# Price discovery with fallible choice

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**Publication date**
2018

**Document Version**
Other version

**License**
Other

Link to publication

# Appendix A

# Market failure

Initial simulations often resulted in a premature end of trading; this appendix investigates why. Section A.1 illustrates that there are different reasons for market failure and provides a characterization. Next, section A.2 estimates the probability of market failure. It derives a predicted probability of 0.72 of observing market failure in case of Zero-Intelligence trading, that is corroborated by actual simulations. Simulating expected utility maximization with the same model leads to an even higher frequency of market failure. Section A.3 demonstrates that this is largely due to the way that price expectations are initialized. Section A.4 reflects while section A.5 concludes.

## A.1 Possible end states of the simulation of Scarf economies

Market failure occurs if an inefficient allocation does not give rise to further trading. Economic theory attributes this to information asymmetries, incentives that promote non-competitive behavior, or externalities. However, there may be other reasons as well. Pareto improvements may be difficult to detect or impossible to implement under the institutional arrangements. Suppose that traders in the Scarf stable economy have reached one of the following allocations:[1]

$$\mathbf{W}^1 = \begin{pmatrix} 220 & 120 & 60 \\ 0 & 3 & 7 \\ 10 & 0 & 10 \end{pmatrix}; \mathbf{W}^2 = \begin{pmatrix} 40 & 325 & 35 \\ 0 & 0 & 10 \\ 2 & 0 & 18 \end{pmatrix}.$$

In either case, there exist Pareto improvements. In $\mathbf{W}^1$, for instance, traders 1 and 3 could give their excess amounts of money and of commodity 2 to trader 2. However, that does not qualify as trading. In this case, the trading process has

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[1]Recall that the preferences of each type of trader with respect to commodities are as follows:

$$\begin{aligned} u_1(\mathbf{x}_1) &= \min(\tfrac{x_{11}}{400}, \tfrac{x_{31}}{20}) \\ u_2(\mathbf{x}_2) &= \min(\tfrac{x_{12}}{400}, \tfrac{x_{22}}{10}) \\ u_3(\mathbf{x}_3) &= \min(\tfrac{x_{23}}{10}, \tfrac{x_{33}}{20}). \end{aligned}$$

In the stable Scarf economy, each trader owns the commodity he does not derive utility from.

taken the economy to an inefficient allocation, which cannot be improved within the given institutional setting. In $\mathbf{W}^2$, trader 2 wants to have more of commodity 2, and is able and willing to pay for it. Moreover, trader 3 has an excess amount of commodity 2. However, trader 3 may argue that instead of having an excess amount of commodity 2, he does not have enough of commodity 3. In this case, the deadlock is less severe than in $\mathbf{W}^1$, but it could just as well be persistent. If trader 3 believes that there is no more supply of commodity 3 forthcoming, then there is also no point in selling commodity 2 and obtaining more money.[2] If trader 3 offers a high price for commodity 3, then trader 1 may be tempted into arbitrage. However, if he is aware that commodity 3 is scarce then he also has to consider the risk of being unable to buy back what he is about to sell. Clearly, market failure is a possible outcome (even in large economies) because at some point the remaining Pareto improvements can be difficult or even impossible to implement given the behavior of traders and / or the market protocol.

Cases like $\mathbf{W}^2$ are particularly relevant for robot trading, because some algorithms exhibit rather rigid behavior. For instance, maximizing expected utility given expected prices would make $\mathbf{W}^2$ a steady state if trader 3 expects $p_3 < 17\frac{1}{2}$.

There may have been market failure in the experiments of Anderson et al. (2004), e.g. at the end of period 6 of the counter clockwise treatment, c.f. table A.1.

**Table A.1** – Excess quantities per participant after session 511 / period 6

| good | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 5 | 15 | 31 | 47 | 36 | 16 | 4 | 52 | 0 | 0 | 2 | 104 | 50 | 4 | 14 |
| 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 4.5 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 4 | 0 | 0 |

*Endowments that do not contribute to utility, by commodity and trader. Anderson et al. (2004) has labeled agents 0,..,14; agents 11 and 12 are both type III. Excess quantities are not necessarily integer-valued. The endowments of trader 12 are* $\mathbf{w}_{12} = (104, 7, 5)$*, implying an excess supply of 4.5 units of commodity 2. Given ample money holdings, traders 11 and 12 are more likely to demand commodities 3 and 2 respectively (instead of selling commodities 2 and 3). If so, there is market failure due to inconsistent plans.*

When feasible Pareto improvements are nearly exhausted, price formation becomes unstable and hence, non-convergent. If traders 11 and 12 were to discover the beneficial trade that exists between them, they could trade at prices close to the CE prices, $\mathbf{p}^* = (1, 40, 20)$, or at prices which are closer to $p_2 = p_3$. It depends on which commodity is exchanged first.

