Why is it so Difficult to Find
an Effect of Exchange Rate Risk on Trade?

by Franc Klaassen *

Department of Economics
University of Amsterdam

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Abstract
It is commonly argued that exchange rate risk hampers international trade. However, the large literature on this subject has not yet provided conclusive evidence. This paper analyzes why it is so difficult to obtain a clear answer from time series analyses. We use data on bilateral aggregate U.S. exports to the other G7 countries. The results show that, as far as the exchange rate concerns, export decisions are mostly affected by the probability distribution of the about one-year-ahead rate. The riskiness of the exchange rate at such a long horizon appears fairly constant over time with only short-term fluctuations. This makes it difficult to discover the true effect of exchange rate risk on trade from the time series data that are typically available.

Key words: Exports, risk measurement, imperfect substitutes, distributed lags

*Department of Economics, University of Amsterdam, Roetersstraat 11, 1018 WB Amsterdam, the Netherlands; tel: +31-20-5254191; fax: +31-20-5254254; E-mail: Klaassen@fee.uva.nl.
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1 Introduction

It is commonly argued that exchange rate risk has a negative effect on international trade. The standard argument is that greater exchange rate risk increases the riskiness of trade profits, leading risk averse traders to reduce trade.

This widespread view has been essential in various economic policy discussions. For instance, it was one of the main economic arguments for European Monetary Unification (see EU Commission (1990)). Moreover, within a floating exchange rate system the supposedly negative effect provides a rationale for foreign exchange interventions to reduce exchange rate fluctuations (Edison (1993)).

Surprisingly, however, the literature has not provided conclusive evidence for a negative effect (see the survey articles by Côté (1994) and McKenzie (1999) and the many references therein). In the theoretical literature the separation theorem shows that, with forward exchange markets and an exogenous forward rate, the optimal export level is independent of exchange rate risk (Ethier (1973)). However, if the determination of the forward rate is taken into account, exchange rate risk can affect trade via its effect on the forward rate, and this effect can be positive or negative (Viaene and De Vries (1992)). Therefore, the true effect is mainly an empirical question. But the voluminous empirical literature has not provided conclusive evidence either. Many papers report insignificant estimates. Hence, there is a tendency among researchers to believe that the effect is absent, or at least small.

Insignificance, however, does not necessarily imply that the effect is absent. Insignificance can be the result of a particular research methodology, even if there is an effect in reality. Due to the surprising conflict between the common idea of a negative effect and the inconclusive empirical results, it is important to find out whether the commonly used research approach is responsible for the insignificance.

Our paper addresses this issue. The aim is to explain why it has been so difficult to find an effect in the empirical literature so far. The explanation will lead to suggestions for future research needed to find the true effect of exchange rate risk on trade.

We examine the question of interest as follows. Since we concentrate on the existing literature, we follow the vast majority of papers by employing time series data on developed countries. More specifically, monthly bilateral aggregate U.S. exports to the other G7 countries from 1978 to 1996 are used. We find that, as far as the exchange rate is concerned, export decisions are mostly affected by the probability distribution of the about one-year-ahead rate. Second, we show that the riskiness of the exchange rate at such a long horizon is fairly constant over time with only short-term fluctuations. Since exports have much more variation over time and exhibit long-term swings, exchange
rate risk can explain only a minor part of the development of exports through time. Hence, even if risk affects exports, this effect will be overshadowed by the effect of unobserved export determinants and random noise on exports. This makes it difficult to find a significant effect of risk on trade, no matter whether the true effect is zero or not.

Hence, in our opinion, the use of time series data on developed countries may well be responsible for the insignificant results in many papers. As a guideline for future research we propose to use different data, for instance, data on countries with more time variation in exchange risk, such as developing countries (Arize, Osang and Slottje (2000)), or data with cross-sectional variation in exchange risk (De Grauwe and Verfaille (1988)). The results in those papers are encouraging, as they show that the effect may be negative after all.

Besides our explanation, there may be additional reasons for the insignificant exchange rate risk effect. We discuss three of them (see Côté (1994) and McKenzie (1999) for other potential reasons). First, the effect of exchange rate risk on trade may be truly absent, for instance, because firms can avoid all exchange risk by hedging. However, Wei (1999) finds no support for this hedging argument. The absence of any effect would also be in contrast with the widespread view of a negative effect.

A second possible reason is that the empirical tests may have methodological problems (see also Bini-Smaghi (1991)). One concerns the measurement of exchange rate risk. In principle, the (subjective) risk for goods traders is not observed. This is one reason why it is difficult to find an effect of risk on trade. It is, however, possible to address the issue in an indirect manner by using the variance of the future exchange rate conditional on past rates as a proxy for risk. This is the most popular approach in the literature, and since we concentrate on the existing literature, we follow that.\footnote{We do not claim that the conditional variance is the best proxy for risk. Future research is needed to find out what is the best proxy. Such research goes beyond the scope of this paper, because we intend to explain why it has been so difficult to find an effect of exchange rate risk on trade in the existing literature, and there risk is proxied by the conditional variance.}

To measure risk we use the idea of Merton (1980) and Andersen and Bollerslev (1998) to compute monthly volatility by cumulating squared daily exchange rate changes in the month, instead of taking a single squared monthly change. Then we use the forecasts from an autoregressive model of order two (AR(2)) on the monthly volatilities to derive the risk measure. The data show that our AR measure provides a substantially better indication of future volatility than the moving variance and generalized autoregressive conditional heteroskedasticity (GARCH) measures used so far.\footnote{The moving average measure is used in, for instance, Chowdury (1993) and Arize, Osang and Slottje (2000). GARCH risk measures are used in Kroner and Lästrapes (1993), among others. The}
the estimated effect of risk on exports more accurate. Despite this methodological improvement, we still find no significant effect of risk on trade.

A third potential reason for the ambiguous results in the literature concerns the dynamic specification of trade models. If one does not properly account for lagged effects of, for instance, the exchange rate, then this will bias the estimated effect of exchange rate risk on trade. To model the dynamics, we employ a distributed lag model and introduce a new way to impose structure on the lag coefficients. It separates the total effect of a regressor from the distribution of the effect over time and uses the Poisson probability function for the latter. We argue that the “Poisson lag structure” is more appropriate than the commonly used geometric and polynomial lag structures. The insignificant effect of risk on trade, however, is also robust to this methodological improvement.

