Interface transparency of superconductor/ferromagnetic multilayers
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I. INTRODUCTION

In combining a superconductor (S) with a ferromagnet (F) rather than with a normal metal, various effects have been predicted to occur. One is the modification of Andreev reflections at the S/F interface, which would introduce spin selectivity in the conductance of an SF junction, with strong implications for devices at mesoscopic length scales.2 Another is the possibility of a phase difference $\Delta \phi = \pi$ over an S/F/S junction,3 resulting in an oscillatory behavior of the superconducting transition temperature $T_c$, with F-layer thickness $d_F$ of S/F multilayers.4–6 An oscillation in $T_c$ was recently reported for Nb/Gd,7 but its origin is still controversial.8 All such effects concern the behavior of the superconducting order parameter near the S/F interface, and in that sense they form part of the general issue of the proximity effect, well known for the S/N case, but hardly investigated for the S/F case. Apart from the spin dependence, the main parameter which discerns an F metal from an N metal is the coherence length in the normal metal $\xi_N$. In the S/F case, the exchange energy is much larger than the superconducting transition temperature, which makes $\xi_F$ virtually temperature independent. We will come back to this below.

Going one step further, it may be asked how $\xi_F$ can be varied. Control is clearly by the exchange splitting $\Delta E_{ex}$, defined as the effective energy difference for electrons at the Fermi level with spins parallel and antiparallel to the magnetization. It is connected to the magnetic moment $\mu_F$ of the host ion by

$$\Delta E_{ex} = I_{eff} \mu_F,$$

with $I_{eff}$ an effective exchange integral. Thus, it is to be expected that $\xi_F$ can be increased by lowering $\mu_F$. Surprisingly, these simple concepts have never yet been investigated. It is the purpose of this paper to report such systematic research, and to show that the above-sketched picture misses one essential ingredient, namely the transparency of the S/F interface for Cooper pairs. We present measurements on S/F...
TABLE I. Experimental values of the Fe moment $\mu_F$, the magnetically dead layer $d_{nd}$, the decoupling thickness $d_{dc}^F$, the critical thickness $d_{cr}^S$, and the specific resistivity $\rho$ at 6 K for alloys V$_{1-x}$Fe$_x$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$\mu_F$ (µB)</th>
<th>$d_{nd}$ (nm)</th>
<th>$d_{dc}^F$ (nm)</th>
<th>$d_{cr}^S$ (nm)</th>
<th>$\rho$ (µΩ cm)</th>
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<td>10.6</td>
</tr>
</tbody>
</table>

multilayers, where the F metal is a ferromagnetic alloy with a moment which can be varied over almost an order of magnitude by changing the alloy composition. We determine $d_{dc}^F$ and $d_{cr}^S$ for different $\mu_F$, and find a surprising nonmonotonic behavior for the latter. From analysis of the data using proximity effect theory, it is found that by including the interface transmission coefficient (or transparency) $T$ as a parameter, we are able to account for the experimental results.

We find that $T$ strongly decreases with increasing $\mu_F$. This may well be due to the spin splitting in the ferromagnet, which leads to partial reflection of Cooper pairs at the S/F interface as discussed for the conduction in Ref. 1.

II. EXPERIMENTAL

The multilayers were grown by dc magnetron sputtering as described previously. They consist of V layers [$T_c = 5.1$ K, Ginzburg-Landau coherence length $\xi_{GL}(0) = 13.9$ nm] and V$_{1-x}$Fe$_x$ alloy layers. The case $x = 1$ (V/Fe) was already studied. In bulk V-Fe alloys, the average moment per Fe atom $\mu_F$ changes from 2.2$\mu_B$ for pure Fe to 0 for $x = 0.3$. The main reason for choosing an alloy is to have different magnetic moments with the least changes of disorder at the interfaces. The V/Fe interface is well behaved, with a lattice mismatch of only 5% and with disorder confined to the two atomic planes on each side. The alloys have even smaller lattice mismatches with V, so that the favorable situation with respect to compositional disorder will remain.