Indeed, figures 3.1 and 3.2 both show signs of price instability near the end of some trading periods, particularly in the unstable economies. The relevance of market failure, therefore, is not only that trading comes to a halt (i.e. that it is a steady state), but also that price formation may become unstable when market failure is imminent.

[2] For achieving a higher level of efficiency, trader 3 should exhibit other-regarding behavior instead of being self-interested.

In experiments market failure can be avoided by timely resetting of endowments. However, that seems artificial: why would endowments be replenished (if at all) before the set of Pareto improvements is exhausted?[3]

Let traders be allowed to exchange commodities, provided that they exchange goods ($j = 2, 3$) for money ($j = 1$). Trading is voluntary; this is taken to mean that parties involved should be at least as well off after an exchange.[4] For simplicity's sake, assume that voluntary trading will exhaust the set of feasible, beneficial exchanges. That is, complex Pareto improvements (difficult to detect or requiring coordination) will also be implemented by the traders. Therefore, if trading comes to a halt it will be either because no further Pareto-improvements exist (i.e. the final allocation lies in the core), or else because none of the remaining Pareto-improvements can be implemented due to the market protocol.[5]

Consider any one of the three Scarf economies (c.f. 3.2.2). Let $\mathbf{W}$ be an arbitrary allocation of endowments, and consider taking away quantities of commodities from the traders, in such a way that their utility levels do not deteriorate. If it's possible to take away $v_{ji}$ of commodity $j$ from trader $i$ without lowering his utility level, then $v_{ji}$ will be called an excess quantity. The matrix $\mathbf{V}(\mathbf{W}) \in \mathbb{R}^3_+ \times \mathbb{R}^3_+$ consists of all excess amounts $v_{ji}$, which exist in allocation $\mathbf{W}$. Let $\mathbf{e} = \sum_i \mathbf{v}_i$ be the vector with cumulative excess quantities per commodity. In the initial allocation, all utility levels are equal to zero and hence $\mathbf{e} = (400, 10, 20)'$.

**Proposition A.1.** *Consider a Scarf economy, in which bilateral trading occurs if both parties are at least as good off after the exchange as before, with an arbitrary allocation* $\mathbf{W}$*, and let* $\mathbf{e}$ *be the corresponding vector of cumulative excess quantities: (i) if* $\mathbf{e}$ *does not contain any zeros, then there exists a Pareto-improvement, which can also be implemented through trading; (ii) if* $\mathbf{e}$ *contains two zeros, then the allocation is in the core; (iii) if* $\mathbf{e}$ *contains three zeros, then the allocation is the competitive equilibrium; and (iv) if* $\mathbf{e}$ *contains one zero, then there is market failure unless traders, who prefer the combination of commodities that are in excess supply, have money to spare.*

*Proof.* First note that in case of no zeros (i.e. case (i)) there exists a Pareto-

[3] The speed with which the set of feasible Pareto improvements is exhausted depends (among other things) on the complexity of trading. The final allocation in session 511 / period 6 was obtained after 8 minutes, 231 offers and 66 transactions. Period 6 in session 414, on the other hand, lasted 10 minutes, 411 offers and 162 transactions and was at that time two trades removed from market failure. Trading in the stable Scarf is more complex than trading in the unstable economies (in which two out of three traders operate in one market only).

[4] If behavior is more sophisticated then they may accept temporary decreases of utility and market failure would occur less frequently.

[5] If one would impose the law of one price in case the allocation lies in the core then at least one agent will be rationed. The same applies to allocations that qualify as market failure. Both core allocations and market failure can therefore be interpreted as so-called fix-price equilibria with endogenous rationing and no trade, c.f. Benassy (1982)). Fisher (1983) does not consider such quantity constrained equilibria as part of disequilibrium theory proper, because previous research did not show how the economy gets to a fix-price equilibrium. Obviously, one cannot expect traders to know in advance which constraints will prevail ex post, but the notion of a no-trade fix-price equilibrium based on observable rationing constraints does clarify the sense in which a core allocation or market failure in sequential trading is an equilibrium in the minds of economic agents (making it acceptable for them to stop trading). Convergence to the core or market failure, therefore, could be an answer to Fisher's criticism.

improvement: simply award $\mathbf{e}$ to any trader. There also exists a Pareto-improvement which can be implemented by exchanging a commodity for money, because there exists a buyer ($e_1 > 0$), and this buyer prefers one or two other commodities, which are also in excess supply ($e_j > 0,\ j = 2, 3$). The buyer can be made strictly better off: if his type is $i = III$ then he needs to buy commodities 2 and/or 3, which are both available for sale, and if $i \neq III$ then buying either commodity 2 or 3 suffices to make the buyer strictly better off. Anyone of these trades implements a strict Pareto-improvement.

