Essays on markets over random networks and learning in Continuous Double Auctions

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Chapter 3

Information and Efficiency in Thin Markets over Random Networks

3.1 Introduction

In this chapter we consider a market in which transactions only occur between linked traders. These links occur as in a bipartite random network where every link is realised with the same probability, independently of each other. Regular random graphs have been introduced by Erdős and Rényi (1960, 1961). The spot foreign exchange market is studied by Gould et al. (2013a) and is an example of a market in which trade occurs through Bilateral Trading Agreements. Traders provide a block list containing trading partners with whom they prefer not to trade, to protect themselves against adverse selection and to control counterparty risk. In such a market a transaction between two traders only takes place if both are not part of the other's block list. We use the model of Gould et al. and additionally assume that links are realised with the same probability and independently of each other.

Markets over networks have been studied in various settings. Corominas-Bosch (2004) and Chatterjee and Dutta (1998) consider a market in which side by side traders submit an offer which the other traders accept or reject. In Corominas-Bosch (2004) all buyers have the same valuation and sellers the same cost; this allows the network to be split into different subgraphs.

In every subgraph the short side extracts all the possible surplus. We show that under partial information about the network structure, or under incomplete information about valuations and costs, not all the surplus is necessarily extracted. Spulber (2006) and Kranton and Minehart (2001) study simultaneously ascending-bid auctions in which sellers jointly raise their ask until supply equals demand, and then trade occurs. Easley and Kleinberg (2010) and Blume et al. (2009) introduce intermediaries who act strategically and profit from trade. The power of a trader in a network is formalised in Calvó-Armengol (2001) by considering the number of linked traders and their links. A higher market power is achieved when a trader is linked to more traders and when linked traders have fewer links themselves.

For bilateral trading Myerson and Satterthwaite (1983) and Chatterjee and Samuelson (1983) study Nash equilibrium strategies that monotonely transform valuations and costs into offers and exhibit an equilibrium in which they are piecewise linear. We restrict attention to linear markup and markdown strategies where the intensity of the markup or markdown depends on the information set that is available to the trader. These strategies have been introduced by Zhan and Friedman (2007); Cervone et al. (2009) discuss a version that is symmetric between buyers and sellers.

We consider thin markets with few traders, who trade only over existing links in a bipartite graph. These links are formed independently with the same probability p in $(0, 1)$, forming a bipartite random graph à la Erdős-Rényi. Traders behave strategically, and we derive equilibrium configurations depending on the information about the network structure that is available to traders.

Three nested information sets about the realisation of the network are compared. Under no information, traders place orders without knowing which links materialise, but simply the probability p that each link may exist. With partial information, traders know their own links and the probability p that links may exist between other market participants. Under full information, the entire structure of the network is common knowledge.

We study the effect of the quantity of information available to traders on allocative efficiency. We show that this effect is non-monotonic. Furthermore, switching from complete to incomplete information about traders' valuations flips the shape of this non-monotonicity. Under complete information about traders' valuations, we show that for any value of p both no information and full information lead to full allocative efficiency, while the partial information regime is weakly dominated. However, under a more realistic assumption of incomplete information about traders' valuations, this ranking is reversed. If traders use linear markup strategies, partial information strongly dominates full and no information for any value of p.

The organisation of this chapter is as follows. The model and the trading mechanism are described in Section 3.2, together with the markup and markdown strategies and the information sets. Efficiency under complete information about traders' valuations is studied in Section 3.3, followed by incomplete information in Section 3.4. Finally, Section 3.5 concludes.

3.2 The model

Let us consider a market over a bipartite Erdős-Rényi network. In such a network every buyer b_i and every seller s_j are connected with probability p in $(0, 1)$ independently of other links. Trade is possible only if a link exists. An example of such a market is the spot exchange market studied in Gould et al. (2013a). In comparison with this market we add the assumption that every pair of traders is linked with the same probability and independently of other links. Furthermore, in the spot exchange market trade is only possible when both traders do not include the other in their blocklist. E.g. this may occur when the trading partner does not exceed some risk requirement and therefore the network structure is considered exogenous. Nevertheless, a bijective transformation exists from the probability of a link in the spot exchange market to the probability of a link in this chapter. The probability of a link in the spot exchange market is the square of the latter probability.

A buyer desires to obtain one unit of a good and a seller seeks to sell one unit. Under complete information valuations equal one and costs equal zero, whereas under incomplete information the valuations v_i of buyers and costs c_j of sellers are uniformly distributed on the interval $[0, 1]$. This distribution is public information but the realisations are private information. The profit of a buyer is equal to his valuation minus the transaction price if he trades and zero otherwise. The profit of a seller equals the transaction price minus his cost after a trade and zero otherwise.

The probability of a link influences the expected allocative efficiency as absence of links makes some trades impossible. Furthermore, expected efficiency is reduced by strategic behaviour of traders, that could prevent feasible trades. Expected allocative efficiency is defined as the expected total realised surplus from trade divided by the expected maximal total surplus, i.e. the expected total profit of all traders divided by the expected total maximal profit.

We show our results for a market with two buyers and two sellers. In this market the maximal expected surplus equals 2 under complete information and $\frac{2}{5}$ under incomplete information of valuations and costs. The maximal expected surplus under incomplete information is derived for a full network. From the point of view of buyer b_1, he has the highest valuation with probability v_1 since the valuation of the other buyer is uniformly distributed. This results in a trade with the seller with the lowest cost if this trade is feasible. The density function of the lowest cost is given by $2 - 2c_{\text{min}}$ and this trade results in a surplus of $v_1 - c_{\text{min}}$. Similarly buyer b_1 has the lowest valuation with probability $1 - v_1$ and he trades with the seller with the highest cost, with density function $2c_{\text{max}}$, if this trade is feasible. Hence the maximal expected surplus of $\frac{2}{5}$ for a full network is obtained from
$$2\left[\int_0^1 \int_{c_{\text{min}}}^1 (v_1 - c_{\text{min}})(2 - 2c_{\text{min}})v_1 \mathrm{d}v_1 \mathrm{d}c_{\text{min}} + \int_0^1 \int_{c_{\text{max}}}^1 (v_1 - c_{\text{max}})2c_{\text{max}}(1 - v_1)\mathrm{d}v_1 \mathrm{d}c_{\text{max}}\right].$$

However, due to absence of links the maximal expected surplus is reduced, depending on the value of p. For complete and incomplete information about valuations and costs, the ratios between maximal expected surplus given the random network structure and maximal expected surplus of the full network are given by

$$\mathbb{E}(\mathrm{AE}_{\mathrm{p}}^{\mathrm{C}}) = \frac{1{\cdot}4p(1-p)^3+1{\cdot}4p^2(1-p)^2+2{\cdot}2p^2(1-p)^2+2{\cdot}4p^3(1-p)+2{\cdot}p^4}{2},$$

$$\mathbb{E}(\mathrm{AE}_{\mathrm{p}}^{\mathrm{I}}) = \frac{\frac{1}{6}{\cdot}4p(1-p)^3+\frac{1}{4}{\cdot}4p^2(1-p)^2+\frac{1}{3}{\cdot}2p^2(1-p)^2+\frac{43}{120}{\cdot}4p^3(1-p)+\frac{2}{5}{\cdot}p^4}{\frac{2}{5}}.$$

In Fig. 3.1 it is shown that a difference in reduction of efficiency, due to restrictions of the network structure, exists between complete and incomplete information. This is due to the difference in distribution of valuations and costs. Hence, under complete information every trade results in the same surplus while under incomplete information extra links not only increase the expected number of links, but also decrease the expected surplus per trade. Hence the difference between both ratios of efficiency increases.

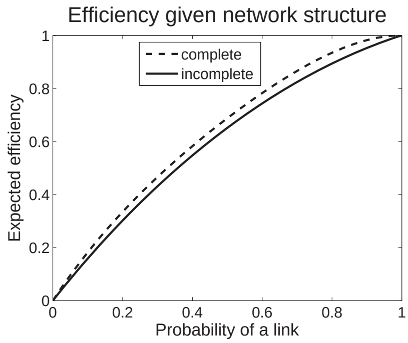


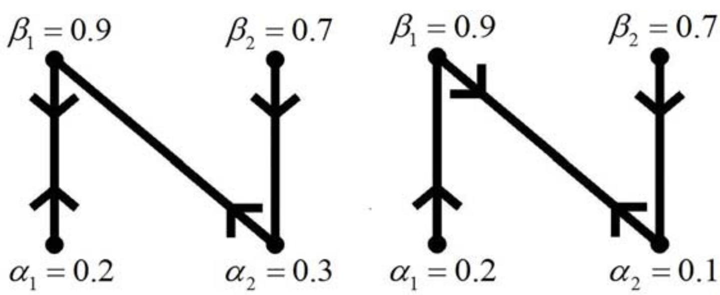
Figure 3.1: Ratios of efficiency given the probability of a link for complete and incomplete information about valuations and costs.

