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Pre-emptive Policy for Systemic Banking Crisis

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Abstract

Systemic banking crises often result from widespread imprudent lending, driven by strong incentives for risk taking and connected lending. This paper identifies a counterbalancing incentive for individual banks to act prudently in the face of widespread risk taking among its competitors. In general, the value of a banking charter is enhanced by reduced competition. Hence a deliberate policy of promoting takeovers of weaker institutions by solvent banks has the effect of increasing the charter value of solvent banks, and grants the managers of better banks an incentive to pursue less profitable but safer lending strategies, thus breaking down the strategic externality of risk-taking strategies. A temporary phase of concentration in banking can thus reinforce stability and pre-emptive closures may in fact reduce the risk of a systemic banking crisis. We also address the case where banking authorities face pressure for an ex post bailout in a context where many banks are in trouble at the same time.
1 Introduction

Economists appreciate competition as a powerful source for efficiency. This in principle applies to the financial sector as well as any other industry. There is no question that increasing competition has played a large role in reducing the costs of financial intermediation. The process of global financial integration has led to direct and indirect entry of new intermediaries and sources of finance and helped reduce the cost of capital. However, the recent experience has seen banking crises arise in many developed countries and developing countries following liberalization and/or deregulation, both of which result in greater competition and entry (Sweden, Finland, Russia, as well as many countries in South East Asia and Latin America). Caprio and Klingebiel (1996) argue that the frequency of banking crisis has been increasing. This has raised concerns that rapid increases in competition in banking may have undesirable consequences due to the special characteristics of the industry. Increasingly, arguments have been raised for limiting the pace of liberalization, especially in developing countries.\(^1\)

To understand the argument it is useful to consider the nature of credit transactions. Banks lend at an agreed price above their cost of funding, but profits are realized only after a delay, contingent on future circumstances. This delay has two effects. The first is that it is difficult for regulators to monitor bank lending decisions ex ante. The second is that bank assets are illiquid in the short term, unlike their liabilities, exposing them to the risk of (self-fulfilling) bank runs (Diamond and Dybvig, 1986).

\(^1\)The early history of banking in Western countries offers some parallels. The UK, France and the US all had a monopolistic bank in their early year of financial development; in the US at the beginning of the last century there were only four chartered banks, and the general view was that limited competition was essential for stability. Arguably, in those years the institutional capacity for regulation and enforcement were much weaker also in these countries.
The regulatory response to the short term illiquidity risk caused by bank runs has been the development of deposit insurance as well as the role of the central bank as a lender of last resort. This has freed banks from the risk of bank runs but has created a countervailing moral hazard problem, namely, the temptation for banks to take excessive risks in lending without suffering increased funding costs.

Certainly, if regulators were to allow individual bank failures while bailing out only depositors (and standing as lenders of last resort to avoid a confidence crisis among other banks), there would be a partial disciplining effect discouraging risk taking. In itself, this is not sufficient. Risk taking is attractive for shareholders of highly leveraged firms even when they may face bankruptcy (Jensen and Meckling, 1976). Banks are highly leveraged institutions managing a resource, credit, which is very easily misallocated in the short term with consequences which become visible only in the medium term. In this paper we focus on circumstances where prudential supervision is ineffective, due to enforcement or information problems. When direct supervision fails, indirect measures may need to be implemented. Hellman, Murdoch, and Stiglitz (1999) argue that limits on deposit rates may be the most effective way to ensure significant charter value while at the same time reducing the marginal incentive for risk taking.

The literature on regulatory monitoring and intervention in banking usually models an ex ante optimal policy which is worse than worst best. Both Aghion et al. (1998) and Mitchell (1999) argue that there is an optimum degree of regulatory intervention. The reason is that an excessively tough intervention policy leads to bankers hiding problem loans: rolling over of credit to insolvent companies causes a further deterioration of collateral value. An excessively soft approach on the other hand leads to a lack of incentives for
monitoring and thus expensive rescues. Thus it may be better to choose an intermediate degree of regulatory intervention to increase the rate of monitoring or the disclosure of poor performance.\(^2\) A general feature of this approach is that it describes intervention in an individual bank default.

In circumstances when regulatory intervention in an individual bank failure creates external effects, the optimal policy should take into account the structure of the banking system. While the ultimate form of supervisory discipline is to allow a bank to go bankrupt, in practice, it is rare to observe bank closures. Central Banks typically intervene to bail out failing institutions, claiming the risk of a domino effect: if one bank is allowed to go under, depositors of other banks (or alternatively, uninsured bank lenders on the interbank market) may panic and demand immediate repayment of their deposits (or their outstanding loans and lines of credit), precipitating a liquidity and ultimately a confidence crisis.\(^3\) On the other hand, this approach too often bails out shareholders in addition to depositors, and reinforces the basic moral hazard risk.

A second reason to take a broader view is that bank closures and mergers affect future competition. Clearly, a more competitive environment reduces rents and thus make the risk-taking option more attractive. The literature has recognized this effect of ex ante competition. If the cost of risk-taking is the risk of failure, it would lead to the loss of the rents associated with a banking license (i.e. its charter value); see Suarez (1999), Matutes and Vives (1995 and 1996). Yet the value of the charter value depends on future

\(^2\)Aghion et al. show that the transfer to a bank which declares insolvency should be nonlinear to discourage dissimulation. It would be interesting to study the best policy if some insolvent banks choose to escape intervention and/or the transfer is insufficient (i.e. the banks ultimately fail).