The paper is organized as follows. In section 2 we use an economic model to examine which variables are important for the empirical work. In section 3 we describe the data. Section 4 presents the econometric model and pays special attention to the new Poisson lag structure and to the specification of the risk measure. In section 5 we report the empirical results, show their robustness regarding various model assumptions, and derive the main conclusion of the paper. Section 6 concludes.

2 Economic Model for Exports

In this section we develop an economic model for the determination of exports. It will motivate the choice of explanatory variables in the econometric model in section 4.

The economic model is based on the popular two-country imperfect substitutes model, which considers domestic exports and goods produced abroad as imperfect substitutes (see Goldstein and Khan (1985) and Rose (1991)). We extend that model by explicitly accounting for the lag between the time of the trade decision and the time of the actual trade flow and payment. This time lag is an important characteristic of international trade, as Goldstein and Khan (1985) and Sawyer and Sprinkle (1997) argue. It makes the traders forward-looking and motivates the relevance of exchange rate risk as a potential determinant of trade.

Let \( t \) denote the time (month) of observing a nominal export flow \( X_t \) from the home to the foreign country, expressed in domestic currency. Exports are, supposedly, the result of a contract signed \( m \) months earlier, stating both the export quantity \( Q_t \) and price \( P_{xt} \). Because in the empirical part we study U.S. exports, which are almost

GARCH literature is surveyed by Bollerslev, Chou and Kroner (1992).
completely invoiced in U.S. dollars (see Page (1981)), we assume for simplicity that the price $P_{xt}$ is specified in the home currency, so that $X_t = Q_t P_{xt}$.

The focus variable is (the logarithm of) the real value of exports, using the price $P_t$ of domestically produced goods as deflator:

$$x_t = q_t + p_{xt},$$

where $x_t = \log(X_t / P_t)$, $q_t = \log(Q_t)$ and $p_{xt} = \log(P_{xt} / P_t)$. We concentrate on the real value $x_t$ rather than the quantity $q_t$, which is often analyzed in the literature, because we study bilateral exports for which $x_t$ is observable, while we have no observations of the bilateral export prices $P_{xt}$ needed to derive $q_t$ from $x_t$.

The determinants of exports follow from the assumptions regarding export supply and demand. Supply is an unknown function $q_s$ of the relative price of exports:4

$$q_s^t = q_s^*(p_{xt}).$$

Foreign demand for domestic exports, $q_d^t$, depends on two components. The first one is the relative price for foreigners. Since exports are invoiced in domestic currency, the nominal price for foreigners is $P_{xt}/S_t$, where $S_t$ is the nominal (spot) exchange rate, that is, the domestic currency price of one unit of foreign currency. The relative price is therefore $(P_{xt}/S_t)/P_t^*$, where $P_t^*$ is the foreign currency price of foreign produced goods. In logarithms, this equals $\log(P_{xt}/P_t \cdot P_t/(S_t P_t^*)) = p_{xt} - s_t$, where $s_t = \log(S_t P_t^*/P_t)$ is the real exchange rate.

Although it is implicitly assumed that $P_t$ and hence $p_{xt}$ are perfectly forecastable at time $t-m$ when the trade decision is made, such an assumption is not realistic for $s_t$, at least not for floating exchange rates. Hence, we account for the randomness of $s_t$ at time $t-m$. As usual in the trade literature, we assume that the mean and variance of $s_t$, conditional on information $I_{t-m}$ consisting of past real exchange rates $s_{t-m}, s_{t-m-1}, \ldots$, are sufficient to capture the effects of exchange rates on export demand.5 Let $E_{t-m}\{s_t\}$ and $V_{t-m}\{s_t\}$ denote the conditional mean and variance, respectively.

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3 The model can be extended to allow for invoicing in foreign currency as well. In that case, $X_t$ also depends on the contemporaneous nominal exchange rate, which converts the foreign currency invoiced part of exports into domestic currency. It can be shown that the collection of export determinants in the final model equation (5) should then be extended by the contemporaneous real exchange rate.

4 The export price is relative to the price of domestic output in month $t$, because we assume that the exporter receives payment in the same month as the delivery of the goods, which is month $t$. This assumption is a reasonable approximation, as Stokman (1995) reports that payments peak in the month of delivery and decline rapidly in subsequent months.

5 For simplicity, we abstract from the existence of a forward market to hedge exchange rate risk. With a forward market, export demand would also depend on the forward exchange rate. The forward exchange rate is highly dependent on the mean and variance of the future spot rate (see Vlaene and De Vries (1992)), which we both take account of. Hence, the benefits from also including the forward rate are likely small.
The second determinant of foreign demand is based on the orders that the foreign importer expects from his customers between the contract date \( t - m \) and the date of delivery \( t \). We represent this order flow by a single order at some intermediate time \( t - k \) \((k < m)\). The order quantity is determined by (the logarithm of) real foreign income \( y^*_{t - k} \) at that time. For simplicity and to concentrate on the randomness of the exchange rate only, we assume that \( y^*_{t - k} \) is known at time \( t - m \) and thus let \( y^*_{t - k} \) capture the effect of income on foreign demand.

Combining the price and income components just discussed, we specify the demand for domestic exports as

\[
q^d_t = q^d (y^*_{t - k}, p_{xt} - E_{t - m} \{s_t\}, V_{t - m} \{s_t\}) .
\] (3)

The market for domestic exports is in equilibrium if

\[
q_t = q^s_t = q^d_t .
\] (4)

Solving (2)-(4) for \( p_{xt} \) and \( q_t \) and substitution in (1) then yields

\[
x_t = x (y^*_{t - k}, E_{t - m} \{s_t\}, V_{t - m} \{s_t\}) .
\] (5)

Hence, the determinants of real (domestic output) exports are real foreign income (with an expected positive effect), the expected real exchange rate level (positive effect, since a U.S. dollar depreciation generally lowers the foreign currency price of U.S. exports, thereby increasing the quantity and the dollar value of exports) and real exchange rate risk (unknown effect). The inclusion of foreign income and the exchange rate is standard in trade models. The fact that the exchange rate does not appear directly but rather through its conditional mean and variance originates from the time lag between contract date \( t - m \) and payment date \( t \). This lag makes the traders forward-looking agents, and because the exchange rate at \( t \) is unknown at \( t - m \), the exchange rate effect goes via the two conditional moments. Finally, the model has a shorter time lag for foreign income than for both exchange rate moments, as \( k < m \); this is also realistic from an empirical point of view (Goldstein and Khan (1985)).