3.2.1 Trading mechanism

The symmetric trading mechanism consists of simultaneous submission of bids and asks by all traders after which the offers are made public. A buyer ranks his connected sellers by their asks, and a seller his linked buyers by their bids. Trades respect such preferences: preferred buyer-seller pairs are matched with each other. As long as further trades are possible, such a preferred pair naturally exists. Every seller desires to trade with the buyer with the highest bid which ensures that this buyer can trade with his preferred connection. The trade is executed at a price that is equal to the average of bid and ask and this is repeated until no further trades are possible. In contrast to some related literature, this trading mechanism gives equal power to both sides of the market.

If this trading mechanism does not lead to a unique outcome, as a result of traders that do not have a unique preferred trading partner, the trading mechanism selects the outcome that maximises total surplus. Under complete information about valuations and costs this is conservative towards our result, under incomplete information this occurs only in a nullset.

A possible realisation of the bipartite Erdős-Rényi network with bids β_i and asks α_j is given in Fig. 3.2. In the first example buyer b_1 and seller s_1 trade after which b_2 and s_2 trade; this coincides with the social optimum. In the second example however b_1 and s_2 trade. Hence the most profitable trade occurs first and therefore a social optimum is not necessarily reached.



(a) The social optimum is reached. (b) The social optimum is not reached.

Figure 3.2: Example of the trading mechanism.

3.2.2 Markup and markdown strategies

There is an incentive for traders to act strategically and bid below their valuation and ask above their cost to obtain a higher profit. Under complete information about valuations and costs traders choose a unique strategy given the available information about the network. Under incomplete information traders choose a strategy that is depending on the realisation of their valuation or cost. We assume that traders use linear markup and markdown strategies symmetric on $[0, 1]$ from Cervone et al. (2009). These strategies transform the valuations and costs as follows:

A buyer with valuation v_i bids $\beta_i = v_i(1 - m_i^d)$.

A seller with cost c_j asks $\alpha_j = c_j + m_j^u(1 - c_j)$.

The values m_i^d and m_j^u denote the intensity of the markdown of buyer i and the markup of seller j. The higher these values, the further away bids and asks are from the valuations and costs. The Nash equilibrium markdown and markup strategies are determined on the basis of the distribution of valuation and cost of others, not on the realisation of it. Moreover, we need to take into account the information set of a trader. Hence the markdown and markup strategies will not be a simple transformation of the valuation or cost, but will also depend on the information that is available to traders about the network structure.

3.2.3 The information sets

We study the Nash equilibrium markdown and markup strategies depending on the information set available to traders. Under complete information the valuations and costs are known; under incomplete information only their distribution. Moreover, the number of traders on both sides of the market is known. We consider the following nested sets of information about the network structure, which are all common knowledge:

- No information: The probability of a link is known.
- Partial information: The probability of a link is known as well as the realisation of the own links.
- Full information: The realisation of the entire network is known.

Under no information only the minimal amount of information is available to traders. The probabilities of all networks can be calculated and hence the equilibrium strategy depends only on the probability of a link. Partial information allows a trader to base the strategy on the number of own links and hence the equilibrium strategy depends on the number of a player's own links and the probability that other links are realised. With full information the entire network is known and the equilibrium strategy is based on the realisation of all links.

We show the partitions of the possible networks of the different information sets, for two buyers and two sellers, in Figs. 3.3-3.5. Networks that are not distinguishable are shown in the same partition.

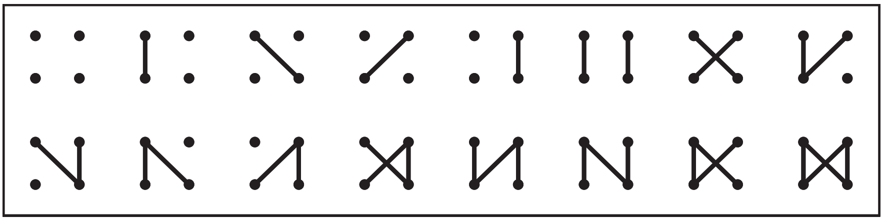


Figure 3.3: Partition under no information.

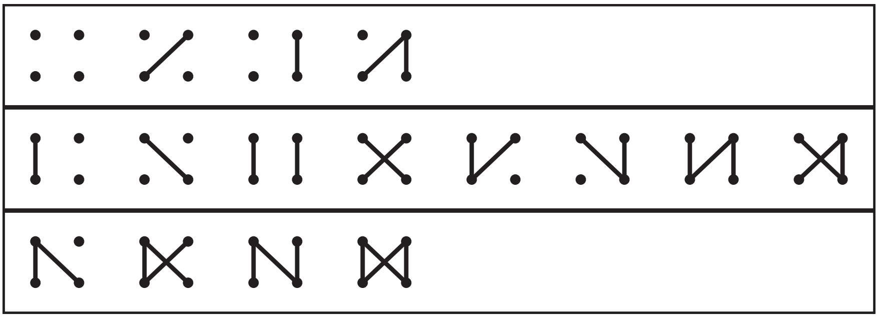


Figure 3.4: Partitions with partial information from the perspective of the top left node.

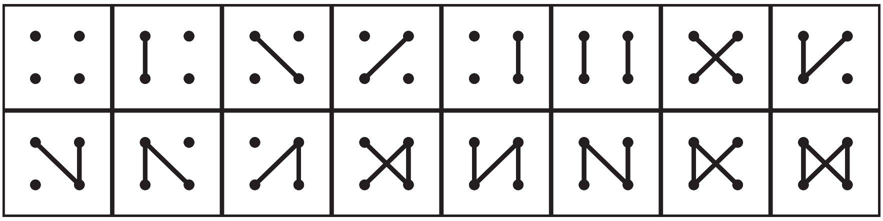


Figure 3.5: Partitions under full information from the perspective of the top left node.

3.3 Complete information about valuations and costs

To compare the expected efficiency given different information sets we consider a market with two buyers and two sellers. Under complete information we assume that valuations and costs are equal to one respectively zero, which is common knowledge. For each information set we calculate the symmetric Nash equilibrium strategies from the profit functions given in Appendix 1. These profit functions are a multiplication of the profit of trade and an indicator function that attains the value one if the trade is feasible.

No Information

Under no information traders have no knowledge about the realisation of links; but they know the probability that they occur. All the traders have the same information and thus they use the same deterministic strategy. Naturally, bids can be decreased and asks increased until they are equal. Hence in the unique symmetric Nash equilibrium buyers bid $\frac{1}{2}$ and sellers ask $\frac{1}{2}$. Given the limitations of p full efficiency is reached; i.e. strategic behaviour does not reduce efficiency.

Partial Information

For computational reasons we restrict offers of traders to the grid $[0, \frac{1}{2k}, ..., 1]$. Traders with one link may prefer to be less aggressive to outcompete other traders. This requires them to increase their bid or decrease their ask by $\frac{1}{2k}$. With a rougher grid this becomes less attractive. For a rough grid with $k < 5$ buyers bid and sellers ask $\frac{1}{2}$ in equilibrium and hence full efficiency is reached.

Below we show the equilibrium strategies as a function of p for $k = 5$, the roughest grid that does not always lead to full efficiency. First given is the markup of a trader with one link, second the markup of a trader with two links. For mixed strategies the probabilities are given by ρ_i. In the range $\frac{1}{\sqrt{11}} < p < \frac{1}{3}$ the latter equilibrium is unstable with respect to the strategy of traders with two links. If one trader deviates to the stable equilibrium it is optimal for other traders to deviate also, because this allows for trades between agents with two links.

$$
\begin{array}{lll}
0 & < p < \frac{5-\sqrt{5}}{10}: & [\frac{1}{2}], [\frac{1}{2}]. \\
\frac{5-\sqrt{5}}{10} & < p < \frac{\sqrt{17}-3}{4}: & [\rho_1 \frac{2}{5}, (1-\rho_1)\frac{3}{5}], [\frac{3}{5}], \text{where } \rho_1 = \frac{-4}{2p^2+3p-5}. \\
\frac{\sqrt{17}-3}{4} & < p < \frac{1}{\sqrt{11}}: & [\frac{2}{5}], [\frac{3}{5}]. \\
\frac{1}{\sqrt{11}} & < p < \frac{1}{3}: & [\frac{2}{5}], [\frac{1}{2}] \text{ stable and } [\frac{2}{5}], [\frac{3}{5}] \text{ unstable.} \\
\frac{1}{3} & < p < \frac{5+\sqrt{5}}{10}: & [\rho_1 \frac{3}{10}, \rho_2 \frac{2}{5}, (1-\rho_1-\rho_2)\frac{1}{2}], [\frac{1}{2}], \\
 & & \text{where } \rho_1 = -\frac{-8+45p-45p^2}{41(p-1)p} \text{ and } \rho_2 = -2\frac{9-25p+25p^2}{41(p-1)p}. \\
\frac{5-\sqrt{5}}{10} & < p < 1: & [\frac{1}{2}], [\frac{1}{2}].
\end{array}
$$

For $k = 5$ we find that full efficiency is not attained for $\frac{5-\sqrt{5}}{10} < p < \frac{1}{\sqrt{11}}$. When the grid is sufficiently dense full efficiency is not reached for every value of p. For denser grids this area increases and hence the result may also hold without the assumption of a grid of strategies. The subset of p for which traders with one link become less aggressive increases. As k goes to infinity these traders use mixed strategies over an infinite number of strategies. When the probability p is sufficiently small, traders with two links will become more aggressive when traders with one link are less aggressive. This does not hold when the probability p is relatively large, because this will cause a profit of zero in a fully connected network. For a subset of p, strategic behaviour reduces efficiency.