Samples were grown with alloy compositions $x = 1$, 0.88, 0.77, 0.53, 0.38, and 0.34. Three different sets of multilayers were prepared. One set was used to determine $\mu_F$, built as follows: $d_{v}^{\text{out}} / N \times (d_{v}^{\text{out}} / d_F)$ for the outer V layers $d_{v}^{\text{out}}$ are for protection, typically 10–40 nm. The inner V layer $d_{v}^{\text{in}}$ is typically 3 nm; it is not superconducting but meant to increase the number of interfaces, in order to obtain a realistic picture of the F layer magnetism. The F layer $d_F$ is varied in thickness, typically between 0.5 and 5 nm, while the number of repetitions $N$ is adapted to the strength of the moment. For Fe, $N = 3$ suffices, while $N = 20$ for V$_{0.6}$Fe$_{0.4}$. The magnetization $M$ was measured with a magnetometer based on a superconducting quantum interference device at 5 or at 10 K.

In all cases, $M$ versus $d_F$ could be described with a straight line, yielding the effective magnetic moment per Fe atom $\mu_F$ and the magnetically dead layer per interface $d_{nd}$ (see Ref. 9, 10). They are given in Table I, while a comparison of $\mu_F(x)$ with values found in bulk alloys (from Ref. 11) is given in Fig. 1. Films and alloys show some differences; near $x = 1$, the values in the films are slightly lower than in the bulk while below $x = 0.75$ the films show higher values. We assume that this is due to the different morphologies of film and bulk material. Furthermore, $d_{nd}$ is relatively low in all cases. Values stay below about 0.3 nm or roughly one atomic layer, in clear contrast to the findings in the case of Co and Ni.

The second sample set was used to determine $d_{cr}^S$ by the variation of $T_c$ as function of $d_F$. This is done with samples built with five layers (although three would suffice): $d_F / d_{cr}^S / d_F / d_{cr}^S / d_F$. This curve is shown for two different values of $d_{cr}^S$, namely 40 and 55 nm. Both curves show a steep descent above 1 nm, and level off to values near $T_c$. Incidentally, neither curve shows an oscillation in $T_c$, as might be found if $\pi$ coupling were

FIG. 1. The effective magnetic moment per Fe atom $\mu_F$ versus concentration $x$ for alloys V$_{1-x}$Fe$_x$; ○ indicates bulk alloys, data taken from Ref. 11; ● indicates films, this work. The drawn line is meant to guide the eye.
interface transparency of

define
clearly shift to higher
dF
they should. Furthermore, the steepest descent of the curves

dresults. Values for
as using the 50% point, turned out to give very similar re-


tensity against

the curves for both thicknesses

played in

for all alloy concentrations.

v~

behaves as described by Eqs.

intercalated to

present. We will briefly come back to the issue of π coupling
at the end of the Discussion.

In Fig. 3, the same transitions have been plotted, but
scaled to $T_c^{\text{high}}, T_c^{\text{low}}$, and for all concentrations. For $x=0.34$
the curves for both thicknesses $d_F$ essentially coincide, as
they should. Furthermore, the steepest descent of the curves
clearly shift to higher $d_F$ upon decreasing $x$ or $\mu_F$. We now
define $d_{cr}$ by extrapolating the steepest slope in the transition
curve to the $d_F$ axis (see Fig. 3). Different definitions, such
as using the 50% point, turned out to give very similar results.
Values for $d_{cr}$ are given in Table I. We plot this quantity
against $\mu_F^{-1}$ in Fig. 4 and find a reasonably linear relation.
Making the identification $d_{cr}^2/2 = \xi_F$, it follows that $\xi_F$
behaves as described by Eqs. (1,2). Given the small thick-
nesses involved, such clean-limit behavior could be expected,
but the linear behavior also implies that the quantity
$\nu_F/\xi_{cr}$ basically remains constant with varying $x$.