### 3 Data Characteristics

#### 3.1 Data

The export data are monthly bilateral aggregate U.S. exports to the six other G7 countries, namely Canada, France, Germany, Italy, Japan and the U.K. We use bilateral
instead of the often used multilateral data to avoid the difficult construction of multi-
country explanatory variables. Moreover, by considering several flows we can provide
some insight into the robustness of our results. The fact that we use aggregate instead
of product-specific trade data is not important for the main conclusion of the paper.

The export time series span January 1978 through November 1996, leading to 227
monthly observations. For the other variables we have a longer history, which is useful
given the lags in (5); they are available from April 1974 through November 1996.

The data source for the export values is the U.S. Bureau of the Census. To convert
the nominal dollar exports into real (domestic output) exports $x_t$, we use the U.S.
wholesale price index from the OECD Main Economic Indicators. Because real national
income is only available at the quarterly frequency, we proxy $y_t^*$ by monthly foreign
industrial production, as usual; the data are from the OECD. The monthly nominal
exchange rate is taken from the IMF International Financial Statistics and the wholesale
price index from the OECD is used to convert it into the real exchange rate $s_t$ (except
for the French real exchange rate, which is based on French and U.S. consumer price
indices, because French WPI is not available).

3.2 Nonstationarity and Cointegration

As usual, we assume that exports $x_t$ and foreign industrial production $y_t^*$ have a unit
root. On the other hand, Andersen, Bollerslev, Diebold and Labys (1999) demonstrate
that the conditional variance of nominal exchange rates is stationary, so that it is likely
that also the real exchange rate variance $V_{t-m}\{s_t\}$ is stationary. Finally, we assume
that the expected real exchange rate $E_{t-m}\{s_t\}$ is stationary. Part of the literature
on long-run relative purchasing power parity (PPP), in other words stationarity of the
real exchange rate, supports this; for instance, Koedijk, Schotman and Van Dijk (1998).
However, there is no general consensus on the validity of PPP (Engel (2000), among
others). The main conclusion of our paper, however, also holds if PPP is not valid, as
it concerns the risk measure, which is stationary (see also footnote 7).

Next, we check for cointegration between $x_t$ and $y_t^*$. From an economic point of
view, one expects that they are cointegrated. This is confirmed by empirical results in
Sawyer and Sprinkle (1997), among others. However, obtaining statistical evidence for
our data is not straightforward and requires a cointegration analysis that goes beyond
the scope of this paper. Instead, we follow an indirect approach. First, we simply
assume cointegration and specify an econometric model using $x_t$ and $y_t^*$ in levels. Then

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\footnote{For details we refer to the discussion paper version of the paper, CentER for Economic Research
Tilburg University Discussion Paper No. 9973.}
we estimate that model and examine the residuals. We will show in subsection 5.1 that
these are stationary. Hence, given the stationarity of $E_{t-m}\{s_t\}$ and $V_{t-m}\{s_t\}$, it is very
likely that $x_t$ and $y_t^*$ are cointegrated, as economic intuition suggests.

4 Bivariate Econometric Model

This section develops a bivariate econometric model, to be estimated in section 5. The
model consists of an export equation (described in subsection 4.1) and an equation for
exchange rate volatility, which is needed to specify the risk term in the export equation
(see 4.4). The model is completed by the restrictions on the dynamic structure of
the export equation in subsection 4.2 and by the specification of the exchange rate
expectation (see 4.3).

4.1 Export Equation

The equation for exports $x_t$ is based on the variables that appear in economic model
(5), that is, foreign income $y_t^{* - k}$ and the exchange rate expectation $E_{t-m}\{s_t\}$ and
variance $V_{t-m}\{s_t\}$. The economic model takes explicit account of the dynamic nature
of international trade by specifying the determinants of the export of a good in month
$t$ when the export contract was signed $m$ months before. However, our trade data are
aggregated across many products and it is likely that for different products the lags are
different. To account for this in our empirical model, we use a distributed lag model,
where the effect of a change in a regressor is allowed to be distributed over time.

We thus specify real exports as

$$x_t = \beta_0 + \sum_{l=1}^{\infty} (\beta_{y} y_{t-l}^{* - k} + \beta_{E} E_{t-l}\{s_t\} + \beta_{V} V_{t-l}\{s_t\}) + \varepsilon_t, \quad (6)$$

where the lag structure on the $\beta$, the expectation $E_{t-l}\{s_t\}$ and variance $V_{t-l}\{s_t\}$ will be
specified in the next three subsections. The disturbance term $\varepsilon_t$ is allowed to follow an
AR(2) process with autoregressive coefficients $\theta_1$ and $\theta_2$ and with conditionally normal

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7 Equation (6) is in the levels of the variables, despite the nonstationarity of $x$ and $y^*$. The reason is
that we need the residuals of the levels equation to test for cointegration between $x$ and $y^*$, as explained
at the end of subsection 3.2. In case of cointegration, one could also analyze the export equation in error-
correction form. However, Sims, Stock and Watson (1990) argue that such a stationarity transformation
by difference and cointegration operators is unnecessary if the parameters of interest relate to stationary
regressors, because the estimates of these parameters have standard (normal) asymptotic distributions,
unaffected by the nonstationarity of the other variables. This special case applies to our setup, as the
parameters of interest $\beta_{V}$ concern exchange rate risk, which is stationary (see subsection 3.2). Since
an equation in levels is also easier to interpret, we use the levels form only.
innovations having mean zero and variance $\sigma^2$. Though not explicitly mentioned, we include eleven monthly dummies to allow for seasonal differences.  

