Price discovery with fallible choice

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Chapter 3

Experimental trading in the Scarf economies

3.1 Introduction

Anderson et al. (2004) reports experiments in which human subjects traded in the famous examples of Scarf (1960). It demonstrates that human traders are able to approximate the Walrasian equilibrium in the stable version of the Scarf economy, while failing to obtain equilibrium in the unstable versions. These results are in line with the predictions of tâtonnement theory. The research of Anderson et al. is described in section 3.2. Sections 3.3 and 3.4 describe and analyze the data that were kindly provided by Prof. Anderson. Section 3.5 reflects on the experiments of Anderson et al., and on disequilibrium theory and section 3.6 concludes.

3.2 The experiments of Anderson et al.

3.2.1 Research questions

The central question of Anderson et al. (2004) is whether price formation conforms to tâtonnement theory: (i) does experimental price formation converge in the stable example; (ii) do these prices orbit in the unstable examples and (iii) if so, do they orbit in the predicted direction (clockwise or counter-clockwise, depending on the initial allocation)?

3.2.2 Parameters

Anderson et al. have changed the parameters of the Scarf economies to make price formation more challenging for human traders. Scarf's original equilibrium prices, $\mathbf{p}^* = (1, 1, 1)$, are easy to guess and that could distort the price discovery process (c.f. section 2.1). Furthermore, in an experimental setup it is convenient to have indivisible commodities, and also more than one unit of each commodity. Especially so, if one commodity takes on the role of money.

The Scarf economies consist of three traders, $i = I, II, III$. In each experiment, there are 15 traders; this is achieved by replicating the basic economy five times. Let traders 1,4,7,10,13 be of type $i = I$; traders 2,5,8,11,14 be of type $i = II$ and the others of type $i = III$.[1] In addition, there are three commodities, $j = 1, 2, 3$, the first of which is money. Money is the *numéraire*, i.e. $p_1 = 1$. A commodity bundle is denoted by $\mathbf{x} \in \mathbb{R}^3_+$, with the first subscript of x_{ji} referring to a commodity, and the second to a (type of) trader. The preferences of each type of trader with respect to commodities are described by Leontief utility functions:

$$\begin{array}{rcl} u_1(\mathbf{x}_1) & = & \min(\frac{x_{11}}{400}, \frac{x_{31}}{20}) \\ u_2(\mathbf{x}_2) & = & \min(\frac{x_{12}}{400}, \frac{x_{22}}{10}) \\ u_3(\mathbf{x}_3) & = & \min(\frac{x_{23}}{10}, \frac{x_{33}}{20}) \end{array} . \tag{3.2.1}$$

The examples differ in how endowments are allocated to types of traders, in particular (commodities in rows, types of traders in columns):

$$\mathbf{W}^A = \begin{pmatrix} 0 & 0 & 400 \\ 10 & 0 & 0 \\ 0 & 20 & 0 \end{pmatrix}; \mathbf{W}^B = \begin{pmatrix} 400 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 20 \end{pmatrix}; \mathbf{W}^C = \begin{pmatrix} 0 & 400 & 0 \\ 0 & 0 & 10 \\ 20 & 0 & 0 \end{pmatrix}.$$

Here, $\mathbf{W}^A$ is the initial allocation of the basic stable version of the Scarf economy; $\mathbf{W}^B$ and $\mathbf{W}^C$ are the initial allocations of the basic clockwise and the counter-clockwise unstable versions of the Scarf economy. Demand, given endowments $\mathbf{W}^A$, is equal to

$$\begin{array}{ll} \hat{x}^A_{11} = \dfrac{200p_2}{20 + p_3}; & \hat{x}^A_{31} = \dfrac{10p_2}{20 + p_3}; \\ \hat{x}^A_{12} = \dfrac{800p_3}{40 + p_2}; & \hat{x}^A_{22} = \dfrac{20p_3}{40 + p_2}; \\ \hat{x}^A_{23} = \dfrac{400}{p_2 + 2p_3}; & \hat{x}^A_{33} = \dfrac{800}{p_2 + 2p_3}. \end{array}$$

Cases $\mathbf{W}^B$ and $\mathbf{W}^C$ are similar. All three examples have the same equilibrium, i.c.

$$\{\mathbf{p}^*, \mathbf{X}^*\} = \left\{ (1, 40, 20), \begin{pmatrix} 200 & 200 & 0 \\ 0 & 5 & 5 \\ 10 & 0 & 10 \end{pmatrix} \right\}.$$

Note that replication does not alter the equilibrium. The Walrasian equilibrium has the equal treatment property, which says that traders of the same type receive the same allocation in a Walrasian equilibrium. End states of the experimental economy obviously do not have the equal treatment property, because agents make their own decisions, depending on their own expectations and opportunities.

3.2.3 Market protocol

Anderson et al. allow trading at all prices. That means that trading is not postponed until the equilibrium has been determined. Instead, an exchange occurs if two traders both agree to it. With hindsight, traders may regret previous decisions, because they

[1] Our labeling of traders and commodities differs from Anderson et al. (2004).

have paid dearly or sold too cheap. Such "errors" can cause the equilibrium to shift.[2] However, without trading at non-equilibrium prices traders cannot learn.

Money performs its role as a medium of exchange: commodities 2 and 3 can only be traded in exchange for money. In the stable Scarf economy, agents 1 and 2 have to sell before they can buy. In the clockwise unstable economy, trader 3 has to sell part of his endowment of commodity 3 in return for money, even though he does not derive any consumptive utility from it. If they choose to, agents can also buy / sell commodities with the intention of selling / buying them at a later time for a better price.

Trading occurs in a Continuous Double Auction (CDA). In this auction, both buyers and sellers can submit offers. Traders can propose an offer, e.g. sell one unit of commodity 2 in exchange for 35 units of money, or they can accept a pending offer (or floor offer). An acceptance of a pending offer can be partial. If five units of commodity 2 are up for sale, then a buyer may agree to accept one unit at the going price. In that case, the acceptance gets executed and the quantity for sale of the floor ask is updated, but its ask price remains unchanged. In addition to proposing a bid or an ask or accepting a pending offer, owners can also cancel a pending offer.[3]

An offer to buy is called a bid and an offer to sell is called an ask. A floor offer is the currently best bid or ask, with the unit price determining which offer is best. The depth of the order book is one, which means that offers that have been replaced as the currently best offers, are lost. If trader 1 offers to sell commodity 2 at 50 units of money, and trader 4 is prepared to sell at 45 then the offer of trader 1 does not reappear after trader 4's ask has been accepted. If trader 1 still wants to sell at 50, then he has to submit a new offer.

Trading is asynchronous in the sense that there is no prescribed sequence of submitting offers. Traders can propose in either market, as they see fit. Furthermore, new bids and asks can be entered before the last offer has been accepted. Limited synchronization is provided by the auctioneer informing all traders simultaneously about changes in floor offers.

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[2]The Scarf economies are special in the sense that the equilibrium will shift only in prices, but not in allocation. Internal consistency requires for each trader that his notional demand is equal to $\mathbf{x}_i^* = \beta_i \mathbf{q}_i$ where $\beta_i \geq 0$ is determined by trader i's budget and $\mathbf{q}_i$ depends on the parameters of the Leontief utility function alone. As a matter of fact

$$\mathbf{q}_1 = \begin{pmatrix} 400 \\ 0 \\ 20 \end{pmatrix}, \mathbf{q}_2 = \begin{pmatrix} 400 \\ 10 \\ 0 \end{pmatrix}, \mathbf{q}_3 = \begin{pmatrix} 0 \\ 10 \\ 20 \end{pmatrix}.$$

Solving for external consistency, i.e. for the equality of demand and supply, is enough to identify the β_i: they are all equal to 0.5. Therefore, the equilibrium allocation is unique and equal to $\mathbf{X}^*$. Whether a Walrasian equilibrium exists for an arbitrary initial allocation $\mathbf{W} \in \mathbb{R}_+^3 \times \mathbb{R}_+^3$, depends on the existence of prices that make $\mathbf{X}^*$ affordable, given $\mathbf{W}$. Therefore, if trading changes the allocation from, say, $\mathbf{W}^A$ to $\tilde{\mathbf{W}}$ then the equilibrium shifts from $\{\mathbf{p}^*, \mathbf{X}^*\}$ to $\{\tilde{\mathbf{p}}, \mathbf{X}^*\}$, provided a suitable $\tilde{\mathbf{p}}$ exists.