Next we turn to the behavior of $d_{cr}$. For all alloys, the

$T_c(d_F)$ curves are similar to the one presented in Fig. 2(a).
The scatter in the individual points is small enough to find
values for $d_{cr}$ with good accuracy. All values for $d_{cr}$
are collected in Table I. Especially interesting is the behavior
near $x=1$, which is reproduced in Fig. 5. There, $T_c(d_F)$ is
plotted on a somewhat expanded scale for the three systems
with the highest moments ($x=1, 0.88, 0.77$). The behavior
for $x=1, 0.77$ is very smooth; for $x=0.88$, the scatter in
points is quite large, actually the largest by far of all sets
measured [compare also Fig. 2(a) for $x=0.34$]. Even then,
the plot unequivocally shows that the curves shift to higher
thickness with decreasing $x$. This behavior is quite unex-
pected, and comprises the main issue of our research, to be
discussed below. Figure 6 shows the full behavior of $d_{cr}^5(\mu_F)$. A clear maximum is found between $x=0.77$
and $x=0.53$, before a slow decrease sets in. The value at $x=0.34$, where the magnetic moment has decreased by a factor
8, is actually equal to the value for $x=1$ (Fe). To make the
point in another way, we plotted in Fig. 6 the results of earlier measurements with Co and Ni as the F metal (open circles), where $d_{cr}$ is found to be much lower at the same values for the magnetic moment. Next to $\mu_F$, another factor
must play a major role in determining the physics. We will
now argue that this factor is the interface transparency.
and Lukichev, following earlier work on the clean-limit Green's functions at the interface as derived by Kuprianov with boundary conditions for the normal and anomalous transport properties of tunnel junctions. The model is based on the Usadel equations, which represents the limit of small temperature and critical fields, but did not incorporate interfacial imperfections, which play an important role. It is also useful to remark that the transparency discussed here is conceptually equal to the barrier strength parameter $g$ in the Usadel model.

As a matter of fact, the single parameter $e$ of that model is, in general, not suited to describe proximity effect and transparency in an independent way, although it turns out to be possible in the limiting case of F/S/F trilayers with $d_F \gg \xi_F$. This point is quite important for the correct description of experimental results and therefore the model and this issue in particular will be discussed in the Appendix. Here, we continue with showing some of the results of the model calculations, which will serve to illustrate the analysis of the experiments. For the underlying data, we need the dependence $T_c(d_S)$, for different values of the proximity effect parameter $\gamma$ and the transparency parameter $\gamma_b$, defined as (see the Appendix):

$$\gamma = \frac{\rho_S \xi_S}{\rho_F \xi_F}, \quad \gamma_b = \frac{R_B}{\rho_F \xi_F},$$

with $\rho_i$ the normal-state resistivity of metal $i$, and $R_B$ the normal-state boundary resistance times its area. The connection between $\gamma_b$ and the transparency $T$ is roughly given by

$$T = \frac{1}{1 + \gamma_b}.$$

Figures 7 and 8 show two types of results from the calculations. In Fig. 7, $T_c(d_S)$ is given for an F/S/F trilayer with $d_F = 10 \xi_F$, $\rho_S = \rho_F$, $\xi_S / \xi_F = 10$ ($\gamma = 10$) and complete transparency ($\gamma_b = 0$).

IV. DATA ANALYSIS BY PROXIMITY EFFECT THEORY

A. Theory; a brief description

Scattering of a normal electron or quasiparticle on a potential barrier at an interface will lower its transmission coefficient $T$. In S/N structures, one source for this is the potential step due to the difference in lower band-edge energies. Defects can also cause potential scattering, and are usually modeled as a $\delta$ function with a certain strength. Theoretically, the consequences of reduced $T$ for different quantities such as the superconducting density of states or the superconducting order parameter are meant to guide the eye.

In the Appendix, starting with McMillan’s tunneling model for bilayers, the critical temperature, has long been a subject for investigation, with a good overview of the early work can be found in Ref. 12. Experimentally, this may not be surprising, since interface imperfections play an important role. It is also useful to remark that the transparency discussed here is conceptually equal to the barrier strength parameter $Z$ in the Blonder-Tinkham-Klapwijk model for transport properties of tunnel junctions.

Recently, model calculations were performed on the proximity effect in S/N multilayers for arbitrary transparency of the interface. The model is based on the Usadel equations, with boundary conditions for the normal and anomalous Green’s functions at the interface as derived by Kuprianov and Lukichev, following earlier work on the clean-limit case by Zaitsev. The model can be easily adapted to the S/F case by noting that the coherence length in the F metal is determined by $\Delta E_{cr}$ and therefore independent of temperature. In an earlier analysis of results on V/Fe multilayers, a similar model was used (due to Radović et al.), which could well describe the behavior of critical temperatures and critical fields, but did not incorporate interface transparency explicitly. As a matter of fact, the single parameter $\epsilon$ of that model is, in general, not suited to describe proximity effect and transparency in an independent way, although it turns out to be possible in the limiting case of F/S/F trilayers with $d_F \gg \xi_F$. This point is quite important for the correct description of experimental results and therefore the model and this issue in particular will be discussed in the Appendix. Here, we continue with showing some of the results of the model calculations, which will serve to illustrate the analysis of the experiments. For the underlying data, we need the dependence $T_c(d_S)$, for different values of the proximity effect parameter $\gamma$ and the transparency parameter $\gamma_b$, defined as (see the Appendix):

