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### Price discovery with fallible choice

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# Chapter 2

## Price Formation

This chapter reviews theories of price formation, ranging from tâtonnement to price setting. The presentation largely draws on stability theory, because of its direct relation to the general equilibrium model.<sup>1</sup> Section 2.2 discusses price formation with an auctioneer; it also proposes a new price adjustment process based on a Cobb-Douglas approximation of preferences (section 2.2.2.5). Section 2.3 considers alternative approaches, in which agents themselves propose prices. To provide some context, section 2.1 presents a short summary of general equilibrium theory. Section 2.4 briefly considers the contributions of experimental economics and agent-based modeling (ABM). Section 2.5 reflects on disequilibrium theory, the Sonnenschein-Mantel-Debreu result and monopolistic competition. Section 2.6 concludes.

### 2.1 General equilibrium theory

General equilibrium theory, which dates back to 1874, to Walras' *Eléments d'économie politique pure*, received its formal expression in the second half of the 20-th century, with major contributions from Arrow, Debreu, and McKenzie, c.f. Walras (1984), Arrow and Debreu (1954), McKenzie (1959), Debreu (1959). Its fundamental questions concern the existence, uniqueness, optimality and stability of Walrasian equilibria.

Consider an exchange economy,  $\xi$ , consisting of  $n$  agents or traders and  $m$  commodities. Each trader  $i$  has preferences that can be represented by a continuous, quasi concave, monotone utility function,  $u_i : \mathbb{R}_+^m \rightarrow \mathbb{R}$ . Traders also have non-negative endowments,  $\mathbf{w}_i \in \mathbb{R}_+^m$ . By assumption prices,  $\mathbf{p}$ , are non-negative and add up to 1,  $\mathbf{p} \in S^{m-1} = \left\{ \mathbf{p} \in \mathbb{R}_+^m \mid \sum_j p_j = 1 \right\}$ . Taking these prices as given, the agents maximize their utility subject to a budget constraint:

$$\begin{aligned} \hat{\mathbf{x}}_i &= \arg \max u_i(\mathbf{x}_i) \\ \text{s.t.} \quad & \mathbf{p} \cdot \mathbf{x}_i \leq \mathbf{p} \cdot \mathbf{w}_i. \end{aligned}$$

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<sup>1</sup>Other branches of economics, finance in particular, have also made significant contributions to the study of price formation. Finance, among other things, offers models for the valuation of financial assets, while market micro-structure theory studies the impact of trading rules and trading systems on information and price formation.

Here,  $\mathbf{p} \cdot \mathbf{x}_i$  represents the inner product of the  $(1 \times m)$  vector  $\mathbf{p}$  and the  $(m \times 1)$  vector  $\mathbf{x}_i \in \mathbb{R}_+^m$ , i.e.  $\mathbf{p} \cdot \mathbf{x}_i = \sum_j p_j x_{ji}$ . The first subscript of  $x_{ji}$  refers to a commodity, the second to a trader. If  $\hat{x}_{ji}(\mathbf{p}) > w_{ji}$  then  $i$  plans to buy commodity  $j$  at the given prices; if  $\hat{x}_{ji}(\mathbf{p}) < w_{ji}$  then  $i$  is a seller of commodity  $j$ . Since (i)  $u_i$  is continuous, and (ii) the set of commodity bundles that satisfy the budget constraint is compact, the optimization problem of the consumer has a solution. If the utility function is strictly quasi concave then this solution is unique. In that case,  $\hat{\mathbf{x}}_i(\mathbf{p}) \in \mathbb{R}_+^m$  is a vector which denotes trader  $i$ 's notional demand at prices  $\mathbf{p}$ . If the utility function is quasi concave then there can be multiple solutions and  $\hat{\mathbf{x}}_i$  is a convex set of such vectors.

Individual demand always satisfies the Weak Axiom of Revealed Preference (WARP). Let  $\mathbf{p}, \mathbf{q} \in S^{m-1}$ ,  $\mathbf{p} \neq \mathbf{q}$  and  $\hat{\mathbf{x}}_i(\mathbf{p}) \neq \hat{\mathbf{x}}_i(\mathbf{q})$ ; then we have  $\mathbf{p} \cdot (\hat{\mathbf{x}}_i(\mathbf{q}) - \mathbf{w}_i) \leq 0 \Rightarrow \mathbf{q} \cdot (\hat{\mathbf{x}}_i(\mathbf{p}) - \mathbf{w}_i) > 0$ . In words, if  $\hat{\mathbf{x}}_i(\mathbf{q})$  and  $\hat{\mathbf{x}}_i(\mathbf{p})$  are both affordable at  $\mathbf{p}$  and  $\hat{\mathbf{x}}_i(\mathbf{p})$  is chosen over  $\hat{\mathbf{x}}_i(\mathbf{q})$ , then  $\hat{\mathbf{x}}_i(\mathbf{p})$  is not affordable whenever  $\hat{\mathbf{x}}_i(\mathbf{q})$  is chosen. Certain (but not all) individual demand functions also satisfy the gross substitution property (GS); this is the case if for all  $k \neq j$ ,  $\mathbf{q} = \frac{\mathbf{p} + \alpha \mathbf{e}_j}{1 + \alpha}$ ,  $\alpha > 0$  and  $\mathbf{e}_j$  the  $j$ -th unit vector we have

$$\begin{aligned} \hat{x}_{ji}(\mathbf{q}) &< \hat{x}_{ji}(\mathbf{p}) \\ \hat{x}_{ki}(\mathbf{q}) &> \hat{x}_{ki}(\mathbf{p}). \end{aligned}$$

**Definition 2.1.** A competitive or Walrasian equilibrium of an exchange economy is a duplet  $(\mathbf{p}^*, \mathbf{X}^*) = (\mathbf{p}^*, (\mathbf{x}_1^*, \dots, \mathbf{x}_n^*))$  characterized by two consistency requirements: (i) for each trader  $i$ , notional demand at  $\mathbf{p}^*$  is consistent with his preferences, i.e.  $\mathbf{x}_i^* \in \hat{\mathbf{x}}_i$  (internal consistency); and (ii) the plans of traders are mutually consistent, i.e.  $\sum_i \mathbf{x}_i^* \leq \sum_i \mathbf{w}_i$  (external consistency).

