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Published in:
Physical Review. B, Condensed Matter

DOI:
10.1103/PhysRevB.56.2779

Citation for published version (APA):

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Interface transparency of superconductor/ferromagnetic multilayers

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(Received 5 March 1997)

We have investigated the behavior of the superconducting transition temperature $T_c$ in superconducting/ferromagnetic (S/F) multilayers, as a function of the different layer thicknesses and for varying magnetic moment $\mu_F$ of the F-layer atoms. The system studied consists of superconducting V and ferromagnetic $V_{1-x}Fe_x$ alloys with $x$ such that $\mu_F$ on the Fe atom is varied between 2 and 0.25$\mu_B$. We determined the superconducting coherence length in the F layer $\xi_F$, which is found to be inversely proportional to $\mu_F$. We also determined the critical thickness of the S layer, above which superconductivity appears. This thickness is found to be strongly nonmonotonic as function of the Fe concentration in the alloys. By analyzing the data in terms of the proximity-effect theory, we show that with increasing $\mu_F$, the increasing pair breaking in the F layer by the exchange field is counteracted by a decreasing transparency of the S/F interface for Cooper pairs.

I. INTRODUCTION

In combining a superconductor (S) with a ferromagnet (F) rather than with a normal metal, various effects have been predicted to occur. One is the modification of Andreev reflections at the S/F interface, which would introduce spin selectivity in the conductance of an SF junction, with strong implications for devices at mesoscopic length scales. Another is the possibility of a phase difference $\Delta \phi = \pi$ over an S/F/S junction, resulting in an oscillatory behavior of the superconducting transition temperature $T_c$ with F-layer thickness $d_F$ of S/F multilayers. An oscillation in $T_c$ was recently reported for Nb/Gd, but its origin is still controversial. All such effects concern the behavior of the superconducting order parameter near the S/F interface, and in that sense they form part of the general issue of the proximity effect, well known for the S/N case, but hardly investigated for the S/F case. Apart from the spin dependence, the main parameter which discards an F metal from an N metal in the framework of the proximity effect is the coherence length $\xi_F$, which measures the penetration depth of a Cooper pair into the ferromagnet. This length is supposed to be small, as can be estimated from the simple clean-limit expression

$$\xi_F = \hbar v_F / \Delta E_{ex}. \quad (1)$$

With $v_F$ a typical Fermi velocity of $10^6$ m/s and $\Delta E_{ex}$ a typical exchange splitting of 1 eV, $\xi_F$ is of the order of 1 nm, much smaller than the typical superconducting coherence length $\xi_S \approx 10$ nm. In consequence, the F layer thickness $d_{F, crit}$, needed to decouple two S layers (meaning that the order parameter in F is fully depressed), is very small. Furthermore, the order parameter on the S side will be profoundly influenced, since it must bend almost to zero at the interface. Experimentally, this translates into the fact that one S layer between two F layers needs a minimum or critical thickness $d_{F, crit}$ for superconductivity to develop, $d_{F, crit}$ being governed by both $\xi_S$ and $\xi_F$. Of course, the concept of a critical thickness is not peculiar to the S/F problem. In S/N systems it may be encountered as well, but the behavior of $T_c$ with $d_F$ is more complicated because of the temperature dependence of the coherence length in the normal metal $\xi_N$. In the S/F case, the exchange energy is much larger than the superconducting transition temperature, which makes $\xi_F$ virtually temperature independent. We will come back to this below.

Going one step further, it may be asked how $\xi_F$ can be varied. Control is clearly by the exchange splitting $\Delta E_{ex}$, defined as the effective energy difference for electrons at the Fermi level with spins parallel and antiparallel to the magnetization. It is connected to the magnetic moment $\mu_F$ of the host ion by

$$\Delta E_{ex} = I_{eff} \mu_F. \quad (2)$$

with $I_{eff}$ an effective exchange integral. Thus, it is to be expected that $\xi_F$ can be increased by lowering $\mu_F$. Surprisingly, these simple concepts have never yet been investigated. It is the purpose of this paper to report such systematic research, and to show that the above-sketched picture misses one essential ingredient, namely the transparency of the S/F interface for Cooper pairs. We present measurements on S/F...
TABLE I. Experimental values of the Fe moment $\mu_F$, the magnetically dead layer $d_{md}$, the decoupling thickness $d_c^F$, the critical thickness $d_e^F$, and the specific resistivity $\rho$ at 6 K for alloys $V_{1-x}Fe_x$.