If $\mathbf{e}$ contains two zeros (case (ii)), let commodity $j$ be in excess supply, and suppose that there exists a trader $i$ who can be made strictly better off by awarding him $\mathbf{e}$. In that case, commodity $j$ must be binding for trader $i$ and we have $\alpha_{ji}w_{ji} = u_i$. In addition to $j$ trader $i$ also prefers another commodity, say $k$; if $\alpha_{ki}w_{ki} > \alpha_{ji}w_{ji}$ then $v_{ki} > 0$ and there would have been two goods in excess supply; because there is only one we also must have $\alpha_{ki}w_{ki} \leq \alpha_{ji}w_{ji}$; but from this it follows that the utility of trader $i$ will not increase when he is awarded all of the excess amount of commodity $j$; so, contrary to the assumption, no trader can be made strictly better off, and therefore the allocation is in the core.

If $\mathbf{e} = \mathbf{0}$ (case (iii)), then no trader has excess quantities, $\forall i, j : \ u_i = \alpha_{ji}w_{ji}$. This is precisely the internal consistency requirement of competitive equilibrium. External consistency is guaranteed by the fact that (a) in this case we have a no trade equilibrium and (b) all traders together own all commodities, so demand also equals supply.

Finally, in case (iv) ($\mathbf{e}$ contains one zero) there exists a Pareto-improvement: by giving $\mathbf{e}$ to the trader who prefers the combination of commodities in excess supply, that trader will be made strictly better off without hurting the others. What needs to be established is which Pareto-improvements can be implemented if commodities have to be exchanged for money. First, suppose that money is not in excess supply, $e_1 = 0$, then there is no trading because there are no buyers, who have money to spare. Next, assume $e_1 > 0$: in this case, only traders who prefer the combination of excess commodities and who have excess money can implement the improvement. To see this, suppose also $e_2 > 0$ and consider traders 1 and 3, who prefer either commodities 1 and 3 or 2 and 3; trader 1 does not derive any utility from commodity 2, so he cannot be made better off; for trader 3 we have $w_{33}/20 \leq w_{23}/10$ (because otherwise also $e_3 > 0$) which implies that he cannot be made better off by increasing his stock of commodity 2. Therefore, if a Pareto-improvement can be implemented in this case, it is because trader 2, who prefers the combination of commodities in excess supply, has money to spare. If trader 2 has no money to spare, i.e. if $w_{12}/400 \leq w_{22}/10$, then there is market failure. □

## A.2 The likelihood of market failure

Voluntary trading in the Scarf economies has an inherent direction. Consider an agent who does not derive utility from, say, commodity 2. If he has a positive endowment of commodity 2, then this stock will be available for selling, and it will not be replenished. If the agent initially owns nothing of commodity 2, he will not buy anything of it. Therefore, it is reasonable to expect that this agent's endowment of commodity 2 will ultimately be equal to zero. For simplicity's sake, assume that this argument also applies to money, then each trader at some point will have positive amounts of at most two commodities.[6] If traders are active, then one of these commodities will be binding in terms of utility; put differently, each column of matrix $\mathbf{V}\,(\mathbf{W})$, will after some time contain two zeros.

In the excess matrices below, a $\cdot$ represents a positive amount; underscores mark quantities that do not contribute to a trader's level of utility (preferences as given in section 3.2.2). The first two matrices represent market failure ($F$), because none of the traders has money to spare, while there still exist Pareto improvements: if traders 1 and 2 give their excess amounts to trader 3, the latter will be strictly better off, while the level of utility of traders 1 and 2 does not deteriorate. The third allocation allows further trading (trader 1 can sell commodity 3 to trader 2 and be strictly better off, while the level of utility of trader 2 stays the same). This branch is split into two sub-branches: one in which the quantity of money has become binding for trader 2 (upper sub-branch) and one in which the quantity of commodity 3 has become binding for trader 1 (lower sub-branch).

$$\begin{pmatrix} 0 & 0 & \underline{0} \\ \underline{0} & \cdot & \cdot \\ \cdot & \underline{0} & 0 \end{pmatrix} F, \quad \begin{pmatrix} 0 & 0 & \underline{0} \\ \underline{0} & \cdot & 0 \\ \cdot & \underline{0} & \cdot \end{pmatrix} F$$