If the utility functions are strictly quasi concave then we have  $\mathbf{x}_i^* = \hat{\mathbf{x}}_i$ ; furthermore, if they are strictly monotone then we have  $\sum_i \mathbf{x}_i^* = \sum_i \mathbf{w}_i$ . In this case, equilibrium prices are a zero of the aggregate excess demand function,  $\mathbf{z}(\mathbf{p}) = \sum_i (\hat{\mathbf{x}}_i(\mathbf{p}) - \mathbf{w}_i)$ .

According to the Sonnenschein-Mantel-Debreu (SMD) result,  $\mathbf{z}(\mathbf{p})$  is subject to very few restrictions (Sonnenschein (1973); Mantel (1974, 1976); Debreu (1974)). For any arbitrary function,  $f$ , that is (i) continuous for relative prices bounded away from zero, and (ii) homogenous of degree zero,  $f(\lambda \mathbf{p}) = f(\mathbf{p})$ , and (iii) satisfies Walras' Law,  $\mathbf{p} \cdot f(\mathbf{p}) = 0$ , there exists an economy that has  $f$  as its aggregate excess demand function.

If all individual demand functions satisfy GS then the aggregate excess function also satisfies GS. This condition is sufficient but not necessary. WARP, however, does not aggregate. That is, we do not necessarily have:  $\mathbf{p} \cdot \mathbf{z}(\mathbf{q}) \leq 0$  and  $\mathbf{p} \neq \mathbf{q} \Rightarrow \mathbf{q} \cdot \mathbf{z}(\mathbf{p}) > 0$ . If there are three commodities or less then GS does imply that aggregate excess demand satisfies WARP (c.f. Kehoe and Mas-Colell (1984)). Positing that GS or WARP holds at the aggregate level amounts to a quite strong assumption.

The stability of an equilibrium is defined relative to a price adjustment process: basically an equilibrium is stable if there exists a meaningful price adjustment process that converges to it.<sup>2</sup>

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<sup>2</sup>Formally, an equilibrium price vector  $\mathbf{p}^*$  is locally stable if the trajectories of the price adjustment

The main results of general equilibrium theory concern the existence, uniqueness and optimality of Walrasian equilibria. If utility functions are quasi concave, then at least one Walrasian equilibrium exists. If the aggregate excess demand function satisfies GS, then the Walrasian equilibrium is unique. Walrasian equilibrium allocations are Pareto-efficient provided that preferences are locally non-satiated, i.e. one cannot strictly improve the utility of one agent without decreasing the utility levels of one or more other agents. The relevance of these results depends on stability. After all, if a Walrasian equilibrium cannot be attained reasonably fast, or if prices move away from it after being slightly perturbed then what has been established?<sup>3</sup>

Insofar as stability theory focuses on the stability of Walrasian equilibria, it is unduly restrictive. One has to assume that all trading is postponed until the equilibrium prices have been determined. Otherwise, with trading at all prices (or "false" prices), the equilibrium itself is a moving target. Suppose that someone has bought a commodity at a price, that with hindsight is too high; then wealth has been transferred from the buyer to the seller, and that may shift the initial equilibrium. However, finding equilibrium prices without the benefit of trading requires a substantial degree of coordination and / or a lot of information. Indeed, price dynamics that is governed by  $\mathbf{z}(\mathbf{p})$  can be as "bad" as desired, due to the SMD result: it may exhibit periodic cycles, or even chaotic fluctuations. Finding zeros of such functions is a task that can be accomplished, but mainly at the expense of economic plausibility. Fortunately, with the advent of the so-called Edgeworth and Hahn/Negishi processes, the focus of stability theory has shifted to price adjustment processes (see below). In the next sections, we adopt the process-centric rather than the equilibrium-centric perspective.

However, before discussing various theories of price formation we like to make matters concrete by considering three examples, that were put forward by Scarf (1960). These examples have become the litmus test of choice in stability theory, because of their historical significance and because of the challenges they pose to price adjustment processes. In each case, there are three traders,  $i = 1, 2, 3$ , and three commodities,  $j = 1, 2, 3$ . The (Leontief) preferences of the agents with respect to commodities can be described as follows:

$$\begin{aligned} u_1(\mathbf{x}_1) &= \min(x_{11}, x_{31}) \\ u_2(\mathbf{x}_2) &= \min(x_{12}, x_{22}) \\ u_3(\mathbf{x}_3) &= \min(x_{23}, x_{33}) \end{aligned} \quad (2.1.1)$$

The examples differ in how endowments are allocated, in particular (commodities in rows, traders in columns):

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process of all initial price vectors in some small neighborhood of  $\mathbf{p}^*$  converge to  $\mathbf{p}^*$ . The price vector  $\mathbf{p}^*$  is globally stable if the trajectories of all initial price vectors converge to  $\mathbf{p}^*$ . If a price adjustment process converges for all starting points and for all economies, then it is called an effective price mechanism, c.f. Saari and Simon (1978). An economy with multiple equilibria is called "system stable", if the trajectories of all price vectors converge to some equilibrium price vector. Instability can also be local or global: an equilibrium is locally totally unstable if the trajectories of all initial price vectors other than  $\mathbf{p}^*$  in some small neighborhood of  $\mathbf{p}^*$  lead away from  $\mathbf{p}^*$ ; global total instability applies when the trajectories of all initial price vectors other than  $\mathbf{p}^*$  lead away from  $\mathbf{p}^*$ .