Full Information

Under full information traders have full knowledge about the realisation of the network. When both sides of the market have the same size, bids and asks equal one half in the symmetric equilibrium. For traders with one link it is not profitable to be less aggressive. When one side of the market is thinner, it extracts all the possible surplus. Agents with one link are less aggressive to outcompete the other trader on the same side of the market. Given the limitations of p full efficiency is reached, i.e. efficiency is not reduced by strategic behaviour.

No and full information lead to full efficiency, given the limitations of the network structure. For a grid of possible strategies we show that under partial information the strategic behaviour of traders decreases efficiency for a subset of p. Moreover, we argue that with a denser grid efficiency is decreased for a larger range of values for p. Under no and full information, restricting strategies of traders to a grid has no effect. Hence a non-monotonicity occurs and partial information is weakly dominated. Under complete information about traders' valuations and costs it is optimal when traders either receive all or no information about the network structure.

3.4 Incomplete information about valuations and costs

Under incomplete information, valuations and costs are uniformly distributed on $[0, 1]$, where the distribution is common knowledge but the realisations are private information. To compare the expected efficiency given different information sets we again consider a market with two buyers and two sellers. The necessary calculations for every information set are shown in Appendix 2. As an example, the best response functions under full information are given below for a network where buyer b_1 is connected with both sellers. We solve these to find the Nash equilibrium strategies and calculate expected allocative efficiency, volume and profit.

Example

Network $b_1 \leftrightarrow s_1$ & $b_1 \leftrightarrow s_2$

In equilibrium it holds that $m_1^u = m_2^u = m^u$ and thus buyer b_1 trades with the seller with the lowest cost $c_{\text{min}} = \min(c_1, c_2)$, which has pdf $2 - 2c_{\text{min}}$. We denote the profit of a buyer with bid β_i trading with a seller with ask α_j as $\pi(\beta_i, \alpha_j)$ and similar for sellers. For simplicity we disregard in this notation that the offers are a function of both the strategy and the valuation or cost. The integration limits are set to indicate the region of valuations and costs in which trade occurs:

$$\frac{\partial}{\partial m_1^d} \int_0^{\frac{1-m_1^d-m^u}{1-m^u}} \int_{\frac{c_{\text{min}}+m^u(1-c_{\text{min}})}{1-m_1^d}}^1 \pi(\beta_1, \alpha_{\text{min}})(2 - 2c_{\text{min}})\mathrm{d}v_1\mathrm{d}c_{\text{min}} = 0.$$

Seller s_1 only trades when its ask is lower than the ask from seller s_2:

$c_1 + m_1^u(1 - c_1) < c_2 + m_2^u(1 - c_2)$. For a given cost c_1 this happens with probability $\mathbb{P}(\text{trade}) = 1 - \frac{(1-m_1^u)c_1+m_1^u-m_2^u}{1-m_2^u}$:

$$\left[\frac{\partial}{\partial m_1^u} \int_0^{\frac{1-m_1^d-m_1^u}{1-m_1^u}} \int_{\frac{c_1+m_1^u(1-c_1)}{1-m_1^d}}^1 \pi(\alpha_1, \beta_1)\mathbb{P}(\text{trade})\mathrm{d}v_1\mathrm{d}c_1\right]_{\{m_2^u=m_1^u\}} = 0.$$

Solving these best response functions gives the Nash equilibrium strategies

$m^u = m_1^u = m_2^u \approx 0.110$ and $m_1^d \approx 0.341$.

The expected efficiency given the reductions invoked by absence of links is given by

$$\mathbb{E}(\text{AE}) = \frac{\int_0^{\frac{1-m_1^d-m^u}{1-m^u}} \int_{\frac{c_{\text{min}}+m^u(1-c_{\text{min}})}{1-m_1^d}}^1 (v_1-c_{\text{min}})(2-2c_{\text{min}})\mathrm{d}v_1\mathrm{d}c_{\text{min}}}{\int_0^1 \int_{c_{\text{min}}}^1 (v_1-c_{\text{min}})(2-2c_{\text{min}})\mathrm{d}v_1\mathrm{d}c_{\text{min}}} \approx 0.858.$$

The ratio between the expected number of trades and the maximal number of trades gives the expected volume:

$$\mathbb{E}(\text{Volume}) = \frac{1}{2}\int_0^{\frac{1-m_1^d-m^u}{1-m^u}} \int_{\frac{c_{\min}+m^u(1-c_{\min})}{1-m_1^d}}^1 1\cdot(2-2c_{\min})\mathrm{d}v_1\mathrm{d}c_{\min} \approx 0.204.$$

Similarly to the best response functions above we calculate the expected profit for a trader having one link, respectively two links:

$$\mathbb{E}(\Pi^1) = \int_0^{\frac{1-m_1^d-m_1^u}{1-m_1^u}} \int_{\frac{c_1+m_1^u(1-c_1)}{1-m_1^d}}^1 \pi(\alpha_1,\beta_1)(1-c_1)\mathrm{d}v_1\mathrm{d}c_1 \approx 0.038,$$

$$\mathbb{E}(\Pi^2) = \int_0^{\frac{1-m_1^d-m^u}{1-m^u}} \int_{\frac{c_{\min}+m^u(1-c_{\min})}{1-m_1^d}}^1 \pi(\beta_1,\alpha_1)(2-2c_{\min})\mathrm{d}v_1\mathrm{d}c_{\min} \approx 0.138.$$

Comparisons

No information outperforms full information in terms of expected efficiency for values of p smaller than the benchmark $c \approx 0.106$, but for large values of p the opposite holds, as shown in Fig. 3.6. As the available information has no effect on the efficiency reduction due to absence of links, we emphasise solely the effect of strategic behaviour. Hence the ratio is shown between the realised efficiency and the maximal efficiency given the network structure. The maximum differences are reached at $p \approx 0.070$ and $p \approx 0.729$.

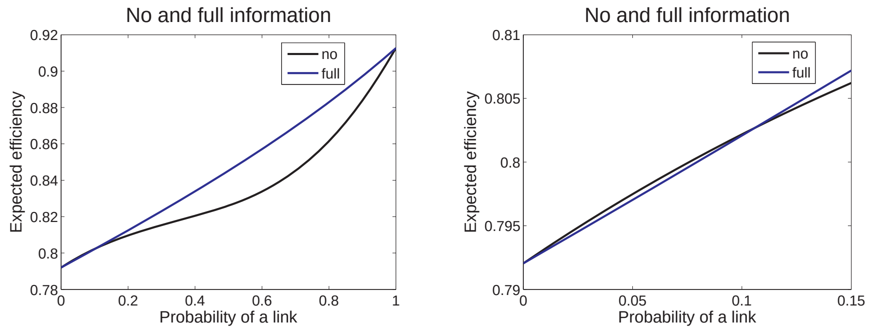


Figure 3.6: Efficiency no and full information.