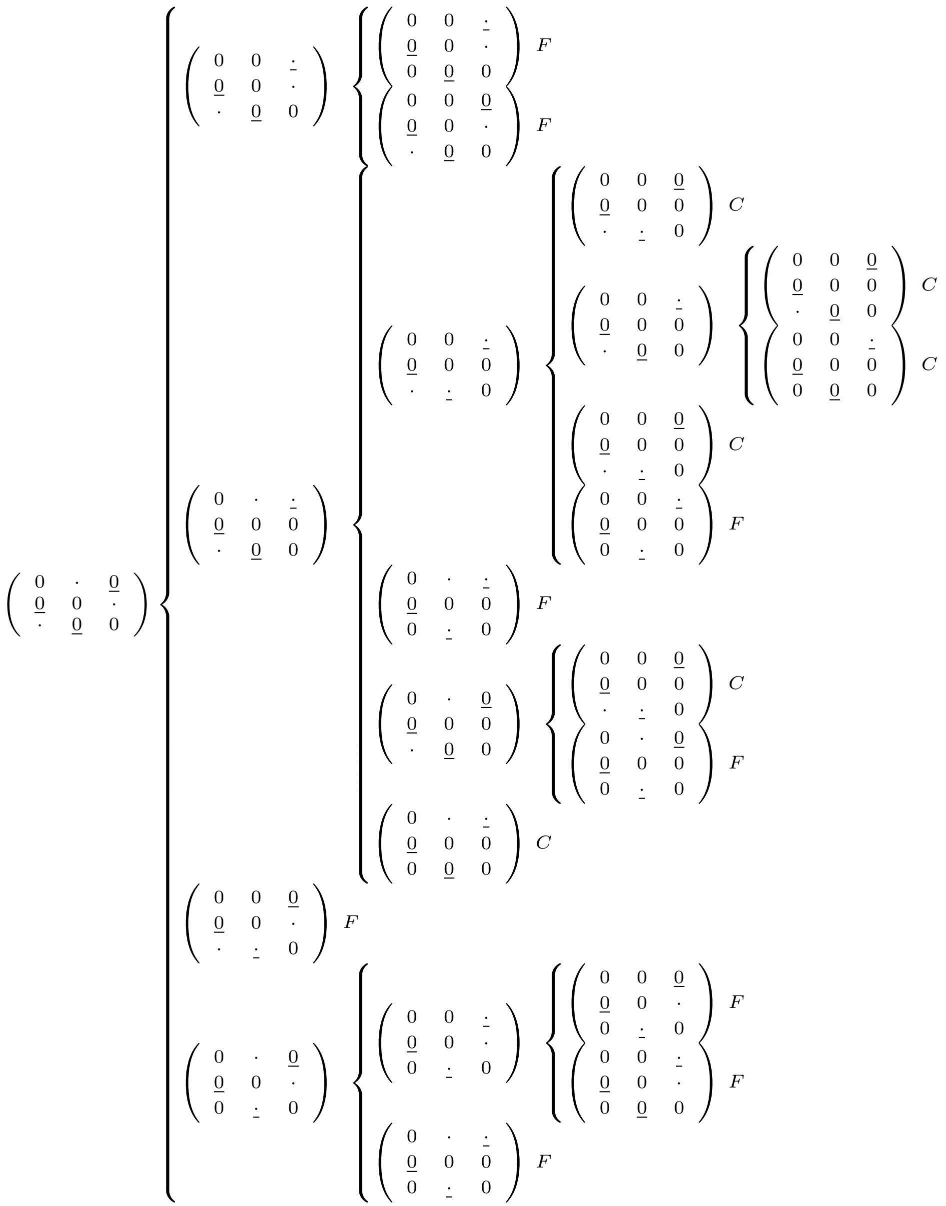
$$\begin{pmatrix} 0 & \cdot & \underline{0} \\ \underline{0} & 0 & 0 \\ \cdot & \underline{0} & \cdot \end{pmatrix} \begin{cases} \begin{pmatrix} 0 & 0 & \underline{0} \\ \underline{0} & 0 & 0 \\ \cdot & \underline{\cdot} & \cdot \end{pmatrix} & C \\ \begin{pmatrix} 0 & \cdot & \underline{0} \\ \underline{0} & 0 & 0 \\ 0 & \underline{\cdot} & \cdot \end{pmatrix} & F \end{cases} \qquad \begin{pmatrix} \cdot & 0 & \underline{0} \\ \underline{0} & \cdot & \cdot \\ 0 & \underline{0} & 0 \end{pmatrix} \begin{cases} \begin{pmatrix} 0 & 0 & \underline{0} \\ \underline{\cdot} & \cdot & \cdot \\ 0 & \underline{0} & 0 \end{pmatrix} & C \\ \begin{pmatrix} \cdot & 0 & \underline{0} \\ \underline{\cdot} & 0 & \cdot \\ 0 & \underline{0} & 0 \end{pmatrix} & F \end{cases}$$