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TABLE II. Values for the coherence lengths $\xi_\rho$, for the specific resistivity $\rho$ at 6 K, for the proximity-effect parameter $\gamma$, and for the transparency parameter $\gamma_b$, for alloys V$_{1-x}$Fe$_x$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$\xi_\rho$ (nm)</th>
<th>$\rho(\mu\Omega/cm)$</th>
<th>$\gamma$</th>
<th>$\gamma_b$</th>
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As has been discussed above, a full description of the $T_c$ variation in a proximity effect system needs five parameters: the S bulk layer critical temperature $T_{c0}$, the thicknesses $d_S$ and $d_F$, the proximity-effect strength $\gamma$ and the transparency parameter $\gamma_b$. Starting with $\gamma$, it can be seen from Eq. (3) that this parameter is fully determined by measurable quantities. We take $\xi_\rho$ from the linear relation between $d_{th}^\rho/\xi_\rho$ and $\mu_F^{-1}$, shown in Fig. 3, rather than from the actual values of $d_{th}^\rho$. The values used are given in Table II. For $\xi_\rho$ we use 8.8 nm, corresponding to $\xi_{Gd}(0)=13.9$ nm. The normal-state resistivities $\rho_{S,F}$ are also known. They were measured on thin films of 50, 100, and 250 nm, down to 6 K for all compositions and for V. The averaged values are given in Table II. Due to the use of alloys, $\rho_F$ actually increases considerably (about 2 $\mu\Omega$ cm/at %) up to $x=0.5$, thereby lowering the resistivity ratio in $\gamma$ from 1.7 to 0.06. Values for $\gamma$ can now be calculated, and they are found (see Table II) to decrease monotonically with decreasing moment. Note that this is due to a decrease in both the factors $\rho_S/\rho_F$ and $\xi_S/\xi_F$, and neither factor therefore can be the cause of the measured increase of $d_{th}^\rho$. With the values for $\gamma$, we calculate theoretical values $d_{th}^{\rho b}$ under the assumption that $\gamma_b=0$. The numbers, plotted as squares in Fig. 9(a), do not mimic the experimental results, shown as filled circles, in two respects. They do not go through a maximum, as was already anticipated from the monotonic behavior of $\gamma$, but also, the measured values are much lower than the calculated ones. Especially for Fe, a low value for $\xi_\rho$ and an also relatively low value for $\rho_F$ lead to a very high $\gamma$ and a theoretical critical thickness which is close to the asymptotic limit of about 6 $\xi_\rho$.

The simple fact that $d_{th}^\rho$ is much smaller than expected for the case $\gamma_b=0$, already indicates reduced interface transparency; a value for $T<1$ ($\gamma_b>0$) leads to smaller $d_{th}^\rho$ (see Fig. 8); for $T=0$ the superconductor will behave as an isolated film ($d_{th}^\rho \rightarrow 0$). The next step therefore is to use the model calculations in order to find the value of $\gamma_b$ needed to reproduce the measured values for $d_{th}^\rho$. $T$ is then simply found from Eq. (4). The results, plotted in Fig. 9(b), show a simple relation: $T$ is low for the case of Fe, increases more or less linearly with decreasing $\mu_F$ or $x$, and reaches the order of 1 for low Fe concentration. The observed maximum in $d_{th}^\rho$ is therefore due to the competition of three ingredients: on the side of high Fe concentration, the increasing $\xi_F$ and decreasing $\rho_S/\rho_F$ lead to smaller $d_{th}^\rho$; on the low Fe side, the change in interface transparency has become less important, and the change in $d_{th}^\rho$ is as expected from the change in $\gamma$.