### 4.2 Lag Structure

Unrestricted estimation of export equation (6) is infeasible because of the infinite number of parameters. Therefore, we now introduce restrictions on $\beta_{yl}$, $\beta_{El}$ and $\beta_{Vl}$.

There exist several ways to restrict the $\beta_l$ in (6) (in this subsection $\beta_l$ is shorthand notation for $\beta_{yl}$, $\beta_{El}$ or $\beta_{Vl}$). A popular method is the geometric lag specification. It implies that the $\beta_l$ are decreasing over $l$. However, this seems inappropriate for the expected exchange rate effects $\beta_{El}$, because their pattern may well be hump shaped, as Sawyer and Sprinkle (1997) claim. A second popular lag structure is the polynomial or Almon lag specification. It assumes that the $\beta_l$ fall on a polynomial of a prespecified order. Though more flexible than the geometric model, the polynomial structure may force some $\beta_l$ to be positive and others to be negative, which is difficult to justify theoretically (see Goldstein and Khan (1985)).

To avoid the disadvantages just described, we introduce an alternative approach. Let us suppose that all $\beta_l$ have the same sign. Then, one can write $\beta_l = \beta \cdot w_l$, where $w_l \geq 0$ and $\sum_{l=1}^{\infty} w_l = 1$. Hence, $\beta$ gives the total, long run effect of the regressor, while the $w_l$ describe how the total effect is distributed over time; by definition, the $w_l$ form a probability function with support $\{1, 2, 3, \ldots\}$. The parameters $\beta$ and $w_l$ thus represent two clearly distinct features of the lag structure, which is attractive from an estimation point of view.

Besides the interpretation of $\beta$ and the $w_l$, the main attractive feature of the proposed class of probability function based lag specifications is its flexibility. One can choose any probability function for the $w_l$, depending on the specific needs. For instance, the approach encompasses the geometric lag specification as the special case where the $w_l$ follow the (translated) geometric probability function $w_l = \gamma \cdot (1 - \gamma)^{l-1}$. It can also capture hump shaped or bimodal lag patterns.

Within the general class of lag specifications, we take “Poisson lags” for our export model (subsection 5.2 shows the robustness of the central conclusion of the paper to this choice). That is,

$$\beta_{il} = \beta_i \cdot \frac{(\lambda_i - 1)^{l-1}}{(l-1)!} \exp[-(\lambda_i - 1)], \quad \text{for } \lambda_i \geq 1 \text{ and } i = y, E, V. \quad (7)$$

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8For Canada we also allow for a break in $\beta_y$ from 1991 onwards to account for the increase in trade openness due to the Free Trade Agreement between the U.S. and Canada. Moreover, for Canada we use an AR(3) instead of AR(2) process to capture extra autocorrelation in the disturbance term.
Note that we have to translate the Poisson probability function one unit to the right, because \( l \) starts at one instead of zero. The parameter \( \lambda \) is close to the mode of the translated Poisson distribution.\(^9\) Hence, we give \( \lambda \) the convenient interpretation of the lag at which the maximal effect occurs, that is, the lag with the largest coefficient \( \beta_l \). Figure 1 illustrates the Poisson lags for \( \lambda = 3.40 \) and \( \lambda = 12.72 \) (with \( \beta = 2.21 \) and \( \beta = 0.65 \), respectively; the numbers are based on the estimation results below).

The economic model of section 2 implies that \( \lambda_y \) is smaller than \( \lambda_E \) and \( \lambda_V \) (as \( k < m \) in (5)) and that \( \lambda_E = \lambda_V \). Hence, we impose the latter equality (the data do not reject it; see subsection 5.2) and call the joint parameter \( \lambda_{EV} \).

The Poisson lag structure (7) is very parsimonious. This is at the cost of flexibility. However, Poisson lags can capture a declining lag structure as well as a hump shaped one and imply that all \( \beta_l \) have the same sign. In this sense, Poisson lags improve on the geometric and polynomial lags discussed above. We can let the data decide whether a declining or hump shaped lag structure is more appropriate and how long it takes for industrial production and exchange rates to have the strongest effect on exports, an issue that is also unresolved in the literature (see Sawyer and Sprinkle (1997)). The exchange risk horizon that is relevant for goods traders will be an important element in the derivation of the main conclusion of the paper.

### 4.3 Specification of Real Exchange Rate Expectation

The export equation (6) also requires a proper specification of the expected real exchange rate, \( E_{t-l}\{s_t\} \). Since the paper concentrates on the real exchange rate variance, we take a simple but reasonable model for the expectation:

\[
E_{t-l}\{s_t\} = s_{t-l}.
\]

As Mark and Choi (1997) argue, such a forecasting rule is difficult to beat at horizons of, say, 3 to 12 months. For longer horizons, such as 48 months, Mark and Choi report some extra predictability in real exchange rates using fundamentals. Although such horizons are possibly beyond the horizon relevant for goods traders, even then forecasting rule (8) appears reasonable.

### 4.4 Specification of Real Exchange Rate Risk

To complete the econometric model, we have to specify real exchange rate risk \( V_{t-l}\{s_t\} \). We first present two risk measures that are commonly used in the trade literature.

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\(^9\) The exact mode of the translated Poisson distribution with parameter \( \lambda \) is the largest integer \( l \) less than \( \lambda \); if \( \lambda \) itself is an integer, then \( l = \lambda - 1 \) and \( l = \lambda \) are tie modes.
Then we introduce an alternative measure, based on AR(2) forecasts of monthly real exchange rate volatilities, where monthly volatility is computed as the sum of squared daily changes in that month.

The measures used so far are typically one-period-ahead volatility measures, that is, \( V_{t-l}\{s_t\} \) for \( l = 1 \). Hence, in case of monthly data it is one-month-ahead risk and for quarterly data it is one-quarter-ahead risk that is allowed to affect trade flows. We do not a priori impose such a specific time lag and present \( V_{t-l}\{s_t\} \) also for \( l > 1 \).