This split ignores the possibility that trader 2 has such a large amount of money, that after the exchange, trader 1, or possibly both traders 1 and 2, have money to spare. The effect of trading is that excess quantities decrease if commodities end up in the hands of traders who need them. By waiting long enough before applying this analysis, the assumption that either one or the other commodity will become binding will be justified.[7] In the next step, after the third allocation, trading will come to halt because either the allocation is in the core (upper sub-branch, denoted by $C$; here each trader has an excess amount of commodity 3, and there is no way to make any trader strictly better off) or there is market failure (lower sub-branch; here trader 2 needs an additional amount of commodity 2, but he can only buy commodity 3; there exists a Pareto improvement because if both traders 2 and 3 were to give their excess amounts to trader 1 the latter would be strictly better off).

Below we give the genealogy of all other possible end states and we characterize them as either a core allocation or as market failure. By making two bold assumptions, we can assign probabilities to a particular end state occurring and derive a crude estimate of the likelihood that market failure will obtain: (i) each of the eight possible configurations of two zeros per column can occur with equal probability; and (ii) after a split, either of the two sub-branches can occur with equal probability. If trading

[6]Money could be an exception, because it provides future freedom of action.

[7]The possibility that both quantities become binding at the same time will be ignored (because it is negligible); this explains why the competitive equilibrium is not an end state in this analysis.



is voluntary and otherwise random then there is no reason why the market would prefer one sub-branch over the other.[8] Under these assumptions, the probability of

[8]If behavior of traders would be less random, then certain sub-branches could become irrelevant. Suppose that traders want to be strictly better off when they spend money. In that case, the third allocation would represent market failure, because trader 2 would refuse to buy commodity 3, which does not add to his level of utility (c.f. mode II behavior below).

market failure is $\frac{371}{512} \approx 0.72$. This initial assessment is corroborated by simulations: with ZI-traders, 722 out of 1000 runs resulted in market failure.

$$
\begin{pmatrix} \cdot & 0 & \underline{0} \\ \underline{0} & \cdot & 0 \\ 0 & \underline{0} & \cdot \end{pmatrix}
\left\{\begin{array}{l}
\begin{pmatrix} 0 & 0 & \underline{\cdot} \\ \underline{0} & \cdot & 0 \\ 0 & \underline{0} & \cdot \end{pmatrix}
\left\{\begin{array}{l}
\begin{pmatrix} 0 & 0 & \underline{0} \\ \underline{0} & \cdot & 0 \\ 0 & \underline{0} & \cdot \end{pmatrix} F \\
\begin{pmatrix} 0 & 0 & \underline{\cdot} \\ \underline{0} & 0 & 0 \\ 0 & \underline{0} & \cdot \end{pmatrix} F
\end{array}\right. \\
\begin{pmatrix} \cdot & 0 & \underline{\cdot} \\ \underline{0} & \cdot & 0 \\ 0 & \underline{0} & 0 \end{pmatrix}
\left\{\begin{array}{l}
\begin{pmatrix} 0 & 0 & \underline{\cdot} \\ \underline{\cdot} & \cdot & 0 \\ 0 & \underline{0} & 0 \end{pmatrix}
\left\{\begin{array}{l}
\begin{pmatrix} 0 & 0 & \underline{0} \\ \underline{\cdot} & \cdot & 0 \\ 0 & \underline{0} & 0 \end{pmatrix} C \\
\begin{pmatrix} 0 & 0 & \underline{\cdot} \\ \underline{\cdot} & 0 & 0 \\ 0 & \underline{0} & 0 \end{pmatrix} F \\
\begin{pmatrix} 0 & 0 & \underline{0} \\ \underline{\cdot} & \cdot & 0 \\ 0 & \underline{0} & 0 \end{pmatrix} C \\
\begin{pmatrix} 0 & 0 & \underline{\cdot} \\ \underline{\cdot} & 0 & 0 \\ 0 & \underline{0} & 0 \end{pmatrix} F
\end{array}\right. \\
\begin{pmatrix} \cdot & 0 & \underline{\cdot} \\ \underline{\cdot} & 0 & 0 \\ 0 & \underline{0} & 0 \end{pmatrix} F
\end{array}\right. \\
\begin{pmatrix} 0 & 0 & \underline{0} \\ \underline{\cdot} & \cdot & 0 \\ 0 & \underline{0} & \cdot \end{pmatrix} F \\
\begin{pmatrix} \cdot & 0 & \underline{0} \\ \underline{\cdot} & 0 & 0 \\ 0 & \underline{0} & \cdot \end{pmatrix}
\left\{\begin{array}{l}
\begin{pmatrix} 0 & 0 & \underline{\cdot} \\ \underline{\cdot} & 0 & 0 \\ 0 & \underline{0} & \cdot \end{pmatrix}
\left\{\begin{array}{l}
\begin{pmatrix} 0 & 0 & \underline{0} \\ \underline{\cdot} & 0 & 0 \\ 0 & \underline{0} & \cdot \end{pmatrix} F \\
\begin{pmatrix} 0 & 0 & \underline{\cdot} \\ \underline{0} & 0 & 0 \\ 0 & \underline{0} & \cdot \end{pmatrix} F
\end{array}\right. \\
\begin{pmatrix} \cdot & 0 & \underline{\cdot} \\ \underline{\cdot} & 0 & 0 \\ 0 & \underline{0} & 0 \end{pmatrix} F
\end{array}\right.
\end{array}\right.
$$