[3]Canceling an offer can be accomplished in two ways, either by submitting an order to cancel, or by accepting one's own offer. The latter opens up the possibility of sending false signals. Actually, in one experimental session (session 414) there was one agent who repeatedly accepted his own offers, but his actions didn't move the market and he didn't gain from it.

At any time, agents can see the currently best offers for buying and selling commodities 2 and 3. This includes their types (bid / ask), quantities and unit prices. The identity of the owner is not visible, so trading is anonymous.

Prices are positive and integer-valued because agents exchange indivisible units of commodities for units of money. If agents propose a new offer, its price must improve upon the price of the floor offer (if any). An improving feasible offer replaces the floor offer as the current best offer. Feasibility demands that traders can only sell commodities that they actually own, and that they can buy only if they have money. Feasibility is checked when an offer is submitted and before a trade is executed. Because the agents can operate in two markets at the same time, it is possible that a buyer's money holdings are insufficient when a seller decides to accept a previously feasible bid. In this case, an acceptance is executed to the extent that it is feasible.

After a floor offer is completely executed, or canceled, or after it has been declared unfeasible, it is removed as the floor offer. A floor offer is also cleared if it is superseded by an acceptance. Suppose there is a best bid and a best ask in a particular market, then an acceptance of the floor bid also clears the pending ask (and vice versa). The reasoning behind this is that an acceptance of a bid is also an ask, and one that necessarily improves upon the floor ask (if any).

3.2.4 Other design considerations

There were separate sessions for the different Scarf economies: six for the counterclockwise treatment, and two for the clockwise and the stable treatment each. Sessions consisted of instruction, one training period (for letting the participants get acquainted with the market and with the software), and several periods of trading (eight or more).[4] Each period started with the same initial allocation, that depended on the treatment.

Subjects, Caltech undergraduate and graduate students, were randomly assigned to their type. After the training period, they were incentivized with a financial stimulus that depended on their subsequent performance. On average, a participant earned $29 dollar for a 3 hour experiment. Many of the subjects had participated in unrelated experiments, but they didn't necessarily have any background in economics. Two sessions were characterized as consisting of experienced participants. No subject participated in more than one of the inexperienced trials.

3.2.5 Methodology

Anderson et al. had to determine whether prices converge, and (if not) whether prices orbit around the equilibrium. For $p > 0$ the function

$$\mathcal{L}(p) = \frac{1}{3}p^3 - (p^*)^2 p + \frac{2}{3}(p^*)^3$$

is strictly positive, unless $p = p^*$; $\mathcal{L}(p)$ will be called a Lyapunov function. Changes in $\mathcal{L}(p(t))$ depend on changes in $p(t)$. If $\mathcal{L}(p(t))$ is continuously decreasing then the

[4] Participants used a software program called MUDA (Multiple Units Double Auction) for entering their offers; it is described in Plott and Gray (1990).

price adjustment process must be converging. Anderson et al. apply the dynamics of a tâtonnement process in continuous time to

$$\tilde{\mathcal{L}}(\mathbf{p}(t)) = \frac{\sum_i w_{2i}}{\lambda_2}\mathcal{L}(p_2(t)) + \frac{\sum_i w_{3i}}{\lambda_3}\mathcal{L}(p_3(t))$$

with $\lambda_2, \lambda_3 > 0$ the speeds at which prices adjust and they show global stability of their version of the stable Scarf economy. Here, $\sum_i w_{ji}$ is the aggregate amount of commodity j. Human price formation, obviously, is different from tâtonnement and one cannot assume that successive values of $\mathcal{L}(p_2(t))$ and $\mathcal{L}(p_3(t))$ are all decreasing. Whether they have a tendency to decrease is tested by Anderson et al. by estimating

$$\tilde{\mathcal{L}}(\mathbf{p}(t)) = \alpha + \beta t \Rightarrow$$

$$\sum_i w_{3i}\mathcal{L}(p_3(t)) = \alpha\lambda_3 - \frac{\lambda_3}{\lambda_2}\sum_i w_{2i}\mathcal{L}(p_2(t)) + \lambda_3\beta t.$$

Here, the sensitivity to changes in prices over time determines whether there is convergence ($\beta < 0$), divergence ($\beta > 0$) or perhaps orbiting ($\beta = 0$) in the stable Scarf economy. The sign of β is inferred from the sign of the estimate of $\lambda_3\beta$.

The analysis of orbiting requires a time series of synchronized transaction prices. In a Continuous Double Auction, agents observe transaction prices in distinct markets, e.g. p_2^1, p_2^2, p_3^3, p_2^4, .. and so on (superscripts refer to the passage of time). These data can be converted into a time series of synchronized prices by successively updating elements:

$$\begin{pmatrix} 1 \\ p_2^0 \\ p_3^0 \end{pmatrix}, \begin{pmatrix} 1 \\ p_2^1 \\ p_3^0 \end{pmatrix}, \begin{pmatrix} 1 \\ p_2^2 \\ p_3^0 \end{pmatrix}, \begin{pmatrix} 1 \\ p_2^2 \\ p_3^3 \end{pmatrix}, \begin{pmatrix} 1 \\ p_2^4 \\ p_3^3 \end{pmatrix}, \ldots$$

For detecting the direction of orbits two methods were devised, the clock hand model (which measures the change in the angle of subsequent observations relative to the equilibrium prices) and the quadrant model (which registers the quadrant of a new observation relative to the current observation). Prices orbit clockwise if the clock hand, fixed at the equilibrium, pointing at two successive observations moves clockwise, or if the new observation falls in the next clockwise quadrant. The latter depends on the position of the current observation relative to the equilibrium. Suppose that the current observation is in the first quadrant relative to the equilibrium prices, that is $p_2 > 40$ and $p_3 > 20$, then the movement is clockwise if the new observation falls into the IV-th quadrant relative to the current observation.

In addition to looking at trade-by-trade data, Anderson et al. (2004) also analyzes average prices per period. Here, two models are used to predict the direction of price changes: movement (i) toward the equilibrium, or (ii) in the direction offsetting excess demand.

3.2.6 Results

Anderson et al. (2004) finds that the experimental results are in line with the predictions of tâtonnement theory, i.e. there is convergence in the stable Scarf economy but not in the unstable versions. Furthermore, prices orbit in the correct direction. Figures 3.1 and 3.2 show the time series of observed trades in different experiments (not synchronized). These graphs exclude data from the training period. At times = 1,2,.. endowments are reset and a new period starts. In the stable economy, trading prices stay near their Walrasian equilibrium values most of the time. The counter-clockwise data show a long fluctuation, with strong divergence taking place in the beginning of the experiment. In session 420 a clockwise treatment was applied (figure 3.2); here, divergence mainly occurs in the second half of the experiment. In session 419, prices are quite stable, with slightly diverging trends. However, by looking at period-to-period data, Anderson et al. did detect signs of clockwise orbiting in the case of session 419 as well. Hirota et al. (2005), in replicating the results of the unstable Scarf economies, finds clockwise orbiting to be much more pronounced.[5] Goeree and Lindsay (2016) finds a similar long term fluctuation in the counter-clockwise unstable Scarf economy.

3.3 Data on individual offers

Prof. Anderson has kindly provided the offers, that were submitted by human traders during two sessions. This allows a detailed comparison of algorithmic and human moves, conditional on the same information set (see chapter 4 and appendix B). Before using this dataset, we first describe and analyze it.

In addition to some files containing the time series of synchronized trading prices of all experimental sessions, we have received two files with individual offers. These refer to sessions 414 (a stable treatment) and to session 511 (a counter-clockwise treatment). The data files consist of a trader's action (new bid / ask, acceptance, cancellation), timing (period, time left), commodity, quantity, price and the identity of the proposer and (if applicable) of the counter-party. Despite scant documentation, we were able to fully reconstruct the time series of synchronized trading prices from the raw data.[6]

Unfortunately, it was not possible to cluster the observations. Apparently, in MUDA (the software which was used in the experiments) it was just as easy to

[5]Hirota et al. (2005) makes several additional observations: (i) markets adjust at different speeds; (ii) percentage price changes, rather than absolute prices, follow the tâtonnement model

$$\begin{pmatrix} \dot{p}_1/p_1 \\ \dot{p}_2/p_2 \end{pmatrix} = \begin{pmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{pmatrix} \cdot \begin{pmatrix} z_1(p_1,p_2) \\ z_2(p_1,p_2) \end{pmatrix} = \mathbf{A}\cdot\mathbf{z}(\mathbf{p})$$

(that is, the estimate of reaction matrix $\mathbf{A}$ can be described as constant and diagonal; $\dot{p}_j$ is the time derivative of p_j); (iii) the experimental data reject the (generalized) Newton algorithm of Smale (which would have produced convergence instead of orbits, c.f. section 2.2.2.2).