<table>
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<tr>
<th>$x$</th>
<th>$\mu_F$ ($\mu_B$)</th>
<th>$d_{md}$ (nm)</th>
<th>$d_c^F$ (nm)</th>
<th>$d_e^F$ (nm)</th>
<th>$\rho$ ($\mu\Omega$ cm)</th>
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multilayers, where the F metal is a ferromagnetic alloy with a moment which can be varied over almost an order of magnitude by changing the alloy composition. We determine $d_c^F$ and $d_e^F$ for different $\mu_F$, and find a surprising nonmonotonic behavior for the latter. From analysis of the data using proximity effect theory, it is found that by including the interface transmission coefficient (or transparency) $T$ as a parameter, we are able to account for the experimental results. We find that $T$ strongly decreases with increasing $\mu_F$. This may well be due to the spin splitting in the ferromagnet, which leads to partial reflection of Cooper pairs at the S/F interface as discussed for the conduction in Ref. 1.

II. EXPERIMENTAL

The multilayers were grown by dc magnetron sputtering as described previously. They consist of V layers ($T_c = 5.1$ K, Ginzburg-Landau coherence length $\xi_{GL}(0) = 13.9$ nm) and $V_{1-x}Fe_x$ alloy layers. The case $x = 1$ ($V/Fe$) was already studied. In bulk V-Fe alloys, the average moment per Fe atom $\mu_F$ changes from 2.2$\mu_B$ for pure Fe to 0 for $x = 0.3$. The main reason for choosing an alloy is to have different magnetic moments with the least changes of disorder at the interfaces. The V/Fe interface is well behaved, with a lattice mismatch of only 5% and with disorder confined to the two atomic planes on each side. The alloys have even smaller lattice mismatches with V, so that the favorable situation with respect to compositional disorder will remain.

Samples were grown with alloy compositions $x = 1$, 0.88, 0.77, 0.53, 0.38, and 0.34. Three different sets of multilayers were prepared. One set was used to determine $\mu_F$, built as follows: $d_c^F$, $N$ x ($d_v^F/d_F$)/$d_v^{out}$. The outer V layers $d_v^{out}$ are for protection, typically 10–40 nm. The inner V layer $d_v^{in}$ is typically 3 nm; it is not superconducting but meant to increase the number of interfaces, in order to obtain a realistic picture of the F layer magnetism. The F layer $d_F$ is varied in thickness, typically between 0.5 and 5 nm, while the number of repetitions $N$ is adapted to the strength of the moment. For Fe, $N = 3$ suffices, while $N = 20$ for V$_{60}Fe_{40}$. The magnetization $M$ was measured with a magnetometer based on a superconducting quantum interference device at 5 or at 10 K. In all cases, $M$ versus $d_F$ could be described with a straight line, yielding the effective magnetic moment per Fe atom $\mu_F$ and the magnetically dead layer per interface $d_{md}$ (see Ref. 9, 10). They are given in Table I, while a comparison of

$\mu_F(x)$ with values found in bulk alloys (from Ref. 11) is given in Fig. 1. Films and alloys show some differences; near $x = 1$, the values in the films are slightly lower than in the bulk while below $x = 0.75$ the films show higher values. We assume that this is due to the different morphologies of film and bulk material. Furthermore, $d_{md}$ is relatively low in all cases. Values stay below about 0.3 nm or roughly one atomic layer, in clear contrast to the findings in the case of Co and Ni.

The second sample set was used to determine $d_e^F$ by the variation of $T_c$ as function of $d_v$. This is done with samples built with five layers (although three would suffice): $d_F/d_v/d_F/d_v/d_F$, with $d_F$ fixed at around 5 nm (enough to represent a “half-infinite” layer) and $d_v$ variable. The final set was used to determine $d_c^F$ by the variation of $T_c$ with $d_F$; now five layers are needed: $d_v^{out}/d_v/d_v^{in}/d_v^{out}/d_v^{out}$. The outer F layers are again of order 5 nm and meant to create a symmetric situation for the V layers when $d_v^{out}$ is varied from 0 to 5 nm (essentially infinity); $d_v$ has to be chosen differently for each alloy concentration which is best illustrated by some results.

