The central question that this thesis addresses is how economic agents learn to form price expectations, which are a crucial element of macroeconomic and financial models. The thesis applies a Genetic Algorithms model of learning to previous laboratory experiments, explaining the observed heterogeneity of individual forecasting behavior. It also studies the effect of information networks in this model, showing that information sharing may lead to more volatile price dynamics. Finally, the thesis reports on an experiment in which subjects either trade an asset or predict its price. The former turns out to be more difficult for the subjects than the forecasting task, which leads to repeated price bubbles.

Tomasz A. Makarewicz (1984) holds a MA degree in economics and philosophy from Warsaw University and a MSc degree in economics from the Tinbergen Institute. In 2011 he joined the Center for Nonlinear Dynamics in Economics and Finance to write his PhD thesis. His main interests are individual learning and heterogeneous price expectations in experiments and agent-based models.
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Chapter 1

Introduction

1.1 Learning, rationality and markets

Learning is one of the defining capacities of man, a fact that has been recognized by most of social sciences and philosophy. The 20th century witnessed advances in modern logic, specifically inquiries about what can be represented and proven within boundaries of a formal language. Since such formal languages stand as a necessary backbone of rationality, these logical investigations gave important insights into the matter of learning itself, with the most important applications in the field of artificial intelligence and computer sciences (Negnevitsky, 2005). On the other hand, psychologists, neuroscientists and social scientists in general have been trying to understand how learning is realized within our minds (Carey et al., 2014). The human brain continues to fascinate with the ability to solve complex problems in a matter of seconds, which seemingly contradicts its affinity for systematic biases and quirks (Kahneman, 2011).

Economic sciences stand as an interesting exception. The bulk of economic literature focuses on the paradigm of perfect rationality, in which economic agents see through the underlying market structure, understand the strategic nature of their interaction and possess mutual knowledge of each other’s (and common) rationality. The resulting rational equilibrium is ‘perfect’ in the sense of, broadly speaking, self-consistent and optimal response to the market constraints and decisions of other agents.

Learning is often mentioned as a justification for perfect rationality, nevertheless, it appears seldom as an explicit feature of typical mainstream economic models, especially in the context of finance or macroeconomics (see Evans and Honkapohja, 2001; Sargent, 1993, for a discussion). In contrast to what is implied by modern logic (Binmore, 1987) economists typically take the learning as automatically perfect and thus having no important dynamics of its own. This practice follows the classical arguments of
CHAPTER 1. INTRODUCTION

Friedman (1953); Lucas Jr. (1972); Lucas Jr (1986); Muth (1961). It is difficult to find a contemporary explication of the rational paradigm’s virtues (or validity), since its proponents take it as a methodological standard, a modeling tool that requires no further justification or explanation. An exception can be found in the concluding remarks of Blundell and Stoker (2005). How does the rational approach fare in modeling practice?

One of the key instances of learning in the economic context, and the focus of this thesis, are price expectations, namely learning to forecast prices. In a plethora of economic problems, agents need to decide on a specific action in the present, whereas the profitability of this action depends on future prices. For example, firms may face a production lag: they have to set up the produced quantity today, but they will sell it tomorrow, with tomorrow’s demand and price. Another important case are investors who buy financial assets, like stocks, in the hope that these assets later gain more value. In order to make their decisions in a reasonable fashion, both the producers and investors need somehow to forecast future prices.

The framework of perfect rationality emphasizes Rational Expectations (RE), understood as model-consistent price forecasts. In the example of the firms, every producer will optimize production based on a price forecast, that in turn follows the accurate belief of common mutually optimal responses to the underlying market clearing mechanism and fundamentals. The latter include distribution of individual market power, demand structure and production technologies. This leads to market clearing (aggregate production is equal to the demand), where price forecasts are self-fulfilling and no firm could improve its profit. The example of financial markets has a similar RE equilibrium: investors optimize their trades conditional on what turns out to be the realized price given these individual trades.

One can show that a RE equilibrium exists under mild assumptions about the market structure. However, the claim of the perfectly rational framework is that the RE equilibrium is the actual state of empirical markets. As mentioned above, this strong belief relies on a particular explication of individual learning. The argument goes that if agents would make poor price forecasts, they would loose profit in comparison with other agents. In a rational response, they would adjust their expectations. Therefore, agents will not make systematic mistakes and RE appears as a fixed point of the aggregate learning process (Muth, 1961). However, proponents of RE typically do not take this reasoning as an explicit model feature. Instead, they would think that

1In addition, one can use a similar evolutionary argument based on so called social learning: agents that make mistakes are replaced by smarter agents. Therefore RE is selected, or ‘learned’, in aggregate terms (Friedman 1953).
1.1. Learning, rationality and markets

Learning pushes the agents to the RE equilibrium, which can therefore be used as a valid approximation of market dynamics. In other words, learning to forecast is perfect: it comes as a crucial foundation of the model, yet it does not have to be investigated on its own.

This approach has two conceptual problems, however. First of all, it assumes what remains an empirical issue. We know that human learning is bounded (Kahneman, 2011), also by the very nature of limitations of logical languages (Binmore, 1987). It is therefore unclear a priori whether learning to forecast can end up with a ‘perfect’ outcome. This issue becomes even more severe if the economic environment is complex and evolves over time.

The second conceptual issue is that the RE equilibrium, as a fixed point of the individual learning and decisions, is inherently static. In the example of the producers economy, the structure of the RE equilibrium remains intact if the production problem becomes repeated: every period has the same realized (market clearing) price. Furthermore, every agent knows that from the beginning. Instead of trading repeatedly on a period-to-period basis, agents in the first period could simply sign a ‘social contract’ that would cover all their actions in all future periods (regardless of the time horizon). And nothing would change in the realized market dynamics.\(^2\)

Under RE, the market price can change only if there is a shock to the underlying fundamentals, such as production technology, preferences or asset dividend. Such shocks are purely exogenous: they are not anticipated and do not follow from anything in the model itself, but rather appear *deus ex machina*. Furthermore, they cannot push the economy to any meaningful dynamics. For example, an exogenous productivity shock would simply make the firms re-optimize their production portfolios, jump to the new market-clearing equilibrium and stay there forever (or until a new shock hits the economy). Similarly, DSGE models, which stand as a backbone of modern macroeconomics, explain the business cycle purely by such exogenous shocks (see An and Schorfheide, 2007, for a theoretical example together with an empirical application). These models do impose additional frictions (e.g., on price adjustment) in order to match the empirical persistence of macro dynamics; but their spirit remains intact.

The static nature of RE equilibria stands in contrast to what one intuitively thinks

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\(^2\)Assuming complete markets, the same holds for a (dynamic model) with uncertainty which leads to the Arrow–Debreu equilibrium. Here we leave aside the issue of multiplicity of RE equilibria. The agents in such a case could coordinate on jumping between the possible equilibria, even on a basis of period-to-period random signal. Nevertheless the reasoning stays intact, as again in the very first period the agents could sign an appropriate contract that would specify the period-to-period coordination mechanism conditional on the realized state of the economy. See Benhabib and Nishimura (2012) for an example of these ‘sunspot equilibria’.
of learning. Namely, agents learn by a process of trial-and-error, experiment with different strategies or wait until they have enough data for appropriate statistical inference, which then they update every time they access new relevant observations. And even if the agents are able to eventually learn the RE equilibrium, a shock to the economy would force them to re-evaluate their knowledge (such as their beliefs about productivity or fundamentals). Learning is therefore a ‘sticky’ and largely unpredictable process, which would impose additional inertia in economic variables that the RE approach cannot acknowledge, or does not want to acknowledge (Sims, 1980). Therefore, learning implies bounded rationality (Simon, 1972).

1.2 Evidence from markets and experiments

In the context of price expectations, how important is the fact that learning is a dynamic and possibly imperfect process? There is no a priori theoretical answer to this fundamentally empirical question. Instead, we can judge the relevance of learning to forecast through studies of market data and experiments.

Learning is not easily identifiable in market surveys, as these rarely accord with the assumption of ceteris paribus and hence can serve only as an indirect proof. Over the last three decades, however, economists gathered substantial evidence that the empirical price expectations are much more complicated than the rational framework implies. A natural alternative is thus to interpret the data as a signal of bounded rationality and learning dynamics. Specific examples include:

- **Consumer inflation expectations**: studies of micro-data surveys clearly indicate that households retain heterogeneous price expectations and most likely learn only from their individual experience, disregarding the full history of macro data (Malmendier and Nagel, 2009). They can be subject to systematic biases like ignoring the business cycle (Thomas Jr., 1999);

- **Financial markets**: asset prices exhibit patterns that are ‘paradoxical’ from the RE perspective (see De Long et al., 1990, for a discussion), including the following three most famous examples:

  1. difficulties in explaining asset prices with bare fundamentals (Israel and Moskowitz, 2013);
  2. investor overreaction to market signals (Bondt and Thaler, 2012);
Furthermore, in the recent two decades we experienced several financial meltdowns (Kindleberger and Aliber, 2011; Reinhart and Rogoff, 2008), which undermined the conviction of financial markets’ efficiency and rationality. Among the most recent crises were:

1. ‘Black Wednesday’ event, which caused the UK to leave the European Exchange Rate Mechanism (Söderlin, 2000),
2. East Asian crisis of the late 1990ties (Best, 2010; Mendoza, 2010);
3. ‘dot-com’, or internet technologies bubble (Griffin et al., 2011; Morris and Alam, 2012);
4. financial meltdown of 2007 (Erkens et al., 2012; Goodhart, 2008; Reinhart and Rogoff, 2009; Shiller, 2008), with an interesting case study of Iceland (Aliber and Zoega, 2011);
5. bubbles in the markets of the so called ‘cryptocurrencies’, like Litecoin, Dogecoin or Bitcoin (Yermack, 2013).

- Housing market: house prices in the USA experienced a bubble due to systematic over-evaluation of the fundamentals before 2007 (Case and Shiller, 2003), and the signs of this can be traced back even to 1990ties (Goodman Jr. and Ittner, 1992). Similar patterns can be observed in other countries, see Ambrose et al. (2013) for a three and a half century data set of the housing market in Amsterdam;

- Producers inflation expectations: survey inflation forecasts fail to comport rational expectations (Mavroeidis et al., 2014), while at the same time explaining an important part of firms’ decision making (Nunes, 2010a).

Laboratory experiments serve as an alternative to market surveys in evaluating human behavior (Smith, 2010). They offer a controlled setting in which researchers can directly setup the structure, fundamentals, information feedback and incentive schemes of the investigated market.\(^3\)

The most popular economic experiments are built over Game Theory settings, with simple benchmark games such as ‘p-beauty contest’ (Duffy and Nagel, 1997; Ho et al., 1998), ultimatum game (Güth et al., 1982), centipede game (McKelvey and Palfrey, 1992) and prisoner’s dilemma (Andreoni and Miller, 1993; Fehr and Gächter, 2000).\(^4\)

\(^3\)For the sake of fairness, it must be emphasized that the use of experiments for testing financial and macroeconomic models does raise some methodological issues of the so called external validity, see Guala and Mittone (2005) for a discussion.

\(^4\)Apart from the question of rationality, the first experiments on Game Theory sparked a discussion about the so-called other-regarding or social preferences, see Fischbacher and Gächter (2010) for literature overview and an example.
A more sophisticated applications, often based on oligopoly games, became popular in Industrial Organization (Huck et al., 1999; Offerman et al., 2002). The rational solutions to these games, understood as (refined) Nash Equilibria, are not necessarily the unique experimental outcome, with costly punishment in the public good game being the most famous example of a robust non-rational finding (Fehr and Gächter, 2000). And even if subjects converge to a Nash Equilibrium, they may need time to do so (Smith, 2010). For the example of the Industrial Organization setting, Offerman et al. (2002) investigate a simple oligopoly game. The authors show that subjects learn different equilibria (perfectly competitive, Cournot-Nash and collusive) depending on the information framing, despite the same underlying production and demand functions. These findings constitute strong evidence in support for the importance of individual learning.

An interesting example is the above mentioned ‘p-beauty contest’, which models a positive feedback between predictions and realization. This captures the spirit of financial markets, with self-fulfilling investor sentiments (Sonnemans and Tuinstra, 2010). In addition, the subjects are given perfect information about the underlying strategic structure of the game and therefore have all means to compute the self-consistent equilibrium already in the very first period of the game. However, experiments show that subjects require repeated interaction before they converge to the Nash Equilibrium. An even more striking finding is that the subjects remain largely heterogeneous and seem to use strategies of diversified levels of sophistication (compare with the survey on consumer’s inflation expectations by Malmendier and Nagel, 2009). This suggests that they undergo a process of learning, which remains ‘imperfect’ from the classical perspective.

Another class of financial market experiments comes with the seminal work by Smith et al. (1988). The authors investigated a market, in which subjects could trade an asset with a declining fundamental price. The result was consistent miss-pricing: subjects would overprice the asset for a substantial number of periods, and the resulting bubbles eventually crashed when the asset fundamental value approached zero. Follow-up studies focused on testing a number of suggested explanations of the experimental bubbles: uncertainty about rationality of other market participants, or ‘rational’ attempts to outsmart other subjects and play out the bubble (Lei et al., 2001); problems with understanding the declining fundamental (Huber and Kirchler, 2012; Kirchler

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5The outcome of Offerman et al. (2002) shows another issue with the rational framework. In the example of the producers game, equilibrium price and welfare crucially depend on whether the firms are price-takers or play a Nash Game. Since the rational framework contains no explicit learning, it cannot explain why in some markets agents realize that they have significant market power, while in other they consider themselves price-takers.
1.2. Evidence from markets and experiments

et al., 2012; Noussair et al., 2001); lack of trading experience (Dufwenberg et al., 2005, Smith et al. 1988); even ‘psychological irrationalities’ like trading to avoid boredom (Active Participation Hypothesis; Lei et al., 2001). It seems that these factors at best only partially explain the experimental bubbles (Noussair and Tucker, 2013).

The trading experiments suggest that the subjects indeed learn (for instance the role of experience is clear), but they learn behavior of a very different nature than what the rational framework postulates. Instead of the perfect, model-consistent price expectations, subjects are rather trying to come up with simple behavioral ‘rules of thumb’ that are good enough so far. As a result, their behavior is much more backward-looking and ‘imperfect’ than expected.

Since price forecasts are not directly observed in trading decisions, the natural step for experimentalists was to set up laboratory studies in order to control the forecasting itself (Marimon et al., 1993). In these Learning-to-Forecast experiments (see Assenza et al., 2014a; Hommes, 2011, for a comprehensive literature overview), subjects play the role of forecasting advisers to computer agents, such as financial investors or producers. Subjects give a price forecast, which the computer agents then use to optimize their decisions. This leads, through a specific market clearing condition, to the realized price and the subjects are paid based only on their forecasting performance.

As a result, we obtain an economically founded feedback between prices and price predictions that can be used to study individual learning conditional on the specific structure of the feedback. The two most important cases turned out to be negative and positive expectations feedback. In the earlier example of the producers, if they expect a high price, they overproduce and the realized price will be low. This is a case of negative feedback. In contrast, financial markets are characterized by self-fulfilling moods: if the investors are optimistic, they will buy more assets, which drives up the asset prices, yielding a positive feedback between expectations and realized market prices.

Heemeejer et al. (2009) show the significant differences of the two types of feedback. The authors report that in their simple linear setup, under the negative feedback treatment subjects quickly converge to the fundamental price, whereas the positive feedback treatment can induce significant price oscillations (see Sonnemans and Tuinstra, 2010).

\footnote{This experimental design has been criticized as simplistic, because in real markets agents are directly asked for trading decisions instead of price expectations. Among critics of experimental economics prevails a misconception that laboratory experiments should be as close to real economic environments as possible. From this perspective, any simplification of the experimental economy is perceived as a design flaw. This is a misunderstanding of the role of experiments. Their goal (and what natural scientists realized already in 16th century) is to isolate one specific factor in a setting simple enough that we can study it directly, without a need of dissecting it from other phenomena, unlike in real markets (see Smith, 2010, for some discussion).}
for a discussion of positive feedback). A follow-up study by Bao et al. (2012) confirms that these dynamics are robust against large and unanticipated changes to the fundamental price. Next to the sign, the complexity of the feedback plays an important role. Hommes et al. (2007) investigate a negative feedback system based on a nonlinear cobweb economy and find ‘excess volatility’: prices fluctuate irregularly around the fundamental price. The more the economy is unstable under the assumption of naive expectations, the more volatile the market becomes. As an example of a complex positive feedback, Hommes et al. (2005) study a two-period ahead non-linear asset pricing market. Their subjects coordinate on price oscillations of varied amplitude and period, which can furthermore change their pattern in a single 50-period session.

In line with findings of the other experiments, Learning-to-Forecast experiments indicate a high level of heterogeneity between the subjects, even at the level of their behavioral rules (Anufriev and Hommes, 2012; Heemeijer et al., 2009; Hommes, 2011). Again, this can be interpreted as a sign of independent learning, which is furthermore sensitive to the specific market environment. The RE solution clearly does not fit these learning dynamics. A pattern that seems to emerge instead is that the subjects learn, in rough terms, adaptive type of expectations under negative and trend following behavior under positive feedback.

The most important limitation of Learning-to-Forecast experiments is that they are based on markets in which the computerized agents behave optimally conditional on the subjects’ price forecasts.\footnote{As explained, this assumption is also the cornerstone of the rational framework. See Smith (2010) for a discussion about other underlying assumptions of rational behavior that do not survive laboratory testing.} In reality this assumption does not necessarily hold. Economists would typically take it for granted, however, and so there are only few studies that directly challenge it. A notable exceptions is a recent experiment by Bao et al. (2013), in which the authors ask their subjects both to (1) forecast prices and (2) setup the production (Learning-to-Optimize experimental design) in a negative feedback producers economy. They identify behavior that seems to contradict the assumption of optimal trading conditional on price forecasts. However, the design of the feedback eventually pushes the subjects to the fundamental solution. The non-optimal behavior may be more relevant in less stable markets with positive or non-linear negative feedback feature, but to our best knowledge this has not been systematically examined (see Assenza et al., 2014a, for a recent overview of the existing literature on the topic). We will investigate this in detail in Chapter 4.
1.3 Models of learning

1.3.1 Rational learning

The bulk of the economic literature did not respond to the criticism of perfect rationality and retains this framework to this day. As a result, theoretical models in economics seldom contain explicit learning features. For instance, this is visible in the design of typical DSGE models, which serve as the workhorse of the theoretical policy-oriented macroeconomics. However, there are some cases of ‘rational learning’. The most popular (especially in Game Theory and finance) is Bayesian updating: agents have a prior belief that they update conditional on unveiling market signals. This can lead to interesting dynamics, such as information cascades (Anderson and Holt, 1997). A more recent development are models of rational inattention (Sims, 2003, 2010): agents cannot perfectly process market information and instead optimize a noisy reaction to the noisy market environment. The approach of rational inattention is somewhat uncommon due to the technical challenges involved in solving such models, but applications can be found in macroeconomics (Maćkowiak and Wiederholt, 2009) and market organization literature Willems (2012).

Neither Bayesian updating nor rational inattention can exhaust the full meaning of learning, however, since learning ought to be understood as something more fundamental, active and insecure than just being subject to an information noise, which is processed in an optimal fashion.

Some proponents of the rational paradigm followed this notion of learning and tried to use it to defend perfect rationality. One important case is the work by Guesnerie (1992) on Eductive Learning. It is based on an idea that agents, following the belief that no agent will use dominated strategies, can iteratively reduce their strategy sets by eliminating those strategies that became dominated after previous iteration of this algorithm. Eductive Learning can be applied to varied economic models (including macro and finance models) and often supports rational solution as the limiting case of the belief of common rationality (Guesnerie, 2002).

Another attempt to use learning as a defense of the rational paradigm comes with so called adaptive learning (Evans and Honkapohja, 2001), which has important applications in macroeconomics (see Evans and Honkapohja, 2009, for extensive overview and examples). Empirical agents, regardless of their rationality, in practice face a task of econometric nature, such as estimating relevant price or production elastici-

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3 Willems (2012) gives a very interesting example of a monopoly firm that over time has to learn its demand of a linear form. The author notes that the model becomes intractable once the learning involves a noisy ‘rationally inattentive’ response to both the slope and the constant of the demand. This striking result casts serious doubts about the empirical relevance of rational inattention models.
ties (Sargent, 1993). Under adaptive learning, agents perceive a law of motion of the economy and learn its parameters over time, e.g. through recursive least squares. A natural question is where such a feedback can lead, and the literature on adaptive learning takes prime interest in the conditions for stability and uniqueness of the RE solution (Evans and Honkapohja, 2001). Additionally, adaptive learning can be used as a selection criterion in case of multiplicity of RE equilibria (Evans and Honkapohja, 1999). Finally, in reality agents presumably have to learn not only the parameters of the law of motion of the economy, but its specific functional form as well. This can lead to non-fundamental dynamics, especially in the case of significant non-linearities, and sparked a literature on restrictive perception equilibria (Branch, 2004; Bullard, 1994; Hommes and Sorger, 1998; Hommes and Zhu, 2014).

The example of Eductive Learning and adaptive learning are similar in spirit. They serve as the natural and unquestionably involved explication of the original, somewhat informal argument of Muth (1961) for the RE. However, due to the intentions of their authors, they cannot escape the criticism of the perfect rationality paradigm that they were supposed to counter in the first place. Both approaches, without any regard to the empirical evidence, simply assume a high level of individual rationality as a necessary characteristic of learning. As a result, Eductive Learning is unable to explain why in the experiments the subjects require repeated interaction to converge to the rational solution (Duffy and Nagel, 1997; Sutan and Willinger, 2004). On the other hand, adaptive learning is at odds with the heterogeneity and ‘irrationality’ of the household and experimental price forecasts (Malmendier and Nagel, 2009).

1.3.2 Learning and experiments: EWA and HSM

The issues with rational learning convey that we must not construct learning models in order to satisfy or explicate some a priori given theoretical model of prediction formation (such as RE), since we cannot have one in a reliable fashion. To the contrary, the learning to forecast remains an empirical phenomenon and hence we have to use empirical data to construct and judge learning models. As discussed earlier, experiments with their controlled settings can serve here as an ideal starting point. The best known examples of such a methodological approach come from Game Theory. First, models from evolutionary game theory can be understood as a form of social learning. Gale et al. (1995) applied it to the experimental findings on the Ultimatum Game. Bowles

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9Guesnerie (2002) openly acknowledges that the Eductive Learning requires ‘two extreme rationality assumptions’, namely Bayesian updating and common agreement on individual rationality.

10Eductive Learning is interpreted an instantaneous mental process, which proceeds the actual game that unfolds according to the rational solution.
and Gintis (2011) provide an extensive review of the example of evolution of human cooperation, whereas Vriend (2000) discusses social versus individual learning in a market context. Next, experiments on specific games with ‘unintuitive’ Nash Equilibria (such as the above mentioned p-beauty contest and the ultimatum game) sparked a large literature on how the agents can learn selecting strategies (see Rand et al., 2013, for a study of the ultimatum game). Important cases include attraction-based learning: reinforcement learning (Roth and Erev, 1995) and fictitious learning (Fudenberg, 1998), together with their generalization in the form of Experience-Weighted Attraction model (EWA; Camerer and Ho, 1999; Ho et al., 2008).

EWA models found successful applications to various experimental data, which stands as an important lesson of how economic experiments can be used for selecting and tuning theory (see Kocher and Sutter, 2005, for an example and literature overview). In the context of this thesis, the main problem of these models is that they were designed specifically to benchmark games, typically of static nature with a small strategy space. In comparison, firms or financial investors face dynamic problems with continuous action space (like a production choice based on price expectations). Even the most general EWA model cannot be directly adapted to such settings, and therefore these models play a limited role in behavioral finance or macro.

Once we reinterpret ‘strategy’ as a prediction rule instead of a point prediction, however, reinforcement learning can be applied to price-expectations feedback. As mentioned above, the type of market feedback seems to explain the subjects’ choice of forecasting heuristics. This leads to a class of Heuristic Switching Models (HSM; Brock and Hommes, 1997), in which agents switch between simple prediction rules (like adaptive, anchor and adjustment or trend extrapolation expectations) depending on their historical ability to forecast prices. A virtue of HSM is that it explicitly models behavioral heterogeneity, and thus can be easily calibrated to experimental data (Hommes, 2013). Among recent examples, Anufriev et al. (2013) use a two-heuristic HSM to evaluate the differences between the positive and negative feedback treatments of Heemeljer et al. (2009), while Anufriev and Hommes (2012) apply a four-heuristic HSM to the nonlinear asset pricing experiment of Hommes et al. (2005). In both cases HSM outperformed benchmark homogenous models, including RE, in replicating heterogeneous, diversified dynamics of these experiments. Furthermore, the HSM approach proved a successful approximation of dynamics of empirical foreign exchange rates (Dieci and Westerhoff, 2010), house prices (Bolt et al., 2011), macroeconomics (De Graauwe, 2011; Massaro, 2012) and asset prices (Boswijk et al., 2007; Westerhoff and Reitz, 2003).

Nevertheless, HSM is limited in one important aspect. It is natural to expect people
CHAPTER 1. INTRODUCTION

to use and switch between different behavioral rules, which remains the cornerstone of HSM. However, there is an infinite space of forecasting heuristics, while HSM is typically based on an \textit{a priori} selection of a handful of prediction rules. Furthermore, this selection in practice is different in different settings. For instance, Westerhoff and Reitz (2003) consider interaction only between fundamentalists and chartists, whereas Anufriev and Hommes (2012) study a model with four heuristics that do not contain the pure fundamental rule. As a result, HSM cannot fully explain the empirical heterogeneity of price forecasting, and constrains the underlying forecasting heuristics in an unsatisfactory fashion.

1.3.3 Agent-based models of learning

Another approach comes with the so called agent-based models (ABM). For the sake of analytical tractability, economists typically use a notion of representative agents to describe the vast number of real market participants, as if the economy was populated by a handful of characteristic individuals (Hartley, 2002). This does not exclude some degree of an underlying heterogeneity, both with the assumption of RE (Heathcote et al., 2009) or without it (with an example of HSM). Nevertheless, such a representation requires restrictive assumptions on the agents’ nature (for example homothetic utility functions; see Kirman, 1992, for a discussion), as well as on the agents’ interaction and market structure (Colander et al., 2008). ABMs are based on alternative, bottom-up approach (Delli Gatti et al., 2011): the model keeps track of \textit{every} individual, which is left as an explicitly independent decision and learning routine. Hence, the market aggregate outcomes are computed directly from the local interactions (Delli Gatti et al., 2010; Tesfatsion and Judd, 2006).  

ABM approach became popular in 1990ties (Farmer and Foley, 2009), with seminal works of Arifovic (1995) and Kirman and Vriend (2001). By their complex nature, ABMs offer the perfect testing and modeling ground for explicit asymmetries, non-trivial local interaction, bounded rationality and the emergent aggregate properties of these (LeBaron and Tesfatsion, 2008; Lengnick, 2013). In addition, ABMs allow for a direct evaluation of dynamical properties of complex systems, instead of relying on the more traditional equilibrium approach (Arthur, 2006). ABMs found successful

\footnote{Sometimes the term ABM is used to refer to any model in which agents of the same type retain heterogeneous beliefs and decisions, even if it is possible (\textit{e.g.} through the methods of statistical physics) to aggregate the individuals into a set of representative agents (for example, see Hommes, 2013: Lux and Marchesi 1999). Hence, some authors prefer the name of Agent-based Computational Economics (ACE) to signify a model that does not use representative agents at all (Kirman, 1992). This thesis will not follow this practice.}

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1.3. Models of learning


There is no free lunch, however. A potential downside of the complexity of the ABM approach is that such models can only be investigated through numerical simulations, e.g. Monte Carlo experiments (Metropolis et al., 1953; Metropolis and Ulam, 1949). Therefore, a typical ABM cannot yield an explicit formulation of the emergent equilibrium conditions (like an equilibrium market clearing solution) or the emergent individual behavior (such as the Euler equation in an infinite horizon optimization problem).\textsuperscript{12} A further implication is that ABMs often generate dynamics and properties that are \textit{ex ante} unpredictable and can be difficult to interpret \textit{ex post}.\textsuperscript{13} This led many economists to believe that ABMs are unintelligible ‘black-boxes’, a methodological attitude that originates from the problem of ‘wilderness of bounded rationality’ (Conlisk, 1996; Sims, 1980).

The specific challenge for ABMs in this context is that these models remain sensitive to the assumed individual behavior (LeBaron, 2006). A common practice is to select the behavioral skeleton of an ABM in such a way that its aggregate outcomes match the relevant stylized facts (like a distribution of prices or production decisions). This approach, however, may fail the Lucas critique: it is possible that a model has aggregate dynamics similar to a particular data set only ‘by chance’ and hence is useless for evaluating different economies, or the effects of larger fundamental shocks or policy changes. This issue has been widely recognized (LeBaron, 2001) and led to a rising popularity of calibrating ABMs with experimental data (Duffy, 2006) in the hope of providing these models with sound \textit{empirical} micro-foundations (see also Macy and Willer, 2002, for a sociology sciences perspective). Chapter 2 of this thesis is inspired by this research agenda.

Most of the ABM studies are focused directly on the formation of individual actions

\textsuperscript{12}For a typical ABM, the relevant rational (Walrasian, RE or Nash) equilibrium is likely to be a part of the set of the model’s fixed points. However, it is impossible to assess uniqueness or derive stability conditions of this set. See Chapter 3 for an exemplary financial market in which introducing a network unexpectedly distorts the stability of the RE solution.

\textsuperscript{13}Many economist take as natural that the learned equilibria should have ‘smart’, rational properties (Sims, 1980), whereas ABMs often lead to ‘irrational’ outcomes. For instance, the model of Assenza et al. (2014b) yields an endogenous business cycle without need of additional fundamental shocks. This may fit better the stylized behavior of the empirical GDP, but also means that the agents within the model do not converge to a ‘perfect’ behavior of any type. As discussed above, there is strong evidence against relevance of the ‘perfect equilibrium’, and we leave this part of ABM criticism for other discussions.
(like trading decisions), thus ABM literature on explicit expectation learning is scarce. An interesting exception comes with the works of Arifovic (1995, 1996), who uses Genetic Algorithms to model social learning of price expectations (see Haupt and Haupt (2004) for technical introduction and Dawid (1996) for discussion and examples of economic applications of Genetic Algorithms). Hommes and Lux (2013) combine this approach with the insight of HSM, namely that economic agents operate on the level of heuristics instead of point predictions. The authors consider a model in which every agent learns independently by optimizing a general forecasting rule with Genetic Algorithms. The authors show that this model beats the RE in explaining aggregate price dynamics and individual behavior in the experiment by Hommes et al. (2007). Chapter 2 of this thesis follows applies this approach to a diversified set of Learning-to-Forecast experiments.

1.4 Thesis outline

The main question of this thesis is how do agents learn to forecast in diversified market environments? This includes the role of price-expectations feedback, heterogeneity, and the relation between forecasting and trading. The thesis consists of three related but independently written chapters, which are built around the themes discussed earlier in this introduction.

Chapter 2 adapts the ABM design by Hommes and Lux (2013) of the Genetic Algorithms (GA) based learning and apply it to various experimental settings. For the sake of robust microfoundations, we follow the insight of Heemeijer et al. (2009) and have our agents use a mixture of adaptive and trend following expectations heuristics. We focus on the model’s empirical fit to four different Learning-to-Forecast experiments, which we investigate through novel Monte Carlo simulation studies of one-period and 50-period ahead out-of-sample prediction performance. The specific experiments include: linear positive and negative feedback (Heemeijer et al., 2009); linear positive and negative feedback with large unanticipated shocks to the fundamental (Bao et al., 2012); non-linear cobweb economy (Hommes et al., 2007; van de Velden, 2001); and two-period ahead non-linear asset pricing market (Hommes et al., 2005). We show that for all four experiments, our GA model outperforms rational expectations, simple homogenous expectation model and the simple HSM, both at the aggregate and the individual level. In addition, we show that HSM is a good stylized approximation of the experimental dynamics.

14This chapter is based on joint work with Mikhail Anufriev and Cars Hommes.
1.4. Thesis outline

The goal of Chapter 3 is to study the effect of information networks on price stability and individual forecasting coordination in financial markets. We take the GA model in the setting of the two-period ahead non-linear asset pricing market, and add information networks. Next to extrapolating the trend, agents can also learn to trust (or not) the recent mood of their friends. We find that the information networks destabilize the market. Agents remain well coordinated in terms of their price expectations. Nevertheless, they also learn contrarian strategies, trying to outsmart their friends, whose past decisions lag behind the market price cycle. This result confirms and explains the experimental studies on herding/contrarian strategies (Cipriani and Guarino, 2009; Drehmann et al., 2005). Interestingly, we show that the specific network architecture or size does not seem to play a significant role in the emergent market dynamics.

Chapter 4 discusses an experimental study of a simple linear asset pricing market, in which subjects are asked to predict price, directly trade the asset, or do both. Our investigation offers a link between Learning-to-Forecast and Learning-to-Optimize experimental designs for a positive feedback type of economy. Non-fundamental dynamics occur in all of three treatments, with the forecasting treatment being the most stable. Subjects use price trend or asset return extrapolating type of rules and can coordinate on large price oscillations, but can also converge to a non-fundamental price. Regardless of the treatment, large behavioral heterogeneity persists. This confirms that the subject behavior, which was observed in previous Learning-to-Forecast experiments, is robust against task specification. In fact, the Learning-to-Forecast setting, surprisingly, can result in the most stable learning dynamics, whereas Learning-to-Optimize and Mixed treatments are more unstable and may lead to a cycle of repeated bubbles and crashes. Moreover, the mixed treatment indicates that trading consistently with the price expectations is difficult for the subjects, and only a quarter of them properly optimize their behavior. A general conclusion from this chapter is that the learning to optimize seems more difficult for the subjects than the learning to forecast.

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15This chapter is based on joint work with Te Bao and Cars Hommes, available online as CeNDEF Working Paper 14-01.
Chapter 2

Learning-to-Forecast with Genetic Algorithms

2.1 Introduction

Expectations are a cornerstone of many economic models, because economic agents often operate in a dynamic context. Consumers have to organize their life-time work and consumption paths, while companies decide on how to build up future production capabilities. In either case, the agents must forecast how the uncertain future may unfold. What makes modeling predictions difficult is that they typically form a feedback with the realizations through agents decisions. For instance, if producers expect an increased price of their consumption good, they will rise production. If the demand stays constant, this implies lower market clearing price in the future. It is therefore likely that agents would alter their predictions, leading to a new realized price.

Even if the agents know the structure of the economy, the price-expectation feedback can lead to non-trivial dynamics (Grandmont, 1985; Hommes, 2013; Tuinstra and Weddepohl, 1999). This picture becomes more complicated if the agents furthermore have to learn this structure (Bullard, 1994; Grandmont, 1998). Agents do want to form good price expectations, but how would they cope with this complexity?

The traditional literature (after Muth, 1961) emphasizes the Rational Expectations (RE) hypothesis, which states that in equilibrium the predictions have to be model consistent. Most economists would interpret RE as an ‘as-if’ approximation — real markets behave as if a representative agent was perfectly rational, because real people are rational enough to learn to avoid systematic, correlated errors.\(^\text{1}\) However, this is

\(^{1}\)One interesting and straightforward explication of this approach can be found in the concluding section of Blundell and Stoker (2005).
CHAPTER 2. LEARNING-TO-FORECAST WITH GENETIC ALGORITHMS

not confirmed by the data. A recent example comes from the housing market in the US before the latest economic crisis, where people systematically misjudged the long-term value of their houses (Benítez-Silva et al., 2008; Case and Shiller, 2003; Goodman Jr. and Ittner, 1992). In a broader context of macroeconomics, inflation expectations formed by the ‘Jones’ are far from the RE predictions (Adam, 2007; Assenza et al., 2013; Charness et al., 2007; Pfajfar and Zakelj, 2011) and can be subject to cognitive biases (Malmendier and Nagel, 2009). Many firms similarly fail to use RE (see e.g. Nunes, 2010b, for a discussion about the Phillips Curve and survey expectations).

The failure of RE made many economists look for an alternative model with explicit learning. But alternatives face the so called ‘wilderness of bounded rationality’ problem: there is a myriad of possible learning mechanisms with varied restrictions on human memory and computational capabilities. These range from simple linear heuristic models (see Evans and Ramey, 2006, for a discussion of adaptive expectations), through adaptive or statistical learning (Evans and Honkapohja, 2001), through heuristic switching type of models (Brock and Hommes, 1997) to evolutionary learning mechanisms (Arifovic et al., 2012). Typically these mechanisms lead to different dynamics: for example Bullard (1994) and Tuinstra and Wagener (2007) show that for a standard OLG economy, where the agents use OLS learning for price forecasting, adaptive learning may lead both to stable and complicated, chaotic dynamics.

Learning-to-Forecast (LtF) experiments (Hommes, 2011; Hommes et al., 2005) offer a simple laboratory testing ground for learning mechanisms. These controlled experimental economies have a straightforward and unique fundamental (RE) equilibrium. As in real markets, subjects observe the realized prices and their own past individual predictions, but not the history of other subjects’ predictions, and are not informed about the exact law of motion of the economy. Many LtF laboratory experiments contradict the RE hypothesis. The subjects can coordinate on oscillating and serially correlated time series, and the exact dynamics depend greatly on the specific feedback structure of the experimental economy. Convergence to the fundamental equilibrium happens only under severe restrictions on the underlying law of motion (Hommes, 2011). Another important finding in the experiments is heterogeneity: within the same experimental group, subject tend to use different forecasting rules, see Heemeijer et al. (2009) (henceforth HHST09).

The most successful attempt to explain the LtF experiments comes with the so-called Heuristic Switching Model (HSM; Brock and Hommes, 1997). The basic idea of the model is that the agents have a (small) set of simple forecasting heuristics (rules of thumb like adaptive or trend extrapolating expectations) and choose those that had a better relative past performance. The HSM has successfully been used
to explain different types of aggregate behavior — convergence and oscillations in various experimental settings (Anufriev and Hommes, 2012; Anufriev et al., 2013). A disadvantage of the HSM however is the small set of heuristics, which cannot fully account for the individual heterogeneity. Furthermore different experiments require the HSM to utilize differently specified sets of heuristics. It is unclear why the subjects would use only these particular forecasting rules and how they would learn them.

The purpose of this chapter is to explain LtF experiments with Genetic Algorithms (GA). The basic idea of the model is that the agents forecast prices using a possibly large set of heuristics from a simple but general class. The agents then independently use GA to update and select the heuristics based on their relative success. This results in an agent-based model with explicit individual learning and endogenous heterogeneity. We will argue that our model is able to capture the dynamics at both the aggregate and the individual level for different experimental settings. Furthermore, the model has a clear link to the existing behavioral literature. GA are a flexible optimization procedure, thus the GA-based model retains a basic economic interpretation. Agents, who use GA, have to rely on second-best forecasting rules. Nevertheless, they learn to use them efficiently. For example, if it is profitable to harvest speculative trade revenues, the agents will update their forecasting rule’s parameters with GA, in the direction of stronger trend extrapolation. As a result, the GA model in spirit resembles the HSM, but is more flexible in the specification and evolution of forecasting heuristics.

Dawid (1996) provides a good overview of the first GA applications to economic problems. GA was initially applied in its social learning form to explore stylized facts from experimental data, outperforming the RE hypothesis (Arifovic, 1995), with the examples of the exchange rate volatility (Arifovic, 1996; Lux and Schornstein, 2005) or production level choices in a cobweb producers economy (Dawid and Kopel, 1998). More recently, Hommes and Lux (2013) investigate a model, in which agents use GA to optimize a forecasting heuristic (instead of directly optimizing a prediction) and, much like the actual subjects in LtF experiments, cannot observe each others behavior or strategies. The authors replicate the distribution of the predictions and prices (mean, variance and autocorrelations) of the cobweb experiments by Hommes et al. (2007) and van de Velden (2001) (henceforth HSTV07 and V01 respectively).

We follow Hommes and Lux (2013) in the basic design of our model, but the novelty of this chapter is fourfold. The first is that we will use a different heuristic space, based on the so called first order rule, which is a mixture of adaptive and trend extrapolating heuristics. This gives the model better micro-foundations, as HHST09 find this forecasting rule to well describe the individual expectations in their experiment. The GA agents then learn to optimize the parameters of this simple heuristic.
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The second novelty is that our model allows for a simultaneous explanation of different LtF experiments, based on positive and negative feedback markets with or without breaks in the fundamental price, or with highly non-linear price-expectations feedback. Application to different market settings will demonstrate the generality of our model. In particular, we will look at four market settings: (1) the simple, linear feedback systems from HHST09; (2) the linear price-expectations feedback system with unexpected large shocks to the fundamental price (Bao et al. 2012) (henceforth BHST12); (3) a cobweb producers economy (HSTV07; V01), used also by Hommes and Lux (2013); and (4) a non-linear positive feedback asset pricing economy, where the subjects are asked for two-period ahead predictions (Hommes et al., 2005) (henceforth HSTV05).

The third novelty is that we explain the individual behavior observed in the LtF experiments. We will show with Monte Carlo studies that our model can replicate the long-run behavior of the data, both at the aggregate and individual level. Next we will evaluate the out-of-sample predictive power of the model by means of a simple Sequential Monte Carlo technique. We find that depending on the experiment, our model is comparable or better than the HSM in terms of predicting both the prices and the individual price forecasts one period ahead. This is an important contribution to the literature on agent-based models, which usually focuses on a model’s fit to the aggregate stylized facts.

Finally, the fourth novelty is that the Monte Carlo studies of the GA model enable us to characterize the median forecasting behavior, together with its corresponding confidence bounds, in various experimental settings. The GA simulations thus provide a solid motivation for (1) describing the LtF experimental dynamics in terms of simple heuristics, and (2) for the specific choice of these heuristics for a particular experimental market. This yields natural micro-foundations for models such as HSM.

The chapter is organized as follows. In Section 2, we present the setup and findings of the LtF experiments, and briefly discuss the HSM by Anufriev et al. (2013). In the third section, we introduce our GA model and fit it to the experimental setup by HHST09. In the fourth section we move to the remaining three experiments. Finally, a concluding section gives an overview of the results and suggestions for future research. A number of supplementary issues, including discussions on the initialization of the model and the forecasting rules based on a long-run anchor, are presented in the appendix.
2.2 Learning to Forecast and Heuristic Switching

Consider a market with a number of subjects \( i \in \{1, \ldots, I\} \), who are asked at each period \( t \) to forecast the price of a certain good. The subjects are explicitly informed that they act as forecasting consultants for firms and are rewarded only for the accuracy of the predictions.