<sup>3</sup>In the light of this consideration, one would expect stability theory to be the major concern of economists. Unfortunately, however, this subject has not received the attention it deserves. Many economists were (and still are) convinced that comparative statics serves their interests sufficiently well.

$$\mathbf{W}^A = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}; \mathbf{W}^B = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}; \mathbf{W}^C = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}.$$

Demand, given the allocation  $\mathbf{W}^A$ , is equal to

$$\begin{aligned} \hat{x}_{j1}^A &= \frac{p_2}{p_1 + p_3}, j = 1, 3; \\ \hat{x}_{j2}^A &= \frac{p_3}{p_1 + p_2}, j = 1, 2; \\ \hat{x}_{j3}^A &= \frac{p_1}{p_2 + p_3}, j = 2, 3. \end{aligned}$$

Cases  $\mathbf{W}^B$  and  $\mathbf{W}^C$  are similar. All three examples have the same equilibrium, i.e.

$$\{\mathbf{p}^*, \mathbf{X}^*\} = \left\{ (1, 1, 1), \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} \end{pmatrix} \right\}.$$

## 2.2 Price formation with an auctioneer

Diamond (1987) describes the role of the auctioneer as (i) making everyone aware of prices and trading opportunities, at no cost; (ii) trivializing the problem of pairwise matching of buyers and sellers; and (iii) giving everyone a single budget constraint by coordinating all trading simultaneously. In addition, we propose that the presence of an auctioneer also serves to justify two unlikely assumptions: price taking behavior on the part of economic agents and price formation depending on knowledge of the aggregate excess demand function.<sup>4</sup>

The classification of algorithms as representing price formation with or without an auctioneer to a certain extent is arbitrary. For instance, the Edgeworth process is usually interpreted as uncoordinated trading. However, to assume that trading will always occur if there exists a Pareto-improvement, i.e. to assume that traders will always recognize Pareto-improvements no matter how complicated, is not credible. One may rightly object that little is gained by assuming that the auctioneer can learn anything that the traders themselves don't know. However, by realistically modeling what traders (can) know or learn and by assuming that missing information comes from the auctioneer at least it becomes clear what needs to be improved in modeling trading behavior.

Most of the price adjustment processes in this section assume that trading takes place at equilibrium prices; of these, tâtonnement theory is the oldest. Its failure to

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<sup>4</sup>Schinkel (2001) objects to the auctioneer, characterizing it as an homunculus explanation, i.e. an explanation that assumes what needs to be explained. If that were the case, then there would be no need for the auctioneer to engage in iterative procedures. Research into price formation with an auctioneer seems primarily focused on determining the informational requirements of convergence to the Walrasian equilibrium instead of explaining price formation. In our view, the essence of the auctioneer hypothesis is to make knowledge of the aggregate excess demand function available for the explanation of price formation. Whether that leads to a plausible exposition of price formation is another matter. We fully agree with Schinkel that price formation requires a behavioral explanation, instead of a "macro" explanation, based on aggregate excess demand.

converge in Scarf's unstable examples sparked off research into the computability of Walrasian equilibria (which is considered first) and more importantly, research into trading at all prices (to be considered after the computational approaches). This section ends with a (game-theoretic) process in which the assumption of price taking behavior is slightly relaxed and with a mixed model, in which some traders listen to the auctioneer while others learn by imitating successful trading behavior.

### 2.2.1 Tâtonnement

The natural starting point for an economically sound price adjustment process is the law of demand and supply, which maintains that prices adjust in the direction of their own excess demand. That is, if there is excess supply (demand), then the price must decrease (increase). This condition, together with postponing trade until equilibrium prices have been determined, is the defining characteristic of the so-called tâtonnement price adjustment process. It can be modeled as a difference or as a differential equation.<sup>5</sup> For  $j = 1, \dots, m$ :

$$\begin{aligned} p_j^{t+1} - p_j^t &= F_j(p_j^t, z_j(\mathbf{p}^t)); \\ \frac{dp_j(t)}{dt} &= F_j(p_j(t), z_j(\mathbf{p}(t))). \end{aligned}$$

with  $F_j(p_j, z_j)$  a sign-preserving function in  $z_j$  that satisfies  $F_j(p_j, 0) = 0$ . A sufficient condition for the process in continuous time to converge globally to the static competitive equilibrium is that the aggregate excess demand function satisfies the GS property (after Arrow and Hurwicz (1958) studied local stability, Arrow et al. (1959) addressed global stability through the use of Lyapunov's second method; see also Uzawa (1960); Hahn (1958); Negishi (1958)).<sup>6</sup> It is also sufficient if WARP holds at the aggregate level (c.f. Mas-Colell et al. (1995)). These restrictions, however, are quite severe. If GS does not apply, then tâtonnement can exhibit exotic price dynamics. The examples of Scarf (1960) (see above) were the first to demonstrate that without condition GS tâtonnement may fail to converge. As a matter of fact, allocation  $\mathbf{W}^A$  leads to stability, but  $\mathbf{W}^B$  and  $\mathbf{W}^C$  do not. Here, prices orbit around their equilibrium values, clockwise in the case of  $\mathbf{W}^B$  and counter-clockwise in the case of  $\mathbf{W}^C$ . Gale (1963) shows that tâtonnement can be globally totally unstable: depending on the initially quoted price, the tâtonnement process approaches one of two corner solutions, in which either the sellers or the buyers are practically giving away their endowments, c.f. section 6.1.

