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Lifetime of magnetically trapped ultracold atomic deuterium gas

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We have calculated the magnetic dipolar spin-relaxation rates of magnetically trapped doubly-spin-polarized atomic deuterium gas. The results are expressed in a simple closed formula describing the dipolar-relaxation-limited stability in the Boltzmann ($T \gg T_F$), the degeneracy ($T \ll T_F$), and the crossover ($T \simeq T_F$) regimes. In the degeneracy regime we typically find lifetimes of some hours. We also discuss the dipolar decay of magnetically trapped spin-polarized fermionic alkali-metal atoms.

I. INTRODUCTION

In the past decade dramatic progress has been made in the stabilization of spin-polarized atomic hydrogen. The observation of several exceptional properties in a large number of experiments on this new quantum gas[1] strongly overshadowed the few experimental results[2] on its fermionic counterpart: spin-polarized deuterium. The reason that deuterium has come into prominence to a much lesser amount is that the standard scheme to stabilize hydrogen,[1] i.e., confinement at high magnetic field in a cell with liquid-helium covered walls, is rather inefficient when applied to deuterium: even on superfluid-helium surfaces deuterium atoms undergo a strong adsorption and subsequent recombination.

Recently, we made an analysis[3] of the hyperfine population dynamics of surface-free confined deuterium atoms in a trap similar to those used for confining laser-cooled spin-polarized alkali-metal atoms[4] and spin-polarized hydrogen atoms.[5] It appears that fast two-body spin-exchange processes acting on the trapped electron spin-up deuterium atoms (D↑) lead to the formation of a gas of atoms in one single hyperfine state in which also the nuclear spins are polarized. This gas of doubly-spin-polarized deuterium atoms (D↑↑) is extremely stable at low temperatures. Not only are surface recombination processes eliminated but also collision processes leading to a decay of the deuterium atom gas are strongly suppressed thanks to the Pauli exclusion principle. In contrast to the case of the magnetically trapped boson gas H↑↑ where two-body dipolar relaxation is predicted to be very fast and, to lowest order, independent of temperature,[6] the stability of the fermion gas D↑↑ against dipolar relaxation is expected to grow with decreasing temperature.[3] Applying an evaporative cooling scheme similar to that proposed for H↑↑,[7] will ultimately lead to an ultrastable low temperature D↑↑ gas. The dipolar-relaxation-limited stability of D↑↑ is estimated in Ref. 3 for temperatures well above the Fermi temperature T_F.

Here we describe a more general theory for the dipolar relaxation of the D↑↑ gas which also applies to the degeneracy regime ($T \ll T_F$) and the crossover regime ($T \simeq T_F$). The results are given in the form of a simple closed expression which, in the relevant regimes, describes the temperature and magnetic-field dependence of the effective decay rates within a few percent. This expression also determines the dipolar decay rates of magnetically trapped fermionic alkali-metal atoms. For a degenerate ($T \ll T_F$) D↑↑ gas with density 10^{14} atoms/cm^3 at magnetic field $B = 0.1$ T we find a lifetime as large as 5×10^3 s. This figure even grows with increasing magnetic field. Therefore, magnetically trapped D↑↑ offers a unique possibility for the experimental observation of gas-phase degenerate quantum behavior.

II. DECAY RATES

In principle, a wealth of processes may cause a gas of doubly-spin-polarized magnetically trapped atoms to flip an electronic or nuclear spin. Such spin-depolarization processes lead to a decay of the gas as atoms are formed which are directly ejected out of the trap (because they acquire the released energy in the form of kinetic energy and also because they are formed in hyperfine states which are repelled from the minimum-B-field trap) or because atoms are formed in hyperfine states which disappear out of the trap due to a subsequent strong two-body spin-exchange process. In view of the extreme diluteness of trapped atom gases [highest density achieved up to now: $\sim 3 \times 10^{14}$ atom/cm^3 with H↑↑ (Ref. 5)] the first type of spin-depolarization processes to discuss are single-atom processes. These are commonly considered to be negligible or at least to be made so by judicious selection of the experimental parameters. For instance, the lifetime due to spontaneous emission at hyperfine frequencies is enormous, and nonadiabatic spin-flips (Majorana transitions) can be reduced below any desired rate by a careful trap design.