The feedback relationship between the prices and predictions is summarized by a law of motion of the form

\[
(2.1) \quad p_t = F(p_{1,t}^e, \ldots, p_{I,t}^e),
\]

where the realized price \( p_t \) is a function of all individual forecasts \( p_{i,t}^e \). The mapping \( F(\cdot) \) is generated from market clearing, with aggregate supply and demand derived from optimal choices of firms, consumers or investors, given the subjects’ individual forecasts. Define the fundamental price \( p^f \) as the steady state RE outcome, the self-consistent prediction: \( p^f = F(p^f, \ldots, p^f) \). In all examples below the RE fundamental price is unique.

Unlike the RE agents, subjects in the experiment have limited information about the market. They are informed that their predictions affect the prices, but they are given only a qualitative story about this feedback. Moreover, they are not explicitly informed about the fundamental price.\(^2\)

One important example investigated by HHST09 uses a linear version of (2.1):

\[
(2.2) \quad p_t = A + B \left( \frac{\sum_{i=1}^{I} p_{i,t}^e}{I} - A \right) = A + B (\bar{p}_t^e - A),
\]

where \( \bar{p}_t^e = \frac{\sum_{i=1}^{I} p_{i,t}^e}{I} \) is the average prediction of all individuals at period \( t \) and \( A = p^f \) is the fundamental price. There are two important cases: \( B > 0 \) (positive feedback) and \( B < 0 \) (negative feedback). A typical example of positive feedback is a stock exchange: optimistic investors will buy more stock and due to increased demand the stock price will go up. In this sense the investor sentiments are self-fulfilling (although not perfectly if \( B \neq 1 \)). Negative feedback arises e.g. in a supply driven market where producers face a lag in production. If they expect a high price in the future, they will increase production and so the market clearing price will go down.

\(^2\)Usually it is possible to infer it from the experimental instructions. Anecdotal evidence suggests that even economics students, including graduate students, fail to realize it.
HHST09 used two simple linear treatments:

\[
\text{Positive feedback: } p_t = 60 + \frac{20}{21}(\bar{p}_t - 60) + \varepsilon_t; \quad (2.3)
\]
\[
\text{Negative feedback: } p_t = 60 - \frac{20}{21}(\bar{p}_t - 60) + \varepsilon_t, \quad (2.4)
\]

where \(\varepsilon_t \sim NID(0,0.25)\) is a small noise term. The experiment runs for 50 periods for each group of \(I = 6\) subjects. The two linear treatments are symmetrically opposite. They have the same unique fundamental price \(p^f = 60\) and the same absolute dampening factor \(|B| = \frac{20}{21}\) (but with opposite signs). The dampening factors were chosen so that under naive expectations (i.e., \(\bar{p}_t = p_{t-1}\)), the fundamental price for both treatments is a (unique) stable steady state, but the system would still require some time to converge.

The two feedback treatments resulted in very different aggregate price behavior, illustrated in Figure 2.1a and 2.1b. Under the negative feedback after a short volatile phase of \(7-8\) periods, the price converges to the fundamental value \(p^f = 60\), after which the subjects forecasts coordinate on the fundamental as well. In most of the positive feedback groups, persistent price oscillations arise (Figure 2.1b), where the price overshoots and undershoots \(p^f\); if the price converges to the fundamental at all, it does so only towards the end of the experiment (which happened for two out of seven cases). In spite of the price oscillations, the subjects’ forecasts coordinate within \(2-3\) on a common value (different from the fundamental value) and remain so until the end of the experiment. In positive feedback markets, subjects’ forecasts are thus strongly coordinated, but on a non-RE price.

To describe the subjects’ forecasting behavior, HHST09 use the first-order rule (FOR):

\[
p_{i,t}^e = \alpha_1 p_{t-1} + \alpha_2 p_{i,t-1} + \alpha_3 p^f + \beta (p_{t-1} - p_{t-2}), \quad (2.5)
\]

for \(\alpha_1, \alpha_2, \alpha_3 \geq 0, \alpha_1 + \alpha_2 + \alpha_3 = 1, \beta \in [-1,1]\). Rule (2.5) is an anchor and adjustment rule, extrapolating a price change from an anchor, which is given by a weighted average of past price, individual forecast and the fundamental price \(p^f = 60\).\(^3\) HHST09 estimated this simple rule separately for each subject, based on their predictions from the last 40 periods. It described well the forecasting behavior of around 60% individuals.

HHST09 find that the individual forecasting rules varied between the subjects, even within the same treatment. The authors also report significant differences between

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\(^3\)Under RE, the FOR in (2.5) should be specified with \(\alpha_1 = \alpha_2 = \beta = 0\), which implies that the subjects always predict the fundamental price, \(p_{i,t}^e = p^f = 60\).
the two treatments. Under positive feedback, subjects focused on trend extrapolation ($\beta > 0$) and the estimated weight of the fundamental price $\alpha_3$ coefficients were typically insignificant. Under the negative feedback, the reverse holds: trend extrapolation is barely used ($\beta \approx 0$), while the weight for the fundamental price ($\alpha_3 > 0$) is significant. This shows that a model with a homogeneous forecasting rule (RE, but also linear heuristics like trend extrapolation or naive expectations) may explain one of the two treatments, but not both at the same time. Moreover, a homogenous rule contradicts the significant differences between the subjects within each treatment.

This led Anufriev et al. (2013) to investigate the Heuristic Switching Model (HSM), in which the subjects are endowed with two prediction heuristics:

**adaptive expectations:** $p_{i,t}^e = \alpha p_{i,t-1} + (1 - \alpha)p_{i,t-1}$ with $\alpha \in [0, 1]$,

**trend extrapolation:** $p_{i,t}^e = p_{i,t-1} + \beta(p_{t-1} - p_{t-2})$ with $\beta \in [-1, 1]$,

where the authors used $\alpha = 0.75$ and $\beta = 1$. Notice that both heuristics are a special case of the first-order rule. The idea of the HSM model is that the subjects can at any time use any of the two heuristics, but tend to focus on the one with the higher relative past performance. Under positive feedback, agents easily coordinate their predictions, for example below the fundamental, close to the first observed price, but (by the construction of the positive feedback equation) the next realized price is then slightly higher than the average prediction. The trend extrapolation heuristic captures this gradual increase of initial prices and so becomes popular among the agents. This reinforces the trend and leads to persistent price oscillations. In contrast, under negative feedback there is no possibility of coordination of individual forecasts, unless the agents coordinate on the fundamental price. Under negative feedback the trend extrapolating rule performs poorly and agents switch to adaptive expectations, thus causing the price to converge to the fundamental.

HSM captures the essence of the aggregate forecasting behavior and successfully replicates the results of HHST09 in a stylized fashion. The drawback of the model is that the authors assume a limited number of only two heuristics, without explaining where these heuristics come from, that is, how the subjects are able to learn the two heuristics in the first place. Moreover, the HSM cannot account for heterogeneity of rules among subjects and hence does not explain the experiment at the individual level. To overcome these drawbacks, we will introduce a model with explicit individual learning through Genetic Algorithms.
CHAPTER 2. LEARNING-TO-FORECAST WITH GENETIC ALGORITHMS

2.3 The Genetic Algorithms model

2.3.1 Genetic Algorithms

Genetic Algorithms (GA) forms a class of numerical stochastic maximization procedures that mimic the evolutionary operations with which DNA of biological organisms adapts to the environment. GA were introduced to solve ‘hard’ optimization problems, which may involve non-continuities or high dimensionality with complicated interrelations between the arguments. They are flexible and efficient and so are often used in computer sciences and engineering (Haupt and Haupt, 2004). See e.g. Dawid (1996) for applications in economics.

A GA routine starts with a population of random trial solutions to the problem. Individual trial arguments are encoded as binary strings (strings of ones and zeros), or chromosomes. They are retained into the next iteration with a probability that increases with their relative performance, which is defined directly in terms of a functional value (‘fitness’). This so called procreation operator means that with each iteration, the population of trial arguments is likely to have a higher functional value, i.e. be ‘fitter’. On top of the procreation, GA use three evolutionary operators that allow for an efficient search through the problem space: mutation, crossover and election, where the last operator was introduced in the economic literature (Arifovic, 1995).

Mutation At each iteration, every entry in each chromosome has a small probability to mutate, in which case it changes its value from zero to one and vice versa. The mutation operator utilizes the binary representation of the arguments. A single change of one bit at the end of the chromosome leads to a minor, numerically insignificant change of the argument. But with the same probability a mutation of a bit at the beginning of the chromosome can occur, which changes the argument drastically. With this experimentation, GA can easily search through the whole parameter space and have a good chance of shifting from a local maximum towards the region containing the global maximum.

Crossover Pairs of arguments can, with a predefined probability, exchange predefined parts of their respective binary strings. In practice, the crossover is set to exchange bits corresponding to a subset of the arguments. For example, if the objective function has two arguments, crossover would swap the first argument between pairs of trial arguments. This allows for experimentation in terms of different mixtures of arguments.

Election The election operator is meant to screen inefficient outcomes of the experi-
2.3. The Genetic Algorithms model

In the genetic algorithms model, the procreation routine and the three evolutionary operators have a straightforward economic interpretation for a situation, in which the agents want to optimize their behavioral rules, e.g., price forecasting heuristics. The procreation means that — as in the case of HSM — people focus on better solutions (or heuristics). The mutation and crossover are experimentation with the heuristics’ specifications, and finally the election ensures that the experimentation does not lead to suboptimal heuristics.

An important additional condition for a GA routine is that it requires a predefined interval for each parameter. For the above example of updating behavioral rules through GA, it means that we confine them to some predetermined, finite grid of heuristics. The specific formulation of our GA is given in Appendix A. For the technical discussion refer to Haupt and Haupt (2004).

2.3.2 Model specification

We consider a set of $I = 6$ GA agents in the price-expectation feedback economy (2.1). GA agents use a general forecasting rule which requires exact parameter specification, and each agent is endowed with $H = 20$ such specifications. In order to give our model robust empirical micro-foundations, we follow the estimations by HHST09, as well as the simple model discussed by Anufriev et al. (2013) and focus on the first order rule (FOR).\(^4\)

To be specific, in order to predict price $p_t$ agent $i \in \{1, \ldots, I\}$ focuses on $H = 20$ linear prediction rules given by

\[
\begin{align*}
\hat{p}_{i,h,t} &= \alpha_{i,h,t} p_{t-1} + (1 - \alpha_{i,h,t}) \hat{p}_{i,t-1} + \beta_{i,h,t} (p_{t-1} - p_{t-2}),
\end{align*}
\]

where $\hat{p}_{i,h,t}$ is the prediction of price $p_t$, formulated by agent $i$ conditional on using the rule $h$ at the beginning of period $t$, and $\hat{p}_{i,t-1}$ is the prediction by agent $i$ of the price $p_{t-1}$, which the agent submitted to the market in period $t - 1$. Rule (2.6) is a simplified case of the general FOR (2.5) (with $\alpha_3 = 0$).\(^5\)

\(^4\)Cf. Hommes and Lux (2013), who use the anchor-and-adjustment rule $\hat{p}_{i,t} = \alpha + \beta (p_{t-1} - \alpha)$.

\(^5\)We experimented with the full FOR with the anchor ($\alpha_3 > 0$), namely with an anchor equal to (1) the fundamental price $p_f = 60$; and to (2) the average realized price so far. The two specifica-
CHAPTER 2. LEARNING-TO-FORECAST WITH GENETIC ALGORITHMS

Heuristic FOR (2.6) depends on two parameters only, namely on $\alpha_{i,h,t}$ (price weight) and $\beta_{i,h,t}$ (trend extrapolation coefficient). We emphasize that these parameters are time dependent, because the agents want to fine-tune the FOR (2.6) for their specific market. For example, in an asset pricing market it may pay off to focus on the trend of the asset price. The agents would like to find the optimal degree of trend following, by experimenting with different trend extrapolation coefficients $\beta_{i,h,t}$. This learning is embodied as a GA algorithm and constitutes the novel insight of our model, compared to HSM or any homogenous expectations model.

Define $H_{i,t}$ as the set of heuristics of agent $i$ at time $t$. Each agent has $H = 20$ heuristics which are specified as a pair of parameters $(\alpha_{i,h,t}, \beta_{i,h,t}) \in H_{i,t}$. Each such pair is represented as a chromosome, a binary string of length 40, 20 bits per coefficient. This means that the coefficients have to be bound to a finite interval. Price weight simply spans a simplex $\alpha_{i,h,t} \in [0, 1]$. For the trend extrapolation coefficient $\beta_{i,h,t}$, we report two specifications, namely with $\beta \in [-1, 1]$ (contrarian rules allowed) and $\beta \in [0, 1]$ (contrarian rules not allowed).\(^6\)

The chromosomes are updated independently for each agent by GA evolutionary operators. We focus on the same set of parameters as Hommes and Lux (2013), see Table 2.1. The updating of the heuristics is based on their relative forecasting performance, specifically on mean squared error (MSE). Let

\[ \text{(2.7)} \quad \text{MSE}_{i,h,t} = (p_{e,i,t} - p_t)^2. \]

Define the normalized performance (or fitness) measure as:

\[ \text{(2.8)} \quad \Pi_{i,h,t} = \frac{\exp(-\text{MSE}_{i,h,t})}{\sum_{j=1}^{H} \exp(-\text{MSE}_{i,j,t})}, \]

which is a logit transformation of MSE. The normalized performance measure (2.8) can be directly interpreted as the probability attached to each heuristic $h$ by agent $i$ at time $t$.\(^7\)

\(^6\)Experimentation has led us to take 1.1 as the upper bound of $\beta$, see Appendix C for a discussion of the role of trend extrapolation under positive feedback. As for the lower bound, experimental data suggests that subjects tend to avoid contrarian strategies (HHST09 report only two subjects with such rules) for the sake of completeness we report both specifications.

\(^7\)Notice that (2.8) is also independent between the agents, because they have different sets of heuristics. Measure (2.8) is different from the experimental payoff, which was used by Hommes and
2.3. The Genetic Algorithms model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Notation</th>
<th>Value</th>
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</tr>
<tr>
<td>Number of heuristics per agent</td>
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<td>Number of parameters</td>
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<td>$[0, 1.1]$</td>
</tr>
<tr>
<td>Specification 2</td>
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<td>$[0, 1.1]$</td>
</tr>
<tr>
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<td>${20, 20}$</td>
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<td>Crossover rate</td>
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<tr>
<td>Performance measure</td>
<td>$U(\cdot)$</td>
<td>$\exp(-MSE(\cdot))$</td>
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Table 2.1: Parameter specification used by the Genetic Algorithms agents.

The timing of the model is as follows. Before the market starts to operate, the agents’ heuristics are initialized at random from a ‘uniform’ distribution: agent $i$ samples 800 initial bits (twenty initial heuristics with two parameters, each encoded by twenty bits) independently as 0 or 1 with equal probability. In some initial periods the agents cannot use their heuristics, as these require past prices and predictions. Here the agents sample random predictions from a predefined distribution which we take as exogenous (for instance this can mean the experimental initial predictions). Once the agents have enough observations to use their heuristics, the timing of the market at period $t$ is as follows:

1. The market price $p_t$ is realized according to (2.1) and agents observe it;

2. Agents independently update their heuristics using one GA iteration, where the GA criterion function is $\Pi_{i,h,t}$ (forecasting performance). To be specific, agent $i$ uses four evolutionary operators:

   (a) procreation: agent samples $H$ child heuristics from $H_{i,t}$ with $\Pi_{i,h,t}$ as the corresponding probabilities;

   (b) mutation: each bit of each child heuristic has probability $\delta_m$ to switch its value;

Lux (2013) in their GA model. We decided to use logit transformation of MSE to have a clear link with HSM literature and to keep this model feature independent from the experimental design.
(c) **crossover:** each pair of child heuristics has probability $\delta_c$ to swap last twenty bits, which corresponds to exchanging $\beta$’s.

(d) **election:** each child heuristic is compared with the parent heuristic in terms of $\Pi_{i,h,t}$: a child heuristic becomes part of the set $H_{i,t+1}$ if it outperforms its parent, else its parent is passed to $H_{i,t+1}$.

3. With the new $H_{i,t+1}$, period $t + 1$ starts.

4. Each agent $i$ picks one particular heuristic $i, h, t + 1$, which is based on the hypothetical MSE of heuristics $H_{i,t+1}$ in predicting the last observed price $p_t$ (with probabilities $\Pi_{i,h,t}$). Agent $i$ uses the chosen heuristic to generate her prediction $p_{i,t+1}$.

5. New period $t + 1$ starts: the market price $p_{t+1}$ is realized according to (2.1).

In the first period when the heuristics can be used, their hypothetical past performance is still undefined, and so the agents pick one with equal probabilities. For the HHST09 experiment, GA agents start to use the first-order rule in the second period (one at random) and start to update their heuristics in the third period (for a more detailed discussion of the initialization, see Appendix B).

The last step — the heuristic choice — is the same as in the HSM, but there are two important differences between HSM and our GA model. First, the heuristics evolve over time with $H_{i,t} \neq H_{i,t+1}$. As a result, we obtain a HSM in which the heuristics have time varying parameters, adapted to the specific market dynamics. Second, this learning operates through a stochastic GA procedure, and is independent between the agents. In practice thus the agents will learn different heuristics and remain heterogeneous with $H_{i,t+1} \neq H_{j,t+1}$, which gives us an agent-based counterpart for HSM.

### 2.3.3 50-period ahead simulations

The first test for the fit of our model to the experimental data are 50-period ahead simulations for the HHST09 experiment.\(^8\) We take the feedback equations (2.3) and (2.4) and simulate our model for 50 periods, without any information from the experiment after period 1, and hence compare the realized long-run model dynamics with the experimental data.\(^9\)

---

\(^8\)All simulations were written in Ox matrix algebra language (Doornik, 2007) and are available upon request.

\(^9\)In one of the positive feedback treatment groups, one of the subjects ‘out of the blue’ predicted ten times higher price than both his previous forecast and the realized market price. This destabilized the whole market for a number of periods. In the following analysis, we follow Anufriev et al. (2013)
The model requires exogenous predictions for the first period. This is important, since in the experiment the average initial prediction affected the group dynamics under the positive feedback treatment (cf. Anufriev et al., 2013). In the first Monte Carlo (MC) exercise, we sample initial prediction from a distribution calibrated by Diks and Makarewicz (2013), in order to obtain a general picture of the model dynamics. We resample the model 1’000 times, including new initial predictions and realizations of the learning algorithm, to obtain a satisfactory MC distribution. The median of 1’000 GA simulations, with 95% confidence intervals (CI), for the model with contrarian rules $\beta \in [-1.1, 1.1]$ are shown in Figure 2.2. See also Figure 2.1 for sample experimental prices and predictions and realized 50 period ahead GA simulation of a representative group for each feedback treatment.

Figure 2.1: HHST09: experimental groups and sample 50-period ahead simulations of the GA model (with $\beta \in [-1.1, 1.1]$ and random initial predictions). Black line represents the price and green dashed lines are the individual predictions.

Figure 2.2 shows the MC simulations of the realized prices (top panel) and the degree of coordination, that is the standard deviation of six individual forecasts (bottom panel). The model replicates the experimental outcomes well. Under negative feedback (left panels), prices are quickly pushed close to the fundamental, but individual heterogeneity of GA agents is visible until period 15, consistent with the experimental and omit this group and hence focus on six positive feedback and six negative feedback treatment groups.
data. Under positive feedback, GA agents coordinate their forecasts in less than five periods, but the distribution of realized prices does not collapse into the fundamental even after 50 periods, when the 95% CI of prices is as wide as [55, 75]. The median price resembles the experimental oscillations, including the typical amplitude and turning points. Overall, the 95% CI for our GA model captures 65% (81%) of the experimental prices and 81.33% (71.67%) of the degree of coordination for the negative (positive) feedback treatment. This means that we are able to evaluate roughly 75% of the long-run (50-period ahead) behavior of the experimental groups, both at the aggregate level (prices) and the individual level (coordination of individual predictions).

Which heuristics were learned by our GA agents? Figure 2.3 reports the median (with 95% and 90% CI) for the MC simulations of the price weight $\alpha$ and the trend extrapolation coefficient $\beta$. Large heterogeneity of individual rules persists, but there are clear differences between the two treatments. Under the positive feedback treatment, the median GA agent quickly converges towards

$$p_{t+1}^i \approx 0.9p_t + 0.1p_{t-1}^i + 0.6(p_t - p_{t-1}).$$
2.3. The Genetic Algorithms model

This median rule is close to a pure trend-following rule (i.e. with anchor $p_t$), but has a coefficient $\beta \approx 0.6$, smaller than the coefficient $\beta = 1$ that Anufriev et al. (2013) are using in their 2-type HSM. Furthermore, 72.35% of the GA agents would never use a negative $\beta$ in the last 30 periods (see the green star-line in Figure 2.3d for 27.65% percentile); and in the last period, the chosen $\beta$ has a negatively skewed distribution (see Figure 2.10a).

On the other hand, under negative feedback the median GA agent learns a rule close to

$$(2.10) \quad p_{i,t+1}^c \approx 0.5p_t + 0.5p_{i,t}^c$$

(with median $\beta$ trend coefficient close to 0). This median rule for negative feedback is adaptive expectations with equal coefficient 0.5; Anufriev et al. (2013) are using adaptive expectations with coefficient 0.75 on price in their 2-type HSM. Our learning dynamics therefore confirm the results by HHST09 and Anufriev et al. (2013), albeit with slightly different parametrization.

In the second MC study, we focus on how well our GA model can replicate long-
run dynamics of a specific experimental group. We take initial predictions of each group and use them as initialization for 50-period simulations of the GA model. We investigate the realized prices and individual forecasts. Following Anufriev et al. (2013) we define the GA model expected individual price forecast as

\[
\hat{\pi}_{e,GA}^{i,t} = \frac{1}{H=20} \sum_{h=1}^{H} \prod_{t=1}^{H} p_{i,h,t}^{e,t}.
\]

For each sample model time path, we compute its mean squared error (MSE) in predicting the experimental data (both prices and individual price forecasts) for the last 47 periods (excluding the initialization phase)

\[
MSE_{X}^{\text{prices}} = \frac{1}{47} \sum_{t=4}^{50} \left( p_{Gr X}^{t} - p_{GA}^{t} \right)^2,
\]

\[
MSE_{X}^{\text{price forecasts}} = \frac{1}{6 \times 47} \sum_{i=1}^{6} \sum_{t=4}^{50} \left( \hat{p}_{Gr X}^{i,t} - \hat{\pi}_{e,GA}^{i,t} \right)^2,
\]

where \( p_{Gr X}^{t} \) and \( \hat{p}_{Gr X}^{i,t} \) denote the realized price and the price forecast of subject \( i \) at period \( t \) in an experimental group \( X \) and \( p_{GA}^{t} \) and \( \hat{\pi}_{e,GA}^{i,t} \) are the price and the price forecast of agent \( i \) at period \( t \) predicted by the GA model for the group \( X \).

Table 2.2 reports MSE averaged over the six groups for each treatment, with 1’024 sample GA model paths per experimental group. We also include results for a number
of benchmark models, including simple homogenous expectation rules, RE and HSM (two heuristics specification by Anufriev et al. (2013)). In terms of the long-run, 50 period ahead, predictions, RE and two simple models, adaptive and contrarian expectations, perform the best under negative feedback, as they correctly predict the agents to converge to the fundamental price. Our GA model performs only slightly worse. Under positive feedback, RE, contrarian and adaptive expectations still predict convergence, in contrast to the experimental oscillations. HSM, trend extrapolation and naive expectations perform comparatively well, but surprisingly they are not better than RE. The reason is that the price oscillations predicted by these three models at the longer time horizon fall out of phase with the experimental oscillations. The best fit is achieved by our GA model, especially the one without contrarian rules $\beta \in [0, 1.1]$. We conclude that all benchmark models are able to capture the long-run dynamics of possibly one feedback treatment, but not of two treatments at the same time. Only our GA model successfully evaluates both treatments.

2.3.4 One-period ahead predictions

A good indicator of the model’s fit is the precision of its one-period ahead predictions (Anufriev et al., 2013): how well the model predicts experimental outcomes in period $t + 1$, conditional on the data until period $t$, in terms of MSE. For deterministic models such as HSM and the homogeneous expectations benchmark models, computing one period-ahead MSE is straightforward. For our GA model with its evolutionary operators, however, evaluating MSE is more complicated. Our model is both stochastic and highly non-linear: it evolves according to an analytically intractable period-to-period distribution. To address this issue, we compute the expected MSE using the Sequential Monte Carlo (SMC) approach.\footnote{We checked the robustness of our results by Sequential Importance Sampling (with Resampling) technique called Auxiliary Particle Filter (Doucet et al., 2000), see Appendix E, and obtained comparable results. One can also show that SMC is a restricted version of APF, and so can be used if the results are similar.}

Our SMC is designed in the following way. For each experimental group $X$, we run simultaneously $1'024$ independent GA model simulations. We associate one GA agent with one subject, and in each period $t \geqslant 2$ every GA agent $i$ (1) retains her heuristics from the previous period and (2) is given the experimental prices and the price forecasts of subject $i$ until the previous period $t - 1$. GA agents use the experimental data to update their heuristics and forecast the price $p_t$ in the usual way, which gives us the GA’s price forecasts (2.11) and realized prices (2.1) for period $t$. We evaluate the fit of

\footnote{For the definition of the benchmark rules, please refer to Appendix D.}
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<table>
<thead>
<tr>
<th>MSE</th>
<th>Negative feedback</th>
<th>Positive feedback</th>
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</thead>
<tbody>
<tr>
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<td>HSM</td>
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<tr>
<td>GA: $\beta \in [0, 1.1]$</td>
<td>4.496</td>
<td>25.012</td>
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Table 2.3: HHST09: one-period ahead predictions. MSE of the experimental prices and forecasts, for the Trend Extrapolation, Adaptive, Contrarian, Naive and Rational Expectations, Heuristic Switching Model and Genetic Algorithms models (with $\beta \in [-1.1, 1.1]$ and $\beta \in [0, 1.1]$). MSE averaged over six negative feedback and six positive feedback groups.

the model to the experimental group by computing the average MSE (2.12) over 1’024 GA simulations.

The results are similar to the 50-period ahead simulations, see Table 2.3. Under negative feedback, RE, HSM adaptive, contrarian and naive expectations all capture the convergence of prices and forecasts to the fundamental price, slightly outperforming our GA model. Under positive feedback, all these models (with the exception of HSM) lose their predictive power and under-estimate the experimental oscillatory behavior of individual forecasts. The GA model has the best fit for the positive feedback treatment and outperforms RE by a factor of 10.

These MC simulations show that our model captures both the aggregate and individual behavior in the LtF experiment reported by HHST09, both in terms of short and long-run dynamics. Moreover our model is the only one that captures the observed degree of heterogeneity of individual behavior between the experimental subjects, as measured by the coordination of the contemporary price forecasts (Figure 2.2c and 2.2c), together with the persistent heterogeneity of the forecasting heuristics.

2.4 Evidence from other experiments

Our GA model fits the HHST09 well. We will now move from the simple linear feedback to more complicated experimental settings. To be specific, we look at three other experiments that offer a hierarchy of challenges for the GA model:
2.4. Evidence from other experiments

1. BHST12: linear feedback with large and unanticipated shocks to the fundamental price;

2. HSTV07, V01: nonlinear (cobweb) negative feedback economy, investigated with a GA model by Hommes and Lux (2013);

3. HSTV05: non-linear positive feedback economy, with two-period ahead predictions;

2.4.1 Large shocks to the fundamental price

BHST12 report a LtF experiment with the same structure as HHST09: positive and negative feedback with the linear structure given by (2.2) and the same dampening factor $|B| = \frac{20}{21}$. In this experiment however there are two large and unanticipated shocks to the fundamental price $A$: it changes from $p_f = 56$ to $p_f = 41$ in period $t = 21$ and then to $p_f = 62$ in period $t = 44$ until the last period $t = 65$.

![Negative feedback](a) Experiment group 8

![Positive feedback](b) Experiment group 8

![Sample GA with $\beta \in [-1.1, 1.1]$](c) (d) Sample GA with $\beta \in [-1.1, 1.1]$

**Figure 2.4:** BHST12 experimental groups and sample 65-period ahead simulations of the GA model (with $\beta \in [-1.1, 1.1]$ and random initial predictions). Black line represents the price and green dashed lines are the individual predictions.

The results of this experiment are similar to HHST09 and typical time paths are shown in Figure 2.4. Under negative feedback (Figure 2.4a), a shock to the fundamental breaks the subjects’ coordination and is followed by a quick convergence to the
new fundamental price. Under positive feedback (Figure 2.4b), shocks do not break the coordination, and the predictions and prices fluctuate smoothly towards the new fundamental, eventually over- or undershooting it.

Figure 2.5: BHST12: 65-period ahead Monte Carlo simulation (1000 markets) for the GA model with $\beta \in [-1.1, 1.1]$. Realized price (top) and degree of coordination (standard deviation of individual predictions; bottom) over time. Green dashed line and black pluses represent the experimental median and group observations; red line is the median and blue dotted lines are the 95% confidence interval for the GA model. Left panel displays the negative feedback, right panel the positive feedback.

Figure 2.5 shows 65-period ahead MC simulations of prices, individual price forecasts and the degree of coordination.\textsuperscript{12} Our model replicates well the experimental price dynamics for both treatments, as well as the impact of the shocks to the fun-

\textsuperscript{12}We estimate the distribution of the initial predictions as in Diks and Makarewicz (2013) and sample directly from it, see Appendix B.
2.4. Evidence from other experiments

damental price on individual coordination. For the $\beta \in [-1.1, 1.1]$ specification, the 95% CI of our GA model contain 65.58% (84.23%) of the experimental prices and 84.04% (66.73%) of the standard deviation of individual forecasts under negative (positive) feedback. Overall, we can replicate around 75% of the experimental data with 65-period ahead simulations.

![Figure 2.6: BHST12. 65-period ahead Monte Carlo simulation (1000 markets) for the GA model with $\beta \in [-1.1, 1.1]$. The price weight $\alpha$ and the trend extrapolation $\beta$ chosen by the agents over time. Red line is the median, blue dotted lines are 95% CI, purple dashed are 90% CI for the GA model. Left panel displays the negative feedback, right the positive feedback.](image)

Figure 2.6 illustrates the time evolution of the price weight $\alpha$ and trend extrapolation coefficient $\beta$, which were chosen by the GA agents in the 65-period ahead simulations. The median behavior is similar to that from the experiment without large shocks HHST09. Under the negative feedback, the median GA agent learns the same adaptive expectations rule $p_{i,t+1}^{e} \approx 0.5p_t + 0.5p_{i,t}^{e}$. Under the positive feedback, the median GA agent converges to a heuristic

$$p_{i,t+1}^{e} \approx 0.95p_t + 0.05p_{i,t}^{e} + 0.9(p_t - p_{t-1}),$$

which is a trend following rule with the trend extrapolation coefficient $\beta \approx 0.9$. This trend coefficient is significantly larger than the coefficient 0.6 in rule (2.9) used by the median GA agent under the positive feedback from the experiment without large
shocks HHST09. The 95% CI for the trend extrapolation $\beta$ becomes significantly positive towards the end of the experiment (see also Figure 2.10b for the histogram of $\beta$’s chosen in period 65). Hence, due to the large, unanticipated shocks in the positive feedback treatment, GA agents become strong trend followers.

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<th>Positive feedback</th>
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Table 2.4: BHST12 65-period ahead predictions. MSE of the experimental prices and forecasts, for Trend Extrapolation, Adaptive, Contrarian, Naive and Rational Expectations, Heuristic Switching Model and GA models (with $\beta \in [-1.1, 1.1]$ and $\beta \in [0, 1.1]$). MSE averaged over eight negative feedback and eight positive feedback groups.

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<th>MSE</th>
<th>Negative feedback</th>
<th>Positive feedback</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Prices</td>
<td>Forecasts</td>
</tr>
<tr>
<td>Trend extr.</td>
<td>114.061</td>
<td>121.329</td>
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<tr>
<td>Adaptive</td>
<td>3.689</td>
<td>10.332</td>
</tr>
<tr>
<td>Contrarian</td>
<td>5.92</td>
<td>12.534</td>
</tr>
<tr>
<td>Naive</td>
<td>9.979</td>
<td>16.81</td>
</tr>
<tr>
<td>RE</td>
<td>13.871</td>
<td>20.923</td>
</tr>
<tr>
<td>HSM</td>
<td>38.309</td>
<td>45.679</td>
</tr>
</tbody>
</table>

Table 2.5: BHST12 one-period ahead predictions. MSE of the experimental prices and forecasts, for the Trend Extrapolation, Adaptive, Contrarian, Naive and Rational Expectations, Heuristic Switching Model and Genetic Algorithms models (with $\beta \in [-1.1, 1.1]$ and $\beta \in [0, 1.1]$). MSE averaged over eight negative feedback and eight positive feedback groups.

Table 2.4 reports the MSE for the 65-period ahead simulations initialized with
the experimental initial predictions (1024 simulated markets per group for the GA models). We observe that the adaptive expectations have a good fit to the negative feedback treatment, while naive expectations perform well for the positive feedback. Interestingly, RE are poor for both treatments: they cannot explain oscillations of the positive feedback and the short spells of volatility that follow shocks to the fundamental under the negative feedback treatment. Also, HSM seems below average. In terms of long-run forecasting, our GA model is again second best for the negative feedback and the best for positive feedback.

We also use the SMC approach to compute the GA model’s one-period ahead predicting power, reported in Table 2.5. The results are consistent with the 65-period ahead simulations. For both treatments, the GA model (especially without contrarian rules, $\beta \in [0, 1.1]$) is the best among all reported models.

### 2.4.2 Cobweb economy

V01 and HSTV07 report an LtF experiment in a Cobweb economy setting. HSTV07 investigate 18 markets with six subjects each, divided into three treatments of 6 groups: with stable, unstable (on the verge of stability) and (strongly) unstable parametrization under the assumption of homogenous naive expectations. This is a follow-up on V01 who investigates the strongly unstable treatment with 12 subjects. The experiment resulted in average price equal to the RE fundamental price. However, the realized prices were excessively volatile, but — in contrast to positive feedback experiments — also non-persistent (with weak autocorrelation structure). Hommes and Lux (2013) study this experimental data set with the GA model based on an anchor-and-adjustment (AR1) forecasting rule. It therefore constitutes an important benchmark case for our GA model.

As a first test for our model, we conduct a MC exercise in the vein of Hommes and Lux (2013). For each treatment, we compute six 50-period ahead simulations with different random numbers.\(^{13}\) Next we compute the mean and standard deviation of the realized prices and the individual price forecasts. We repeat this procedure 1’000 times and thus obtain a distribution (including 95% CI) of the realized means and variances of prices and price forecasts. We report the results in Table 2.6 for the two GA model specifications.

Our 50-period ahead simulations explain well the experimental data and perform significantly better than RE. The 95% CI of our GA model with $\beta \in [-1.1, 1.1]$ and

\(^{13}\)We estimate the distribution of the initial predictions as in Diks and Makarewicz (2013), see Appendix B.
CHAPTER 2. LEARNING-TO-FORECAST WITH GENETIC ALGORITHMS

<table>
<thead>
<tr>
<th></th>
<th>Mean(p)</th>
<th>Var(p)</th>
<th>Mean(p*)</th>
<th>Var(p*)</th>
</tr>
</thead>
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<tr>
<td><strong>Stable</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Experiments</td>
<td>5.64†</td>
<td>0.36†</td>
<td>5.56†</td>
<td>0.087*</td>
</tr>
<tr>
<td>GA: AR1</td>
<td>5.565*</td>
<td>0.326*</td>
<td>5.576*</td>
<td>0.1</td>
</tr>
<tr>
<td>GA: FOR $\beta \in [-1, 1]$</td>
<td>5.628</td>
<td>0.372</td>
<td>5.571</td>
<td>0.082</td>
</tr>
<tr>
<td>95% CI</td>
<td>[5.613, 5.643]</td>
<td>[0.359, 0.389]</td>
<td>[5.553, 5.59]</td>
<td>[0.065, 0.101]</td>
</tr>
<tr>
<td>GA: FOR $\beta \in [0, 1]$</td>
<td>5.649</td>
<td>0.353</td>
<td>5.548</td>
<td>0.0565</td>
</tr>
<tr>
<td>95% CI</td>
<td>[5.631, 5.667]</td>
<td>[0.341, 0.371]</td>
<td>[5.527, 5.57]</td>
<td>[0.043, 0.077]</td>
</tr>
</tbody>
</table>

| **Unstable**             |         |        |          |         |
| Experiments              | 5.85†   | 0.63†  | 5.67†    | 0.101†  |
| GA: AR1                  | 5.817   | 0.647  | 5.645*   | 0.16†   |
| GA: FOR $\beta \in [-1, 1]$ | 5.792   | 0.598  | 5.705    | 0.103   |
| 95% CI                   | [5.744, 5.841] | [0.525, 0.746] | [5.667, 5.739] | [0.067, 0.171] |
| GA: FOR $\beta \in [0, 1]$ | 5.825   | 0.557  | 5.694    | 0.079   |
| 95% CI                   | [5.786, 5.863] | [0.487, 0.658] | [5.67, 5.719] | [0.052, 0.122] |

| **Strongly unstable**    |         |        |          |         |
| Experiments              | 5.93†   | 2.62*  | 5.73     | 0.429*  |
| GA: AR1                  | 6.2‡    | 2.161  | 5.434    | 0.769   |
| GA: FOR $\beta \in [-1, 1]$ | 5.809   | 2.172  | 5.832    | 0.345   |
| 95% CI                   | [5.693, 5.908] | [1.626, 2.875] | [5.735, 5.918] | [0.182, 0.598] |
| GA: FOR $\beta \in [0, 1]$ | 5.962   | 1.487  | 5.807    | 0.206   |
| 95% CI                   | [5.876, 6.045] | [1.188, 1.834] | [5.75, 5.858] | [0.113, 0.347] |

| **Strongly unstable, large group** |         |        |          |         |
| Experiments              | 5.937†  | 1.783* | 5.781†   | 0.204†  |
| GA: AR1                  | 6.183‡  | 1.571  | 5.515‡   | 0.5‡    |
| GA: FOR $\beta \in [-1, 1]$ | 5.812   | 1.699  | 5.852    | 0.194   |
| 95% CI                   | [5.731, 5.892] | [1.368, 2.157] | [5.779, 5.918] | [0.122, 0.338] |
| GA: FOR $\beta \in [0, 1]$ | 5.972   | 1.316  | 5.804    | 0.173   |
| 95% CI                   | [5.918, 6.026] | [1.118, 1.553] | [5.768, 5.843] | [0.111, 0.253] |

Table 2.6: HSTV07: 50-period ahead MC results for GA simulations for four treatments, stable, unstable and strongly unstable with 6 or 12 subjects. Average price and prediction, and their variances. Mean experimental statistics; GA simulations with AR1 prediction rule for mutation rate $\delta_m = 0.01$ (mean statistics); GA simulations with FOR with or without contrarian rules (median statistics with 95% confidence intervals). * and † denote experimental statistic which falls into 95% CI of GA FOR with $\beta \in [-1, 1]$ and $\beta \in [0, 1]$ respectively. ‡ denotes Hommes and Lux (2013) statistics which fall outside the 95% CI for GA model with $\beta \in [0, 1]$ when these CI contain the experimental statistics. 

$\beta \in [0, 1]$ replicate 12 and 11 out of 16 experimental statistics respectively. Among the 11 cases successful for the GA model based on FOR rule (2.6) with $\beta \in [0, 1]$, 9 statistics reported by Hommes and Lux (2013) are outside 95% CI of our model. That means that we can replicate around three quarters of experimental descriptive statistics.
## 2.4. Evidence from other experiments

<table>
<thead>
<tr>
<th>Treatments</th>
<th>Stable MSE</th>
<th>Stable Forecasts</th>
<th>Unstable MSE</th>
<th>Unstable Forecasts</th>
<th>Strongly unstable MSE</th>
<th>Strongly unstable Forecasts</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trend extr.</td>
<td>13.3</td>
<td>71.1</td>
<td>16.33</td>
<td>89.59</td>
<td>16.55</td>
<td>89.07</td>
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<tr>
<td>Adaptive</td>
<td>0.117</td>
<td>0.339</td>
<td>7.206</td>
<td>3.272</td>
<td>16.45</td>
<td>7.822</td>
</tr>
<tr>
<td>Contrarian</td>
<td>0.093</td>
<td>0.308</td>
<td>1.746</td>
<td>0.834</td>
<td>13.95</td>
<td>5.282</td>
</tr>
<tr>
<td>Naive</td>
<td>1.076</td>
<td>1.724</td>
<td>14.67</td>
<td>16.18</td>
<td>16.55</td>
<td>18.55</td>
</tr>
<tr>
<td>RE</td>
<td>0.048</td>
<td>0.248</td>
<td>0.364</td>
<td>0.385</td>
<td>2.257</td>
<td>1.844</td>
</tr>
<tr>
<td>HSM</td>
<td>0.178</td>
<td>0.422</td>
<td>7.446</td>
<td>3.431</td>
<td>16.46</td>
<td>7.885</td>
</tr>
<tr>
<td>GA: AR1</td>
<td>0.05742</td>
<td>0.3759</td>
<td>0.3552</td>
<td>0.6596</td>
<td>2.838</td>
<td>2.64</td>
</tr>
<tr>
<td>GA: $\beta \in [-1.1, 1.1]$</td>
<td>0.088</td>
<td>0.356</td>
<td>0.346</td>
<td>0.631</td>
<td>3.445</td>
<td>3.261</td>
</tr>
<tr>
<td>GA: $\beta \in [0, 1.1]$</td>
<td><strong>0.043</strong></td>
<td><strong>0.275</strong></td>
<td><strong>0.223</strong></td>
<td><strong>0.449</strong></td>
<td><strong>2.376</strong></td>
<td><strong>2.114</strong></td>
</tr>
</tbody>
</table>

Table 2.7: HSTV07: 50-period ahead predictions. MSE of the experimental prices and forecasts, for Trend Extrapolation, Adaptive, Contrarian, Naive and Rational Expectations, Heuristic Switching Model and GA models (FOR with $\beta \in [-1.1, 1.1]$ and $\beta \in [0, 1.1]$). MSE averaged over six groups for each treatment (stable, unstable, strongly unstable).