[6]The data of Anderson et al. consist of 5281 observations from session 414 (stable treatment) and 4741 observations from session 511 (counter-clockwise treatment). These observations contain computer-generated actions, related to initializing and finalizing periods. There are 5194 + 4605 = 9799 valid observations representing human decisions. The raw data appear to be not always in chronological order. That is why they have been treated by changing the order of a few observations (72 in session 414 and 14 in session 511): this increased the number of synchronized trading prices that could be successfully reconstructed (a file with synchronized trades from all sessions was also provided; this file was used as a benchmark to verify our interpretation of the raw data). Synchronization critically depends on the ordering of observations, c.f. section 3.2.5. References to specific observations, e.g. observation 115 of session 414, assume the original ordering of the raw data.

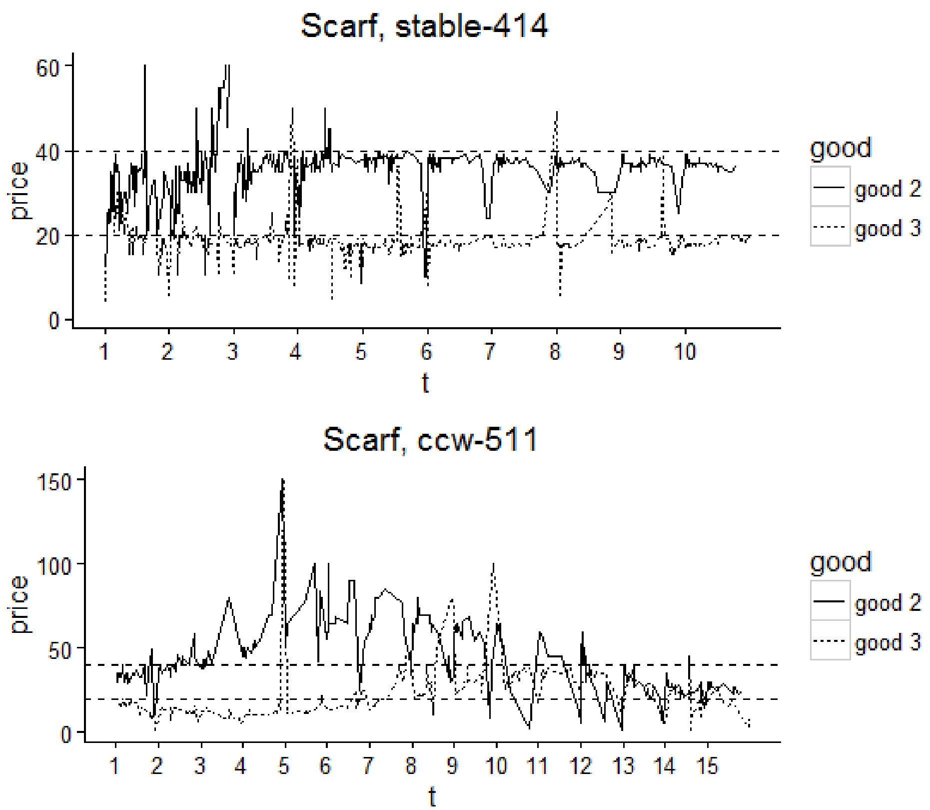

Figure 3.1 – *Experimental trading prices in the stable and counter-clockwise Scarf economies (note that the scales of the y-axes are different). The stable economy has prices near their Walrasian equilibrium values for some time, while the unstable ccw-economy exhibits long fluctuations.*

accept offers by specifying the number of units, say n, as it was to accept n times one unit. Quite often, traders have selected the latter method. For instance, observations 1707-1715 of session 414 refer to nine acceptances to buy one unit of good 3 at price 17. Most likely, these offers (which were submitted by the same trader within the span of three seconds) were the result of a single decision. That would call for clustering. However, clustering would also distort the course of trading. Suppose that trader b bids 16 for good 3 while trader a is still submitting successive acceptances at price 17, then b's bid is canceled because an acceptance at 17 trumps a bid at 16. If the acceptances are clustered, then b's bid would stand and we would have a different continuation of the experiment. We do cluster acceptances for studying quantity decisions (see section 3.4.3), but for the replication we keep the original data. Note, however, that it matters how the data are recorded: (i) for assessing convergence, it will make a difference if there are nine successive trades at price 17 instead of just one; furthermore (ii) algorithms that ignore quantities will overestimate the likelihood of buyers accepting an ask price of 17; and finally (iii) robot traders with limited memory may lose track of previous trades and of the spread of trading prices.

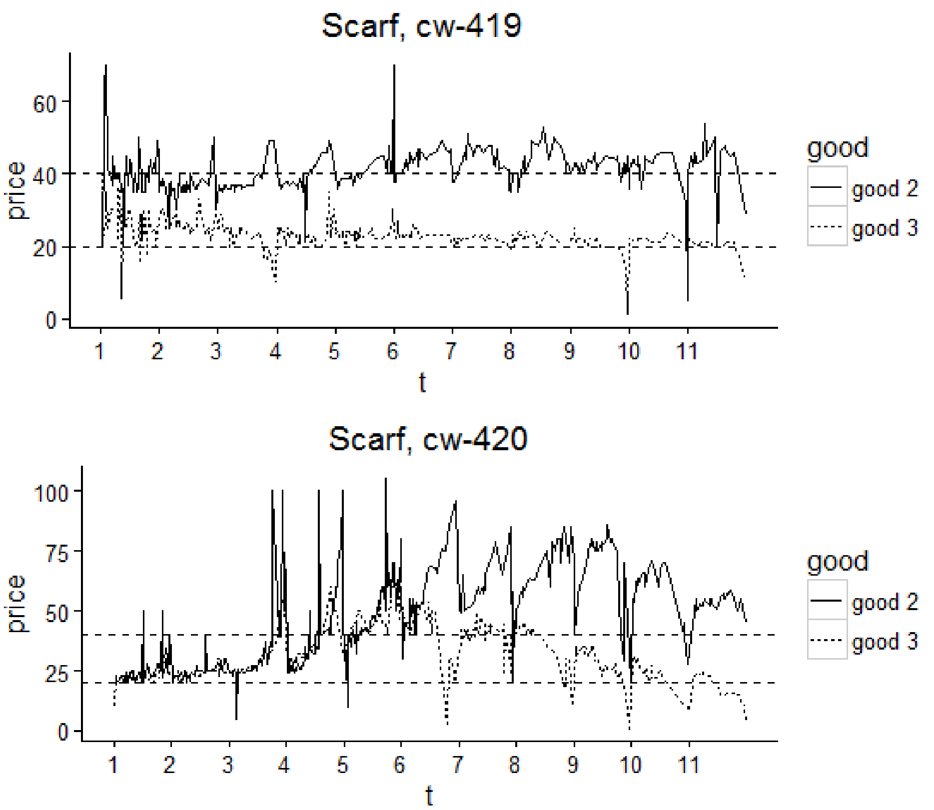

Figure 3.2 – *Experimental trading prices in the clockwise unstable Scarf economy. The data from session 419 look fairly stable while those of session 420 suggest fluctuations per period.*

3.4 Stylized facts of human trading

3.4.1 Overview of actions

In a particular market, available actions consist of proposing to buy or sell and of accepting a bid or an ask. If a trader owns the offer he is accepting then that constitutes a cancellation. Table 3.1 presents an overview of actions, as taken by human traders during session 414, i.e. in playing the stable Scarf economy.[7] It shows that human traders engage in arbitrage. This is most obvious where traders of types I and II accept to buy commodities from which they do not derive utility. However, arbitrage also occurs if traders offer to sell commodities which they must buy back later if they want to restore their level of utility.

Table 3.2 shows the actions in sessions 414 and 511 that can be classified as arbi-

[7] Actions in the counter clockwise treatment, session 511, are similar. The biggest difference is that traders of type III are more active (+30%), while the other traders submit fewer offers (-30%). Furthermore, traders of type III submit fewer strategic offers (less than half compared to the stable treatment) while the other traders relatively submit slightly more strategic offers.

