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Spin exchange and dipolar relaxation rates in atomic hydrogen: Lifetimes in magnetic traps

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We have rigorously calculated all the possible two-body rate constants associated with population dynamics of the hyperfine levels of atomic hydrogen as a function of magnetic field at $T=0$ K. These results are important in view of the recently suggested use of magnetic traps to obtain Bose-Einstein condensation.

Recently, there has been a proposal[1] for magnetic traps of neutral atoms, and Migdall *et al.*[2] have succeeded in laser cooling and magnetically trapping sodium atoms. Such developments have implications for spin-polarized atomic hydrogen where efforts have been expended to achieve Bose-Einstein condensation (BEC) below a critical temperature, $T_c=(3.31\hbar^2/mk_B)(n/g)^{2/3}$ (n is the density and g the spin degeneracy). Until now these fruitless efforts have been aimed at achieving high densities to push T_c into a comfortable temperature range (for $n=1.6\times10^{19}$ cm^{-3} and $g=1$, $T_c=100$ mK). The principal impediment arises from recombination heating of high-density samples of spin-polarized atomic hydrogen to $T>>T_c$. Samples are usually partially confined by a magnetic field ($B\simeq10$ T). However, since a static magnetic field cannot have a maximum in free space,[3] but rather a saddle point, physical walls are required to confine the sample. At the conditions for BEC, recombination of hydrogen on the walls dominates the heating. Recently, Hess[4] and Lovelace, Mehanian, Tommila, and Lee[5] have proposed magnetic traps for hydrogen which remove the necessity of physical walls. In this paper we present as a function of magnetic field the results of exact, coupled-channel calculations of the two main modes of decay from these traps: magnetic dipole relaxation and spin exchange. As the traps are designed for very low temperatures, $T\lesssim30$ mK, the results presented will be limited to $T=0$ K.

To facilitate discussion of the traps we show the hyperfine states of hydrogen as a function of magnetic field in Fig. 1. In high field the two lower states labeled a and b are predominantly electron spin down, whereas c and d are predominantly electron spin up. A gas comprised only of states a and b is referred to as H$\downarrow$. These atoms have the lowest energy in the highest fields and are high-field seekers. A gas of c and d states only is called H$\uparrow$; these atoms are low-field seekers. Hess[4] has proposed a static trap for H$\uparrow$ which confines the c and d atoms in a trap with a field minimum. Lovelace *et al.*[5] propose to trap H$\downarrow$ using an ac trap. Although this is most appealing, as the low-density H$\downarrow$ samples can have lifetimes of days, as limited by relaxation and recombination, it is the most difficult to realize experimentally as ac fields of $B\simeq1$ T at 1–10 kHz frequencies and refrigeration temperatures of $T\simeq1$ mK are required. The H$\uparrow$ trap is conceptually easier to build, but, as we shall show, the lifetimes for densities of order 10^{13}–10^{14}

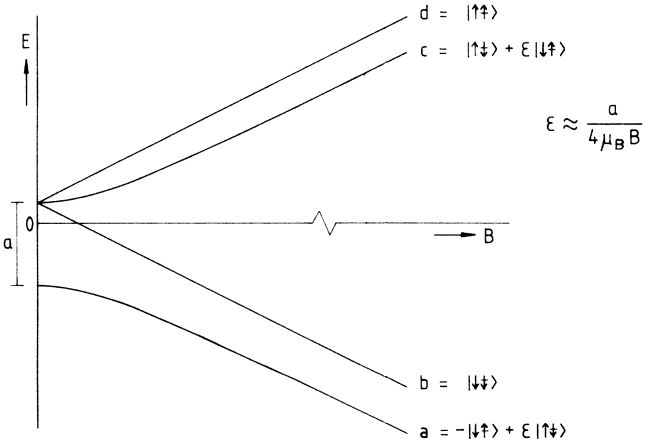

FIG. 1. Energies of the hydrogen hyperfine states as a function of magnetic field. The symbols ↑ and ↑ are the projections of the electronic and nuclear spins, respectively. a is the hyperfine constant and μ_B the Bohr magneton.

cm^{-3} ($T \simeq 7$–34 μK) are limited to tens of seconds. Although volume recombination is not important at these densities, inelastic spin exchange and electronic magnetic dipole relaxation are. A trap initially filled with c- and d-state atoms will be rapidly depleted by spin-exchange events in which binary collisions yield $cc \rightarrow aa$, $cc \rightarrow ac$, and $cc \rightarrow bd$ (processes at $T=0$ K in low fields); the resultant a- and b-state atoms will be ejected from the trap leaving d-state atoms only. These atoms can then relax via a binary-collision magnetic dipole mechanism to electron-spin-reversed states which are ejected until the trap is empty. Dipolar relaxation is mainly due to the electronic magnetic dipoles, and the following channels contribute initially: $dd \rightarrow aa, ad$ and $cd \rightarrow ab, bd, ac$.