<table>
<thead>
<tr>
<th>Treatments</th>
<th>Stable MSE</th>
<th>Stable Forecasts</th>
<th>Unstable MSE</th>
<th>Unstable Forecasts</th>
<th>Strongly unstable MSE</th>
<th>Strongly unstable Forecasts</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trend extr.</td>
<td>1.176</td>
<td>1.997</td>
<td>2.122</td>
<td>3.719</td>
<td>5.856</td>
<td>14.39</td>
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<tr>
<td>Adaptive</td>
<td>0.108</td>
<td>0.328</td>
<td>0.434</td>
<td>0.549</td>
<td>2.784</td>
<td>2.863</td>
</tr>
<tr>
<td>Contrarian</td>
<td>0.102</td>
<td>0.318</td>
<td>0.414</td>
<td>0.497</td>
<td>2.929</td>
<td>2.729</td>
</tr>
<tr>
<td>Naive</td>
<td>0.196</td>
<td>0.448</td>
<td>0.577</td>
<td>0.788</td>
<td>3.095</td>
<td>3.731</td>
</tr>
<tr>
<td>RE</td>
<td><strong>0.048</strong></td>
<td><strong>0.248</strong></td>
<td><strong>0.364</strong></td>
<td><strong>0.385</strong></td>
<td><strong>2.257</strong></td>
<td><strong>1.844</strong></td>
</tr>
<tr>
<td>HSM</td>
<td>0.212</td>
<td>0.474</td>
<td>0.52</td>
<td>0.732</td>
<td>3.065</td>
<td>3.691</td>
</tr>
<tr>
<td>GA: AR1</td>
<td>0.054</td>
<td>0.36</td>
<td>0.51</td>
<td>0.674</td>
<td>5.36</td>
<td>3.432</td>
</tr>
<tr>
<td>GA: $\beta \in [-1.1, 1.1]$</td>
<td>0.13</td>
<td>0.393</td>
<td>0.866</td>
<td>0.795</td>
<td>5.547</td>
<td>3.25</td>
</tr>
<tr>
<td>GA: $\beta \in [0, 1.1]$</td>
<td><strong>0.07</strong></td>
<td><strong>0.31</strong></td>
<td><strong>0.25</strong></td>
<td><strong>0.531</strong></td>
<td><strong>3.079</strong></td>
<td><strong>2.358</strong></td>
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</tbody>
</table>

Table 2.8: HSTV07: one-period ahead predictions. MSE of the experimental prices and forecasts, for the Trend Extrapolation, Adaptive, Contrarian, Naive and Rational Expectations, Heuristic Switching Model and Genetic Algorithms models (with $\beta \in [-1.1, 1.1]$ and $\beta \in [0, 1.1]$). MSE averaged over six groups for each treatment (stable, unstable, strongly unstable).
most of which with a significantly higher precision than the GA model specification in Hommes and Lux (2013).\footnote{14\textsuperscript{4}}

We also check the 50-period ahead dynamics of the model conditional on the initial predictions from particular groups from HSTV07, see Table 2.7. Homogeneous expectation models, as well as HSM for the two unstable treatments are outperformed by RE. The dynamics of this experiment (in contrast to the linear experiments) resemble a white noise around the fundamental price. As a result, predicting the mean of these close-to-chaotic dynamics (as RE do) is better than trying to capture them with simplistic models. Only our GA model, in particular the one with $\beta \in [0, 1.1]$, keeps up with RE, and performs better than Hommes and Lux (2013) GA specification based on an AR1 rule.

The next exercise is the one-period ahead forecasting of the model with SMC approach for the 18 groups from HSTV07. Table 2.8 gives the summary results. It is apparent that the less stable the treatment, the worse fit has any model. As for the 50 period ahead forecasts, the clear winners are RE and our GA model, which are able to explain the data well also for the strongly unstable treatment.\footnote{15\textsuperscript{5}} Our specification again prevails over the AR1 GA model of Hommes and Lux (2013).

We conclude that the cobweb experiments result in unstable, non-persistent prices, and simpler models like homogenous heuristics, but also HSM, miss-identify here any structure. As a result, their point predictions are so poor that it is better to predict the mean price, as in RE. Only our GA model (with $\beta \in [0, 1.1]$) comes close to RE in terms of this task. It furthermore allows to explain the volatility of the experimental markets, which RE cannot account for. Finally, it is clear that the use of experimental micro-foundations has an advantageous effect: our GA model has a better fit to the data than the AR1 specification used by Hommes and Lux (2013).

\subsection{Two-period ahead asset pricing}

HSTV05 report an experiment based on a non-linear positive feedback market; an asset-pricing model with a robotic fundamental trader, in which the current price depends on the subjects’ expectations about the price in the next period: $p_t = F(p_{e_{1, t+1}}, \ldots, p_{e_{6, t+1}})$.\footnote{The GA model simulations are also closer to the experimental data in terms of the autocorrelation of the prices. RE always predicts zero autocorrelation, whereas benchmark models predict high autocorrelation up to the third lag. The experimental data exhibited weak autocorrelation, which is replicated by all three GA model specifications with comparable performance. See Table 2.16 in Appendix 2.F for the results.}

\footnote{Notice that the scale of the prices in this experiment is $[0, 10]$ in contrast with the two previous settings, where the prices belonged to $[0, 100]$ intervals. The highest possible MSE in the linear experiments is 100 times higher than in the cobweb experiment.}
There were two treatments with the difference in the fundamental price: seven markets were based on $p_f = 60$ and three on $p_f = 40$. Subjects coordinated both on stable outcomes and diversified oscillations.

Notice that in this experiment the subject’s decisions are based on a different information set than in the previous one-period ahead experiments. Upfront it is difficult to predict how this will influence subject behavior, specifically whether they will use more complicated strategies or extrapolate the trend to a different degree. After some experimentation, we decided that the two period ahead version of our GA model should be based on the following specification. Define the prediction of price from period $t + 1$ by the GA agent $i$ based on her rule $h$ as

$$
(2.14) \quad p_{i,h,t+1}^e = \alpha_{i,h} p_{t-1} + (1 - \alpha_{i,h}) p_{i,t-1}^e + \beta_{i,h} (p_{t-1} - p_{t-2}).
$$

Once $p_t$ is realized, the agents can evaluate their rules based on the hypothetical performance of predicting $p_t$ two periods ago. GA agents focus on $(p_t - p_{i,h,t,t}^e)^2$, where $p_{i,h,t}^e$ is function of $p_{i,t-2}^e$, $p_{t-2}$ and $p_{t-3}$.

This specification is the most straightforward translation of the baseline one-period ahead forecasting heuristic (2.6). Again, there is no evidence that we need an anchor (see Appendix C). In the baseline simulations, we look at the allowed trend specified as before (with $\beta \in [-1.1, 1.1]$ and $\beta \in [0, 1.1]$). HSTV05 report that many of their subjects use very strong trend extrapolation, thus for the sake of completeness we will also report the results of our model with $\beta \in [-1.3, 1.3]$ and $\beta \in [0, 1.3]$.

In the seven treatment groups with the fundamental price $p_f = 60$, HSTV05 observe groups which have converged to this fundamental, as well as groups with oscillations of different amplitude and frequency. Figure 2.7 displays three typical simulated markets.


**Figure 2.8:** HSTV05: sample 2000-period ahead simulation (b) and its first 500 periods (a) of the GA model with $\beta \in [-1.3, 1.3]$ with fundamental price $p_f = 60$ and random initial predictions. The green lines are individual predictions, the black line is the realized price and the purple dashed line is the fundamental price.

<table>
<thead>
<tr>
<th>MSE</th>
<th>Prices</th>
<th>Forecasts</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trend extr.</td>
<td>178.2</td>
<td>174.9</td>
</tr>
<tr>
<td>Adaptive</td>
<td>96.12</td>
<td>145.9</td>
</tr>
<tr>
<td>Contrarian</td>
<td>157</td>
<td>146.8</td>
</tr>
<tr>
<td>Naive</td>
<td>95.29</td>
<td>144.6</td>
</tr>
</tbody>
</table>

Table 2.9: HSTV05: 50-period ahead predictions. MSE of the experimental prices and forecasts, for Trend Extrapolation, Adaptive, Contrarian, Naive and Rational Expectations, Heuristic Switching Model and GA models (with $\beta \in [-1.1, 1.1]$ and $\beta \in [0, 1.1]$). MSE averaged over all experimental groups.

The GA agents can both converge to the fundamental price (Figure 2.7a) as well as coordinate on unruly oscillations (Figure 2.7c). Furthermore, sometimes both outcomes are present at the same time. Figure 2.7b shows a sample simulation, in which the price seemingly stabilizes at the fundamental value between periods 18 and 20, but then resumes to oscillate mildly.

To further stress the volatile behavior of this market structure, we report one long run simulation for the GA model with $\beta \in [-1.3, 1.3]$. Figure 2.8 displays its first 500 periods.

---

16 See Appendix B for initialization.
2.4. Evidence from other experiments

Figure 2.9: HSTV05: 50-period ahead Monte Carlo simulation (1000 markets) for the GA model with $\beta \in [-1.1, 1.1]$ (left panel) and $\beta \in [-1.3, 1.3]$ (right panel). The price weight $\alpha$ and the trend extrapolation $\beta$ chosen by the agents over time. Red line is the median, blue dotted lines are 95% CI, purple dashed are 90% CI for the GA model.

(Figure 2.8a) and 2'000 (Figure 2.8b) periods. Oscillations of different amplitude are persistent and can reappear even if the market settles on the fundamental price for some time, as seen in Figure 2.8b around period 800 or after period 1200. This means that in the system the fundamental price is not a unique attractor.

To explain this outcome, we take a closer look at the trend extrapolation chosen by the GA agents. Figure 2.9 shows results for MC 50-period ahead simulations for two GA model specifications, with $\beta \in [-1.1, 1.1]$ and $\beta \in [-1.3, 1.3]$. If the agents are allowed to experiment with higher $\beta$, the median price has a very similar oscillatory shape. The difference is seen in the 95% CI: both specifications are likely to generate two price
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MSE Prices Forecasts
Trend extr. 17.4527 55.0898
Adaptive 44.125 25.3157
Contrarian 59.3905 30.8646
Naive 31.6864 20.8416

RE 96.0328 145.998
HSM (4 heuristics) 6.798 —

Table 2.10: HSTV05: one-period ahead predictions. MSE of the experimental prices and forecasts, for Trend Extrapolation, Adaptive, Contrarian, Naive and Rational Expectations, 4-type Heuristic Switching Model (source: Anufriev and Hommes, 2012) and GA models (with $\beta \in [-1.1, 1.1]$ and $\beta \in [0, 1.1]$). MSE averaged over all experimental groups.

Figure 2.10: Positive feedback treatments: HHST09, BHST12 and HSTV05 with $p^f = 60$: 50-period ahead predictions. Distribution of trend extrapolation coefficient $\beta$ chosen by the agents in the last period $t = 50$ across the whole MC sample for each treatment, and two $\beta$ specifications for HSTV05.

bubbles within 50 periods, but the model with $\beta \in [-1.3, 1.3]$ has larger potential oscillations (Figure 2.9b), and the second bubble can be even bigger than the first one (unlike in the linear positive feedback). Regardless, the median GA agent converges
2.4. Evidence from other experiments

to a strong trend extrapolation rule, close to $p_{t+1}^e = p_{t-1} + (p_{t-1} - p_{t-2})$, which is consistent with the behavior of our model in the previous experiments. Nevertheless, the 95% CI of the chosen trend coefficient remain wide and the distribution of this variable in period 50 (Figures 2.10c and 2.10d) is bimodal, with a relatively large mass centered around zero (i.e. weak or no trend extrapolation).

We interpret this finding in the following way. If the price is sufficiently stable and close to the fundamental value, the robotic fundamental trader is powerful enough to mitigate additional price deviations. This discourages GA agents to extrapolate the insignificant trend, and so the price stability becomes self-reinforcing. However, if the trend in prices is sufficiently large, the stabilizing effect of the robotic trader can be counter-weighted by the GA agents coordinating on trend extrapolation. The non-linear and the two-period ahead price feedback amplifies the realized price oscillations (which become self-reinforcing), but also allows their specific shape to be diversified. For this reason we speculate that the two period ahead feedback entails two types of attractors in our model, which corresponds well to the diversified dynamics observed in the experiment.

We note that the model does not predict fast price oscillations, and more than two bubbles within 50 periods are rarely observed (in contrast to the experiment). This is independent from the allowed trend extrapolation and cannot be explained by adding an anchor to the forecasting rule (2.14) (see Appendix C). We speculate that one should experiment with higher order rules to replicate all the oscillations from HSTV05, but we leave this for future investigations.\footnote{HSM does explain these faster oscillations with higher order AR2 rule (Anufriev and Hommes, 2012).}

Even though our GA model leaves space for improvement, it is the only one which is comparatively good in predicting the experimental results of HSTV05 both in the long- and the short-run. Table 2.9 reports the MSE of 50-period ahead simulations initialized with the experimental initial predictions. These are comparatively poor for all models. The best three models are naive, adaptive and RE, though our model (with 1.1 as the upper bound for trend extrapolation) yields similar results. Table 2.10 shows the MSE of one-period ahead predictions for our GA model and benchmark models. The GA model is now among the best, especially in terms of predicting the experimental prices. Surprisingly, the models that did well in 50-period ahead predictions are poor now, while trend extrapolation is comparable with our model. Anufriev and Hommes (2012) investigated the HSTV05 experiment with a four-heuristics HSM, which is a richer model than the two-heuristic HSM we used as a benchmark for the previous experiments. Interestingly, only our GA model (specifically with $\beta \in [0, 1.1]$) is able
to compete with this richer HSM in terms of predicting experimental prices.

This outcome resembles the results for the cobweb economy Hommes et al. (2007). A natural interpretation is given by our GA model, which predicts that the dynamics of HSTV05 economy are unruly price oscillations. It follows that one can successfully predict the subjects forecasts and prices in the short run by a trend extrapolation heuristics. However, the oscillations can arbitrarily change shape, which together with the co-existence of two attractors renders any long-run forecasting virtually impossible. It also means that, despite potentially oscillatory dynamics, no single trend extrapolation model can replicate the long-run dynamics of this experiment. We leave it open for other research whether any structural model can cope with such an environment.

2.5 Conclusions

In this chapter we discuss a model in which agents independently use Genetic Algorithms to optimize a simple forecasting heuristic. We argue that our model is able to replicate many findings from different Learning-to-Forecast experiments, both at the aggregate and individual level.

In Learning-to-Forecast experiments, subjects are asked to forecast prices, while the realized price depends on their predictions. This mimics many well studied economic environments, such as asset pricing markets or cobweb economies. These experiments can be used as a controlled setting to study how the human subjects try to adapt to the price-predictions feedback. Their major insight is that the market converges to the rational expectations equilibrium only if the relationship between the average price expectation and the realized price is linear and negative (Heemeijer et al., 2009). In the case of markets in which this relationship is positive, subjects may coordinate on extrapolating observed price trends, which reinforces price oscillations (Hommes et al., 2005).

The most successful attempt to replicate these dynamics comes from the Heuristic Switching Models (Anufriev and Hommes, 2012). The main intuition of this approach is that among different prediction heuristics, the agents focus on those that have a relatively good hypothetical past performance. On the other hand, Heuristic Switching Models cannot explain the full degree of observed individual heterogeneity, nor does it explain how the agents could learn their heuristics.

Hommes and Lux (2013) explicitly model such individual learning with Genetic Algorithms. We enhance the GA model of Hommes and Lux (2013) with the empirical micro-foundations identified by Heemeijer et al. (2009). Our GA agents use a first-
2.5. Conclusions

order heuristic (a mixture of adaptive and trend extrapolating expectations) to forecast the prices. They independently optimize the parametrization of their heuristics with Genetic Algorithms, thus learning to fine-tune their forecasting rules of thumb to the specific market conditions. This gives an agent-based model of explicit learning-to-forecast with strong empirical motivation for the particular forecasting behavior.

We use our Genetic Algorithms model to investigate four Learning-to-Forecast experiments. The simple linear setting of the experiment reported by Heemeijer et al. (2009) enables us to set up the model. The experiment reported by Bao et al. (2012) adds large and unanticipated shocks to the basic linear structure of Heemeijer et al. (2009). The third experiment, reported by van de Velden (2001) and Hommes et al. (2007) focuses on a non-linear cobweb economy, and is an important benchmark already investigated by Hommes and Lux (2013). Finally, the asset pricing experiment reported by Hommes et al. (2005) introduces two-periods ahead feedback between the predictions and the realized prices.

We evaluate the out of sample one-period ahead and 50 period ahead prediction accuracy of our model in comparison with benchmark models: rational expectations, a number of simple homogenous expectations models (including adaptive and naive expectations) and the Heuristic Switching Model. To our best knowledge, this chapter is the first to present an explicit econometric evaluation of how a full fledged agent-based model performs in explaining experimental data, including the individual level. For the difficult task of one-period ahead model predictions, we develop a Sequential Monte Carlo technique, a special case of the Auxiliary Particle. This is a novelty in the literature, which would rather focus on explaining the aggregate experimental outcomes.

Across the four discussed experiments, we observe a clear pattern in how different models can predict subject behavior. Rational expectations tend to explain comparatively well the negative types of price-predictions feedback. Also, for every experimental economy one can find a simple homogenous expectations model that fits this particular experiment well, but only in the short-run. Typically, contrarian or adaptive expectations have a good one-period ahead fitness to the data under negative feedback, while trend extrapolation or naive expectations outperform other models for positive feedback type of economies. However, there is no single homogenous expectations rule, including rational expectations, that can explain all the experimental economies at the same time. On the other hand, this is where the strength of our Genetic Algorithms model lies, which is able to account for both the aggregate outcomes and the individual behavior across different experiments.

Homogenous expectations models, as well as Heuristic Switching Model, take agents
as using static heuristics, whereas in reality people try to adjust their behavioral rules to the particular circumstances. This means that the simple homogenous models can replicate subject behavior only for very simple experimental economies and typically only in short out-of-sample studies. Across the first three discussed experiments, only our model remains realistically close to the experimental aggregate and individual behavior in the short-run as well as after 50 periods. For the Hommes et al. (2005) data, our model is relatively the best one and directly explains why no model performs well here: this experimental design incurs two type of attractors.

In addition, we conduct a Monte Carlo study of 50-period ahead simulations based on random initialization. With these, we can study the evolution of learning and price dynamics in the four experimental economies, by evaluating the median and 95% confidence intervals of the heuristic coefficients, which were chosen by the agents. Simulations of our model replicate the stylized results by Anufriev et al. (2013) and identify a clear pattern of individual learning. When agents face a negative feedback type of economy, a median agent will rely on adaptive expectations. If the feedback is sufficiently simple, this implies convergence of the market to the fundamental equilibrium. However, strong non-linearities will rather result in near-to-chaotic dynamics around the perfectly rational solution.

On the other hand, positive feedback induces the agents to follow a price trend. Median agent converges to a trend extrapolation rule with little emphasis on her own past predictions, which typically causes price oscillations. The more ‘difficult’ the feedback is (in terms of shocks to the fundamental solution, or non-linear law of motion of the price), the stronger trend extrapolation will be chosen by the median agent. However, positive feedback markets can also settle on the fundamental solution. We emphasize that this is not a sign for support to rational expectations framework. In fact, our model shows that the agents will switch between following price oscillations and settling on the fundamental solution, if they have to act in a complex economic environment, such as the two-period ahead type of expectation-price feedback.

The strength of our model lies in its generality, seen in the good fit to different Learning-to-Forecast experiments. Furthermore its agent-based structure allows for replicating the individual behavior observed in these experiments, by a realistic account of heterogeneity and learning. We therefore argue that it can be used to investigate settings with a more complicated interactions between individual agents. This can include economies with heterogeneous preferences, unequal market power, information networks or decentralized price setting. In any of these cases, heterogeneous price expectations may have important consequences for market efficiency or dynamics. Our Genetic Algorithms model gives a realistic explanation of how such heterogene-
ity between the agents emerges from their individual learning, and what can be the consequences for the aggregate market outcomes.

This contrasts the dominating framework of the perfectly rational expectations. Traditionally, economists assumed that people use sophisticated concepts such as a fundamental price or a long run equilibrium. Economists for a long time disregarded the fact that in the market practice the agents face constraints on their rationality and thus may be forced to use second-best prediction rules. As a result, rational expectation fail to describe economic phenomena, unless these are extremely simple. In the context of price expectations, we propose a model where the agents use simple behavioral rules, but adapt them to the current environment with a smart optimization procedure. This allows for a realistic description of human behavior, which explains the experimental data, both at the individual and the aggregate level, to the degree that was unattainable for the traditional literature.
Appendix 2.A  Formal definition of Genetic Algorithms

In this appendix we present a formal definition of the Genetic Algorithms (GA) version, which served as the cornerstone of our model. It closely follows the standard specification suggested by Haupt and Haupt (2004) and used by Hommes and Lux (2013).

2.A.1 Optimization procedures: traditional and Genetic Algorithms

Consider a maximization problem where the target function \( F \) of \( N \) arguments \( \theta = (\theta^1, \ldots, \theta^N) \) is such that a straightforward analytical solution is unavailable. Instead, one needs to use a numerical optimization procedure.

Traditional maximization algorithms, like the Broyden-Fletcher-Goldfarb-Shanno (BFGS) algorithm, iterate a candidate argument for the optimum of the target function \( F \) by (1) estimating the curvature around the candidate and (2) using this curvature to find the optimal direction and length of the change to the candidate solution. This so called ‘hill-climbing’ algorithm is very efficient in its use of the shape of the target function. On the other hand, it will fail if the target function is ‘ill-behaved’: non-continuous or almost flat around the optima, has kinks or breaks. Here the curvature cannot be reliably estimated. Another problem is that the BFGS may perform poorly for a problem of large dimensionality.

The Genetic Algorithms are based on a fundamentally different approach and therefore can be used for a wider class of problems. The basic idea is that we have a population of arguments which compete only in terms of their respective function value. This competition is modeled in an evolutionary fashion: mutation operators allow for a blind-search experimentation, but the probability that a particular candidate will survive over time is relative to its functional value. As a result, the target function may be as general as necessary, while the arguments can be of any kind, including real numbers, integers, probabilities or binary variables. The only constraint is that each argument must fall into a predefined dense interval \( a_n, b_n \).

2.A.2 Binary strings

A Genetic Algorithm (GA) uses \( H \) chromosomes \( g_{h,t} \in \mathbb{H} \) which are binary strings divided into \( N \) genes \( g^n_{h,t} \), each encoding one candidate parameter \( \theta^n_{h,t} \) for the argument
2.A. Formal definition of Genetic Algorithms

A chromosome $h \in \{1, \ldots, H\}$ at time $t \in \{1, \ldots, T\}$ has predetermined length $L$ and is specified as

$$g_{h,t} = \{g_{h,t}^1, \ldots, g_{h,t}^N\},$$

such that each gene $n \in \{1, \ldots, N\}$ has its length equal to an integer $L_n$ (with $\sum_{n=1}^N L_n = L$) and is a string of binary entries (bits)

$$g_n^{n} = \{g_{h,t}^{n,1}, \ldots, g_{h,t}^{n,L_n}\}, \quad g_{h,t}^{n,j} \in \{0, 1\}$$

for each $j \in \{1, \ldots, L_n\}$. The relation between the genes and the arguments is straightforward. An integer $\theta_n$ is simply encoded by (2.16) with its binary notation. Consider now an argument $\theta_n$ which is a probability. Notice that $\sum_{l=0}^{L_n-1} 2^l = 2^{L_n} - 1$. It follows that a particular gene $g_{h,t}^{n}$ can be decoded as a normalized sum

$$\theta_{h,t} = \sum_{l=1}^{L_n} g_{h,t}^{n,l} \frac{2^l - 1}{2^{L_n} - 1}.$$

A gene of zeros only is therefore associated with $\theta_n = 0$, a gene of ones only – with $\theta_n = 1$, while other possible binary strings cover the $[0, 1]$ interval with an $\frac{1}{2^{L_n-1}}$ increment. Any desired precision can be achieved with this representation. Since $2^{-10} \approx 10^{-3}$, the precision close to one over trillion ($10^{-12}$) is obtained by a mere of 40 bits.

A real variable $\theta_n$ from an $[a_n, b_n]$ interval can be encoded in a similar fashion, by a linear transformation of a probability:

$$\theta_{h,t} = a_n + (b_n - a_n) \sum_{l=1}^{L_n} g_{h,t}^{n,l} \frac{2^l - 1}{2^{L_n} - 1}$$

where the precision of this representation is given by $\frac{b_n - a_n}{2^{L_n-1}}$. Notice that one can approximate an unbounded real number by reasonably large $a_n$ or $b_n$, since the loss of precision is easily undone by a longer string.

2.A.3 Evolutionary operators

The core of GA are evolutionary operators. GA iterates the population of chromosomes for $T$ periods, where $T$ is either large and predefined, or depends on some convergence criterion. First, at each period $t \in \{1, \ldots, T\}$ each chromosome has its fitness equal to a monotone transformation of the function value $F$. This transformation is defined as
\( V(\mathcal{F}(\theta_{h,t})) \equiv V(h_{k,t}) \rightarrow \mathbb{R}^+ \cap \{0\} \). For example, a non-negative function can be used directly as the fitness. If the problem is to minimize a function, a popular choice is the exponential transformation of the function values, similar to the one used in the logit specification of the Heuristic Switching Model (Brock and Hommes, 1997).

Chromosomes at each period can undergo the following evolutionary operators: procreation, mutation, crossover and election. These operators first generate an offspring population of chromosomes from the parent population \( t \) and therefore transform both populations into a new generation of chromosomes \( t + 1 \) (notice the division of the process).

**Procreation**

For the population at time \( t \), GA picks subset \( \mathbb{X} \subseteq \mathbb{H} \) of \( \chi \) chromosomes and picks \( \kappa < \chi \) of them into a set \( \mathbb{K} \). The probability that the chromosome \( h \in \mathbb{X} \) will be picked into \( \mathbb{K} \) as its \( z \)-th element (where \( z \in \{1, \ldots, \kappa\} \)) is usually defined by the power function:

\[
Prob(g_z = g_{h,t}) = \frac{V(g_{h,t})}{\sum_{j \in \mathbb{X}} V(g_{j,t})}.
\]

This procedure is repeated with differently chosen \( \mathbb{X} \)'s until the number of chromosomes in all such sets \( \mathbb{K} \)'s is equal to \( H \). For instance, the roulette is procreation with \( \chi = H \) and \( \kappa = 1 \): GA picks randomly one chromosome from the whole population, where each chromosome has probability of being picked equal to its function value relative to the function value of all other chromosomes. This is repeated exactly \( H \) times.

So called tournaments are often used for the sake of computational efficiency. Here, \( \chi << H \). For instance, GA could divide the chromosomes into pairs and sample two offspring from each pair.

Procreation is modeled as the basic natural selection mechanism. We consider subsets of the original population (or maybe the whole population at once). Out of each such a subset, we pick a small number of chromosomes, giving advantage to these which perform better. We repeat this procedure until the offspring generation is as large as the old one. Thus the new generation is likely to be ‘better’ than the old one.

**Mutation**

For each generation \( t \in \{1, \ldots, T\} \), after the procreation has taken place, each binary entry in each new chromosome has a predefined \( \delta_m \) probability to mutate: ones turned into zeros and vice versa. In this way the chromosomes represent different numbers
and may therefore attain better fit.

The mutation operator is where the binary representation becomes most useful. If the bits, which are close to the beginning of the gene, mutate, the new argument will be substantially different from the original one. On the other hand, small changes can be obtained by mutating bits from the end of the gene. Both changes are equally likely! In this way, GA can easily evaluate arguments which are both far away from and close to what the chromosomes are currently encoding. As a result, GA efficiently converges to the maximum, but are also likely not to get stuck on a local maximum. This is clearly independent of the initial conditions, which gives GA additional advantage over hill-climbing algorithms (like BFGS), where a good choice of the initial argument can be crucial to obtain the global maximum.

**Crossover**

Let $0 \leq C_L, C_H \leq \sum_{n=1}^{N} L_n = L$ be two predefined integers. The crossover operator divides the population of chromosomes into pairs. If $C_L < L - C_H$, it exchanges the first $C_L$ and the last $C_H$ bits between chromosomes in each pair with a predefined probability $\delta_c$. Otherwise, the crossover operator exchanges $\max\{C_L, C_H\}$ bits in each pair of chromosomes with this predefined probability $\delta_c$. This operator facilitates experimentation in a different way than the mutation operator. Typically, it is set to exchange whole arguments, that is there are $0 \leq \nu_L \leq \nu_H \leq N$ such that $C_L = \sum_{n=1}^{\nu_L} L_n$ and $C_H = \sum_{n=\nu_H}^{N} L_n$. This allows the chromosomes to experiment with different compositions of the individual arguments, which on their own are already successful.

**Election**

The experimentation done by the mutation and crossover operators does not need to lead to efficient binary sequences. For instance, a chromosome which actually decodes the optimal argument should not mutate at all. To counter this effect, it is customary to divide the creation of a new generation into two stages. First, the chromosomes procreate and undergo mutation and crossover in some predefined order. Next, the resulting set of chromosomes is compared in terms of fitness with the parent population. Thus, offspring will be passed to the new generation only if it strictly outperforms the parent chromosome. In this way each generation will be at least as good as the previous one, what in many cases facilitates convergence.
Appendix 2.B  Initialization of the model

In this appendix we discuss the initialization of the GA model for the 50-period Monte Carlo simulations, which we use to show that our model replicates experimental stylized facts. Initialization is crucial, since in the experiments the initial individual predictions influenced later outcomes, such as appearance and characteristics of oscillations, or dynamics of coordination. Two examples can be given for HHST09. Under negative feedback, the individual price forecasts coordinated only after the price itself has already converged; in our simulations we want to start with a similar degree of non-coordination between the agents, to show that disappears in the same way as happened in the experiment. Anufriev et al. (2013) suggest that under positive feedback, price oscillations require the subjects to start relatively far from the fundamental price, as was also the case for their HSM. Therefore, proper distribution of initial predictions of the experimental subjects is a crucial aspect of model calibration; without a realistic initialization, the model will not fit the data well.

Diks and Makarewicz (2013) investigate this issue in a systematic fashion for the case of the HHST09 experiment. They argue that the initial subject predictions can be regarded as a sample from a common distribution, which they next estimate. We use their methodology to calibrate the initial period of our model to all the other experiments, that we investigate for our GA model. In each MC simulation, we sample the initial predictions from the distribution calibrated to the respective experimental data.

**HHST09**

For this experiment we use the estimated Winged Focal Point (WFP) reported by Diks and Makarewicz (2013), which is given by

\[
p_{i,1}^\epsilon = \begin{cases} 
\epsilon^1_i \sim U(9.546, 50) & \text{with probability } 0.45739, \\
50 & \text{with probability } 0.30379, \\
\epsilon^2_i \sim U(50, 62.793) & \text{with probability } 0.23882.
\end{cases}
\]

With WFP we replicate the observed behavior of the subjects in the first period. Around 1/3 would predict 50, a mid-point of the suggested interval for the initial price forecast [0, 100]. Others were evenly spread around this focal point, with more people choosing < 50 and almost nobody choosing > 60. Hence the distribution is a composite of a unit mass at 50 and two ‘wings’, uniform distributions preading from the focal point. See Figure 2.11 for a visualization of the density function for this distribution.

**BHST12**
2.B. Initialization of the model

Figure 2.11: Density function of winged focal point distribution for HHST09. Initial prediction will be equal to $p_{i,1} = 50$ with probability $P = 0.30379$ (mass point); with probability $P = 0.45739$ it will fall into the left wing, where its value is drawn from $Uniform(9.546, 50)$; with probability $P = 0.23882$ it will fall into the right wing, where its value is drawn from $Uniform(50, 62.793)$. The size of the wings is scaled to their masses and lengths.

We reestimate WFP model for the data reported by BHST12 using the same methodology as reported by Diks and Makarewicz (2013). This leads to WFP specified as

$$
{p^e_{i,1}} = \begin{cases}
\varepsilon^1_i \sim U(16.406, 50) & \text{with probability 0.32296}, \\
50 & \text{with probability 0.35159}, \\
\varepsilon^2_i \sim U(50, 70.312) & \text{with probability 0.32296}.
\end{cases}
$$

HSTV07; V01

In the case of the cobweb economy experiment, the subjects were asked to predict prices in the $[0, 10]$ interval. Interestingly, the initial predictions still have the WFP form, with a large proportion equal to the midpoint 5 and the rest (not necessarily rounded to a full integer) distributed around this new focal point. To account for that, we reestimate the WFP and obtain

$$
{p^e_{i,1}} = \begin{cases}
\varepsilon^1_i \sim U(1.875, 5) & \text{with probability 0.17983}, \\
5 & \text{with probability 0.36344}, \\
\varepsilon^2_i \sim U(5, 7.5) & \text{with probability 0.45673}.
\end{cases}
$$

HSTV05

In this experiment, the predictions are two-period ahead, hence the subjects would have to give two initial predictions, $p^e_{i,1}$ and $p^e_{i,2}$. First period forecasts are similar to those from the other experiments. As for the second period, one can notice that 2/3 of the subjects, who would predict $p^e_{i,1} = 50$ the focal point in the first period, would do the same in the second period; otherwise they would again draw predictions resembling WFP, but with a substantially small weight on the focal point 50. Hence we follow
Diks and Makarewicz (2013) and get the following estimations for the first period:

\[
\begin{align*}
\hat{p}_{1,1}^e &= \begin{cases} 
\varepsilon_1^1 \sim U(4.712, 50) & \text{with probability 0.31306,} \\
5 & \text{with probability 0.45536,} \\
\varepsilon_1^2 \sim U(50, 64.062) & \text{with probability 0.23158.}
\end{cases}
\end{align*}
\]

Define the auxiliary draw

\[
\begin{align*}
\hat{p}_{1,1}^{\text{aux}} &= \begin{cases} 
\varepsilon_1^1 \sim U(3.125, 50) & \text{with probability 0.44958,} \\
5 & \text{with probability 0.018761,} \\
\varepsilon_1^2 \sim U(50, 67.227) & \text{with probability 0.53166.}
\end{cases}
\end{align*}
\]

Thus, the second period predictions are given by

\[
\begin{align*}
\hat{p}_{1,2}^e &= \begin{cases} 
\hat{p}_{1,2}^{\text{aux}} & \text{always if } \hat{p}_{1,1}^e \neq 50, \\
\hat{p}_{1,2}^{\text{aux}} & \text{with probability } 1/3 \text{ if } \hat{p}_{1,1}^e = 50, \\
50 & \text{with probability } 2/3 \text{ if } \hat{p}_{1,1}^e = 50.
\end{cases}
\end{align*}
\]

**Appendix 2.C Parametrization of the forecasting heuristic**

In this appendix, we will address two issues. First, following HHST09 we will look on the importance of the anchor, for the said experiment and the two-period ahead Hommes et al. (2005) setting. Second, we study the proper degree of allowed trend extrapolation, based on the linear feedback from HHST09.

### 2.C.1 Is the anchor important for HHST09?

HHST09 show that most of their subjects (around 60%) use First-Order prediction rule with heterogeneous parameter specification:

\[
\hat{p}_{1,t}^e = \alpha_1 p_{t-1} + \alpha_2 \hat{p}_{1,t-1}^e + \alpha_3 60 + \beta (p_{t-1} - p_{t-2})
\]
where the fundamental price 60 serves as an anchor\(^{18}\) the three \(\alpha_i\) span a simplex and \(\beta\) is the trend extrapolation coefficient. Our rule (2.6) is a special case of (2.26) with the restriction that \(\alpha_3 = 0\), which implies that fixed anchor is not used by the agents.

\[ \text{Negative feedback} \quad \text{Positive feedback} \]

(a) Distance from the fundamental (b) Distance from the fundamental
(c) Predictions standard deviation (d) Predictions standard deviation

**Figure 2.12:** HHST09: 50-period ahead Monte Carlo simulation (1000 markets) for the GA model with anchored-FOR and \(\beta \in [-1.1, 1.1]\). Realized price and coordination over time. Green dashed line and black pluses represent the experimental median and group observations; red line is the median and blue dotted lines are the 95% confidence interval for the GA model. Left panel displays the negative feedback, right the positive feedback.

Experimental literature suggests that in general anchors and focal points are important in explaining human behavior. However, HHST09 report that the anchor weight \(\alpha_3\) is typically significant for the subjects under negative feedback treatment, while most of the subjects under positive feedback treatment would not use it. Furthermore, under negative feedback prices and predictions converge to the vicinity of 60, which in practice makes the coefficients \(\alpha\) sample-unidentifiable; and could also make redundant the anchor itself. When designing our GA model, we therefore investigated whether the anchor has any additional explanatory power.

To simplify econometric issues, the authors specify the anchor as the fundamental price 60, which however was not directly observed by the subjects. It is more plausible that they used the average price so far as an anchor, \(p_t^a = p_t^f \equiv \sum_{s=1}^t p_s\). We will use

\[ 18 \text{Notice that what is the anchor, can be a matter of interpretation. One may think of the (2.6) rule as an anchor-based rule as well, since it can be rewritten as a rule that adjusts the previous price forecast with the latest observed price and trend.} \]
thus anchored-FOR specified as

\[
p_{i,t}^f = \alpha_1 p_{t-1} + \alpha_2 p_{i,t-1} + \alpha_3 \left( \sum_{s=1}^{t} p_s \right) + \beta (p_{t-1} - p_{t-2}).
\]

We consider the Monte Carlo (MC) simulations exactly as in the first part of Section 3.3, but for the GA model based on (2.27) with \( \beta \in [-1.1, 1.1] \). The results are presented on Figure 2.12. We observe for the positive feedback that, in contrast to our restricted model without an anchor, the GA model based on FOR as in (2.27) does not predict oscillations at all, but rather a sluggish convergence towards the fundamental. This is seen in the stable median price, bounded by relatively narrow 95% CI. This specification misses most of the dynamics observed in half of the experiment. We conclude that there is no evidence for a need of an anchor, specified as a long-run average of the observed prices, in our GA model.

![Figure 2.13: HSTV05 with \( p^f = 60 \): 50-period ahead Monte Carlo simulation (1000 markets) for the GA model with anchored-FOR and \( \beta \in [-1.1, 1.1] \) and \( \beta \in [-1.3, 1.3] \). Realized price over time: red line is the median and blue dotted lines are the 95% confidence interval for the GA model.](image)

**Figure 2.13:** HSTV05 with \( p^f = 60 \): 50-period ahead Monte Carlo simulation (1000 markets) for the GA model with anchored-FOR and \( \beta \in [-1.1, 1.1] \) and \( \beta \in [-1.3, 1.3] \). Realized price over time: red line is the median and blue dotted lines are the 95% confidence interval for the GA model.

### 2.C.2 Anchor and HSTV05

The HSTV05 non-linear, two-period ahead LtF asset pricing market resulted in much more unruly oscillations than those observed in the simple linear experiment HHST09 under positive feedback. One could therefore think that some kind of a long-run anchor might have been important for the subjects, even though they would not use it in one-period ahead forecasting setting. Furthermore, in the experiment the oscillations typically unraveled around the fundamental price, which again suggests that the subjects tried to extrapolate the trend around it. To address this issue, we run the 50-period ahead MC simulation like in Section 4.3, but with FOR (2.14) replaced by the anchored-FOR rule (2.27) adapted for the two-period ahead setting, assuming that
the fundamental price is \( p^f = 60 \).

Results for two specifications (with allowed trend extrapolation \( \beta \in [-1.1, 1.1] \) and \( \beta \in [-1.3, 1.3] \)) are presented on Figure 2.13. Just as in the case of HHST09, we find that the GA model with anchored-FOR rule generates sluggish convergence towards the fundamental price from below. Indeed, in contrast to HHST09, the 95% CI of the GA model’s prices do not include the fundamental \( p^f = 60 \) even after 50 periods. This indicated that adding an anchor to the GA model would decrease its fitness to the experimental data.

2.C.3 Degree of trend extrapolation

Recall that the GA requires a predefined finite interval for the optimized parameters. In the case of our GA model based on (2.6), the price weight is confound to \( \alpha \in [0, 1] \), but \textit{prima facie} there is no ‘natural’ bound for the trend extrapolation \( \beta \in [\beta_L, \beta_H] \), since \textit{a priori} we do not know the degree of trend extrapolation that people consider while forecasting prices. As mentioned in Section 3, we argue that the model performs well if we specify the (2.6) rule to use 1.1 as the upper bound to the trend.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure2_14}
\caption{HHST09: 50-period ahead Monte Carlo simulation (1000 markets) for the GA model with \( \beta \in [-1.5, 1.5] \). Realized price and coordination over time. Green dashed line and black pluses represent the experimental median and group observations; red line is the median and blue dotted lines are the 95\% confidence interval for the GA model. Left panel displays the negative feedback, right the positive feedback.}
\end{figure}
It turns out (not surprisingly) that the allowed trend extrapolation interval has little effect on the behavior of our GA model under the negative feedback. However, the larger the interval $\beta \in [\beta_L, \beta_H]$ is, the bigger the amplitude of the price fluctuations generated under the positive feedback. Thus we experimented with different $\beta$'s, trying to calibrate the model to the the experimental oscillations. We used the same Monte Carlo experiments as in the first part of Section 3.3.

Allowing for a high trend extrapolation $\beta \in [-1.5, 1.5]$ results in a model with huge possible oscillations and little predictive power, see Figure 2.14. On the other hand, specification with $\beta \in [-0.5, 0.5]$ has narrow CI, but predicts small oscillations, see Figure 2.15. We found the model with $\beta \in [-1.1, 1.1]$ to be the best trade-off between fit and explanatory power of the experiment.

![Negative feedback](image1)

![Positive feedback](image2)

(a) Distance from the fundamental  
(b) Distance from the fundamental  
(c) Predictions standard deviation  
(d) Predictions standard deviation

**Figure 2.15:** HHST09. 50-period ahead Monte Carlo simulation (1000 markets) for the GA model with $\beta \in [-0.5, 0.5]$. Realized price and coordination over time. Green dashed line and black pluses represent the experimental median and group observations; red line is the median and blue dotted lines are the 95% confidence interval for the GA model. Left panel displays the negative feedback, right the positive feedback.

This result reflects the experimental findings. HHST09 find that under positive feedback, four out of twenty estimated rules had $\beta > 0.9$ and further five rules had $\beta > 0.75$. Nevertheless, HHST09 in their estimations impose a restriction that $\beta \in [-1, 1]$. Our GA model suggests that such a restriction is inconsistent with the degree of experimental price oscillations.
Appendix 2.D  Definition of forecasting rules

Table 2.11 gives the exact specification for all the prediction rules used in the one-period ahead forecasting exercises for the four experiments. The full HSM specification can be found in Anufriev et al. (2013).