### 4.4.1 Moving Variance Risk Measure

The first popular risk measure is the moving sample variance of past real exchange rate changes. The window width is prespecified and is often about two years. For illustrative purposes, let us thus assume that the window width is 24 months. The moving variance risk measure for \( l = 1 \) is

\[
V_{t-1}\{s_t\} = \frac{1}{24} \sum_{i=1}^{24} (s_{t-i} - s_{t-i-1})^2,
\]

and for \( l > 1 \) it is \( V_{t-l}\{s_t\} = l \cdot V_{t-l}\{s_{t-l+1}\} \). One can interpret this measure as first approximating volatility in month \( t \) by \((s_t - s_{t-1})^2\) and then smoothing by taking the average over 24 months.

The main characteristic of the moving variance measure (9) is that it implies a high (24 months) persistence of real exchange rate shocks and, therefore, suggests considerable serial correlation in risk. To illustrate this, figures 2A and 3A plot measure (9) for the two most important trading partners of the US, namely Canada and Japan (for the other countries the graphs yield a similar message); the thin lines plot \( V_{t-1}\{s_t\} \), while the thick lines plot \( V_{t-12}\{s_t\} \). There are some long swings in the risk measure, particularly for Japan. In subsection 4.4.3 we will check whether the high autocorrelation is real or spuriously induced by definition (9).

### 4.4.2 GARCH Risk Measure

The second commonly used risk measure also approximates volatility in month \( t \) by \((s_t - s_{t-1})^2\), but then uses a GARCH model for smoothing. In case of GARCH(1,1), one specifies the risk measure for \( l = 1 \) as

\[
V_{t-1}\{s_t\} = \omega_0 + \omega_1 (s_{t-1} - s_{t-2})^2 + \omega_2 V_{t-2}\{s_{t-1}\};
\]

for \( l > 1 \) one can compute \( V_{t-l}\{s_t\} \) recursively from \( V_{t-l}\{s_{t-l+1}\} \), as in Engle and Bollerslev (1986).
The main characteristic of measure (10) for our monthly data is illustrated by figures 2B and 3B for both \( l = 1 \) and \( l = 12 \). They suggest that there is no persistence of shocks in risk. The reason for this becomes clear from table 1. The top half of that table presents the first-order autocorrelation \( \rho_1 \) and the Box-Pierce combination \( Q_{10} \) of the first ten autocorrelations of \((s_t - s_{t-1})^2\) for the U.S. versus the other G7 countries. It suggests that squared real exchange rate changes exhibit no autocorrelation at the monthly frequency (for all tests in the paper we use a significance level of 5%). This result is well-known from the GARCH literature (see Bollerslev, Chou and Kroner (1992)) and suggests that the GARCH parameters \( \omega_1 \) and \( \omega_2 \) in (10) are zero, resulting in the lack of persistence of shocks in the monthly GARCH risk measures.

### 4.4.3 AR Risk Measure

The zero serial correlation in risk implied by GARCH measure (10) is not consistent with the high correlation implied above by the moving variance measure (9). Since the main conclusion of the paper will be based on the variation of risk over time, it is important to know what the true degree of serial correlation is.

We start from an idea presented by Merton (1980) and formalized by Andersen and Bollerslev (1998). They argue that the ex-post squared change in a period, although unbiased, is a very noisy indicator for the latent volatility in that period. To explain this, we split a month into days and write the monthly change \( s_t - s_{t-1} \) as the sum of the daily changes \( s_d - s_{d-1} \) over the days \( d \in D_t \), where \( D_t \) is the set of days in month \( t \). This implies \((s_t - s_{t-1})^2 = \sum_{d \in D_t}(s_d - s_{d-1})^2 + \sum_{d, \delta \in D_t, d \neq \delta}(s_d - s_{d-1})(s_\delta - s_{\delta-1})\). Because real exchange rate changes have a conditional mean (close to) zero, the cross products are on average zero. In addition, since the conditional skewness of real exchange rate changes is (close to) zero (see Bollerslev et al. (1992)), the variance of \((s_t - s_{t-1})^2\) equals the variance of \( \sum_{d \in D_t}(s_d - s_{d-1})^2 \) plus the variance of the sum of the cross products. Therefore, the cross products do not affect the mean but raise the variance, so that they are a source of the noise in \((s_t - s_{t-1})^2\).

To obtain a more accurate measure of latent monthly volatility, we remove the cross products. We thus take the sum of squared daily real exchange rate changes over all days in the month:\(^{10}\)

\[
v_t = \sum_{d \in D_t} (s_d - s_{d-1})^2. \tag{11}\]

\(^{10}\)Using daily data is the best we can do, because we have no intraday data. The daily real exchange rates are based on daily nominal exchange rates from Datastream. Because price ratios \( P_t^*/P_t \) are not available on a daily basis, we linearly interpolate the monthly ratios and use the constructed daily price levels to measure daily real exchange rates (given the stability of the price ratios, this will yield a good proxy for the true daily real exchange rates).
This measure is unbiased, just as \((s_t - s_{t-1})^2\), and the reduction in the variance compared to \((s_t - s_{t-1})^2\) is large, as Merton (1980) and Andersen and Bollerslev (1998) show. Note that the daily data only serve to measure monthly volatility; below, we will only work with the constructed monthly volatility, so that the complete analysis will be on a monthly frequency.

We now re-examine the serial correlation in monthly volatility with the more accurate measure \(v_t\). The second half of table 1 shows that there is clear evidence of serial correlation, which is in line with Andersen et al. (1999). This casts serious doubt on the GARCH based claim of no autocorrelation.

To analyze whether the serial correlation in volatility is high, we estimate a second order autoregressive model (AR(2)) for \(v_t\):

\[
v_t = \mu_v + \alpha_1(v_{t-1} - \mu_v) + \alpha_2(v_{t-2} - \mu_v) + \nu_t, \tag{12}
\]

where the innovation \(\nu_t\) is conditionally normally distributed with mean zero, constant variance and uncorrelated with the disturbance \(\varepsilon_t\) in the export equation (6).\(^\text{11}\) As table 2 demonstrates, there is no residual autocorrelation, so that the order of two is sufficient. Moreover, the estimates of \(\alpha_1\) and \(\alpha_2\) are moderate. Hence the suggestion of high persistence of shocks from the moving variance measure (9) is not correct either.