We believe this to be the first demonstration of a barrier transparency which is changed in a continuous (and controlled) fashion, and over a large part of the full range. Of course, the given values for $T$ should not be taken too literally. They depend on the way in which $\xi_F$ is extracted from the $T_{c0}(d_F)$ curves, on the measured values of $\rho_{S,F}$ (which may be somewhat different in multilayers or in single films), and on the approximation used to go from $\gamma_b$ to $T$. Especially a near-zero value for Fe may be too low. On the other hand, a seriously reduced $T$ is needed to explain the low value for $d_{th}^\rho$, while a serious concentration or moment dependence of $T$ is needed to explain the increase in $d_{th}^\rho$. This point leads to the question of the cause of the low value and its change. It is possible that $T$ depends on $x$ as a result of the changing compositional disorder or the changing lattice parameters (strain). It is more probable, however, that $\mu_F$, meaning the ferromagnetism and the spin-dependent band structure, play a role. One mechanism may well be the reduction of Andreev scattering due to the exchange splitting.
since this would translate to a reduced transparency through the use of the boundary conditions for the Usadel equations (see the Appendix). The effect is linear in $\Delta E_{ex}/\epsilon_F$, with $\epsilon_F$ the Fermi energy, and might therefore be appreciable, of the order of 30–50%. Another mechanism can be spin dependence in the normal-state reflection at the interface, such as now investigated in view of giant magnetoresistance effects (see, e.g., Ref. 17). It would take reflections in only one spin channel to lower the transparency for Cooper pairs. Both effects can be present at the same time; from this viewpoint, low transparency looks quite feasible. Interestingly, the few reported values for $d^2\delta/\delta \xi^2$ are much below the upper limit of 6. For Nb/Gd, for instance, the value is 4.2.18 For Nb/Er, the value appears to be between 2 and 3.19 Low transparency may prove to be a general phenomenon in S/F multilayers.

C. On the issue of $\pi$ coupling

In the discussion of the results on the decoupling behavior, we already noted that no oscillatory behavior of $T_c$, and therefore no indication of $\pi$ coupling is found with varying thickness of the magnetic layer for any alloy concentration or magnetic moment. This may not be very surprising. In the original description of a possible mechanism which changes the phase of a Cooper pair by $\pi$, the transfer of the pair through a barrier containing localized moments is accompanied by two virtual spin flips of that moment. Given the strong and itinerant nature of the magnetism in the 3$d$ transition metals under consideration, the spin flips would take the form of spin-wave excitations. This process will have a small probability in view of the large energy denominator involved. In principle, a system with strongly localized (e.g., 4$f$) moments, might offer a better chance for finding $\pi$ coupling. Still, we do not believe that the oscillationlike changes in $T_c$ which were found recently in Nb/Gd (Ref. 7) are actually due to this mechanism. Rather, transparency may again play an important role, as can also be inferred from a report on oscillatory $T_c$’s in Nb/Fe by Mühge et al.,8 who investigated (essentially) trilayer samples with a single superconducting layer. The key observation in both Nb/Gd and Nb/Fe is that $T_c$ increases at the onset of ferromagnetism in the thin F layer. In the spirit of the model used above, we would describe this in the following way: at thicknesses below the onset, strong paramagnetic fluctuations will still act as pair breakers of a strength comparable to the one in the ferromagnet and $T_c$ goes down with increasing $d_F$. At the onset, a static exchange splitting sets in, decreasing the interface transparency and increasing $T_c$. Since this jump will be superimposed on a falling $T_c(d_F)$ curve, it is entirely feasible that $T_c$ decreases again with further increase of $d_F$. Also, the fact that these very thin films have not yet reached their bulk Curie temperature will still change $\xi_F$ and $\Delta E_{ex}$ beyond the transition to ferromagnetism. It is interesting to speculate that in the results on Nb/Gd reported by Strunk et al.,18 the onset of ferromagnetism occurs where the plateau in $T_c(d_{Gd})$ begins, rather than at the downward jump. The reason that no clear plateau-like effects are seen in the measurements presented here is then that for the Fe-rich alloys magnetism already sets in at very small $d_F$ where the resolution is poor, whereas on the dilute side the transparency change has become small, with a correspondingly small effect on $T_c$.

V. SUMMARY

In summary, we investigated decoupling in S/F/S structures upon varying the magnetic moment of the F layer atoms. Indications of $\pi$ coupling in the form of $T_c$ oscillations are not observed. Identifying $d^2\delta/\delta \xi^2$ with $\xi_F$ we find a simple and reasonable dependence $\xi_F \approx \mu_F^{-1}$. We also measured the critical thickness $d^2_\xi$ in F/S/F structures. Here we find a surprising and nonmonotonous behavior as function of $\mu_F$. By analyzing the data in terms of a proximity effect theory, we conclude that this behavior is due to the competing effects of increasing attenuation depth ($\xi_F$) of the order parameter in the F material, and of also increasing transparency of the S/F interface for the penetration of Cooper pairs. More insight in this effect should come from a better understanding of the spin dependence of the different scattering mechanisms at the interface.