\(^3\)Freixas (1999) argues that the optimal policy should be ambiguous, i.e. the authorities should follow a mixed strategy; this approach is usually termed "constructive ambiguity". We obtain a similar result in our optimal market structure strategy.
competition, which depends on the form of the ex post policy intervention.

In this paper we take the view that it is less the initial degree of competition in the industry at the time when the loans are made, as much as the anticipated future degree of competition, that affects the critical risk-taking decision. In practice, banking regulators often force mergers of bailed out banks, implicitly preferring stability over competition (or the liquidation of all trouble banks). A possible explanation is that outright bank closures may destroy informational capital on borrowers (Mailath and Mester, 1994). It is also possible that central bankers, which are often in charge of banking supervision as well, may prefer stability over competition for their own sake.

In our model the bank regulator, in the face of bank insolvency, may use a deliberate bank closures and merger policy as an incentive structure to discourage speculative banking. We consider an oligopolistic banking industry in which there are incentives for risk-taking. We identify two policy instruments: the choice of the long term regulatory framework, which affects the rate of potential entry and thus the future degree of competition; and the ex post merger policy of allowing takeovers of failed banks by solvent institutions, which comes at the cost of reduced competition.

In bad states of the economy, banks which choose for speculative lending will become insolvent. The closure and banking competition policy affects then not just a banker’s incentive to lend safely, but also his competitor’s. The banking supervisory authorities trade off the deadweight losses of (temporary) monopoly against the gains in dissuading speculation.

The results indicate that an active use of competition policy can have a powerful effect on the stability of the banking system. While we believe that this results applies best to countries with weak regulatory frameworks, we are convinced that its application extends to the established practice in
developed countries as well.\footnote{In many cases, bailouts are often conducted by influencing the determinants of profitability for the whole industry, as when the central bank sharply decreases short term rates to reduce funding costs for the whole industry (one much cited example was the Fed policy in the US recession of 1991-92). In this case the losses are socialized in a less visible manner, and they come to benefit also solvent banks. For these reasons we view this approach as inferior.}

In the model we focus explicitly on bank competition in the deposit market. While competition in lending is also important, its effects are very model-sensitive, as they depend on the informational barriers to entry and the appropriability of information itself.\footnote{We are also convinced that in practice bank mergers (including those among solvent banks) are accompanied by branch closures and consolidation for the twin purpose of cost reduction and increased market power.} Increases in competition may destroy incentives for ex ante investment by banks in monitoring and information gathering. Caminal and Matutes (1999) show that some degree of market power is needed to ensure proper monitoring of borrowers. Anand and Galeâ€™s (and that only a (collusive) oligopolistic market can support information gathering (1997). These results have received support in the empirical evidence (e.g. Petersen and Rajan, 1995). Yet Schnitzer (1998) shows that competition does not reduce the bank’s ex ante incentive to monitor (screening) in the case in which the information gathered remains private.\[There are papers (Broecker, 1990; Riordan, 1993) that in the context of common value auctions with independent signals have shown that competition can be damaging for the market for loans since it exacerbates the winner’s curse problem.\]

The effect commented here goes in a direction opposite to the effect modelled so far. The strategic interaction so far makes a bank more willing to take risk the less risky its opponent is. That is, risk taking decisions are strategic substitutes. This is why the risk-taking game has a unique equilibrium that typically involves mixed strategies. A additional idea that...
we plan to model addresses the recognition that the incentive to take risk exists not only at the level of individual banks, but may actually be reinforced at the level of industry. In other words, it is often the case that risky lending (which we take to include rescheduling of doubtful loans) is more desirable for an individual bank when other banks are expected to be doing the same.

There are at least two possible sources of this strategic externality. The rst is the ability of bankers to postpone recognition of bad loans to announce them simultaneously to avoid being singled out for poor performance (Rajan, 1994). The second cause is what is described in Mitchell (1998) as the “too many too fail” effect, or TMTF. This phenomenon was studied rst in the context of transition economies by Perotti (1998) and Mitchell (1998). The basic idea is that when many institutions (either borrowers or lenders) face pressure for costly adjustment, their incentive to comply may depend on the expected strategy by others, since authorities may be unable to force a very large number of defaulters into bankruptcy. This inability may either arise because of logistical limits to enforcement, as it may have been the case for bankruptcy reform in Hungary in the early 90s, or because of the political pressure exercised by an united front, as in the case of trade and bank arrears in Eastern Europe at the beginning of transition or in the banking crises of Russia, Mexico and South East Asia. In some other cases, it is the fear of fire sale liquidation of collateral which could further affect the solvency of other banks which makes the threat to close not credible ex post.

The structure of the paper is as follows. The rst section outlines the basic model and presents results on the optimal long term regulatory and intervention policy. The second section offers some extensions.
2 The model

Time is continuous and indexed by $t$: All agents are risk neutral and in nitely lived, and discount time at the rate $r$.

We consider a banking industry made up of two bank branches. At any point in time there may be either one or two active bankers extracted from a large population of potential bankers. With two active bankers, each owns and manages one branch, so the industry is a duopoly, while with a single active banker the industry is a monopoly.