Previously, calculations of spin exchange in zero field have been made by Berlinsky and Shizgal;[6] dipolar relaxation has been calculated only in the high-field regime for states a-b (nuclear relaxation)[7,8] and b-c (electronic relaxation).[9] Here we present calculations relevant to the traps for $B=0$–10 T. We obtained the volume two-body rates by means of a coupled-channel calculation.[8,10] This takes into account (a) triplet and singlet potentials according to Kolos and Wolniewicz,[10] (b) electron-electron and electron-proton magnetic dipole interactions, (c) intra-atomic and interatomic hyperfine interactions (Fermi contact terms). Considering the low values of the relevant temperatures, we present only the rates in the $T=0$ K limit. For the dipolar transitions this implies that orbital angular momentum l vanishes for the upper channel and equals two in the lower channel. For the exchange transitions, both initial and final l values are zero. Taking into account Bose symmetry, we therefore confine ourselves to the ten symmetrized spin states aa, $(ab+ba)/\sqrt{2}, \ldots, dd$. We calculated the event rates for all downward transitions.

As usual, in a coupled-channel calculation the above-mentioned potentials and couplings are treated to all orders. In the interior region, where the triplet and singlet potentials are of importance ($r < R_0$), we expand the total scattering wave function in SM_SIM_I spin states and solve the set of coupled equations for the coefficient functions which are equivalent to Schrödinger's equation. At R_0 the S matrix due to the interactions in the interior region is calculated and transformed orthogonally to the asymptotic a,b,c,d bases. Beyond R_0 only the long-range dipolar and the intra-atomic hyperfine interactions are taken into account. The former are included there to first order. In calculating these first-order amplitudes the distortion of the radial wave functions due to the couplings and potentials in the interior region $r < R_0$ are propagated into the exterior region. This is done by defining a "local" S matrix, i.e., an r-dependent scattering matrix for the case in which channel couplings are cut off at r. The local S matrix obeys a first-order radial differential equation from which the S matrix at infinity can be derived. By this procedure the weakness of the long-range $1/r^3$ magnetic dipole interactions is exploited to avoid a time-consuming integration of coupled equations beyond R_0. The value for R_0 is chosen so large that the sum of the S-matrix contributions from the interior and exterior regions has converged. Usually a value of $30a_0$ is sufficient. Calculations of the dipolar relaxation rates with an excluded-volume approximation gave very similar results (differences less than 10% at low field). Calculations performed for finite temperature show that thermal changes of relevant rates only become important for $T \gtrsim 1$ K.

We now consider the lifetimes in the H↑ trap. For small fields the exchange process is dominant by orders of magnitude. This enables us to make a simplifying separation of the rate equations. Using the notation n_c and n_d for the densities of states c and d and $G^e_{h_1h_2,h_3h_4}$ and $G^d_{h_1h_2,h_3h_4}$ for the intrinsic exchange and dipolar event rates, respectively, in the case of a transition $h_1h_2 \rightarrow h_3h_4$ among the hyperfine states, we find

$$\dot{n}_c = -(2G^e_{cc,aa} + G^e_{cc,ac} + 2G^e_{cc,bd})\, n_c^2 \equiv -G^e n_c^2 \ , \qquad (1)$$

as the rate at which the c-d trap decays to a d-state trap. This leads to a lower limit for the rate, as in the $G^e_{cc,ac}$ process the atom in the final state c may have gained enough energy to escape the trap. Thereafter the d state decays as

$$\dot{n}_d = -(2G^d_{dd,aa} + G^d_{dd,ad})\, n_d^2 \equiv -G^d n_d^2 \ . \qquad (2)$$

We have calculated the event rates $G^e_{h_1h_2,h_3h_4}$ and $G^d_{h_1h_2,h_3h_4}$ for all hyperfine states. The loss rate in Eqs. (1) and (2) is obtained from an event rate by multiplying by the number of atoms lost per event. Thus for the decay of d-state atoms, the factor is 2 for $dd \rightarrow aa$ and 1 for $dd \rightarrow ad$.