Given the central role of exchange rate risk in our study, we propose an alternative risk measure to improve on the moving variance and GARCH measures. Using (8), we have \(V_{t-1}\{s_t\} = E_{t-1}\{(s_t - s_{t-1})^2\} = E_{t-1}\{\sum_{d \in D_t} (s_d - s_{d-1})^2\} = E_{t-1}\{v_t\}\), so that we can use the AR(2) model (12) to measure real exchange rate risk. The measure, which we call AR, is thus

\[
V_{t-1}\{s_t\} = E_{t-1}\{v_t\} \tag{13}
\]

for one-month-ahead risk \((l = 1)\), where \(v_t\) is defined in (11) and modeled by (12). For general \(l > 1\), \(V_{t-l}\{s_t\} = E_{t-l}\{v_{t-l+1}\} + \ldots + E_{t-l}\{v_t\}\), where each expectation is a standard multi-period-ahead AR(2) forecast, which can easily be obtained in a recursive manner. Figures 2C and 3C illustrate the AR measure (using the parameter estimates from table 2 for \(\mu_v, \alpha_1\) and \(\alpha_2\) in (12)).

We now compare the quality of the proposed AR measure with that of the moving variance (9) and GARCH approach (10). First, as already seen, the AR measure takes account of the serial correlation in monthly volatilities in a better way than the other two. Second, we compare the explanatory power of the three risk measures \(V_{t-l}\{s_t\}\) for future volatility. To measure future volatility, we use the monthly volatilities obtained

\(^\text{11}\) Although the volatility implied by the right-hand-side of (12) is not necessarily positive, this is no problem in our empirical work where we will only use \(E_{t-l}\{v_t\}\), for which all estimates are positive.
from daily data, although the conclusions are essentially the same when using the less accurate squared monthly changes. Thus, the volatility over the \( l \) months from \( t - l + 1 \) to \( t \) is \( v_{t-l+1,t} = \sum_{i=1}^{l} v_{t-l+i} \). A natural choice to quantify explanatory power would be the \( R^2 \) of a regression of \( v_{t-l+1,t} \) on each \( V_{t-l}\{s_t\} \) separately. However, as we are interested in the quality of \( V_{t-l}\{s_t\} \) instead of a linear combination of it, we compute the \( R^2 \) under the restriction of a zero intercept and a slope coefficient of unity. In formula, we use \( 1 - V\{v_{t-l+1,t} - V_{t-l}\{s_t\}\}/V\{v_{t-l+1,t}\} \). Not surprisingly, table 3 shows that for both the one month and one year horizons the explanatory power is highest for the AR measure. It is interesting to see that the explanatory power is even negative for the moving variance measure, indicating that the variance of the forecast error is higher than that of future volatility itself. We conclude that the AR measure outperforms the moving variance and GARCH measures, and we therefore use it to measure real exchange risk in the remaining part of the paper.

Two characteristics of the AR measure will play a crucial role. First, figures 2C and 3C show that real exchange rate risk is time-varying, but that shocks do not persist very long in risk. Second, for long horizons, the time variation in \( V_{t-l}\{s_t\} \) is small relative to its level; for \( l = 12 \) the standard deviation of risk is only 4% of the mean risk level for Canada and 5% for Japan (for the other countries it is 7% (France), 6% (Germany), 8% (Italy), and 11% (UK)).

This completes the econometric model we use to estimate the effect of exchange rate risk on exports. In summary, it is a bivariate model consisting of export equation (6) and volatility equation (12), where the latter is used to generate the exchange rate risk term in the export equation (see (13)). The model has a Poisson lag structure (7) and specifies the exchange rate expectation by (8).

## 5 Empirical Results

This section presents the estimates for the econometric model and examines the robustness of the results (see subsections 5.1 and 5.2, respectively). The results provide the ingredients for subsection 5.3, which derives the main conclusion of the paper.

### 5.1 Estimation Results

We estimate the model with maximum likelihood (ML) on each of the six country pairs separately. The export equation (6) and volatility equation (12) are estimated jointly. This avoids the bias in the standard errors for the export parameters that would result from the often used two-step estimation method, where one first estimates the risk
measure and then uses it as a generated regressor in the export equation (see Pagan (1984)). We have longer series for volatility than for exports (the extra months are April 1974 through December 1977). For efficiency reasons we use these extra months in estimation, so that the bivariate likelihood contribution for the extra months equals the marginal likelihood contribution of the volatility observations only.

Table 4 presents the results for the export equation (the volatility estimates are not reported, since they are virtually the same as the univariate estimates in table 2). First, we consider the disturbance \( \varepsilon_t \) in (6) to address the issue of cointegration between exports and foreign industrial production raised in subsection 3.2. Since the estimates of the autoregressive parameters \( \theta_1 \) and \( \theta_2 \) for \( \varepsilon_t \) are positive and their sum is well below unity, the estimated AR process is stationary (see Hamilton (1994, p. 57)). Stationarity is also confirmed by plots of the residuals. As argued in subsection 3.2, this implies that exports and industrial production are cointegrated.

Table 4 demonstrates that foreign industrial production has the expected positive effect on the real (domestic output) value of U.S. exports. This holds for all six series. The average estimate of \( \beta_y \) is 2.21.\(^{12}\)

An attractive implication of the Poisson lag specification (7) is that we can directly estimate the time lag \( \lambda_y \) between a change in industrial production and the maximal change in exports. Table 4 shows that the maximal effect occurs after about one quarter (the average estimate of \( \lambda_y \) is 3.40, ignoring the outlying estimate for the U.K.). Hence, the effect of foreign income on U.S. exports goes quite rapidly; this corroborates the results in Goldstein and Khan (1985) and Sawyer and Sprinkle (1997). The dots in figure 1 illustrate the implication of the average \( \lambda_y \) for the distribution of the average \( \beta_y \) over the lags.

