34}$ erg s$^{-1}$ (Rutledge et al. 2007; Tomskick et al. 2011). Spectral analysis demonstrates that some systems are still very hard in quiescence with power-law indices near 1 or even lower (similar to what often is seen in outburst; Rutledge et al. 2007), while others are softer with indices even up to 2.6 (e.g. Campa et al. 2002b).

Although the quiescent data are usually not of very high quality, several sources do not show pulsations in quiescence which might indicate that indeed the accretion down to the surface has halted in those systems (Campa et al. 2002b; Wilson et al. 2005). However, in a few other systems pulsations could still be detected in quiescence demonstrating that in those systems either some of the matter still reaches the neutron-star surfaces or the pulsations are in some way caused by the interaction between the magnetic field (which is rotating with the neutron star) and the accretion of matter down to the magnetosphere (Mukherjee & Paul 2005; Rutledge et al. 2007).

For some systems it has been suggested (Campa et al. 2002b) that indeed the emission we observe is due to the cooling of the neutron star and not due to some sort of accretion process; however, the evidence is not conclusive due to the statistical quality of the data. Furthermore, the possibility that they might harbour a neutron star with a hybrid crust was not considered. Tomskick et al. (2011) discussed possible reasons why the candidate Be/X-ray transient IGR J01363+6610 could not be detected with Chandra in its quiescent state (e.g. the system containing a black hole instead of a neutron star), but in light of the above discussion we suggest that the possibility should be considered that this source might still harbour a neutron star but one with a hybrid crust which inhibits significant heating of the neutron star.

The situation for SFXTs is similar to that of the Be/X-ray transients with only a handful of SFXTs studied in quiescence. Also those systems show a variety in quiescent behaviour (see e.g. in’t Zand 2005; Bozzo et al. 2010, 2012). A systematic and homogenous study of many more HMXB transients (both SFXTs and Be/X-ray transients) in quiescence is needed to understand fully how they can be used to study the deep-crustal heating model. A survey (using Chandra) of 16 confirmed neutron-star Be/X-ray transients in their quiescent state has recently been accepted (PI: Wijnands) which will give more insight into this issue.

### 4.2 VFXTs in globular clusters

Many faint X-ray sources have been found in the Galactic globular clusters, and a large number are likely associated with neutron-star X-ray transients (see e.g. Verbunt, Elson & van Paradijs 1984). But the lack of a significant number of outbursts from those sources has led to suggestions that maybe those sources are associated with VFXTs whose outbursts were missed by the all-sky monitors (Wijnands 2008). As estimated in Section 3.2, the quiescent X-ray luminosity of VFXTs in the standard deep-crustal heating model would be in the range $10^{31} - 33$ erg s$^{-1}$ and, indeed, if the standard heating and cooling processes occur, a large fraction of the candidate quiescent LMXBs could be associated with VFXTs. However, as also explained in sections 3.2 and 3.3 the quiescent luminosity of VFXTs could be significantly lower than expected in the standard model and therefore it is unclear if this conclusion still holds. Moreover, even in the standard model it is unlikely that the VFXTs are associated with the candidate quiescent LMXBs in globular clusters.

To demonstrate this, we rewrite the time-averaged accretion rate into

$$\langle M \rangle = \frac{(M_o)\tau_o + (M_q)\tau_q}{\tau_o + \tau_q} \approx \frac{(M_o)}{\tau_o + \tau_q} = \langle M_o \rangle DC \quad (13)$$

with $\langle M_q \rangle$ the time-averaged accretion rate in outburst and $\langle M_q \rangle$ the time-averaged accretion rate in quiescence. In Equation (13) we assume $\langle M_o \rangle t_o > (M_q) t_q$, which is usually true but might not be if $t_q > t_o$, thus for systems with a extremely low DC. Using equations (2) and (13), and assuming $L_q = \langle H \rangle$ (thus standard, slow cooling) one obtains

$$DC = \frac{L_q}{10^{33} \text{erg s}^{-1}} \frac{10^{-11} \text{M}_\odot \text{yr}^{-1}}{(M_o)} \frac{1.5 \text{MeV}}{Q_{\text{nuc}}}$$

$$> \frac{L_q}{3 \times 10^{34} \text{erg s}^{-1}} \frac{1.5 \text{MeV}}{Q_{\text{nuc}}} \quad (15)$$

which assumes $\langle M_q \rangle < 3 \times 10^{-10}$ $\text{M}_\odot \text{yr}^{-1}$ (the limit set by the all-sky monitors).