Each bank branch manages one unit of insured deposits taken from neighboring depositors. These funds can be invested in either safe lending or risky lending. In a duopoly, safe lending yields a profit of $\frac{1}{4}$ per branch and unit of time. In contrast, in a monopoly, safe lending yields an extra profit of $\frac{1}{2} \times $ per branch and unit of time, where $\frac{1}{2} > 0$ captures the rents left by the absence of competition in deposit taking. We assume that these rents come at a cost in terms of depositors’ surplus of $(1 + \zeta)\frac{1}{2}$ per branch and unit of time, where $\zeta > 0$.6

Under any market structure, risky lending adds an extra profit return of $0.1\%$ per branch and unit of time in solvent periods, but leaves the bank exposed to solvency shocks. Solvency shocks occur randomly according to a Poisson process with arrival rate $\lambda$ and generate capital losses on risky lending equivalent to a fraction $\frac{3}{4} < 1$ of the managed funds. We assume that

$$0.1\%; \quad \frac{3}{4} < 0;$$

so the incremental expected return from risky lending over safe lending is negative.

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6In Perotti and Suarez (1999), such profits are endogenized as a result of spatial competition. For simplicity, we adopt here a more streamlined reduced form.
Whenever a bank becomes insolvent, a banking authority intervenes, dismisses the failed banker, and contributes \( \frac{1}{4} \) per failed branch so as to fully pay back to its depositors. At that point the authority must decide who will own and manage the branches of the failed bank. We assume that when all the incumbent bankers fail, the authorities will choose for duopoly, since in this case there is no scope for rewarding either banker and more competition produces a higher social return. In contrast, we consider the possibility that when only one of the duopolist banks fails, the supervisory authorities will allow the solvent banker to take over the failed branch as a reward for solvency. This can take the form of a merger with management passing to the stronger bank.

We denote by \( \pi \) the probability of such a policy converting the survivor into a (temporary) monopolist. We next analyze whether this policy parameter \( \pi \) can be a useful “carrot” in encouraging bankers to lend safely.

We think of monopoly, however, as an intrinsically transitory market structure whose high rents will eventually lead to further entry. We assume, in particular, that in monopoly the entry of a new banker occurs according to a Poisson process with arrival rate \( \lambda \). When entry occurs, the incumbent banker loses one of its branches in favor of the entrant and the industry becomes a duopoly again. In contrast, we view a solvent duopoly as a stable long-run market structure of the industry, that is, such that rents are low enough for no further entry to take place.

Clearly, entry is in part the result of regulatory policy. We think of both \( \pi \) and \( \lambda \) as long term policy parameters: we assume that supervisory authorities can credibly commit to such a policy as the outcome of a stable, long term regulatory policy.

The first parameter \( \pi \) relates to rescue practices: we argue that repeated
crises resolution allow authorities to develop a reputation for rewarding the solvent incumbents with a given probability. The second parameter relates to entry and, more generally, competition policies. In an environment where potential new bankers face uncertain entry costs, bank authorities may affect the entry rate through the stringency of regulatory entry requirements or through their (in)tolerance towards incumbents’ entry deterrence strategies.

3 Equilibrium

The ingredients described above define a simple, stochastic game in continuous time. At any date $t$ the market structure that prevails in the banking industry is indicated by the state variable $s_t = M; D$; where $M$ denotes monopoly and $D$ denotes duopoly. In monopoly dates, the single banker plays against nature, deciding how to lend the deposits managed by his two branches. In duopoly dates, there are two bankers, one at each branch, deciding how to lend their respective deposits. These simple stage games are repeated until the arrival of a solvency shock, at any date, or an entrant, in a monopoly date, produces the failure of one of the existing banks and/or modifies market structure. When a bank fails, the banker is dismissed and exits the game. In the next period, the game continues with the survivor banker and/or the new bankers who replace the failing ones.

In the analysis of the dynamic game, we restrict attention to Markov strategies, that is, we assume that the past influences current play only through its effect on the state variable $s_t$. This is consistent with our interest in a steady-state policy framework in which the credibility of the supervisory policy has been established.

The state variable $s_t$ summarizes the effect of history on payoff functions and action spaces. The Markov lending strategy of a banker is a time-
invariant pair \((m; d) \in [0; 1] \times [0; 1]\) that (allowing for mixed strategies) specifies the probability that the banker gets involved in risky lending while in monopoly and duopoly, respectively. Given the time-invariant nature of the problem, all time indices are dropped hereafter.

Let \(v_M\) and \(v_D\) denote the value of being a monopolist and a duopolist, respectively. Then the instantaneous return from being a monopolist is given by the Bellman equation:

\[
rv_M = \max_{m \in [0; 1]} \left[ 2 \left( 1 + \frac{1}{2}\cdot m \right) \frac{1}{4} \cdot v_M m \right] 
\]

Notice that the first terms in the RHS collects the stage profits from (safe or risky) lending, the second represents the expected capital losses due to dismissal if the solvency shock arrives, and the third accounts for the expected capital loss from becoming a duopolist if an entrant arrives. The multiplication by two reflects the fact that the monopolist banker owns two bank branches.

To derive a similar expression for \(v_D\); let \(d^*\) denote the lending strategy of the competitor in duopoly. Then

\[
rv_D = \max_{d \in [0; 1]} \left[ (1 + d) \frac{1}{4} \cdot v_D d + d^*(1 - d) \cdot (v_M m - v_D) \right]
\]

where the first and second terms can be interpreted exactly as in (2), whereas the third accounts for the expected capital gain that the duopolist obtains from becoming a monopolist if, at the arrival of a solvency shock, his competitor fails, but he survives and gets control of the failed branch.