III. RESULTS

Figure 2 shows a compilation of results for the alloy with $x = 0.34$, having $\mu_F = 0.25\mu_B$ (Fe atom), the smallest moment in the series. First we consider $T_c(d_v)$, shown in Fig. 2(a). The asymptotic value of 5.1 K for bulk V is reached above 150 nm. Below 50 nm, $T_c$ starts to drop sharply, and $d_e^F$ is reached around 28 nm. Also shown are measurements of $d_c^F$. For this, $d_v$ is chosen from the $T_c(d_v)$ curve, such that the single film $T_c$ is in the range 2–3 K, well below the bulk value. This is then the measured $T_c$ for the decoupling sample in the limit of large $d_v^{in}$, called $T_c^{low}$. Decreasing $d_F$ leads to increasing $T_c$, when the superconducting order parameters leaking out of the V layers start to overlap. At $d_F = 0$, $T_c$ reaches the value corresponding to 2 $d_v$ in the $T_c(d_v)$ curve [dotted lines in Fig. 2(a)], which is called $T_c^{high}$. In Fig. 2(b), such transition curves are shown for two different values of $d_v$, namely 40 and 55 nm. Both curves show a steep descent above 1 nm, and level off to values near $T_c^{low}$ above 2 nm. Incidentally, neither curve shows an oscillation in $T_c$, as might be found if $\pi$ coupling were
FIG. 2. Data for V_{0.34}Fe_{0.66}. (a) Critical temperature $T_c$ versus V thickness $d_V$. Different symbols represent different sample sets. The dashed line shows the bulk $T_c$ for V. The drawn line represents the model calculations, with $\gamma$ and $\gamma_b$ given in Table II. The dotted lines show the range of $T_c$ values covered by the experiments displayed in (b). Also indicated is $d^{cr}_V$, (b) $T_c$ versus $d_F$ for two values of $d_V$. The dashed lines show the limiting values as follow from the trilayer data in (a).

We will briefly come back to the issue of $\pi$ coupling at the end of the Discussion.

In Fig. 3, the same transitions have been plotted, but scaled to $T_c^{high}-T_c^{low}$, and for all concentrations. For $x=0.34$ the curves for both thicknesses $d_F$ essentially coincide, as they should. Furthermore, the steepest descent of the curves clearly shift to higher $d_F$ upon decreasing $x$ or $\mu_F$. We now define $d_{cr}^{S}$, by extrapolating the steepest slope in the transition curve to the $d_F$ axis (see Fig. 3). Different definitions, such as using the $50\%$ point, turned out to give very similar results. Values for $d^{cr}_S$ are given in Table I. We plot this quantity against $\mu_F^{-1}$ in Fig. 4 and find a reasonably linear relation. Making the identification $d^{S}_{cr}/2=\xi_F$, it follows that $\xi_F$ behaves as described by Eqs. (1,2). Given the small thicknesses involved, such clean-limit behavior could be expected, but the linear behavior also implies that the quantity $u_F/\xi_F$ basically remains constant with varying $x$.

Next we turn to the behavior of $d^{cr}_S$. For all alloys, the $T_c(d_V)$ curves are similar to the one presented in Fig. 2(a). The scatter in the individual points is small enough to find values for $d^{cr}_S$ with good accuracy. All values for $d^{cr}_S$ are collected in Table I. Especially interesting is the behavior near $x=1$, which is reproduced in Fig. 5. There, $T_c(d_V)$ is plotted on a somewhat expanded scale for the three systems with the highest moments ($x=1$, 0.88, 0.77). The behavior for $x=1$, 0.77 is very smooth; for $x=0.88$, the scatter in points is quite large, actually the largest by far of all sets measured [compare also Fig. 2(a) for $x=0.34$]. Even then, the plot unequivocally shows that the curves shift to higher thickness with decreasing $x$. This behavior is quite unexpected, and comprises the main issue of our research, to be discussed below. Figure 6 shows the full behavior of $d^{S}_{cr}(\mu_F)$. A clear maximum is found between $x=0.77$ and $x=0.53$, before a slow decrease sets in. The value at $x=0.34$, where the magnetic moment has decreased by a factor 8, is actually equal to the value for $x=1$ (Fe). To make the point in another way, we plotted in Fig. 6 the results of earlier measurements with Co and Ni as the F metal (open circles), where $d^{S}_{cr}$ is found to be much lower at the same values for the magnetic moment. Next to $\mu_F$ another factor must play a major role in determining the physics. We will now argue that this factor is the interface transparency.

FIG. 3. Change of critical temperature $T_c$ with F layer thickness $d_F$, scaled according to $t=\varepsilon_T/(T_c^{high}-T_c^{low})$. The lines are meant to guide the eye. The construction for the determination of $d^{S}_{cr}$ is indicated for $x=0.53$. The arrows show the values of $d^{cr}_S$ for all alloy concentrations.