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<sup>5</sup>Samuelson (1941) first formalized Walras' idea of tâtonnement as a set of differential equations. This specification has been criticized because it adjusts prices simultaneously, c.f. Negishi (1962). Van der Hoog (2005) investigates the difference between simultaneous and sequential price adjustment, and concludes that their stability properties are similar.

<sup>6</sup>Gross substitution also implies the uniqueness of the equilibrium and the so-called Hicks conditions for stability, that started the subject of stability theory, c.f. Hicks (1939). Although Samuelson claimed that the Hicks conditions were neither sufficient nor necessary for dynamic stability, McFadden (1968) has shown that these conditions are sufficient if markets differ in their speed of adjustment. Interestingly, Hicks commented on Samuelson's differential equations that "... for the understanding of the economic system we need something more, something which does refer back, in the last resort, to the behavior of people and the motives of their conduct", c.f. Hicks (1939, p. 337).

For the discrete case, GS is not sufficient; there will be no convergence to an equilibrium if the speed of adjustment is too high even if GS holds (c.f. Goeree et al. (1998)). Tuinstra (1999) shows that stability may also depend on which price normalization rule is selected. Other researchers have found complicated dynamic behavior of prices in discrete time as well, c.f. Bala and Majumdar (1992); Day and Pianigiani (1991); Saari (1985); Weddepohl (1995).

Scarf's examples have stimulated research into the computation of equilibria (c.f. section 2.2.2) and into trading at all prices (c.f. section 2.2.3). The latter processes may attain a rest point, and that rest point itself may be a (no-trade) competitive equilibrium, but most likely it is not the initial Walrasian equilibrium.

## 2.2.2 The computational approach

### 2.2.2.1 Scarf

The field of computation of economic equilibria started off with the publication of Scarf and Hansen (1973). Its algorithm divides the price simplex into sub-simplices, the vertices of which can be labeled. For instance, one can assign the name of the commodity to the vertex, which at that point is in greatest excess demand. With such a labeling rule Sperner's lemma applies, which holds that there exists a sub-simplex that is completely labeled, i.e. with all vertices having different labels. A completely labeled sub-simplex contains the equilibrium; by moving in the direction of each of its vertices, different commodities become in greatest excess demand. The algorithm of Scarf and Hansen moves from one sub-simplex to a neighboring one without returning to sub-simplices that it has visited before and its path is guaranteed to end in the completely labeled sub-simplex. The granularity of the sub-simplices determines the accuracy of the approximation.

### 2.2.2.2 Smale's algorithm, PID

Smale (1976) shows that the generalized Newton process

$$J(\mathbf{p}) \cdot \frac{d\mathbf{p}}{dt} = -\lambda \mathbf{z}(\mathbf{p}) \quad (2.2.1)$$

converges for almost all aggregate excess demand functions, subject to some boundary and regularity conditions. Here,  $J(\mathbf{p})$  is the Jacobian of the aggregate excess demand function; the sign of  $\lambda$  depends on  $\det J(\mathbf{p})$ . The price dynamics of equation (2.2.1) is not in line with the law of demand and supply, because price adjustments depend on excess demand in multiple markets and on all partial derivatives with respect to prices. While in reality such a dependency is possible, the dynamics of equation (2.2.1) can be quite complicated. Keenan (1981) shows that there may exist an open set of initial prices for which the algorithm of Smale (1976) does not converge; put differently, it is not globally convergent and therefore not an effective price mechanism. Furthermore, having to know the aggregate excess demand function and its derivatives for any  $\mathbf{p}$  is a strong assumption. Saari and Simon (1978) demonstrates that there is very little scope for easing these informational requirements: an effective price mechanism requires at least the knowledge of most elements of the Jacobian.

Kumar and Shubik (2004) proposes a "proportional integral-derivative (PID) controller". In addition to the aggregate excess demand function, changes in prices are also driven by one or more derivatives of the aggregate demand function:

$$\frac{dp_j(t)}{dt} = \lambda_0 z_j(\mathbf{p}(t)) + \lambda_1 \frac{dz_j(\mathbf{p}(t))}{dt} + \lambda_2 \frac{d^2 z_j(\mathbf{p}(t))}{dt^2}. \quad (2.2.2)$$

Depending on the choice of parameters, the price dynamics can be stable or unstable. For instance,  $\lambda = (\lambda_0, \lambda_1, \lambda_2) = (0.2, 0.275, 0)$  yields convergence to the competitive equilibrium of the unstable clockwise Scarf economy (which uses  $\mathbf{W}^B$ ). By adding the second derivative term, the speed of convergence can be improved. In a simulation of (2.2.2),  $\lambda = (0.2, 0.275, 0)$  requires more than 250 iterations, while a process with  $\lambda = (0.2, 0.275, 0.2)$  converges in about 150 iterations.<sup>7</sup>

### 2.2.2.3 Van der Laan/Talman

Van der Laan and Talman (1987) presents a globally and universally convergent, i.e. an effective, price adjustment process. Here, the auctioneer successively eliminates aggregate excess demand in all markets, by applying the law of supply and demand while keeping markets equilibrated in which demand already equals supply. This process depends on the value of aggregate excess demand and on the location of the corresponding price vector relative to the initial price vector. Herings (1997) has proved that convergence for this process is global and universal under standard conditions on utility functions and consumption sets.