<table>
<thead>
<tr>
<th>Rule</th>
<th>Prediction</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Homogeneous rules</strong></td>
<td></td>
</tr>
<tr>
<td>Trend extr.</td>
<td>$p_{t}^e = p_{t-1} + (p_{t-1} - p_{t-2})$</td>
</tr>
<tr>
<td>Adaptive</td>
<td>$p_{t}^e = 0.75p_{t-1} + 0.25p_{t-1}^e$</td>
</tr>
<tr>
<td>Contrarian</td>
<td>$p_{t}^e = p_{t-1} - 0.5(p_{t-1} - p_{t-2})$</td>
</tr>
<tr>
<td>Naive</td>
<td>$p_{t}^e = p_{t-1}$</td>
</tr>
<tr>
<td>RE</td>
<td>$p_{t}^e = p^f$</td>
</tr>
</tbody>
</table>

| **Heterogeneous rules**                                   |
| HSM          | two heuristic model (trend extrapolation vs. adaptive expectations) |
| GA model     | $p_{i,t}^e = \alpha_{i,t}p_{t-1} + (1 - \alpha_{i,t})p_{i,t-1}^e + \beta_{i,t}(p_{t-1} - p_{t-2})$ |

$\beta \in [-1.1, 1.1], \alpha_{i,t} \in [0, 1] \text{ and } \beta_{i,t} \in [-1.1, 1.1]$  
$\beta \in [0, 1.1], \alpha_{i,t} \in [0, 1] \text{ and } \beta_{i,t} \in [0, 1.1]$  

| Table 2.11: Specification of the forecasting rules $p_{t}^e$ for one-period ahead forecasting environment. |

Appendix 2.E  APF for the GA model

In this appendix we discuss how Auxiliary Particle Filter can be used to estimate the mean squared error of one-period ahead predictions of our GA model. This is complicated task, since the model operates on unobservable evolution of heuristics. Thus what it predicts for period $t$ is in fact a distribution, which is a function of all periods until and including $t - 1$. The following discussion explains formally how it can be represented as a Sequential Importance Sampling problem. Next we show how exactly one can estimate the period-to-period distribution of the heuristic evolution through a simple Monte Carlo integral. The final part of this appendix presents the estimated MSE for the four discussed experiments, along with exemplary price and price forecasts from the experiments and their one-period ahead predictions of our GA model.
2.E.1 General specification

We introduce the following notation. Let \( a_t \) denote the state of the model at the beginning of period \( t \). Specifically, we mean the set of the six sets of chromosomes \( H_{t-1}, \) which correspond to the six lists of twenty heuristics \( H_{i,t-1} \) of each agent. Be the beginning of the period \( t \) we mean that the price \( p_{t-1} \) is already observed, but the heuristics are not yet updated (hence the subscript of \( H_{t-1} \)). Consider an experimental group \( X \), for which we can observe its underlying chromosomes only indirectly, through the realized prices and predictions picked by the subjects (observational variables).

Both the state and observed variables are evolving according to a distribution \( q(\cdot) \) over time \( t \in \{1, \ldots, 50\} \). Denote also \( p_{t,GRX}^e = \{p_{1,GRX}^e, \ldots, p_{6,GRX}^e\} \) as the set of six individual predictions from period \( t \) in the experimental group \( X \). Henceforth \( t \) in the superscript denotes history of the variable, so \( p_{t-1,GRX}^e = \{p_{1,GRX}^e, \ldots, p_{t-1,GRX}^e\} \) and \( p_{t-1,GRX,t-1}^e = \{p_{1,GRX}^e, \ldots, p_{t-1,GRX}^e\} \).

Our problem is to define the baseline distribution \( q(p_{t,GRX}^e, a_{t-1}) \), that is, to evaluate the distribution of the real predictions \( p_{t,GRX}^e \) given the predictions from the period \( t-1 \) and what they signal could have been the chromosomes \( H_{t-1} \). This is a typical state-space model problem. Essentially, \( q(\cdot) \) can be decomposed as

\[
q(p_{t,GRX}^e, a_t) = q(p_{t,GRX}^e | a_t) \times q(a_t)
\]

\[
= q(a_t | p_{t,GRX}^e) \times q(p_{t,GRX}^e),
\]

\[
q(p_{t,GRX}^e | a_{t-1}) = q(p_{t,GRX}^e | a_t) \times q(a_t | a_{t-1}),
\]

\[
q(a_t | p_{t-1,GRX}^e) = q(a_t | a_{t-1}) \times q(a_{t-1} | p_{t-1,GRX}^e),
\]

\[
q(a_t | a^{t-1}) = q(a_t | a_{t-1}),
\]

(2.28)

\[
q(p_{t,GRX}^e | p_{t-1,GRX,t-1}^e) = q(p_{t,GRX}^e | p_{t-1}^e).
\]

The first three equalities are a simple consequence of the structure of our model, whereas the two last ones are implied by it being a Markovian process.

Given the information structure of the experiment we can assume that in each period, conditional on the history until \( t-1 \) and the chromosome set \( H_t \), the individual predictions are independent between the agents, and their joint density is a simple
product of the marginal densities of individual forecasts:

\[
    p^e_{t,GRX} \sim q \left( p^e_{t,GRX} | a_t, p^{GRX,t-1}, p^{e,GRX,t-1} \right) \\
    = \Pi_{i=1}^6 q \left( p^e_{t,GRX} | a_t, p^{GRX,t-1}, p^{e,GRX,t-1} \right). 
\]  

(2.29)

Unfortunately, \( q(a_t | a_{t-1}) \) is not that simple to work with. As explained earlier, it is not feasible to represent this problem analytically or to linearize it. Nevertheless, it is fairly simple to simulate \( a_t \) conditional on \( a_{t-1} \). Therefore, we focus on SISR technique known as Auxiliary Particle Filter (APF) (Johansen and Doucet, 2008).\(^{19}\) APF works on two distributions (baseline \( q(\cdot) \) and importance \( g(\cdot) \)), which are used to generate MC weights and updates for the particles, and these we will approximate with simple MC integrals.

The general APF algorithm is discussed by Johansen and Doucet (2008) and below we provide its interpretation to our problem. The starting point of the APF is the question: what is \( q(a_t | p^e_{GRX,t}) \) the distribution of possible heuristics throughout the experiment, conditional on the observed individual predictions from group \( X \), until some period \( t = 1, \ldots, T \). We need this to estimate \( MSE_{X,t} \), MSE of the model predictions specifically for period \( t \), which for brevity we will denote in this appendix simply as \( MSE^q_{X,t} \). To be specific,

\[
    MSE^q_{X,t} = \mathbb{E}_q \left\{ MSE^e_X (a^t) | p^e_{GRX,t} \right\} \\
    = \int MSE^e_X (a^t) q (a^t | p^e_{GRX,t}) \, da^t, 
\]

(2.30)

the expected MSE of the GA model conditional on the observed price forecasts in the group \( X \). The subscript \( q \) in \( \mathbb{E}_q \) signals that the expectation is taken in respect to the \( q(\cdot) \) density. It turns out that in practice this estimator has better properties if we use importance sampling approach (the specific reason is that we obtain a better approximation of the tails of the distribution, see Johansen and Doucet, 2008). Denote the importance distribution as \( g(\cdot) \) and assume that \( g(a_t | a_{t-1}) = q(a_t | a_{t-1}) \) (which is a standard assumption for APF). It follows that \( g(p^e_{GRX,t} | a_{t-1}) \) can be decomposed in the same manner as the baseline distribution in equation (2.28). Hence we focus

\(^{19}\)It is extremely difficult, also in conceptual terms, to define a reverse distribution of the model at period \( t \) conditional on period \( t + 1 \), given the complexity of GA operators. As a result, we leave open the question whether econometrically more efficient filtering-smoothing techniques can be used for the case of our model.
instead

\[
MSE^e_{X,t} = \int MSE^e_X \left( a^t \right) \frac{q \left( a^t \mid p^{e,GRX,t} \right)}{g \left( a^t \mid p^{e,GRX,t} \right)} g \left( a^t \mid p^{e,GRX,t} \right) da^T
\]

\[
= \mathbb{E}_g \left\{ MSE^e_X \left( a^t \right) \frac{q \left( a^t \mid p^{e,GRX,t} \right)}{g \left( a^t \mid p^{e,GRX,t} \right)} \right\}
\]

\[
\equiv \mathbb{E}_g \left\{ MSE^e_X \left( a^t \right) w_t \right\},
\]

where \( g(\cdot) \) is the importance density and \( w_t \) denote the importance weights.

This expected value can be approximated with a MC integral, based on samples from \( g(\cdot) \). It still not easy to work with. With (2.31) we estimate the MSE of the model’s one period-ahead predictions for a specific period \( t \). Since we do it conditional on the whole history until this period, we would need to sample all \( t \) periods conditional on \( t \) periods of the data. This means that MSE is a function of the contemporary heuristics, but the MC weights are computed as probabilities for the whole history of the heuristics’ evolution!

The next step is to represent this integral in a recursive way. To be specific, we can transform density \( q(\cdot) \) from period \( t \) as

\[
q \left( a^t \mid p^{e,GRX,t} \right) = \frac{q \left( a^t \mid p^{e,GRX,t} \right)}{q \left( p^{e,GRX,t} \right)}
\]

\[
= \frac{q \left( a^t \mid p^{e,GRX,t} \right) q \left( a^{t-1} \mid p^{e,GRX,t} \right)}{q \left( p^{e,GRX,t} \right)}
\]

\[
= q \left( a^t \mid p^{e,GRX,t-1} \right) q \left( a^{t-1} \mid p^{e,GRX,t-1} \right),
\]

(2.32)

where the last equality holds because the formation of heuristics at period \( t \) is independent from further forecasting at that period, that is \( q \left( a^{t-1} \mid p^{e,GRX,t} \right) = q \left( a^{t-1} \mid p^{e,GRX,t-1} \right) \) (the relation goes the other way around). In the same way we can decompose

\[
g \left( a^t \mid p^{e,GRX,t} \right) = g \left( a^t \mid p^{e,GRX,t-1} \right) g \left( a^{t-1} \mid p^{e,GRX,t-1} \right).
\]

(2.33)

It follows that

\[
w_t = \frac{q \left( a^t \mid p^{e,GRX,t} \right)}{g \left( a^t \mid p^{e,GRX,t} \right)}
\]

\[
= \frac{q \left( a^t \mid p^{e,GRX,t-1} \right) q \left( a^{t-1} \mid p^{e,GRX,t-1} \right)}{g \left( a^t \mid p^{e,GRX,t-1} \right) g \left( a^{t-1} \mid p^{e,GRX,t-1} \right)}
\]

\[
= \frac{q \left( a^t \mid p^{e,GRX,t-1} \right)}{g \left( a^t \mid p^{e,GRX,t-1} \right)} w_{t-1}.
\]

(2.34)
Next denote $\tilde{w}_t \equiv q \left(p^{e,GRX,t} \right) w_t$. The Markovian nature of both $q(\cdot)$ and $g(\cdot)$, together with the decomposition (2.28) and (2.34) gives us

$$
\tilde{w}_t = \frac{q(\alpha^t, p^{e,GRX,t})}{g(\alpha^t | p^{e,GRX,t})}
= \tilde{w}_{t-1} \frac{q(\alpha_t | \alpha_{t-1}) q \left( p_t^{e,GRX} | \alpha_t \right)}{g(\alpha_t | \alpha_{t-1}) g \left( p_t^{e,GRX} | \alpha_{t-1} \right)}
= \tilde{w}_{t-1} \frac{q \left( p_t^{e,GRX} | \alpha_t \right)}{g \left( p_t^{e,GRX} | \alpha_{t-1} \right)}
$$

(2.35)

where the last equality follows from the APf assumption that $q(\alpha_t | \alpha_{t-1}) = g(\alpha_t | \alpha_{t-1})$.

These lengthy derivations can now be put to a good use. If the MC integral for period $t-1$ is based on a good sample of potential market outcomes, we can simulate them for one extra period and hence use (2.35) recursion to reevaluate the fit of each such market for period $t$. This is the basic idea of APF: every counter-factual market is a particle that follows the data. If our model is well specified, the particles should have little problems doing so. This is measured through two things. MSE is the direct signal of the particle about the reliance of the model. Second, APF weights $\tilde{w}_t$ show a relative reliance of the particle itself. The former depends only on the current period. Strictly speaking, the latter depends on the whole history, but the recursion (2.35) allows to express it as a straightforward recursive process.

In order to keep the pool of particles representative, it is customary in the literature to resample them. This is done based on the APF weights, since we want to have more of the ‘reliable’ particles. Thus, the whole algorithm becomes an iterative procedure over $b \in \{1, \ldots, B\}$ particles:

1. Compute resample weights $g \left( p_t^{e,GRX} | a_t^b \right) w_{t-1}^b$, normalize them, and use them as probabilities to resample (with replacement) $B$ particles $a_{t-1}^{bs}$ from the existing $a_{t-1}^b$.

2. Draw $a_t^b$ from $a_{t-1}^{bs}$, compute $\tilde{w}_t^b$ from (2.35), normalize them and use them to compute MSE for period $t$.

Specifically for our model, we follow Anufriev et al. (2013) and we assume that the baseline distribution of the six realized predictions is given by standard normal
distribution. To be specific, we represent it as

\[
(2.36) \quad p^e_{GRX} \sim q(p^e_{GRX} | a_t, p^{GRX}_{t-1}, p^{e,GRX}_{t-1}) = \prod_{i=1}^{6} N(\hat{p}^e_{i,t}, 1).
\]

We simplify the notation by suppressing \( p^e_{GRX} \) and \( q(p^e_{GRX} | a_t) \). For a general case, (2.36) density of the experimental predictions \( p^e_{GRX} \) is just a product of normal standard densities centered around the forecasts predicted by a model.

For \( g(p^e_{GRX} | a_t) \) we use (a product of six) Student-t with one degree of freedom. This density is again analytically straightforward and compares \( \hat{p}^e_{i,t} \) with \( \hat{p}^e_{i,t} \) price forecasts predicted by the GA model as in equation (2.11). We define \( \hat{p}^e_{i,t} \) individual price forecasts and predicted realized price \( \hat{p}^e_{i,t} \) as in the previous subsection.

We use 128 particles \( a_t <b> \) for \( b \in \{1, \ldots, 128\} \) (128 sets of six heuristic sets for six agents), with full resampling. The core problem of our investigation lies with the \( q(a_t | a_{t-1}) \) and \( g(a_t | a_{t-1}) \) distributions, which cannot be tracked analytically. We approximate them with a MC integral in the following fashion. Consider first the baseline distribution. At the beginning of each iteration \( t > 1 \) of AFP, for each particle \( a_t <b> \) (that is, for each set of six agents and their chromosomes), we simulate 256 counter-factual iterations, with each market iteration \( s \in \{1, \ldots, 256\} \) of the following structure (see section 3 for details of the model):

1. Heuristics evolve with the GA operators based on the experimental group data, which is observable until period \( t - 1 \).
2. Agents report their forecasts \( p^e_{i,<b,s>} \), which hence also generates the counter-factual price \( p^e_{i,<b,s>} \).

The counter-factual price forecasts \( \hat{p}^e_{i,<b,s>} \) (again, we look at the expected price forecasts, see formula (2.11)) and prices are evaluated against the actual experimental prices and forecasts, which gives the baseline density

\[
(2.37) \quad q(p^e_{GRX} | a_{t-1}^<b>) = \frac{1}{256} \sum_{s=1}^{256} \prod_{i=1}^{6} N(\hat{p}^e_{i,t} - \hat{p}^e_{i,t}, 1).
\]

The importance distribution is generated in a symmetric way. For each period \( t \), we simulate 256 counter-factual two-period market iterations for each particle \( b \), that is for each counter-factual market \( s \in \{1, \ldots, 256\} \) we repeat the above iteration twice. As

\footnote{For APF, the choice of variance of the distributions is not important. We use standard normal for the sake of computational efficiency.}

\footnote{Larger particle samples are computationally involved and do not seem to increase the efficiency of the estimates.}
one can expect, for period $t$ the importance density is based on the data until period $t - 2$, and the simulated counter-factual prices and forecasts from period $t - 1$ are used by the GA agents to act in period $t$. We use these to compute price $\hat{p}_{t, \text{importance}}^{<b,s>}$ and $\hat{p}_{t,i, \text{importance}}^{<b,s>}$ individual price forecasts predicted by the model. We therefore define
\begin{equation}
(2.38) \quad g(p_{t,GRX}^e|a_{t-1}^b) = \frac{1}{256} \sum_{s=1}^{256} \prod_{i=1}^{6} T_1 \left( \hat{p}_{t,i, \text{importance}}^{<b,s>} - p_{t,i,GRX}^e \right),
\end{equation}
where $T_1$ denotes Student-t distribution with one degree of freedom.\(^{22}\)

We use the baseline (2.36) and the importance (2.38) distributions for the APF updating as described above.\(^{23}\) We run a separate APF for each of the twelve investigated experimental groups. Like in the 50-period ahead simulations, the chromosomes (or the particles) are initialized at random from the ‘uniform’ distribution defined above. Notice, however, that in this case we do not have the problem of the initial predictions or prices, since the APF works on the period-to-period basis, independently for each experimental group.

For deterministic models like RE or HSM or homogeneous heuristic models, the APF effectively reduces to the procedure reported by Anufriev et al. (2013), since all the particles would be the same and had the same weights. Moreover, APF procedure reduces to the simple SMC described in section 3, if for the importance distribution we use standard normal instead of Student-t.

For each experimental group, we focus on seven variables in total, which we obtain by using the APF weighting of the particles. For each of the last 47 periods in each group, we look at the (mean) one-period ahead prediction of the price, as well as at the (mean) one-period ahead predictions of individual price forecasts. Notice that we compute the expected APF MSE’s, instead of MSE’s for the expected prices or predictions. We compute these variables for each group, and average them separately over the two treatments to obtain the average MSE of the model’s prediction of the prices and individual forecasts.

The drawback of this method is the computational time. For a dual-core Pentium

\(^{22}\)For the sake of computational efficiency, we perform the resampling $a_t \sim a_{t-1}$ of particle $b$ in the following way. We pick at random one counter-factual $s$ market from the importance MC density for period $t + 1$ and use the chromosomes as they are at the end of the first iteration for this counter-factual market — that is, one of the 256 chromosome sets updated based on the data until $t - 1$ in the first step of the importance distribution estimation.

\(^{23}\)Both the baseline and importance densities are a product of six independent densities, which can take very low values in the first periods for some experimental groups. To ensure numerical stability, we multiply both joint densities by $10^{60}$ (or each of the six marginal distributions by $10^{10}$) and truncate them at $10^{-100}$. 69
with 2.7GHz clock and 3.21 GB RAM, a shot of APF for one experimental group from HHST09 takes approximately 50 minutes for a relatively small number of 32 particles.

2. E. 2 Results for the four experiments

Results for the four experiments are presented in this subsection: Table 2.12 and Figure 2.16 for HHST09; Table 2.12 and Figure 2.17 for BHST12; Table 2.14 and Figure 2.18 for HSTV07; and Table 2.15 and Figure 2.19 for HSTV05.

<table>
<thead>
<tr>
<th>MSE</th>
<th>Negative feedback</th>
<th>Positive feedback</th>
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<tbody>
<tr>
<td></td>
<td>Prices</td>
<td>Forecasts</td>
</tr>
<tr>
<td>Trend extr.</td>
<td>21.101</td>
<td>35.648</td>
</tr>
<tr>
<td>Adaptive</td>
<td>2.3</td>
<td>14.912</td>
</tr>
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<td>Contrarian</td>
<td><strong>2.249</strong></td>
<td><strong>14.856</strong></td>
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<tr>
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</tr>
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<td>HSM</td>
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<td>17.106</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Genetic Algorithm model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline Sequential Monte Carlo</td>
</tr>
<tr>
<td>( \beta \in [-1.1, 1.1] )</td>
</tr>
<tr>
<td>( \beta \in [0, 1.1] )</td>
</tr>
<tr>
<td>Auxiliary Particle Filter</td>
</tr>
<tr>
<td>( \beta \in [-1.1, 1.1] )</td>
</tr>
<tr>
<td>( \beta \in [0, 1.1] )</td>
</tr>
</tbody>
</table>

Table 2.12: HHST09. one-period ahead predictions. MSE of the experimental prices and forecasts, for the Trend Extrapolation, Adaptive, Contrarian, Naive and Rational Expectations, Heuristic Switching Model and Genetic Algorithms models (with \( \beta \in [-1.1,1.1] \) and \( \beta \in [0,1.1] \)). MSE averaged over 6 negative feedback and 6 positive feedback groups.
Figure 2.16: HHST09: APF one-period ahead predictions of the GA model with $\beta \in [-1.1, 1.1]$ for prices and price forecasts of subject 1 from sample groups from each treatment. Black line denotes the experimental variable and red boxes display the model predictions.
Table 2.13: BHST12: one-period ahead predictions. MSE of the experimental prices and forecasts, for the Trend Extrapolation, Adaptive, Contrarian, Naive and Rational Expectations, Heuristic Switching Model and Genetic Algorithms models (with $\beta \in [-1.1, 1.1]$ and $\beta \in [0, 1.1]$). MSE averaged over 8 negative feedback and 8 positive feedback groups.

<table>
<thead>
<tr>
<th>Genetic Algorithm model</th>
<th>Negative feedback</th>
<th>Positve feedback</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td>Forecasts</td>
</tr>
<tr>
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</tr>
<tr>
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<td><strong>10.332</strong></td>
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<tr>
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<td>5.92</td>
<td>12.534</td>
</tr>
<tr>
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<td>16.81</td>
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<td>RE</td>
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</tr>
<tr>
<td>HSM</td>
<td>38.309</td>
<td>45.679</td>
</tr>
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</table>

Figure 2.17: BHST12: APF one-period ahead predictions of the GA model with $\beta \in [-1.1, 1.1]$ for prices and price forecasts of subject 1 from sample groups from each treatment. Black line denotes the experimental variable and red boxes display the model predictions.
### 2.E. APF for the GA model

<table>
<thead>
<tr>
<th>Treatments</th>
<th>Stable Prices</th>
<th>Stable Forecasts</th>
<th>Unstable Prices</th>
<th>Unstable Forecasts</th>
<th>Strongly unstable Prices</th>
<th>Strongly unstable Forecasts</th>
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<tbody>
<tr>
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<td>Contrarian</td>
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<td>0.414</td>
<td>0.497</td>
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<tr>
<td>Naive</td>
<td>0.196</td>
<td>0.448</td>
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<td>0.788</td>
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<td>3.731</td>
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<tr>
<td>RE</td>
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<td><strong>0.364</strong></td>
<td><strong>0.385</strong></td>
<td><strong>2.257</strong></td>
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<td>0.474</td>
<td>0.52</td>
<td>0.732</td>
<td>3.065</td>
<td>3.691</td>
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<table>
<thead>
<tr>
<th><strong>Genetic Algorithm model</strong></th>
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<tbody>
<tr>
<td>Baseline Sequential Monte Carlo</td>
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<tr>
<td>$\beta \in [-1.1, 1.1]$</td>
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<td>$\beta \in [0, 1.1]$</td>
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<th>Auxiliary Particle Filter</th>
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<td>AR1</td>
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<tr>
<td>$\beta \in [0, 1.1]$</td>
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</tbody>
</table>

**Table 2.14:** HSTV07: one-period ahead predictions. MSE of the experimental prices and forecasts, for the Trend Extrapolation, Adaptive, Contrarian, Naive and Rational Expectations, Heuristic Switching Model and Genetic Algorithms models (with $\beta \in [-1.1, 1.1]$ and $\beta \in [0, 1.1]$). MSE averaged over 6 groups for each treatment (stable, unstable, strongly unstable).
Figure 2.18: HSTV07. APF one-period ahead predictions for GA model with $\beta \in [-1.1, 1.1]$ and $\beta \in [0, 1.1]$, for prices from the third group of each treatments (stable, unstable, strongly unstable). Black line denotes the experimental variable and red boxes display the model predictions.
2.E. APF for the GA model

MSE Prices Forecasts
Trend extr. 17.4527 55.0898
Adaptive 44.125 25.3157
Contrarian 59.3905 30.8646
Naive 31.6864 20.8416
RE 96.0328 145.998

Genetic Algorithm model

Baseline Sequential Monte Carlo
\( \beta \in [-1.1, 1.1] \) 42.224 74.95
\( \beta \in [0, 1.1] \) 5.934 30.341

Auxiliary Particle Filter
\( \beta \in [-1.1, 1.1] \) 19.794 39.226
\( \beta \in [-0, 1.1] \) 17.899 39.256
\( \beta \in [-1.3, 1.3] \) 20.55 45.274
\( \beta \in [0, 1.3] \) 17.104 39.291

Table 2.15: HSTV05: one-period ahead predictions. MSE of the experimental prices and forecasts, for Trend Extrapolation, Adaptive, Contrarian, Naive and Rational Expectations, Heuristic Switching Model and GA models (with \( \beta \in [-1.1, 1.1] \) and \( \beta \in [0, 1.1] \)). MSE averaged over all experimental groups.

Group 8, GA: \( \beta \in [-1.1, 1.1] \)

(a) Price
(b) Price forecasts of subject 1

Group 8, GA: \( \beta \in [0, 1.1] \)

(c) Price
(d) Price forecasts of subject 1

Figure 2.19: HSTV05: APF one-period ahead predictions of the GA model with \( \beta \in [-1.1, 1.1] \), for prices and price forecasts of subject 1 from group 8. Black line denotes the experimental variable and red boxes display the APF one-period ahead predictions.
Appendix 2.F  Price autocorrelation in the cobweb experiment

Table 2.16 gives the first three autocorrelations of the experimental groups in HSTV07 and the 50-period ahead simulations of the GA and benchmrak models.

<table>
<thead>
<tr>
<th>Treatments</th>
<th>Stable</th>
<th>Unstable</th>
<th>Strongly unstable</th>
</tr>
</thead>
<tbody>
<tr>
<td>Autocorrelation</td>
<td>$\rho_1$</td>
<td>$\rho_2$</td>
<td>$\rho_3$</td>
</tr>
<tr>
<td>Experiment</td>
<td>-0.1878</td>
<td>0.06323</td>
<td>-0.12</td>
</tr>
<tr>
<td>Trend extr.</td>
<td>-0.9661</td>
<td>0.9423</td>
<td>-0.9209</td>
</tr>
<tr>
<td>Adaptive</td>
<td>-0.5996</td>
<td>0.3446</td>
<td>-0.3078</td>
</tr>
<tr>
<td>Contrarian</td>
<td>-0.257</td>
<td>-0.3006</td>
<td>0.1604</td>
</tr>
<tr>
<td>Naive</td>
<td>-0.9043</td>
<td>0.837</td>
<td>-0.7911</td>
</tr>
<tr>
<td>RE</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>HSM</td>
<td>-0.6528</td>
<td>0.4224</td>
<td>-0.3438</td>
</tr>
<tr>
<td><strong>GA: AR1</strong></td>
<td>-0.1161</td>
<td>0.008603</td>
<td>-0.1253</td>
</tr>
<tr>
<td><strong>GA: $\beta \in [-1.1, 1.1]$</strong></td>
<td>-0.1102</td>
<td>-0.3232</td>
<td>0.002674</td>
</tr>
<tr>
<td><strong>GA: $\beta \in [0, 1.1]$</strong></td>
<td>-0.2955</td>
<td>0.1059</td>
<td>-0.171</td>
</tr>
</tbody>
</table>

Table 2.16: HSTV07: 50-period ahead predictions. First three autocorrelations in the experimental groups, and in the 50-period ahead simulations of the Trend Extrapolation, Adaptive, Contrarian, Naive and Rational Expectations, Heuristic Switching Model and GA models (FOR with $\beta \in [-1.1, 1.1]$ and $\beta \in [0, 1.1]$). Autocorrelations averaged over six groups for each treatments (stable, unstable and strongly unstable).
Chapter 3

Networks of heterogeneous expectations in an asset pricing market

3.1 Introduction

In this chapter, we study the effect of networks on learning in the context of a non-linear asset pricing market. We consider a model, in which the agents apply Genetic Algorithm (GA) to optimize a simple price forecasting rule. The agents learn whether to trust the observed price trend and/or the former trading decisions of their friends. The main questions of the chapter are: how do networks affect individual behavior of the agents and the emerging aggregate market dynamics? Does the network facilitate convergence to the fundamental equilibrium? To what extent will the network promote coordination or herding?

One of the fundamental debates of contemporary economics is whether economic agents can learn rational expectations (RE), that is model-consistent predictions of future market prices. Among other evidence, experiments suggest that people use simple forecasting heuristics (Heemeyer et al., 2009; Hommes, 2011; Hommes et al., 2005). In the case of asset pricing economies, this leads to price oscillations that repeatedly over- and undershoot the fundamental (RE) equilibrium. Nevertheless, many economists question the empirical validity of such experiments. One of the criticism is that these experiments are based on economies with no or limited information flows between the agents. An informal (yet popular) belief is that in real financial markets the agents can share knowledge about efficient and inefficient trading strategies, and so an information network facilitates convergence towards RE.
Informal information sharing is indeed an important market phenomenon (Bollen et al., 2011; Nofsinger, 2005). Being closer to the core of an information network leads to higher profits (Cohen et al., 2008), but some researchers have also argued that networks are a cause of herding (Shiller and Pound, 1989). The latter argument became popular after the 2007 crisis in the non-academic discussion and in behavioral economics (for example Akerlof and Shiller, 2009, refer to animal spirits as the driving force of financial bubbles).

Herding has no single definition in the economic literature, but is typically understood as behavior such that individuals, facing uncertainty, prefer to follow the ‘view of the others’ (the mood of their friends or general market opinion) rather than basing their decision only on individual data. In the context of financial trading, this can be a rational strategy for agents who are endowed with noisy information about the asset, which ‘averages out’ for the whole public (Park and Sabourian, 2011). An information network can further facilitate such herding (Acemoglu and Ozdaglar, 2011). However, herding can also lead to coordination on bubble paths if the individual information is correlated (Panchenko et al., 2013). This leads back to the two conflicting views on the rationality of financial agents, which can be associated with Keynes (Akerlof and Shiller, 2009) and Muth (Muth, 1961). How important is therefore herding for market stability, and is herding propagated by information networks? There is no clear answer neither from theoretical nor from empirical work.

From the RE perspective, market price should contain all necessary information about the stocks, hence perfect rationality rarely leaves room for herding or networking. The exception would be the case of significant private information (Ozsoylev and Walden, 2011; Park and Sabourian, 2011) or sequential trading (with the famous example of information cascades, see Anderson and Holt, 1997, for a discussion), but neither approach has a clear empirical motivation.

Alternatively, models that depart from rational expectations often investigate some form of social learning. The seminal paper by Kirman (1993) stands as the benchmark for the studies of economic herding (see Alfarano et al., 2005, for a more recent example and a literature review). In this ‘ant-model’, agents are paired at random and imitate each others choices with some exogenous probability, which leads to interesting herding dynamics. The problem with this approach, however, is that individual imitation is assumed as given, instead of being learned by the agents.

Another approach comes with the classical Brock-Hommes Heuristic Switching Model (HSM; Brock and Hommes, 1997), in which agents coordinate on price prediction heuristics that have a better past forecasting performance. A more general, agent-based counterpart of HSM comes with Genetic Algorithm based models of so-
cial learning (see Arifovic et al., 2013, for example and a good literature overview). This approach can explicitly account for learning (agents switch to better strategies), but does not fit our intuition of herding, which is understood as following others, instead of using similar trading or forecasting strategies (see Panchenko et al., 2013, for interesting discussion; see also Section 2.6).

Empirical studies give ambiguous results on the existence or importance of herding, with the main issue being that such behavior cannot be directly observed in market data. Chiang and Zheng (2010) show that the stock indices between industries are sometimes more correlated than the fundamentals would imply, which can be understood as a sign of herding. However, this effect is absent in Latin America and US data, and its interpretation is subject to debate. An alternative approach is to use experiments, where the information structure is controlled by the researcher, who can thus directly observe herding. This leads to a surprising result, however: experiments suggest that contrarian (understood as anti-herding) behavior is more popular than herding. Two such studies were reported by Drehmann et al. (2005) and Cipriani and Guarino (2009). This is important evidence, since both studies include professional traders in their experiments.

As a result, existence of herding (or contrarian) behavior is not a clear-cut fact. Furthermore, it is not clear whether herding would bring economic agents closer to the rational outcome, or rather to volatile price dynamics (Shiller and Pound, 1989). Therefore, there is a need for theoretical inquiries into how herding or contrarian behavior may be learned in information networks.

Theoretical studies of networks typically focus on static equilibria, or simple behavioral rules of imitation, since adding explicit and realistic learning features into such models easily makes them analytically intractable (see Jadbabaie et al., 2012, for a discussion). An additional conceptual problem is that the RE models focus on an alleged fixed point of the postulated underlying learning process, while being agnostic about its dynamics. Therefore, if taken seriously in the context of networking in asset pricing markets, these models predict that networks, as was the case of herding, are relevant only in the presence of important individual information, like private signals about the fundamental price. This is difficult to defend without implicit assumptions on (bounds of) individual rationality.

To our best knowledge, Panchenko et al. (2013) (henceforth PGP13) is the only paper which conduct a full-fledged theoretical study of the effect of the information network on learning in an asset pricing market. The authors use a HSM model to show that prices are not tamed by the presence of the network. Their interesting paper is however subject to some limitations. For example, the agents can choose between two
forecasting heuristics, and are placed on a random network. As a result, it may be difficult to directly use their results for specific markets or experimental studies.

The goal of our chapter is to investigate a much more involved learning structure. We will use the GA model discussed in Chapter 2, which explains well the individual forecasting heterogeneity of Learning to Forecast experiments. This approach has two advantages: (1) we will work with a realistic, experimentally tested model that explains well financial bubbles and (2) we will obtain further insight into the original experiments: to what extent their results (such as the price bubbles) depend on the lack of networking. This model will also serve as an ideal benchmark for further asset pricing experiments with more involved information networks.

The GA model from Chapter 2 is an agent-based model (ABM) based on the work of Hommes and Lux (2013). Its idea is that agents, who are asked to predict a price, follow a simple linear forecasting rule, which is a mixture of adaptive and trend extrapolation expectations. This rule requires specific parametrization, and each agent is endowed with a list of possible specifications of the general heuristic. The agents then observe the market prices and update the list of rules with the use of the GA stochastic evolutionary operators. For instance, if the market generates persistent price oscillations, the agents will experiment with higher trend extrapolation coefficients. Since the agents use the GA procedure independently, the model allows for explicit individual learning. In Chapter 2 we showed that the model replicates well the experimental degree of individual heterogeneity, as well as aggregate price dynamics.

In this chapter we extend the GA model of Chapter 2 to include an information network. Agents observe the past trading behavior of their friends and can learn whether to trust it, just as they learn whether to extrapolate the price trend. The model by its ABM structure can evaluate the effects of different, also asymmetric, networks on price dynamics. Furthermore, the model explicitly accounts for individual learning, and so we can also study the formation of herding/contrarian behavior at the individual level.

The chapter is organized in the following way. Section 2 will introduce the theoretical agent-based model, describing a two-period ahead non-linear asset pricing market with GA agents and robotic trader, with GA agents forecasting prices conditional on past realized prices and trades of their friends. The third section will present the parametrization of the model, including the investigated network structures, and the setup of the Monte Carlo numerical study of the model. Section 4 will be devoted to small networks of six agents, with which we will highlight the emerging properties of individual learning and resulting price dynamics. The fifth section will move to large networks of up to 1’000 agents. Finally, the last section will sum up our results and
3.2 Theoretical model

In this section we present the building blocks of the model: an asset pricing market with heterogeneous expectations and an information network. The model is based on the standard two-period ahead asset pricing market, used for example by Hommes et al. (2005). For the sake of presentation, most of the analytical results concerning the rational solution of the model are given in Appendix 3.A.

3.2.1 Market

Consider $I$ myopic mean-variance agents who invest on a period to period basis. They can choose between a safe bond with a gross return $R = 1 + r$ (with $r > 0$), or a risky asset that in period $t$ can be bought at price $p_t$ and gives a stochastic dividend $y_t$. It is commonly known that $y_t \sim NID(y, \sigma_y^2)$. Then the expected return on a unit of the asset bought at $p_t$ is

$$
E_{i,t} \{ \rho_{t+1} \} \equiv \rho_{i,t+1} = E_{i,t} \{ p_{t+1} \} + y - Rp_t,
$$

where $E_{i,t}\{\cdot\}$ stands for the individual expectation, which do not have to coincide with the (true) conditional expected value operator $E\{\cdot\}$.

Denote the agent $i$’s expectation of the price in the next period $t+1$ as $E_{i,t}(p_{t+1}) = p_{i,t+1}$. We assume that the agent perceives the variance of one unit of the asset return as a constant, $Var_t(\rho_{t+1}) = \sigma_a^2$. Conditional on the realized price in the current period $t$, the agent’s $i$ risk adjusted utility at period $t$ is given by

$$
U_{i,t+1} = U_{i,t} + z_{i,t} E_{i,t} (\rho_{t+1}) - \frac{a}{2} Var (z_{i,t} \rho_{t+1})
$$

$$
= U_{i,t} + z_{i,t} \left( \rho_{i,t+1}^e + y - Rp_t \right) - \frac{a}{2} \sigma_a^2 z_{i,t}^2,
$$

where $a$ is the risk-aversion factor. Hence, define agents’ $i$ optimal demand at period $t$ as $z_{i,t}$, which becomes a linear demand schedule of the form

$$
z_{i,t}(p_t) \equiv \frac{p_{i,t+1}^e + y - Rp_t}{a \sigma_a^2} = \frac{p_{i,t+1}^e + y}{a \sigma_a^2} - \frac{R}{a \sigma_a^2} p_t.
$$

\[1\] One easily find $\sigma_a^2$ such that in the RE solution, the perceived and realized variances of the asset return coincide. Namely, $\sigma_a^2 = (1 + R)^2 \sigma_0^2 + \sigma_y^2$. 

indicate potential extensions.
CHAPTER 3. NETWORKS OF HETEROGENEOUS EXPECTATIONS

For simplicity we assume that the agents face no further liquidity constraints. Agents can take short positions, so \( z_{i,t} < 0 \) is viable.

The market operates in the following fashion. At the beginning of every period \( t \), each agent \( i \) has to provide her demand schedule \( z_{i,t} \) (3.3). Notice that because the agents are asked for a demand schedule, they do not have to forecast the contemporaneous price \( p_t \). Next to the agents’ demands, there is no additional exogenous supply/demand of the asset. The market clears if the following equilibrium condition is fulfilled:

\[
\text{Demand}_t = \sum_{i=1}^I z_{i,t} = 0 = \text{Supply}_t.
\]

Denote the average prediction of the agents of the price at \( t+1 \) as \( \bar{p}_{t+1} \equiv \frac{1}{I} \sum_{i=1}^I p^e_{i,t+1} \). Substituting the demand schedules (3.3) into the market equilibrium condition (3.4), we have that the realized market clearing price is given by

\[
p_t = \max \left\{ 0, \frac{\bar{p}_{t+1} + y}{R} + \eta_t \right\},
\]

where \( \eta_t \sim \text{NID}(0, \sigma^2_\eta) \) is a small idiosyncratic price shock.\(^2\) The price cannot be negative, so it is capped at zero.

This market has a straightforward RE stationary solution such that \( p^e_{t,t+1} = \mathbb{E}\{p_{t+1}\} = p^f \) with

\[
p^f = \frac{y}{r}.
\]

Without any additional assumptions, the model could explode even under Rational Expectations (RE). Following the experimental design of Hommes et al. (2005), we introduce a robotic trader to act as a stabilizing force on the market.

Robotic trader at period \( t \) trades as if the next price would be at the fundamental, i.e. his price forecast is \( p^e_{\text{ROBO},t+1} = p^f \). He becomes the more active the farther the market is from the fundamental solution. Define

\[
n_t = 1 - \exp \left( -\phi |p_{t-1} - p^f| \right)
\]

as the relative trading share of the robotic trader, which depends on his sensitivity \( \phi \).

---

\(^2\) One can provide microfoundations for \( \eta_t \) with additional supply shocks or demand of noise traders.
3.2. Theoretical model

Denote

\[ \hat{p}_{t+1}^e \equiv n_t p^f + (1 - n_t) \bar{p}_{t+1}^e = p^f n_t + \frac{1 - n_t}{I} \sum_{i=1}^{I} p_{i,t+1}^e, \]  

that is \( \hat{p}_{t+1} \) denotes the ‘total’ market price expectations, averaged over the robotic trader and the GA agents. Then the actual realized price including the robotic trader becomes

\[ p_t = \frac{(1 - n_t) \hat{p}_{t+1}^e + n_t p^f + y}{R} + \eta_t \]

\[ = \frac{\hat{p}_{t+1}^e + y}{R} + \eta_t \]

which is a two-period ahead nonlinear price-expectations feedback system. Notice that the introduction of the robotic trader does not change the steady state RE solution, nor does it exclude a possibility of an explosive rational solution (see Appendix 3.A for a discussion). Nevertheless, the stabilizing effect of the robotic trader will appear strong enough to prevent the model simulations from diverging.

3.2.2 Network

The agents are not fully isolated. Instead, they are positioned on an unweighted symmetric, irreflexive and a-transitive information network \( I \). Let \( I_i \) denote the set of friends of agent \( i \) and \( \hat{I}_i \equiv |I_i| \) denote the number of her friends (or the size of her friend set \( I_i \)). Throughout the Chapter, we assume that for a particular market this information network is fixed and exogenous. A natural extension is to allow the agents learn how to link with other agents. See Albert and Barabási (2002); Goyal (2002); Newman (2003) for a general introduction into networks and Bala and Goyal (2000) for endogenous network formation. In the next section we will discuss the specific networks used in the numerical simulations of the model.

Within the network, an agent cannot directly observe the price expectations of her friends, but she knows whether in the recent past they were buying or selling the asset. This resembles reality, where the market participants are likely to share only qualitative information (‘this stock is profitable, I just bought it!’). We emphasize that an agent cannot see the contemporary trade decisions of her friends, and moreover agents have no private information about the future price shocks \( \eta_t, \eta_{t+1}, \eta_{t+2}, \ldots \). Consider agent
and define

\[
    m_{j,s} = \begin{cases} 
        +1 & \text{if } z_{j,s} > 0, \\
        0 & \text{if } z_{j,s} = 0, \\
        -1 & \text{if } z_{j,s} < 0 
    \end{cases}
\]

a simple sign function of her realized demand in period \( s \).