Under these circumstances, two atom processes determine the lifetime of doubly-spin-polarized trapped atom gases. As in the doubly-spin-polarized state the electronic spins of two colliding atoms are parallel, spin-exchange plays no role in the decay of the gas. Precession of the

spins in the dipolar magnetic fields of other atoms, however, does lead to spin flipping, which results in a decay of the sample. Spin depolarization due to the magnetic dipole-dipole interaction is the dominant loss mechanism for trapped H↑ ‡ (Ref. 6) and D↑ ‡ (Ref. 3) gases. For magnetically trapped doubly-spin-polarized alkali-metal atoms other decay processes such as resonance recombination are important.[8] At very low temperatures, however, also for trapped alkali-metal atoms dipolar relaxation may be the dominant decay process.

From now on we restrict ourselves to the case of dipolar relaxation in doubly-spin-polarized gases of deuterium atoms and assume the relevant collision energies to be small compared to the splittings between the various hyperfine energy levels. We denote the one atom hyperfine states α, β, γ, δ, ϵ, and ζ in order of increasing energy (see inset Fig. 1), so that the trapped doubly-polarized state is denoted by ζ. Assuming all atoms formed in dipolar relaxation events either to be in a hyperfine state with predominantly electron spin-down and hence to be ejected from the minimum-B-field trap, or to have acquired enough energy to escape from the trap and stick to a wall outside the trap region, we find for the decay of the ζ-atom gas

$$\frac{d}{dt}n_\zeta = -2n_\zeta^2 \sum_{\{xy\}}{}' G_{\zeta\zeta\to xy}\,, \tag{1}$$

with n_ζ denoting the ζ-atom density, and the summation $\sum'_{\{xy\}}$ running over all distinct pairs of hyperfine states not equal to $\zeta\zeta$. Starting from the quantum-mechanical Bogoliubov-Born-Green-Kirkwood-Yvon (BBGKY) hierarchy we find that the relaxation rate $G_{\zeta\zeta\to xy}$ can be expressed as an ensemble average of the relative velocity $2\hbar k/m$ times an effective cross section $\sigma_{\zeta\zeta\to xy}$

$$G_{\zeta\zeta\to xy} = (2\pi)^{-3}\int_0^\infty 4\pi k^2 dk F_{\zeta\zeta}(k)\frac{\hbar k}{m/2}\sigma_{\zeta\zeta\to xy}(k)\,. \tag{2}$$

In this equation $F_{\zeta\zeta}(k)$ is the distribution function for the relative wave number k defined as

$$F_{\zeta\zeta}(k) \equiv (2\pi)^{-3}\int d\mathbf{K} f_\zeta(|\tfrac{1}{2}\mathbf{K}+\mathbf{k}|) f_\zeta(|\tfrac{1}{2}\mathbf{K}-\mathbf{k}|)\,, \tag{3}$$

in which $f_\zeta(k)$, the distribution function for the single atom wave numbers, is assumed to be isotropic and normalized as $(2\pi)^{-3}\int d\mathbf{k} f_\zeta(k) = 1$. In Eq. (2) no Pauli blocking terms of the form $(1-f_x)(1-f_y)$ appear because the atoms in hyperfine state x and y formed in a dipolar relaxation event undergo no blocking effects as the internal states x and y are unequal to ζ or otherwise because they have acquired enough kinetic energy to prevent any blocking effect to occur.