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APPENDIX

We consider a multilayered structure consisting of alternating F and S layers of thickness $d_F$ and $d_S$, respectively, and with a finite transparency of the FS boundary. The S layer has a bulk critical temperature $T_{c0}$. We assume dirty-limit conditions for both F and S metals: $l_{F,S} \ll \xi_{F,S}$, where $l_{F,S}$ and $\xi_{F,S}$ are the mean free paths and coherence lengths in the F(S) layers. Due to the translational symmetry of the problem it is sufficient to consider an elementary unit cell with period $\Lambda=(d_F+d_S)/2$. With these assumptions the proximity effect in the system can be described within the framework of the Usadel equations for the S and F layers. Near $T_c$ these equations can be linearized and written in the form5,6

$$\xi_S^2 \frac{\pi T_c}{|\omega|} \frac{d^2}{dx^2} \Phi_S^\pm - \Phi_S^\mp = 2 \Delta \delta^\pm, \quad 0 < x < d_S, \quad (A1)$$

$$\xi_F^2 \frac{d^2}{dx^2} \Phi_F^\pm + i \Phi_F^\pm = 0, \quad -d_F < x < 0, \quad (A2)$$

$$\Delta \ln \frac{T_c}{T_{c0}} + \pi T_c \sum_{\omega > 0} \frac{(2 \Delta - \Phi_S^\pm/\omega)}{\omega} = 0. \quad (A3)$$

Here $\Phi_{F,S}^\pm = \Phi_{F,S}(\omega) \pm \Phi_{F,S}(-\omega)$ are the anomalous Green’s functions integrated over energy and averaged over the Fermi surface, $\Delta$ is the order parameter in the S layer, $\delta^+ = 1$, $\delta^- = 0$, and $\omega = \pi T(2n+1)$ for $n=0,1,2,\ldots$ are the Matsubara frequencies. Note that the functions
\( \Phi_{F,S}(\omega) \) are not symmetric with respect to sign reversal of the energy \( \omega \), i.e., \( \Phi_{F,S}(\omega) \neq \Phi_{F,S}(-\omega) \). This symmetry is restored in the more conventional case of proximity effect in an NS sandwich: \( \Phi_{N,S}(\omega) = \Phi_{N,S}(-\omega) \), which results in \( \Phi_{N,S}^+ = 2\Phi_{N,S} \) and \( \Phi_{N,S}^- = 0 \). Another important difference between the NS and FS cases is that \( \xi_N \) is \( \omega \) dependent, whereas \( \xi_F \) is constant. Some specific phenomena which result from these peculiarities of FS systems were pointed out previously in Refs. 4–6. Here we are interested in the effects of the intrasurface transparency of an FS interface. Similar to the case of an NS sandwich, Eqs. (A1) and (A2) must be supplemented with the following boundary conditions in the middle of the layers:

\[
\frac{d}{dx} \Phi_{F}^{\pm}(x = d_F/2) = 0, \quad \frac{d}{dx} \Phi_{F}^{\pm}(x = -d_F/2) = 0, \tag{A4}
\]

as well as at the FS boundary\(^{15} \)

\[
\xi_S \frac{d}{dx} \Phi_{S}^{\pm} = \gamma \xi_F \frac{d}{dx} \Phi_{F}^{\pm}, \tag{A5}
\]

where

\[
\gamma = \frac{\rho_S \xi_S}{\rho_F \xi_F}, \quad \gamma_B = \frac{R_B}{\rho_F \xi_F}. \tag{A6}
\]