We can now formally prove that \(v_M > v_D\): Suppose, on the contrary, that \(v_M < v_D\): Then, given the signs of the third terms in the RHS of (2) and (3), we would have

\[
rv_M > \max_{m \in [0; 1]} \left[ 2 \left( 1 + \frac{1}{2}\cdot m \right) \frac{1}{4} \cdot v_M m \right] > \max_{m \in [0; 1]} \left( (1 + m) \frac{1}{4} \cdot v_M m \right)
\]
and

$$r v_D \max_{d \in [0;1]} [(1 + \delta d)^{1/4} \cdot v_D d] < \max_{d \in [0;1]} [(1 + \delta d)^{1/4} \cdot v_M d]:$$

But this implies $$r v_M > r v_D$$; which is a contradiction.

In what follows we will constrain attention to the unique symmetric Markov Perfect Equilibrium (MPE) of our game.\(^7\)

**Definition 1** An equilibrium is a lending strategy \((m; d)\) that solves the Bellman equations (2) and (3) for \(d^* = d\):

In the remaining of this section we characterize such an equilibrium in a constructive way.

### 3.1 Monopoly

The contribution of risky lending to the value of being a monopolist is captured by the terms multiplied by \(m\) in (2). The trade-off is between the excess return \(2 \xi\) and the expected capital loss \(\nu_M\) that associate with risky lending. The term \(\nu_M\) captures the usual effect of charter values on risk-taking: the fact that in case of failure the banker loses his bank. The optimal choice of \(m\) can then be described as

$$m = \begin{cases} 0 & \text{if } 2^{\sigma_4} \nu_M < 0; \\ [0; 1] & \text{if } 2^{\sigma_4} \nu_M = 0; \\ 1 & \text{if } 2^{\sigma_4} \nu_M > 0; \end{cases}$$

(5)

Of course \(\nu_M\) is endogenous and determined simultaneously with \(v_D\): For the time being, however, consider \(v_D\) as given. It follows from (2) that if \(m = 0\) then

$$v_M = v_M^0(v_D) \cdot \frac{2(1 + \frac{1}{4} \cdot 4 + \nu_D)}{r + \nu}$$

(6)

\(^7\)This equilibrium concept involves a symmetric Nash equilibrium in every proper subgame.
while if \( m = 1 \) then
\[
v_M = v_0^1(v_D), \quad 2(1 + \frac{1}{2} + \omega) \frac{1}{2} + \frac{v_D}{r + \pm +},
\]

Moreover, the conditions \( 2^\omega \frac{1}{4} \), \( v_M^0(v_D) = 0; 2^\omega \frac{1}{4} \), \( v_M^1(v_D) = 0; \) and \( v_M^0(v_D) = v_M^1(v_D) \) are equivalent and define a unique critical value
\[
v^\omega = 2^{\frac{1}{4}}(r + \pm \omega i, (1 + \frac{1}{2})},
\]
such that, by (5),
\[
\begin{align*}
8 & = 0; \quad \text{if } v_D > v^\omega; \\
m & = 2 [0; 1] \quad \text{if } v_D = v^\omega; \\
\cdot & = 1; \quad \text{if } v_D < v^\omega;
\end{align*}
\]
This fully characterizes the equilibrium choice of \( m \) conditional on \( v_D \):

3.2 Duopoly

The contribution of risky lending to the value of being a duopolist is measured by the terms multiplied by \( d \) in (3). As in the case of monopoly, there is a trade-off between the excess return \( \zeta \) and the expected capital loss \( v_D \) that associate with risky lending.

The critical feature of the rescue policy is that with some probability \( \xi \) it leads the surviving bank to become a monopolist: this introduces a strategic interaction between the lending strategies of the competing banks. The larger is the difference \( v_M - v_D \) and the riskier is the lending strategy \( d^\omega \) followed by the competitor, the greater are the incentives of a duopolist to remain safe. Hence the risk-taking decisions of duopolists are strategic substitutes. Specifically,
\[
\begin{align*}
8 & = 0; \quad \text{if } \zeta \ i, v_D \ i, d^{\rho} (v_M \ i, v_D) < 0; \\
d & = 2 [0; 1] \quad \text{if } \zeta \ i, v_D \ i, d^{\rho} (v_M \ i, v_D) = 0; \\
\cdot & = 1; \quad \text{if } \zeta \ i, v_D \ i, d^{\rho} (v_M \ i, v_D) > 0;
\end{align*}
\]
Recall that we have to account for the simultaneous determination of \( v_M \) and \( v_D \): From (3), however, \( v_D \) can only take two values. Clearly, if \( d = d^a = 0 \) then
\[
v_D = v_D^0 \cdot \frac{\sqrt{4}}{r},
\]
while if \( d = d^a = 1 \) then
\[
v_D = v_D^1 \cdot \frac{(1 + \circ) \sqrt{4}}{r + \circ}.
\]
Moreover, the linearity of the maximand in (3) implies that, for a given \( d^a \), if some \( d \in (0; 1) \) is optimal, then any other \( d \) is also optimal. But this includes \( d = 1 \); which leads to \( v_D = v_D^1 \): Hence in (mixed strategy) equilibria with \( d = d^a \in (0; 1) \) we will have \( v_D = v_D^1 \):

3.3 Equilibrium for low \( \circ \)

Let \( \circ^0 \) denote the maximum value of \( \circ \) that, given (10) and (11), is compatible with having \( d = d^a = 0 \):
\[
\circ^0 \cdot \frac{\sqrt{4}}{r};
\]
(13)
We can prove the following result.