In a dilute gas of identical fermions in one single internal state the antisymmetrization requirement prevents the fermions to approach one another at distances much smaller than the thermal wavelength. As the thermal wavelength scales with the inverse square root of the temperature, it follows that at low temperatures fermions experience only the long-range parts of their mutual interactions. Therefore, in our calculation the short-ranged triplet interaction is neglected and the dipolar interaction, from which only the very weak long-range part contributes, is taken into account to lowest order. We find

$$\sigma_{\zeta\zeta\to xy}(k) = \frac{\pi^3 m^2 k'}{\hbar^4 k}\int_{4\pi} d\hat{k}\int_{4\pi} d\hat{k}' |V_{\zeta\zeta,\{xy\}}(\mathbf{k},\mathbf{k}')|^2\,. \tag{4}$$

In this equation $\mathbf{k}' = k'\hat{k}'$ denotes the final-state wave vector, and the V matrix element is the antisymmetrized momentum representation of the dipolar interaction

$$V_{\zeta\zeta,\{xy\}}(\mathbf{k},\mathbf{k}') \equiv \int d\mathbf{r}\,\tfrac{1}{2}\{\exp[i(\mathbf{k}-\mathbf{k}')\mathbf{r}] - \exp[i(\mathbf{k}+\mathbf{k}')\mathbf{r}]\} V_{\zeta\zeta,\{xy\}}(\mathbf{r})\,, \tag{5}$$

with $\{xy\}$ denoting a symmetrized and normalized spin state $(xy+yx)/[2(1+\delta_{xy})]^{1/2}$. Using the fact that the hyperfine energy splittings are much larger than typical kinetic energies so that $\hbar k \ll \hbar k' \simeq [m(2\epsilon_\zeta - \epsilon_x - \epsilon_y)]^{1/2}$ with ϵ_x the hyperfine energy of x-state atoms, and taking into account only the electron-electron part of the dipolar interaction, which by far dominates over the electron-proton and proton-proton contributions, we find

$$\sigma_{\zeta\zeta\to xy}(k) = \frac{a_{ee}^2}{90\pi}\frac{k}{k'}|\langle\zeta\zeta|\Sigma^{ee}|\{xy\}\rangle|^2\,. \tag{6}$$

In this equation the electronic-dipolar length scale

$$a_{ee} \equiv m\mu_0\mu_e^2/\hbar^2\,, \tag{7}$$

which when the deuterium atom mass is substituted for m takes the value $a_{ee} = 3.26\times10^{-11}$ m, and the spin matrix element $\langle\zeta\zeta|\Sigma^{ee}|\{xy\}\rangle$ describes the coupling of the electron spin-operators to a spin-operator of rank two. This

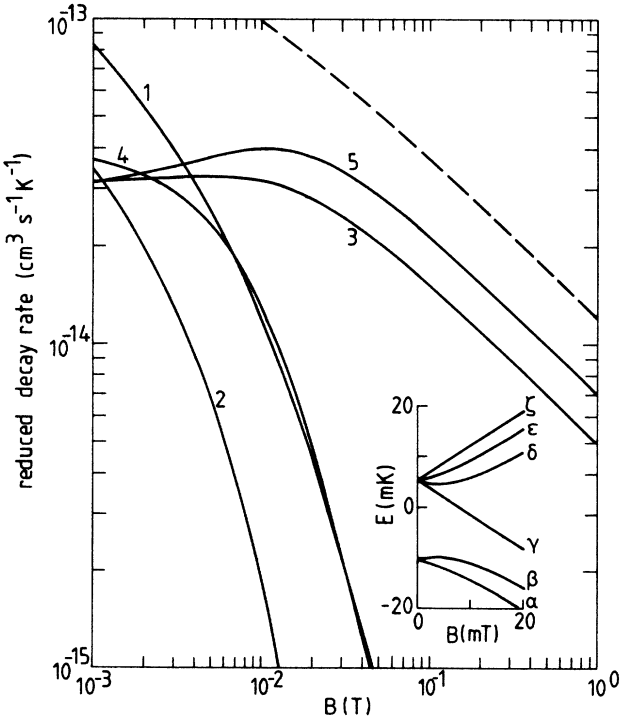

FIG. 1. The reduced dipolar relaxation rates $g_{\zeta\zeta\to xy}$ as functions of magnetic field. The curves correspond to $xy = 1{:}\zeta\epsilon$, $2{:}\epsilon\epsilon$, $3{:}\zeta\alpha$, $4{:}\epsilon\alpha$, $5{:}\alpha\alpha$. The dashed curve represents the sum of the five rates. Inset: Ground-state hyperfine energy levels of atomic deuterium.