Table 3.1 – Overview of human actions in session 414 (stable Scarf economy)

description	traders *I*		traders *II*		traders *III*		total
	regular	strategic	regular	strategic	regular	strategic	
propose bid 2		59	451	2	256	6	774
propose bid 3	412	6		44	133	16	611
propose ask 2	613		60	33	31	61	798
propose ask 3	75	92	732		49	67	1015
accept bid 2	178		6	10	4	15	213
accept bid 3	9	20	58		13	12	112
accept ask 2		36	162	9	245	7	459
accept ask 3	429	6		44	582	14	1075
cancel bid 2	21		3		2		26
cancel bid 3	6				1		7
cancel ask 2	21		2		3		26
cancel ask 3	71		4		3		78
total	1835	219	1478	142	1322	198	5194

Human actions have been classified by type and as either regular or strategic. Strategic offers include arbitrage (e.g. traders of type I accepting asks for commodity 2), but for instance also asks from buyers which are intended to drive down the ask price.

trage.[8] It is interesting to see the variation in the willingness to engage in arbitrage; this is most pronounced in the practice periods (20% - 25% of all the actions is characterized as an arbitrage). When trading "for real" the number of identifiable arbitrage actions quickly decreases (to around 7% of the actions). Since prices in the counter clockwise treatment are more volatile, there are more opportunities for arbitrage. Yet, the propensity to engage in arbitrage here is slightly lower: 6% of all actions versus 7.5% in the stable economy.

If a buyer already owns something of a commodity, he can also offer to sell it in order to drive down ask prices. Depending on the ask price, the buyer may even run little risk of his ask being accepted. Such offers to sell, which are not intended to be accepted, are strategic.[9] Since we do not actually know the intentions and

[8] Contrary to table 2 of Anderson et al. (2004), session 414 consisted of 1 + 10 periods (instead of 1 + 11).

[9] There are some subtleties involving the classification of offers. Whether or not an offer is treated as strategic, but not as arbitrage depends on the trader's endowments: for an offer to be strategic, the trader must have a stock of the commodity whose price he is trying to improve. For example, a trader of type *I* can submit an offer to buy commodity 2; if he has stock of commodity 2, then the offer is treated as strategic, otherwise it will be considered an arbitrage. Furthermore, an action

Table 3.2 – Arbitrage in sessions 414 (stable) and 511 (ccw)

period \ trader	stable				ccw			
	I	*II*	*III*	total	*I*	*II*	*III*	total
0	25	44	29	98	53	61	12	126
1	18	14	21	53	15	7	2	24
2	15	2	21	38	19	1		20
3	7	9	8	24	11	1		12
4	11	14	10	35	4		5	9
5	21	2		23	14	1	1	16
6	4	9	8	21	6		5	11
7	5	4	11	20	6		1	7
8	6	8	6	20	3	10	2	15
9	4	7	4	15	6		2	8
10	6	1	4	11	2	1	3	6
11					4	1	5	10
14						4	1	5
15							1	1
total	122	114	122	358	143	87	40	270

Actions identified as arbitrage (see text for criteria). The willingness to engage in arbitrage appears to vary with periods (practice versus playing "for real") and treatments. The decreasing number of arbitrages in the stable treatment reflects the shortening of the length of subsequent periods. In the counter clockwise experiment the amount of arbitrage tends to fluctuate.

Traders of type III have substantially fewer arbitrages in the counter clockwise treatment, because they have to sell before they can buy.

expectations of individual traders it may well be that there are more strategic offers (including arbitrage) than what is suggested by table 3.1. Interestingly, buyers are prepared to put in strategic asks, but sellers are reluctant to submit strategic bids (c.f. table 3.3). Since buyers and sellers are the same agents in different roles, it is not very likely that buyers are better than sellers in assessing whether prices are plausible or not. We propose that traders consider strategic bids riskier than strategic asks.

Behavior, in more than one way, affects the perception of feasible opportunities. Traders ignore arbitrages that are deemed unfeasible because of the associated risk. They do consider options that lower their current level of utility: in all, 731 out of 5194 offers would decrease utility levels if accepted. For instance, traders of type *I* with

may be a regular proposal or an arbitrage depending on the quantity. Robot traders offer limited amounts, but human traders sometimes propose to exchange two or three units. If such an offer would be accepted then it may need to be reversed, while the same offer with quantity one can be accepted without the need to reverse arising.

Table 3.3 – Strategic bids and asks

description	traders I	traders II	traders III	total
bid 2	23	(2)		25
bid 3	(3)	3	(3)	9
ask 2		23	34	57
ask 3	71		39	110
total	97	28	76	201

Offers that refer to arbitrage have been excluded. This explains the difference between for instance the 67 strategic bids for commodity 2 in table 3.1 and the 25 offers here. There are more strategic asks than bids. This is partly due to the fact that some strategic bids can only occur after traders overshoot their optimal endowments. Figures between brackets refer to such strategic bids. Ignoring those bids, it is clear that buyers are more likely than sellers to submit a strategic offer. Of the 23 strategic bids for commodity 2 from traders of type I, thirteen are due to a single trader, who is an exception to the rule.

endowments like $\mathbf{w} = (25, 8, 1)'$ are prepared to submit a bid for commodity 3, rather than first selling more of commodity 2. Clearly, they have enough of commodity 2 to sell so that they can replenish their money holdings at a later time. Furthermore, a trader of type III with endowments like $\mathbf{w} = (350, 0, 2)'$ is prepared to bid for commodity 3 even though his stock of commodity 3 is not "tight" (i.e. the stock of commodity 3 does not effectively determine his utility level). Hence, traders are not necessarily greedy.

3.4.2 Expected prices as reservation prices

Expected prices can serve as reservation prices. After all, why buy now if one expects prices to decrease, or why wait with selling if the price is expected to decline? If price expectations of human traders are anchored by some kind of average of observed prices, then their reservation prices will be much more concentrated than would otherwise be the case. Put differently, by focusing on averages traders can reduce uncertainty. Are expected prices of human traders (as revealed by their acceptances) aligned with average prices as estimated at the time of acceptance?

Figure 3.3 plots the trading prices of commodities 2 and 3 in sessions 414 and 511. The upper half shows prices as they were proposed. The lower half gives the same prices, divided by an exponential moving average (with random weights) at the time of the exchange. That moving average is here interpreted as an expected price. Apparently, it is more difficult for human traders to gauge expected prices in session 511, but the relative prices are nevertheless still centered around 1.

Table 3.4 compares trading behavior relative to three different estimates of average prices, i.c. exponential moving averages of offered prices and of trading prices (EMA(o) and EMA(t) respectively) and expected prices derived from Gjerstad-Dickhaut (GD-) beliefs. Table 3.4 reports proposed prices provided there is no floor offer. Both buyers and sellers seem to accept offers that are slightly worse than expected prices based on a weighted average of observed prices. For instance, they accept to buy a commodity if its price is less than, say, 105% of the expected price.

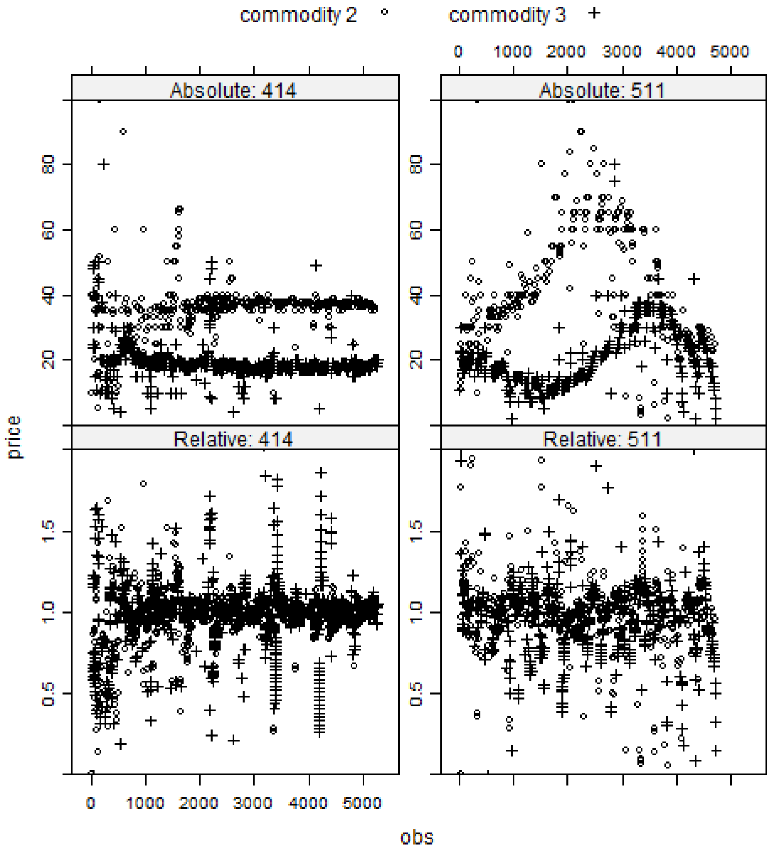

Figure 3.3 – *Experimental trading prices. The top-panels show absolute prices as observed in sessions 414 (stable) and 511 (ccw). The bottom panels give the absolute prices divided by an estimate of the average price that was current at the time of the trade. In both the stable and the counter-clockwise treatment, relative prices are clustered around 1, suggesting that expectations are anchored by averages.*

On average, sellers accept bids which are at 93% of their expected prices. Possibly, traders use reservation prices $p = (1 + \mu)\,\bar{p}$, where μ is a markup (positive for buyers, negative for sellers) and where $\bar{p}$ is the expected price.