Proposition 1 Suppose \( \circ \leq \circ^0 \). Then:

1. If \( \pm 2r \sqrt{2} \) the equilibrium features \((m; d) = (0; 0)\):

2. Otherwise, there is a critical value \( \circ^0 \) such that the equilibrium features \((m; d) = (0; 0)\) for \( \circ \leq \circ^0 \) and \((m; d) = (1; 0)\) for \( \circ > \circ^0 \):

\[
\circ^0 \geq \frac{2r + 2r \sqrt{2} \pm \sqrt{2r + 2\pm}}{2r + 2\pm} < 1
\]
(14)
Proof By construction $\beta = 0$ implies $d = 0$ and $v_D = v_D^0$: Determining the equilibrium value of $m$ requires comparing $v_D^0$ with $v^\alpha$: It follows from (8) and (11) that $v_D^0$, $v^\alpha$ is equivalent to $\beta = 0$: Hence, when $\beta = 1$; (9) implies $m = 0$ for all $\beta < 0$: Otherwise, (9) implies $m = 0$ for $\beta = 0$ and $m = 1$ for $\beta < 0$.

Proposition 1 says, in words, that whether the duopolist bankers take risk or not depends basically on $\beta$; while for low $\beta$ whether the monopolist banker takes risk or not depends on how much entry he faces.

3.4 Equilibrium for high $\beta$

With $\beta > 0$ the equilibrium necessarily involves $d > 0$: Hence $v_D = v_D^1$ and the equilibrium value of $m$ can be immediately characterized using (9).

Lemma 1 Suppose $\beta > 0$: Then

1. If $\beta > 2r^{1/2}$ the equilibrium features $m = 1$:

2. Otherwise, there is a critical value

$$\bar{\beta} = \left(2r + 2r^{1/2} + r\pm r^2 + 1\right) + r^2 > 1$$

such that the equilibrium features $m = 0$ for $\beta < 0$ and $m = 1$ for $\beta > 0$.

Proof Given (9), determining the equilibrium value of $m$ requires comparing $v_D^0$ with $v^\alpha$: It follows from (8) and (12) that $v_D^0$, $v^\alpha$ is equivalent to $\beta = 0$: When $\beta > 2r^{1/2}$ we have $\beta = 1$ so $m = 1$ for all $\beta < 0$: Otherwise, $\beta > 1$ so $m = 0$ for $\beta < 0$ and $m = 1$ for $\beta > 0$.

The discussion can now be split in two cases, depending on whether the combination of $\beta$ and $\beta$ makes the monopolist bank willing or not to take risk.
The following result characterizes the equilibrium for the case in which \( \bar{\theta} > \max f^{0,0} \).

**Proposition 2 (risky monopolist)** Suppose \( \bar{\theta} > \max f^{0,0} \). Then, there is a critical value
\[
x = \frac{(r + \bar{\theta} + \bar{\gamma})(r^0 \bar{\theta} \bar{\gamma})}{\bar{\gamma} (r + \bar{\theta})(1 + 2^{1/2} + \bar{\gamma})}
\]
where \( \bar{\gamma} \) is

\[
y = \frac{(r + \bar{\theta}) (r^0 \bar{\theta} \bar{\gamma})}{\bar{\gamma} (r + \bar{\theta})(1 + 2^{1/2} + \bar{\gamma})}
\]

such that the equilibrium lending strategy is \((m; d) = (1; \min f^0; 1g)\):

**Proof** With \( \bar{\theta} > \max f^{0,0} \) we necessarily have \( d > 0 \); \( vD = vD^3 \) and \( m = 1 \) (see Lemma 1). To find the equilibrium value of \( d \); let \( x \) denote the unique solution to the equation
\[
\beta^0 = 0
\]
whose explicit expression appears in (16). Notice that \( \bar{\theta} > 0 \) implies \( \beta^0 > 0 \), and \( vD^3 > 0 \) which, together with \( vM(vD) \); \( vH > 0 \); guarantees \( x > 0 \). Yet \( x \) can be greater or smaller than 1: If \( x > 1 \); we can substitute \( d^u = 1 \) in equation (10) and complete the proof that the equilibrium of our Markov game is \((m; d) = (1; 1)\). Otherwise, the equilibrium is \((m; d) = (1; x)\); since neither \( d = d^u = 0 \) nor \( d = d^u = 1 \) satisfy (10), while \( d^u = x \) makes the duopolists indifferent towards the different possible choices of \( d \); including \( d = x \).

The last result in this section completes the characterization of the equilibrium by considering the case in which \( \theta^0 < \theta \).