$$
\begin{pmatrix} \cdot & \cdot & \underline{0} \\ \underline{0} & 0 & 0 \\ 0 & \underline{0} & \cdot \end{pmatrix}
\left\{\begin{array}{l}
\begin{pmatrix} 0 & \cdot & \underline{\cdot} \\ \underline{0} & 0 & 0 \\ 0 & \underline{0} & \cdot \end{pmatrix} F \\
\begin{pmatrix} \cdot & \cdot & \underline{\cdot} \\ \underline{0} & 0 & 0 \\ 0 & \underline{0} & 0 \end{pmatrix} C
\end{array}\right.
\qquad
\begin{pmatrix} \cdot & \cdot & \underline{0} \\ \underline{0} & 0 & \cdot \\ 0 & \underline{0} & 0 \end{pmatrix}
\left\{\begin{array}{l}
\begin{pmatrix} \cdot & 0 & \underline{\cdot} \\ \underline{0} & 0 & \cdot \\ 0 & \underline{0} & 0 \end{pmatrix} F \\
\begin{pmatrix} \cdot & \cdot & \underline{\cdot} \\ \underline{0} & 0 & 0 \\ 0 & \underline{0} & 0 \end{pmatrix} C
\end{array}\right.
$$

Without any specific assumptions with respect to expectation formation, other algorithms may be expected to do worse to the extent that agents refuse to trade excess quantities. Simulations with robot traders, maximizing their expected utility given expected prices, gave a frequency of approximately 90% of ending in market failure. This type of behavior places extra restrictions on the set of Pareto improvements which can be implemented through trading, because some traders refuse to acknowledge that they have excess quantities. Instead of supplying their excess quantities, they demand more of another commodity which they consider to be cheap and which would give them a higher level of utility.

Market failure, therefore, is a distinct possibility when replicating the experiments of Anderson et al. (2004) with robot traders. If the trading process quickly removes opportunities for further trading then (i) the economy will be trapped in market failure sooner rather than later, (ii) traders will have had little chance to learn prices, and (iii) while in market failure, learning will go astray because all offers will be rejected.

The likelihood of market failure presumably depends on price expectations, because with perfect foresight the probability of obtaining market failure would be zero. The next section considers the impact of initial price expectations.

## A.3 Sensitivity to initial price expectations

Let's assume that trader 3 maximizes utility given expected prices. Furthermore, suppose that initially expected prices are uniformly distributed on the simplex. Given these assumptions, it is possible to simulate the probability that trader 3's initial offer will land him with all of either commodity 2 or commodity 3. Based on 2500 trials, this probability is estimated to be 0.85. Suppose that after trader 3's initial offer we have

$$\mathbf{W} = \begin{pmatrix} x & 0 & 400 - x \\ 0 & 0 & 10 \\ 0 & 20 & 0 \end{pmatrix}.$$

In the next step, traders 1 and 3 will bid for commodity 3. As soon as all of commodity 3 has been divided between traders 1 and 3, then we have market failure due to inconsistent plans provided that trader 3 (i) has some money left and (ii) he perceives his situation as not having enough of commodity 3. Clearly, there is a fair chance that market failure occurs in as few as two or three steps. If so, then quantity setting and / or the assumption that prices are uniformly distributed on the simplex must be at fault, because in this case there nothing else that can explain market failure. Is there a better alternative for the latter assumption?