Note that an estimated markup $\hat{\mu} \neq 0$ can be spurious: if traders estimate average prices differently, and if pessimistic buyers and sellers are likely to trade sooner rather than later, then it may seem that $\mu \neq 0$, while in reality $\mu = 0$ for everyone.[10] By simulating the distribution of exponential moving averages with random weights, we can calculate log likelihood values for different markups, c.f. table 3.5. The calculation is based on the likelihood of observing either an acceptance or a counter-

[10]Uneven learning offers a plausible explanation for different reservation values. Although traders observe the same history, they can have different initial expectations, or use different learning rules; some traders may discard old observations while others don't, et cetera.

Table 3.4 – Offers compared with expected prices

	session 414		session 511	
	mkt 2	mkt 3	mkt 2	mkt 3
buyer proposes, no floor				
EMA(t)	0.75	0.75	0.65	0.68
EMA(o)	0.74	0.73	0.66	0.67
GD	0.76	0.79	0.76	0.94
seller proposes, no floor				
EMA(t)	1.27	1.30	1.28	1.34
EMA(o)	1.21	1.27	1.39	1.35
GD	1.24	1.44	1.51	1.91
seller accepts				
EMA(t)	0.94	0.93	0.93	0.92
EMA(o)	0.92	0.89	0.93	0.91
GD	0.97	0.95	1.03	1.31
buyer accepts				
EMA(t)	1.07	1.00	1.07	1.04
EMA(o)	1.02	0.97	1.07	1.03
GD	1.04	1.05	1.26	1.37

Prices at which proposals and acceptances have occurred, relative to a current estimate of the average price. The results of the moving averages based on trading prices, EMA(t), and offered prices, EMA(o), are quite similar. GD-beliefs suggest that behavior is different in session 511. However, this is an artifact, due to the fact that our implementation of GD-beliefs is based on all observations (c.f. appendix B). With each new observation receiving relatively less weight than the previous one, this estimate is less flexible and has more difficulty in picking up the long term fluctuation in session 511.

offer, conditional on the next offer and on the floor offer (if any). For example, suppose that the next offer is a bid for commodity 2, and that there is a floor ask price of 45 for commodity 2. Assume that every potential buyer is equally eager to submit an offer, and that traders accept a floor offer if it compares favorably to their reservation price, $p = (1 + \mu)\,\bar{p}$. Then, if there are five potential buyers and three of them willing to accept the ask price, the probability of observing an acceptance is $3/5$, conditional on the next offer being a bid for commodity 2. The probability of observing a counter-offer instead would be $2/5$. If there is no floor ask, then the observed bid for commodity 2 has conditional probability 1. In this way, we can calculate conditional log likelihoods for different values of μ.[11] The table

[11]The simulation consists of 1,000 runs. Initial price expectations per robot trader per run per commodity are uniformly distributed in [1,200]. After each new observation, traders select a random weight from the interval $(0.95, 1.00)$ for mixing (this value is assigned to their current exponentially weighted average). The same weighing procedure was also used for creating figure 3.3 and table 3.4.

Conditioning on the type of the next offer serves to avoid the problem that rejections are not properly defined. Recall that the protocol of Anderson et al. stipulates asynchronous trading: buyers (sellers) accept a pending ask (bid) by offering the ask (bid) price. If a floor bid is improved upon by a buyer, then we consider it to be a rejection of the floor bid by the sellers, because the latter were not quick enough to accept it (c.f. appendix B). Thus, competition among buyers can lead to rejections, that are not necessarily the explicit result of meditated choice on the part of

Table 3.5 – Simulated conditional log likelihoods

case	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
stable	-2825	<u>-2641</u>	-2995	-3470	-3908	-4402	-5175	-5901	-6965	-8083
ccw	-2338	-2248	<u>-2200</u>	-2269	-2413	-2500	-2777	-3000	-3427	-3629

Log likelihoods are based on the probability of observing either an acceptance or a counter-offer, conditional on the type of the next offer. Suppose that the next offer is a bid for, say, commodity 2. If there is no floor ask for commodity 2, then the probability of the offer being a proposal is one. If there is a pending ask for commodity 2, then the probability of seeing an acceptance depends on the number of buyers with $(1 + \mu)\bar{p}_2$ exceeding the ask price, relative to the total number of buyers of commodity 2 at that time. These simulations suggest $\mu \neq 0$. We have assumed that buyers and sellers have the same value of μ, where it is understood that sellers use $(1 - \mu)\bar{p}_2$.

shows that the markup most likely is not spurious. Given the conservative calculation (see footnote 11), deviations between trading prices and expected prices cannot be exclusively explained by acceptance of offers by sufficiently pessimistic traders.

A markup $\mu \neq 0$ can have different explanations. It could be the result of competitive pressure. If everybody else is accepting asks above the expected price, then a buyer may stand to lose if he doesn't follow suit.[12] Alternatively, it could also be a rule of thumb for dealing with uncertainty. Competitive pressure may be expected to vary over time, across markets and across experiments, suggesting a variable markup. If the markup would be more or less constant, then that could suggest it is due to a rule of thumb.[13]

Figure 3.4 shows the decrease of uncertainty over time in session 414, as measured by the entropy of GD-beliefs. Here, entropy measures the unpredictability of bid and

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sellers. For instance, observations 114 and 115 of session 414 were preceded by various trades in commodity 2 at a price of 39; then, two traders virtually at the same time submitted ask prices of 12 and 5 respectively. While an ask price of 12 is already surprising, its "rejection" by buyers is unlikely for any method of expectation formation.

Still, there are certain observations that pose a problem for the calculation. Suppose that a human seller accepts to sell at 15, and that there are no robot sellers willing to do the same if $\mu \geq -0.10$, and that there a few willing to sell at 15 starting from $\mu = -0.11$. One approach would be to set the loglikelihoods of $\mu \geq -0.10$ to $-\infty$; another is to ignore such observations (including them for some μ's and not for others would introduce bias). Implausible acceptances can be due to human subjects wanting to contribute to the success of the experiment by keeping trading alive (i.e. to the Active Participation Hypothesis, c.f. Lei et al. (2001)). Setting loglikelihoods to $-\infty$ would then overestimate the size of the markup; we have chosen to be conservative and have ignored "impossible" events.

[12] We have simulated a payoff matrix showing that a markup $\mu \neq 0$ can be a Nash equilibrium. In this experiment, consisting of 10,000 runs in the stable treatment, one trader of type III considers different values for the markup, ranging from $\mu = 0$ to $\mu = 0.1$. All other traders use a markup of $\mu = 0.05$ (buyers accept if the floor ask compares favorably to $(1 + \mu)\, p^e$ and sellers accept if the floor bid is better than $(1 - \mu)\, p^e$, with p^e the expected price). The best choice for the trader of type III is to play $\mu = 0.05$ as well; this leads to a higher average level of utility than the other available strategies.

[13] If agents use rules of thumb to avoid costs of decision making, they may be slow to recognize differences in uncertainty between markets or over time.