The expected real exchange rate has the anticipated significantly positive effect for all six export flows. The average estimate of \( \beta_E \) is 0.65. It is remarkable that our estimates are so consistent across countries given the wide range of estimated price elasticities of the quantity of exports in the literature, as analyzed by Marquez (1999). Some differences in the estimates are to be expected, for instance because the commodity composition of U.S. exports differs across the six partner countries and because price elasticities may well differ across commodities. However, all export flows have one common source and the countries of destination are not very different, so that the true export processes are probably also somewhat similar. Therefore, we view the

\(^{12}\)The estimates for \( \beta_y \) are not directly comparable with the income elasticities of U.S. exports that are typically reported in the literature, since the endogenous variable in (6) is the value of exports expressed in domestic output, not the export quantity, and because the explanatory variable is industrial production, not real national income.
relative consistency of our estimates as an indication of the robustness of the analysis.

The central parameter of the paper is \( \beta_V \), the total impact of real exchange rate risk on exports. We find that the estimates of \( \beta_V \) are insignificant for all countries and have different signs (t-values are 1.47 (Canada), 1.17 (France), -0.05 (Germany), 0.81 (Italy), 0.82 (Japan) and -0.95 (UK)). This inconclusiveness confirms the results in many earlier papers using similar data (time series of developed countries). For example, concentrating on studies that also analyze data on post Bretton Woods bilateral US exports, Baum, Caglayan and Ozkan (2001) find that 15 out of 17 estimates are insignificant, and Cushman (1988) reports both significantly negative and positive effects, as well as insignificant effects. The interesting aspect of the estimates in the current paper is that our careful examination of the lag structure and the measurement of risk does not alter that common finding.

The Poisson lags yield an average estimate of 12.72 for \( \lambda_{EV} \). This parameter gives the lag distribution for both the expected exchange rate \( E_{t-l}\{s_t\} \) and the risk term \( V_{t-l}\{s_t\} \), as argued below (7). Concerning the expected exchange rate, we thus find that in the short-run the effect of changes in the exchange rate on exports is small, while in the longer run there is a clear effect, with the maximal effect occurring after about one year. This supports the view of a hump shaped instead of a declining lag pattern, so that it helps solve the question on the true lag pattern for exchange rates raised by Goldstein and Khan (1985). The asterisks in figure 1 illustrate the distribution of the average \( \bar{\beta}_E \) over the lags as implied by the average \( \bar{\lambda}_{EV} \). For exchange rate risk, the estimates of \( \lambda_{EV} \) indicate that, if there is an effect of risk on trade, only the longer-term risk is relevant for goods traders, in particular the about one-year-ahead risk.

Comparing the estimated \( \lambda_{EV} \) and \( \lambda_y \), we conclude that exchange rate changes need more time to affect exports than changes in income (\( \lambda_{EV} > \lambda_y \)). This is in line with our economic model and with conventional wisdom (Goldstein and Khan (1985)).

5.2 Diagnostics and Sensitivity Analysis

Our model – like all models – is imperfect. Hence, before trying to explain the insignificant exchange rate risk effect, it is important to examine its robustness to possible model imperfections.

The first support for robustness is that the estimation results are fairly similar across the six export flows (see table 4) and across the six exchange rate volatility equations (table 2). As argued in the previous subsection, the true export processes are probably also somewhat similar. The same holds for the exchange rate volatility series, because they have a common dollar component. Therefore, we regard the consistency of our
estimates across the six models as positive.

Second, though the Poisson lags (7) have several appealing properties, we also estimate the model with polynomial lags for $\beta_{yl}$, $\beta_{El}$ and $\beta_{Vl}$ in export equation (6); see the notes to table 5 for details on the polynomials. The insignificance of the estimates for $\beta_V$ is robust to this, as appears from the insignificant t-values for $\beta_V$ on the top row of table 5. In addition, the lags with the maximal effects are (1,26), (5,10), (1,10), (2,10), (1,13) and (8,12), which are in line with the $(\lambda_y, \lambda_{EV})$ pairs obtained from the Poisson structure. In particular, the maximal effect of the exchange rate also occurs after about one year.

Third, we check the restriction $\lambda_E = \lambda_V$ that the Poisson lags for the exchange rate expectation and risk are equal (apart from scale), as suggested by our economic model. Table 5 shows that likelihood ratio tests do not reject it. Moreover, even if it is not imposed, the estimates for $\beta_V$ remain insignificant.

The fourth issue concerns the AR risk measure (13). The GARCH approach (10) instead of AR yields a constant risk measure, so that it is impossible to find an effect of risk on trade using time series data due to a lack of identification. Using the moving variance measure (9) leads to four significant t-values for $\beta_V$. Two of them are positive and two are negative, so that there is no consistent effect of risk on export. Given the problematic explanatory power of the moving variance for future volatility (table 3), we do not view the change in significance of $\beta_V$ as a sign of non-robustness of our model, but rather as another indication of the problematic quality of the moving variance as a measure of risk in trade equations.

Fifth, table 5 presents tests for autocorrelation in the export residuals and their squares. There is no serious autocorrelation in the levels, and further inspection of the few significant autocorrelation statistics for the squares shows that there is no reason to extend the model.

Finally, we test the zero correlation between the export and volatility disturbances $\varepsilon_t$ and $\nu_t$, that is, $\sigma_{\varepsilon\nu} = 0$. Unrestricted estimation yields t-values for $\sigma_{\varepsilon\nu}$ that are far from significant. Moreover, the t-values of $\beta_V$ are very similar to the ones assuming zero correlation.

In summary, the results of subsection 5.1 are robust, in particular the insignificance of the effect of real exchange rate risk on trade and the about one year period between a change in the exchange rate and its maximal effect on trade.
5.3 Why is it so Difficult to Find an Effect of Exchange Risk on Exports?

The previous two subsections have shown that the common finding of insignificant effects of exchange rate risk on trade is also relevant for our data and that our methodological improvements do not change that. Hence, given the type of data that are used, it is indeed difficult to find a significant effect.

The insignificance reported above may indicate that there is no effect of risk on exports. As a large number of papers report insignificant results, there is a tendency among researchers towards this view, even though the absence of an effect is in contrast with the public view of a negative effect.