The magnetic field dependence of G^e and G^d at $T=0$ K is shown in Fig. 2. With this we can determine the lifetime in the H↑ trap. For the c state the half-life is $\tau^e_{1/2} = 1/n_{c0}G^e$, where n_{c0} is the initial c density; for the d state $\tau^d_{1/2} = 1/n_{d0}G^d$, where n_{d0} is the density at time $t=0$. (Note that in an exact solution of the coupled rate equations the important process $G^e_{cc,bd}$ feeds c atoms into the d state.) For a field $B=0.1$ T and for c-state densities of 10^{14} cm^{-3} we find $\tau^e_{1/2} = 1\times10^{-1}$ s ($G^e = 1\times10^{-13}$ $cm^3 s^{-1}$); for $\tau^d_{1/2}$ we find 4.2 s for these conditions ($G^d = 2.39\times10^{-15}$ $cm^3 s^{-1}$). In Figs. 3(a) and 3(b) we present the results for all hyperfine rates. We have split these up into the faster predominantly exchange and electronic dipolar rates [Fig. 3(a)] and the slower rates [Fig. 3(b)]. With these data lifetimes of all other trapped states can be obtained.

In conclusion, we have shown that H↑ traps with densities of order 10^{14} cm^{-3} rapidly decay to d-state traps which can have reasonably long lifetimes.

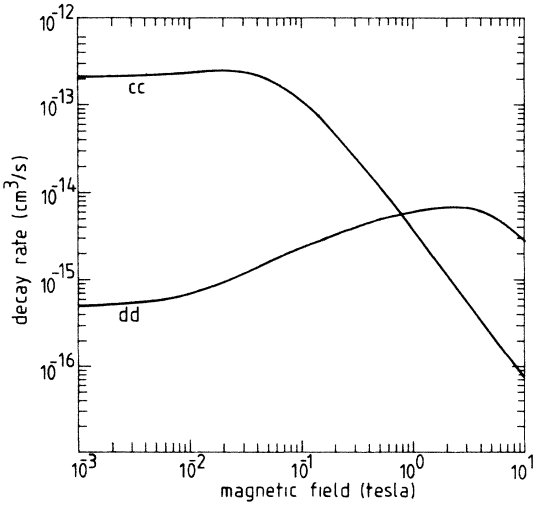

FIG. 2. The decay rates G^e and G^d defined in Eqs. (1) and (2) as a function of magnetic field at $T=0$ K. At higher fields, other channels become important and should be included.