Retaining partial equilibria while balancing another market requires the auctioneer to know the aggregate excess demand function and its derivatives everywhere. This is in line with Saari and Simon (1978).

### 2.2.2.4 Demand schedules

Goeree and Lindsay (2016) introduces a market in which traders submit demand and supply schedules rather than simple offers.<sup>8</sup> The schedules are based on demand and supply at a limited number of prices; interpolation is used to calculate demand and supply at intermediate prices. This permits the auctioneer to determine equilibrium prices using the algorithm of Smale (1976). The authors show that traders have an incentive to submit competitive schedules: i.e. it is a weakly dominant strategy to submit a set of quantities that maximize utility, taking prices as given.

The schedules market was subjected to laboratory tests, using the counter-clockwise unstable Scarf economy as implemented by Anderson et al. (2004).<sup>9</sup> There were experiments in which traders had to submit offers, and others in which the agents submitted demand schedules. In the schedules market prices do not oscillate (as they

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<sup>7</sup>Based on visual inspection of the graphs of Kumar and Shubik (2004). Although equation (2.2.2) suggests that time is assumed to be continuous, its application to examples is discrete.

<sup>8</sup>The schedules market as proposed by Goeree and Lindsay (2016) is procedural rather than computational. However, it is discussed here because of its conceptual relation with Cobb-Douglas approximation, c.f. section 2.2.2.5.

<sup>9</sup>These experiments will be discussed in chapter 3.

do when traders submit simple offers); instead, they quickly converge to the equilibrium. Furthermore, the schedules market is claimed to be much more efficient and more egalitarian than the benchmark. We think these results may be explained as follows. In the experimental set-up of Anderson et al. (2004), trading is subject to the condition that commodities be exchanged for money. Apart from reducing efficiency, this constraint also creates unequal opportunities for different types of traders. The schedules market doesn't impose that condition.

Eliciting the information in the form of demand schedules to a certain extent circumvents the result of Saari and Simon (1978), because the auctioneer approximates the value of the aggregate excess demand by means of interpolation. The next algorithm is also related to the idea of a schedules market.

### 2.2.2.5 Cobb-Douglas approximation

Consider the plight of an auctioneer, who doesn't know the preferences of the agents, but who does know the allocation of endowments. He can *estimate* demand schedules by *assuming* that each trader has Cobb-Douglas preferences. Let  $\mathbf{p}^k$  be the prices in the  $k$ -th iteration. Individual demand at  $\mathbf{p}^k$  suffices to identify the hypothetical preferences. By calculating expenditure per commodity as a percentage of each agent's budget, the auctioneer can derive approximate demand schedules:

$$x_{ji}(\mathbf{p}|\mathbf{p}^k) = \frac{p_j^k x_{ji}(\mathbf{p}^k)}{\mathbf{p}^k \cdot \mathbf{w}_i} \frac{\mathbf{p} \cdot \mathbf{w}_i}{p_j}.$$

This allows the auctioneer to calculate prices that equilibrate the associated Cobb-Douglas economy, and which feed into the next iteration. Can this price adjustment process, call it  $\mathcal{P}$ , help the auctioneer?

Appendix C provides some insights into the dynamics of Cobb-Douglas approximation. It offers a proof of global convergence for a set of economies in which traders have preferences that can be represented by CES utility functions ranging from Leontief to Cobb-Douglas functions, and it also applies the process to the Scarf economies.

## 2.2.3 Trading at all prices

### 2.2.3.1 The Edgeworth process

The Edgeworth process assumes that trading will take place if and only if there exists a coalition with a Pareto-improving exchange among the members of the coalition. That is, each Pareto improving trade will actually take place (c.f. Uzawa (1962)).<sup>10</sup> In the absence of production this is a gradient process, that converges to a Pareto-efficient allocation: as long as trading continues utilities are monotonically non-decreasing; and by construction the process can only terminate in a Pareto-efficient allocation. While utility levels follow an orderly dynamics, this does not necessarily apply to prices. As the set of Pareto-improving transactions gets exhausted, price formation may become unstable (c.f. chapter 4).

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<sup>10</sup>If there are institutional constraints, e.g. commodities have to be traded for money, then the Edgeworth process can only run for as long as there are Pareto improvements that can be implemented, c.f. appendix A.

Are traders aware of all the opportunities that may exist? Pareto improving coalitions can be very large, and as a consequence difficult to detect. A complex Pareto improvement, may require agents to submit offers in a particular order; and some of these offers possibly will not directly benefit one or even both parties. Therefore, the Edgeworth process practically requires the coordination of an auctioneer.<sup>11</sup> In the absence of an active auctioneer, no one is aware of aggregate excess demand and as a result it can be almost impossible to determine whether trading should stop or not. Schmeidler has proved that if there is a mutually advantageous trade, then there is one involving at most as many traders as there are commodities (c.f. Fisher (1983, p.30, footnote 18)). If the same good or service at different times, or in different circumstances, is considered to be a different commodity then the number of commodities can easily exceed the number of traders. A stronger result is due to Madden (1978): if each trader has positive amounts of each commodity, then the existence of a Pareto-improving transaction implies the existence of a mutually beneficial bilateral trade. However, the assumption that endowments are strictly positive is rather strong.