matrix has nonvanishing off-diagonal elements for $xy=\zeta\epsilon$, $\epsilon\epsilon$, $\zeta\alpha$, $\epsilon\alpha$, and $\alpha\alpha$, with absolute values $2\sqrt{3}\sin(\theta_+)$, $2\sqrt{6}\sin^2(\theta_+)$, $2\sqrt{3}\cos(\theta_+)$, $2\sqrt{3}\sin(2\theta_+)$, and $2\sqrt{6}\cos^2(\theta_+)$, respectively. Here, the mixing angle θ_+ is determined by the relation $2\sqrt{2}\cot(2\theta_+)=2(\mu_e+\mu_n)B/a_{\rm hf}+1$, in which μ_e (μ_n) is the electronic (nuclear) magnetic moment, and $a_{\rm hf}$ is the hyperfine constant (for deuterium: $a_{\rm hf}=1.446\times10^{-25}$ J).

Using the indentity

$$(2\pi)^{-3}\int 4\pi k^2 dk\frac{\hbar^2k^2}{m}F_{\zeta\zeta}(k) = (2\pi)^{-3}\int 4\pi k^2 dk\frac{\hbar^2k^2}{2m}f_\zeta(k)$$

in the right-hand side of which we recognize $\langle E_{\rm kin}\rangle$, the average kinetic energy per atom, we find

$$G_{\zeta\zeta\rightarrow xy}=g_{\zeta\zeta\rightarrow xy}\langle E_{\rm kin}\rangle , \tag{8}$$

with the reduced relaxation rate depending only on the magnetic-field strength B given by

$$g_{\zeta\zeta\rightarrow xy}=\frac{1}{45\pi}\frac{a_{ee}^2}{\hbar k'}|\langle\zeta\zeta|\Sigma^{ee}|\{xy\}\rangle|^2 . \tag{9}$$

The average kinetic energy per atom depends on the precise form of the distribution function f_ζ. At low kinetic energies the dipolar interaction is much more effective for elastic collisions ($\zeta\zeta\rightarrow\zeta\zeta$) than for inelastic ones ($\zeta\zeta\rightarrow xy$), as the energy released in a hyperfine transition reduces the overlap between initial and final states. As a result, the translational degrees of freedom may be assumed to be in thermal equilibrium so that $f_\zeta(k)$ is given by the Fermi-Dirac distribution which depends on temperature and density. For practical purposes the mean free particle energy for a Fermi-Dirac distribution can be parametrized as

$$\langle E_{\rm kin}(n_\zeta,T)\rangle\simeq[(\tfrac{3}{5}k_BT_F)^{1.7}+(\tfrac{3}{2}k_BT)^{1.7}]^{1/1.7} , \tag{10}$$

with k_B the Boltzmann constant and

$$T_F\equiv\hbar^2(6\pi^2n_\zeta)^{2/3}/(2mk_B)$$

the Fermi temperature. This parametrization reduces to the well-known expressions in the Boltzmann limit ($T\gg T_F$) and in the degeneracy limit ($T\ll T_F$), and is accurate within 2% over the whole range of T/T_F values.

Equation (8) together with the expression (9) for the reduced relaxation rate and the parametrization (10) for the mean kinetic energy per atom constitute a simple closed expression for the relaxation rates in a deuterium atom gas.

III. IMPLICATIONS FOR MAGNETICALLY TRAPPED D↑ ↑

An important feature of the above results is that the fermionic character of the deuterium atoms manifests itself in the relaxation rates as a proportionality to the mean kinetic energy of the atoms [Eq. (8)] so that the stability grows with decreasing temperature. This is not so for hydrogen atoms which behave like bosons. The same analysis applied to bosons in essence yields an equation similar to Eq. (8) but with $\langle E_{\rm kin}\rangle$ replaced by the final-state energy $\hbar^2k'^2/m$, yielding relaxation rates independent of temperature. Hence, we expect the fermion D↑ ↑ to be much more stable than the boson H↑ ↑ at temperatures which are small compared to their hyperfine level splittings.