FIG. 5. Change of critical temperature $T_c$ with S layer thickness $d_S$ for alloys with $x=1$, 0.88, and 0.77 (two sample sets). The lines are the results of the calculations with the parameters given in Table II.
IV. DATA ANALYSIS BY PROXIMITY EFFECT THEORY

A. Theory; a brief description

Scattering of a normal electron or quasiparticle on a potential barrier at an interface will lower its transmission coefficient $T$. In S/N structures, one source for this is the potential step due to the difference in lower band-edge energies. Defects can also cause potential scattering, and are usually modeled as a $\delta$ function with a certain strength. Theoretically, the consequences of reduced $T$ for different quantities such as the superconducting density of states or the critical temperature, have long been a subject of investigation, starting with McMillan’s tunneling model for bilayers, which represents the limit of small $g$, defined as (see the Appendix):

$$T_c = \frac{\rho_s \xi_s}{\rho_F \xi_F} \frac{R_B}{\rho_F \xi_F}, \quad \gamma_b = \frac{R_B}{\rho_F \xi_F}$$

with $\rho$ the normal-state resistivity of metal $i$, and $R_B$ the normal-state boundary resistance times its area. The connection between $\gamma_b$, the transparency parameter, and the proximity effect parameter $g$, is roughly given by

$$\gamma \approx \frac{\rho_s \xi_s}{\rho_F \xi_F} \frac{R_B}{\rho_F \xi_F}$$

Figures 7 and 8 show two types of results from the calculations. In Fig. 7, $T_c(d_S)$ is given for an F/S/F trilayer with $d_F = 10 \xi_F$, $\rho_s = \rho_F$, $\xi_S / \xi_F = 10$ ($\gamma = 10$) and complete transparency ($\gamma_b = 0$).
TABLE II. Values for the coherence lengths $\xi_F$, for the specific resistivity $\rho$ at 6 K, for the proximity-effect parameter $\gamma$, and for the transparency parameter $\gamma_b$, for alloys $V_{1-x}Fe_x$.

<table>
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<tr>
<th>$x$</th>
<th>$\xi_F$ (nm)</th>
<th>$\rho (\mu \Omega \ cm)$</th>
<th>$\gamma$</th>
<th>$\gamma_b$</th>
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parity ($\gamma_b=0$), and an N/S/N trilayer with the same parameters. The difference between both curves is quite small and only clearly visible below $T_T/T_{c0}=0.5$, where the temperature dependence of $\xi_N$ becomes important. In the F case, it is easy to define a value for the critical thickness, $d_{cr}^F$, for which we take the thickness at $T_T/T_{c0} = 0.01$. Figure 8 shows the behavior of $d_{cr}^F$ for the F/S/F case as function of the parameters $\gamma$ and $\gamma_b$. The plot demonstrates some general features of proximity effect systems. In the large-$\gamma$ limit, $d_{cr}^F/\xi_F \to \pi \sqrt{2} \gamma_F \approx 6$, with $\gamma_F$ the Euler constant. This limit is hard to reach in S/N systems, where $\xi_N$ is of the same order of magnitude, but is easily met in S/F systems with $\rho_S/\rho_F$ of order one, and with $\xi_F$ an order of magnitude larger than $\xi_F$. Also, if $\gamma$ is large and therefore ‘proximity leak’ is small, it takes a very high barrier (large $\gamma_b$, small $T$) to lower $d_{cr}^F$.

B. Discussion of the results

As has been discussed above, a full description of the $T_c$ variation in a proximity effect system needs five parameters: the S bulk layer critical temperature $T_{c0}$; the thicknesses $d_S$ and $d_F$, the proximity-effect strength $\gamma$ and the transparency parameter $\gamma_b$. Starting with $\gamma$, it can be seen from Eq. (3) that this parameter is fully determined by measurable quantities. We take $\xi_F$ from the linear relation between $d_{cr}^F$ and $\mu_F^{-1}$, shown in Fig. 3, rather than from the actual values of $d_{cr}^F$. The values used are given in Table II. For $\xi_F$ we use 8.8 nm, corresponding to $\xi_{GL}(0) = 13.9$ nm.¹⁰ The normal-state resistivities $\rho_{S,F}$ are also known. They were measured on thin films of 50, 100, and 200 nm, down to 6 K for all compositions and for V. The averaged values are given in Table II. Due to the use of alloys, $\rho_F$ actually increases considerably (about 2 $\mu \Omega$ cm/atom) up to $x=0.5$, thereby lowering the resistivity ratio in $\gamma$ from 1.7 to 0.06. Values for $\gamma$ can now be calculated, and they are found (see Table II) to decrease monotonically with decreasing moment. Note that this is due to a decrease in both the factors $\rho_S/\rho_F$ and $\xi_S/\xi_F$, and neither factor therefore can be the cause of the observed increase in $d_{cr}^S$. With the values for $\gamma$, we calculate theoretical values $d_{cr}^S$ under the assumption that $\gamma_b = 0$. The numbers, plotted as squares in Fig. 9(a), do not mimic the experimental results, shown as filled circles, in two respects. They do not go through a maximum, as was already anticipated from the monotonic behavior of $\gamma$, but also, the measured values are much lower than the calculated ones. Especially for Fe, a low value for $\xi_F$ and an also relatively low value for $\rho_F$ lead to a very high $\gamma$ and a theoretical critical thickness which is close to the asymptotic limit of about 6 $\xi_S$.