**Proposition 3 (safe monopolist)** Suppose \( \theta^0 < \theta \); \( 2r^{1/2} \) and \( \theta^0 < \theta \). Then, there is a critical value
\[
y = \frac{(r + \bar{\theta}) (r^0 \bar{\theta} \bar{\gamma})}{\bar{\gamma} (r + \bar{\theta})(1 + 2^{1/2} + \bar{\gamma})}
\]

such that the equilibrium lending strategy is \((m; d) = (0; \min f^0; 1g)\):
Proof With $\pm 2r^{1/2}$ and $\circ^{0} < \circ^{-0}$ we necessarily have $d > 0$; $v_D = v_D^{1}$ and $m = 0$ (see Lemma1). To find the equilibrium value of $d$; let $y$ denote the unique solution to the equation

$\circ^{0} v_D^{1} y + y^{4} [v_M^{0}(v_D^{1}) - v_D^{1}] = 0$;

whose explicit expression appears in (17). Notice that $\circ^{0} < \circ^{-0}$ implies $\circ^{1/2} v_D > 0$ and $v_M^{0}(v_D^{1}) - v_D^{1} > 0$; guaranteeing $y > 0$. Yet $y$ can be greater or smaller than 1: If $y > 1$; we can substitute $d^{\alpha} = 1$ in equation (10) and complete the proof that the equilibrium is $(m; d) = (0; 1)$. Otherwise, the equilibrium is $(m; d) = (0; y)$; since neither $d = d^{\alpha} = 0$ nor $d = d^{\alpha} = 1$ satisfy (10), while $d^{\alpha} = y$ makes the duopolists indifferent towards the different possible choices of $d$; including $d = y$.

It is worth noting that both $x$ and $y$ may or may not take values larger than one over the relevant parameter configurations. For this reason we do not emphasize the corner solutions involving $d = 1$.

3.5 Equilibrium regimes

Figure 1 depicts four different areas in the $\pm \circ$ space, each corresponding to qualitatively different equilibrium regimes. Notice that the parameter space is divided vertically by the line $\pm = 2r^{1/2}$ and horizontally by the line $\circ^{0} = \circ^{0}$. The resulting NW and SE quadrants are further subdivided by the curves $\circ^{0} = \circ^{0}$ and $\circ^{0} = \circ^{-0}$; where notice that $\circ^{0}$ and $\circ^{-0}$ are, from (14) and (15), decreasing functions of $\pm$ that pass through the point $(2r^{1/2}, \circ^{0})$: The values of $(m; d)$ assigned to each region directly come from Propositions 1-3.

The most interesting areas in Figure 1 are those above the $\circ^{0} = \circ^{0}$ line. Elsewhere we have $d = 0$ no matter the value of $\pm$ so an ideal combination of competition among bankers and safe lending might be achieved by either starting from an ever-lasting duopoly state or by fixing a high entry rate that
quickly moves the banking sector from, say, an initial monopoly state to an ever-lasting duopoly state. With $\gamma > \gamma^0$; however, duopolist bankers take some risk and, if either $\pm$ or $\gamma$ are sufficiently large, monopolist bankers do as well ($m = 1$). With $d > 0$ intervention and competition policies have a non-trivial impact on both risk taking and competition: they affect duopolists’ lending policies, $x$; as well as the frequency with which duopoly and monopoly emerge as a state of the banking industry. Hereafter we focus on cases with $\gamma > \gamma^0$: The following table summarizes the impact of the various parameters of the model on the equilibrium value of $d$ when $d \in (0; 1)$:

$$r \gamma^0 \frac{1}{2} \pm \frac{1}{i}$$

The signs reported correspond to those of the partial derivatives of $x$ and $y$ (which always coincide) with respect to the specified parameter. The intuition behind them is quite obvious.
4 Optimal competition and rescue policies

Competition and rescue policies are described by a pair \((±; ¹) \in [0; 1] \times [0; 1]\). Every pair \((±; ¹)\) induces a unique symmetric equilibrium \((m; d)\) in the game described in the previous section. An optimal policy \((±; ¹)\) will minimize the social loss function \(L\) that adds up (i) the deadweight losses associated with the incidence of monopoly, (ii) the expected return losses associated with risky-lending. Relative to a duopoly, a monopoly produces a flow of deadweight losses of \(±_{1/4}\) per branch and unit of time. Relative to safe lending, risky lending involves an extra flow return of \(0_{1/4}\) per branch and unit of time and an expected capital loss of \(0_{3/4}\) per branch and unit of time, so it associates with net expected losses of \(0_{3/4} - 0_{1/4}\) per branch and unit of time.

Denote by \(′_M\) and \(′_D\) the expected durations of states \(M\) and \(D\); respectively. These are simply obtained from the inverse of the Poisson rates at which the transition between states occurs. The industry will switch from \(M\) to \(D\) if the monopolist becomes insolvent or the entry of a competitor occurs, so

\[
′_M = \frac{1}{m + ±}.
\]

The industry will switch from \(D\) to \(M\) if one of the duopolist fails but the other survives and gets the branch of the failing one, so

\[
′_D = \frac{1}{2^{1, (1 i d)}}.
\]

With these durations the relative frequency of state \(M\) along the history of the industry will be

\[
A = \frac{′_M}{′_M + ′_D} = \frac{2^{1, (1 i d)}}{2^{1, (1 i d) + (m + ±)}}.
\]

Notice that the frequency of state \(M\) is decreasing in \(m\) and increasing in \(d\) if \(d < 1/2\) and decreasing in \(d\) if \(d > 1/2\): Actually, with either \(d = 1\) or \(d = 0\); \(D\) is an absorbing state of the industry.
We can now write down an expression for the social loss function $L$:

$$L = 2\frac{1}{2}A + 2\left(\frac{3}{4}i - \frac{1}{4}\right)[A + (1 - A)d].$$

(18)

Thus, the policy parameters $\pm$ and $^1$ affect $L$ through $m$, $d$, and $A$: Moreover,

$$\frac{\partial L}{\partial m} = 2\left(\frac{3}{4}i - \frac{1}{4}\right)A > 0;$$

$$\frac{\partial L}{\partial d} = 2\left(\frac{3}{4}i - \frac{1}{4}\right)(1 - A) > 0;$$

$$\frac{\partial L}{\partial A} = 2\frac{1}{2} + 2\left(\frac{3}{4}i - \frac{1}{4}\right)(m - d);$$

Notice that if $m = 1$ then $\frac{\partial L}{\partial A} > 0$; else $\frac{\partial L}{\partial A} > 0$ if and only if $d < \frac{1}{2} \frac{1}{4} \frac{1}{2}$.