Typically quiescent LMXB candidates in globular clusters have bolometric X-ray luminosities of $10^{32} - 33$ erg s$^{-1}$ with the brightest being $\sim 3 \times 10^{33}$ erg s$^{-1}$ (Heinke et al. 2003, about a third of the sources listed in that paper have an X-ray luminosity $> 1 \times 10^{33}$ erg s$^{-1}$). This results (using equation 15) in limits on the DC of $> 0.0025-0.005$ for $L_q = 1 \times 10^{32}$ erg s$^{-1}$, DC $> 0.025-0.05$ for $L_q = 1 \times 10^{33}$ erg s$^{-1}$ and DC $> 0.1-0.2$ for $L_q = 3.6 \times 10^{33}$ erg s$^{-1}$. The range in DC is due to the fact that we have assumed

$$\sim 0.0025-0.005$$

$$0.025-0.05$$

$$0.1-0.2$$
that $Q_{\text{acc}}$ must be between 1 and 2. Putting reliable observational constraints on the DC of possible VFXTs in globular clusters is difficult. However, currently there are about 30 quiescent LMXB candidates identified for which no outbursts\(^5\) have been seen yet [see Heinke et al. (2003) for a list; several additional sources have been found since that publication; e.g. Lugger et al. (2007); Guillot et al. (2009, 2011); Maxwell et al. (2012)]. During the $\text{Chandra}$ and $\text{XMM–Newton}$ observations which detected those sources, they were not in outburst. Therefore, assuming that all sources are from the same population of transient type, the DC of those systems is $<1/30 = 0.033$ (see also Heinke 2010). We note that a number of clusters have multiple $\text{Chandra}$ and $\text{XMM–Newton}$ observations during which those sources were not seen in outbursts. However, those additional observations do not constrain the DC further because it is quite possible that the quiescent duration is significantly longer than the sampling time-scale meaning that the observations probe the same quiescent period and therefore are not statistically independent (basically many outburst-quiescent cycles must have passed for the sampling, if performed with random time-intervals between the observations, to become independent). We note that the uncertainties on the DC inferred from observations is likely to be large, but the DC has to be $>10$ per cent to explain the brightest sources (in the standard deep-crustal heating model) which seems unlikely. Furthermore, similar low DCs were inferred for VFXTs near the Galactic centre (Degenaar & Wijnands 2010), so it is quite possible that they indeed have such low DCs.

The conclusion from the above exercise is that VFXTs can still be associated with some of the quiescent LMXB candidates, but only with the faintest sub-set of the group and likely not with the brightest objects ($>10^{35}$ erg s\(^{-1}\); especially not those of a few times $10^{33}$ erg s\(^{-1}\); note that this assumed standard heating and cooling). This is supported by the fact that the three VFXTs currently known in globular clusters (M15 X-3, NGC 6440 X-2 and NGC 6388; Heinke, Cohn & Lugger 2009b; Heinke et al. 2010; Bozzo et al. 2011) all have quiescent luminosities (well) below $10^{32}$ erg s\(^{-1}\) (only M15 X-3 has been detected; Heinke et al. 2009b). It seems that such faint transients are indeed present in globular clusters, but that their quiescent luminosities are really low. It is worth noting that those are the only systems for which useful constraints have been set on the quiescent properties of neutron-star VFXTs. The Galactic disc sources are usually too much absorbed (see also Section 4.3) for any useful constraints indicating that globular clusters are prime targets to study quiescent VFXTs. Sensitive monitoring programs (see Altamirano et al., in preparation; Wijnands et al. 2012) are needed to detect the very-faint outbursts of VFXTs in globular clusters and combined with rapid follow-up observations using $\text{Chandra}$ (Altamirano et al., in preparation) to determine the exact position; the quiescent counterpart can then be studied in archival $\text{Chandra}$ data or in newly proposed $\text{Chandra}$ observations.