The simple fact that $d_{cr}^S$ is much smaller than expected for the case $\gamma_b=0$, already indicates reduced interface transparency; a value for $T<1 (\gamma_b>0)$ leads to smaller $d_{cr}^S$ (see Fig. 8); for $T=0$ the superconductor will behave as an isolated film ($d_{cr} \to \infty$). The next step therefore is to use the model calculations in order to find the value of $\gamma_b$ needed to reproduce the measured values for $d_{cr}^S$. $T$ is then simply found from Eq. (4). The results, plotted in Fig. 9(b), show a very simple relation: $T$ is low for the case of Fe, increases more or less linearly with decreasing $\mu_F$ or $x$, and reaches the order of 1 for low Fe concentration. The observed maximum in $d_{cr}^S$ is therefore due to the competition of three ingredients: on the side of high Fe concentration, the increasing $\xi_F$ and decreasing $\rho_S/\rho_F$ will lower $d_{cr}^S$, but the increasing interface transparency will increase $d_{cr}^S$, and wins; on the low Fe side, the change in interface transparency has become less important, and the change in $d_{cr}^S$ is as expected from the change in $\gamma$.

We believe this to be the first demonstration of a barrier transparency which is changed in a continuous (and controlled) fashion, and over a large part of the full range. Of course, the given values for $T$ should not be taken too literally. They depend on the way in which $\xi_F$ is extracted from the $T_{c0}(d_{cr})$ curves, on the measured values of $\rho_{S,F}$ (which may be somewhat different in multilayers or in single films), and on the approximation used to go from $\gamma_b$ to $T$. Especially a near-zero value for Fe may be too low. On the other hand, a seriously reduced $T$ is needed to explain the low value for $d_{cr}^S$, while a serious concentration or moment dependence of $T$ is needed to explain the increase in $d_{cr}^S$. This point leads to the question of the cause of the low value and its change. It is possible that $T$ depends on $x$ as a result of the changing compositional disorder or the changing lattice parameters (strain). It is more probable, however, that $\mu_F$, meaning the ferromagnetism and the spin-dependent band structure, play a role. One mechanism may well be the reduction of Andreev scattering due to the exchange splitting.¹
since this would translate to a reduced transparency through the use of the boundary conditions for the Usadel equations (see the Appendix). The effect is linear in $\Delta E_{ex}/\epsilon_F$, with $\epsilon_F$ the Fermi energy, and might therefore be appreciable, of the order of 30–50%. Another mechanism can be spin dependence in the normal-state reflection at the interface, such as now investigated in view of giant magnetoresistance effects (see, e.g., Ref. 17). It would take reflections in only one spin channel to lower the transparency for Cooper pairs. Both effects can be present at the same time; from this viewpoint, low transparency looks quite feasible. Interestingly, the few reported values for $d^2\xi/\xi$ are much below the upper limit of 6. For Nb/Gd, for instance, the value is 4.2. For Nb/Er, the value appears to be between 2 and 3.19 Low transparency may prove to be a general phenomenon in S/F multilayers.

C. On the issue of $\pi$ coupling

In the discussion of the results on the decoupling behavior, we already noted that no oscillatory behavior of $T_c$, and therefore no indication of $\pi$ coupling is found with varying thickness of the magnetic layer for any alloy concentration or magnetic moment. This may not be very surprising. In the original description of a possible mechanism which changes the phase of a Cooper pair by $\pi$, the transfer of the pair through a barrier containing localized moments is accompanied by two virtual spin flips of that moment. Given the strong and itinerant nature of the magnetism in the 3$d$ transition metals under consideration, the spin flips would take the form of spin-wave excitations. This process will have a small probability in view of the large energy denominator involved. In principle, a system with strongly localized (e.g., 4$f$) moments, might offer a better chance for finding $\pi$ coupling. Still, we do not believe that the oscillationlike changes in $T_c$ which were found recently in Nb/Gd (Ref. 7) are actually due to this mechanism. Rather, transparency may again play an important role, as can also be inferred from a report on oscillatory $T_c$’s in Nb/Fe by Mühge et al., who investigated (essentially) trilayer samples with a single superconducting layer. The key observation in both Nb/Gd and Nb/Fe is that $T_c$ increases at the onset of ferromagnetism in the thin F layer. In the spirit of the model used above, we would like to thank P. Koorevaar for early contributions to this work, and J.A. Mydosh, P.H. Kes, and C.J.M. Beenakker for discussions. This work is part of the research program of the “Stichting voor Fundamenteel Onderzoek der Materie” (FOM), which is financially supported by NWO.