We find that the amount of information available to traders has a non-monotonic effect on efficiency; irrespective of the probability of a link, partial information leads to the highest expected efficiency. Moreover, we observe that switching from complete to incomplete information reverses the shape of the non-monotonicity. We conclude that in terms of efficiency the following order of information sets holds:

$$0 < p < c : \mathbb{E}(\mathrm{AE}_{\text{partial}}) > \mathbb{E}(\mathrm{AE}_{\text{no}}) > \mathbb{E}(\mathrm{AE}_{\text{full}}),$$
$$c < p < 1 : \mathbb{E}(\mathrm{AE}_{\text{partial}}) > \mathbb{E}(\mathrm{AE}_{\text{full}}) > \mathbb{E}(\mathrm{AE}_{\text{no}}).$$

The maximum difference between information sets is reached near $p = \frac{2}{3}$ where the probability of having one respectively two links is equal. At this point uncertainty about the network structure is the most reduced by additional information. Fig. 3.7 shows the efficiency under strategic behaviour given the restrictions of the network structure, between the different information sets:

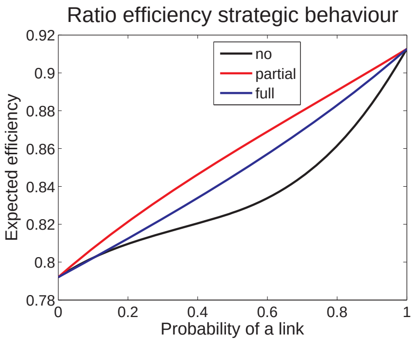
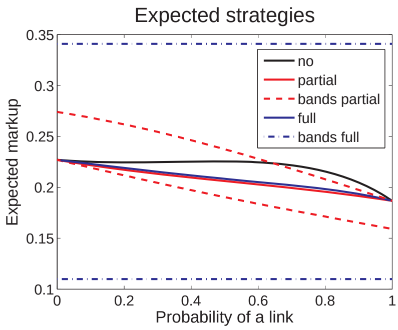
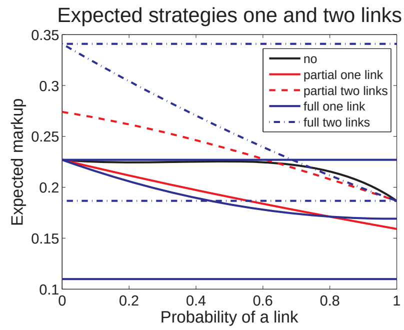


Figure 3.7: Comparison between ratios between efficiency under strategic behaviour and the maximal efficiency given the limitations of the network structure.

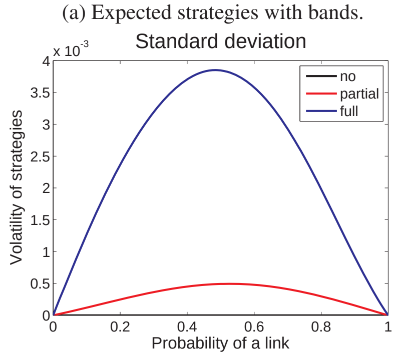
These results can be explained by the equilibrium strategies for which the average value for having one link, respectively two links, with their bands and the volatility for every value of p are displayed in Fig. 3.8. The no information strategies are the highest, but are not subject to volatility. The average partial and full information strategies are similar albeit the volatility is significantly larger in the latter case. A higher volatility in observable market power results in a



(a) Expected strategies with bands.



(b) Strategies for one and two links with bands.



(c) Volatility of strategies.

Figure 3.8: Distribution of strategies.

higher volatility of strategies, since the higher the observable market power the more aggressive offers the trader submits.

The expected value of the bargaining power measure, as in Calvó-Armengol (2001), can be calculated on the basis of the available information about the market. This measure takes on values in the interval $[0, 1]$ and is increasing in market power. For example with no information the expected bargaining power is always equal to one half. Under partial information, having two links results in an expected bargaining power larger than one half, in which case the trader will aim for a higher profit. For this trader it results in possible large profits but reduces the probability of trading. Having one link results in an expected bargaining power less than one half. Under full information certain networks lead to an even higher dispersion between traders' expected market powers.

Volatility of strategies has a negative effect; lower markups cause a slightly higher efficiency whereas higher markups may result in absence of trade. Partial information leads to the highest expected efficiency and the negative effects of higher markups for no information and high volatility for full information are similar.

For example for $p = \frac{1}{2}$, under no information a trader will always use the markup strategy 0.224. Under partial information a trader with one link uses 0.190 and a trader with two links 0.237. Under full information the markup does not only depend on the own number of links but also on the links of others. If a trader has one link his strategy ranges from 0.110 to 0.227, with two links from 0.187 to 0.341.

Volume

In Fig. 3.9 the expected number of trades, i.e. the volume, shows a similar comparison as the expected efficiency. For $p > 0.030$ we find that full information leads to a higher volume than no information, for small p the opposite holds. For every value of the probability of a link, partial information leads to the highest expected volume.

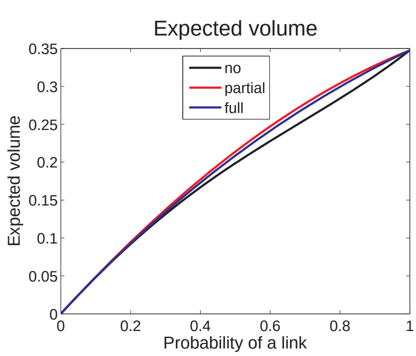


Figure 3.9: Expected volume for all information sets.

Expected profit

The expected profit for a trader that has one link, respectively two links, is shown in Fig. 3.10. A trader with one link has the highest expected profit under partial information; for $p < 0.408$ the lowest under full information, and otherwise the lowest under no information. A trader with two links has the highest expected profit under full information, the lowest under no information. Comparing partial and full information, a trader with one link has a higher expected profit under partial information and a trader with two links under full information. For any value of p the latter is dominated and hence partial information leads to the highest expected efficiency.

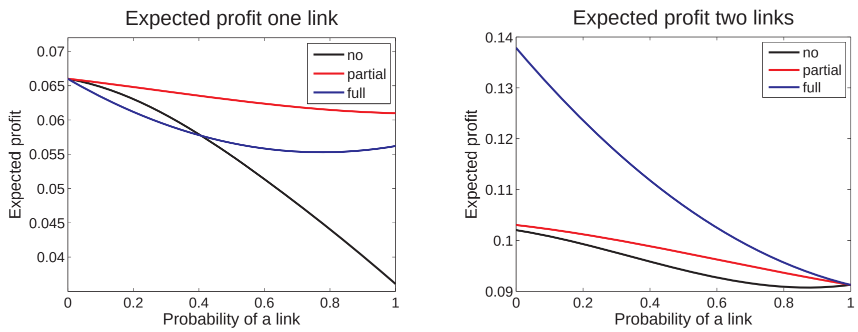


Figure 3.10: Expected profit for having one (left) and two links (right).

3.5 Concluding remarks

In a bipartite Erdős-Rényi market agents only trade in case they are linked to each other. The trading mechanism allows preferred trades to occur, not necessarily the socially optimal allocation of trades. In such a market three ordered sets of information about the network structure are considered; no, partial and full information. These information sets are compared under complete and incomplete information about valuations and costs.

With no information only the probabilities of all networks can be calculated and hence the equilibrium strategy depends on the probability of a link. Partial information allows a trader to base the strategy on the number of his own links and hence the equilibrium strategy depends on the number of a player's own links and the probability that other links are realised. With full information the entire network is known and the equilibrium strategies are based on the realisation of all links.

Under complete information about traders' valuations, in a market with two buyers and two sellers, no and full information lead to attain full efficiency for every probability of a link. Due to strategic behaviour of traders, under partial information allocative efficiency might be reduced. Hence we found that partial information is weakly dominated by no and full information and it is optimal if either everything or nothing of the realisation of the network structure is revealed to traders.

Under incomplete information about valuations and costs, expected efficiency given no and full information are comparable, when we assume that traders use markup and markdown strategies. For a small probability of a link no information outperforms and the opposite holds for a large probability. Partial information leads to the highest expected efficiency, since markups in no information and volatility of strategies in full information are higher, and thus strongly dominates no and full information. Higher markups and a larger volatility increase the probability of absence of trades and hence decrease the expected efficiency. Knowledge of the own links rather than only of the probability distribution improves efficiency, but adding knowledge of the links of others decreases efficiency. It is optimal, when only the realisation of own links is known. Therefore, more information does not necessarily lead to a higher expected allocative efficiency. Furthermore, the expected volume and the expected profit for traders when they have one link, respectively two links, are compared.

We demonstrated that the effect of the quantity of information available to traders on the allocative efficiency is non-monotonic. Moreover, the shape of this non-monotonicity flips over when we switch from complete to incomplete information about traders' valuations.

Appendix A: Profit functions complete information about valuations and costs

The profit functions of buyer b_1 under complete information about valuations and costs are shown below. These are used to calculate the Nash equilibrium markup and markdown strategies. Some best response functions are symmetric and hence we find symmetric markups, and simplifying assumptions about the strategies of others can be made.

Network 1: $b_1 \leftrightarrow s_1$

Buyer b_1 trades if $1 - m_1^d \geq m_1^u$, which results in a profit of $\pi(m_1^d, m_1^u) = 1 - \frac{1-m_1^d+m_1^u}{2}$:
$\mathbb{E}(\Pi_{b_1}) = \pi(m_1^d, m_1^u)\mathbb{1}_{\{1-m_1^d \geq m_1^u\}}$.