It might be possible that we have underestimated the amount of energy released per accreted nucleon (see e.g. Steiner 2012) and therefore the systems should be more luminous per accreted nucleon than assumed (e.g. evidence has been reported that in some systems extra heat in shallower layers in the crust must be liberated to explain the quiescent properties of those systems; Brown & Cumming 2009; Degenaar, Brown & Wijnands 2011b). However, this should also be true for the other quiescent LMXBs, which do not require more energy per accreted nucleon to explain the base quiescence level and most are actually too cold to be explained using the standard model.

Therefore, we conclude that there are indeed VFXTs in globular clusters, but that they can likely not be associated with the brightest amongst the quiescent LMXB candidates ($>10^{35}$ erg s\(^{-1}\)) and likely also not with the slightly fainter ones ($10^{32}–33$ erg s\(^{-1}\)). It is more likely that those quiescent LMXB candidates are associated with bright transients similar to the transients IGR J17480–2446 and Swift J174805.3–244637 in Terzan 5 which were as well identified previously as quiescent LMXB candidates (Heinke et al. 2006) before they exhibited their bright outbursts (Degenaar & Wijnands 2011a; Wijnands et al. 2012). However, also those transients should have low DCs in order to be consistent with the observations of the lack of outbursts (either very-faint or bright). More and more evidence becomes available that there exist a class of transients which might have indeed very low DCs. This might not be unexpected because there is a strong selection effect in favour of discovering new transients with a high DC.

4.3 Final remarks

The above discussion has demonstrated that VFXTs could be extremely faint in quiescence but some of them could reach quiescent X-ray luminosities of $10^{32}$ to even $10^{33}$ erg s\(^{-1}\) depending on their accretion rate history and the amount of matter accreted. So, it would be interesting to observe quiescent VFXTs and determine their quiescent properties. Sadly, two main factors hamper the study of the thermal cooling emission of quiescent neutron-star VFXTs. First of all, many quiescent LMXBs exhibit, besides the thermal component, a hard, non-thermal component above 2 keV. The origin is not well understood but its presence inhibits the most accurate study of the thermal component and in some systems only this hard component can be detected. In particular, there seems to be a trend which indicates that the fainter a source is in quiescence, the larger the contribution is of this non-thermal component to the 0.5–10 keV X-ray luminosity (Jonker et al. 2004). If this trend is also valid for the quiescent state of VFXT, then this will make it difficult to maybe even impossible to put significant constraints on the thermal component in those systems.

In addition, VFXTs are difficult to discovery because their peak X-ray luminosity is below the sensitivity limits of all-sky X-ray instruments in orbit and, therefore, most outbursts are missed. Often, only using pointed observations of more sensitive instruments, those outbursts can be detected. But usually, those pointed observations have a very narrow field-of-view and mostly are pointed towards the Galactic centre and the Galactic bulge. Consequently, the interstellar absorption is usually rather high for those systems with values between $10^{22}$ cm\(^{-2}\) and $10^{23}$ cm\(^{-2}\). With such high column densities and the expected low surface temperature of the neutron stars in VFXTs, it is very difficult or impossible to detect the thermal component. Even more so if also the non-thermal component discussed above is present in the quiescent spectrum as well.

Despite that, the situation described above looks very bleak; it might still be possible to use quiescent VFXTs as tests for the deep-crustal heating model. As described in Section 4.2, many X-ray transients are expected to be present in Galactic globular clusters among which are many VFXTs (three sources are already currently known). Typically for many clusters the column density is not very

\(^5\)This excludes the two sources (IGR J17480–2446, Swift J174805.3–244637) in Terzan 5 which were previously identified as candidate quiescent LMXBs (Heinke et al. 2006) but which have now been shown to be associated with bright transients. However, the final conclusions do not depend very sensitively on how many quiescent LMXBs are currently known but have not been associated yet with outbursts.
high and the distances are typically quite accurately known. Continuous monitoring of those clusters with sensitive X-ray instruments would be crucial to catch VFXTs in outburst so that later they can be studied in quiescence (Heinke et al. 2009b, 2010).