Prices being uniformly distributed on the simplex is an appropriate assumption for an uninformed external observer. Suppose that we adopt the point of view of individual traders and ask how information, which is available to them, can be used to obtain an intuition with respect to prices. If prices are externally given, most traders can determine (i) which level of utility is feasible and (ii) how to allocate their endowments to buying the commodities they want. Suppose that traders can reverse this reasoning and derive price expectations from:

- an expected level of utility;
- a budget, in which endowments are earmarked for obtaining certain preferred commodities.

The human traders in the experiments of Anderson et al. also knew beforehand that prices were integer-valued. For instance, trader 3 would spend 1 to 399 units of money on obtaining commodity 2, and the remainder on commodity 3. In the subjective approach, the knowledge that prices are integer-valued, can also be factored into the price expectations. Let $u^*$ be the expected level of utility and let $0 < \frac{\theta}{10} < 1$, $\theta = 1, 2, .., 9$ be the way how trader 1 splits his endowments between buying commodities

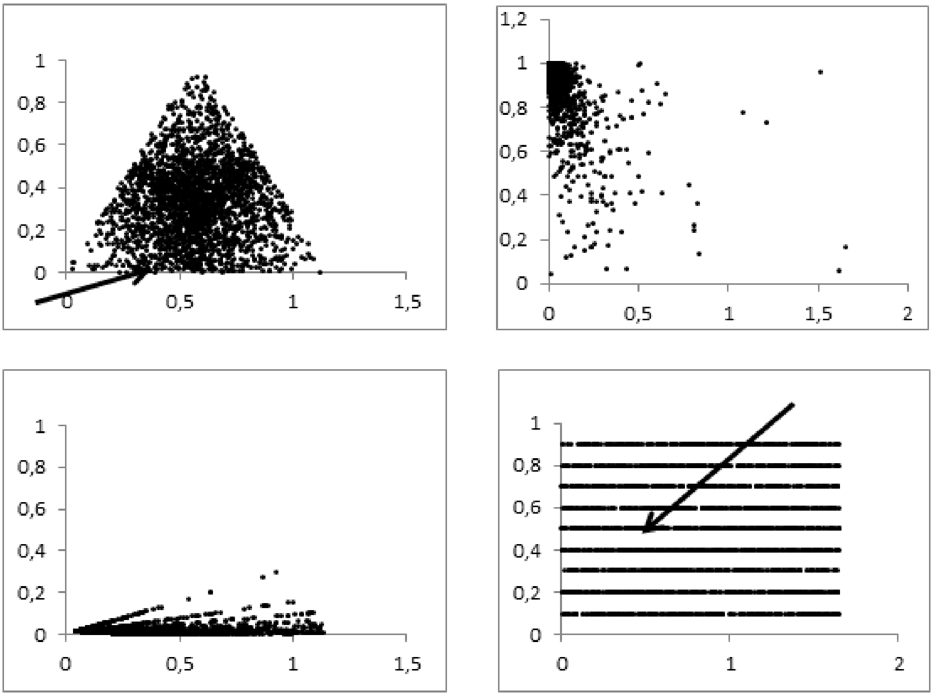



**Figure A.1** – *The objective (top) and subjective approach (bottom) to initial price expectations. The graphs on the left show prices in a simplex; the graphs on the right the relative split of the budget between two commodities (y-axis) by expected utility (x-axis). In the top-left panel, prices are selected at random from the simplex. The arrow points to the Walrasian equilibrium prices. The top-right panel shows the corresponding bias in expected utility and expenditure. In the subjective approach causation runs in the other way. Given indivisible commodities, traders expect discrete budget splits. Combined with random levels of expected utility this leads to expected prices that are relatively close to the Walrasian equilibrium prices.*

1 and 3. Assuming that trader 1's expectations are consistent, then one easily finds his expected prices to be

$$\mathbf{p}_1^{exp} = (1, \frac{400u^*}{\theta}, \frac{20(10-\theta)}{\theta}).$$

For other traders a similar reasoning applies. Figure A.1 compares the two approaches to initializing price expectations. The top-left graph shows 2500 points sampled uniformly from the simplex, and next to it the implied values of $\frac{\theta}{10}$ (y-axis) and $u^*$ (x-axis). The lower panel contains the corresponding graphs for the subjective approach. Here, causation runs in the opposite direction: values for $\frac{\theta}{10}$ and $u^*$translate into expected prices (bottom-left graph). The arrows point to initial expectations which would lead to the competitive equilibrium.