Network 2: $b_1 \leftrightarrow s_1$ & $b_1 \leftrightarrow s_2$

Buyer b_1 trades with the seller with the lowest ask $m^u = m_1^u = m_2^u$, if $1 - m_1^d \geq m^u$:
$\mathbb{E}(\Pi_{b_1}) = \pi(m_1^d, m^u)\mathbb{1}_{\{1-m_1^d \geq m^u\}}$.

Network 3: $b_1 \leftrightarrow s_1$ & $b_2 \leftrightarrow s_1$

b_1 only trades when its bid is higher than the bid from b_2, $1 - m_1^d > 1 - m_2^d$, or with probability one half if they are equal, if $1 - m_1^d \geq m_1^u$:
$\mathbb{E}(\Pi_{b_1}) = \pi(m_1^d, m_1^u)\mathbb{1}_{\{1-m_1^d \geq m_1^u\}}(\mathbb{1}_{\{1-m_1^d > 1-m_2^d\}} + \frac{1}{2}\mathbb{1}_{\{m_1^d = m_2^d\}})$.

Network 4: $b_1 \leftrightarrow s_1$ & $b_2 \leftrightarrow s_2$

Buyer b_1 trades if $1 - m_1^d \geq m_1^u$:
$\mathbb{E}(\Pi_{b_1}) = \pi(m_1^d, m_1^u)\mathbb{1}_{\{1-m_1^d \geq m_1^u\}}$.

Network 5: $b_1 \leftrightarrow s_2$, $b_2 \leftrightarrow s_1$ & $b_2 \leftrightarrow s_2$

Buyer b_1 is only connected to s_2. Unless b_2 and s_2 prefer to trade with each other he trades with s_2, if $1 - m_1^d \geq m_2^u$. This happens unless $m_2^d < 1 - m_1^d$ and $m_2^u < m_1^u$:
$\mathbb{E}(\Pi_{b_1}) = \pi(m_1^d, m_2^u)\mathbb{1}_{\{1-m_1^d \geq m_2^u\}}(1 - \mathbb{1}_{\{m_2^d < m_1^d\}}\mathbb{1}_{\{m_2^u < m_1^u\}})$.

Network 6: $b_1 \leftrightarrow s_1$, $b_1 \leftrightarrow s_2$ & $b_2 \leftrightarrow s_2$

If $1 - m_1^d \geq m_1^u$ and $1 - m_1^d \geq m_2^u$, buyer b_1 trades with s_1 except when he and s_2 both prefer to trade with each other. The latter happens when $1 - m_1^d > 1 - m_2^d$ and $m_2^u < m_1^u$:

$$\mathbb{E}(\Pi_{b_1}) = \pi(m_1^d, m_2^u)\mathbb{1}_{\{1-m_1^d \geq m_2^u\}}\mathbb{1}_{\{1-m_1^d > 1-m_2^d\}}\mathbb{1}_{\{m_2^u < m_1^u\}}$$
$$+ \pi(m_1^d, m_1^u)\mathbb{1}_{\{1-m_1^d \geq m_1^u\}}(1 - \mathbb{1}_{\{1-m_1^d > 1-m_2^d\}}\mathbb{1}_{\{m_2^u < m_1^u\}}).$$

Network 7: $b_1 \leftrightarrow s_1$, $b_1 \leftrightarrow s_2$, $b_2 \leftrightarrow s_1$ & $b_2 \leftrightarrow s_2$

In the symmetric equilibrium both sellers use the strategy $m^u = m_1^u = m_2^u$. Buyer b_1 can trade as long as his bid exceeds the askprice of the sellers:

$$\mathbb{E}(\Pi_{b_1}) = \pi(m_1^d, m^u)\mathbb{1}_{\{1-m_1^d \geq m^u\}}.$$

Appendix B: Efficiency under incomplete information about valuations and costs

The possible realisations of the network, without permutations, are given below. For each network we show the best response function of buyer b_1 under incomplete information about valuations and costs. Some best response functions are symmetric and hence we find symmetric markups, and simplifying assumptions about the strategies of others can be made. We denote the profit of a buyer with bid β_i trading with a seller with ask α_j as $\pi(\beta_i, \alpha_j)$ and similar for sellers. For simplicity we disregard in the notation that the offers are a function of both the strategy and the valuation or cost. The integration limits are set to indicate the region of valuations and costs in which trade occurs.

Network 1: $b_1 \leftrightarrow s_1$

$$\left[\frac{\partial}{\partial m_1^d} \int_0^{\frac{1-m_1^d-m_1^u}{1-m_1^u}} \int_{\frac{c_1+m_1^u(1-c_1)}{1-m_1^d}}^1 \pi(\beta_1, \alpha_1) \mathrm{d}v_1 \mathrm{d}c_1\right]_{\{m_1^u=m_1^d\}} = 0$$

Network 2: $b_1 \leftrightarrow s_1$ & $b_1 \leftrightarrow s_2$

In equilibrium, $m_1^u = m_2^u = m^u$ and thus b_1 trades with the seller with the lowest cost $c_{\text{min}} = \min(c_1, c_2)$ which has pdf $2 - 2c_{\text{min}}$:

$$\frac{\partial}{\partial m_1^d} \int_0^{\frac{1-m_1^d-m^u}{1-m^u}} \int_{\frac{c_{\text{min}}+m^u(1-c_{\text{min}})}{1-m_1^d}}^1 \pi(\beta_1, \alpha_{\text{min}})(2 - 2c_{\text{min}}) \mathrm{d}v_1 \mathrm{d}c_{\text{min}} = 0.$$

Network 3: $b_1 \leftrightarrow s_1$ & $b_2 \leftrightarrow s_1$

b_1 only trades when its bid is higher than the bid from b_2, $v_1(1 - m_1^d) > v_2(1 - m_2^d)$:

$$\left[\frac{\partial}{\partial m_1^d} \int_0^{\frac{1-m_1^d-m_1^u}{1-m_1^u}} \int_{\frac{c_1+m_1^u(1-c_1)}{1-m_1^d}}^1 \pi(\beta_1, \alpha_1) \frac{v_1(1-m_1^d)}{(1-m_2^d)} \mathrm{d}v_1 \mathrm{d}c_1\right]_{\{m_2^d=m_1^d\}} = 0.$$

Network 4: $b_1 \leftrightarrow s_1$ & $b_2 \leftrightarrow s_2$

The network is split into two separate markets; the best response function of b_1 is the same as in network 1.

Network 5: $b_1 \leftrightarrow s_2$, $b_2 \leftrightarrow s_1$ & $b_2 \leftrightarrow s_2$

b_1 is only connected to s_2 and unless b_2 and s_2 prefer to trade with each other he can trade with s_2. This happens with probability $\mathbb{P}(\text{trade}) = 1-(1-\min\{1, \frac{v_1(1-m_1^d)}{1-m_2^d}\})(1-\frac{(1-m_2^u)c_2+m_2^u-m_1^u}{1-m_1^u})$. We disregard the possibility that the latter term is negative, since in equilibrium the markup of a trader with two links is higher than the markup of a trader with one link:

$$\left[\frac{\partial}{\partial m_1^d}\int_0^{\frac{1-m_1^d-m_2^u}{1-m_2^u}}\int_{\frac{c_2+m_2^u(1-c_2)}{1-m_1^d}}^1 \pi(\beta_1,\alpha_2)\mathbb{P}(\text{trade})\mathrm{d}v_1\mathrm{d}c_2\right]_{\{m_1^u=m_1^d, m_2^u=m_2^d\}} = 0.$$

Network 6: $b_1 \leftrightarrow s_1$, $b_1 \leftrightarrow s_2$ & $b_2 \leftrightarrow s_2$

b_1 trades with s_1 except when he and s_2 both prefer to trade with each other. This happens with probability $\frac{v_1(1-m_1^d)}{1-m_2^d}(1-\frac{(1-m_2^u)c_2+m_2^u-m_1^u}{1-m_1^u}) = \frac{v_1(1-m_1^d)}{1-m_2^d}\cdot\max\{0, \frac{(1-m_1^u)c_1+m_1^u-m_2^u}{1-m_2^u}\}$, where we disregard the possibility that the first term is negative:

$$\left[\frac{\partial}{\partial m_1^d}\int_0^{\frac{1-m_1^d-m_1^u}{1-m_1^u}}\int_{\frac{c_1+m_1^u(1-c_1)}{1-m_1^d}}^1 \pi(\beta_1,\alpha_1)(1-\frac{v_1(1-m_1^d)}{1-m_2^d}\cdot\max\{0,\frac{(1-m_1^u)c_1+m_1^u-m_2^u}{1-m_2^u}\})\mathrm{d}v_1\mathrm{d}c_1\right.$$
$$\left.+\int_0^{\frac{1-m_1^d-m_2^u}{1-m_2^u}}\int_{\frac{c_2+m_2^u(1-c_2)}{1-m_1^d}}^1 \pi(\beta_1,\alpha_2)\frac{v_1(1-m_1^d)}{1-m_2^d}(1-\frac{(1-m_2^u)c_2+m_2^u-m_1^u}{1-m_1^u})\mathrm{d}v_1\mathrm{d}c_2\right]_{\{m_1^u=m_2^d, m_2^u=m_1^d\}}$$
$$=0.$$