Within each of the regions in Figure 1, $m$ is constant. Moreover, Figure 1 is invariant to $^1$: Hence, constraining attention to the cases with $^0 > ^0$; only changes in $\pm$ that lead to crossing the line $^0$ will change $m$ from 0 to 1 or vice versa. Such changes associate with jumps in $L$ and may produce a corner solution for $\pm$ at its largest value compatible with $m = 0$ (that is, at the point of the line $^0$ that corresponds to the prevailing $^0$).

For constant $m$, the marginal effects of the policy parameters on the loss function can be decomposed as

$$\frac{dL}{d\pm} = \frac{\partial L}{\partial A} \frac{\mu A}{\partial \pm} + \frac{\partial L}{\partial d} \frac{\partial d}{\partial \pm} + \frac{\partial L}{\partial A};$$

$$\frac{dL}{d^1} = \frac{\partial L}{\partial A} \frac{\mu A}{\partial ^1} + \frac{\partial L}{\partial d} \frac{\partial d}{\partial ^1} + \frac{\partial L}{\partial A};$$

The first term in each equation collects the effects that operate through the frequency of state $M$; i.e., both the direct and the indirect (via $d$) effects of the corresponding policy parameter on $A$: The second terms collect the effects that operate through the lending policy of the duopolists.

\footnote{It is worth noting that if we constrain attention to policies with $\pm > 2r^{1/2}$ we always get equilibria with $(m, d) = (1; \min x; 1) g$; where $x$ is given by equation (16). Hence changing $\pm$ in the range $(2r^{1/2} 1)$ and $^1$ in the range $[0; 1)$ does not involve any discontinuous change in $L$.}
The results in Propositions 2 and 3 imply that \( \frac{\partial d}{\partial \pm} \geq 0 \) and \( \frac{\partial d}{\partial \bar{\chi}} \leq 0 \): Hence restricting competition by means of decreasing \( \pm \) and rewarding solvency by means of increasing \( \bar{\chi} \) decreases \( d \) and, through it, \( L \). On the other hand, decreasing \( \pm \) or increasing \( \bar{\chi} \) has a direct effect on \( \tilde{A} \) which will typically increase \( L \). This effect is reinforced (if \( d > 1-\bar{\chi} \)) or weakened (if \( d < 1-\bar{\chi} \)) by the effect on \( \tilde{A} \) of the aforementioned change in \( d \):

Obtaining analytical results about the optimal values of \( \pm \) and \( \bar{\chi} \) is difficult since the expressions involved do not lead to closed-form solutions and corner solutions cannot be generally ruled out. Numerical examples show, however, that the solution for \( \bar{\chi} \) tends to be at a corner (typically \( \bar{\chi} = 1 \)), while the solution for \( \pm \) tends to be either interior or at the maximum value compatible with \( m = 0 \):

We can illustrate the trade-offs involved in the design of the optimal policy by considering the parameterization given in the following table:

<table>
<thead>
<tr>
<th>( r )</th>
<th>( \frac{1}{4} )</th>
<th>( \frac{3}{4} )</th>
<th>( \frac{1}{2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 0:06 )</td>
<td>( 0:03 )</td>
<td>( 1:33 )</td>
<td>( 0:06 )</td>
</tr>
</tbody>
</table>

Figure 1 depicts as a bold solid line the different pairs \((d; \tilde{A})\) that describe the equilibria which can be induced by fixing \( \bar{\chi} = 1 \) and allowing \( \pm \) to vary from \( 0:05 \) to \( 1 \) (all of which involve \( m = 1 \)). It also depicts as a dashed line the pairs induced when \( \bar{\chi} \) is reduced to \( 0.5 \). Finally it shows one of the level curves of the loss function \( L \) in the \((d; \tilde{A})\) space, where higher levels are always attained by moving towards the origin. With fixed \( m = 1 \); the upper level sets in this space are always nicely convex, since the slope of the level sets is, from the differentiation of (18):

\[
\frac{d\tilde{A}}{dd} = i \left( \frac{\frac{3}{4}}{2} \right) + \left( \frac{\frac{3}{4}}{2} \right) \left( 1 \frac{1}{i} \right) < 0;
\]

whose absolute value increases as \( d \) increases and \( \tilde{A} \) decreases.
The trade-off for the choice of $\pm$ is very clear: a larger $\pm$ means a larger $d$ and a lower $\hat{A}$: Moreover, the combinations $(d; \hat{A})$ induced with $\mu = 1$ dominate those induced with $\mu = 0.5$ and, actually, with any other $\mu$, so it is optimal to $\mu = 1$: In this example the optimal value of $\pm$ is 0.29; which leads to $(m; d) = (1; 0.36)$ and $\hat{A} = 0.073$; implying that bank branches fail at an average Poisson rate of 0.025. For comparison, notice that with $\mu = 0$; the solution would involve $(m; d) = (1; 1)$ and $\hat{A} = 0$; and bank branches would fail at an average Poisson rate of 0.060:

