Phase instabilities in absorptive optical bistability

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PHASE INSTABILITIES IN ABSORPTIVE OPTICAL BISTABILITY

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We report that for absorptive optical bistability in a Fabry-Pérot cavity fluctuations in the phases of the fields can generate side-mode instabilities. These phase instabilities can occur in the one-atom branch of the bistability curve. Our results are based on the linearized Maxwell-Bloch equations for a Fabry-Pérot cavity with non-ideal mirrors. They strongly depend on the value of the mirror transmission coefficient and the ratio of the medium damping coefficients.

The Maxwell-Bloch theory of absorptive optical bistability in a one-directional ring cavity predicts that fluctuations in the phases of the fields cannot be responsible for instabilities in the output signal [1,2]. Only for the two-photon case so-called phase instabilities have been found [3]. In the present paper we shall demonstrate that if the cavity is of the Fabry-Pérot type the picture alters completely: then phase instabilities exist in a large part of parameter space.

Our discussion is based on an earlier article [2] in which we performed a linear stability analysis for absorptive optical bistability in a non-ideal Fabry-Pérot cavity filled with a medium of homogeneously broadened two-level particles. It was shown that the linearized Maxwell-Bloch equations give rise to two separate eigenvalue problems, one for the amplitudes and another for the phases of the fields. Each of these can be formulated as follows [2]

$$\frac{\mathrm{d}}{\mathrm{d}\zeta}\begin{pmatrix}\Delta f\\ \Delta b\end{pmatrix}=\begin{pmatrix}H_{11} & H_{12}\\ H_{21} & H_{22}\end{pmatrix}\begin{pmatrix}\Delta f\\ \Delta b\end{pmatrix}, \quad (1)$$

$$\Delta f(0)=R^{1/2}\Delta b(0), \quad \Delta f(1)=R^{-1/2}\Delta b(1). \quad (2)$$

The quantities Δf and Δb are deviations of the amplitudes or the phases of the forward and the backward electric field. The matrix elements H_{ij} depend on the eigenvalue λ and the stationary forward and backward electric fields, which we shall denote by f and b, respectively. These fields themselves are a function of the longitudinal spatial variable $\zeta=z/L$, with L the length of the cavity. Spatial variations in the transverse direction have not been taken into account. The boundary conditions (2) on the deviations Δf and Δb are imposed by the cavity mirrors with reflectivity $R=1-T$. Only for specific values of λ the conditions (2) can be satisfied. We have demonstrated lately [2] that $\lambda=0$ cannot be a solution for the phase eigenvalue problem; hence, the cavity mode which is in resonance with the laser signal does not exhibit phase instabilities.

To prove that side modes do generate phase instabilities we write the stability problem (1)-(2) in a closed form by introducing the ration $Q=\Delta f/\Delta b$ and choosing as a new independent variable $\chi=2f^2+2b^2+2K-\frac{1}{2}$ instead of ζ. The constant K is defined as

$$4K=-x^2(1+R)+[T^2x^4+2x^2(1+R)+1]^{1/2}, \quad (3)$$

with x the amplitude of the stationary transmitted field. The result of these steps reads

$$\mathrm{d}Q/\mathrm{d}\chi=H_1(Q^2+1)+H_2Q, \quad (4)$$

$$Q(\chi_1)=R^{-1/2}, \quad Q(\chi_0)=R^{1/2}. \quad (5)$$

In the literature [4] the differential equation (4) is called a Riccati equation. It must be integrated along the interval $[\chi_1, \chi_0]$, which is given by $\chi_1=\frac{1}{2}x^2(1+R)-A^2$ and

$$[(\chi_0^2-A^2)^{1/2}+\chi_0]\exp[2(\chi_0^2-A^2)^{1/2}] = (x^2-A^2)\exp(2CT+Tx^2). \quad (6)$$

Here A stands for the square root $(\frac{1}{2}-2K)^{1/2}$ and C is the cooperation parameter. For the phase eigenvalue problem the quantities H_i are determined by [2]

$$H_1 = \frac{(1+\lambda_\perp^{-1})(2\chi+1)G-4\lambda_\perp^{-1}A^2}{8A(\chi^2-A^2)^{1/2}(\chi+A^2+1)^{1/2}}, \quad (7)$$

$$H_2 = \frac{\lambda_\perp^{-1}+\hat{\lambda}(2\chi+1)}{(\chi^2-A^2)^{1/2}} - \frac{(1+\lambda_\perp^{-1})(2\chi+1)(\chi+A^2)G}{4A^2(\chi^2-A^2)^{1/2}(2\chi+A^2+1)}, \quad (8)$$

with

$$G = -1+\frac{2\chi+1}{1-\lambda_p} + \frac{[(2\lambda_p^{-1}\chi+1)^2+4\lambda_p^{-1}A^2(1-\lambda_p^{-1})]^{1/2}}{1-\lambda_p^{-1}}. \quad (9)$$