The agents have memory length \( \tau \) and consider the simplest possible index \( \text{Mood}_{j,t-1} \), defined as the average mood of agent \( j \) in periods \([t - \tau, t - 1]\)

\[
    \text{Mood}_{j,t-1} = \frac{1}{\tau} \sum_{s=t-\tau}^{t-1} m_{j,s}.
\]

For instance, if during the last \( \tau \) periods the agent \( j \) was always buying (selling) the asset, her index is +1 (−1). If she was more likely to buy (sell) the asset, her mood index is positive (negative) and so a positive (negative) index means that the agent \( j \) remained optimistic (pessimistic) about the asset profitability. A special case of short memory is \( \tau = 1 \) when the mood index becomes the sign of the agent’s \( j \) very last transaction at period \( t - 1 \).

By assumption, the mood of agent \( j \) \( \text{Mood}_{j,t-1} \) is visible to all of her friends, that is agent \( i \) can access all \( \text{Mood}_{j,t-1} \) for which \( j \in I_i \). However, in order to study the effect of herding, we assume that the agents do not distinguish between their friends and instead rely on the mood of their peers, that is the mood index (3.11) averaged among all \( \tilde{I}_i \) of their friends:

\[
    \text{Peer}_{i,t-1} \equiv \frac{1}{\tilde{I}_i} \sum_{j \in \tilde{I}_i} \text{Mood}_{j,t-1}
\]

\[
    = \frac{1}{\tau \tilde{I}_i} \sum_{j \in \tilde{I}_i} \sum_{s=t-\tau}^{t-1} m_{j,s}.
\]

We will refer to (3.12) as the measure of agent \( i \)’s friends’ mood. It has a straightforward interpretation as the average mood of \( i \)’s friends. Hence if the agents learn to herd, their price expectations should depend positively on the peer mood.

### 3.2.3 Fundamental solution benchmark

We define the fundamental solution to our model as a rational expectations (RE) equilibrium, that is as a set of model-consistent demand schedules such that for every agent
3.2. Theoretical model

At every period \( t \) it holds that \( p_{i,t+1}^e = E_t \{ p_{t+1} \} \) (where \( E_t \) denotes the (mathematical) conditional expected value). This is equivalent to \( E_{i,t} (p_{t+1}) = E_t \{ p_{i,t+1} \} \), that is the expected returns on the asset are model consistent.

As already mentioned, the RE steady state solution is unique. Notice that the network has no effect on the RE solution. In fact, its presence or particular structure makes no difference on the behavior of the agents in equilibrium. Therefore, the RE framework gives a strong prediction that adding a network into a model will result in the same (long-run) dynamics. See Appendix 3.A for a discussion.

3.2.4 Experimental and Genetic Algorithms benchmark

Our model is based on the experimental market investigated by Hommes et al. (2005), with almost the same parametrization (see later discussion). The authors report that the subjects follow price trend extrapolation rules and that the prices were unlikely to settle on the fundamental value, and often oscillated instead around the fundamental in an irregular fashion. Robotic agent prevented large bubbles, however. Hommes et al. (2008), who studied an experimental two-period ahead asset pricing market without the robotic trader, found their prices to explode instead of oscillating.

In Chapter 2 we have already investigated the model without the networks and shown that the GA agents learn to use trend following heuristics in a similar fashion to the experimental subjects. Furthermore, long-run simulations of the model revealed that it contains two attractors. The market could switch between settling on the fundamental steady state and oscillations of varying amplitude. A basic question is whether the introduction of a network will change this outcome, and in particular whether both attractors are robust against networks.

3.2.5 Price expectations and learning

Agents consider themselves as price-takers and so their task is simply to try to predict the next price \( p_t \) as accurately as possible, conditional on the market events and friends’ behavior until and including period \( t - 1 \). The GA agents are not perfectly rational and instead rely on a simple rule of thumb, a linear heuristic of the form

\[
\begin{align*}
   p_{i,t+1}^e &= \alpha_{i,t} p_{t-1} + (1 - \alpha_{i,t}) p_{i,t}^e + \beta_{i,t} (p_{t-1} - p_{t-2}) + \gamma_{i,t} \Gamma_{Peer_{i,t-1}} \\
   &= \underbrace{\alpha_{i,t} p_{t-1}}_{\text{adaptive expectations}} + \underbrace{(1 - \alpha_{i,t}) p_{i,t}^e}_{\text{trend extrapolation}} + \underbrace{\beta_{i,t} (p_{t-1} - p_{t-2})}_{\text{herding/contrarian}} + \underbrace{\gamma_{i,t} \Gamma_{Peer_{i,t-1}}}_{\text{network}}.
\end{align*}
\]

The first two elements of the rule, adaptive and trend extrapolation expectations, come directly from the baseline GA model discussed in Chapter 2. The new part of the above
rule is the last term, the peer effect, which is a weighted sum of the moods of all friends of agent \(i\).\(^3\)

The linear heuristic (3.13) of agent \(i\) depends on the specific parameters chosen at period \(t\): the price weight \(\alpha_{i,t} \in [0,1]\), the trend coefficient \(\beta_{i,t} \in [-1.3, 1.3]\) and finally the trust index \(\gamma_{i,t}\). The trust index \(\gamma_{i,t} \in [-1,1]\) can be interpreted as the importance agent \(i\) attaches towards her friends decisions. If \(\gamma_{i} > 0\), agent \(i\) believes it is worth to follow her friends’ past trades, and that the past optimism of her friends signals the price \(p_t\) to increase even more than just due to current trend. Conversely, \(\gamma_{i} < 0\) implies that the agent \(i\) behaves in contrast to her friends. In other words, \(\gamma_{i,t} > 0\) means herding behavior, while \(\gamma_{i,t} < 0\) implies contrarian behavior.\(^4\)

The specific value of the trust index \(\gamma_{i,t}\) is multiplied by a sensitivity parameter \(\Gamma\), which remains constant and homogeneous across agents. We use the multiplicative form of the herding/contrarian term, with these two factors separated, for the sake of an easier display and interpretation of the results.

The agents do not use the same heuristic over time. Depending on the market conditions, the heuristic (3.13) should be based on different parameters. For instance, in the periods of strong price oscillations, agents should switch from low or negative trend extrapolation to strong trend extrapolation. Furthermore, the goal of the chapter is to identify circumstances in which the agents learn herding or contrarian strategies.

We follow Chapter 2 and model the time evolution of the heuristic (3.13) by Genetic Algorithms. This results in individual learning and heterogeneity of forecasting behavior, similar to the experimental findings. See Chapter 2 for a technical discussion and parametrization. The intuition of the model is the following.

The agents are endowed with a small set of different parameter triplets \((\alpha_{i,t}, \beta_{i,t}, \gamma_{i,t})\), which correspond to a small list of specific forecasting heuristics (3.13). At every period \(t\), every agent chooses one particular heuristic \(p_{\text{est},t+1}\) to predict the next price \(p_{t+1}\), according to their relative forecasting performance in the previous period. Next, the price \(p_t\) is realized and the agents observe how well their heuristics would forecast \(p_t\) in the previous period (conditional on the relevant past information set). Based on this information, every agent independently updates her heuristics using four evolutionary operators: procreation (better heuristics replace worse), mutation and crossover.

\(^3\)RE are a special case of (3.13). To be specific, with \(\alpha_{i,t} = \beta_{i,t} = \gamma_{i,t} = 0\) and \(p_{\text{est},2} = p^f\), every next forecast of \(p_{t+1}\) will also be equal to \(p^f\)\(^{+1}\).

\(^4\)In the literature on heterogeneous price expectations, one can often find the label ‘contrarian’ in an alternative use, namely as a heuristic according to which a positive (negative) price trend means expected price decrease (increase). For (3.13), this means \(\beta_{i,t} < 0\). In line with Chapter 2, the agents in our model will converge to strong trend following rules with \(\beta_{i,t} >> 0\) and \(\beta_{i,t} < 0\) will not play a significant role. We will thus never use the term ‘contrarian’ in this sense.
(experimentation with heuristic specification) and election (screening ineffectual experimentation). Afterward, the next period \( t + 1 \) starts and the procedure is repeated. See Chapter 2 for technical presentation.

3.2.6 Coordination versus herding

We emphasize the difference between coordination and herding. Herding (contrarian) means that the agents directly follow (contrast) the decisions of their friends, which in this model is embodied by relatively high positive (negative) values of the trust index \( \gamma \). On the other hand, coordination is the similarity of agents’ behavior: in this context similarity of individual price forecasts and forecasting rules. We follow Heemeijer et al. (2009) and express this value as the standard deviation of the individual price forecasts, a measure of dis-coordination of the form

\[
D_t = \sqrt{\frac{1}{6} \sum_{i=1}^{6} \left( p_{i,t+1}^e - \frac{1}{6} \sum_{j=1}^{6} p_{j,t+1}^e \right)^2} \in [0, \infty[.
\]

If \( D_t = 0 \), then all agents at period \( t \) have same the forecast of the next price, while higher values of (3.14) imply larger dispersion of the contemporary individual price forecasts.

It is easy to see that coordination and herding are two different things. Regardless of the particular network, agents interact indirectly through the market price. Experiments and previous work on the GA model, reported in Chapter 2, show that such an indirect interaction can be sufficient to impose a large degree of coordination in asset markets, even though the agents cannot observe each other. One of the goals of this chapter is to investigate whether herding further can help coordination.

3.3 Monte Carlo studies

3.3.1 Parametrization of the model

Following the design of Hommes et al. (2005), the parameters are set in the following way.\(^5\) Regardless of the network, the gross interest rate is set to \( R = 1 + r = 0.05 \) and the dividend to \( y = 3 \), which gives the fundamental price \( p_f = 60 \). The standard deviation of the price shocks in (3.9) is set to \( \sigma_\eta = 0.1 \), which implies that under RE the price should be normally distributed and approximately in the \([59.75, 60.25]\) interval.

\(^5\) All the simulations were written in Ox matrix algebra language (Doornik, 2007) and can be provided at request.
for 99% number of periods. It will appear that these small idiosyncratic price shocks play no significant role in the system.

The only difference with Hommes et al. (2005) comes with the parametrization of the robotic trader. Recall the definition of the relative weight of robotic trader (3.7), which depends on the sensitivity parameter $\phi$. For instance, the robotic trader will take over exactly a quarter of the market ($n_t = 0.25$) if the absolute price deviation is

$$
|p_{t-1} - p^f| = -\frac{\ln 0.75}{\phi}.
$$

Hommes et al. (2005) set $\phi = 1/200$, which means that the robotic traders takes over 25% if the price is close to the minimum allowed price $p_t = 0$ or the maximum allowed price $p_t = 100$.\(^6\) In this chapter we want to study the impact of the networks on market stability and hence we want to scale down possible price oscillations. Hence, we set $\phi = 1/104.281784903$, which implies that the robotic trader will take over a quarter of the market if the price will reach $p_{t-1} = 90$ or $p_{t-1} = 30$, that is if it deviates from the fundamental by a factor of 50% (100% in the setup of Hommes et al. (2005)).

The GA model requires parametrization in two respects. First, the specification for the evolutionary operators we take directly from Chapter 2. In comparison with Chapter 2, the GA agents in this chapter optimize three parameters instead of two, because of the extra trust index $\gamma$. This parameter is associated with a gene of 20 bits, which is the same as genes representing the two other coefficients of the agent heuristic (3.13).

Second, the GA agents can optimize the heuristic parameters only from a prede
dined interval (see Chapter 2 for a discussion). The price weight $\alpha \in [0, 1]$ has to span a simplex; the trend weight we take from Chapter 2 to be $\beta \in [-1.3, 1.3]$. We assume that all agents have their total herding/contrarian effect equal to 6 (in absolute terms), which corresponds to 10% of the fundamental price. This implies that the trust sensitivity is set as $\Gamma = 6$, given that the trust index has to be chosen from a unit inteerval $\gamma \in [-1, 1]$. Finally, the heuristic (3.13) is based on the mood index (3.11), for which we take memory $\tau = 5$.\(^7\)

---

\(^6\)Specifically, $n_t = 25\%$ happens if the price deviation is $|p_{t-1} - p^f| \approx 57.5$. There were two treatments with $p^f = 60$ and $p^f = 40$, for which thus $n_t \approx 25\%$ if the price hits $p_t = 0$ and $p_t = 100$ respectively.

\(^7\)See the next two sections for additional information on the robustness of the parameters $\Gamma$ and $\tau$. 

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3.3.2 Initialization

In Chapter 2 we note that the GA model is sensitive to initial conditions. First, the agents require some initial heuristics. Here we follow the previous chapter and sample them at random. Second, the heuristics require past data from at least two periods, including the previous price forecasts itself. In our numerical simulations, the agents in the very first period always predict the fundamental price and only start using their heuristics in the second period, assuming no trend. Specifically

\[
\begin{align*}
    p_{i,2}^c &= p^f = 60, \\
    p_{i,3}^c &= \alpha_{i,3} p_1 + (1 - \alpha_{i,3}) p_{i,2}^c \\
    &= 60 + \alpha_{i,3} \eta_1 \quad \text{for every } i \in I,
\end{align*}
\]

which implies that the market is initialized at the fundamental value (lest the price shock), \( p_1 \approx p_2 \approx p^f \).

This may seem as a surprising design feature, since ABM’s are often used to study convergence. It will turn out that the model in this chapter is inherently unstable. Indeed, if initialized in the fundamental, most likely it will diverge at some point!

3.3.3 Small networks of six agents

In the first part of our study, we will look at networks of six agents. This is the typical number of subjects in the LtF experiments, which can thus serve as a reference point for the following results. Furthermore, it is easy to trace the behavior of only six agents, while this number of nodes is sufficient to have networks with interesting properties, such as regular lattice or asymmetric positioning. We will therefore use these networks to gather basic intuitions of how the network structure and placement affects individual behavior, herding and coordination, before moving to large networks.

For six identical agents, one can arrange them on 156 different networks. For the sake of presentation, we focus on six specific: no network, circle, fully connected, two connected clusters, core-periphery and star. See Figure 3.1 for a visualization of these networks, and Table 3.1 for summary statistics.

The model with no network serves as an natural benchmark, a setting studied with the LtF experiments and in Chapter 2. The circle and fully connected networks are important to evaluate the network effect on price stability, coordination, learning and herding. The three other networks represent asymmetric positioning of the agents,
which we will show to have an interesting effect on herding.\textsuperscript{8}

\begin{table}[h]
\centering
\begin{tabular}{|l|c|c|c|c|c|}
\hline
\textbf{Network} & \textbf{Clusters} & \textbf{Diameter} & \textbf{Closeness} & \textbf{Density} & \textbf{Transitivity} \\
\hline
No network & 6 & N/A & 0 & 0 & 0 \\
Circle & 1 & 3 & 0.6667 & 0.4 & 0 \\
Fully connected & 1 & 1 & 1 & 1 & 1 \\
Two connected & 1 & 3 & 0.6889 & 0.4667 & 0.7778 \\
Core-periphery & 1 & 3 & 0.7556 & 0.5333 & 0.5 \\
Star & 1 & 2 & 0.6667 & 0.3333 & 0 \\
\hline
\end{tabular}
\caption{Properties of the small networks used in Section 3.3, see Appendix 3.D for definition.}
\end{table}

In the later analysis the model turns out to be unstable in terms of the realized prices, but also in terms of individual coordination and strategies. In repeated simulations of the small networks of six agents, a clear median pattern will emerge, but individual simulations will exhibit oscillatory behavior. In order to understand these dynamics, we will focus on three types of evidence.

Every network will be simulated 1'000 times, based on the same price shocks $\eta$.\textsuperscript{9}

\textsuperscript{8}Another interesting topic is whether clustering has any effect on individual coordination. We investigated three additional networks (with three and two symmetric clusters, and with four fully connected agents and two unconnected outsiders). However, these networks indicate that the clustering does not have any meaningful effect on the market, so for the sake of presentation we leave them out of the chapter. The Monte Carlo results are available in the online supplementary material.

\textsuperscript{9}Because the price shocks are normally distributed with small variance, the simulation dynamics
but on differently realized learning via different random numbers given to the GA procedure. Every market is simulated for 2'500 periods, including the initial 100 periods which can be interpreted as a learning phase. This offers two ways to represent the data.\(^\text{10}\)

First, for each of the 1'000 simulated markets one can look at the realized statistics of that market (for periods 101 till 2'500, so excluding the learning phase), which we will refer to as long-run statistics. This gives a distribution of 1'000 long-run statistics, and one of particular importance is the distribution of the long-run market price standard deviation. The latter can be used directly as an indicator of market volatility.

Second, one can look at realized market variables across the 1'000 markets in a particular period. We will focus on the median and 95% confidence intervals (CI) and how they evolve over time. The variables of interest are price, dis-coordination measure (3.14) and the coefficients chosen by the agents to specify the forecasting heuristic (3.13).

Finally, the two above Monte Carlo (MC) measures show general patterns of market behavior. It will be clear that the model does not converge, hence it may be difficult to understand the emergent properties of individual learning based solely on this MC exercise. In order to interpret the results, we will also present some sample time paths of the realized market and learning variables.\(^\text{11}\)

### 3.3.4 Large networks

Large networks are empirically more relevant than the ones based on six agents. However, one can identify a myriad of possible and interesting large network topologies, inducing a need for a selection for the numerical study. We will base ours on Panchenko et al. (2013) and the above mentioned small networks. Specifically, we will focus on networks of size \(I \in \{50, 100, 250, 500, 1000\}\), with 10 different architectures that range from regular lattice to small-world network. Due to the limit of the study, we will leave large-scale networks for future studies (cf. PGP13).

The non-regular networks are defined through a non-deterministic generating process (see below for details for random and small world networks). In principal one could obtain a distribution of results based on a repeated sampling of these non-regular nets-\(^\text{are largely independent from them.}\)

\(^\text{10}\)It is impossible to have the same random numbers for learning, since in the model with no network the agents do not optimize the trust index. However, in all other five networks indeed all the agents optimize this coefficient, and their learning is based on the same set of random numbers.

\(^\text{11}\)The full results of the MC study for all 6 small networks, and three additional, three and two clusters and two outsiders, which include 330 graphs grouped into 77 panels, as well as tables for prediction correlation, constitute a 52 page supplement online material.
CHAPTER 3. NETWORKS OF HETEROGENEOUS EXPECTATIONS

works. This in turn would be obtained by a proper MC exercise per every realized network (in line with the MC study for the networks of 6 agents). However, a single sample network of many agents (with 1’000 as the maximum market size in this chapter) is already numerically involved, and difficult in terms of presentation. Furthermore, we will see that the behavior of sample large networks is consistent with the behavior observed in the MC study for the small networks. As a result, a full-fledged MC exercise could offer only little additional understanding. Instead, for every network size and architecture, we will focus on one sample realized simulation, with longer time horizon of 25’000 periods. We will present the long-run outcomes of these networks, including the distribution of trust and price stability.

We will study four regular networks, with architectures that are based on their 6-agent counterparts:

**No network** — every agent has an empty set of friends.

**Fully connected** — every agent is befriended with every other agent.

**Regular** — every agent has exactly 4 friends. This market can be represented by a circle such that every agent is linked with two agents to the left and two to the right.

**Star** — one central (or star) agent, who is connected to all other agents, whereas other edge agents are connected only with the star agent.

Random network with probability $\pi_l$ is typically defined as a random graph, in which every two agents are linked with probability $\pi_l$. Such an architecture is analytically tractable, and, relevant for our study, can offer a wide distribution of links between the agents. We will focus on two random networks:

**Random(4)** — sparse random with $\pi_l$ set in such a way that on average there are exactly 4 links per agent, which gives the same density as for the regular network defined above.

**Random(16)** — dense random with $\pi_l$ such that on average there are exactly 16 links per agent, i.e. with density four times larger than for the regular network defined above.

Empirical social networks often have characteristics of the so-called small world networks (SM): small density (few links between the agents), together with large transitivity (‘cliquishness’, high probability that a friend of my friend is also a friend of me), but also small characteristic path (average distance between the agents). In practice
3.4 Networks of six agents

such networks look like semi-independent clusters that are connected with each other by infrequent ‘bridge agents’ (see the network in Figure 3.1d for a simplest example, in which agents 3 and 4 serve as such a bridge between two clusters). It is impossible to obtain networks with such properties through random graphs. Watts and Strogatz (1998) propose the following algorithm: start with a regular lattice network of \( K \) links per node and rewire every link with probability \( \pi_r \). Rewired networks based on well chosen parameters \( K \) and \( \pi_r \) have SM properties. In order to obtain networks of a similar density as the random ones, while maintaining a connection to the study of PGP13, we will use the following four rewired networks:

\[
\text{Rewired}(4, 0.01) \quad \text{— on average four links per agent and rewiring probability of } \pi_r = 1\%. \quad \text{This gives a SM network for the largest market } I = 1'000.
\]

\[
\text{Rewired}(4, 0.1) \quad \text{— on average four links per agent and rewiring probability of } \pi_r = 10\%. \quad \text{This gives a SM network for the intermediate market } I = 100.
\]

\[
\text{Rewired}(16, 0.01) \quad \text{— on average 16 links per agent and rewiring probability of } \pi_r = 1\%.
\]

\[
\text{Rewired}(16, 0.1) \quad \text{— on average 16 links per agent and rewiring probability of } \pi_r = 10\%.
\]

The representation of the realized non-regular networks for all five possible network sizes, together with a table of basic characteristics, is available in Appendix 3.E.

3.4 Networks of six agents

This section consists of three parts. First, we will look on the model without a network (or an empty network). This is essentially the same setup as in Chapter 2. However, that Chapter had a different focus than this study, namely it considered four different LtF experiments and how the model is able to replicate the behavior of the subjects. We will supplement Chapter 2 by an in-depth analysis of the emergent properties of the GA model for the two-period ahead asset pricing market, specifically what drives the coordination on the oscillatory prices paths and how does this link with the individual learning. Second, we will introduce two regular lattice networks (circle and fully connected; see Figure 3.1 for a visualization) in order to study whether the agents learn to herd or rather to contrast the behavior of their friends. Finally, we will consider three asymmetric networks (two connected clusters, core-periphery and star) in order to check if such asymmetries have any effect on herding, as would seem natural.
3.4.1 Benchmark model without network

Observation 1. Without a network, there are two types of attractors in the model: coordination on the fundamental price and oscillations around the fundamental.

In line with the findings of Chapter 2, we find that there are two possible outcomes in the model if there is no network. First, the GA agents can coordinate on the fundamental price and stay there (see Figure 3.2b for a sample market). This is a self-confirming steady state, since if the price stays at the fundamental level, the forecasting rule (3.13) effectively reduces to forecasting the fundamental as well. The trend term $\beta$ is irrelevant and retains wide distribution centered around zero (weak trend following; see below for a further analysis). The price shocks $\eta_t$ are too small to push the agents from this equilibrium.

However, if by chance the agents were on average considering sufficiently strong trend extrapolation (sufficiently large $\beta$), then they can pick up a sufficiently large shock $\eta_t$ as a sign for price trend, outweigh the robotic trader and impose a regime of price oscillations (see Figure 3.2c for a sample market). This is again a self-confirming regime: if there is significant trend, agents effectively learn to follow it, which amplifies the trend despite the stabilizing effect of the robotic trader. We observe that across 1’000 simulations, slightly more than 50% of the markets slipped into significant oscillations.

\footnote{Recall that the simulations for a specific network are based on the same series of price shocks but}
price oscillations. This is visible in the bimodal distribution of the long-run standard deviation of the price, as visible on Figure 3.2a. It also corresponds to the results of Hommes et al. (2005), who report that among the seven experimental groups with \( p_f = 60 \), four exhibited oscillations, two monotonic convergence and one an intermediary case.

**Observation 2.** *GA agents, without any direct link, can coordinate well by learning to follow the price trend, which can differ across time and between markets.*

Disregarding the initial 100 periods, the predictions of GA agents are sharply correlated with an average correlation coefficient of 98.7% (average from 1’000 simulated markets).\(^\text{13}\) Across the simulations, the median dis-coordination (3.14) remains low, below 0.1 (see Figure 3.3b for the median and 95% CI over time). Nevertheless, large outliers are possible and the upper bound of the 95% CI exceeds 2. This implies that the agents are typically well coordinated, but momentary breaks of this coordination occur from time to time.

The reason that the agents remain well coordinated despite no direct links between them is that they learn similar behavior, and so their predictions react to specific market condition (current price and price trend) in the same manner. This in turn is reinforced by the positive feedback nature of the market. For example, if the GA agents have similar optimistic price forecasts, the realized price will indeed be high and the agents have no incentive to change their behavior. If the prices remain stable, a specific parametrization of the forecasting (3.13) is in practice irrelevant. On the other hand, the median GA agent converges to the following strong trend rule

\[
p_{t+1}^e \approx 0.9p_t - 0.1p_{t-1} + 0.5\Delta p_{t-1},
\]

as presented at the bottom panel of Figure 3.3. Notice that this rule is similar to the median rule reported in Chapter 2 for the same market. In the unstable markets GA agents learn to coordinate on the price trend. The trend coefficient \( \beta \) retains a wide distribution even towards period 2’500, including its upper 95% CI bound being far away from the median, which suggests that the specific trend is realized differently on different markets. Indeed, the long-run standard deviation of the unstable markets has a wide distribution, which means that different markets are unstable to a different degree, or the market instability (i.e. the specific price trend) changes over time.

**Observation 3.** *In unstable markets, the robotic trader is responsible for the reversal different realized learning.*

\(^{13}\)Specifically, correlation of predictions for any pair of agents across 1’000 markets is equal to 98.7%.
Regardless of whether the market was stable or not, its long-run average price has a degenerate distribution at approximately the fundamental level. Furthermore, the price oscillations are centered around the fundamental and 95% of times are approximately contained within the interval [45, 75], that is $75\% - 125\%$ of the fundamental price (see Figure 3.3a). This corresponds to a relative weight of the robotic trader (3.7) $n_t \approx 13.4\%$ at the peak of oscillations.

Consider again the unstable market presented in Figure 3.2c. Figure 3.4 displays a number of variables from that market in periods $1'001$ till $1'025$. Within these 25 periods, the market experienced one bubble, a subsequent crash, a period of crisis and then recovery towards a new bubble. The turning points of the bubble and crisis happened in periods $1'005$ and $1'016$ respectively (which means that one full cycle took
3.4. Networks of six agents

Figure 3.4: Sample market without network, periods 1’001 till 1’025. Realized market variables in these periods and individual forecasts (including the corresponding heuristic specifications) of price from these periods.

around 20 periods, see Figure 3.4a). What caused these market turn-overs?

Throughout periods 1’000 – 1’025 the price shocks were not larger than 0.2 (i.e. their two standard deviation) in absolute terms. The cycle of bubbles and crashes arises endogenously, without a need for large exogenous shocks. Before the bubble turning point in period 1’006, GA agents used the highest possible trend extrapolation (Figure 3.4f) and remained well coordinated. Notice that in the period of bubble build-up, the agents undershoot the price, but slowly converge towards it. Indeed, their error approaches zero at the turning point (Figure 3.4e). It means that GA agents are ‘catching up’ with the bubble. This is possible, because they use a constant trend coefficient, whereas the actual price trend \((p_{t-1} - p_{t-2})\) looses on its momentum (see Figure 3.4d for the price trend observed by the agents). The latter is due to the robotic trader: the more the bubble builds up, the more he becomes active (Figure 3.4c). The

\footnote{Compare with standard RE models.}
maximum price trend happens three periods before the turning point, after which the robotic trader becomes twice as active (with his relative weight increasing from around 5% to 10%).

At some moment, the robotic trader becomes influential enough to outweigh the GA agents and to halt the price trend completely.\(^{15}\) Specifically this happens in period 1\(^{′}\)006, the last period for the price trend to remain positive (albeit already close to zero, see Figure 3.4d). GA agents do not realize that yet, but rather observe that in the past it paid off to follow the trend (indeed they just ‘caught up’ with the growing price) and so follow its latest observed value \(p_{1005} - p_{1004}\) (which is only mildly positive) when forecasting the price in the next period 1\(^{′}\)007. On the other hand, in period 1\(^{′}\)007 the robotic trader does not care for the price trend to have lost momentum. Instead he considers only the absolute deviation of the price from the fundamental and remains highly active. The total effect is that the realized price \(p_{1007}\) is slightly smaller than the previous price. GA agents momentarily realize that the strong trend following just lost its potency and start to experiment with lower coefficients \(\beta\) of (3.13) for predicting price in the next period \(p_{1008}\) (see Figure 3.4f). This results in a moment of dis-coordination (Figure 3.4b), and also means that the stabilizing effect of the robotic trader becomes even more important and the price starts to drop in a more pronounced way.

This does not lead the price to converge to the fundamental, what the robotic trader is trying to achieve. Instead, GA agents notice the downward trend and immediately follow it (chosen trend coefficients \(\beta\) were diverging from their allowed upper bound of 1.3 for only two periods), which reinforces the crash. The market undershoots the fundamental value and a symmetric sequence of events happens around period 1\(^{′}\)015.

**Finding 1.** *Without a network, the market can be both stable and unstable. Instability occurs if GA agents learn to follow the current price trend, which generates a high level of coordination during the build-up of bubbles and crises. However, a constant price trend cannot be sustained forever because of the robotic trader. When the trend loses enough momentum, the GA agents experiment with their forecasting heuristics. As a result, a tipping point of bubbles and crashes emerges, in which agents are dis-coordinated before a price trend reversal, which renews coordination on extrapolating the reversed price trend.*

These findings are not driven by the nature of the robotic trader *per se*, but by the fact that a stable price trend is unattainable because of the robotic trader. In

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\(^{15}\)This does not require a substantial negative price shock. Instead, it relies more on the decreasing price trend, growing weight of the robotic trader and the feedback coefficient in the price equation (3.9) being less than unity due to a positive interest rate.
3.4. Networks of six agents

real markets, a number of factors can have a corresponding effect of breaking up the price trends: liquidity constraints, regulations on prices (such as a maximum allowed price change in a day); and finally ‘common knowledge’ that bubbles cannot build up forever. If there is uncertainty of how exactly the price trend will be broken (for example because of strategic uncertainty, or because the legal constraints are imprecise), market participants find it easy to follow price bubbles and crashes, but are not that skilled at playing out the markets’ tipping points. The same mechanism emerges in our model as summarized in Finding 1. Just as in real life, the GA agents follow the market, which reinforces its growth, until it reaches its natural limit and despite the agents being overly optimistic, it starts to stagnate. The agents panic and the bubble bursts without the optimism it was built upon. A period of crisis thus emerges, when agents’ pessimism reinforces the price drop.

3.4.2 Contrarian strategies induced by networks

Observation 4. Networks have a destabilizing effect on the market.

Introducing any type of network into the model brings forth additional price instability. MC results for two regular networks, circle and fully connected, are presented in Figure 3.5. In comparison with the model without a network, coordination on the fundamental price does not constitute a likely attractor anymore. Only about 5% of the markets exhibit long-run price’s standard deviation close to zero for the circle network, and virtually none for the denser fully connected network (middle panel of Figure 3.5). On the other hand, in almost all markets price oscillations arise. Furthermore, these oscillations are stronger, implying larger market volatility. The prices still fluctuate around the fundamental value, but the 95% CI of the realized price across time are approximately twice as wide as without network, specifically they constitute an interval close to [30, 90], i.e. [50%, 150%] of the fundamental price. This implies price oscillations, that start almost immediately (left panel of Figure 3.5). In the unstable simulations, typical price long-run standard deviation is also twice as large for both the circle and the fully connected networks than without network. Sample simulations (right panel of Figure 3.5) confirm that the price oscillations have a more pronounced amplitude.

Observation 5. Agents learn to use heuristics with strong price trend extrapolation, and also a contrarian attitude towards other agents. Despite the contrarian strategies, they remain well coordinated.

In the remaining of this subsection, we will focus on the fully connected network.
Results for the circle network are essentially the same. The first observation is that just as was the case for the model without a network, the GA agents exhibit strong trend following behavior. Figure 3.6 presents the time distribution of the price weight $\alpha$ and trend coefficient $\beta$ chosen by the agents across 1,000 simulated fully connected markets. The median agent quickly converges to a strong trend following rule

$$p^e_{i,t+1} \approx p_{t-1} + 1(p_{t-1} - p_{t-2}) - 0.9\Gamma_{Peer_i,t-1},$$

which is slightly stronger in terms of trend following coefficient $\beta$ than for the case of no network (even though the 95% CI have approximately the same width). Notice furthermore that the weight on the previous forecast $1 - \alpha$ is close to zero, which is
3.4. Networks of six agents

![Graphs of parameters](image)

(a) Price weight $\alpha$
(b) Forecast weight $1 - \alpha$
(c) Trend coefficient $\beta$

**Figure 3.6:** Monte Carlo study (1000 markets) of fully connected network: time evolution of heuristic (3.13) parameters chosen by the agents to forecast prices. Median represented by a red line and 95% CI represented by blue lines.

consistent with the findings from Chapter 2: large price instability induces learning of stronger trend, but also less ‘conservative’ behavior.

Interestingly, the agents learn contrarian rules with median $\gamma \approx -0.9$. If the friends of an agent were buying in the past (remained optimistic), she would decrease her price forecast. Figure 3.7 shows the 95% CI of the trust agent 1 puts into her friends and trust she receives herself over time.\(^{16}\) The median agent quickly learns a strong contrarian heuristic and is furthermore distrusted by her friends. However, the 95% CI for both variables remain wide and approach zero from below, which means that the agents are still experimenting with the specific strength of the contrarian strategies. Indeed, at all time the agents seem to have similar trust coefficients $\gamma$, but there are periods of significant differences between them in their level of trust, as seen in the standard deviation of that variable between the agents (low median, but wide 95% CI, see Figure 3.7c). Finally, despite the contrarian behavior, GA agents remain well coordinated, as measured by the correlation of their price forecasts equal to 96.8%.

What is the reason for these two facts, and how are they connected with the increased price volatility?

**Observation 6.** Agents can only look at the past behavior of their friends, but the market is unstable. Therefore, trades that were optimal in the past are inconsistent

\(^{16}\)Because the network is symmetric, the results for the other five agents are indistinguishable. This shows that the MC sample of 1’000 gives a sufficient picture of the model’s behavior.
Figure 3.7: Monte Carlo study (1000 markets) of fully connected network: time evolution of $\gamma_1$ trust index of agent 1; $\frac{1}{5} \left( \sum_{i=2}^{6} \gamma_i \right)$ normalized trust index received by agent 1 from her friends (all other five agents); and standard deviation of the trust indices of all six agents. Median represented by a red line and 95% CI represented by blue lines.

with contemporary market conditions. As a result, observed mood of friends is ‘sticky’, which induces the agents to learn contrarian strategies.

To study the reason of the contrarian behavior, we will focus on a sample cycle of boom and crisis presented in Figure 3.8, for one market with the fully connected network. We observe that this cycle is similar to the one from a sample market without a network, specifically the oscillations have similar period (but a higher amplitude, as discussed in Observation 4) and the timing of the bubble-crisis cycle is symmetric. During a build-up period of the bubble, the agents are ‘chasing’ the bubble with strong trend following rules, until the robotic trader curbs the price trend. Afterwards, there is a small phase when the agents experiment with their rules, what together with the influence of the robotic trader causes a trend reversal, which is quickly picked up by the agents. The evolution of the trust index $\gamma$ is the opposite to that of the trend coefficient: it typically stays close to its lower boundary ($\gamma \approx -1$), but agents experiment with higher trust during the market reversals.

To interpret these results, the following lemma is useful (see Appendix 3.B for the proof):

**Lemma 1.** Disregarding the price shocks, agent $i$ buys (sells) if her price forecast is higher (lower) than the average market expectation (3.8), that is if $p^e_{i,t+1} > \bar{p}_{t+1}$
(3.4. Networks of six agents)

\[ p_{t+1}^e < \hat{p}_{t+1}^e \]. Similarly, robotic trader buys (sells) if the average market expectation (3.8) is above (below) the fundamental price, i.e., robotic trader’s forecast. This implies that the GA agents and the robotic trader can buy (sell) even though they expect negative (positive) price trend.

This lemma follows from the two-period ahead structure of the market and has a simple interpretation. Consider a market at period \( t \) with only two agents, no price shocks (\( \eta_t = 0 \)) or robotic trader (\( n_t = 0 \)). Let both agents expect the price to increase in the next period \( t + 1 \) in comparison with the last observed \( p_{t-1} \). Their demand functions (3.3) however depend not only on their expectations of \( p_{t+1} \), but also on the contemporary price \( p_t \), which follows the market clearing condition (3.4). If one agent is more optimistic about the next period (say \( p_{1,t+1}^e > p_{2,t+1}^e \)), then she will take relatively longer position. Because the market has to clear, she will buy the asset from the other agent, \( z_{1,t} = -z_{2,t} > 0 \). In a general case of many agents, price shocks and the robotic trader, the market clearing condition implies that a GA agent buys the asset if she is relatively optimistic, not if she is optimistic in absolute terms.

The important consequence of the lemma is that, because of the robotic trader, GA agents are likely to be buying (selling) the asset even after they realize a market reversal. For an illustration consider the sample network of six fully connected agents (Figures 3.8 and 3.9). In this market there was a bubble with a peak in period 1015, with a negative price trend afterward. The agents realized that after observing \( p_{1016} \)
and their subsequent prediction $p^e_{1017}$ for price in period 1017 decreased in comparison with their previous forecast $p^e_{1016}$, but was still highly above the fundamental price. Agents noticed that the price gained a negative trend, but did not expect it to fall to or below the fundamental price immediately. On the other hand, the robotic trader is always trading as if the next price will be at the fundamental level. As a result, the GA agents after the reversal of the bubble became pessimistic in absolute terms, but still optimistic relatively to the robotic trader. Figure 3.8d shows the sign of the robotic trades.\footnote{Notice that this is not the index based on five previous trade signs.}

The construction of the market is that the agents forecast two-period ahead, and their information set spans until the previous period. This means that they form their forecast of $p_{1015}$, i.e. the peak of the bubble, based on the friends’ behavior until period 1013, which roughly corresponds to the moment when the price (following the previous crisis) surpasses the fundamental level and the GA agents finally become relatively more optimistic than the robotic trader and start to buy the asset. Furthermore, the peer effect is based on an index with a non-trivial memory (of 5 periods). We observe that the agents’ mood indices becomes positive only around the moment of the bubble burst (Figure 3.9c), that is the agents acquire reputation of full optimism when the market already crashes or is about to crash. This lag is apparent in Figure 3.9d, which shows for every GA agents the mood index averaged between her friends she observes.
at period $t$, that is averaged over friends and over periods $t-6$ until $t-1$, which she uses to predict the next price $t+1$. The GA agents can trade quite efficiently, but their reputation is ‘sticky’ and hence reflects the past, not the contemporary market conditions. A contrarian attitude is therefore natural.

To what extent is this driven by the robotic trader? Because the robotic trader has such a firm belief about the next price, the GA agents are likely to take similar (long or short) positions once the market is far from the fundamental: their individual price forecasts remain heterogeneous, on ‘the same side’ (below or above the fundamental). On the other hand, without the robotic trader the GA agents would be more likely to trade in a more diversified fashion (some would buy, some would sell), and on average their reputation could be less ‘sticky’. Nevertheless, agents who have many friends would still likely observe ‘sticky’ mood (notice the difference between individual mood indices (Figure 3.9c) and the observed ones (Figure 3.9d)). Furthermore, even without the robotic trader the lag of the index mood is apparent (especially after the market reversals).\textsuperscript{18} It means that unless the price oscillations can take a relatively long period, the agents will simply never have time to acquire a positive (negative) mood index during the bubble (crisis) build-ups.\textsuperscript{19} Notice that the current model generates fast oscillations despite the stabilizing influence of the robotic trader. We leave it for future inquiries to study the robustness of this phenomenon in alternative market structures.

Observation 7. **Contrarian strategies add momentum to the trend reversal around the tipping points of bubbles and crises. Through interaction with agents’ strong trend following behavior, this causes price oscillations with larger amplitude.**

As discussed for the markets without a network, the reason for price trend reversals is that once the price diverges sufficiently away from the fundamental, the robotic trader halts its current trend. This causes dis-coordination and a tipping point: agents start to experiment with their heuristics, while the robotic trader insists on pushing the price back to the fundamental. Therefore, the prices turn around and agents quickly pick the new trend up, causing a new phase of the bubble-crisis cycle.

The contrarian strategies work in the same direction. For example, during a bubble build-up the agents slowly become optimistic. As discussed, this happens not fast enough to make the agents learn herding strategies, since the agents become fully optimistic close to the tipping point of the bubble. Thus their heuristics remain strictly

\textsuperscript{18}Simulations, in which a cap on price change replaced the robotic trader as a stabilizing factor, also indicate strong contrarian behavior; this seems to be caused by the discussed lag of the mood index.

\textsuperscript{19}Sample simulations indicate that in this setup the specific value of the memory length $\tau$ plays little role.
contrarian, but the optimism build-up around the tipping point plays a crucial role in the bubble crash.

Once the robotic trader stops the price growth, we see in the price expectation heuristic (3.13) that the price trend becomes unimportant, while the negative $\gamma$ trust index together with the newly established optimism among friends means that the agents are likely to forecast lower price (observed optimism times the contrarian attitude yields an additional negative element in the pricing forecast heuristic). Given the positive feedback between the predictions and price, the initial price drop is therefore more severe relatively to the case without a network, in which no such contrarian attitude can emerge.

One can observe this by comparing the first price drop after the bubble burst: $p_{1006} - p_{1015} \approx -4.65$ for the sample fully connected market (Figure 3.8a) is much larger than $p_{1007} - p_{1006} \approx -0.67$ for the sample no network market (Figure 3.4a), which results for a sharper decrease of the corresponding price forecasts. The increased (in absolute terms) initial trend after bubbles burst (or crises finish) makes the agents predict larger price change, which reinforces the size of the trend. This can be mitigated by the robotic trader only once the price deviation is sufficiently larger in comparison with the market without a network, which makes the realized oscillations wider.

Notice that this further confirms the discussed intuition of the contrarian strategies. Agents around the peak of the bubble (crisis) are considered optimistic (pessimistic). They also use contrarian strategies, which reinforces the market reversal (in contrast to the observed mood of the friends), and makes the contrarian attitude self-fulfilling.

The general conclusion about the network effect on the market is therefore only partially in line with the popular belief. The GA agents, if endowed with additional information about friends’ behavior, learn contrarian beliefs. This actually implies larger bubbles and crashes, while not disturbing the high level of forecasting coordination — the two phenomena that some economists would in fact associate with herding (Shiller and Pound, 1989).