APPENDIX

We consider a multilayered structure consisting of alternating F and S layers of thickness $d_F$ and $d_S$, respectively, and with a finite transparency of the FS boundary. The S layer has a bulk critical temperature $T_{c0}$. We assume dirty-limit conditions for both F and S metals: $l_{F,S},\xi_{F,S},$ where $l_{F,S}$ and $\xi_{F,S}$ are the mean free paths and coherence lengths in the F(S) layers. Due to the translational symmetry of the problem it is sufficient to consider an elementary unit cell with period $\Lambda=(d_F+d_S)/2$. With these assumptions the proximity effect in the system can be described within the framework of the Usadel equations for the S and F layers. Near $T_c$, these equations can be linearized and written in the form:

$$\frac{\pi T_c}{|\alpha|} \frac{d^2}{dx^2} \Phi_S^\mp - \Phi_S^\pm = 2\Delta \delta^\pm, \quad 0 < x < d_S,$$  \hspace{1cm} (A1)

$$\frac{\pi T_c}{|\alpha|} \frac{d^2}{dx^2} \Phi_F^\mp + i\Phi_F^\pm = 0, \quad -d_F < x < 0,$$  \hspace{1cm} (A2)

$$\Delta \ln \frac{T_c}{T_{c0}} + \pi T_c \sum_{\omega > 0} \left[ (2\Delta - \Phi_S^\pm) / \omega \right] = 0.$$  \hspace{1cm} (A3)

Here $\Phi_S^\pm = \Phi_{F,S}(\omega) \pm \Phi_{F,S}(-\omega)$ are the anomalous Green’s functions integrated over energy and averaged over the Fermi surface, $\Delta$ is the order parameter in the S layer, $\delta^+ = 1, \delta^- = 0,$ and $\omega = \pi T (2n+1)$ with $n=0,\pm 1,\pm 2,\ldots$ are the Matsubara frequencies. Note that the functions
\( \Phi_{F,S}^{\pm}(\omega) \) are not symmetric with respect to sign reversal of the energy \( \omega \), i.e., \( \Phi_{F,S}^{\pm}(\omega) \neq \Phi_{F,S}^{\pm}(-\omega) \). This symmetry is restored in the more conventional case of proximity effect in an NS sandwich: \( \Phi_{N,S}^{\pm}(\omega) = \Phi_{N,S}^{\pm}(-\omega) \), which results in \( \Phi_{N,S}^{\pm} = 2 \Phi_{N,S} \) and \( \Phi_{N,S} = 0 \). Another important difference between the NS and FS cases is that \( \xi_N \) is \( \omega \) dependent, whereas \( \xi_F \) is constant. Some specific phenomena which result from these peculiarities of FS systems were pointed out previously in Refs. 4–6. Here we are interested in the effects of the intransparency of an FS interface. Similar to the case of an NS sandwich, Eqs. (1) and (2) must be supplemented with the following boundary conditions in the middle of the layers:

\[
\frac{d}{dx} \Phi_S^\pm(x = d_S/2) = 0, \quad \frac{d}{dx} \Phi_F^\pm(x = -d_F/2) = 0, \quad (A4)
\]
as well as at the FS boundary\(^{15}\)

\[
\xi_S \frac{d}{dx} \Phi_S^\pm = \gamma \xi_F \frac{d}{dx} \Phi_F^\pm, \quad (A5)
\]

where

\[
\gamma = \frac{\rho_S \xi_S}{\rho_F \xi_F}, \quad \gamma_b = \frac{R_B}{\rho_F \xi_F}. \quad (A6)
\]