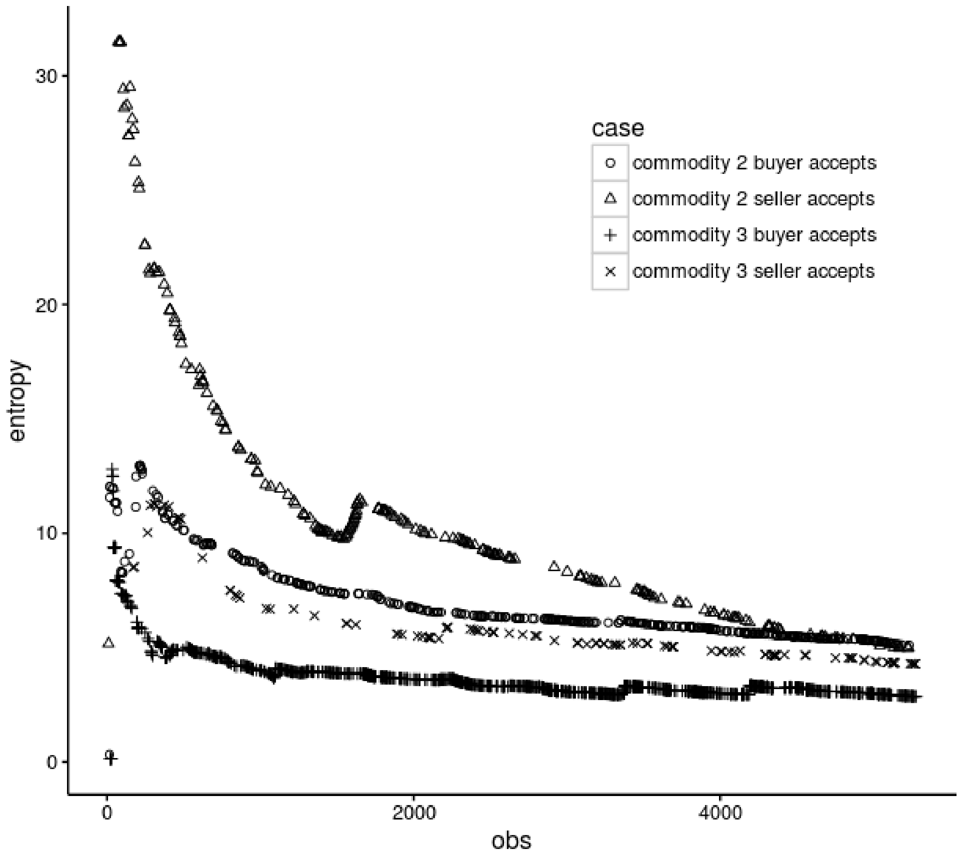

Figure 3.4 – *Evolution of uncertainty over time in session 414, as measured by the entropy of Gjerstad-Dickhaut beliefs. Interestingly, in either market sellers face more uncertainty than buyers. There is less uncertainty in market 3 compared to market 2 for both buyers and sellers, because there are more transactions in market 3.*

ask prices. There are more transactions in market 3 than in market 2; that could make agents feel more confident about their expected prices for commodity 3. Does lower uncertainty translate into smaller markups?

On this point the evidence is mixed: table 3.4 shows that average markups for buyers are smaller in market 3 (e.g. for EMA(t) it is 1.00 versus 1.07 in session 414 and 1.04 versus 1.07 in session 511), but for market 2 there is no difference between sessions 414 and 511 (e.g. for EMA(t) 1.07 in both session 414 and 511, while the latter is more uncertain, c.f. figure 3.3); sellers apply consistent markups across markets and sessions ($1 - \mu$ is approximately 0.93 with exception of EMA(o) in market 3 of session 414 where it is 0.89). Traders of type *III* (who buy both commodities 2 and 3) also apply similar markups in both markets (c.f. figure 3.5).

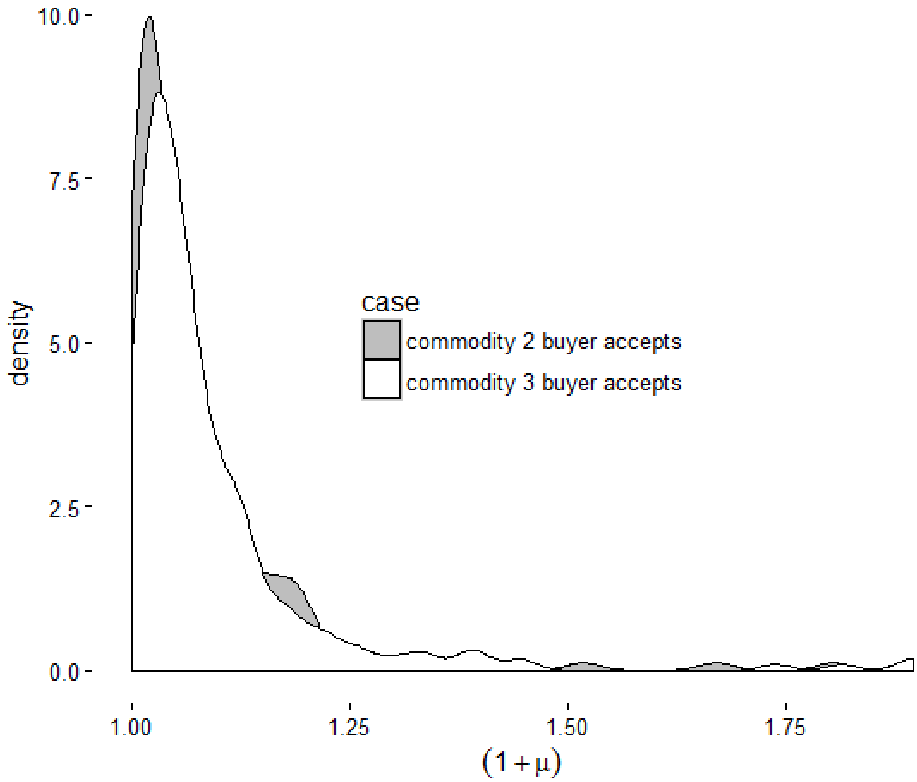

Figure 3.5 – *Acceptance of asks by type III buyers. The figure shows cases in which the ask price exceeds the expected price. The difference between commodities is negligible.*

3.4.3 Quantity setting

Quantities proposed and accepted depend on what a trader needs at expected prices, but also on what is feasible and on uncertainty with respect to price expectations. By offering to buy or sell small amounts traders can limit losses due to trading at unfavorable prices. While quantities proposed can be observed directly from the offers, decisions with respect to quantities accepted are more difficult to observe. The average quantity traded is one unit in both markets; but this is due to traders often submitting n acceptances of one unit, instead of once accepting n units.[14] We have tried to reconstruct the underlying decisions by clustering acceptances from the same trader, for the same commodity and the same price, provided that a subsequent submission occurred at most two seconds later than its predecessor.[15]

[14] Note that decisions with respect to quantities are also censored: at the going price a trader may have decided to accept all of the five units on offer; however, if another trader is quicker to accept three units then the data contain at most two acceptances of the former trader.

[15] The flow of time in the experiments was recorded in integer-valued seconds. Since we do not know if time was rounded or truncated one second may have been too restrictive.

Table 3.6 – Trading behavior by quantity decisions

case	mkt	action	≤ 1	≤ 2	≤ 3	≤ 4	≤ 5	≤ 10	≤ 20
stable	2	propose bid	65	82	90	95	99	100	100
		propose ask	47	78	91	92	95	100	100
		accept ask	87	98	99	100	100	100	100
		accept bid	75	95	99	99	100	100	100
	3	propose bid	48	75	85	90	95	100	100
		propose ask	56	70	77	81	90	94	100
		accept ask	86	93	98	99	100	100	100
		accept bid	71	88	92	96	97	100	100
ccw	2	propose bid	64	74	85	89	95	98	99
		propose ask	69	86	93	95	99	100	100
		accept ask	85	91	98	99	100	100	100
		accept bid	82	94	97	98	100	100	100
	3	propose bid	35	55	74	82	97	100	100
		propose ask	34	45	56	62	80	98	100
		accept ask	50	70	82	94	100	100	100
		accept bid	70	85	92	95	99	100	100

Cumulative percentages of actions with a quantity $\leq n$, per treatment, market and action. Quantities accepted reflect underlying decisions (see text). As expected, quantities proposed in market 3 tend to be greater than quantities proposed in market 2. Interestingly, in the market for commodity 3 in the unstable ccw treatment, quantities proposed and accepted are greater than the corresponding quantities in the stable treatment. In the market for commodity 2 in the ccw treatment, buyers propose greater quantities than in the stable treatment, while sellers propose relatively smaller quantities and accepted quantities are fairly similar.

Table 3.6 illustrates that traders try to limit the adverse consequences of trading at unfavorable prices by proposing and especially accepting, smaller amounts.

Table 3.7 gives average prices per quantity decided. It shows that larger quantities proposed tend to occur at prices that better resemble the Walrasian equilibrium prices. This is especially the case for ask prices. We assume that this is due to a greater confidence in expected prices. For quantities accepted, the relation with prices is less pronounced. Average trading prices (i.e. accepted prices) of commodity 3 are fairly close the Walrasian equilibrium value of 20. Most likely, the spread between accepted bid and ask prices of commodity 2 either reflects or causes orbiting in the ccw treatment.