**Finding 2.** In markets with a relatively fast bubble-crisis cycle, the history of agents’ trading decisions lags behind the contemporary market conditions. As a result, agents have an incentive to learn contrarian strategies. This has a negative impact on price stability. Because the agents’ reputation catches up with the market conditions just before the tipping point, the contrarian strategies imply that the turning points of the price cycle generates stronger price reversal. This larger initial price trend is in turn reinforced by the agents’ trend extrapolation behavior, which makes it more difficult for the robotic trader to stabilize the market.
3.4. Networks of six agents

Central agents:

Agent 3

Agent 1

Agent 1

Non-central agents:

Agent 1

Agent 3

Agent 3

Two connected clusters
core-periphery
Star

Figure 3.10: Monte Carlo study (1000 markets) of two connected clusters, core-periphery and star networks: time evolution of trust index $\gamma$ of specified agents. Median represented by a red line and 95% CI represented by blue lines.

3.4.3 Learning in asymmetric networks

Observation 8. An asymmetric position in the network has no direct effect on herding. Instead, agents with fewer friends experiment with relatively weaker contrarian strategies.

Intuition suggests that agents with a unique position in a network — like a center of a star network — should receive more attention and thus play an important role in coordination. However, our model predicts so only for some networks. Consider three networks: two connected clusters, core-periphery and star. In all of them, one can distinguish central and non-central agents with interesting asymmetric positions. In the two connected clusters, agents 1 and 3 belong to the same cluster, but agent 3 also links to the second cluster (see Figure 3.1d). In the core-periphery network, a reverse case holds: agent 1 lies in the core and links to a periphery agent, while agent 3 is such a periphery agent (see Figure 3.1e). The most extreme is the situation of the star network, where agent 1 is the hub of the star, while agent 3 is a typical edge of the star (see Figure 3.1f).
Figure 3.10 shows MC results for the trust given by these agents across time. Again the 95% CI are wide (demonstrating erratic behavior of these networks), but there is a clear pattern in terms of the median agent across the simulations. Regardless of the network, the central agent is always likely to use strict contrarian strategies, just as was the case of the agents from regular networks. Non-central agents are also contrarian. However, their median trust index can be much larger: instead of a low $\gamma \approx -0.9$ for the case of the fully connected network, the median non-central agent 3 in star (core-periphery) uses a weaker contrarian $\gamma \approx -0.6$ ($\gamma \approx -0.75$).\(^{20}\) This is not the case for the non-central agent 1 in the two connected clusters network, who uses a strong contrarian strategy with $\gamma \approx -0.9$. This indicates that a more central position on its own does not guarantee higher levels of received trust.

What we observe instead is agents experimenting with relatively higher trust levels if they have fewer friends. Specifically, the edge agents in the star network, and the periphery agents in the core-periphery network, have only one friend; and their median trust level is visibly higher (even if still negative) in contrast to other agents.\(^{21}\) This result will be more apparent in the large networks, and has a natural intuition.

In the setup of our model, the agents cannot distinguish between their friends and look only at the average sign of their friends’ mood. On the other hand, the agents remain heterogeneous (see the previous discussion): despite in general similar price forecasts, they can have quite different realized moods over time (see lemma 1). It means that a ‘popular’ agent (with many friends) often has to wait longer to observe ‘sharp’ consensus among her friends, whereas ‘unpopular’ agents are more likely to observe outliers, which can be useful around the tipping points. For instance, consider the sample fully connected network (Figure 3.8) and compare the sharply changing individual mood index (Figure 3.9c) with much smoother observed friends’ mood index (that is, the average mood index of five friends, Figure 3.9d). Therefore the agents with fewer friends are typically contrarian, but are also willing to experiment more when bubbles and crises regimes brake down.

A clear suggestion for future studies follows. First, the above reasoning does not have to hold when the agents can distinguish between their friends.\(^{22}\) Second, this is an important insight when studying models with endogenous network formation. Both

\(^{20}\)In practice this means that the agents have low typical value of $\gamma$, but are willing to temporarily experiment with it to a larger extent than other agents.

\(^{21}\)One can see that as well by analyzing networks with small clusters, which are not presented in this chapter.

\(^{22}\)As a robustness check, we run some simulations where agent $i$ can attach different trust levels $\gamma_{i,j}$ to her different friends $j$. This does not seem to change the qualitative outcome of contrarian strategies being learned by the agents, but further studies should investigate this manner in a systematic fashion.
Figure 3.11: Monte Carlo study (1000 markets) of two connected clusters, core-periphery and star networks: time evolution of dis-coordination measure (3.14) (median represented by a red line and 95% CI represented by blue lines); distribution of the long-run standard deviation.

issues demand further theoretical and experimental work.

Observation 9. Overall coordination is the same regardless the shape of the network, including whether it is symmetric or not.

Figure 3.11 presents the MC results for the stability of the three asymmetric networks: the two connected clusters, the core-periphery and the star, namely the 95% CI and median dis-coordination over time (top panel) and SD of the long-run prices over 1000 markets for each network. There is a clear pattern visible. No difference emerges in terms of coordination (which also looks like the coordination in any market with a symmetric network). However, across the three networks the star market is

\[23\] The typical correlation of the individual forecasts is also comparable between all considered net-
likely to experience higher long-run SD of prices, while the two other networks are much more similar. Furthermore, the star network never stays in the stable attractor, which can happen in the case of other asymmetric networks. This indicates that the star network generates unique dynamics in comparison with all the other non-empty networks, which yield more comparable results. We will see a similar pattern in the large networks.

Finding 3. In asymmetric networks, more herding can emerge. This is driven by the fact that agents with fewer links find it easier or more useful to experiment with relatively higher trust. With the exception of the unique dynamics of the star network, this does not influence the overall market stability or coordination.

3.4.4 Profits and utility

Under RE, the expected profits are equal to zero. The intuition of this result follows from the arbitrage argument: the fundamental price $p'$ balances the asset revenue (the dividend $y$ and the resale gain $p_{t+1}$) and the opportunity cost ($R p_t$). Formally, under RE if $p'_{t+1} = \mathbb{E}\{p_{t+1}\} = p'$ for every period $t$ and disregarding the price shock $\eta_t$, the asset return (3.1) becomes

$$\mathbb{E}\{p_{t+1}\} = p' + y - R p' = y - r p' = 0,$$

where the last equality holds because $p' = y/r$. This further implies that the individual demands (3.3) are also equal to zero, since $z_{i,t} = \rho_{i,t+1}/(a \sigma_a^2)$.

The GA model predicts price oscillations and forecasting heterogeneity. This, in contrast to RE, implies non-trivial trades and asset returns. Notice that furthermore the GA agents are maximizing a trade-off between the asset return and the risk. It is therefore important to understand the model profit and utility distribution.

Figure 3.12 shows the MC distribution of average profits for three networks, circle, fully connected and star, for the GA agents (left panel) and the robotic trader (right panel). Around 95%.

Notice that the law of motion of the economy and hence the realized prices, as well as the GA agents learning problem (forecasting efficiency) do not depend on the relative risk aversion factor $a \sigma_a^2$. Thus we have not specified its value in the previous discussion. On the other hand, as evident from equations (3.2) and (3.3), the total demand and the utility of GA agents are both a linear function of the inverse of the risk aversion factor. The specific choice of $a \sigma_a^2$ therefore can only scale the numerical value of the realized profits and utility, without changing their qualitative behavior. In the presented simulation outcomes, we used normalized $a \sigma_a^2 = 1$. 

works and around 95%.

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works and around 95%.
Figure 3.12: Monte Carlo study (1000 markets) with circle, fully connected and star networks: time evolution of average profit of the GA agents and the robotic trader (median represented by a red line and 95% CI represented by blue lines).

Panel). Average profit of agent $i$ is defined as

$$
\bar{\pi}_{t,t} = \frac{1}{t} \sum_{s=1}^{t} \rho_{t+1} z_{i,t}.
$$

Because the robotic trader has a constant price expectation and the market cycle is symmetric, his average profit quickly converges to zero, with narrow 95% CI. On the other hand, GA agents seem to be trading quite poorly: regardless of the network, their average profit quickly becomes negative (including the upper bound of 95% CI).

Notice that the profits in a given period do not sum up to the dividend, because we include the opportunity cost of the forfeit secure interest rate into the asset return. In fact, the total economic profit cannot be positive unless the market diverges to an infinite price, which resembles the transversality condition in the RE infinite time-horizon solution.
This does not mean that the GA agents are irrational, however. Their trading is dictated by aversion towards risk. The pricing equation (3.9) was defined under the assumption that the agents have myopic mean-variance preferences, trading conditional on their beliefs about the subsequent price. Therefore, the same utility based on the robotic trader’s decisions can be used as a reference point for the performance and learning efficiency of the GA agents. Specifically, we focus on the average realized utility

\[ \bar{U}_{i,t} = \frac{1}{t} \sum_{s=1}^{t} \rho_{t+1} z_{i,t} - 0.5 z_{i,t}^2. \]

Figure 3.13 shows that the GA agents, regardless of the network, obtain much higher...
utility than the robotic trader. In fact, after around 1000 period the lower bound of the 95% CI of their average utility is higher than the median average utility of the robotic trader.

This observation has a simple interpretation. Because of his constant price expectations, the robotic trader has on average zero profit, but also takes extremely risky positions around the market reversals, *i.e.* when he also becomes the most active. GA agents forfeit part of the profit and hence increase their utility by avoiding excess risk. This is particularly clear for the case of without networks. Figure 3.14 shows the evolution of average profits and utilities in the sample unstable no network market, which was presented before in Figures 3.2c and 3.4. We observe that initially the prices are stable, which corresponds to inactive robotic trader and individual trades and profits close to zero. However, once the market switches to the unstable attractor, robotic trader tries to push the price back to the fundamental, while GA agents follow price trends. As a result, the relative average profit of the robotic trader increases at the expense of his relative utility.

**Finding 4.** *GA agents, in comparison with the robotic trader, obtain lower economic profits, but also higher utility. The reason is that instead of predicting the fundamental price, they follow the market cycle and thus avoid substantial risk.*

### 3.5 Large networks

In this section we present the results for the sample simulations of the large network (with 50 to 1000 agents). We first show the aggregate dynamics and then discuss individual learning. One of the most important findings of this analysis is that the large networks (even with hundreds of agents) generate similar market dynamics as
the small ones, which suggests that the specific network architecture or size is less significant than the existence of links between the agents in the first place.

3.5.1 Impact of the network on price stability

**Observation 10.** Network size has a stabilizing effect on the prices only for relatively small networks and past a certain threshold plays no significant role.

Figure 3.15 shows the long-run price SD of regular and non-regular networks as a function of their size (see section 3.3 for definition). In comparison with the networks of six agents, large regular networks are marginally more stable (with the markets without a network being the sole exception). For example, the star network of 50 agents has price SD equal to $SD_p = 15.657$ (Figure 3.15a), which is below the price SD for the bulk of star networks with 6 agents (see bottom right panel on Figure 3.11). Above 100 agents, however, the network size hardly has an effect on price volatility.

![Figure 3.15: Standard deviation of price in periods 101 – 25,000 for different network architecture and size.](image)

**Observation 11.** Specific network architecture (including its density) plays little role in price volatility, and is important only for extreme cases, namely no network markets and star networks.

Another interesting observation from Figure 3.15 is that, with the exception of markets without a network, the long run price SD is similar between the network structures (both regular and non-regular), as it is between networks of the same structure and different size (in fact it is difficult to distinguish individual networks in this Figure, since the relevant lines almost overlap). In all these cases, the price SD falls into a narrow interval $SD_p \in [14.5, 15.5]$, despite the networks having different density and other relevant measures. We will see below that the reason for this is that the agents
from different networks learn comparable behavior, especially in terms of price trend extrapolation.

As mentioned above, the exception is the no network case. It has price SD $SD_p \approx 8$ half in magnitude of other networks, which is consistent with the findings for the small networks. This shows that the contrarian behavior, absent in the case without a network, gives an additional momentum to the price trends, implying larger oscillations. The GA model without a network is nevertheless unstable in comparison with the RE benchmark, according to which $SD_p = 0.1 \ll 6$.

**Finding 5.** Specific network size and architecture has a negligible effect on price stability for large enough networks. The exception is the case without a network, which is more stable than markets with any type of a network.

### 3.5.2 Impact of the network on individual behavior

**Observation 12.** Regardless of the network size, agents focus on strong trend extrapolation rules. They continue to experiment with the specific trend coefficient and this learning does not settle down.

In almost all considered networks (both in terms of architecture and size), agents on average use high trend extrapolation coefficients with $\beta \approx 0.85$ (where the specific value does not seem to differ substantially between networks). Figure 3.16 gives the average $\beta$ for all networks and network sizes. Furthermore, there is no convergence. The SD of $\beta$ is close to 0.55 in all networks, consistent with the small networks outcomes: agents typically use very high $\beta$, but during market reversals they experiment with lower values of the trend extrapolation coefficient. A noticeable exception are agents from the no network markets of small size (up to 100 agents), who do focus on higher $\beta \approx 1$. It means that in the positive feedback type markets, specific network structure has little effect on the emerging trend following behavior.

**Observation 13.** Agents in general use strong contrarian strategies, regardless of the specific network architecture or size. However, the less friends an agent has, the more she is willing to experiment with relatively higher trust levels.

Across all networks, the average trust given by agents is low, with (roughly) $\gamma \in [-0.7, 0.9]$. Nevertheless, there is some variability across time and between agents, as seen in relatively high standard deviations of the relevant trust measures. Furthermore, the number of connected friends has a clear and negative effect on the trust level. The most striking illustration of this effect can be seen in the case of star networks, were the
central agent (regardless of the actual network size) on average uses $\gamma \approx -0.8$, but the other agents prefer $\gamma \approx -0.3$, which is close to a neutral strategy (see Figure 3.17a).

In all other networks with a diversified number of friends per agent, more popular agents have lower average trust, though this effect becomes negligible once agents have more than a dozen of friends. Figure 3.18 shows the average trust index $\gamma$ for the sample non-regular networks, as a function of network size and agent’s number of friends. We observe that the effect of network size is small, but the number of friends is important. Furthermore, Figure 3.17b shows the average $\gamma$ as a function of the number of friends for these networks with 1’000 agents. The clear pattern is that the more popular agents have lower average trust index, with a decreasing marginal effect of the number of friends. Another interesting observation is that there is little difference between the networks. Indeed, the lines describing this effect for different network architectures almost coincide on one hyperbola, and further MC would be likely to show that the relevant distributions of the trust index are indistinguishable.

We argue that the reason for this outcome is in line with Finding 3. Namely, as discussed in the previous section, the more friends an agent has, the smoother over time
becomes the average friends mood she observes. As a result, it more closely follows the market cycle with a substantial lag. At any point, the mood index of many agents is more likely to represent the overall market mood in the past, before the latest market reversal. Therefore, the agent with more friends has a higher incentive to remain conservative, unwilling to experiment with strong contrarian attitude. The results for large networks are consistent with this interpretation.

**Finding 6.** Agents tend to use strong price trend extrapolation and contrarian heuristics. They experiment with the specific heuristic parametrization after the market reversals. Furthermore, agents with fewer friends are more likely to experiment with trust level, since they are more likely to observe outlier behavior.

To sum up, embedding agents in a network has a strong effect on the aggregate outcomes: stronger price oscillations occur. However, specific network architecture is not important, because the emerging learning is similar between the networks.

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**3.5. Large networks**
3.6 Conclusions

In this chapter we investigate a financial market, in which agents need to predict the price of an asset two-periods ahead. They are placed in a fixed information network, and use a simple general forecasting heuristic, which contains adaptive and trend extrapolation expectations, and an additional term of (dis-)trust towards friends’ past trading decisions. The agents independently optimize the specification of their heuristics by Genetic Algorithms. This gives a model of endogenous learning of price forecasting and herding/contrarian behavior. Our main findings are: (i) networks destabilize the markets; (ii) the Genetic Algorithm agents learn to extrapolate the trend; and (iii) they learn to use contrarian strategies, because the observed pessimism/optimism of their friends lags behind the cyclical market dynamics.

Information networks play a crucial role for real financial investors, but it is not clear how they affect market stability and efficiency. From the perspective of the Rational Expectations framework, information flows can help the agents to converge to the fundamental equilibrium, but if the agents have no private information, networks should play no role in the equilibrium itself. On the other hand, many behavioral economists, in line with the popular opinion, identify networks as one of the reasons for herding and ‘animal spirits’, which enables the agents to coordinate on non-fundamental, self-reinforcing price oscillations. Here herding is understood as following the opinion of friends (or maybe the general market opinion) instead of ones private beliefs or information. Empirical investigations add a twist to the theoretical puzzle: market data gives no clear indication whether herding is a popular strategy, while experiments identify contrarian strategies as a more common behavioral pattern.

In order to shed some light on this puzzle, we design an agent based network model for a simple market of financial asset. Every agent can independently learn whether to follow the observed price trend and in addition, whether to trust the past decisions of her friends (increase or decrease price forecast, if her friends were optimistic). We investigate small networks of six agents to obtain basic understanding of such learning dynamics, and hence we study networks of fifty to one thousand agents with architectures ranging from regular through random to small world topologies.

The main outcomes of the model are the following. First, information networks destabilize the market. Without information flows, the model exhibits two types of attractors, the fundamental solution and erratic price oscillations around the fundamental. Once the agents are positioned in an information network of any architecture or size, the stable fundamental steady state attractor disappears and market repeatedly over- and under-prices the asset in a smooth cycle of bubbles and crashes. Second,
agents learn to extrapolate the price trend regardless of the network, which makes their price forecasts well coordinated. Third, despite the large degree of coordination in terms of realized price forecasts, agents learn contrarian heuristics. This is because the mood of friends, which the agents observe, is ‘sticky’: it represents past decisions that were made during a different part of the market cycle, and so are different from what is rational in the present. For example, after a bubble crashes, agents should expect the price to decrease, but they remember their friends being optimistic just before the market collapsed. Fourth, the Genetic Algorithms (in comparison with the robotic trader) take less risky trading position, which implies lower profit, but higher realized utility. Fifth, the specific network architecture plays little role in market volatility (past network size of mere hundred agents) or emerging learning. The heuristics do not settle, since during market reversals the agents experiment with their specification. These dynamics resemble real markets: during bubbles and crashes financial agents just follow the current trend, but ‘panic’ around bubble/crash tipping points. Finally, agents with fewer links experiment with relatively higher trust during market reversals, as a natural consequence of the fact that they cannot distinguish between their friends.

The results of our model offer a good interpretation to many empirical and experimental findings. First, the agents learn contrarian behavior, in line with the findings from experiments. Second, the agents remain well coordinated despite the contrarian attitude, which explains why indirect measures for market data may (mistakenly) point towards herding. It also shows that the popular belief about herding (that it is a driving factor of price oscillations) may be wrong: coordination occurs despite agents’ lack of trust towards each other, since they converge to similar forecasting rules of thumb.

Our investigation is based on the specification of two crucial elements: asset market and networks. These choices are independent from the Genetic-Algorithms-based learning itself. In principle, our model can be used for other financial regimes or information networks. Indeed, many of our results should be tested in other economic environments. For example, contrarian behavior emerges because the agents are not able to acquire reputation of optimism (pessimism) before the market bubble crashes (crisis ends). This relies on the fact that the financial market, which we have used, allows for relatively fast price oscillations. Furthermore, the outcomes of our simulations seem to be robust against specification of some key parameters of the model (including the allowed trust), nevertheless further experimental work could fine-tune the model or show its limitations. The model should therefore be thought of as a benchmark for future theoretical and laboratory inquiries.
CHAPTER 3. NETWORKS OF HETEROGENEOUS EXPECTATIONS

Appendix 3.A  Rational solution

Proposition 1. The network \( I \) has no effect on the fundamental RE solution.

Proof. Under RE, at every period \( t \) the agents have homogenous price expectations, and by extension homogenous demand schedules (3.3) and realized demands. The agents know that and so can infer the realized demands (and the mood indices) of all agents in the economy. Since there is no private information, the information set of every agent is the same and publicly known, hence the network cannot provide any additional information.

Proposition 2. Under Rational Expectations, the fundamental price (3.6) defined as
\[
p^f = \frac{y}{r}
\]
is the unique stationary steady state such that the predictions are constant over time (in expected terms) and model consistent, namely \( \hat{p}_{t+1} = E_t\{p_{t+1}\} = E_{t-1}\{p_t\} \) for every period \( t \).

Proof. Recall that the market clearing (3.9) gives the price \( p_t \) as a linear function of price expectations in the next period
\[
p_t = \frac{\hat{p}_{t+1} + y}{R} + \eta_t.
\]
In expected terms, model consistent predictions in a stationary steady state imply thus the predictions must be homogenous and solve the equation
\[
(3.23) \quad p^* = \frac{p^* + y}{1 + R},
\]
which implies \( p^* = \frac{y}{r} \).

Proposition 3. Explosive price paths are possible RE solutions in the model without additional constraints on the price. A price cap \( \Pi > p_t \) reduces the set of RE equilibria to the fundamental solution.

Proof. Assuming model consistent predictions, agents are able to ‘guess’ the next price \( p_{t+1} \) (less the random shock \( \eta_{t+1} \)). Thus, the market clearing equation requires
\[
(3.24) \quad p_t = \frac{(1-n_t)p_{t+1} + n_t p^f + y}{R},
\]
where the share of the robotic trader \( n_t \) is defined by equation (3.7). It is easy to see (cf. Hommes et al., 2005) that without the robotic trader, the prices could lie on an explosive path with growth rate \( R \). However, the presence of the robotic trader
implies that explosive paths have to grow even faster to ‘outweigh’ the robotic trader. Specifically

\[
(3.25) \quad p_{t+1} = \frac{R p_t - (1 - \exp(-\phi |p_{t-1} - p^f|)) \cdot p^f - y}{\exp(-\phi |p_{t-1} - p^f|)}.
\]

Because this non-linear dynamic system is analytically cumbersome and furthermore discontinuous exactly at its steady state,\(^{26}\) we present the following proof of the system’s instability. Recall that by definition of the fundamental solution, \(y = r p^f\). Consider now a case such that \(p_t > p^f\) and furthermore \(p_{t-1} \neq p^f\), which implies \(n_t > 0\). It follows that

\[
(r + n_t)p_t > (r + n_t)p^f \\
[(1 + r) - (1 - n_t)]p_t - y - n_t p^f > 0 \\
R p_t - n_t p^f - y > (1 - n_t)p_t \\
p_{t+1} > p_t.
\]

Symmetric proof shows that if \(p_t < 0\) and \(p_{t-1} \neq 0\), \(p_{t+1} < p_t\). In words, if the price diverges from the fundamental equilibrium under model-consistent predictions for two periods \(t\) and \(t - 1\) (disregarding the price shocks), then the agents predict that the price in next period \(t + 1\) will diverge even more from the fundamental and in the same direction as it happened in period \(t\).

This implies that price time paths are monotonic for every period \(s\) subsequent from \(t - 1\), \(s \geq t\). It is easy to see that in the limit the prices will diverge to infinity or negative infinity, since the term \(\frac{R}{1 - n_t} > 1\) is growing over time. Notice there is an infinite number of such explosive solutions.

The infinite price decline is impossible due to the natural non-negativity constraint. In a similar vein, infinite price growth is curbed if there is an additional price cap \(p_t < \Pi\)

\[\square\]

**Appendix 3.B  Proof of Lemma 1**

**Proof.** Recall that the optimal demand of an agent \(i\) is

\[
(3.26) \quad z_{i,t} = \frac{p^e_{i,t+1} + y}{a_\sigma^2} - \frac{R}{a_\sigma^2} p_t.
\]

\(^{26}\)Specifically, the robotic trader share \(n_t\) is a function of absolute price deviation from the fundamental solution.
In the same manner, demand of the robotic trader is simply

\[(3.27)\quad z_{ROBO,t} = p^f + y - \frac{R}{a\sigma_a^2} p_t.\]

Finally, the realized price setting the price shock to zero is

\[(3.28)\quad p_t = \frac{\hat{p}_{t+1}^c + y}{R} = \frac{(1-n_t)\hat{p}_{t+1}^c + n_t p^f + y}{R}.\]

Substituting (3.28) into (3.26) we obtain

\[(3.29)\quad z_{i,t} = \frac{\hat{p}_{i,t+1}^c + y - \frac{R}{a\sigma_a^2} \hat{p}_{t+1}^c + y}{R} = \frac{\hat{p}_{i,t+1}^c - \hat{p}_{t+1}^c}{a\sigma_a^2}.\]

It follows that the demand of agent \(i\) at period \(t\) \(z_{i,t}\) is positive only if the price forecast for period \(t + 1\) of that agent is larger than the average (including the robotic trader) forecast of \(p_{t+1}\), namely if \(\hat{p}_{i,t+1}^c > \hat{p}_{t+1}^c\). By extension, the robotic trader will also buy the asset if his prediction (which is the fundamental value) is larger than the average market prediction, that is if the average market prediction is below the fundamental.

\(\square\)

**Appendix 3.C  Equivalence of forecasting and trading peer bias.**

In the model, we specified the peer effect and the corresponding herding/contrarian strategies as the bias to the price forecast. In this appendix we will show that this can be reinterpreted as a demand bias. To be specific, following Chapter 2 the agents in our model forecast the prices with adaptive/trend extrapolation rule (optimized with GA procedure) and hence trade optimally; however, they can also change their price forecast if they observe their friends to be optimistic/pessimistic, which is measured by the trust index \(\gamma \in [1,1]\) in (3.13). We defined herding (contrarian) behavior as \(\gamma > 0\) (\(\gamma < 0\)).

However, one may think that herding or contrarian behavior has nothing to do with price forecasting itself, but rather reflects an additional (possibly irrational) bias to the demand. In other words, the agent has a price forecast, which in our case is generated by the GA optimized heuristic, but in principle could follow any other model, ranging
from fundamental to simple adaptive or naive expectations. The agent hence uses this forecast to compute the optimal demand and only then adds or subtracts an additional quantity to the demand if she follows herding or contrarian strategy.

To see that these two interpretations are equivalent, recall the forecasting heuristic (3.13). Substituting it into the optimal demand schedule (3.3) yields

\[ z_{i,t} = \frac{p_{e,Peer}^{t+1} + y - Rp_t}{a\sigma_a^2} = \frac{\alpha p_{t-1} + (1 - \alpha)p_{i,t-1}^e + \beta(p_{t-1} - p_{t-2}) + \gamma \hat{\Gamma}_{Peer,i,t-1} + y - Rp_t}{a\sigma_a^2} \]

\[ \equiv \frac{p_{e,NoPeer}^{t+1} + y - Rp_t}{a\sigma_a^2} + \gamma \hat{\Gamma}_{Peer,i,t-1}, \]

where \( \hat{\Gamma} = \Gamma / s\sigma_a^2 \) is a constant measuring agents’ sensitivity to the peer effect, \( \gamma \in [-1,1] \) is the trust index as in the main body of the chapter and

\[ p_{e,NoPeer}^{t+1} = \alpha p_{t-1} + (1 - \alpha)p_{i,t-1}^e + \beta(p_{t-1} - p_{t-2}) \]

is the original adaptive/trend extrapolation forecasting heuristic from Chapter 2.

We can see that the demand written as (3.30) exemplifies the above mentioned interpretation: agents forecast the next price with the typical heuristic, which is agnostic to the friends’ behavior; but then the agents have an additional demand bias depending on the trust index \( \gamma \). Consider now a GA model in which the agents use (3.31) to obtain price forecasts \( p_{e,t+1}^{Peer} \) and hence use the herding/contrarian biased demand (3.30). Next, they update \( \alpha \) and \( \beta \) the parameters of the forecasting heuristic (3.31) and the bias \( \gamma \) with the GA procedure, where the performance of a heuristic specification is measured in terms of logit transformation of the hypothetical utility (3.2). To be specific,

\[ V_{i,h,t} = U_{i,h,t+1} + z_{i,t}(p_{e,NoPeer}^{i,h,t+1}) (p_{t+1} + y - Rp_t) - \frac{a}{2\sigma_a^2} z_{i,t}(p_{e,NoPeer}^{i,h,t+1})^2. \]

Now define the realized return on the asset as

\[ \rho_{t+1} \equiv p_{t+1} + y - Rp_t. \]
the chapter (forecasting bias and no quantity bias), and so using (3.33) we can rewrite
\[ V_{i,h,t} - U_{i,h,t+1} = \frac{p_{i,h,t+1}^\text{Peer} + y - Rp_t}{a\sigma_a^2} (p_{t+1} + y - Rp_t) - \frac{a\sigma_a^2}{2} \left( \frac{p_{i,h,t+1}^\text{Peer} + y - Rp_t}{a\sigma_a^2} \right)^2 \]
\[ = \frac{(p_{i,h,t+1}^\text{Peer} - p_{t+1} + \rho_{t+1})\rho_{t+1}}{a\sigma_a^2} - \frac{(p_{i,h,t+1}^\text{Peer} - p_{t+1} + \rho_{t+1})^2}{a\sigma_a^2} \]
\[ = \frac{(p_{i,h,t+1}^\text{Peer} - p_{t+1})\rho_{t+1} + \rho_{t+1}^2}{a\sigma_a^2} \]
\[ - \frac{(p_{i,h,t+1}^\text{Peer} - p_{t+1})^2 + 2(p_{i,h,t+1}^\text{Peer} - p_{t+1})\rho_{t+1} + \rho_{t+1}^2}{2a\sigma_a^2} \]
\[ = \frac{\rho_{t+1}^2}{2a\sigma_a^2} - \frac{(p_{i,h,t+1}^\text{Peer} - p_{t+1})^2}{2a\sigma_a^2}. \]

(3.34)

It follows that the performance of the quantity-biased demand (3.30) is equal to a constant term minus MSE of the forecast-biased heuristics (3.13). Therefore, in practice the two models: in which the peer effect bias appears in the price forecast or directly in the demand, are equivalent, if one properly chooses the sensitivity parametrization of the logit transformation.

Appendix 3.D  Definition of network properties

Consider \( I \) agents, who are placed within an unweighted, symmetric and a-transitive network of friends \( I \). First recall that degree between two agents is defined as the shortest path (sequence of linked agents) between them. The following measures are commonly used to describe the architecture of the network \( I \):

**Number of clusters** We define cluster as a subset of the network such that (1) all the agents in the cluster are pairwise connected (there exists a path of a finite degree between them) and (2) none of the agent is connected with any agent that does not belong to the cluster. In some of the analyzed networks, the agents form ‘non-trivial’ subsets such that there is no link between them. On the other hand, these agents will still interact indirectly, through the market clearing price.

**Diameter** Also denoted as the characteristic degree or the longest path, simply measures what is the longest path between nodes in the network.\(^ {27} \)

**Closeness** This measure is typically set as the average degree: how far away agents are

\(^{27}\) Sometimes unconnected nodes are said to have path equal to infinity. We will define diameter disregarding such unconnected nodes.
on average. The drawback of the average path length is that it cannot properly cope with networks of disconnected clusters, and hence we will follow suggestion of Newman (2003) and use the closeness measure instead. First recall that the network is anti-reflexive, or that we disregard whether an agent is linked with herself or not. Instead, we are interested in what is the typical distance between the agent and the remaining $I - 1$ agents. Thus define

$$\text{(3.35)} \quad Cl = \frac{1}{0.5I(I-1)} \sum_{i=1}^{I-1} \sum_{j=i+1}^{I} d_{ij}^{-1},$$

where $d_{ij}^{-1}$ is the inverse of the shortest path length (degree) between agents $i$ and $j$, or 0 if these two agents are not connected. The closeness measure $Cl$ is an index with a straightforward interpretation: if all the agents are connected (disconnected), $Cl = 1$ ($Cl = 0$).

**Density** Denote the number of links between the agents as $E$. Density is defined as

$$\text{(3.36)} \quad DE = \frac{E}{0.5I(I-1)},$$

which is simply the ratio of realized to potential links: how dense the network is.

**Transitivity** Often described as ‘cliquishness’ or clustering, shows whether the nodes in the network form triangles (‘cliques’): friends of my friends are also friends of mine. Among many formal definitions of this measure, we follow Watts and Strogatz (1998) and specify transitivity as computationally efficient index of the form

$$\text{(3.37)} \quad Tr = \frac{1}{I} \sum_{i=1}^{I} \frac{\text{number of triangles connected to vertex } i}{\text{number of triples centered on vertex } i}.$$)

If the agents are always (never) connected with the friends of their friends, $Tr = 1$ ($Tr = 0$).28

Notice that (3.37) is a local, not a global measure. It shows how close are friends of friends, but not whether the network can be divided into significantly differentiated clusters of ‘sub-networks’. In order to avoid confusion we decided to adopt the name ‘transitivity’ instead of the widely used ‘clustering’.

28Notice that if a vertex $i$ is not a center of any triple, it cannot be a part of a triangle either. In such a case, the element in the sum would be a ratio of zero to zero. Instead, it is simply defined as zero.
Appendix 3.E  Large networks characteristics

![Networks of different sizes](image)

(f) Network properties

**Figure 3.19:** Realized random(4) networks.
### 3.E. Large networks characteristics

**Figure 3.20:** Realized random(16) networks.

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<thead>
<tr>
<th>Network size</th>
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<th>Closeness</th>
<th>Density</th>
<th>Transitivity</th>
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(a) 50 agents
(b) 100 agents
(c) 250 agents
(d) 500 agents
(e) 1000 agents

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(f) Network properties

**Figure 3.21:** Realized rewired(4, 0.01) networks.
3.E. Large networks characteristics

(a) 50 agents  
(b) 100 agents

(c) 250 agents  
(d) 500 agents

(e) 1000 agents

<table>
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<th>Network size</th>
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<th>Closeness</th>
<th>Density</th>
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(f) Network properties

**Figure 3.22**: Realized rewired(4, 0.1) networks.
Figure 3.23: Realized rewired(16, 0.01) networks.
Figure 3.24: Realized rewired(16, 0.1) networks.
Chapter 4

Bubble Formation and (In)Efficient Markets in Learning-to-Forecast and -Optimize Experiments

4.1 Introduction

This chapter investigates the price dynamics and bubble formation in an experimental asset pricing market with a price adjustment rule. The purpose of the study is to address a fundamental question about the origins of bubbles: do bubbles arise because agents fail to form rational expectations or because they fail to optimise their trading quantity given their expectations?

We design three experimental treatments: (1) subjects make a forecast only, and are paid according to forecasting accuracy; (2) subjects make a quantity decision only, and are paid according to the profitability of their decision; (3) subjects make both a forecast and a quantity decision, and are paid by their performance of either of the tasks with equal probability. Under perfect rationality and perfect competition, these three tasks are equivalent and should lead the subjects to an equilibrium with a constant fundamental price. In contrast, we find none of the experimental markets to show a reliable convergence to the fundamental outcome. The market price is relatively most stable, with an upward trend in the treatment where the subjects make a forecast only. There are recurring bubbles and crashes with high frequency and magnitude when the subjects submit both a price forecast and a trading quantity decision.

Asset bubbles can be traced back to the very beginning of financial market, but has not been investigated extensively by modern economics and finance literature. One possible reason is that it contradicts the standard theory of rational expectations
(Lucas Jr., 1972; Muth, 1961) and efficient markets (Fama, 1970). Recent finance literature however has shown growing interest in bounded rationality (Farmer and Lo, 1999; Shiller, 2003) and ‘abnormal’ market movement such as over- and underreaction to changes in fundamentals (Bondt and Thaler, 2012) and excess volatility (Campbell and Shiller, 1989). The recent financial crisis and precedent boom and bust in the US housing market highlight the importance of understanding the mechanism of financial bubbles in order for the policies makers to design policies/institutions to enhance market stability.

It is usually difficult to identify bubbles using data from the field, since people may substantially disagree about the underlying fundamental price of the asset. Laboratory experiments have an advantage in investigating this question by taking full control over the underlying fundamental price. Smith et al. (1988) are among the first authors to reliably reproduce price bubbles and crashes of asset prices in a laboratory setting. They let the subjects trade an asset that pays a dividend in each of 15 periods. Therefore the fundamental price at each period equals the sum of the remaining expected dividends and follows a decreasing step function. The authors find the price to go substantially above the fundamental price after the initial periods before it crashes towards the end of the experiment. This approach has been followed in many studies i.e. Dufwenberg et al. (2005); Haruvy and Noussair (2006); Lei et al. (2001); Noussair et al. (2001). A typical result of these papers is that the price boom and bust is a robust finding despite several major changes in the experimental environment.

Nevertheless, Huber and Kirchler (2012); Kirchler et al. (2012) argue that the non-fundamental outcomes in this type of experiments are due to misunderstanding: subjects may be simply confused by the declining fundamental price. They support their argument by showing that no bubble appears when the fundamental price is not declining or when the declining fundamental price is further illustrated by an example of ‘a depletable gold mine’. Another potential concern is that in these experiments, due to typically short horizon (15 periods), one cannot test whether financial crashes are likely to be followed by new bubbles. It is very important to study consequent boom-bust cycles in asset prices, for example to understand the evolution of the asset prices between the dot-com and the 2007 crises.

The Smith et al. (1988) experiment are categorised as ‘learning to optimise’ (henceforth LtO) experiments (see Duffy, 2008, for an extensive discussion). Besides this approach, there is ‘learning to forecast’ (henceforth LtF) experimental design introduced by Marimon et al. (1993) (see Hommes, 2011, for a comprehensive survey). Hommes

\[ \text{For survey of the literature, see Noussair and Tucker (2013); Sunder (1995).} \]
et al. (2005) run an experiment where subjects act as professional advisers (forecasters) for a pension fund: they submit a price forecast, which is transformed into a quantity decision of buying/selling by a computer program based on optimization over a standard myopic mean-variance utility function. Subjects are paid according to their forecasting accuracy. The fundamental price is defined as the rational expectation equilibrium and remains constant over time. The result of this chapter is twofold: (1) the asset price fails to converge to the fundamental, but oscillates and forms bubbles in several markets; (2) instead of having rational expectations, most subjects coordinate on a price trend following strategies (cf. Bostian and Holt, 2009). Heemeijer et al. (2009) and Bao et al. (2012) investigate whether the non-convergence result is driven by the positive expectation feedback nature of the experimental market in Hommes et al. (2005). Positive/negative expectation feedback means that the realised market price increases/decreases when the average price expectation increases/decreases. The results show that while negative feedback markets converge quickly to the fundamental price, and adjust quickly to a new fundamental after a large shock, positive feedback markets usually fail to converge, but under-react to the shocks in the short run, and over-react in the long run.

The subjects in Hommes et al. (2005) and other ‘learning to forecast’ experiments do not directly trade, but are assisted by a computer program to translate their forecasts into optimal trading decisions. A natural question is what will happen if they submit explicit quantity decisions, i.e. if the experiment is based on the ‘learning to optimise’ design. Are the observed bubbles robust against the LtO design or are they just an artifact of the computerised trading in the LtF design?

In this chapter we design an experiment, in which the fundamental price is constant over time (as in Hommes et al., 2005), but the subjects are asked to directly indicate the amount of asset they want to buy/sell. Different from the double auction mechanism in the Smith et al. (1988) design, the price in our experiment is determined by a price adjustment rule based on excess supply/demand (Beja and Goldman, 1980; Campbell et al., 1997; LeBaron, 2006). Our experiment is helpful in testing financial theory based on such demand/supply market mechanisms. Furthermore, our design allows us to have a longer time span of the experimental sessions, which will enable a test for the recurrence of bubbles and crashes.

The main finding of our experiment is that the persistent deviation from the fundamental price in Hommes et al. (2005) is a robust finding against task design. Based on Relative Absolute Deviation (RAD) and Relative Deviation (RD) as defined by Stöckl et al. (2010), we find that the amplitude of the bubbles in treatment (2) and (3) is much higher than in treatment (1). We also find large heterogeneity in traded quan-
tities than individual price forecasts. These findings suggest that learning to optimise is even harder than learning to forecast, and therefore leads to even larger deviations from rationality and efficiency.

An important finding of our experiment is that in the mixed, LtO and LtF designs we find some repeated ‘super bubbles’, where the price increases to more than 3 times the fundamental price. This was not observed in the former experimental literature. Considering that bubbles in stock and housing prices reached similar levels (the housing price index increases by 300% in several local markets before it decreased by about 50% during the crisis), our experimental design may provide a potentially better test bed for policies that deal with large bubbles.

Another contribution is that, to our best knowledge, we are the first to perform a formal statistical test on individual heterogeneity in forecasting and trading strategies in an asset pricing experiment. In particular, in some trading markets we observe a large degree of heterogeneity in the quantity decision even when the price is rather stable. By examining treatment (3), we also find that many subjects fail to trade at the conditionally optimal quantity given their own forecast.

This chapter is related to Bao et al. (2013) who run an experiment to compare the LtF, LtO and Mixed designs in a cobweb economy. The main difference is that they consider a negative expectation feedback system, for which all markets converge to the RE fundamental price. This chapter is also related to a study by Haruvy et al. (2013) who follow the basic design of Smith et al. (1988), with an additional new issue or repurchase of stocks in order to increase/decrease the supply of stock shares on the market. Theoretically, since the fundamental price in this type of studies is based purely on the dividend process, and irrespective of the size of the share supply, the new issue and repurchase should generate no impact on the asset price. But the results suggest that the price level is actually negative related to the supply of asset. This outcome points in the same direction as the intuition behind the models based on excess supply/demand, which we used in our experiments. The difference is that we keep the asset supply constant in our experiment, and the price change is driven instead by the asset excess demand of the investors (played by subjects).

The chapter is organised as follows: Section 2 presents the experimental design, Section 3 states the research hypothesis of the experiment, Section 4 reports the experimental result, and finally, Section 5 concludes.
4.2 Experimental design

4.2.1 Experimental economy

The experiment is based on an asset market in Brock and Hommes (1998) where the price is determined by excess demand/supply. The agents are assumed to have a simple myopic mean-variance objective function. There are \( I = 6 \) agents, who allocate investment between a risky asset and a risk-free bond. Each agent \( i \) at time \( t \) has an objective function that is increasing in his wealth in the next period \( W_{i,t+1} \), but decreasing with the perceived investment risk \( V_{i,t}(W_{i,t+1}) \). The wealth of agent \( i \) evolves according to

\[
W_{i,t+1} = RW_{i,t} + z_{i,t}(p_{t+1} + y_{t+1} - Rp_t),
\]

where \( R = 1 + r \) is the gross interest rate of the risk-free bond (assumed constant over time), \( z_{i,t} \) is the demand of risky asset by agent \( i \) in period \( t \) (positive sign for buying and negative sign for selling). \( p_t \) and \( p_{t+1} \) are the prices of the risky asset in periods \( t \) and \( t + 1 \) respectively, and \( y_{t+1} \) is the assets dividend paid at the beginning of period \( t + 1 \).