5 Extensions

5.1 Alternative regulatory objectives

Without modification of the bankers’ lending game, the idea is to allow for a regulator whose loss function $L$ does not fully count as a social loss the
declines in industry profits associated with more competition or with less risk-taking. Suppose, in particular, that every expected unit of income of bankers carries a weight of $1 - a$; with a $2 [0; 1]$ in the regulator's social welfare function, while depositors' surplus and the cost of bank rescue packages carries a weight of $1$. Then the expression for $L$ would become:

$$L = 2[(1 + \zeta)(1_i a)]^{\frac{1}{2}} + 2[\frac{3}{4}i (1_i a)^{\frac{1}{2}}[\hat{A}m + (1_i \hat{A})]$$

Thus changing $a$ modifies the weighting of the losses associated with $\hat{A}$ and $d$.

To explore the consequences of this re-weighting, it is worth referring to Figure 1 again. The different pairs $(d; \hat{A})$ which can be induced by varying $\pm$ remain the same. Having $a > 0$ does only affect the level sets of the social loss function $L$. Now their slope becomes

$$\frac{d\hat{A}}{dd} = i \left[ \frac{3}{4}i (1_i a)^{\frac{1}{2}}(1_i \hat{A}) \right] < 0$$

and it is immediate to check that the absolute value of this slope is increasing in $a$ if and only $\frac{3}{4}i (1 + \zeta)^{\frac{1}{2}} < 0$: This condition holds in our example, which implies that increasing $a$ will lead the regulator to reduce $\hat{A}$ at the cost of larger $d$. Intuitively, a larger $a$ leads the regulator to disregard a larger part of the duopolists' gains from risky lending as a social gain, making him more inclined to reduce $d$ at the cost of larger $\hat{A}$: Figure 2 shows how fixing $a = 0.01$ in the above example shifts downwards the regulator's preferred value of $\pm$.

This suggests that in a banking industry tempted to risky lending, a regulator that cares less about banks' profits may actually carry out a more restrictive competition policy than a regulator with a greater concern about
Figure 3: Effects of $a$ on the regulator’s loss function

banks’ profits. The reason for this is that, at the relevant margin, risky lending may be more of a source of profits for banks than the rents due to a less competitive environment.

5.2 Unavailability of alternative bankers

The key equations of the bankers’ game get changed if there is a probability $\alpha$ that no banker can be found to replace a failing one. Think of $\alpha$ as an aggregate shock. If a monopolist fails this shock means he will be bailed-out: to his effects, it is as if the solvency shock had not happened. Hence (2) becomes

$$rv_M = \max_{m \in [0, 1]} \left[ 2(1 + \frac{\alpha}{2} + \delta_m) \right] \left[ \frac{1}{4} \right] (1 - \alpha) v_M m + \delta(v_M + v_D) \right];$$  

so having $\alpha > 0$ is isomorphic to having a lower $\delta$ in the baseline model. When a duopolist fails, the effect of the shock depends on whether there is a
survivor of not and what the value of $^1$ is. To simplify the discussion consider the case $^1 = 1$: Then (3) becomes:

$$rv_D = \max_{d \in [0, 1]} [(1 + \delta d)^{1/4_i} (1_i \cdot d^a) v_D d + (1 \cdot d^a(1_i \cdot d)^1 (v_M \cdot v_D))];$$ (21)

where the factor $(1_i \cdot d^a)$ accounts for the fact that if the competitor also fails and there is no external replacement for the failing banker, he will survive. Here the impact of $^\gamma > 0$ is not isomorphic to having a lower $^\phi$, in the baseline model. Actually $^\gamma > 0$ brings in some strategic complementarity: if $d^a$ is large, the duopolist is more likely to survive a solvency crisis.

My guess is that optimal policies with this effect in place may require lower $^\xi$.

### 5.3 Intrasectoral (or regional) externalities

A rather simple way to reflect them would be to modify the last extension so as to make $^\gamma$ an increasing function of $d^{\text{out}}$; where $d^{\text{out}}$ is the level of risk taken in the other sectors or regions. The underlying assumption is that if other sectors are in trouble, there will be less healthy banks somewhere else from which to “hire” new bankers. The symmetric intrasectoral equilibrium would solve a sector’s problem for given $d^{\text{out}}$; which leads to some pair $(m(d^{\text{out}}); d(d^{\text{out}}))$ and impose that an overall equilibrium is a fixed point where $d(d^{\text{out}}) = d^{\text{out}}$.

I think that the intrasectoral externality here can bring in a sufficiently strong strategic complementarity to sustain multiplicity of equilibria, including “bad” ones that display a too-many-to-fail problem.
6 Conclusions

One general feature of most models of bank insolvency is that the allocation of control rights over banks is too often not modelled, or assumed to be assigned according to bankruptcy rules even in circumstances when enforcement may be not credible. Our focus on the ex post decision on industry concentration is clearly relevant not just for shareholders, but also for banker. The banker who chooses to remain solvent while others go for broke may renounce happily some short term profits in the expectation of the chance of being asked to run a larger bank. In practice, the outcome of many bank mergers is in truth a takeover, where the managers in control of the combined bank are often the bankers who managed to remain (more) solvent.
References