Table 3.7 – Average prices by quantity decisions

case	mkt	action	1	2	3	4	5	$6-10$	$11-20$	>20
stable	2	propose bid	30	30	31	32	30	19		
		propose ask	92	45	39	47	40	38		
		accept ask	34	35	32	37	36			
		accept bid	37	38	37	38	35	38		
	3	propose bid	14	15	17	17	16	13	15	5
		propose ask	41	25	27	22	24	20	18	
		accept ask	17	20	16	17	17			
		accept bid	20	18	19	17	16	18	14	
ccw	2	propose bid	34	40	27	43	32	21	5	1
		propose ask	81	39	39	43	41	48		
		accept ask	37	35	53	26	26			
		accept bid	45	44	42	62	45			
	3	propose bid	16	14	16	17	19	13		2
		propose ask	32	25	26	19	38	25	15	
		accept ask	22	16	16	19	19			
		accept bid	23	19	19	21	15	17		

Average prices by treatment, market, action and quantity decided. In the case of the stable treatment, in the market for commodity 3 one observation with ask price 2018 and quantity 12 has been omitted. Inclusion of this long shot would have resulted in an average price of 49 (instead of 18) for the category $11-20$. *Interestingly, in the stable treatment proposed ask prices at higher quantities are closer to the Walrasian equilibrium values. In the ccw treatment, most proposed ask prices for commodity 3 are well above 20.*

3.4.4 Preference for markets

Sometimes, traders can choose in which market to submit an offer. This decision can depend on endowments, floor offers (if any) and possibly also on relative uncertainty. We propose that the behavior of these traders can be characterized as follows:

- acceptance is driven by opportunity and an aversion of uncertainty;
- own proposals are driven by needs.

Table 3.8 categorizes the circumstances for traders of type III according to needs and opportunities. If his endowment of commodity 2 is tight, then buying additional amounts of commodity 2 will raise the trader's utility level. On the other hand, floor prices may represent favorable opportunities, either because one pending rival bid is low while the other is not; or because an ask price is more favorable than the other (the table refers to situations in which both floor asks or both floor bids exist simultaneously).

Table 3.8 – Choice of market by traders of type III (%)

Short of	floor	acceptance		proposal	
		commodity 2	commodity 3	commodity 2	commodity 3
2	$2 \succ 3$	100		65	35
	$3 \succ 2$		100	88	12
	$2 \approx 3$	34	66	76	24
3	$2 \succ 3$	100		59	41
	$3 \succ 2$		100	33	67
	$2 \approx 3$	31	69	14	86
either	$2 \succ 3$	100		66	34
	$3 \succ 2$		100	78	22
	$2 \approx 3$	34	66	56	44

The preference for a market depends on needs and opportunities. Column "Short of" indicates which endowment is tight; and "floor" shows which market is considered attractive, relative to expected prices. For proposals this depends on pending rival bids, for acceptances on floor ask prices. If market 2 is more attractive than market 3, then this is labeled as $2 \succ 3$. *Markets 2 and 3 are considered equally attractive if both floor asks are less than 90% of their expected value. Traders accept offers in the market that is most attractive. If markets are equally attractive, traders of type 3 show a strong inclination to accept asks in market 3. This is largely explained by the fact that the average size of trades is one in both markets, while the need for commodity 3 is twice the size of the need for commodity 2. If endowments of both commodities 2 and 3 are tight, then traders may still anticipate the greater likelihood of successful trades in the market for commodity 3. In that case, the greater propensity to submit own proposals in market 2 makes sense.*

3.4.5 End of period allocations

Figures 3.1 and 3.2 show the evolution of trading prices. This section focuses on the allocation: how close does human trading in the Scarf examples approximate the Walrasian equilibrium allocation? In figures 3.6 and 3.7 the endowments of traders of the same type have been aggregated. The figures show the relative share of different types of traders in the total amount of each commodity. In every triangle, the distances from each side to an arbitrary point in the interior of the triangle always add up to one. These distances represent the shares of the different types of traders in the total amount of a particular commodity. If all is owned by traders of the same type then the allocation is in a corner of the triangle. Each dot represents the allocation at the end of a period (the training period has been excluded).

In the Walrasian equilibrium allocation the total amount of commodity 1 is shared evenly between traders of type I and II. That is, in the triangle for commodity 1, this allocation lies halfway on the side that connects I and II. Commodities 2 and 3 are evenly shared between types II and III and types I and III respectively.

Although aggregation of endowments per trader type suggests more convergence than there need be, the proximity to the Walrasian equilibrium allocation is remark-

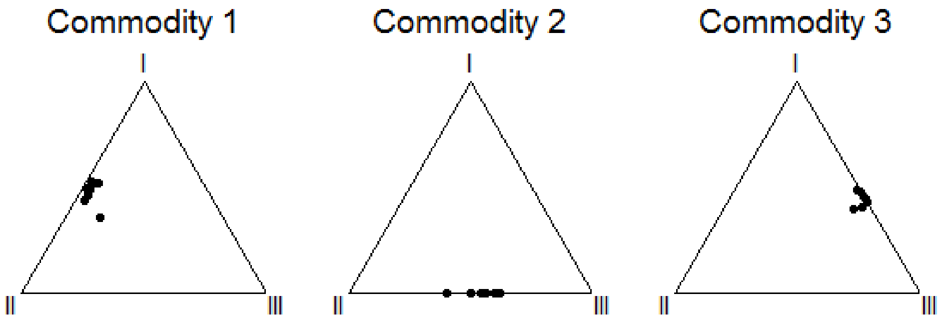

Figure 3.6 – *Shares per trader type in the available commodities in the stable treatment. Each dot represents an allocation at the end of a period (excluding the training period). The experimental allocations (per trader type) are quite close to the Walrasian equilibrium allocation, i.e. the midpoints of the left (commodity 1), bottom (commodity 2) and right side (commodity 3) of the triangles.*

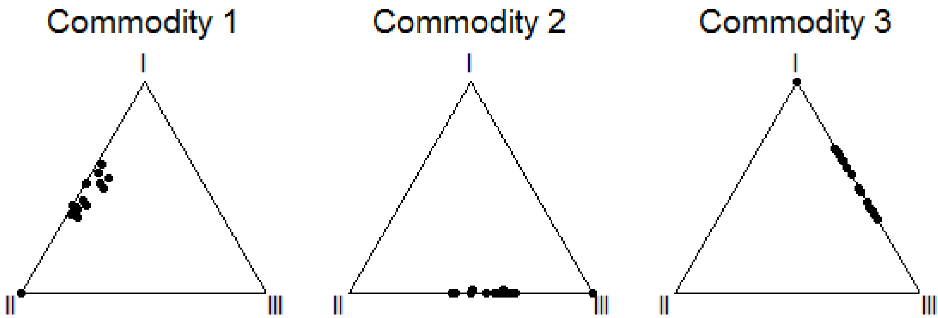

Figure 3.7 – *Shares per trader type in the available commodities in the counter clockwise treatment. Each dot represents an allocation at the end of a period (excluding the training period). Although the allocations in the stable treatment are more concentrated, here the allocations per trader type are also fairly close to the Walrasian equilibrium values.*

able, even in the counter clockwise treatment. This may be due to the fact that the equilibrium allocation in the Scarf economies is fixed (c.f. footnote 2 on page 27).

3.5 Discussion

3.5.1 Anderson et al. (2004)

The experiments of Anderson et al. are a major step forward compared to previous experiments in a single financial market with exogenous reservation prices. The fact that their results agree with tâtonnement theory (and that some of them have been replicated) is remarkable, and very important for stability theory. We have a few critical remarks to offer.

Price formation becomes unstable when the set of Pareto improvements gets exhausted (c.f. appendix A). To various degrees, the graphs in figures 3.1 and 3.2 show signs of unstable price formation near the end of a period. In creating time series of synchronized trading prices, Anderson et al. (2004) circumvents this problem by ignoring observations when one side of a market is not active, i.e. when there were no bids or asks for either commodity 2 or 3. While this decision mainly censors observations at the beginning and at the end of a period, it also censors some observations during a period, thereby distorting the time series of synchronized trading prices. The latter is unfortunate, indeed. However, we do agree that it may make sense to ignore observations near the end of trading when investigating convergence.[16]

Anderson et al. have devised two models for detecting the direction of orbiting, the clock hand and the quadrant model. The latter is said to be more powerful than the clock hand model, because in the quadrant model random predictions are wrong $\frac{3}{4}$ of the time instead of $\frac{1}{2}$ of the time as in the clock hand model. This argument is not relevant. Both the clock hand and the quadrant model require synchronized prices. Whereas the clock hand model can use all observations, the quadrant model must skip observations for getting independent data points. If all observations are used, then every next data point lies on one of the axes. After assigning axes to specific quadrants, the chance of being right with random prices would be the same as in the clock hand model: $\frac{1}{2}$. Skipping every other observation would leave a power of $\frac{3}{4} \times \frac{1}{2}$, which is less than $\frac{1}{2}$.