Observe the contrast between the alternative approaches: while it is very unlikely to be near the equilibrium prices starting from random prices in the simplex, traders will have already guessed one relative price correctly if they provisionally split their endowments equally between the two commodities they prefer! Such a 50-50 split is likely to have a more than fair probability.[9] Given subjective initial price expectations, the probability that trader 3 obtains all of commodity 2 or 3 after his first offer drops from 0.85 to 0.11 (also based on 2500 simulations).

In the experiments of Anderson et al. (2004), we don't see market failure after only a few trades. It is likely, therefore, that the initial price expectations of human traders are more informative than prices being uniformly distributed on the simplex. In the calibration of FACTS and in subsequent simulations, we use the subjective approach above, to initialize price expectations:

$$\begin{aligned}\mathbf{p}_1^{exp} &= (1, \frac{40u_1^*}{\theta}, \frac{20(1-\theta)}{\theta})\\ \mathbf{p}_2^{exp} &= (1, \frac{40(1-\theta)}{\theta}, \frac{20u_2^*}{\theta})\\ \mathbf{p}_3^{exp} &= (1, \frac{40\theta}{u_3^*}, \frac{20(1-\theta)}{u_3^*})\end{aligned}$$

with $u_i^* \sim uniform(0.1, 1)$ and $\theta \sim uniform\,(0.4, 0.6)$. The parameters of these uniform distributions are ad hoc assumptions, reflecting our expectation that it is difficult to estimate $\mathbf{u}^*$ beforehand and that $\theta = 0.5$ is a likely choice. Note that we do not use the fact that prices are integer-valued, for ease of implementing algorithms and for avoiding discontinuities.

## A.4 Discussion

In case of market failure, it is difficult or sometimes even impossible to implement the remaining Pareto improvements, due to the behavior of traders or to institutional requirements. A Walrasian equilibrium may exist, but it may be impossible to achieve it without an auctioneer unless a hypothetical invisible hand steers the economy clear from allocations like $\mathbf{W}^1$. *A priori*, there isn't any reason why that would be the case. Note that this is not due to any adverse structural factors like externalities, or incentives for non-competitive behavior, but instead to the process of trading at all prices.

The relevance of market failure is three-fold: (i) it is a potential steady state of a process with trading at all prices; (ii) when market failure is imminent, price formation may become unstable and (iii) when it obtains, it may distort learning because no offer will be accepted. These considerations apply to both human and robot trading, but the behavior of robot traders is often more rigid which may increase the likelihood of market failure. In the light of (ii), the appropriate notion of convergence to Walrasian prices is that trading prices are sufficiently close to these Walrasian prices for a sufficiently long time, but not necessarily near the end of trading.

Figures 3.1 and 3.2 suggest that market failure several times exerted its influence in the experiments of Anderson et al. (2004), especially in the unstable economies, leading to unstable price formation near the end of a period. It seems that endowments were reset "just in time". In a laboratory context, the length of the next period

[9] As a matter of fact, sophisticated traders can deduce it (c.f. section 3.5.1).

is under the control of experimenters. In real life, however, there is no guarantee that endowments are replenished (if at all) before the set of feasible Pareto improvements is exhausted. For this reason, and out of curiosity, we do not reset endowments in our simulations.

Economies with robot traders are potentially more susceptible to market failure because algorithms usually are less flexible than humans are and typically also less sophisticated. Trading behavior can help to postpone market failure. With arbitrage, or when traders make mistakes, we can have acceptances which otherwise would not have occurred. That puts money in the hands of other traders, opening up new possibilities for trading. This can also happen if human subjects feel committed to making the experiment, in which they participate, a success (this is the so-called Active Participation Hypothesis, c.f. Lei et al. (2001)).[10]

## A.5 Conclusions

In this appendix we have determined the possible end states of an exchange economy with trading at all prices. Market failure, among them, is a state in which Pareto improvements cannot be implemented, due to behavior or to the market protocol. If a market is not sufficiently transparent or if Pareto improvements require complex coalitions then possibly they may not be detected in time. We have analyzed the probability of market failure for ZI-trading and found it to be 0.72. This value is corroborated by simulations. The propensity of market failure was shown to depend on initial price expectations. Selecting price expectations at random from the price simplex is likely to result in market failure. Indirectly, this suggests that endowments and preferences may affect the price expectations of human traders.

[10]Simulations with sharp CES preferences replacing Leontief preferences (for modeling mistakes), and with traders having an urge to accept proposals after seeing too many rejections confirm these intuitions. Arbitrage is considered in chapter 5.