Network 7: $b_1 \leftrightarrow s_1$, $b_1 \leftrightarrow s_2$, $b_2 \leftrightarrow s_1$ & $b_2 \leftrightarrow s_2$

In equilibrium, $m_1^u = m_2^u = m^u$ and thus b_1 trades with the seller with the lowest cost $c_{\text{min}} = \min(c_1, c_2)$ which has pdf $2-2c_{\text{min}}$, if his bid is higher than the bid of b_2. For a given value of $\beta_1 = v_1(1-m_1^d)$ this happens with probability $\frac{v_1(1-m_1^d)}{1-m_2^d}$. b_1 trades with the seller with the highest cost $c_{\text{max}} = \max(c_1, c_2)$ which has pdf $2c_{\text{max}}$, if his bid is lower than the bid of b_2. For a given value of $\beta_1 = v_1(1-m_1^d)$ this happens with probability $1-\frac{v_1(1-m_1^d)}{1-m_2^d}$:

$$\left[\frac{\partial}{\partial m_1^d}\int_0^{\frac{1-m_1^d-m^u}{1-m^u}}\int_{\frac{c_{\text{min}}+m^u(1-c_{\text{min}})}{1-m_1^d}}^1 \pi(\beta_1,\alpha_{\text{min}})(2-2c_{\text{min}})\frac{v_1(1-m_1^d)}{1-m_2^d}\mathrm{d}v_1\mathrm{d}c_{\text{min}}\right.$$
$$\left.+\int_0^{\frac{1-m_1^d-m^u}{1-m^u}}\int_{\frac{c_{\text{max}}+m^u(1-c_{\text{max}})}{1-m_1^d}}^1 \pi(\beta_1,\alpha_{\text{max}})2c_{\text{max}}(1-\frac{v_1(1-m_1^d)}{1-m_2^d})\mathrm{d}v_1\mathrm{d}c_{\text{max}}\right]_{\{m^u=m_2^d=m_1^d\}} = 0.$$

Full Information

In the full information setting, traders have knowledge of the entire realisation of the network. Hence Nash equilibrium strategies are calculated per possible network.

Network: $b_1 \leftrightarrow s_1$

Solving the best response function of network 1 gives the Nash equilibrium strategies $m_1^d = m_1^u \approx 0.227$. This allows us to calculate expected allocative efficiency given the limitations of the network structure, which is the ratio between the expected surplus from trade, divided by the total expected surplus:

$$\mathbb{E}(\mathrm{AE}) = \frac{\int_0^{\frac{1-m_1^d-m_1^u}{1-m_1^u}} \int_{\frac{c_1+m_1^u(1-c_1)}{1-m_1^d}}^1 (v_1-c_1)\mathrm{d}v_1\mathrm{d}c_1}{\int_0^1\int_{c_1}^1 (v_1-c_1)\mathrm{d}v_1\mathrm{d}c_1} \approx 0.792.$$

Moreover, the expected volume, the ratio between the expected number of trades and the maximal number of trades, equals:

$$\mathbb{E}(\mathrm{Volume}) = \frac{1}{2}\int_0^{\frac{1-m_1^d-m_1^u}{1-m_1^u}} \int_{\frac{c_1+m_1^u(1-c_1)}{1-m_1^d}}^1 1\mathrm{d}v_1\mathrm{d}c_1 \approx 0.125.$$

Similarly to the best response function above we calculate the expected profit for a trader that has one link:

$$\mathbb{E}(\Pi^1) = \int_0^{\frac{1-m_1^d-m_1^u}{1-m_1^u}} \int_{\frac{c_1+m_1^u(1-c_1)}{1-m_1^d}}^1 \pi(\beta_1,\alpha_1)\mathrm{d}v_1\mathrm{d}c_1 \approx 0.066.$$

Network: $b_1 \leftrightarrow s_1$ & $b_1 \leftrightarrow s_2$

Solving the best response functions of network 2 and a symmetric version of 3 gives the Nash equilibrium strategies $m^u = m_1^u = m_2^u \approx 0.110$ & $m_1^d \approx 0.341$:

$$\mathbb{E}(\mathrm{AE}) = \frac{\int_0^{\frac{1-m_1^d-m^u}{1-m^u}} \int_{\frac{c_{\min}+m^u(1-c_{\min})}{1-m_1^d}}^1 (v_1-c_{\min})(2-2c_{\min})\mathrm{d}v_1\mathrm{d}c_{\min}}{\int_0^1\int_{c_{\min}}^1 (v_1-c_{\min})(2-2c_{\min})\mathrm{d}v_1\mathrm{d}c_{\min}} \approx 0.858,$$

$$\mathbb{E}(\mathrm{Volume}) = \frac{1}{2}\int_0^{\frac{1-m_1^d-m^u}{1-m^u}} \int_{\frac{c_{\min}+m^u(1-c_{\min})}{1-m_1^d}}^1 1\cdot(2-2c_{\min})\mathrm{d}v_1\mathrm{d}c_{\min} \approx 0.204,$$

$$\mathbb{E}(\Pi^1) = \int_0^{\frac{1-m_1^d-m_1^u}{1-m_1^u}} \int_{\frac{c_1+m_1^u(1-c_1)}{1-m_1^d}}^1 \pi(\alpha_1,\beta_1)(1-c_1)\mathrm{d}v_1\mathrm{d}c_1 \approx 0.038,$$

$$\mathbb{E}(\Pi^2) = \int_0^{\frac{1-m_1^d-m^u}{1-m^u}} \int_{\frac{c_{\min}+m^u(1-c_{\min})}{1-m_1^d}}^1 \pi(\beta_1,\alpha_1)(2-2c_{\min})\mathrm{d}v_1\mathrm{d}c_{\min} \approx 0.138.$$

Network: $b_1 \leftrightarrow s_1$ & $b_2 \leftrightarrow s_2$

This network is similar to the first network, except that expected volume is doubled.

Network: $b_1 \leftrightarrow s_2$, $b_2 \leftrightarrow s_1$ & $b_2 \leftrightarrow s_2$

Solving the best response functions of network 5 and a symmetric version of 6 gives the Nash equilibrium strategies $m_1^u = m_1^d \approx 0.169$ & $m_2^u = m_2^d \approx 0.246$:

$$\begin{aligned}\mathbb{E}(\text{AE}) = & \Big[2\int_0^{\frac{1-m_2^d-m_1^u}{1-m_1^u}} \int_{\frac{c_1+m_1^u(1-c_1)}{1-m_2^d}}^1 (v_2-c_1) \\ & \times(1-(1-\tfrac{v_2(1-m_2^d)}{1-m_1^d})(1-\max\{0,\tfrac{(1-m_1^u)c_1+m_1^u-m_2^u}{1-m_2^u}\}))\mathrm{d}v_2\mathrm{d}c_1 \\ & + \int_0^{\frac{1-m_2^d-m_2^u}{1-m_2^u}} \int_{\frac{c_2+m_2^u(1-c_2)}{1-m_2^d}}^1 (v_2-c_2)\tfrac{v_2(1-m_2^d)}{1-m_1^d}(1-\tfrac{(1-m_2^u)c_2+m_2^u-m_1^u}{1-m_1^u})\mathrm{d}v_2\mathrm{d}c_2] \\ & /[2\int_0^1\int_{c_1}^1(v_2-c_1)(1-(1-v_2)(1-c_1))\mathrm{d}v_2\mathrm{d}c_1 + \int_0^1\int_{c_2}^1(v_2-c_2)v_2(1-c_2)\mathrm{d}v_2\mathrm{d}c_2\Big] \\ & \approx 0.867,\end{aligned}$$

$$\begin{aligned}\mathbb{E}(\text{Volume}) = & \tfrac{1}{2}\Big[2\int_0^{\frac{1-m_2^d-m_1^u}{1-m_1^u}} \int_{\frac{c_1+m_1^u(1-c_1)}{1-m_2^d}}^1 1 \\ & \times(1-(1-\tfrac{v_2(1-m_2^d)}{1-m_1^d})(1-\max\{0,\tfrac{(1-m_1^u)c_1+m_1^u-m_2^u}{1-m_2^u}\}))\mathrm{d}v_2\mathrm{d}c_1 \\ & + \int_0^{\frac{1-m_2^d-m_2^u}{1-m_2^u}} \int_{\frac{c_2+m_2^u(1-c_2)}{1-m_2^d}}^1 1\cdot\tfrac{v_2(1-m_2^d)}{1-m_1^d}(1-\tfrac{(1-m_2^u)c_2+m_2^u-m_1^u}{1-m_1^u})\mathrm{d}v_2\mathrm{d}c_2\Big] \approx 0.293,\end{aligned}$$