Although the authors took care to make price formation more challenging, one can argue that there is still too much symmetry because in an optimum all traders have to divide their budget evenly between two commodities. Such a split of the budget can be a focal point absent a deeper understanding of the preferences.

Sophisticated traders could have deduced the equilibrium prices in the unstable economies, even without having to trade. They would derive information from the fact that they themselves have to be optimized if the economy as a whole is in equilibrium. In particular, sophisticated traders can determine that they have to assign half of

[16]In this thesis, the focus is on replicating human trading behavior. Since the censored offers and trades have been observed by traders, they were part of their information set and they have conditioned subsequent trading. That is why we do not ignore any observation, c.f. chapter 4.

Table 3.9 – Ex ante information of sophisticated traders in the stable Scarf economy

description	general	agent I	agent II	agent III
$\hat{x}_k\ (\alpha_k > 0)$	$\frac{\alpha_j \sum_r p_r w_r}{\alpha_k p_j + \alpha_j p_k}$	$\hat{x}_1 = 5p_2$	$\hat{x}_1 = 10p_3$	$\hat{x}_2 = \frac{200}{p_2}$
$\hat{x}_j\ (\alpha_j > 0)$	$\frac{\alpha_k \sum_r p_r w_r}{\alpha_k p_j + \alpha_j p_k}$	$\hat{x}_3 = \frac{p_2}{4}$	$\hat{x}_2 = \frac{p_3}{4}$	$\hat{x}_3 = \frac{400}{p_2}$
$\hat{u}$	$\frac{\alpha_k \alpha_j \sum_r p_r w_r}{\alpha_k p_j + \alpha_j p_k}$	$\frac{p_2}{80}$	$\frac{p_3}{40}$	$\frac{20}{p_2}$
$\frac{p_j}{p_k}$	$\frac{\alpha_j}{\alpha_k}$	$p_3 = 20$	$p_2 = 40$	$p_3 = \frac{1}{2}p_2$

Sophisticated traders know that they themselves have to be in equilibrium if the economy as a whole is in equilibrium. From this, they can deduce that they have to assign half of their budget to each commodity. In the stable Scarf economy this leads to the above constraints on quantities and prices; in the unstable economies, they can even infer the complete Walrasian equilibrium from this budget rules.

their budget to each commodity.[17] In the unstable economies, this implies that in equilibrium their utility level will be 0.5; and from this piece of information they can infer all equilibrium prices. Judging from the experimental results of the unstable treatments, choices made by the subjects of Anderson et al. are fallible rather than sophisticated. Table 3.9 summarizes the information that is available to sophisticated traders in the stable Scarf economy, based on introspection alone.

3.5.2 Disequilibrium theory

The Scarf examples have a special feature that could explain why trading prices in the stable treatment approach the Walrasian equilibrium prices. The preferences of traders in the Scarf examples are such that a trade at a false price in one commodity does not distort demand for the other commodity. Suppose that traders of type I and III exchange commodity 2 at a price which is too high; then, wealth is transferred from a trader of type III to another trader of type I. Since both spend half of their budget on commodity 3, the decrease in demand for commodity 3 from the type III trader is exactly offset by an increase from the type I trader. Furthermore, in the market for commodity 2, there is now excess supply (relative to the Walrasian equilibrium) that may depress the price for commodity 2 at a later stage. Even though this seems to be a reasonable explanation, robot traders find it easier to learn the Walrasian equilibrium prices if this symmetry in demand is broken (c.f. 4.4.2.1).

Figure 3.1 illustrates that human trading produces trading prices that approach the Walrasian equilibrium values from below. Cliff (1997) reports the same phenomenon and explains it as an artefact. In the ZIP-model, a trade occurs if

$$\begin{aligned}(1+\theta_{seller})\,p_{seller} &\leq (1-\theta_{buyer})\,p_{buyer} \Leftrightarrow \\ \frac{(1+\theta_{seller})}{(1-\theta_{buyer})}p_{seller} &\leq p_{buyer}\end{aligned}$$

[17] Let u be the utility level associated with notional demand. We have $x_j = \frac{u}{\alpha_j}$, $\alpha_j > 0$. Furthermore, let $B = \sum_j p_j w_j$ and θ_j the share of the budget spend on commodity j. With these definitions, we can rewrite the budget constraint, $\mathbf{p} \cdot \mathbf{x} = B$, as

$$\begin{aligned}p_i &= \alpha_i \quad \frac{\theta_i B}{u} \\ p_j &= \alpha_j \quad \frac{(1-\theta_i)B}{u}\end{aligned}$$

with i and j such that $\alpha_i > 0$ and $\alpha_j > 0$ and $0 < \theta_i < 1$. Taking quotients left and right yields:

$$\frac{p_i}{p_j} = \frac{\alpha_i}{\alpha_j}\frac{\theta_i}{1-\theta_i} \Rightarrow \theta_i = \frac{\alpha_i p_j}{\alpha_i p_j + \alpha_j p_i}.$$

Substitution of the expected price expectations in terms of B and u leads to:

$$\theta_i = \frac{\alpha_i p_j}{\alpha_i p_j + \alpha_j p_i} = \frac{\alpha_i \alpha_j \frac{(1-\theta)B}{u}}{\alpha_i \alpha_j \frac{(1-\theta)B}{u} + \alpha_i \alpha_j \frac{\theta B}{u}} = 1 - \theta_i.$$

On the assumption that they will be able to realize their optimal plan, sophisticated traders can deduce that in equilibrium they will spend half their budget on commodity i and the other half on commodity j; and hence also:

$$\frac{p_i}{p_j} = \frac{\alpha_i}{\alpha_j}.$$

with $\theta \geq 0$ the profit margin. From this it follows that the reservation price of buyers, p_{buyer} exceeds the reservation price of sellers, $p_{sellers}$. Therefore, profits of buyers exceed profits of sellers in absolute terms. As a consequence, initial transaction prices should be expected to be below the equilibrium. These initial trading prices then shape subsequent expectations. A markup relative to the expected price for the purpose of deciding an acceptance, however, would correspond to $\theta < 0$. Then, the phenomenon of trading prices approaching the equilibrium prices from below is *not* an artifact.

3.6 Conclusions

We have discussed the experiments of Anderson et al. (2004). These are very relevant for our purpose because they feature trading at all prices and two markets (the stable state therefore can shift away from the Walrasian equilibrium). The results of Anderson et al. agree with tâtonnement theory. This gives us several ways for discriminating between rival behavioral hypotheses: not only is convergence of trading prices to the Walrasian equilibrium values contingent on the initial allocation, but orbiting of trading prices in accordance with tâtonnement theory provides another criterion. Furthermore, and perhaps most impressive, the end-of-period allocations should be quite close to the Walrasian equilibrium allocation.

Trading in the unstable treatments nicely illustrates that human choice is fallible: sophisticated traders would have derived the equilibrium prices of the unstable Scarf economies through introspection, i.e. without having to trade; the subjects of Anderson et al., on the other hand, did not achieve equilibrium in the unstable treatments.

We have analyzed the experimental data that we have received from Prof. Anderson; these pertain to the stable and the counter clockwise treatment. They illustrate that people engage in arbitrage, but mainly in the training periods. While playing "for real", identifiable arbitrage only accounts for about 7% of the actions. For this reason we postpone the analysis of arbitrage to chapter 5; we focus first on "regular" behavior in which traders buy what they need and sell what they can spare. This has the added advantage that it will become clear whether or not arbitrage is necessary for achieving convergence to the Walrasian equilibrium and / or orbiting.

The data also show that reservation prices are anchored by price expectations. Furthermore, traders typically exchange small quantities (presumably to mitigate adverse effects of mistaken price expectations). To demonstrate the latter finding, we have clustered successive acceptances that appear to be due to the same decision. Apparently, the subjects of Anderson et al. found it easier to accept n times one unit of a commodity instead of accepting n units once. Clustering, unfortunately, can distort the integrity of the experiment; therefore we will have to use the unclustered data for comparing robot behavior with trading by human beings.

Hirota et al. (2005) plots cumulative angles of successive trades, as determined by the clock hand model, to demonstrate the difference between clockwise and counter-clockwise orbiting; this is quite effective. We will use this idea in the calibration of price expectations, c.f. chapter 4.