Another aspect worth mentioning here is that, with exception of the Boltzmann ($T\gg T_F$) regime, the relaxation rates $G_{\zeta\zeta\rightarrow xy}$ not only depend on the magnetic-field strength and temperature but via a dependence on T_F also on atom density. For instance, in the degeneracy limit ($T\ll T_F$), the relaxation rates are proportional to $n_\zeta^{2/3}$ yielding a decay $(d/dt)n_\zeta\sim n_\zeta^{8/3}$. This behavior makes it possible to determine the trapped gas temperature, and more specifically to signal degeneracy, simply by monitoring the trapped atom density.

Using Eq. (9) we have calculated the various reduced dipolar-relaxation rates $g_{\zeta\zeta\rightarrow xy}$ and the sum rate $\sum'_{\{xy\}}g_{\zeta\zeta\rightarrow xy}$ for deuterium as functions of magnetic field. The results are displayed in Fig. 1. A prominent feature of this figure is the rapid decrease with increasing magnetic field of the rates having an ϵ-atom in the final state. This is because these rates have Σ^{ee} matrix elements which vanish at high magnetic fields. The rates $\zeta\zeta\rightarrow\zeta\alpha$ and $\zeta\zeta\rightarrow\alpha\alpha$ diminish much more slowly as they are not reduced by their Σ^{ee} matrix elements which go to a constant for $B\rightarrow\infty$, but only by their inverse proportionality to the final state momentum $\hbar k'$. For not too low magnetic fields, the sum rate is proportional to the inverse square root of the magnetic field. For $B\gtrsim0.02$ T we find $\sum'_{\{xy\}}g_{\zeta\zeta\rightarrow xy}\simeq\tilde{g}B^{-1/2}$ with a relative error not exceeding a few percent for $\tilde{g}=1.16\times10^{-14}$ cm^{3}s^{-1}T$^{1/2}$K^{-1}. Using this expression and the parametrization (10) we find for the characteristic decay rate $\Gamma_{\rm dip}\equiv(-1/n_\zeta)(d/dt)n_\zeta$

$$\Gamma_{\rm dip}\simeq2\tilde{g}n_\zeta B^{-1/2}\tfrac{3}{5}T_F[1+(\tfrac{5}{2}T/T_F)^{1.7}]^{1/1.7} . \tag{11}$$

For $n_\zeta=10^{14}$ cm^{-3} and $B=0.1$ T kinetic energies are small compared to the relevant hyperfine level separations up to temperatures of a few tens of millikelvins. At $T=10$ mK we find $\Gamma_{\rm dip}\simeq0.1$ s^{-1}. At the same density and magnetic field but for $T=T_F\simeq39$ μK Eq. (11) yields $\Gamma_{\rm dip}\simeq5\times10^{-4}$ s^{-1}, while for $T\ll T_F$ the decay rate reaches its minimum value $\Gamma_{\rm dip}\simeq2\times10^{-4}$ s^{-1}.

Equations (8) and (9) not only apply to the case of D↑ ↑, but also to any doubly-spin-polarized fermionic alkali-metal atom gas, provided the relevant kinetic energies are small in comparison with the hyperfine energy levels separation. As the reduced relaxation rates depend on the atomic mass m as $\sim m^{3/2}$, we expect the alkali-metal atoms to be more unstable against dipolar relaxation than deuterium. Using the above expressions we find doubly-spin-polarized ^{6}Li to be most stable of all fermionic alkali-metal-atoms with a total reduced dipolar-relaxation rate $\tilde{g}\simeq6.2\times10^{-14}$ cm^{3}s^{-1}T$^{1/2}$K^{-1} for magnetic fields $B\gtrsim0.02$ T. Hence, even without taking into account decay due to resonance recombination, which is absent in D↑ ↑ ^{3} but effective in magnetically trapped doubly-spin-polarized alkali-metal atoms,[8] deuterium is more stable than the alkali-metal-atoms.

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