Here \( \xi_F \) is defined in Eq. (1), \( \xi_S \) is defined as \( \xi_S = 2 \xi_{GL}(0)/\pi, \rho_i \) is the normal-state resistivity of metal \( i \), and \( R_B \) is the normal-state boundary resistance times its area. Equation (A3) is a self-consistency equation for the order parameter in the S layer. The parameters \( \gamma \) and \( \gamma_b \) have a simple physical meaning: \( \gamma \) is a measure of the strength of the proximity effect between the F and S metals, whereas \( \gamma_b \), given by

\[
\gamma_b = (2/3)(1_F/\xi_F)((1 - T(\theta))/(T(\theta))) \quad (A7)
\]
describes the effect of the boundary transparency. The parameter \( T(\theta) \) denotes the transmission coefficient through the interface for a given angle \( \theta \) between the quasiparticle trajectory and interface and \( \langle \ldots \rangle \) denotes the angle averaging over the Fermi surface. The condition \( \gamma_b = 0 \) corresponds to a perfectly transparent boundary, whereas \( \gamma_b = \infty \) corresponds to a vanishingly small boundary transparency. Specific expressions for \( T \) can be obtained for certain models for the potential barrier. The case of a \( \delta \)-potential barrier \( U(x) = U_0 \delta(x - x_0) \) yields \( T(\theta) = 4U_F(\theta)u_F(\theta)/(4U_0^2 + [u_F(\theta) + v_F(\theta)]^2) \), where \( u_F,S(\theta) \) are the projections of the Fermi velocities of F and S metals on the direction perpendicular to the interface. If the exchange splitting in the ferromagnet is the main cause for intransparency, a simple expression for \( T \) was given in Ref. 1. By assuming a Stoner-like model, in which the exchange energy \( h_0 \) results in a potential step for one of the spin subbands, it follows that

\[
T_{SP}(\theta) = \frac{4k_F^2 k_1^2 k_1}{(k_F^2 + |k_1|^2)^2}, \quad (A8)
\]

where the different wave vectors must be projected on the direction perpendicular to the interface, giving the \( \theta \) dependence. For equal and free-electron-like bands: \( k_\parallel \propto \sqrt{\epsilon}, \quad k_\perp \propto \sqrt{\epsilon - h_0}, \quad k_\perp \propto \sqrt{\epsilon + h_0}, \) with \( \epsilon \) the energy of the electron with respect to the Fermi energy, it can easily be shown that \( T = 1 \) when \( h_0 = 0 \), while \( T = 0 \) for \( h_0 = \epsilon_F \), since then no occupied states are present for the \( k_\perp \) subband. In between these limits, \( T(h_0) \) is roughly linear.\(^{1}\)

Equations (A1) and (A2) were discussed extensively in Refs. 20, 21 in connection with the proximity effect in NS sandwiches with thick S layers and thin N layers, which is a particular case of the multilayer problem. It was shown there that solving Eqs. (A1) and (A2) may be reduced to solving Eqs. (A1) and (A3) in the S region with an effective boundary condition for \( \Phi_S^\pm(0) \). Such a boundary condition can be derived for certain limits. For instance, solving the equation for \( \Phi_N \) in the N region and using the boundary conditions of Eq. (A5) in the linear regime under consideration near \( T_c \), one obtains

\[
\Phi_S^\pm(0) = \frac{\gamma A_N(\omega)}{1 + \gamma_b A_N(\omega)} \Phi_S^\pm(0), \quad (A9)
\]

where the parameter \( A_N(\omega) \) is given by the expression

\[
A_N(\omega) = \left( \frac{\omega}{\pi T_c} \right)^{1/2} \tanh \left( \frac{d_N}{2 \xi_N} \left( \frac{\omega}{\pi T_c} \right)^{1/2} \right). \quad (A10)
\]

with \( \xi_N = \sqrt{\nu_N l_N/6 \pi T_c} \).

In the case of an FS sandwich, one needs an effective boundary condition for \( \Phi_F^\pm \), since this function goes into the self-consistency equation (A3). Such a boundary condition was derived in Refs. 5,6 for the case of a fully transparent FS interface and may be straightforwardly generalized for the case of arbitrary transparency using Eq. (A5). The condition is simplified considerably in the most interesting case of a large exchange splitting \( \Delta E_F \); one arrives at an expression similar to Eq. (A9) with \( A_N \) substituted by \( A_F \). The length \( \xi_F \) is independent of temperature, which means that \( A_F(\omega) \) becomes independent of \( \omega \):

\[
A_F = \left[ \sin^2 \frac{d_F}{2 \xi_F} \tanh \frac{d_F}{2 \xi_F} + \cos^2 \frac{d_F}{2 \xi_F} \coth \frac{d_F}{2 \xi_F} \right]^{-1}. \quad (A11)
\]

Relation (A11) leads to the oscillatory dependence of \( T_c \) on F layer thickness discussed theoretically in Refs. 4–6. Furthermore, \( A_F = 1 \) in the limit of thick F layers, \( d_F/2 \xi_F \gg 1 \).