$$\begin{aligned}\mathbb{E}(\Pi^1) = & \int_0^{\frac{1-m_1^d-m_2^u}{1-m_2^u}} \int_{\frac{c_2+m_2^u(1-c_2)}{1-m_1^d}}^1 \pi(\beta_1,\alpha_2) \\ & \times(1-(1-\min\{1,\tfrac{v_1(1-m_1^d)}{1-m_2^d}\})(1-\tfrac{(1-m_2^u)c_2+m_2^u-m_1^u}{1-m_1^u}))\mathrm{d}v_1\mathrm{d}c_2 \\ & \approx 0.056,\end{aligned}$$

$$\begin{aligned}\mathbb{E}(\Pi^2) = & \int_0^{\frac{1-m_2^d-m_1^u}{1-m_1^u}} \int_{\frac{c_1+m_1^u(1-c_1)}{1-m_2^d}}^1 \pi(\beta_2,\alpha_1)(1-\tfrac{v_2(1-m_2^d)}{1-m_1^d}\cdot\max\{0,\tfrac{(1-m_1^u)c_1+m_1^u-m_2^u}{1-m_2^u}\})\mathrm{d}v_2\mathrm{d}c_1 \\ & + \int_0^{\frac{1-m_2^d-m_2^u}{1-m_2^u}} \int_{\frac{c_2+m_2^u(1-c_2)}{1-m_2^d}}^1 \pi(\beta_2,\alpha_1)\tfrac{v_2(1-m_2^d)}{1-m_1^d}(1-\tfrac{(1-m_2^u)c_2+m_2^u-m_1^u}{1-m_1^u})\mathrm{d}v_2\mathrm{d}c_2 \approx 0.099.\end{aligned}$$

Network: $b_1 \leftrightarrow s_1$, $b_1 \leftrightarrow s_2$, $b_2 \leftrightarrow s_1$ & $b_2 \leftrightarrow s_2$

Solving the best response function of network 7 gives the Nash equilibrium strategies $m^u = m_1^u = m_2^u = m_1^d = m_2^d \approx 0.187$:

$$\mathbb{E}(\mathrm{AE}) = \Big[\int_0^{\frac{1-m_1^d-m^u}{1-m^u}} \int_{\frac{c_{\min}+m^u(1-c_{\min})}{1-m_1^d}}^1 (v_1 - c_{\min})(2-2c_{\min})\frac{v_1(1-m_1^d)}{1-m_2^d}\mathrm{d}v_1\mathrm{d}c_{\min}$$
$$+ \int_0^{\frac{1-m_1^d-m^u}{1-m^u}} \int_{\frac{c_{\max}+m^u(1-c_{\max})}{1-m_1^d}}^1 (v_1 - c_{\max})2c_{\max}(1-\frac{v_1(1-m_1^d)}{1-m_2^d})\mathrm{d}v_1\mathrm{d}c_{\max}\Big]$$
$$/\Big[\int_0^1 \int_{c_{\min}}^1 (v_1 - c_{\min})(2-2c_{\min})v_1\mathrm{d}v_1\mathrm{d}c_{\min}$$
$$+ \int_0^1 \int_{c_{\max}}^1 (v_1 - c_{\max})2c_{\max}(1-v_1)\mathrm{d}v_1\mathrm{d}c_{\max}\Big] \approx 0.913,$$

$$\mathbb{E}(\mathrm{Volume}) = \tfrac{1}{2}\Big[2\int_0^{\frac{1-m_1^d-m^u}{1-m^u}} \int_{\frac{c_{\min}+m^u(1-c_{\min})}{1-m_1^d}}^1 1\cdot(2-2c_{\min})\frac{v_1(1-m_1^d)}{1-m_2^d}\mathrm{d}v_1\mathrm{d}c_{\min}$$
$$+2\int_0^{\frac{1-m_1^d-m^u}{1-m^u}} \int_{\frac{c_{\max}+m^u(1-c_{\max})}{1-m_1^d}}^1 1\cdot 2c_{\max}(1-\frac{v_1(1-m_1^d)}{1-m_2^d})\mathrm{d}v_1\mathrm{d}c_{\max}\Big] \approx 0.347,$$

$$\mathbb{E}(\Pi^2) = \int_0^{\frac{1-m_1^d-m^u}{1-m^u}} \int_{\frac{c_{\min}+m^u(1-c_{\min})}{1-m_1^d}}^1 \pi(\beta_1,\alpha_{\min})(2-2c_{\min})v_1\mathrm{d}v_1\mathrm{d}c_{\min}$$
$$+ \int_0^{\frac{1-m_1^d-m^u}{1-m^u}} \int_{\frac{c_{\max}+m^u(1-c_{\max})}{1-m_1^d}}^1 \pi(\beta_1,\alpha_{\max})2c_{\max}(1-v_1)\mathrm{d}v_1\mathrm{d}c_{\max} \approx 0.091.$$

Combining the 4 possibilities of having only one link in the network, the 4 possibilities of having one trader that has two links, the 2 possibilities of having two linked pairs, the 4 possibilities of having 3 links in total, with the possibility of having a fully connected network, gives a function of the expected efficiency in terms of the probability of a link. We show the expected efficiency reduction due to strategic behaviour and the total expected efficiency. The latter is the ratio of efficiency reductions due to strategic behaviour and the limitations of the network:

$$\mathbb{E}(\mathrm{AE_s}) = \frac{0.132\cdot 4p(1-p)^3+0.215\cdot 4p^2(1-p)^2+2\cdot 0.132\cdot 2p^2(1-p)^2+0.311\cdot 4p^3(1-p)+0.366\cdot p^4}{\frac{1}{6}\cdot 4p(1-p)^3+\frac{1}{4}\cdot 4p^2(1-p)^2+\frac{1}{3}\cdot 2p^2(1-p)^2+\frac{43}{120}\cdot 4p^3(1-p)+\frac{2}{5}\cdot p^4},$$

$$\mathbb{E}(\mathrm{AE_{p,s}}) = \frac{0.132\cdot 4p(1-p)^3+0.215\cdot 4p^2(1-p)^2+2\cdot 0.132\cdot 2p^2(1-p)^2+0.311\cdot 4p^3(1-p)+0.366\cdot p^4}{\frac{2}{5}}.$$

The expected volume as a function of p is given by

$$\begin{aligned}\mathbb{E}(\text{Volume}) &= 0.125 \cdot 4(1-p)^3 p + 0.204 \cdot 4(1-p)^2 p^2 + 2 \cdot 0.125 \cdot 2(1-p)^2 p^2 \\ &\quad + 0.293 \cdot 4(1-p)p^3 + 0.347 \cdot p^4 \\ &\approx 0.499 \cdot p - 0.181 \cdot p^2 + 0.039 \cdot p^3 - 0.009 \cdot p^4.\end{aligned}$$

Furthermore we calculate the expected strategy as a function of the probability of a link:

$$\begin{aligned}\mathbb{E}(m) &= 0.227 \cdot 4(1-p)^3 p + \frac{2 \cdot 0.110 + 0.341}{3} \cdot 4(1-p)^2 p^2 + 0.227 \cdot 2(1-p)^2 p^2 \\ &\quad + \frac{0.169 + 0.246}{2} \cdot 4(1-p)p^3 + 0.187 \cdot p^4 \\ &\approx \frac{0.908 \cdot p - 1.522 \cdot p^2 + 1.150 \cdot p^3 - 0.349 \cdot p^4}{1-(1-p)^4}.\end{aligned}$$

Conditioning on having one link, respectively two links, we calculate the expected profit:

$$\begin{aligned}\mathbb{E}(m^1) &= 0.227 \cdot (p(1-p) + (1-p)^2) + 0.110 \cdot p(1-p) + 0.169 \cdot p^2 \\ &\approx 0.227 - 0.117 \cdot p + 0.059 \cdot p^2, \\ \mathbb{E}(m^2) &= 0.341 \cdot (1-p)^2 + 0.246 \cdot 2p(1-p) + 0.187 \cdot p^2 \approx 0.341 - 0.191 \cdot p + 0.036 \cdot p^2.\end{aligned}$$

Moreover, the expected strategy of a trader with one link, respectively two links, is given by

$$\begin{aligned}\mathbb{E}(\Pi^1) &= 0.066 \cdot (p(1-p) + (1-p)^2) + 0.038 \cdot p(1-p) + 0.056 \cdot p^2 \\ &\approx 0.066 - 0.028 \cdot p + 0.018 \cdot p^2, \\ \mathbb{E}(\Pi^2) &= 0.138 \cdot (1-p)^2 + 0.099 \cdot 2p(1-p) + 0.091 \cdot p^2 \approx 0.138 - 0.177 \cdot p + 0.130 \cdot p^2.\end{aligned}$$

Partial Information

In this setting, traders know only the realisation of own links and the probability of the other links. Hence Nash equilibrium strategies for having one link m^1 and for having two links m^2 are found based on the best response functions given below.