The agent solves the myopic optimisation problem:

\[
\text{Max } z_{i,t}E_{i,t}W_{i,t+1} - \frac{a}{2}E_{i,t}V(W_{i,t+1}),
\]

where \( a \) is a parameter for risk aversion. \( E_{i,t}V(W_{i,t+1}) \) is the conditional expectation by the agent on the the conditional variance of the wealth based on publicly available information. The conditional variance equals the \( z_{i,t}^2 \) times the conditional variance of the excess return per share, \( \rho_{t+1} \) defined by

\[
\rho_{t+1} \equiv p_{t+1} + \bar{y} - Rp_t.
\]

We assume that agents have homogeneous and constant expectations on the excess return, i.e. \( E_{i,t}V(\rho_{t+1}) \equiv \sigma^2 \). This leads to \( E_{i,t}V(W_{i,t+1}) = \sigma^2 z_{i,t}^2 \). Let \( \rho'_{i,t+1} = E_{i,t}p_{t+1} \). For simplicity, we assume that the dividend follows an i.i.d. stochastic process,

\footnote{This assumption is made for analytical tractability. Brock and Hommes (1998) noted that the heterogeneity in the price expectations in the expected return of the objective function can lead to heterogeneity in the variance as well, but they ignore it as it is a second order effect. Nelson (1992) provides some justification that in such a model there is typically more disagreement on the mean than the variance of the return. Moreover, this implication assumption is particularly useful in a laboratory experiments, where it would be almost impossible for the subjects to solve the problem if heterogeneity in expected variance is introduced.}
where the unconditional expected value \( E(y_t) = \bar{y} \). The objective function of the agent can be rewritten as:

\[
U_{i,t}(z_{i,t}) = RW_{i,t} + (p_{i,t+1}^e + \bar{y} - Rp_t)z_{i,t} - \frac{a\sigma^2 z_{i,t}^2}{2}
\]

where agents need to choose \( z_{i,t} \) optimally based on their prediction on \( p_{t+1} \). The optimal solution of this quadratic function is shown by

\[
z_{*i,t} = \frac{\rho_{t+1}^e}{a\sigma^2} = \frac{p_{i,t+1}^e + \bar{y} - Rp_t}{a\sigma^2},
\]

where \( \rho_{t+1}^e \) is the conditional expectation on the excess return in the next period.

The market price is set by a market maker using a simple price adjustment mechanism (Beja and Goldman, 1980),

\[
p_{t+1} = p_t + \lambda (Z_D^t - Z_S^t) + \varepsilon_t,
\]

where \( \varepsilon_t \sim N(0,1) \) is a small i.i.d. idiosyncratic shock, \( \lambda > 0 \) is a scaling factor, \( Z_S^t \) is the exogenous supply and \( Z_D^t \) is the total demand. This mechanism guarantees that excess demand/supply increases/decreases the price.

For simplicity, the exogenous supply \( Z_S^t \) is normalised to 0 in all periods. We take \( R\lambda = 1 \), specifically \( R = 1 + r = 21/20 \), \( \lambda = 20/21, a\sigma^2 = 6 \), and \( \bar{y} = 3.3 \). The price adjustment based on aggregate individual demand thus takes the form of

\[
p_{t+1} = p_t + \frac{20}{21} \sum_{i=1}^6 z_{i,t} + \varepsilon_t.
\]

For an optimising agent and the chosen parameters, the individual optimal demand (4.5) equals

\[
z_{*i,t} = \frac{\rho_{t+1}^e}{a\sigma^2} = \frac{p_{i,t+1}^e + 3.3 - 1.05p_t}{6},
\]

Substituting it back into (4.7) gives

\[
p_{t+1} = 66 + \frac{20}{21} (\bar{p}_{t+1}^e - 66) + \varepsilon_t,
\]

where \( \bar{p}_{t+1}^e = \frac{1}{6} \sum_{i=1}^6 p_{i,t+1}^e \) is the average prediction of price \( p_{t+1} \).

\[3\]See e.g. Chiarella et al. (2009) for a survey on the abundant literature about the price adjustment market mechanisms.

\[4\]Heemeeijer et al. (2009) used a very similar price adjustment rule, but the fundamental price is
4.2. Experimental design

temporary equilibrium with point-beliefs of prices. Equation (4.9) represents the price adjustment process as a function of the average individual forecast.

By plugging in the rational expectations condition, namely $ar{p}^{e}_{t+1} = p^f = E(p_{t+1}) = E\left(\frac{20}{21} (\bar{p}^{e}_{t+1} - 66) + \epsilon_t\right)$, $p^f = 66$ is the unique Rational Expectations Equilibrium (REE) of the system. It is also true that $p^f = \bar{y}/r$, namely, the fundamental price is equal to the discounted sum of dividend in infinite horizon. If all the agents have rational expectations, the realised price becomes $p_t = p^f + \epsilon_t = 66 + \epsilon_t$, i.e. the fundamental price plus a white noise, and, on average, the price forecasts are self-fulfilling.

4.2.2 Experimental treatments

Based on the nature of the task and the payoff structure, three treatments are set up:

LtF Classical Learning-to-Forecast experiment. Subjects are asked for one-period ahead price predictions $p^{e}_{i,t+1}$, based on which the realised price is generated according to the price adjustment rule (4.9). The subjects’ reward depends only on the prediction accuracy, defined by (see also Table 4.2 in Appendix 4B)

\[
(4.10) \quad \text{Payoff}_{i,t} = \max\left\{0, (1300 - \frac{1300}{49} (p^{e}_{i,t+1} - p_{t+1})^2)\right\}.
\]

The law of motion of the treatment economy is given by (4.9).

LtO Classical Learning-to-Optimise experiment, where the subjects are asked to decide on the asset quantity $z_{i,t}$. They are not explicitly asked for a price prediction, but can use a built-in calculator in the experimental program to compute the expected asset return $p_{t+1}$ for each price forecast $p^{e}_{i,t+1}$ as in equation (4.3). Subjects are rewarded based on a linear transformation of the realised (profit) utility given by

\[
(4.11) \quad U_{i,t} = \max\left\{0, 800 + 40(z_{i,t}(p_{t+1} + 3.3 - 1.05p_t) - 3z^2_{i,t})\right\},
\]

that is on how close their choice was to the optimal choice regardless of their individual prediction. The law of motion of the LtO treatment is given by (4.7). We add a constant 800 in order to avoid negative payoff.

Mixed Each subject is asked first for his or her price forecast and second for the choice of the asset quantity. In order to avoid hedging, the reward for the whole

\[60 \text{ in a learning to forecast experiment that compares positive versus negative expectation feedback systems.}\]
experiment is based on either Equation (4.10) or Equation (4.11) with equal probability. The law of motion of the treatment economy is given by (4.7), the same as inLtO and does not depend on the submitted price forecasts.

The same payoff scheme for the forecasting task in the LtF and mixed treatments is the same as in the previous LtF experiments. The points in each treatment are exchanged into Euro with the conversion rate 3000 points = 1 Euro.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Notation</th>
<th>Parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Market parametrization</td>
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<td></td>
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<tr>
<td>Subjects</td>
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<td>6</td>
</tr>
<tr>
<td>Risk penalty</td>
<td>$a \sigma_z^2$</td>
<td>6</td>
</tr>
<tr>
<td>Expected value of dividend</td>
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</tr>
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</tr>
<tr>
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<td>Price</td>
<td>$p^f$</td>
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</tr>
<tr>
<td>Excess demand</td>
<td>$z^*(p^f)$</td>
<td>0</td>
</tr>
<tr>
<td>Points per 1 Euro</td>
<td></td>
<td>3000</td>
</tr>
</tbody>
</table>

Table 4.1: Parametrization of the experiment.

Finally, we would like to emphasise that the LtF and LtO treatments are equivalent under the assumption of perfect rationality and perfect competition, because the models of the economy in these two treatments, Equations (4.6) and (4.9) are equivalent.

4.2.3 Liquidity constraints

To limit the effect of extreme price forecasts or quantity decisions in the experiment, we impose the following liquidity constraints on the subjects. For the LtF treatment, price predictions such that $p^e_{t+1} > p_t + 30$ or $p^e_{t+1} < p_t - 30$ are treated as $p^e_{t+1} = p_t + 30$ and $p^e_{t+1} = p_t - 30$ respectively. For the LtO treatment, quantity decisions greater than 5 or smaller than −5 are treated as 5 and −5 respectively. These two liquidity constraints are roughly the same, since the optimal asset demand (4.8) is close to one sixth of the expected price difference. Nevertheless, the liquidity constraint in the LtF treatment was never binding, while under theLtO treatment subjects would sometimes trade at the edges of the allowed quantity interval.\footnote{We also imposed additional constraint that $p_t$ has to be non-negative and not greater than 300. In the experiment, this constraint never had to be implemented.}
4.2.4 Number of observations

Experimental instructions with the computer screen presented to the subjects are shown in Appendix 4.A. The experiment took place on December 14, 17, 18 and 20, 2012 and June 6, 2014 at the CREED Laboratory, University of Amsterdam. 144 subjects were recruited. The experiment employs a group design with 6 subjects in each experimental market. There are 24 markets in total and 8 for each treatment. No subject participates in more than one session. The duration of the experiment is typically about 1 hour for the LtF treatment, 1 hour and 15 minutes for the LtO treatment, and 1 hour 45 minutes for the Mixed treatment.

4.3 Testable Hypotheses

The RE benchmark suggests that the subjects should learn to play the rational expectations equilibrium and behave similarly in all treatments. In addition, a rational decision maker should be able to solve the optimal demand for the asset given his price forecast according to Equation (4.8) in the Mixed treatment. These theoretical predictions can be formulated to the following testable hypotheses:

**Hypothesis 1:** The asset prices converge to the rational expectation equilibrium in all markets;

**Hypothesis 2:** There is no systematic difference between the market prices across the treatments;

**Hypothesis 3:** Subjects’ earnings efficiency (defined as the ratio of the experimental payoff divided by the hypothetical payoff when all subjects play the REE) are independent from the treatment;

**Hypothesis 4:** The quantity decision by the subjects are conditionally optimal to their price expectations in the Mixed treatment;

**Hypothesis 5:** There is no systematic difference between the decision rules used by the subjects for the same task across the treatments;

These hypotheses are further translated into rigorous statistical tests. More specifically, the distribution of Relative (Absolute) Deviation (Stöckl et al., 2010) measures price convergence and differences between the treatments (Hypothesis 1 and 2). Relative earnings can be compared with the Mann-Whitney-Wilcoxon rank-sum test (Hypothesis 3). Finally, we estimate individual behavioral rules for every subject: a simple restriction test will reveal whether Hypothesis 4 is true, while the rank-sum test can again be used to test the rule differences between the treatments.
(Hypothesis 5). Notice that Hypothesis 1 is nested within Hypothesis 2, while Hypothesis 4 is nested within Hypothesis 5.

4.4 Experimental results

4.4.1 Overview

The market prices in each treatment are shown in Figure 4.1 (LtF treatment), Figure 4.2 (LtO treatment) and Figure 4.3 (Mixed treatment). For most of the groups, the prices and predictions remained in the interval \([0, 100]\). The exceptions are the mixed treatment groups 1, 4 and 8 (Figures 4.3a, 4.3d and 4.3h). In the first two of these three groups, prices peaked at almost 150 (more than twice the fundamental price \(p_f = 66\)) and for the last group, the prices reached 225, almost 3.5 times the fundamental price.

As shown by the figures, the market price is the most stable in the LtF treatment, and the most unstable in the Mixed treatment. In the LtF treatment, there is little heterogeneity in the individual forecasts shown by the green dashed lines. In the LtO treatment, however, there is a high level of heterogeneity in the quantity decisions shown by the blue dashed lines. In the Mixed treatment, it is somewhat surprising that the low heterogeneity in price forecasts and the high heterogeneity in quantity decisions coexist.

The dynamics are diversified between the groups and treatments. In addition, convergence to the REE does not seem to occur in any of the treatments. This suggests the hypotheses based on the rational expectations benchmark are likely to be rejected. In the remainder of this section, we will discuss the statistical evidence in detail.

4.4.2 Quantifying the bubbles

We follow Stöckl et al. (2010) to evaluate the size of mispricing and the experimental asset bubbles, using the Relative Absolute Deviation (RAD) and Relative Deviation (RD). These two indices measure respectively the absolute and relative deviation from the fundamental in a specific period \(t\) and are given by

\[
RAD_{g,t} \equiv \left| \frac{p_{g,t} - p_f}{p_f} \right| \times 100\%,
\]

\[
RD_{g,t} \equiv \frac{p_{g,t} - p_f}{p_f} \times 100\%.
\]
Figure 4.1: Groups 1-8 for the Learning to Forecast treatment. Straight line shows the fundamental price $p^f = 66$, solid black line denotes the realised price, while green dashed lines denote individual forecasts.
Figure 4.2: Groups 1-8 for the Learning to Optimize treatment. Each group is presented in two panels. The upper panel displays the fundamental price $p^f = 66$ (straight line) and the realised prices (solid black line), while the lower panel displays individual trades (dashed blue lines) and average trade (solid red line). Notice the different $y$-axis scale for group 7 (picture g).
4.4. Experimental results

Figure 4.3: Groups 1-8 for the Mixed treatment with subject forecasting and trading. Each group is presented in a picture with two panels. The upper panel displays the fundamental price $p^D = 66$ (straight line), the realised prices (solid black line) and individual predictions (green dashed lines), while the lower panel displays individual trades (dashed blue lines) and average trade (solid red line). Notice the different $y$-axis scale for groups 1, 4 and 8 (pictures a, d and h respectively).
where \( p_f = 66 \) is the fundamental price and \( p_g^t \) is the realised asset price at period \( t \) in the session of group \( g \). The average RAD is defined as

\[
RAD_g = \frac{1}{50} \sum_{t=1}^{50} RAD_{g,t},
\]

and it shows the average relative distance between the realised prices and the fundamental, while the average \( \overline{RD}_g \) (defined similarly as in (4.14)) focuses more on the sign of this relationship. Groups with average \( \overline{RD} \) close to zero could either converge to the fundamental (in which case the \( RAD_{g,t} \) is also close to zero) or oscillate around the fundamental (with high \( RAD_{g,t} \)), while positive or negative average \( \overline{RD} \) signals that the group typically over- or underpriced the asset.

The results for average \( \overline{RAD} \) and \( \overline{RD} \) measures are presented in Table 4.4 in the appendix. They confirm that the LtF groups were the closest to, though still quite far from, the REE (with an average \( \overline{RAD} \) of about 9.5%), while Mixed groups exhibited largest bubbles with an average \( \overline{RAD} \) of 36%. Interestingly, LtO groups had significant oscillations (on average high \( \overline{RAD} \) of 24.6%), but centered close to the fundamental price (average \( \overline{RD} \) of 1.4%, compared to average \( \overline{RD} \) of −3% and 16.1% for the LtF and Mixed treatments respectively). LtF groups on average are below the fundamental price and Mixed groups typically overshoot it.

A simple t-test shows that for the LtO and Mixed treatment, as well as for 6 out of 8 LtF groups (exceptions are Markets 7 and 8), the means of the groups’ \( RAD \) measures (disregarding the initial 10 periods to allow for learning) are significantly larger than 3%. Furthermore, for all groups in all three treatments, t-test on any meaningful significance level rejects null of the average price (for periods 11−50, we use the last 40 periods to allow for learning by the subjects in the first 10 periods) being equal to the fundamental value. This result shows negative evidence towards the Hypothesis 1: none of the treatments converges to the REE.

There is no significant difference between the treatments in terms of \( \overline{RD} \) according to Mann-Whitney-Wilcoxon test (henceforth MWWT; \( p \)-value> 0.1 for each pair of the treatments). However, the difference between the LtF treatment and each of the other treatments in terms of \( \overline{RAD} \) is significant at 5% according to MWWT (\( p \)-value= 0.002 and 0.003 respectively), while the difference between the LtO and Mixed is not significant (\( p \)-value= 0.753). This is strong evidence against Hypothesis 2, as it shows that trading and forecasting tasks yield different market dynamics.

\(^6\)3% \( \overline{RAD} \) is equivalent to a typical price deviation of 2 in absolute terms, which corresponds to twice the standard deviations of the idiosyncratic supply shocks, i.e 95% bounds of the REE.
Our results are comparable with the results in Stöckl et al. (2010) (see specifically their Table 4 for the $\overline{RAD}/RD$ measures) in terms of the typical RAD values. Nevertheless, there are some important differences. First, group 8 from the mixed treatment (with $\overline{RAD}$ equal to 120.7%) exhibits the largest relative price bubble among the experimental data. Second, the four experiments investigated by Stöckl et al. (2010) have shorter spans (with sessions of either 10 or 25 periods) and so typically witness one bubble. Our data shows that the mispricing in experimental asset markets is a robust finding. The crash of a bubble does not enforce the subjects to converge to the fundamental, but instead marks the beginning of a ‘crisis’ until the market turns around and a new bubble emerges. This succession of over- and under-pricing of the asset is reflected in our $RD$ measures, which are smaller than the typical ones reported by Stöckl et al. (2010), and can even be negative, despite high $RAD$.

Result 1. Among the three treatments, LtF incurs dynamics closest to the REE. Nevertheless, the average price is still far from the rational expectations equilibrium. Furthermore, in terms of aggregate dynamics LtF treatment is significantly different from the other two treatments, which are indistinguishable between themselves. We conclude that Hypothesis 1 and 2 are rejected.

4.4.3 Earnings efficiency

Subjects’ earnings in the experiment are compared to the hypothetical case where all subjects play according to the REE in all 50 periods. Subjects can earn 1300 points per period for the forecasting task when they play according to REE because they make no prediction errors, and 800 points for the trading task when they play according to the REE because the asset return is 0 and they should not buy or sell. We use the ratio of actual to hypothetical REE payoffs as a measure of payoff efficiency. This measure can be larger than 100% in treatments with the LtO and Mixed Treatments, because the subjects can profit if they buy and the price increases and vice versa. These earnings efficiency ratios, as reported in Table 4.5 in the appendix, are generally high (more than 75%).

The earnings efficiency for the forecasting task is higher in the LtF treatment than in the Mixed treatment (difference is significant at 5% level according to MWWT, $p$-value=0.001). At the same time, the earnings efficiency for the trading task is very similar in the LtO treatment and the Mixed treatment (difference is not significant at 5% level according to MWWT test, $p$-value=0.753).

Result 2. Forecasting efficiency is significantly higher in the LtF than the Mixed treatment, while there is no significant difference in the trading efficiency in treatments LtO
and Mixed. **Hypothesis 3 is partially rejected.**

### 4.4.4 Conditional optimality of forecast and quantity decision in mixed treatment

In the Mixed treatment, each subject makes both a price forecast and a quantity decision. It is therefore possible to investigate whether these two are consistent, namely, whether the subjects’ quantity choices are close to the optimal demand conditional on the price forecast as in Equation (4.8) \((1/6) of the corresponding expected asset return). Figure 4.4 shows the scatter plot of the quantity decision against the implied predicted return (4.3), which we constructed based on the price predictions of each subject, for each period separately.\(^7\) If all individuals made consistent decisions, these points should lie on the (blue) line with slope 1/6.

![Figure 4.4: ML estimation for trading rule (4.15) in the Mixed treatment. Panel (a) is the scatter plot of the traded quantity (vertical axis) against the implied expected return (horizontal axis). Each point represents one decision of one subject in one period from one group. Panel (b) is the scatter plot of a trading rule (4.15) slope (reaction to expected return; horizontal axis) against constant (trading bias; vertical axis). Each point represents one subject from one group. Solid line (left panel)/triangle (right panel) denotes the optimal trade rule \((z_{i,t} = 1/6\rho_{e,i,t})\). Dashed line (left panel)/circle (right panel) denotes the estimated rule under restriction of homogeneity \((z_{i,t} = c + \phi\rho_{e,i,t})\).](fig)

Figure 4.4 brings two interesting observations. First, subjects have some degree of ‘digit preference’, in the sense that the trading quantities are typically round numbers or contain only one digit after the decimal. Second, the quantity choices are far from being consistent with the price expectations. In fact, the subjects sometimes sold

---

\(^7\)Sometimes the subjects submit extremely high price predictions, which in many cases seem to be typos. We exclude these outliers, where the predicted returns on the asset greater than 60 in absolute terms.
(bought) the asset even though they believed its return will be substantially positive (negative).

To further evaluate this finding, we run a series of Maximum Likelihood (ML) regressions based on

\[ z_{i,t} = c_i + \phi_i \rho_{i,t+1} + \eta_{i,t}, \]

with \( \eta_{i,t} \sim NID(0, \sigma^2_{\eta,i}) \). This model has a straightforward interpretation: it takes the quantity choice of subject \( i \) in period \( t \) as a linear function of the implied (by the price forecast) return on the asset. It has two important special cases: homogeneity and optimality (nested in homogeneity). To be specific, subject homogeneity (heterogeneity) corresponds to an insignificant (significant) variation in the slope \( \phi_i = \phi_j (\phi_i \neq \phi_j) \) for any (some) two subjects \( i \) and \( j \). Optimality of individual quantity decisions implies homogeneity with an additional restriction that \( \phi_i = \phi_j = 1/6 \). The constant \( c_i \) shows subject’s \( i \) ‘irrational’ optimism/pessimism bias. Optimality thus corresponds to homogeneity with an additional condition \( c_i = c_j = 0 \) (no agent has a decision bias).

The assumptions of homogeneity and perfect optimisation are tested by estimation of equation (4.15) with the restrictions on the parameters \( c_i \) and \( \phi_i \). These regressions are compared with an unrestricted regression (with \( \phi_i \neq \phi_j \) and \( c_i \neq c_j \)) via a Likelihood Ratio (LR) test. The detailed results can be found in Table 4.8 in Appendix 4.E, but they boil down to one observation: both the assumption of homogeneity and perfect optimisation are rejected by the data. Furthermore, we explicitly tested for \( z_{i,t} = \rho_{i,t}/6 \) when estimating individual rules. Estimations identified 11 subjects (23%) as consistent traders (see footnote 12 for a detailed discussion). We conclude that this is evidence for heterogeneity of individual sophistication. The majority of the subjects identifies the variables that are relevant to their economic decision, but only a minority is able to learn the (mathematically) optimal solution. Instead of optimising in the mathematical sense, most subjects follow simple rules of thumb.

This result has important implications for economic modelling. The RE hypothesis is built on homogeneous and model consistent expectations, which the agents in turn use to optimise their decisions. Many economists find the first element of RE unrealistic: it is difficult for the agents to form rational expectations due to limited

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8We use ML since the optimality constraint does not exclude heterogeneity of the idiosyncratic shocks \( \eta_{i,t} \) and so the model is non-linear. We exclude outliers defined as observations when a subject would predict the asset to have its return higher than 60 in absolute terms. To account for the initial learning, we exclude the first ten periods from the sample. We also drop subjects 4 and 5 from group 6, since they would always pass \( z_{i,t} = 0 \) for \( t > 10 \). Interestingly, these two subjects had non-constant price predictions, which suggests that they were not optimisers.
CHAPTER 4. BUBBLE FORMATION IN LTF AND LTO EXPERIMENTS

understanding of the structure of the economy. But the second part of RE is often taken as a good approximation: agents should make an optimal decision conditional on what they think about the economy, even if their forecast is wrong. Our subjects were endowed with as much information as possible, including an asset return calculator, a table for profits based on the predicted asset return and chosen quantity and the explicit formula for profits; and yet many failed to behave optimally. The design of the randomised payoff excludes risk hedging as a potential reason. The simplest explanation is that individuals in general lack the computation capacity to make perfect optimisations.

**Result 3.** The subjects’ quantity decisions are not conditionally optimal to their price forecasts in the Mixed treatment. We conclude that Hypothesis 4 is rejected.

### 4.4.5 Estimation of individual behavioural rules

Prior experimental work (Heemeijer et al., 2009) suggests that in LtF experiments, subjects use heterogeneous forecasting rules which nevertheless can be typically described a linear First-Order Rule

$$p_{i,t}^e = \alpha_i p_{t-1} + \beta_i p_{i,t-1} + \gamma_i (p_{t-1} - p_{t-2}). \tag{4.16}$$

Two important special cases of (4.16) are pure trend following rule with $\alpha_i = 1$ and $\beta_i = 0$, yielding

$$p_{i,t}^e = p_{t-1} + \gamma_i (p_{t-1} - p_{t-2}), \tag{4.17}$$

and adaptive expectations with $\gamma_i = 0$ and $\alpha_i + \beta_i = 1$, namely

$$p_{i,t}^e = \alpha_i p_{t-1} + (1 - \alpha_i) p_{i,t-1}. \tag{4.18}$$

To explain the trading behaviour of the subjects from the LtO and Mixed treatments, we estimate a general trading strategy in the following specification:

$$z_{i,t} = \begin{cases} \text{constant}_i + \chi_i z_{i,t-1} + \phi_i \rho_{t-1}, & \text{(LtO)} \\ \text{constant}_i + \chi_i z_{i,t-1} + \phi_i \rho_{t-1} + \zeta_i \rho_{i,t+1}, & \text{(Mixed)} \end{cases} \tag{4.19}$$

This rule can capture the most relevant possible elements of the individual trading and has two interesting special cases. First, what we call persistent demand ($\phi_i = \zeta_i = 0$)
4.4. Experimental results

characterised by a simple AR process:

\[
(4.20) \\
z_{i,t} = \text{constant}_i + \chi_i z_{i,t-1}.
\]

A second special case is a return extrapolation rule (with \(\chi_i = 0\)):

\[
(4.21) \\
z_{i,t} = \begin{cases} \\
\text{constant}_i + \phi_i \rho_{t-1} & (\text{LtO}), \\
\text{constant}_i + \phi_i \rho_{t-1} + \zeta_i \rho_{e,t} & (\text{Mixed}). \\
\end{cases}
\]

For every subject from the LtF and LtO treatments, we estimate her behavioral heuristic starting with the general forecasting rule (4.16) or the general trading rule (4.19) respectively. To allow for learning, all the estimations are based on the last 40 periods. Testing for special cases of the estimated rules is straightforward: insignificant variables are dropped until all of the rule coefficients are significant at 5% level.\(^9\)

The same estimation approach is used for the Mixed treatment (now also allowing for the expected return term \(\zeta_i\)).\(^{10}\) The caveat is that the two rules (4.16) and (4.19) are closely linked. Their contemporary idiosyncratic errors are potentially correlated,\(^{11}\) while the trade decision depends on the contemporary expected forecast (if \(\zeta_i \neq 0\)). Since the contemporary trade does not appear in the forecasting rule, the forecast based on the rule (4.16) is assumed to be exogenous and can be estimated independently from the trading rule (4.19). This leaves the potential endogeneity only in the trading heuristic (4.19), and we deal with a simple instrumental variable approach. Estimated price expectations rule (4.16) (again testing for the special cases) yields fitted price forecasts of a subject. Next the trading rule (4.19) is estimated both with the fitted forecasts as instruments; and directly with the reported forecasts. We control for endogeneity by comparing the two estimators with the Hausman test. Finally, the special cases of (4.19) are tested based on reported or fitted price forecasts accordingly to the Hausman test.\(^{12}\)

The estimation results can be found in Appendix 4.E, in Tables 4.6, 4.7 and 4.9 respectively for the LtF, LtO and Mixed treatments.

\(^9\)Adaptive expectations (4.18) impose a restriction \(\alpha \in [0, 1]\) (with \(\alpha = 1 - \beta\)), so we follow here a simple ML approach. If \(\alpha_i > 1\) (\(\alpha_i < 0\)) maximises the likelihood for (4.18), we use the relevant corner solution \(\alpha_i = 1\) (\(\alpha_i = 0\)) instead. We check the relevance of the two constrained models (trend and adaptive) with the Likelihood Ratio test against the likelihood of (4.16).

\(^{10}\)See footnote 7.

\(^{11}\)This can happen e.g. when a subject makes the two decisions at the same time.

\(^{12}\)Whenever the estimations indicated that a subject from the Mixed treatment used a return extrapolation rule (4.21) of a form \(z_{i,t} = \zeta_i \rho_{e,t}\), that is a rule in which only the implied expected return was significant, we directly tested \(\zeta_i = 1/6\). This restriction implies trading consistently with the price forecast, which we could not reject for 11 out of 48 subjects.
In order to quantify whether agents use different decision rules in different treatments, we test the differences of the coefficients in the decision rules with the rank sum test. The LtF treatment can be directly compared to the Mixed treatment according to coefficients in equation (4.16), and the LtO treatment can be compared to the Mixed treatment based on equation 4.19. Since the LtF design implies an optimal trade conditional on the price forecast, one can show that a forecasting rule (4.16) with coefficients \( (\alpha_i, \beta_i, \gamma_i) \) is approximately equivalent to a trading rule with coefficients \( \chi_i = \beta_i \) and \( \phi_i = (\alpha_i + \gamma_i - R)/6 \). Thus, the LtF and LtO treatment are also comparable in terms of adaptiveness or conservatism (former terms), and response to the asset return (latter terms).

First, when the forecasting rules in the LtF and Mixed treatments are compared, we observe rules with a trend extrapolation terms are popular in both treatments (respectively 39 in LtF and 25 in Mixed out of 48). Other subjects would rarely use a pure adaptive rule (4.18) (none and 3 subjects in the LtF and Mixed treatments respectively), but instead a general FOR (4.16) with insignificant \( \gamma_i = 0 \). There were none subjects in the LtF treatment, and only 2 in the Mixed treatment, for whom we could not identify a a significant forecasting rule. The trend coefficients for both treatments are on average close to \( \bar{\gamma} \approx 0.4 \) (i.e. weak trend following, in line with the previous LtF experiments), and not significantly different in terms of distribution (with MWWT \( p \)-value of 0.736). The difference between the two treatments lies in the forecasts conservatism: whereas LtF subjects do prefer an adaptive rule with average coefficient \( \bar{\beta} = 0.56 \), Mixed treatment subjects almost never use their past predictions while forecasting (\( \bar{\beta} = 0.06 \) and the two variables have significantly different distribution with MWWT \( p \)-value close to zero).

Secondly, when the trading rules in the LtO and Mixed treatment are compared, we find that the rules with a term on past or expected return is the dominating rule in both treatments (33 in the LtO and 32 in the Mixed treatment). There are only 12 subjects using a significant AR1 coefficient \( \chi_i \) in the LtO treatment, and 8 in the Mixed treatment. This shows that the majority of our subjects tried to extrapolate the asset return dynamics, which leads to a common behaviour of trend chasing. Nevertheless, no significant trading rule was found for 11 LtO treatment and 8 Mixed treatment subjects. The average demand persistence was equal to \( \bar{\chi} = 0.07 \) and \( \bar{\chi} = 0.006 \), and the average asset extrapolation was equal to \( \bar{\phi} = 0.09 \) and \( \bar{\phi} + \bar{\zeta} = 0.06 \), for the LtO and Mixed treatment respectively.13 The distributions of the two rule coefficients is

13 Notice that the Mixed treatment trading rule (4.19) is a function of both the past and the expected asset return, and the latter is both unobservable in the LtO treatment, while being popular among Mixed treatment subjects. In this sense the two rules are not equivalent. For the sake of comparability,
insignificant with MWWT \( p \)-values of 0.425 and 0.885 for \( \chi_i \) and \( \phi_i / \phi_i + \zeta_i \) respectively.

Finally, in order to evaluate the difference between the LtF and LtO treatments, we compare the implied trading rules in the LtF treatment with the trading rules in the LtO treatment as discussed above. The trading conservatism, with average \( \bar{\beta} = 0.56 \) and \( \bar{\chi} = 0.07 \) for the LtF and LtO treatments respectively, is significantly higher in the LtF treatment (MWWT \( p \)-value close to zero). This further means that the implied reaction to the asset return is weaker in the LtF treatment (average implied \( \bar{\phi} = -0.03 \)) than in the LtO treatment (average \( \bar{\phi} = 0.09 \)), and this difference is again confirmed by MWWT \( p \)-value close to zero. Hence, the results suggest that the LtF treatment is more stable than the other two treatments because agents use a stronger adaptive component in their forecasting rules.

We conclude that the estimated behavioral rules show that most subjects, regardless of the treatment, follow the observed price trend with a weak trend extrapolation type of rules. However, LtF subjects were much more adaptive in their behavior. This explains more stable dynamics under the LtF treatment. In addition, large individual heterogeneity persists within each of the treatment.

**Result 4.** The subjects use similar trading strategies in the LtO and Mixed treatments. While the subjects from the LtF treatment prefer more adaptive type of forecasting strategy, which explains more stability of that treatment. We conclude by rejecting the Hypothesis 5.

### 4.5 Conclusions

The origin of asset price bubbles is an important topic for both researchers and policy makers. This chapter investigates the price dynamics and bubble formation in an experimental asset pricing market with a price adjustment rule. A fundamental question about the origins of bubbles we address is: do bubbles arise because agents fail to learn to forecast accurately or because they fail to optimise their trading? We investigate the occurrence, the magnitude and the recurrence of bubbles in three treatments based on the tasks of the subjects: price forecasting, quantity trading and both. Under perfect rationality and perfect competition, these three tasks are equivalent and should lead the subjects to an equilibrium with a constant fundamental price. In contrast, we find none of the experimental markets to show a reliable convergence to the fundamental outcome, and recurring bubbles and crashes occur with the highest frequency.

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we look at what we interpret as an individual reaction to asset return dynamics: \( \phi_i \) in LtO treatment and \( \phi_i + \zeta_i \) in the Mixed treatment.
and magnitude when the subjects submit both a price forecast and a trading quantity decision.

This result shows that the deviation of market prices from the rational expectations equilibrium in former learning to forecast experiments (Hommes et al, 2005, 2008, and Heemeijer et al. 2009) is a robust phenomenon. Moreover, when the subjects act in a learning to optimise environment or submit both a forecast and a quantity, the deviation or asset bubbles become more severe. In contrast to the learning to forecast experiments, the coordination of individual decisions is lower in the trading treatments, which suggests that homogeneity of beliefs is not a necessary condition for mispricing or bubbles to occur. In particular, we provide a rigorous statistical test result on the individual heterogeneity in forecasting and trading strategies within the same treatment and across treatments. In the mixed treatment, in which we directly observe both the trading decisions and price expectations, there are only a quarter of the subjects being able to submit a trading quantity that is conditionally optimal to their price forecasts. We find that while there is no significant different in the trading strategy in the Mixed and LtO treatment, the subjects in the LtF treatment use adaptive expectations with a higher frequency than their counterparts in the Mixed treatment, which leads to more stabilised price behaviour.

What is the behavioural foundation for the difference in the individual decisions and aggregate market outcomes in the learning to forecast and learning to optimise market? There are several candidate explanations: (1) the quantity decision task is more cognitive demanding than the forecasting task, in partial when the subjects in the LtF treatment are helped by a computer program. Following Rubinstein (2007), we use decision time as a proxy for cognitive load and compare the average decision time in each treatment. It turns out while subjects take significantly longer time in the Mixed treatment than the other two treatments according to MWWT, there is no significant difference between the LtF and LtO treatments. It helps to explain why the markets are particularly volatile in the Mixed treatment, but does not explain why the LtO treatment is more unstable than the LtF treatment. (2) In a LtF treatment, the subjects’ goal is to find the accurate forecast. Only the size of the prediction error matters while the sign does not matter. Conversely, in a LtO market it is in a way more important for the subjects to predict the direction of the price movement right, and the size of the prediction error is important only to a secondary degree. (For example, if the subjects predict the return will be high and decided to buy, he can still make a profit if the price goes up far more than he expected, and his prediction error is large.) Therefore, the subjects may have a natural tendency to pay more attention to the trend of the price, which leads to a higher degree to trend extrapolation in the
4.5. Conclusions

Asset mispricing and financial bubbles can cause serious market inefficiencies, and may become a threat to the overall economic stability, as shown by the 2007 financial and economic crisis. It is therefore crucial to study the origins of assets’ mispricing in order to design regulations on the financial market. Proponents of the rational expectations would often claim that the serious asset pricing bubbles cannot arise, because rational economic agents would efficiently arbitrage against it and quickly push the ‘irrational’ (non-fundamental) investors away from the market. Our experiment suggests otherwise: people exhibit heterogeneous and not necessarily optimal behaviour, but because they are trend-followers, their ‘irrational’ (non-fundamental) beliefs are correlated. This is reinforced by the positive feedback between expectations and realised prices on the asset pricing markets, as stressed e.g. in Hommes (2013). Therefore, price oscillations cannot be mitigated by more rational market investors, and trading heterogeneity persists. As a result, waves of optimism and pessimism can arise despite the fundamentals being relatively stable. A strong policy implication is that the financial authorities should remain skeptical about the moods of the investors: fast increase of asset prices should be considered as a warning signal, instead of a reassuring signal of growth of the economic fundamentals only.

The design of our experiment can be extended to study other topics related to financial bubbles, such as markets with financial derivatives and the housing market. The advantage of our framework is that we can define a constant fundamental with positive dividend process, and the price is easy to calculate, and the same for all participants in the market. However, the subjects in our experiment can short-sell the asset as much as they want in order to profit from the fall of asset price during the market crash, which may not be feasible in real markets. An interesting topic for future research are experimental markets where agents face short selling constraints (Anufriev and Tuinstra, 2013) or the role of financial derivatives in (de)stabilising markets.

Another possible extension is to impose a network structure among the traders, i.e. one trader can only trade with some, but not all the other traders; or traders need to pay a cost in order to be connected to other traders. This design can help us to examine the mechanism of bubble formation in financial networks (Gale and Kariv, 2007), and network games (Galeotti et al., 2010) in general. There has been a pioneering experimental literature by Gale and Kariv (2009) and Choi et al. (2013) that study how network structure influence market efficiency when subjects act as

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14The asset price is usually defined for each transaction in a typical Smith et al. (1988) experiment, but it can also be the same for the whole market if the trading mechanism is a call market system, e.g. Akiyama et al. (2012)).
intermediaries between sellers and buyers. Our experimental setup can be extended to study how network structure influences market efficiency and stability when subjects act as traders of financial assets in the over the counter (OTC) market.
Appendix 4.A Instructions and computer screen

4.A.1 LtF treatment

General information
In this experiment you participate in a market. Your role in the market is a professional Forecaster for a large firm, and the firm is a major trading company of an asset in the market. In each period the firm asks you to make a prediction of the market price of the asset. The price should be predicted one period ahead. Based on your prediction, your firm makes a decision about the quantity of the asset the firm should buy or sell in this market. Your forecast is the only information the firm has on the future market price. The more accurate your prediction is, the better the quality of your firm’s decision will be. You will get a payoff based on the accuracy of your prediction. You are going to advise the firm for 50 successive time periods.

About the price determination
The price is determined by the following price adjustment rule: when there is more demand (firm’s willingness to buy) of the asset, the price goes up; when there is more supply (firm’s willingness to sell), the price will go down.
There are several large trading companies on this market and each of them is advised by a forecaster like you. Usually, higher price predictions make a firm to buy more or sell less, which increases the demand and vice versa. Total demand and supply is largely determined by the sum of the individual demand of these firms.

About your job
Your only task in this experiment is to predict the market price in each time period as accurately as possible. Your prediction in period 1 should lie between 0 and 100. At the beginning of the experiment you are asked to give a prediction for the price in period 1. When all forecasters have submitted their predictions for the first period, the firms will determine the quantity to demand, and the market price for period 1 will be determined and made public to all forecasters. Based on the accuracy of your prediction in period 1, your earnings will be calculated.
Subsequently, you are asked to enter your prediction for period 2. When all participants have submitted their prediction and demand decisions for the second period, the market price for that period, will be made public and your earnings will be calculated, and so on, for all 50 consecutive periods. The information you can refer to at period t consists of all past prices, your predictions and earnings.
Please note that due to liquidity constraint, your firm can only buy and sell up to
CHAPTER 4. BUBBLE FORMATION IN LTF AND LTO EXPERIMENTS

a maximum amount of assets in each period. This means although you can submit any prediction for period 2 and all periods after period 2, if the price in last period is \( p_{t-1} \), and your prediction is \( p^e_t \): the firm’s trading decision is constrained by \( p^e_t \in [p_{t-1} - 30, p_{t-1} + 30] \). More precisely, the firm will trade as if \( p^e_t = p_{t-1} + 30 \) if \( p^e_t > p_{t-1} + 30 \), and trade as if \( p^e_t = p_{t-1} - 30 \) if \( p^e_t < p_{t-1} - 30 \).

About your payoff

Your earnings depend only on the accuracy of your predictions. The earnings shown on the computer screen will be in terms of points. If your prediction is \( p^e_t \) and the price turns out to be \( p_t \) in period \( t \), your earnings are determined by the following equation:

\[
\text{Payoff} = \max \left[ 1300 - \frac{1300}{49} (p^e_t - p_t)^2, 0 \right].
\]

The maximum possible points you can earn for each period (if you make no prediction error) is 1300, and the larger your prediction error is, the fewer points you can make. You will earn 0 points if your prediction error is larger than 7. There is a Payoff Table on your table, which shows the points you can earn for different prediction errors. We will pay you in cash at the end of the experiment based on the points you earned. You earn 1 euro for each 2600 points you make.

4.A.2 LtO treatment

General information

In this experiment you participate in a market. Your role in the market is a Trader of a large firm, and the firm is a major trading company of an asset. In each period the firm asks you to make a trading decision on the quantity \( D_t \) your firm will BUY to the market. (You can also decide to sell, in that case you just submit a negative quantity.) You are going to play this role for 50 successive time periods. The better the quality of your decision is, the better your firm can achieve her target. The target of your firm is to maximize the expected asset value minus the variance of the asset value, which is also the measure by the firm concerning your performance:

\[
(1) \quad \pi_t = W_t - \frac{1}{2} \text{Var}(W_t)^2
\]

The total asset value \( W_t \) equals the return of the per unit asset multiplied by the number of unit you buy \( D_t \). The return of the asset is \( p_t + y - R p_{t-1} \), where \( R \) is the gross interest rate which equals 1.05, \( p_t \) is the asset price at period \( t \), therefore
$p_t - Rp_{t-1}$ is the capital gain of the asset, and $y = 3.3$ is the dividend paid by the asset. We assume the variance of the price of a unit of the asset is $\sigma^2 = 6$, therefore the expected variance of the asset value is $6D_t^2$. Therefore we can rewrite the performance measure in the following way

$$\pi_t = (p_t + y - Rp_{t-1})D_t - 3D_t^2$$

The asset price in the next period $p_{t+1}$ is not observable in the current period. You can make a forecast $p_e$ on it. There is an asset return calculator in the experimental interface that gives the asset return for each price forecast $p_e$ you make. Your own payoff is a function of the value of target function of the firm:

$$\text{Payoff}_t = 800 + 40 * \pi_t$$

This function means you get 800 points (experimental currency) as basic salary, and 40 points for each 1 unit of performance (target function of the firm) you make. If your trades will be unsuccessful, you may lose points and earn less than your basic salary, down to 0. Based on the asset return, you can look up your payoff for each quantity decision you make in the payoff table.