From a modeling perspective, the Edgeworth process is not entirely consistent: although it describes an economy that is out of equilibrium, traders always expect that they can complete their planned trades at the current prices. The Edgeworth process also lacks in realism in so far as it excludes arbitrage. It does not allow traders to engage in trades which lower their level of utility, even if they expect that this loss will be offset at a later time.

### 2.2.3.2 The Hahn/Negishi process

The Hahn/Negishi process assumes that there are sufficiently well-organized markets, so that at any price there is either excess demand or excess supply but not both, c.f. Hahn and Negishi (1962). In that case, buyers who want more of a certain commodity know that they need to offer a higher price. And similarly, sellers who cannot clear their stock will have to lower their asking prices. Or to put it differently, active traders will have to lower their utility targets. The sum of target utilities can be used as a Lyapunov function for proving convergence.

In a Continuous Double Auction (CDA), both buyers and sellers can submit offers at the same time, usually subject to the condition that each new offer must improve upon a standing offer, or floor offer (if any). In such an environment, the basic assumption of the Hahn process is not fulfilled. After a buyer submits a bid, there is no protocol in place which guarantees that all feasible transactions at that bid price are implemented before any other offers at different prices can be submitted. Even when newly offered bid prices (ask prices) are below (above) the previous trading price, the next trading price can be either above or below the previous one depending on haggling (i.e. on bargaining). This violates the assumptions of the Hahn process. Furthermore, it is possible that at the going prices, a prospective buyer first has to sell something of another commodity before he can accept a proposal that is on offer. In that case, there is both unsatisfied demand and supply at the going prices. This is all the more likely when institutional constraints stipulate that commodities must

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<sup>11</sup>According to Schinkel (2001, p. 90), however, the Edgeworth process operates without an auctioneer.

be exchanged for money. Even though the CDA is a centralized and well-organized market, it does not meet the requirements of the Hahn/Negishi process. This shows that this assumption also is rather strong.

## 2.2.4 Hybrid models

### 2.2.4.1 The Cournot-Shubik mechanism

Kumar and Shubik (2004) also considers a strategic bid-offer game as a market mechanism, in which desired quantities are the strategic variables. For commodity  $j$ , buyers offer money,  $b_{ji} \geq 0$ , and sellers offer quantities  $q_{ji} \geq 0$ . If both  $\sum_i b_{ji} > 0$  and  $\sum_i q_{ji} > 0$  then

$$p_j = \frac{\sum_i b_{ji}}{\sum_i q_{ji}}. \quad (2.2.3)$$

otherwise  $p_j = 0$ . Note that  $p_j$  clears the market. By offering to buy more of  $j$ , a trader raises the price  $p_j$  and also increases his share in  $\sum_i q_{ji}$ . Here, agents realize that they have an impact on prices; this gives rise to a variety of monopolistic competition. All traders have to reveal their plans simultaneously; and each tries to formulate a best response to the strategies played by others. In a dynamic version, traders take the strategies of others (as played in the previous iteration) as given. Knowing their impact on prices, traders maximize their utility function. The game is continued (and trading is postponed) until the prices are in equilibrium. Kumar and Shubik (2004) derives the price dynamics of the clockwise unstable Scarf economy as:

$$\begin{aligned} p_1(t) &= \frac{p_3(t-1)(p_1(t-1) + p_2(t-1))}{p_2(t-1)(p_1(t-1) + p_3(t-1))}; \\ p_2(t) &= \frac{p_1(t-1)(p_2(t-1) + p_3(t-1))}{p_3(t-1)(p_1(t-1) + p_2(t-1))}; \\ p_3(t) &= \frac{p_2(t-1)(p_1(t-1) + p_3(t-1))}{p_1(t-1)(p_2(t-1) + p_3(t-1))}; \end{aligned}$$

which produces a fast convergence to the Nash equilibrium, which here coincides with the Walrasian equilibrium. A degree of proximity, similar to the PID controllers above, is reached after 25 iterations.

It is unclear whether and to what extent people behave strategically when the size of the economy increases. But an expected sensitivity of prices to one's own decisions, however slightly or even poorly perceived, does induce preferences over prices. This is taken up in section 2.3.3.

### 2.2.4.2 Private versus public prices

Gintis (2007) considers bilateral trading in the counter clockwise Scarf economy. Traders have reservation prices that can be either private or public. Private prices are generated at random in the initialization phase. Public prices are recalculated each period by the auctioneer, who determines aggregate excess demand, and applies a tâtonnement process to calculate the new prices. Traders meet at random, and trade if they can (according to some simple rules). Every ten periods, a fraction of

traders imitates the reservation price of peers, provided their peer was more successful. If everyone listens to the auctioneer, prices orbit around the static competitive equilibrium, like they do in the example of Scarf. On the other hand, if no one listens then prices converge to the Walrasian equilibrium. In intermediate cases, complex dynamics can arise. With 10% of the traders listening to the auctioneer, initial price instability wears off and prices fluctuate around the static equilibrium. With 40% of the traders listening to the auctioneer, prices exhibit periodic behavior.

Gintis claims to have shown that "the instability of the tâtonnement process is due to the public nature of prices, which leads to excessive correlation in the behavior of economic agents.", c.f. Gintis (2007). This argument is a non sequitur: mixing tâtonnement with another, convergent process does not explain why tâtonnement is divergent. Gintis' other main point seems valid and is important: learning is an alternative for coordination by the auctioneer.

## 2.3 Price formation without an auctioneer

In real life, it is not the auctioneer but agents themselves who propose prices. Firms, in particular, perform this role. Price setting can be based on perfect foresight, rational expectations with more or less insight into demand or supply, on learning or on heuristics.