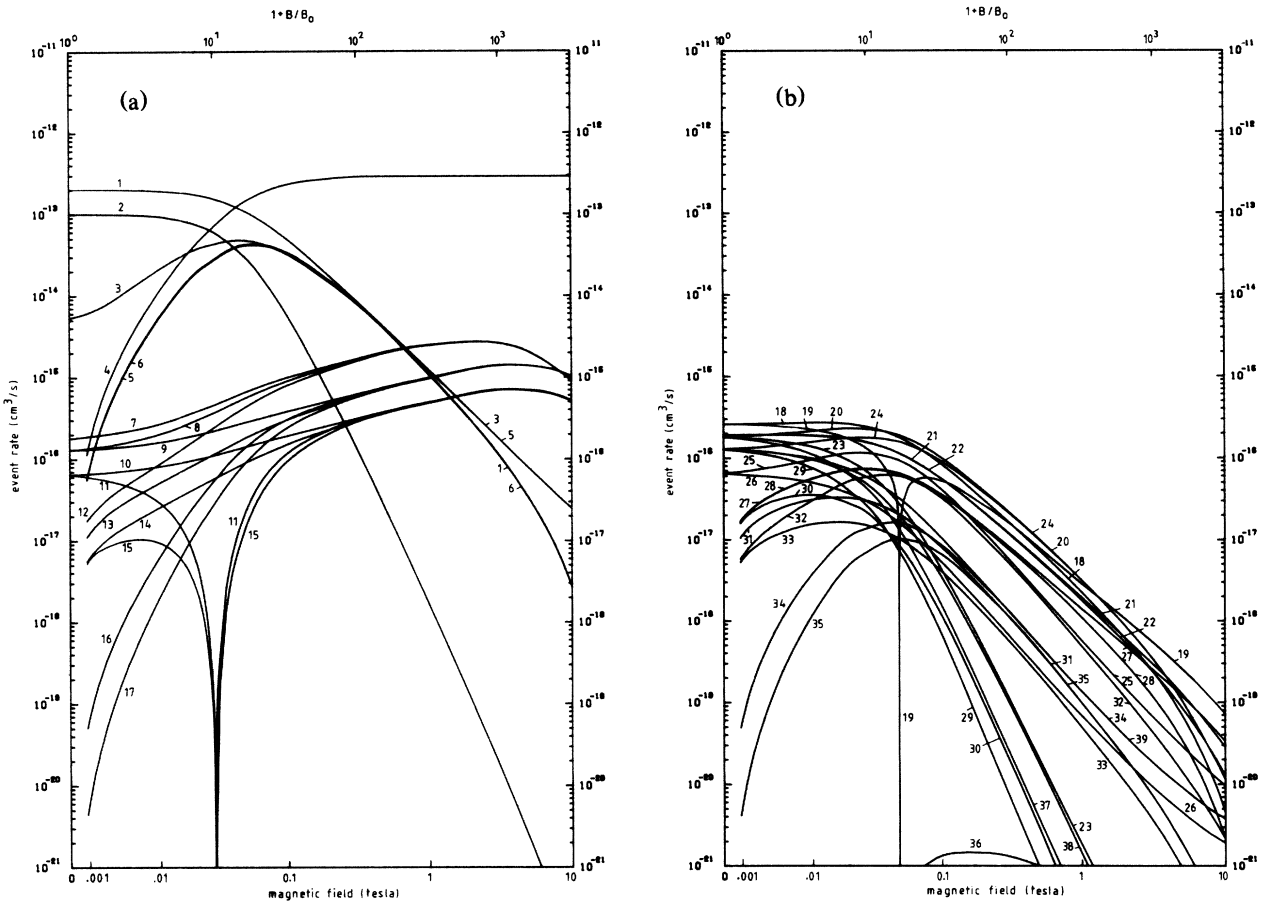

FIG. 3. The $T=0$ K spin exchange ($G^e_{h_1h_2,h_3h_4}$) and dipolar relaxation ($G^d_{h_1h_2,h_3h_4}$) event rates as a function of magnetic field. The horizontal scale shows a gradual transition from a linear to a logarithmic field dependence as we plot $1+B/B_0$ logarithmically. Here $B_0=a/16\mu_B=3.17\times10^{-3}$ T (a is the hyperfine constant, μ_B the Bohr magneton), the factor $\frac{1}{16}$ ensuring a favorable separation in linear and logarithmic parts. The lower horizontal scale shows corresponding values for B. (a) The curves correspond to the following rates; $h_1h_2\rightarrow h_3h_4$: (1) $bd\rightarrow aa$, (2) $cc\rightarrow aa$, (3) $cc\rightarrow bd$, (4) $bd\rightarrow ac$, (5) $cc\rightarrow ac$, (6) $ac\rightarrow aa$, (7) $dd\rightarrow aa$, (8) $cd\rightarrow ab$, (9) $dd\rightarrow ad$, (10) $bd\rightarrow ab$, (11) $cd\rightarrow ac$, (12) $cc\rightarrow bb$, (13) $bc\rightarrow bb$, (14) $cd\rightarrow bd$, (15) $ac\rightarrow ab$, (16) $ad\rightarrow aa$, (17) $cc\rightarrow bc$. (b) The remaining relaxation rates, not shown in (a). Curves correspond to the following rates: (18) $dd\rightarrow ac$, (19) $ac\rightarrow bb$, (20) $cd\rightarrow aa$, (21) $bc\rightarrow ab$, (22) $cd\rightarrow ad$, (23) $bc\rightarrow aa$, (24) $cc\rightarrow ab$, (25) $bc\rightarrow ac$, (26) $bd\rightarrow ad$, (27) $ad\rightarrow ab$, (28) $cd\rightarrow bc$, (29) $bc\rightarrow ad$, (30) $dd\rightarrow cc$, (31) $dd\rightarrow cd$, (32) $ad\rightarrow ac$, (33) $bd\rightarrow bc$, (34) $ab\rightarrow aa$, (35) $cd\rightarrow cc$, (36) $bd\rightarrow bb$, (37) $bb\rightarrow aa$, (38) $cc\rightarrow ad$, (39) $bb\rightarrow ab$.

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