You can of course also calculate your payoff for each given forecast and quantity using equation (2) and (3) directly. In that situation you can ask us for a calculator.

**About the price determination**

The price is determined by the following price adjustment rule: when there is more demand than supply of the asset (namely, more traders want to buy), the price will go up; and when there is more supply than demand of the asset (namely, more people want to sell), the price will go down.

**About your job**

Your only task in this experiment is to decide the quantity the firm will buy/sell. At the beginning of period 1 you determine the quantity to buy or sell (submitting a positive number means you want to buy, and negative number means you want to sell) for period 1. After all traders submit their quantity decisions, the market price for period 1 will be determined and made public to all traders. Based on the value of the target function of your firm in period 1, your earnings in the first period will be calculated. Subsequently, you make trading decisions for the second period, the market price for that period will be made public and your earnings will be calculated, and so on, for
all 50 consecutive periods. The information you can refer to at period t consists of all
previous prices, your quantity decisions and earnings.
Please notice that due to the liquidity constraint of the firm, the amount of asset you
buy or sell cannot be more than 5 units. Which means you quantity decision should be
between $-5$ and 5. The numbers on the payoff table are just examples. You can use
any other number such as 0.01, $-1.3$, 2.15 etc., as long as they are within $[-5, 5]$. if
When you want to submit numbers with a decimal point, please write a “.”, NOT a “,”.

**About your payoff**
In each period you are paid according to equation (3). The earnings shown on the
computer screen will be in terms of points. We will pay you in cash at the end of the
experiment based on the points you earned. You earn 1 euro for each 2600 points you
make.

### 4.A.3 Mixed treatment

**General information**
In this experiment you participate in a market. Your role in the market is a Trader of
a large firm, and the firm is a major trading company of an asset. In each period the
firm asks you to make a trading decision on the quantity $D_t$ your firm will BUY to the
market. (You can also decide to sell, in that case you just submit a negative quantity.)
You are going to play this role for 50 successive time periods. The better the quality
of your decision is, the better your firm can achieve her target. The target of your firm
is to maximize the expected asset value minus the variance of the asset value, which is
also the measure by the firm concerning your performance:

\[ \pi_t = W_t - \frac{1}{2} Var(W_t)^2 \]

The total asset value $W_t$ equals the return of the per unit asset multiplied by the
number of unit you buy $D_t$. **The return of the asset** is $p_t + y - Rp_{t-1}$, where $R$
is the gross interest rate which equals 1.05, $p_t$ is the asset price at period $t$, therefore
$p_t - Rp_{t-1}$ is the capital gain of the asset, and $y = 3.3$ is the dividend paid by the
asset. We assume the variance of the price of a unit of the asset is $\sigma^2 = 6$, therefore the
expected variance of the asset value is $6D_t^2$. Therefore we can rewrite the performance
measure in the following way

\[ \pi_t = (p_t + y - Rp_{t-1})D_t - 3D_t^2 \]
The asset price in the next period $p_{t+1}$ is not observable in the current period. You can make a forecast $p_e^t$ on it. **There is an asset return calculator in the experimental interface** that gives the asset return for each price forecast $p_e^t$ you make. Your own payoff is a function of the value of target function of the firm:

$$ (3) \quad \text{Payoff}_{t} = 800 + 40 \times \pi_t $$

This function means you get 800 points (experimental currency) as basic salary, and 40 points for each 1 unit of performance (target function of the firm) you make. If your trades will be unsuccessful, you may lose points and earn less than your basic salary, down to 0. Based on the asset return, you can look up your payoff for each quantity decision you make in the **payoff table**.

You can of course also calculate your payoff for each given forecast and quantity using equation (2) and (3) directly. In that situation you can ask us for a calculator.

The payoff for the forecasting task is simply a decreasing function of your forecasting error (the distance between your forecast and the realized price). When your forecasting error is larger than 7, you earn 0 points.

$$ (4) \quad \text{Payoff}_{\text{forecasting}} = \max \left[ 1300 - \frac{1300}{49} (p_e^t - p_t)^2, 0 \right] $$

**About the price determination**

The price is determined by the following price adjustment rule: when there is more demand than supply of the asset (namely, more traders want to buy), the price will go up; and when there is more supply than demand of the asset (namely, more people want to sell), the price will go down.

**About your job**

Your task in this experiment consists of two parts: (1) to make a price forecast; (2) to decide the quantity the firm will buy/sell. At the **beginning of period 1 you submit your price forecast between 0 and 100**, and then determine the quantity to buy or sell (submitting a positive number means you want to buy, and negative number means you want to sell) for period 1, and the market price for period 1 will be determined and made public to all traders. Based on your forecasting error and performance measure for the trading task, in period 1, your earnings in the first period will be calculated.

Subsequently, you make forecasting and trading decisions for the second period, the market price for that period will be made public and your earnings will be calculated,
and so on, for all 50 consecutive periods. The information you can refer to at period \( t \) consists of all previous prices, your past forecasts, quantity decisions and earnings. Please notice that due to the liquidity constraint of the firm, the amount of asset you buy or sell cannot be more than 5 units. Which means you quantity decision should always be between \(-5\) and \(5\). The numbers on the payoff table are just examples. You can use any other numbers such as 0.01, \(-1.3\), 2.15 etc. as long as they are within \([-5,5]\).

**About your payoff**

In each period you are paid for the forecasting task according to equation (4) and trading task according to equation (3). The earnings shown on the computer screen will be in terms of points. We will pay you in cash at the end of the experiment based on the points you earned for either the forecasting task or the trading task. Which task will be paid will be determined randomly (we will invite one of the participants to toss a coin). That is, depending on the coin toss, your earnings will be calculated either based on equation (3) or equation (4). You earn 1 euro for each 2600 points you make.

**4.A.4 Computer screen**

An illustration of the computer screens seen by the subjects is shown on Figure 4.5. The screen was divided into 3 mini pages. In the top mini page, subjects were prompted to submit their decisions, \(i.e.,\) their price forecast or the amount they want to trade. After submitting their decisions, they go to a waiting page until all the subjects have made their decisions for this period, and then the price and payoff of this period is calculated, the program goes to next period and the screen is reloaded to show the updated information. In the bottom left mini page there was a graph plotting past market prices (the “Real Price”) and, if they were a forecaster, they also saw their own past price forecast history (“Your Prediction”). Finally, in the bottom right mini page they saw a table reporting the history of realized prices, as well as their own prior decisions and cumulative payoffs. If the subject was a quantity decision maker, he/she was also helped by an imbedded calculator. In each period, the subjects could type in their price forecast and press “calculate”, and the calculator will tell them the asset return for this forecast in this period.

Subjects in LtF/LtO treatment saw the screen for a forecaster/trader only. In a Mixed treatment, the subjects first see the screen of the forecast, and then go to the trading page.
4.A. Instructions and computer screen

Your decision for period 5
What is your prediction for the price in period 5?

<table>
<thead>
<tr>
<th>Period</th>
<th>Your prediction</th>
<th>The realized price</th>
<th>The points you earned in the period</th>
<th>The points you have earned so far</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>26</td>
<td>27.64</td>
<td>1288.64</td>
<td>2038.56</td>
</tr>
<tr>
<td>3</td>
<td>49</td>
<td>49.69</td>
<td>1287.37</td>
<td>2054.42</td>
</tr>
<tr>
<td>2</td>
<td>80</td>
<td>62.31</td>
<td>0</td>
<td>1277.05</td>
</tr>
<tr>
<td>1</td>
<td>35</td>
<td>34.07</td>
<td>1277.05</td>
<td>1277.05</td>
</tr>
</tbody>
</table>

(a) Screen for a forecaster.

Your decision for period 4
What is the quantity you want to produce in period 4?

The return to one unit of asset is 4.24 if the price is 9

Use another forecast to calculate the return.

(b) Screen for a quantity decision maker.

Figure 4.5: Computer screen for subjects in LtF treatment (upper panel) and LtO panel (lower panel).
## Appendix 4.B  Payoff tables

**Table 4.2:** Payoff table for forecasters.

![Payoff Table for Forecasting Task](image)

- Your Payoff = \(\max[1300 - \frac{1300}{\text{error}} \cdot (\text{Your Prediction Error})^2, 0]\)
- 3000 points equal 1 euro

<table>
<thead>
<tr>
<th>error</th>
<th>points</th>
<th>error</th>
<th>points</th>
<th>error</th>
<th>points</th>
<th>error</th>
<th>points</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1300</td>
<td>1.85</td>
<td>1209</td>
<td>3.7</td>
<td>937</td>
<td>5.55</td>
<td>483</td>
</tr>
<tr>
<td>0.05</td>
<td>1300</td>
<td>1.9</td>
<td>1204</td>
<td>3.75</td>
<td>927</td>
<td>5.6</td>
<td>468</td>
</tr>
<tr>
<td>0.1</td>
<td>1300</td>
<td>1.95</td>
<td>1199</td>
<td>3.8</td>
<td>917</td>
<td>5.65</td>
<td>453</td>
</tr>
<tr>
<td>0.15</td>
<td>1299</td>
<td>2</td>
<td>1194</td>
<td>3.85</td>
<td>907</td>
<td>5.7</td>
<td>438</td>
</tr>
<tr>
<td>0.2</td>
<td>1299</td>
<td>2.05</td>
<td>1189</td>
<td>3.9</td>
<td>896</td>
<td>5.75</td>
<td>423</td>
</tr>
<tr>
<td>0.25</td>
<td>1298</td>
<td>2.1</td>
<td>1183</td>
<td>3.95</td>
<td>886</td>
<td>5.8</td>
<td>408</td>
</tr>
<tr>
<td>0.3</td>
<td>1298</td>
<td>2.15</td>
<td>1177</td>
<td>4</td>
<td>876</td>
<td>5.85</td>
<td>392</td>
</tr>
<tr>
<td>0.35</td>
<td>1297</td>
<td>2.2</td>
<td>1172</td>
<td>4.05</td>
<td>865</td>
<td>5.9</td>
<td>376</td>
</tr>
<tr>
<td>0.4</td>
<td>1296</td>
<td>2.25</td>
<td>1166</td>
<td>4.1</td>
<td>854</td>
<td>5.95</td>
<td>361</td>
</tr>
<tr>
<td>0.45</td>
<td>1295</td>
<td>2.3</td>
<td>1160</td>
<td>4.15</td>
<td>843</td>
<td>6</td>
<td>345</td>
</tr>
<tr>
<td>0.5</td>
<td>1293</td>
<td>2.35</td>
<td>1153</td>
<td>4.2</td>
<td>832</td>
<td>6.05</td>
<td>329</td>
</tr>
<tr>
<td>0.55</td>
<td>1292</td>
<td>2.4</td>
<td>1147</td>
<td>4.25</td>
<td>821</td>
<td>6.1</td>
<td>313</td>
</tr>
<tr>
<td>0.6</td>
<td>1290</td>
<td>2.45</td>
<td>1141</td>
<td>4.3</td>
<td>809</td>
<td>6.15</td>
<td>297</td>
</tr>
<tr>
<td>0.65</td>
<td>1289</td>
<td>2.5</td>
<td>1134</td>
<td>4.35</td>
<td>798</td>
<td>6.2</td>
<td>280</td>
</tr>
<tr>
<td>0.7</td>
<td>1287</td>
<td>2.55</td>
<td>1127</td>
<td>4.4</td>
<td>786</td>
<td>6.25</td>
<td>264</td>
</tr>
<tr>
<td>0.75</td>
<td>1285</td>
<td>2.6</td>
<td>1121</td>
<td>4.45</td>
<td>775</td>
<td>6.3</td>
<td>247</td>
</tr>
<tr>
<td>0.8</td>
<td>1283</td>
<td>2.65</td>
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Note that 3000 points of your profit corresponds to €1.
## Appendix 4.C RADs and RDs

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**Table 4.4:** Relative Absolute Deviation (RAD) and Relative Deviation (RD) of the experimental prices for the three treatments, in percentages.
## Appendix 4.D  Earnings Ratios

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**Table 4.5**: Average earnings (in Euro) and earnings efficiency for each market.
Appendix 4.E  Estimation of individual forecasting rules

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Table 4.6: Estimated individual rules for the LtF treatment.
### 4.E. Estimation of individual forecasting rules

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Table 4.6: (continued) Estimated individual rules for the LtF treatment.
### Table 4.7: Estimated individual rules for the LtO treatment. S, N and U denote respectively stable, neutrally stable and unstable rule if all six subjects would use this rule.

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### 4.E. Estimation of individual forecasting rules

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Table 4.7: (continued) Estimated individual rules for the LtO treatment.
### Table 4.8: Mixed treatment: quantities chosen by individuals explained by their contemporaneous expected asset returns. Log-likelihood measures for models with various restrictions on the parameters and parameter heterogeneity. In parenthesis, likelihood ratio test p-values for the restrictions imposed in the estimation on the unrestricted model (reported in first row). Estimation for 46 individuals, unrestricted sample and sample restricted for observations with expected asset return above 60 and 30.

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### Table 4.9: Estimated individual rules for the mixed treatment (system of quantity and predicted price rules).

S, N and U denote respectively stable, neutrally stable and unstable system of rules if all six subjects would use this system. ADA and TRE denote a price prediction rule of a subject that could be classified as adaptive or trend extrapolation expectations respectively.

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**Table 4.9:** Estimated individual rules for the mixed treatment (system of quantity and predicted price rules). S, N and U denote respectively stable, neutrally stable and unstable system of rules if all six subjects would use this system. ADA and TRE denote a price prediction rule of a subject that could be classified as adaptive or trend extrapolation expectations respectively.
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Table 4.9: Estimated individual rules for the mixed treatment (system of quantity and predicted price rules). S, N and U denote respectively stable, neutrally stable and unstable system of rules if all six subjects would use this system. ADA and TRE denote a price prediction rule of a subject that could be classified as adaptive or trend extrapolation expectations respectively.
Appendix 4.F  Rational strategic behaviour

Our experimental results are clearly different from the predictions of the rational expectation equilibrium (REE). Previous sections discussed some evidence that non-fundamental prices and oscillations are caused by bounded rationality and simple individual heuristics of our subjects. However, we also found that they would typically earn high payoffs, implying some sort of successful profit seeking behaviour.

In this appendix, we discuss whether rational strategic behaviour can explain our experimental results. Indeed, different types of rational equilibria may exist depending on agents' perception of the economy. Three cases are discussed: (1) agents are price takers; (2) agents know their market power and coordinate on monopolistic behaviour; (3) agents know their market power but play a non-cooperative game. We show that in the price-taking case, the LtF and LtO treatments are equivalent, with the same rational fundamental solution. If the subjects behave strategically or try to collude, the economy can have alternative rational equilibria, where the subjects collectively ‘ride a bubble’, or jump around the fundamental price. Nevertheless, these rational equilibria predict different outcomes than the individual and aggregate behaviour observed in the experiment.

Without loss of generality, in the case of the non-cooperative game we focus on the one-shot game version of the experimental market to derive our results. More precisely, we look at the optimal decisions that the agents in period $t$ (knowing prices and individual traded quantity until and including period $t$) have to formulate only for the next period $t + 1$. We follow this approach for two reasons. First, by definition agents are myopic and their payoff in $t + 1$ depends only on the realised profit from that period, and not on the stream of future profits from period $t + 2$ onward. Second, the experiment is a repeated game with finitely many repetitions, and subjects knew it would end after 50 periods. Using the standard backward induction reasoning, one can easily show that a sequence of one-period game equilibria forms a rational equilibrium of the finitely repeated game as well.

4.F.1  Price takers

Realised utility of investors in the LtO treatment is given by (4.4) and is equivalent to the following form:

$$U_{i,t}(z_{i,t}) = z_{i,t} (p_{t+1} + y - Rp_t) - \frac{a\sigma^2}{2} z_{i,t}^2,$$

(4.22)
4.F. Rational strategic behaviour

where $z_{i,t}$ is the traded quantity and $U_{i,t}$ is a quadratic function of the traded quantity. As shown in Section 2 discussed that, assuming the agent is a price taker, the optimal traded quantity conditional on the expected price $p_{i,t+1}^e$ is given by

$$z_{i,t}^{PT} = \arg \max_{z_{i,t}} U_{i,t} = \frac{p_{i,t+1}^e + y - Rp_t}{a\sigma_z^2}.$$  

Note that this result relies on the assumption that the subjects do not know the price determination function. We argue that the subjects also have an incentive to minimise their forecasting error. To see that, suppose that the realised market price in the next period is $p_{t+1}$, and the subject makes a prediction error of $\epsilon$, i.e., her prediction is $p_{i,t+1}^e = p_{t+1} + \epsilon$. The payoff function can be rewritten as:

$$U_{i,t}(z_{i,t}) = z_{i,t} (p_{t+1} + y - Rp_t) - \frac{a\sigma_z^2}{2} z_{i,t}^2$$

$$= \frac{(p_{t+1} + \epsilon + y - Rp_t)(p_{t+1} + y - Rp_t)}{a\sigma_z^2} - \frac{(p_{t+1} + \epsilon + y - Rp_t)^2}{2a\sigma_z^2}$$

$$= \frac{(p_{t+1} + y - Rp_t)^2}{2a\sigma_z^2} - \frac{\epsilon^2}{2a\sigma_z^2}.$$  

This shows that utility is maximised when $\epsilon = 0$. Assuming perfect rationality and price taking behaviour (perfect competition), the task of finding the optimal trade coincides with the task of minimising the forecast error. Thus when all agents have rational expectations and are price takers, the market price equals the REE regardless of the task.

**Finding 7.** When the subjects act as price takers, the utility function in the Learning to Optimise treatment is a quadratic function of the prediction error, the same (up to a monotonic transformation) as in the Learning to Forecast treatment. Hence, LtF and LtO treatments have equivalent tasks under Rational Expectations.

4.F.2 Collusive outcome

Suppose now that the agents realise how their trading quantities influence the price and are able to coordinate on a common strategy. This results in a collusive market, similar to a producers’ oligopoly. We assume that in the collusive solution, all agents behave as a monopoly that maximises joint (unweighted) utility; thus the solution is symmetric, that is for each agent $i$, $z_{i,t} = z_t$. In our experiment the price determination
CHAPTER 4. BUBBLE FORMATION IN LTF AND LTO EXPERIMENTS

function is:

\[ p_{t+1} = p_t + 6\lambda z_t, \]

and so the monopoly under perfect rationality maximises

\[
U_t = \sum_{i=1}^{6} U_{i,t}(z_t) = 6 \left[ z_t (p_{t+1} + y - R p_t) - \frac{a\sigma^2 z_t^2}{2} \right]
\]

\[ = 6 \left[ z_t^2 \left( 6\lambda - \frac{a\sigma^2}{2} \right) + z_t (y - rp_t) \right]. \]

(4.26)

Notice that when \( \lambda = 20/21, a\sigma^2 = 6 \), as in the experiment, the coefficient before \( z_t^2 \) is positive, \( 6\lambda - \frac{a\sigma^2}{2} = \frac{19}{7} > 0 \), and thus the profit function is U shaped, instead of inversely U shaped. \(^{15}\) This means that a finite global maximum does not exist (utility goes to +\( \infty \) when \( z_t \) goes to either +\( \infty \) or \( -\infty \)). The global minimum is obtained when

\[ z_{i,t} = \frac{r}{38} (rp_t - y) = \frac{r}{38} (p_t - p_f). \]

In our experiment, the subjects are constrained to choose a quantity from \([-5, 5]\) and the price is bound to the interval \([0, 300]\). Collusive equilibrium in the one-shot game implies that the subjects coordinate on \( z_{i,t} = 5 \) or \( z_{i,t} = -5 \), depending on which is further away from \( \frac{r(rp_t - y)}{38} \) (as (4.26) is a symmetric parabola). Since \( \frac{r(rp_t - y)}{38} > 0 \) when the price is above the fundamental (\( p_t = y/r \)), we can see that the agents coordinate on \(-5\) if the price is higher than the REE (\( p_t > y/r \)). Similarly, rational agents coordinate on \( +5 \) if the price is lower than the REE (\( p_t < y/r \)). If the price is exactly at the fundamental, rational agents are indifferent between \(-5\) and \( 5 \). Notice that in such a case trading the REE quantity \( (z_{i,t} = 0) \) gives the global minimum for the monopoly.

As a consequence, the collusive outcome predicts that the subjects will ‘jump up and down’ around the fundamental. When the price is just below the fundamental, rational agents will buy the asset, which brings the price above the fundamental, and hence the agents in the next period will sell the asset, and so forth. Notice that if the initial price is far below (above) the fundamental, the monopoly will buy (sell) the asset until the price overshoots (undershoots) the fundamental. Then the subjects start to ‘jump up and down’ as described before.

\(^{15}\) If \( 6\lambda - \frac{a\sigma^2}{2} < 0 \), this objective function is inversely U shaped. The maximum point is achieved when \( z_{i,t} = \frac{y}{12\lambda - a\sigma^2} \). This means when \( p_t = y/r \), namely when the price is at the REE, the optimal quantity under collusive equilibrium is still \( 0 \). When the price is higher or lower than the REE, the optimal quantity increases with the difference between the price and the REE. This means there is a continuum of equilibria when the economy does not start at the REE.
4.F. Rational strategic behaviour

Finding 8. When the subjects know the price determination function and are able to form a coalition, the collusive profit function in the LtO treatment is U shaped. Subjects would buy under-priced and sell an over-priced asset. In the long run rational collusive subjects will alternate their trading quantities between $-5$ and $5$ and so the price will alternate around the equilibrium.

Such alternating dynamics would resemble coordination on contrarian type of behaviour, but has not been observed in any of the experimental groups. Instead, our subjects coordinated on trend-following trading rules, which resulted in smooth, gradual price oscillations.

Notice also that the demand at the edge of the liquidity constraint ($z_{i,t} = \pm 5$) would generate rapid price changes, namely $p_{t+1} = p_t \pm (20/21)/(6*5) \approx p_t \pm 28.57$, that is the price would change in every period by around 28.57 in absolute terms. This has not been observed even in the super-bubble group 8 from the Mixed treatment. Indeed, quantity decisions equal to 5 or $-5$ happened only 7 times in the LtO and 44 times in the Mixed treatment (i.e. 0.39% and 1.83% of observations respectively). Typical subject behaviour was much more conservative: 97% and 91% traded quantities in the LtO and Mixed treatments respectively were confined in the interval $[-2.5, 2.5]$. A good example is group 4 from the Mixed treatment, in which the price reached 150, but the individual trades were rarely outside the interval $[-3, 3]$ (see Fig. 4.3).

4.F.3 Perfect information non-cooperative game

Consider a scenario, in which the subjects realise the experimental price determination mechanism, but cannot coordinate their actions and play a symmetric Nash equilibrium (NE) instead of the collusive one. There is a positive externality of the subjects’ decisions: when one subject buys the asset, it pushes up the price and also the benefits of all the other subjects. The collusive equilibrium internalises this externality, but could the same happen if the subjects in the experiment could not coordinate? In other words, would they have an incentive to ‘free-ride’ on the demand of others, and would that push the price back to the REE?

In the case of a non-cooperative one-shot game, we again focus on a symmetric solution. Consider agent $i$, who optimises her quantity choice believing that all other agents will choose $z_{o,t}^i$. This means that the price at $t+1$ becomes

\begin{equation}
(4.27) \quad p_{t+1} = p_t + 5\lambda z_{o,t}^i + \lambda z_{i,t}.
\end{equation}
Agent $i$ maximises therefore

$$U_{i,t} = z_{i,t} (\lambda z_{i,t} + 5\lambda z^o_t + y - rp_t) - \frac{a\sigma^2}{2} z^2_{i,t}$$

$$= z^2_{i,t} \frac{2\lambda - a\sigma^2}{2} + z_{i,t}(5\lambda z^o_t + y - rp_t).$$

(4.28)

Notice that $2\lambda - a\sigma^2 = -86/21 < 0$. This is an inversely U shaped parabola with the unique maximum given by the reaction function

$$z^*_{i,t}(z^0_t) = \frac{5\lambda z^o_t + y - rp_t}{a\sigma^2 - 2\lambda}.$$  

(4.29)

A symmetric solution requires $z^*_{i,t}(z^0_t) = z^0_t$, which implies

$$z^*_{i,t} = \frac{rp_t - y}{7\lambda - a\sigma^2} = \frac{3}{2}(rp_t - y).$$

(4.30)

Furthermore the reaction function $z^*_{i,t}(z^0_t)$ is linear with respect to $z^0_t$, with a slope $\frac{5\lambda}{a\sigma^2 - 2\lambda} = \frac{100}{86} > 1$. Thus, $z^o_t > z^*_{i,t}$ ($<$ and $=)$ implies $z^*_{i,t} > z^0_t$ ($<$ and $=$), or in words, if agent $i$ believes that the other players will buy (sell) the asset, she has an incentive to buy (sell) even more. Then as a best response, the other agents should further increase/decrease their demand, and this is limited only by the liquidity constraints.

The strategy (4.30) thus defines the threshold point between the two corner strategies, i.e. the full NE strategy is defined as

$$z^{NE}_{i,t} = \begin{cases} 
5 & \text{if } z^o_t > z^*_{i,t} \\
z^*_{i,t} & \text{if } z^o_t = z^*_{i,t} \\
-5 & \text{if } z^o_t < z^*_{i,t}.
\end{cases}$$  

(4.31)

The boundary strategies can be infeasible if the previous price is too close to zero or 300. To sum up, as long as the price $p_t$ is sufficiently far from the edges of the allowed interval $[0, 300]$, there are three NE of the one-shot non-cooperative game, which are defined by all players playing $z_{i,t} = -5$, $z_{i,t} = z^*_{i,t}$ and $z_{i,t} = +5$ for all $i \in \{1, \ldots, 6\}$.

If the agents coordinate on the strategy $z_{i,t} = z^*_{i,t}$, the price evolves according to the following law of motion:

$$p_{t+1} = \frac{10p_t - 60y}{7}.$$  

(4.32)

\textsuperscript{16}Notice that we can interpret $z^o_t$ as the average quantity traded by all other agents, besides agent $i$, and the reasoning for NE strategy (4.31) remains intact. This implies that NE has to be symmetric.
In contrast to the collusive game, in the non-cooperative game the fundamental price is therefore a possible steady state, but only if it is an outcome in the initial period. Additional equilibrium refinements may further exclude it as a rational outcome, since it is the least profitable one. Recall that the subjects earn 0 when they play \( z^*_t \) with price at the fundamental (because there is no trade). On the other hand, they may earn a positive profit by coordinating on \(-5\) or \(5\). For example, when all of them buy 5 units of asset, the utility for each of them will be \( (p_t + y + 6\lambda z_{i,t} - (1+r)p_t)z_{i,t} - \frac{\alpha^2}{2}z_{i,t} = (33.3 - 0.05p_t) \cdot 5 - 75 \). This equals 76.5 when \( p_t = 60 \), 16.5 when \( p_t = 300 \) and 75 when the previous price is equal to the fundamental, \( p_t = 66 \). This explains why the payoff efficiency (average experimental payoff divided by payoff under REE) is larger than 100% in some markets in the LtO or Mixed treatments where prices have large oscillations.

Notice that the linear equation (4.32) is unstable, so the NE of the one-shot game leads to unstable price dynamics in the repeated game even if the agents coordinate on \( z_{i,t} = z^*_t \), as long as the initial price is different from the fundamental price. Indeed, if the initial price is 67 or 65 (fundamental price plus or minus one), the price hits the upper cap of 300 or the lower cap of 0 in 16 and 12 periods respectively, and rational non-cooperative agents would be forced to use appropriate corner strategies \((-5\) and \(5\) respectively). Furthermore the agents can switch at any moment between the three one-shot game NE defined by (4.31). This implies that in the repeated non-cooperative game, many rational price paths are possible. This includes many price paths where agents will often coordinate on \(5\) or \(-5\) strategies. Furthermore, notice that the up and down alternating price behaviour around the fundamental, which was the solution for the collusive equilibrium, is a NE as well, and hence this is the Pareto efficient equilibrium for this game.

**Finding 9.** In the non-cooperative game with perfect information, there are two possible types of NE. The fundamental outcome is a possible outcome only if the initial price is equal to the fundamental price. Otherwise, the agents will coordinate on unstable, possibly oscillatory price dynamics, with traded quantities of \(-5\) or \(5\). When they coordinate on a non-zero quantity, their payoff can be higher than their payoff under the REE under the price-taking beliefs.

Altogether, the perfectly rational agents can coordinate on price boom-bust cycles and earn positive profit. However, this would require even stronger assumptions than the fundamental equilibrium, namely that the agents perfectly understand the underlying price determination mechanism. Furthermore, the cycle of bubbles and crashes is suboptimal in comparison with the ‘jumping up and down around the fundament-
tal’ equilibrium. If the agents were rational enough to coordinate, then it remains a mystery why they would coordinate on the less efficient path of bubbles and crashes.

Furthermore, such rational equilibria with price oscillations predict that the subjects to coordinate on homogeneous trades at the edge of the liquidity constraints (traded quantities should often, or even always, be either 5 or −5). The subjects from the LtO and Mixed treatments behaved differently. Their traded quantities were highly heterogeneous (which implied the observed heterogeneity of the estimated trading and forecasting rules), and rarely reached the liquidity constraints, as discussed above.

Therefore, the rational solutions, in particular the ones from the perfect information, non-cooperative games provide some useful insights on why subjects “ride the bubbles” in the LtO and Mixed treatment. However, since the rational solution cannot explain the heterogeneity of the individual decisions and the fact that the subjects shy away from the boundary solutions, the bubbles and crashes we see from the data is probably a result of the joint forces of rational (profit seeking) and boundedly rational behaviour with some coordination on trend-following buy and hold and short sell strategies.
Chapter 5

Summary

Price expectations remain a matter of controversy in the economic literature. Recent market developments, including the 2007 financial meltdown, challenge the traditional view of perfect rationality and market efficiency. Learning-to-Forecast experiments (LtF) indicate that, when facing feedback between expectations and realized prices, subjects remain heterogeneous and follow simple behavioral rules, with their exact choice depending on the nature of the feedback. The most important difference depends upon whether the market has a negative or positive feedback structure. Under negative feedback, for example in a producers economy of a perishable good, the realized price is negatively correlated with expectations. This drives laboratory subjects to adaptive behavior and consequently they coordinate on the rational fundamental solution.

In contrast, financial markets feature a self-fulfilling positive feedback structure. For example, optimistic agents will increase demand for a financial asset, what indeed drives up the price of this asset. LtF experiments demonstrate that human subjects in such a positive feedback environment learn to extrapolate price trends, which results in erratic and, most likely, non-converging price oscillations.

The aim of this thesis is to study learning to forecast behavior. We focus on a Genetic Algorithms based learning model, its fit to LtF experimental data and application to various market settings. We also run an experiment to study the robustness of the LtF design. The thesis consists of three related but independent chapters.

In Chapter 2, we develop a model in which agents use Genetic Algorithms to optimize a general forecasting heuristic to form their price expectations (Hommes and Lux, 2013). We apply the model to four experimental settings:

- linear positive and negative feedback (Heemeijer et al., 2009);
- linear positive and negative feedback with large and unanticipated shocks to the fundamental price (Bao et al., 2012);
• non-linear cobweb economy (Hommes et al., 2007; van de Velden, 2001);
• non-linear two-period ahead asset pricing market (Hommes et al., 2005).

Our model outperforms Rational Expectations, a number of other homogeneous expectation rules (naive, adaptive, trend extrapolation and contrarian), and a simple Heuristic Switching Model with heterogeneous expectations. Specifically, the model has an excellent short-horizon out-of-sample predictive power. In addition, it is the only model able to explain the individual behavior, and successfully predict the experimental dynamics in the long run (fifty periods ahead). It also justifies using the Heuristic Switching Model as a stylized description of experimental dynamics. This is an important contribution to the literature, since in recent years policy makers are looking for economic models with robust behavioral micro-foundations.

In Chapter 3, we use the Genetic Algorithms Learning-to-Forecast model to study information networks in financial markets. The goal is to evaluate the impact of information flows on market stability and individual coordination. We focus on a non-linear two-period ahead asset pricing model, in which the agents can learn whether to extrapolate the observed price trend, but also whether to trust the average mood of the friends that the agents observe through a network. We show that without a network, agents can coordinate either on the fundamental value, or on trend-following behavior. The latter makes the market switch between the fundamental solution and erratic price oscillations. Adding a network of any architecture or size destabilizes the market, which almost never stays in the fundamental solution. This follows from two related observations. First, agents learn to coordinate on stronger price trend following rules. Secondly, agents learn contrarian behavior: they are more optimistic if they observe their friends to be selling in the past and vice versa. The reason for the contrarian behavior is that the agents try to ‘out-smart’ their friends. For example, in the moment when the bubble starts to collapse, an agent realizes the negative trend, as well as the fact that until the peak of the bubble, her friends were buying the asset. She therefore has an incentive to trade against her friends.

The model predicts that, despite the contrarian attitude, the agents learn a similar type of behavior and therefore remain well coordinated. This is an important insight into the literature’s conflicting view on herding. Experiments demonstrate that people use contrarian (anti-herding) strategies, which indeed occurs in our simulations. On the other hand, indirect measures used for the market data suggest some degree of herding. In the model, similar learning of contrarian attitude drives agents to similar behavior, which may be mistaken for herding.

Chapter 4 reports an experiment based on a simple linear asset pricing market,
in which subjects were asked to forecast the prices, trade the asset or do both. The importance of this experiment is that it establishes a link between Learning-to-Forecast and Learning-to-Optimize experiments. It can therefore serve as a useful benchmark for future theoretical and experimental investigations on learning in asset pricing markets.

We find that regardless of the treatment, subjects coordinate on non-fundamental outcomes: stable price far from the fundamental value or persistent price oscillations. The Learning-to-Optimize and mixed treatments are more unstable, with the highest bubble (around 3.5 times the fundamental price at the peak) in one of the groups from the mixed treatment. These results show that the learning to optimize is even more difficult for the subjects than the learning to forecast. Furthermore, the results of the LtF experiments are robust, in the sense that price oscillations are not just an artifact of the forecasting treatment. We also confirm statistically significant heterogeneity of subjects’ behavioral rules.

The most surprising result comes from the mixed treatment, in which both the trades and the corresponding price forecasts are observable. This allowed us to explicitly test their consistency. We found only a quarter of our subjects to trade optimally conditional on the implied expectations of the asset return. This stands as a striking warning against the idea that economic agents are perfect optimizers — an idea popular even among the proponents of boundedly rational price expectations. Further studies shall focus on theoretical models of learning to optimize, and its link to learning to forecast.
Bibliography


Samenvatting (Summary in Dutch)

Prijsverwachtingen blijven een controversiële kwestie in de economische literatuur. Recentte marktontwikkelingen, waaronder de financiële crisis van 2007, trekken de traditionele denkbeelden van perfecte rationaliteit en marktefficiëntie in twijfel. Zogeheten Learning-to-Forecast-experimenten (LtF) tonen aan dat proefpersonen, wanneer ze geconfronteerd worden met feedback tussen verwachtingen en gerealiseerde prijzen, heterogeen blijven en simpele gedragsregels volgen. De precieze keuze voor een gedragsregel hangt af van het soort feedback. Het belangrijkste onderscheid dat gemaakt wordt is tussen markten met een negatieve dan wel een positieve feedbackstructuur. Bij negatieve feedback, zoals in een economie met producenten van een bederfelijk goed, is de gerealiseerde prijs negatief gecorreleerd met de verwachtingen. Dit duwt proefpersonen in de richting van adaptief gedrag, waardoor ze coördineren op de rationele, fundamentele oplossing.

Daarentegen worden financiële markten juist gekenmerkt door een selffulfilling positieve feedbackstructuur. Zo zullen optimistische agenten de vraag naar een financieel product doen toenemen, waardoor de prijs van dit product ook daadwerkelijk stijgt. LtF-experimenten tonen aan dat proefpersonen in een omgeving met dergelijke positieve feedback leren om prijstrends te extrapoleren, hetgeen resulteert in grillige en niet-convergerende prijsoscillaties. Het doel van dit proefschrift is om het Learning-to-Forecast-gedrag, d.w.z. het leren om te voorspellen, nader te bestuderen. We richten ons op een leermodel met Genetische Algoritmen, het fitten hiervan op data uit LtF-experimenten, en de toepassing van het model op uiteenlopende experimentele markten. We voeren ook een experiment uit om de robuustheid van de LtF-opzet te onderzoeken. Het proefschrift bestaat uit drie gerelateerde, maar onafhankelijke hoofdstukken.

In Hoofdstuk 2 ontwikkelen we een model waarin agenten Genetische Algoritmen gebruiken die een algemene voorspellingsheuristiek optimaliseren om prijsverwachtingen te vormen (Hommes en Lux, 2013). We passen het model toe op vier experimentele omgevingen:

- lineaire positieve en negatieve feedback (Heemeijer et al., 2009);
• lineaire positieve en negatieve feedback met grote, onverwachte schokken op de fundamentele waarde (Bao et al., 2012);

• een niet-lineaire varkenscycluseconomie (Hommes et al., 2007; van de Velden, 2001);

• een niet-lineaire aandelenmarkt waarin agenten twee perioden vooruit voorspellen (Hommes et al., 2005).

Ons model presteert beter dan Rationele Verwachtingen, een aantal andere homogene verwachtingsregels (naïeve, adaptieve, trend-extrapolerende en ‘contrarian’-verwachtingen) en een eenvoudig Heuristiek Switchingmodel met heterogene verwachtingen. In het bijzonder heeft het model een zeer hoog voorspellend vermogen wat betreft out-of-sample kortetermijnvoorspellingen. Daarnaast is het het enige model dat in staat is individueel gedrag te verklaren en de experimentele dynamica op de lange termijn (vijftig perioden vooruit) te voorspellen. Het biedt ook een rechtvaardiging voor het gebruik van het Heuristisch Switchingmodel als een gestileerde beschrijving van experimentele dynamica. Dit is een belangrijke bijdrage aan de literatuur, aangezien beleidsmakers de laatste jaren op zoek zijn naar economische modellen met robuuste microfunderingen van gedrag.

In Hoofdstuk 3 gebruiken we het Learning-to-Forecast-model met Genetische Algoritmen voor het bestuderen van informационetwerken in financiële markten. Het doel hiervan is om het effect van informatiestromen op marktstabiliteit te evalueren. We concentreren ons op een niet-lineair model voor aandelen waarin agenten twee perioden vooruit voorspellen. De agenten leren of ze de geobserveerde prijstrend moeten extrapoleren, en ook of ze moeten vertrouwen op de gemiddelde gemoedstoestand van de vrienden die ze observeren in het netwerk. We laten zien dat zonder een netwerk agenten kunnen coördineren op ofwel de fundamentele waarde, ofwel trendvolgend gedrag. Dit laatste doet de markt overschakelen van de fundamentele oplossing naar grillige prijsschommelingen. Het toevoegen van een netwerk, ongeacht de architectuur of grootte, destabiliseert de markt, die vrijwel nooit in de fundamentele oplossing blijft. Dit volgt uit twee gerelateerde observaties. Ten eerste leren de agenten te coördineren op sterkere trendvolgende regels. Ten tweede leren de agenten ‘contrarian’ gedrag: ze zijn optimistisch als ze observeren dat hun vrienden in het verleden verkochten en vice versa. De reden voor het contrarian gedrag is dat de agenten proberen hun vrienden te slim af te zijn. Een voorbeeld hiervan is dat op het moment dat de zeepbel in elkaar begint te klappen een agent zich bewust wordt van een negatieve trend, maar ook van het feit dat haar vrienden tot aan het hoogtepunt van de zeepbel het aandeel hebben.
gekocht. Ze heeft daarom een prikkel om aandelen te verkopen, m.a.w. te handelen in tegenstelde richting van haar vrienden.

Het model voorspelt dat de agenten – ondanks hun contrarian houding – leren een gelijksoortig gedrag aan te nemen en daardoor goed gecoördineerd blijven. Dit is een belangrijk inzicht voor de tegenstrijdige opvattingen die in de literatuur bestaan over kuddegedrag. Experimenten tonen aan dat personen contrarian strategieën gebruiken (d.w.z. anti-kuddegedrag), hetgeen inderdaad overeenkomt met het gedrag in onze simulaties. Aan de andere kant leidt een gelijksoortig leerproces van de *contrarian* houding ertoe dat agenten een *gelijksoortig gedrag* aannemen. Dit kan ten onrechte geïnterpreteerd worden als kuddegedrag.

Hoofdstuk 4 doet verslag van een experiment gebaseerd op een eenvoudig lineaire aandelenmarkt, waarin proefpersonen gevraagd werd prijzen te voorspellen, het aandeel te verhandelen, of beide. Het belang van dit experiment is dat het een verbinding legt tussen Learning-to-Forecast- en Learning-to-Optimize-experimenten. Het kan daarom dienstdoen als een nuttig referentiepunt voor toekomstige theoretische en experimentele onderzoeken naar leren in aandelenmarkten.

We vinden dat in elke variant van het experiment proefpersonen coördineren op niet-fundamentele uitkomsten: ofwel stabiele prijzen ver van de fundamentele waarde, ofwel aanhoudende prijsoscillaties. De Learning-to-Optimize- en gemengde variant van het experiment zijn instabiliseren; de grootste zeepbel (op het hoogtepunt ongeveer 3.5 keer de fundamentele waarde) vond plaats bij een van de groepen in de gemengde variant. Deze resultaten laten zien dat het leren om te optimaliseren voor de proefpersonen nog moeilijker is dan het leren om te voorspellen. Bovendien blijken de resultaten van de LtF-experimenten robuust te zijn, in de zin dat prijsoscillaties niet slechts een artefact zijn van de versie van het experiment waarin proefpersonen voorspellen. We vinden ook bevestiging voor statistisch significante heterogeniteit in de gedragsregels van de proefpersonen.

Het meest verrassende resultaat komt uit de gemengde variant van het experiment waarin zowel de transacties als de corresponderende prijstoetsing geobserveerd worden. Hierdoor kunnen we expliciet toetsen voor hun consistentie. We vinden dat slechts een kwart van de proefpersonen optimaal handelt conditioneel op hun geïmpliceerde verwachtingen van het rendement van het aandeel. Dit is een empirisch bewijs tegen het idee dat economische agenten perfecte optimaliseerders zijn — een populair idee, zelfs onder voorstanders van begrensd rationele prijstoetsing. Nader onderzoek zal zich richten op theoretische modellen van het leren om te optimaliseren, en de verbinding hiervan met het leren om te voorspellen.
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