### 2.3.1 Perfect foresight

The simplest way of abandoning the auctioneer, and also the least satisfactory, is to assume that traders have perfect foresight. They can trade immediately because they already know all equilibrium prices in advance. This hypothesis skips the equilibration process completely, and fails to explain how equilibria may come about. Previous experience, or, in very special cases, introspection may give traders knowledge of equilibrium prices, and allows them to set prices at these values (c.f. section 3.5.1). But this is not realistic in general equilibrium models, especially if there are multiple equilibria. Furthermore, perfect foresight also implicitly assumes the absence of any mistakes.

### 2.3.2 The Fisher process

Fisher has generalized the Hahn process, c.f. Fisher (1983). Instead of assuming that markets are well-organized, Fisher distinguishes markets by commodity, date and seller. By letting sellers set prices and by letting buyers search for the lowest price, the basic assumption of the Hahn process is satisfied: after trading in each market there is either unsatisfied demand or unsatisfied supply, but not both. Simply, because markets are so small. Here, price setting by sellers replaces the auctioneer; it also abandons the law of demand and supply. Agents are aware of rationing and they expect that they may not be able to complete planned trades. As a result, equilibria of the generalized Hahn, or Fisher, process do not have to be Walrasian. Further-

more, Fisher allows money, speculative trading and production and consumption at disequilibrium prices.<sup>12</sup>

In this model, in which agents act on perceived opportunities, strong assumptions are required to guarantee stability. One has to rule out that beliefs by themselves upset an otherwise imminent equilibrium; furthermore, price adjustments have to be sufficiently fast for the economy to reach or approximate equilibrium. To achieve this, Fisher simply assumes that after some time there are no longer any favorable surprises.

"What matters is not whether new opportunities are really there but whether agents believe that they are. Opportunity is in the eye of the beholder and it is the perceptions of agents which lead them to act - even though such perceptions may be wrong. (...) If we are to establish that a competitive economy tends to equilibrium in the absence of a stream of new opportunities, it must be in the absence of a stream of *perceived* new opportunities, real or imagined.", Fisher (1983, p. 89).

Eventually, then, as in the Hahn process expected utility levels are non-increasing and the sum of target utilities can be used as a Lyapunov function for proving convergence.

### 2.3.3 Price setting

The following processes are based on expectations with respect to demand schedules, leading to monopolistic competition. They differ in the way that expectations are constrained. With fewer or less stringent constraints, there are more equilibria. The equilibria themselves typically differ from Walrasian equilibria, nor do they have to be Pareto-efficient.

Negishi (1961) proposes that some firms have expectations with respect to the demand curve they are facing; these monopolistic competitors can set prices by maximizing profit, subject to their conjectures about demand. This leads to many subjective equilibria. The expectations with respect to demand are assumed to be linear functions of own prices, and required to be consistent with currently observed price-quantity combinations. Here, stability is proved, assuming that all goods are gross substitutes.

Gabszewicz and Vial (1972) constrains the conjectures to be completely correct, also at price-quantity combinations that cannot be observed. This raises deep issues: (i) is it plausible that firms can model the economy, in particular that they know the recursive impact of their own expectations?<sup>13</sup> (ii) do shareholders accept profit maximization if a majority of them is better off at slightly lower profits and more favorable prices? The authors do not address the stability of the equilibrium; their main goal is to show that an increasing number of firms leads to more competitive behavior.

Silvestre (1977) is an intermediate between the subjective approach of Negishi (1961) and the objective approach of Gabszewicz and Vial (1972). It requires that

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<sup>12</sup>Money, speculative trading and out-of-equilibrium consumption and are not a part of the Hahn/Negishi process and neither of the Edgeworth process.

<sup>13</sup>The consumption of shareholders (and hence the demand curve) depends on the firm's profits, which in their turn depend on the firm's expectations with respect to demand.

expectations with respect to demand schedules are consistent with observed values, and also that, at observed values, the expected derivative of the demand schedule is correct.

Schinkel et al. (2002) has sellers who expect sales to depend on own prices only. The sellers update their prior beliefs with respect to conjectured demand after observing sales at proposed prices, using Bayes' rule. If initial expectations do not assign zero probability to prices and sales that can actually happen ("No statistical surprise"), then the economy converges to a conjectural equilibrium. Which equilibrium is reached depends on the initial beliefs (trading is postponed until equilibrium prices have been determined). Under "No statistical surprise" this process is globally stable with respect to initial belief-structures.

Kirman (1995) considers how learning affects the coordination between different equilibria of an  $N$ -oligopoly game. Here firms have to learn both the demand schedule they are facing and the strategies of other firms. Kirman (1995) considers three learning rules (OLS for estimating the parameters of a misspecified model of the market, and two rules of thumb). In an evolutionary simulation, the least successful players copied the strategies of the most successful players. This experiment highlights the robustness of the Nash-equilibrium as an attractor.

## 2.4 Experimental economics and ABM

Experimental economists claim that it only takes a few, uninformed traders to achieve equilibrium in an economy that is endowed with a Continuous Double Auction (c.f. Smith (1962); Friedman and Rust (1993); Anderson et al. (2004); Smith (2008)). Most experimental markets, however, are rather simple. Often they consist of a single financial market with one asset and money; these markets are populated by traders who have exogenous reservation prices. The combination of reservation prices and the fact that there is just one market is giving the game away. However, Anderson et al. (2004) and Crockett et al. (2011) provide experimental studies of human trading in several well-known examples of general equilibrium theory, and their research also supports the predictions of tâtonnement theory. The experiments of Anderson et al. in particular are described in detail in the next chapter.