During an outburst, one specific observable property can be very useful to determine whether a VFXT is a primordial system or that it has accreted at a significantly higher rate in the past: detecting millisecond X-ray pulsations. As discussed in Section 4.1 in the context of the accreting millisecond X-ray pulsar SAX J1808.4–3658, detecting such millisecond X-ray pulsations basically guarantees that the neutron star has accreted enough matter to replace the crust. Therefore, those systems cannot be primordial (hence we expect spin periods >10 ms for primordial VFXTs) and the neutron star systems in those systems cannot have a hybrid crust (unless in some way the matter spins up the neutron star but is not eventually accreted on the neutron star itself). The fact that the VFXT NGC 6440 X-2 is a millisecond X-ray pulsar with a spin frequency of 4.8 ms (Altamirano et al. 2010) indicates that this source is not a primordial VFXT and, that is, faint quiescent emission (Heinke et al. 2010) cannot be due to non-standard heating in a hybrid neutron-star crust because the crust in this system should be fully replaced and a different explanation (e.g. enhanced core cooling) is needed. Detailed calculations have to be performed whether other observable properties during outbursts can discriminate systems with a fully accreted crust with those which have a hybrid crust (e.g. type-I X-ray bursts behaviour; super-burst behaviour).

Another possibility is to study those VFXTs which are active for a very long time (years to decades, instead of weeks to months), the so-called quasi-persistent sources, when their outbursts turn off. Several such quasi-persistent VFXTs have been identified (see e.g. Del Santo et al. 2007; Degenaar & Wijnands 2010, albeit that those transients are close Sgr A* and therefore have a large column density making them unsuitable for studying the soft thermal component). It has been found that for normal transients which accrete so long that the neutron-star crust is heated considerably out of thermal equilibrium with the core and for several months to years after the end of their outburst, the observed quiescent temperature tracks the thermal evolution of the crust instead of that of the core (until thermal equilibrium is reached again; see Wijnands et al. 2001, 2002, Rutledge et al. 2002; Cackett et al. 2006, 2008, 2010; Fridriksson et al. 2010, 2011; Degenaar & Wijnands 2011b, 2009; Degenaar et al. 2009, 2011a,b; Díaz Trigo et al. 2011). If a quasi-persistent VFXT can be studied right after the end of its prolonged outburst, the observed crust cooling curve could give significant insight into the heating and thermal processes which occurred in the crust during the outburst. Very likely, a fully accreted crust will exhibit a different cooling profile than a hybrid crust.6 Although this assumes that still a significant amount of heat is produced in the crust during outburst to elevate the thermal emission above the sensitivity limits of the current generation of X-ray instruments.

Obtaining crust-cooling curves for neutron stars in HMXBs (i.e. after the bright and extended type-II outbursts observed from several neutron-star Be/X-ray transients) will also be very interesting because it is expected that those curves could deviate significantly from those obtained so far for the neutron-star LMXBs. Both the effects of hybrid crusts in the neutron stars in HXMBs as well as their high magnetic field strength might produce observational effects on the crust cooling curves.

Primordial VFXTs will have different companions than systems which are only in a VFXT phase but normally they are normal transients or they used to be normal transients in the past (King & Wijnands 2006). So, finding the optical or IR companion star of VFXTs would be very useful to pin-point the most likely accretion rate history of those sources. Again, globular clusters might be the best places to study this because of the low absorption for many of them compared to field targets. However, the companion star for both the primordial and the non-primordial VFXTs can be very faint (Heinke et al. 2009b, 2010) and detecting them will remain a challenge, let alone obtaining spectral observations to confirm the type of companion star. The European Extreme Large Telescope using adaptive optics might be able to detected more systems in the crowded globular clusters and it might be able to take spectra of the brightest targets.

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REFERENCES


6 We note that for the bright transient IGR J17480–2446 in Terzan 5 (which might harbour a neutron star with a hybrid crust; see Section 4.1) the crust cooling curve has been measured (Degenaar et al., in preparation; Degenaar & Wijnands 2011b; Degenaar et al. 2011b) and it did not meet the theoretical expectations. This could be due to a neutron star with a hybrid crust [see Degenaar et al. (in preparation) for an in-depth discussion].