Here \( \xi_F \) is defined in Eq. (1), \( \xi_S \) is defined as \( \xi_S = 2\xi_{GL}(0)/\pi, \rho_i \) is the normal-state resistivity of metal \( i \), and \( R_B \) is the normal-state boundary resistance times its area. Equation (A3) is a self-consistency equation for the order parameter in the S layer. The parameters \( \gamma \) and \( \gamma_B \) have a simple physical meaning: \( \gamma \) is a measure of the strength of the proximity effect between the F and S metals, whereas \( \gamma_B \), given by

\[
\gamma_B = (2/3)(1_F/\xi_F)((1 - T(\theta))/T(\theta)) \tag{A7}
\]
describes the effect of the boundary transparency. The parameter \( T(\theta) \) denotes the transmission coefficient through the interface for a given angle \( \theta \) between the quasiparticle trajectory and interface and \( \langle \ldots \rangle \) denotes the angle averaging over the Fermi surface. The condition \( \theta = 0 \) corresponds to a perfectly transparent boundary, whereas \( \theta = \infty \) corresponds to a vanishingly small boundary transparency. Specific expressions for \( T \) can be obtained for certain models for the potential barrier. The case of a \( \delta \)-potential barrier \( U(x) = U_0 \delta(x-x_0) \) yields \( T(\theta) = 4\pi U_0/\sqrt{\pi} \), where \( \nu_{F,S}(\theta) \) are the projections of the Fermi velocities of F and S metals on the direction perpendicular to the interface. If the exchange splitting in the ferromagnet is the main cause for intrasurface transparency, a simple expression for \( T \) was given in Ref. 1. By assuming a Stoner-like model, in which the exchange energy \( h_0 \) results in a potential step for one of the spin subbands, it follows that

\[
T_{SF}(\theta) = \frac{4k_F^2k_i^2}{(k_F^2 + k_i^2)^2}, \tag{A8}
\]

where the different wave vectors must be projected on the direction perpendicular to the interface, giving the \( \theta \) dependence. For equal and free-electron-like bands: \( k_F \approx \sqrt{\varepsilon}, \quad k_i \approx \sqrt{\varepsilon - h_0}, \quad k_F \approx \sqrt{\varepsilon + h_0}, \) with \( \varepsilon \) the energy of the electron with respect to the Fermi energy, it can easily be shown that \( T = 1 \) when \( h_0 = 0 \), while \( T = 0 \) for \( h_0 = \varepsilon_F \), since then no occupied states are present for the \( k_i \) subband. In between these limits, \( T(h_0) \) is roughly linear.\(^{1} \)

Equations (A1) and (A2) were discussed extensively in Refs. 20,21 in connection with the proximity effect in NS sandwiches with thick S layers and thin N layers, which is a particular case of the multilayer problem. It was shown there that solving Eqs. (A1) and (A2) may be reduced to solving Eqs. (A1) and (A3) in the S region with an effective boundary condition for \( \Phi_{S}(0) \). Such a boundary condition can be derived for certain limits. For instance, solving the equation for \( \Phi_{N} \) in the N region and using the boundary conditions of Eq. (A5) in the linear regime under consideration near \( T_c \), one obtains

\[
\Phi_{N}(0) = \frac{\gamma A_N(\omega)}{1 + \gamma A_N(\omega)} \Phi_{S}(0), \tag{A9}
\]

where the parameter \( A_N(\omega) \) is given by the expression

\[
A_N(\omega) = \left( \frac{\omega}{\pi T(\theta)} \right)^{1/2} \tanh \left( \frac{d_N}{2\xi_N} \right)^{1/2} \left( \frac{\omega}{2\pi T(\theta)} \right)^{1/2}. \tag{A10}
\]

with \( \xi_N = \sqrt{\nu_{N}/6\pi T(\theta)} \).

In the case of an FS sandwich, one needs an effective boundary condition for \( \Phi_{S}(0) \), since this function goes into the self-consistency equation (A3). Such a boundary condition was derived in Refs. 5,6 for the case of a fully transparent FS interface and may be straightforwardly generalized for the case of arbitrary transparency using Eq. (A5). The condition is simplified considerably in the most interesting case of a large exchange splitting \( \Delta E_F \); one arrives at an expression similar to Eq. (A9) with \( A_N \) substituted by \( A_F \). The length \( \xi_F \) is independent of temperature, which means that \( A_F(\omega) \) becomes independent of \( \omega \):

\[
A_F = \left[ \sin^2 \left( \frac{d_F}{2\xi_F} \right) \tan \left( \frac{d_F}{2\xi_F} \right) + \coth^2 \left( \frac{d_F}{2\xi_F} \right) \coth \left( \frac{d_F}{2\xi_F} \right) \right]^{-1}. \tag{A11}
\]