We used the notation $\lambda_p=\lambda_\perp\lambda_\parallel$, with $\lambda_i=1+\gamma_i^{-1}\lambda$ for $i=\perp, \parallel$. Furthermore, $\gamma_\perp$ and $\gamma_\parallel$ are the damping coefficients of the medium. Their ratio $\gamma_\parallel/\gamma_\perp$ will be called d in the following. The symbol $\hat{\lambda}$ was employed to denote the scaled eigenvalue $\lambda L/(cCT)$. In general λ will be complex so that the sign of the root figuring in eq. (9) is to be specified yet. It must be chosen according to the prescription

$$\mathrm{Re}\,\frac{[(2\lambda_p^{-1}\chi+1)^2+4\lambda_p^{-1}A^2(1-\lambda_p^{-1})]^{1/2}}{2\lambda_p^{-1}\chi+2\lambda_p^{-1}A^2+1} > 0\,. \quad (10)$$

In the so-called adiabatic limit, defined by $\tau_i=c/(\gamma_i L)\to 0$ for $i=\perp, \parallel$, the right-had side of eq. (7) becomes zero. The differential equation (4) can be solved analytically in that case; with the help of the boundary conditions (5) it can then be proven that the real part of the eigenvalue λ satisfies

$$R^{1/2}b(0)/f(0) = \exp\left\{\mathrm{Re}\hat{\lambda}\left[4f(0)^2-4b(0)^2-Tx^2 + \log\left(\frac{Rx^2-A^2}{4b(0)^2-A^2}\right)\right]\right\}. \quad (11)$$

In ref. [2] we have given the implicit relations which determine the stationary fields $f(0)$ and $b(0)$ as a function of the parameters T, C and x. From these relations one can see immediately that eq. (11) implies $\mathrm{Re}\hat{\lambda}<0$. Thus phase instabilities only occur if the response times τ_i of the medium take finite values. This statement confirms earlier work [5], carried out for the case of ideal mirrors.

In fig. 1 instability domains in the $(x, \tau_\perp)$-plane are shown for phase instabilities of the first sidemode. The boundaries, which correspond to $\mathrm{Re}\lambda=0$, have been computed from a numerical integration of the Riccati equation at $d=1$ and $C=300$. The first picture shows the case $T=0.05$. For this value of the transmission coefficient phase instabilities only occur in a small part of parameter space. This is not surprising since it has been proven analytically [6] that for vanishing T phase instabilities do not exist

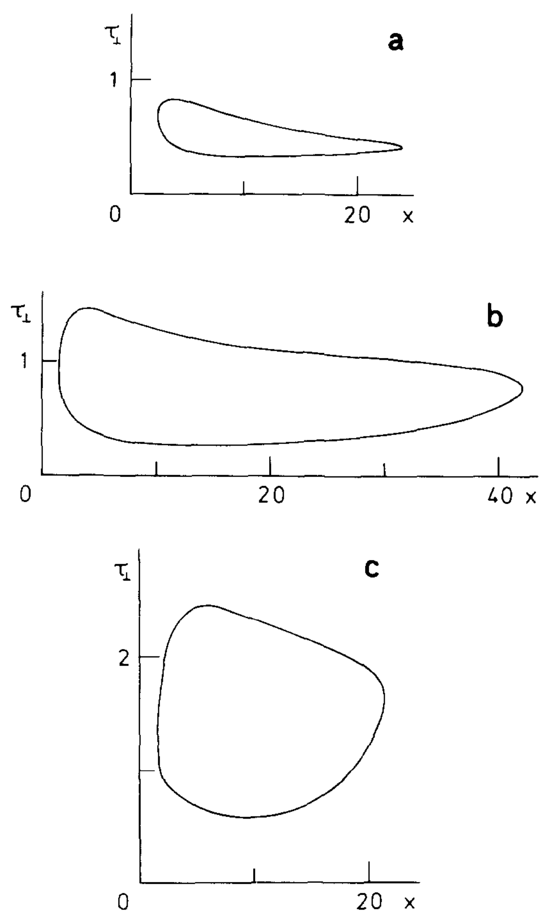

Fig. 1. Nearest-side-mode instability domains for $d=1$ and $C=300$. The first picture (a) shows the case $T=0.05$, the second (b) $T=0.1$ and the third (c) $T=0.3$.

if $d \leqslant 1$. The other pictures of fig. 1 show the instability regions at $T=0.1$ and $T=0.3$, respectively. One can observe that as the transmission coefficient increases the phase of the output field becomes unstable at higher values of the dimensionless medium response time $\tau_\perp$. In the x-direction the instability domain behaves differently as T increases: up to $T=0.1$ the domain grows in that direction as well, whereas for higher values of T it shrinks again.

All domains of fig. 1 give rise to phase instabilities in a substantial part of the upper branch of the bistability curve. For T equal to 0.05 the upper branch starts at $x=16.5$ while at $T=0.1$ and 0.3 the boundary values for x are 15.9 and 13.1, respectively. The response time for the positive-slope instabilities, as given by the real part of the eigenvalue λ, is typically of the order of 10^2 cavity round-trip times. At $T=0.3$ it is twice as long as that for the case $T=0.1$. The frequency difference between the first side-mode and the resonant mode is close to the free spectral range $\pi L/c$ throughout.