Agent Based Modeling has proposed different robot traders, such as Zero-Intelligence, Zero-Intelligence Plus, Gjerstad-Dickhaut and Adaptive Aggressive (AA-) traders (Gode and Sunder (1993); Gjerstad and Dickhaut (1998); Vytelingum (2006); Cliff (1997)).<sup>14</sup> These different types of robots are capable of proposing prices and, to different extents, of learning. One of the drivers of this development is the search for algorithms that outperform human traders. According to De Luca and Cliff (2011) AA-traders have reached that goal. Robot trading can also be applied to the task of

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<sup>14</sup>ABM, of course, has also generated models with an auctioneer, e.g. Van der Hoog (2005) presents a model in which traders are aware of disequilibrium through rationing. It is a dynamic version of disequilibrium models as discussed in Benassy (1982) and Dreze (1987). The markets are coordinated by an auctioneer who is able to observe aggregate excess demand and who communicates rationing constraints to individual traders. Van der Hoog (2005) finds that quantity constraints sometimes let tâtonnement converge to a Walrasian equilibrium when a process without such constraints does not. The additional information provided by quantity constraints doesn't have much impact on the disequilibrium process of pure credit and cash-in-advance economies.

replicating human trading. This is taken up in chapter 4 and in appendix B, using the data of Anderson et al. (2004).

Hommes (2006) and LeBaron (2006) provide extensive overviews of agent-based modeling as applied to behavioral economics and finance. Hommes (2013) focuses on bounded rationality and heterogeneous beliefs, confronting theory with experimental data. Of particular interest is the recognition that agents can employ different strategies when it comes to expectation formation, and that beliefs or expectations are heterogeneous. Switching between different strategies can lead to rational routes to randomness, as agents become more sensitive to the past performance of strategies. It is shown that simple rules of thumb can be ecologically rational, i.e. they can survive the competition with more sophisticated learning strategies.

## 2.5 Discussion

### 2.5.1 Disequilibrium theory

Stability theory should be derived from disequilibrium theory, with the latter being a theory of how agents behave and interact while the economy is out of equilibrium. Or, as Schinkel (2001) puts it, disequilibrium theory is concerned with changes in individual plans as a result of changes in information. As it happened, stability theory came to be largely occupied with the stability of the Walrasian equilibrium instead.

We consider experimental trading at all prices in a Continuous Double Auction to be a fairly realistic. Experimental trading in the unstable Scarf economies therefore is a strong indication that we should not expect real markets to be an effective price mechanism in the sense of Saari and Simon (1978). We propose that disequilibrium theory, among other things, should try to explain the dislocation (if any) of stable states of experimental trading relative to the Walrasian equilibrium.

The CDA is a good setting for studying disequilibrium theory, because it inflicts *essential* uncertainty on traders. Agents have to seize opportunities as they see them. These opportunities spring from differences in behavior and in learning: although agents observe the same history, they still may have different expectations of what the equilibrium prices will be, because of different initial expectations, or because of different learning rules. A CDA provides a certain degree of coordination and transparency, but far less than is assumed in the Hahn/Negishi process. Furthermore, the CDA provides potentially large, integrated, markets as opposed to the artificially small markets in Fisher (1983), that are distinguished by commodity, seller and date. Last, but not least, experimental economics has obtained its remarkable results using a CDA.

### 2.5.2 The Sonnenschein-Mantel-Debreu result

The SMD-result is usually taken to imply that price dynamics can be as exotic as desired. However, that interpretation is vulnerable on several accounts:

- it is unlikely that aggregate excess demand drives price formation:
  - without an auctioneer no agent can observe aggregate excess demand;

- to the extent that aggregate excess demand can exert its influence with trading at all prices it is distorted: at any time, demand and supply are expressed only partially, and (in a monetary economy) agents may even have to sell before they can articulate demand;
- even though the aggregate excess demand *function* may violate WARP, a suitable price adjustment *process* can still deliver orderly prices, c.f. appendix C.<sup>15</sup>

## 2.6 Conclusions

In this chapter we have reviewed different theories of price formation, mainly coming from stability theory. Although many economists ignore stability theory, we consider it paramount.

Fisher (1983) is one of the most advanced contributions to stability theory; it allows trading at all prices and also production and consumption before equilibrium prices have been determined. Fisher, however, is not very optimistic about further progress of stability theory at a high level of generality.

Stability theory is largely focused on the Walrasian equilibrium. As a result, it typically has consumers and producers making comprehensive plans that cover all markets (i.e. all goods, locations, dates and contingencies). In order to keep decision-making manageable, it is assumed that decision-makers act as price-takers or that they have very good expectations. Furthermore, often price formation is assumed to be driven by aggregate excess demand. Comprehensive choice, price taking and a crucial role for aggregate excess demand are unrealistic and unsuitable for our purpose.

We are interested in the discovery of stable states through trading at all prices. We want to know (i) how people cope if they don't know what the equilibrium prices are; (ii) how the end result of trading at all prices can be characterized; (iii) whether or not it is close to the Walrasian equilibrium; and (iv) why? We will therefore approach the subject of stability theory from the perspective of experimental economics and agent-based modeling.

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<sup>15</sup> $\mathcal{P}$  illustrates that such a price adjustment process can even be intelligible. Ackerman (2002, p. 123) is quick to conclude that after the SMD result any algorithm that "reliably converges to equilibrium must be even less realistic, and far more complex, than tâtonnement", using Smale's algorithm to underscore the point.