Relation (A11) leads to the oscillatory dependence of \( T_c \) on F layer thickness discussed theoretically in Refs. 4–6. Furthermore, \( A_F = 1 \) in the limit of thick F layers, \( d_F/2\xi_F \gg 1 \). As a result, in the latter regime the effective parameter in the boundary condition [Eq. (A9)] becomes \( \gamma/(1 + \gamma_B) \), i.e., the transparency can be incorporated in a single parameter. It is then possible to find the correspondence between this single parameter and the parameter \( \epsilon \) from the model of Radović et al.,\(^{3} \) defined as

\[
\epsilon = \frac{\xi_S}{\eta \xi_F}. \tag{A12}
\]

Simple algebraic manipulation shows that, since for full transparency we have \( \gamma = \epsilon^{-1} \), for arbitrary transparency we must have
In this same case of thick F layers, the equations for $T_c$ also reduce to a very simple form:

$$\eta = \frac{\rho_S}{\rho_F} \frac{1}{1 + \gamma_b}.$$  \hspace{1cm} (A13)

It is interesting to note that these equations are nothing else than those from the de Gennes–Werthamer theory, with the effective boundary condition introduced above. Furthermore, it should be remarked that the single parameter description only holds for the linear problem near $T_c$ whereas the behavior of the densities of states in S layers is not simply scaled as $\gamma/(1 + \gamma_b)$.

Finally, it is easy to solve Eqs. (A14) in two limiting cases of weak and strong suppression of $T_c$. In the first regime, where $(T_{c0} - T_c)/T_{c0} \ll 1$, the thickness dependence of $T_c$ has the form:

$$T_c/T_{c0} = 1 - \frac{\pi^2 \xi_S}{2 d_S} \frac{\gamma}{(1 + \gamma_b)/\xi_S}, \quad \gamma/(1 + \gamma_b) \ll 1,$$  \hspace{1cm} (A15)

The critical thickness $d_{cr}^{th}$ is easily found by taking the limit $T_{c0}/T_c \rightarrow \infty$ and using the asymptotic form $\psi(z) = \ln(4 \gamma_b z)$ at $z \gg 1$ in the second part of Eq. (A14) (where $\gamma_b = 1.78$ is the Euler constant). We obtain $\Omega^{SF}/\xi_S = \pi^2 \gamma_b / 2 d_S$ for $\gamma_b = 5.93$ for $\gamma/(1 + \gamma_b) \gg 1$, and $\Omega^{SN}/\xi_S = 4 \gamma_b / (1 + \gamma_b)$ for $\gamma/(1 + \gamma_b) \ll 1$.

The well-known de Gennes result for the critical thickness for SN systems with full transparency, $\gamma_b = 0$, and $\gamma \ll 1$ reads $d_{cr}^{th}/\xi_S = 2 \sqrt{2} \gamma_b / \pi$. Thus, for comparable values of the pair-breaking parameter $\gamma$ the critical thickness in an SN multilayer is somewhat smaller than in an SF one. A comparison of $T_c(d_s/\xi_S)$ curves for SF and SN multilayers was already made in Fig. 7 for two values of $\gamma$ and for $\gamma_b = 0$. In accordance with earlier calculations (see Ref. 4 and references therein) the behavior of $T_c(d_s/\xi_S)$ for SF and SN is most different in the regime of strong pair breaking, $T_c/T_{c0} \ll 1$, where the drop of $T_c$ in the SF case is steeper. Nevertheless, a critical thickness exists both in the SF and SN cases; it is a general property of proximity-effect systems, provided that the N(F) layers are thick. To illustrate this, in Fig. 10 we compare the dependences of $d_{cr}^{th}/\xi_S$ on the interface transparency in the S/F and S/N cases for several values of $\gamma$. In both cases, $d_{cr}^{th}$ was taken at the value where $T_c/T_{c0} = 0.01$. Since $d_{cr}^{th}$ is controlled by the parameter $\gamma/(1 + \gamma_b)$, it decreases with the increase of the transparency parameter $\gamma_b$ and with the decrease of the pair-breaking parameter $\gamma$. The curves in Fig. 10 may be used to estimate critical thicknesses in real multilayer structures.

![FIG. 10. Comparison of the calculated change in critical thickness $d_{cr}^{th}/\xi_S$ for F/S/F trilayers (drawn lines) and N/S/N trilayers (dashed lines) as function of the transparency parameter $\gamma_b$ for different values of the proximity-effect parameter $\gamma$.](image)

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