In the full (nonlinear) Maxwell-Bloch equations the phases and the amplitudes of the fields are coupled to each other. Therefore, not only amplitude instabilities but also instabilities in the phase of the output field can initiate a self-pulsing behaviour in the transmitted intensity. In a recent experiment [7] such behaviour has not been observed for absorptive optical bistability in a Fabry-Pérot cavity. This experimental fact is in accordance with the present formalism: from the stability problem (4)-(5) it follows that the phase eigenvalue has a negative real part at experimental values of the parameters, viz. $T=0.3$, $C=300$, $\tau_\perp=10$ and $d=1$. Furthermore, we have shown in a recent article [8] that the same holds for the amplitude eigenvalue. From these facts one may infer that the output field is perfectly stable under the experimental conditions.

The predictions of the present theory coincide with those of uniform-field theory if the transmission coefficient approaches zero. To verify this point we have drawn in fig. 2 the nearest-side-mode instability domains for $T=0$, $T=0.02$ and $T=0.1$ at $d=2$ and $C=300$. The smallest domain correspond to $T=0$; its bounding curve has been determined with the help of uniform-field theory [6]. If the instability domain for $T=0.001$ is calculated on the basis of the present theory, this curve is recovered with a relative error of less than one percent. Thus our numerical work is in agreement with uniform-field theory for $T\downarrow 0$, as expected. However, already at $T=0.02$ there are important deviations: the domain for $T=0.02$ is considerably larger than its uniform-field counterpart. As a result the length of the interval of unstable x-values differs by 50 percent from the uniform-field prediction.

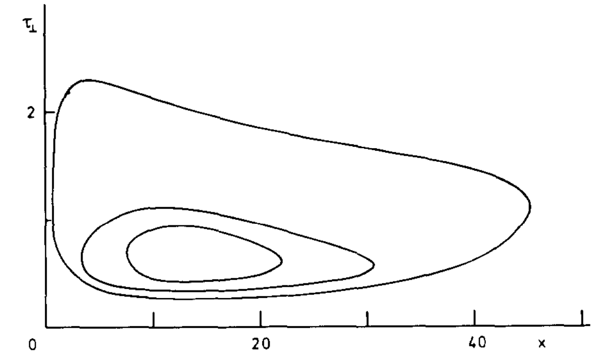

Fig. 2. Nearest-side-mode instability domains for $d=2$ and $C=300$. The smallest domain corresponds to $T=0$, the largest to $T=0.1$, while the domain in between belongs to the case $T=0.02$.

Table 1
Values of the cooperation parameter C at which nearest-side-mode instabilities appear.

T	$d=0.5$	$d=1$	$d=2$
0	∞	∞	231
0.05	486	247	121
0.1	260	138	80.4
0.2	162	95.0	62.5
0.3	172	105	73.4

A comparison between figs. 1 and 2 brings out that at $T=0.1$ the instability domain grows in all directions if d is augmented from 1 to 2. In fact, the stability of the phase of the output field critically depends on the value for the ratio $d=\gamma_\parallel/\gamma_\perp$. This statement is evidenced by table 1, where values for the cooperation parameter are listed at which phase instabilities start developing. Besides the dependence of our results on the parameter d the influence of the transmission coefficient can be assessed as well from table 1. The threshold value for C lowers sharply if T increases from zero to 0.2; if T grows on further it starts rising again.

In this report we have seen that for absorptive optical bistability in a Fabry-Pérot cavity the Maxwell-Bloch theory predicts the existence of side-mode phase instabilities. These phase instabilities come into being in the negative-slope part of the bistability curve and attain the upper branch as the cooperation parameter increases. A comparison with results on amplitude instabilities [8] learns that for transmission coefficients less than 0.1 the positive-slope instabilities for the phase extend towards higher x-values than those for the amplitude. For strongly non-ideal mirrors, with T in the range 0.2 to 0.3, the reverse is true.

References

[1] L.A. Lugiato, in: Progress in optics, XXI, ed. E. Wolf (North-Holland, Amsterdam, 1984) p. 69.

[2] A.J. van Wonderen, B.J. Douwes and L.G. Suttorp, Physica A, to be published.

[3] P. Galatola, L.A. Lugiato, M. Vadacchino and N.B. Abraham, Optics Comm. 69 (1989) 414.

[4] E.L. Ince, Ordinary differential equations (Dover Publications, New York, 1926).

[5] H.J. Carmichael, Optics Comm. 53 (1985) 122.

[6] A.J. van Wonderen and L.G. Suttorp, Physica A, to be published.

[7] B. Ségard, B. Macke, L.A. Lugiato, F. Prati and M. Brambilla, Phys. Rev. A 39 (1989) 703.

[8] A.J. van Wonderen and L.G. Suttorp, Phys. Rev. A, to be published.