If a trader has one link the possible networks are 1, 3, 4 or 5 and hence the best response function is given by

$$\Big[\frac{\partial}{\partial m_1^d}(1-p)p\int_0^{\frac{1-m_1^d-m^1}{1-m^1}}\int_{\frac{c_1+m^1(1-c_1)}{1-m_1^d}}^1 \pi(\beta_1,\alpha_1)\mathrm{d}v_1\mathrm{d}c_1$$
$$+(1-p)p\int_0^{\frac{1-m_1^d-m^2}{1-m^2}}\int_{\frac{c_1+m^2(1-c_1)}{1-m_1^d}}^1 \pi(\beta_1,\alpha_1)\frac{v_1(1-m_1^d)}{(1-m^1)}\mathrm{d}v_1\mathrm{d}c_1$$
$$+(1-p)^2\int_0^{\frac{1-m_1^d-m^1}{1-m^1}}\int_{\frac{c_1+m^1(1-c_1)}{1-m_1^d}}^1 \pi(\beta_1,\alpha_1)\mathrm{d}v_1\mathrm{d}c_1$$
$$+p^2\int_0^{\frac{1-m_1^d-m^2}{1-m^2}}\int_{\frac{c_2+m^2(1-c_2)}{1-m_1^d}}^1 \pi(\beta_1,\alpha_2)$$
$$\times\min\{1,1-(1-\frac{v_1(1-m_1^d)}{1-m^2})(1-\frac{(1-m^2)c_2+m^2-m^1}{1-m^1})\}\mathrm{d}v_1\mathrm{d}c_2\Big]_{\{m^1=m_1^d\}}=0.$$

If a trader has two links networks 2, 6 or 7 are possible and which results in the following best response function:

$$\Big[\frac{\partial}{\partial m_1^d}(1-p)^2\int_0^{\frac{1-m_1^d-m^1}{1-m^1}}\int_{\frac{c_{\min}+m^1(1-c_{\min})}{1-m_1^d}}^1 \pi(\beta_1,\alpha_{\min})(2-2c_{\min})\mathrm{d}v_1\mathrm{d}c_{\min}$$
$$+2(1-p)p[\int_0^{\frac{1-m_1^d-m^1}{1-m^1}}\int_{\frac{c_1+m^1(1-c_1)}{1-m_1^d}}^1 \pi(\beta_1,\alpha_1)\min\{1,1-\frac{v_1(1-m_1^d)}{1-m^1}\frac{(1-m^1)c_1+m^1-m^2}{1-m^2}\}\mathrm{d}v_1\mathrm{d}c_1$$
$$+\int_0^{\frac{1-m_1^d-m^2}{1-m^2}}\int_{\frac{c_2+m^2(1-c_2)}{1-m_1^d}}^1 \pi(\beta_1,\alpha_2)\frac{v_1(1-m_1^d)}{1-m^1}(1-\frac{(1-m^2)c_2+m^2-m^1}{1-m^1})\mathrm{d}v_1\mathrm{d}c_2]$$
$$+p^2\big[\int_0^{\frac{1-m_1^d-m^2}{1-m^2}}\int_{\frac{c_{\min}+m^2(1-c_{\min})}{1-m_1^d}}^1 \pi(\beta_1,\alpha_{\min})(2-2c_{\min})\frac{v_1(1-m_1^d)}{1-m^2}\mathrm{d}v_1\mathrm{d}c_{\min}$$
$$+\int_0^{\frac{1-m_1^d-m^2}{1-m^2}}\int_{\frac{c_{\max}+m^2(1-c_{\max})}{1-m_1^d}}^1 \pi(\beta_1,\alpha_{\max})2c_{\max}(1-\frac{v_1(1-m_1^d)}{1-m^2})\mathrm{d}v_1\mathrm{d}c_{\max}\big]\Big]_{\{m^2=m_1^d\}}=0.$$

The Nash equilibrium strategies are calculated with double precision from these two best response functions. The second derivatives in these points are smaller than zero; ensuring a maximum.

No Information

With no information the only knowledge about the network is the probability that a link occurs. Hence the best response function of b_1 is a weighted average over all seven best response functions given above. Because of symmetry we may assume that all other traders use the same strategy, i.e. $m = m_2^d = m_1^u = m_2^u$:

$$
\begin{aligned}
&\Big[\tfrac{\partial}{\partial m_1^d} 2(1-p)^3 p \int_0^{\frac{1-m_1^d-m}{1-m}} \int_{\frac{c_1+m(1-c_1)}{1-m_1^d}}^1 \pi(\beta_1,\alpha_1) \mathrm{d}v_1 \mathrm{d}c_1 \\
&+(1-p)^2p^2 \int_0^{\frac{1-m_1^d-m}{1-m}} \int_{\frac{c_{\min}+m(1-c_{\min})}{1-m_1^d}}^1 \pi(\beta_1,\alpha_1)(2-2c_{\min}) \mathrm{d}v_1 \mathrm{d}c_{\min} \\
&+2(1-p)^2p^2 \int_0^{\frac{1-m_1^d-m}{1-m}} \int_{\frac{c_1+m(1-c_1)}{1-m_1^d}}^1 \pi(\beta_1,\alpha_1) \tfrac{v_1(1-m_1^d)}{(1-m)} \mathrm{d}v_1 \mathrm{d}c_1 \\
&+2(1-p)^2p^2 \int_0^{\frac{1-m_1^d-m}{1-m}} \int_{\frac{c_1+m(1-c_1)}{1-m_1^d}}^1 \pi(\beta_1,\alpha_1) \mathrm{d}v_1 \mathrm{d}c_1 \\
&+2(1-p)p^3 \int_0^{\frac{1-m_1^d-m}{1-m}} \int_{\frac{c_2+m(1-c_2)}{1-m_1^d}}^1 \pi(\beta_1,\alpha_2)(1-(1-\min\{1,\tfrac{v_1(1-m_1^d)}{1-m}\})(1-c_2)) \mathrm{d}v_1 \mathrm{d}c_2 \\
&+2(1-p)p^3 [\int_0^{\frac{1-m_1^d-m}{1-m}} \int_{\frac{c_1+m(1-c_1)}{1-m_1^d}}^1 \pi(\beta_1,\alpha_1)(1-\min\{1,\tfrac{v_1(1-m_1^d)}{1-m}\}c_1) \mathrm{d}v_1 \mathrm{d}c_1 \\
&+\int_0^{\frac{1-m_1^d-m}{1-m}} \int_{\frac{c_2+m(1-c_2)}{1-m_1^d}}^1 \pi(\beta_1,\alpha_2) \tfrac{v_1(1-m_1^d)}{1-m}(1-c_2) \mathrm{d}v_1 \mathrm{d}c_2] \\
&+p^4 \big[\int_0^{\frac{1-m_1^d-m}{1-m}} \int_{\frac{c_{\min}+m(1-c_{\min})}{1-m_1^d}}^1 \pi(\beta_1,\alpha_{\min})(2-2c_{\min}) \tfrac{v_1(1-m_1^d)}{1-m} \mathrm{d}v_1 \mathrm{d}c_{\min} \\
&+\int_0^{\frac{1-m_1^d-m}{1-m}} \int_{\frac{c_{\max}+m(1-c_{\max})}{1-m_1^d}}^1 \pi(\beta_1,\alpha_{\max}) 2c_{\max}(1-\tfrac{v_1(1-m_1^d)}{1-m}) \mathrm{d}v_1 \mathrm{d}c_{\max} \big]\Big]_{\{m=m_1^d\}} = 0.
\end{aligned}
$$

The Nash equilibrium strategy m_1^d is solved to be the solution in $[0,1]$ of

$$
\begin{aligned}
\text{Root}\big[& -40+45p-41p^2+22p^3 \\
& +(300-340p+203p^2-61p^3)m_1^d \\
& +(-700+875p-254p^2-77p^3){m_1^d}^2 \\
& +(780-1070p+32p^2+306p^3){m_1^d}^3 \\
& +(-460+670p+124p^2-258p^3){m_1^d}^4 \\
& +(120-180p-40p^2+60p^3){m_1^d}^5\big] = 0.
\end{aligned}
$$