As a result, in the latter regime the effective parameter in the boundary condition [Eq. (A9)] becomes \( \gamma/(1 + \gamma_b) \), i.e., the transparency can be incorporated in a single parameter. It is then possible to find the correspondence between this single parameter and the parameter \( \epsilon \) from the model of Radović et al.,\(^{4}\) defined as

\[
\epsilon = \frac{\xi_S}{\eta F}. \quad (A12)
\]

Simple algebraic manipulation shows that, since for full transparency we have \( \gamma = \epsilon^{-1} \), for arbitrary transparency we must have
\[ \eta = \frac{\rho_s}{\rho_F} \frac{1}{1 + \gamma_b}. \quad (A13) \]

In this same case of thick F layers, the equations for \( T_c \) also reduce to a very simple form:

\[ \Omega \tan(\Omega d_s/2\xi_s) = \gamma/(1 + \gamma_b), \quad (A14) \]

It is interesting to note that these equations are nothing else than those from the de Gennes–Werthamer theory, with the effective boundary condition introduced above. Furthermore, it should be remarked that the single parameter description only holds for the linear problem near \( T_c \) whereas the behavior of the densities of states in S layers is not simply scaled as \( \gamma/(1 + \gamma_b) \).

Finally, it is easy to solve Eqs. (A14) in two limiting cases of weak and strong suppression of \( T_c \). In the first regime, where \( (T_c - T_c) / T_c \propto 1 \), the thickness dependence of \( T_c \) has the form:

\[ T_c / T_c = 1 - \frac{\pi^2 \xi_S}{2d_S} \left( \frac{\gamma}{1 + \gamma_b} \right) \approx 1, \quad (A15) \]

The critical thickness \( d_{cr}^{th} \) is easily found by taking the limit \( T_c / T_c \rightarrow \infty \) and using the asymptotic form \( \psi(\zeta) \approx \ln(4\gamma_b \zeta) \) at \( \zeta \approx 1 \) in the second part of Eq. (A14) (where \( \gamma_b = 1.78 \) is the Euler constant). We obtain \( \Omega \approx 1/2\gamma_b \) and then the first part of Eq. (A14) yields \( d_{cr}^{th, SF} / \xi_S = \pi^2 \gamma_b \approx 5.93 \) for \( \gamma/(1 + \gamma_b) \approx 1 \), and \( d_{cr}^{th, SN} / \xi_S = 4 \gamma_b / (1 + 3 \gamma_b) \) for \( \gamma/(1 + \gamma_b) \approx 1 \).

The well known de Gennes result for the critical thickness for SN systems with full transparency, \( \gamma_b = 0 \), and \( \gamma \ll 1 \) reads \( d_{cr}^{th, SN} / \xi_S = 2 \sqrt{2} \gamma_b \gamma \). Thus, for comparable values of the pair-breaking parameter \( \gamma \) the critical thickness in an SN multilayer is somewhat smaller than in an SF one. A comparison of \( T_c(d_s / \xi_S) \) curves for SF and SN multilayers was already made in Fig. 7 for two values of \( \gamma \) and for \( \gamma_b = 0 \). In accordance with earlier calculations (see Ref. 4 and references therein) the behavior of \( T_c(d_s / \xi_S) \) for SF and SN is most different in the regime of strong pair breaking, \( T_c / T_c \ll 1 \), where the drop of \( T_c \) in the SF case is steeper. Nevertheless, a critical thickness exists both in the SF and SN cases; it is a general property of proximity-effect systems, provided that the N(F) layers are thick. To illustrate this, in Fig. 10 we compare the dependences of \( d_{cr}^{th} / \xi_S \) on the interface transparency in the S/F and S/N cases for several values of \( \gamma \). In both cases, \( d_{cr}^{th} \) was taken at the value where \( T_c / T_c \approx 0.01 \). Since \( d_{cr}^{th} \) is controlled by the parameter \( \gamma/(1 + \gamma_b) \), it decreases with the increase of the intransparency parameter \( \gamma_b \) and with the decrease of the pair-breaking parameter \( \gamma \). The curves in Fig. 10 may be used to estimate critical thicknesses in real multilayer structures.

\[ \gamma = \frac{1}{\Omega d_s/2\xi_s} = \gamma/(1 + \gamma_b) \]

\[ \psi(1/2 + \Omega^2 T_c/d_s/2T_c) - \psi(1/2) = -\ln(T_c/T_c) \]

**FIG. 10.** Comparison of the calculated change in critical thickness \( d_{cr}^{th} / \xi_S \) for F/S/F trilayers (drawn lines) and N/S/N trilayers (dashed lines) as a function of the transparency parameter \( \gamma_b \) for different values of the proximity-effect parameter \( \gamma \).