Insignificance, however, does not necessarily imply the absence of an effect. Even if there is an effect in reality, this effect may be hard to find with particular research approaches.

We argue that this is presumably the case. For that, we use two of our empirical results. First, the estimated Poisson parameter $\lambda_{EV}$ (the lag with the maximal exchange rate effect on exports) implies that export decisions are mostly affected by the exchange rate of about one year later (see subsection 5.1). Second, at such a long horizon, the variation of exchange risk over time is fairly small and deviations from average risk are rather short-lived (see the end of subsection 4.4). Since exports exhibit large variation and long-term swings over time, exchange rate risk can explain only a minor part of the development of exports through time. Hence, even if risk affects exports, this effect will be dominated and overshadowed by the effect of unobserved export determinants and random noise on exports. Loosely speaking, risk is too constant to find its effect on exports from time series analyses, no matter whether the true effect is zero or not.

Since the majority of existing papers employ time series data on developed countries, for which risk is presumably also rather constant, we think that the ambiguity of the estimated risk effects is mainly driven by this particular research approach and not necessarily by the absence of an effect.

6 Conclusion

This paper presents an empirical study on the effect of exchange rate risk on exports. Although the common view is that this effect is negative, the existing empirical literature, mainly using time series data of developed countries, reports ambiguous results. The focus of our paper is to explain this contradiction.

To motivate the choice of variables in the econometric export model we develop
an economic model. It extends the standard imperfect substitutes model by explicitly accounting for the time lag between the export decision and the actual trade flow and payment. The model therefore considers traders as forward-looking agents. It implies that not only foreign income and the expected future real exchange rate are important, but also that real exchange rate risk may be relevant for exports.

The econometric model contains two methodological contributions to the trade literature. First, we improve on currently used risk measures by using daily exchange rates to construct more accurate monthly volatilities and then using AR(2) forecasts of these monthly volatilities to compute multi-month-ahead risk. Second, we enhance the dynamic specification of the model by introducing a new and convenient Poisson lag structure for the distributed lag model.

The empirical results are based on monthly bilateral aggregate U.S. exports to the other G7 countries from 1978 to 1996. They demonstrate that, as expected, foreign income affects U.S. exports positively and rather quickly, with the maximal effect occurring after about one quarter. Exports react more slowly to changes in the real exchange rate, as the Poisson lags show that the maximal effect occurs only after about one year. This helps solve the question on the true lag pattern for exchange rates raised by Goldstein and Khan (1985). The expected real exchange rate level has the normal positive effect, but real exchange rate risk has no significant effect.

This last result is in line with previous research. However, we do not consider it as conclusive evidence for the absence of an effect of risk on trade. We have shown that the long-term (about one year) risk that is relevant for goods traders is rather constant over time with only short-term deviations from average risk. Therefore, any time series analysis will most likely lead to an insignificant effect, no matter whether the true effect is zero or not. We thus view the common use of time series data on developed countries as the basic reason why it has been so difficult to find an effect of exchange rate risk on trade.

Our conclusion serves to make researchers aware of this methodological problem. To finally solve the still unsatisfactorily answered question of the effect of exchange risk on trade, we suggest to study the effect using data on countries with much more time variation in exchange rate risk, such as developing countries (as in Arize, Osang and Slottje (2000)). Alternatively, employing cross-sectional variation in exchange risk may be fruitful (De Grauwe and Verfaille (1988)).
References


Table 1: Autocorrelation in monthly real exchange rate volatility

<table>
<thead>
<tr>
<th></th>
<th>U.S. dollar real exchange rate versus currency of</th>
<th>Can</th>
<th>Fra</th>
<th>Ger</th>
<th>Ita</th>
<th>Jap</th>
<th>U.K.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Using monthly data:</td>
<td>$\rho_1$</td>
<td>0.01</td>
<td>0.06</td>
<td>0.11</td>
<td>0.06</td>
<td>0.02</td>
<td>0.12*</td>
</tr>
<tr>
<td></td>
<td>$Q_{10}$</td>
<td>5.41</td>
<td>4.28</td>
<td>6.26</td>
<td>6.21</td>
<td>27.47*</td>
<td>7.48</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[0.06]</td>
<td>[0.06]</td>
<td>[0.06]</td>
<td>[0.06]</td>
<td>[0.06]</td>
<td>[0.06]</td>
</tr>
<tr>
<td>Using daily data:</td>
<td>$\rho_1$</td>
<td>0.24*</td>
<td>0.36*</td>
<td>0.39*</td>
<td>0.35*</td>
<td>0.31*</td>
<td>0.49*</td>
</tr>
<tr>
<td></td>
<td>$Q_{10}$</td>
<td>31.39*</td>
<td>129.73*</td>
<td>111.89*</td>
<td>86.06*</td>
<td>94.34*</td>
<td>217.27*</td>
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<td>[0.00]</td>
<td>[0.00]</td>
<td>[0.00]</td>
<td>[0.00]</td>
<td>[0.00]</td>
</tr>
</tbody>
</table>

Standard errors in parentheses and p-values in square brackets; * is significant at 5% level. The symbol $\rho_1$ denotes the first-order autocorrelation and $Q_{10}$ is the Box-Pierce statistic of order 10.

Table 2: AR(2) estimation results for monthly real exchange rate volatility

<table>
<thead>
<tr>
<th></th>
<th>U.S. dollar real exchange rate versus currency of</th>
<th>Can</th>
<th>Fra</th>
<th>Ger</th>
<th>Ita</th>
<th>Jap</th>
<th>U.K.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>$10^4\mu_v$</td>
<td>2.00*</td>
<td>8.69*</td>
<td>9.39*</td>
<td>8.99*</td>
<td>8.65*</td>
<td>8.36*</td>
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<tr>
<td></td>
<td></td>
<td>(0.14)</td>
<td>(0.78)</td>
<td>(0.75)</td>
<td>(0.98)</td>
<td>(0.66)</td>
<td>(0.91)</td>
</tr>
<tr>
<td>AR coefficients</td>
<td>$\alpha_1$</td>
<td>0.21*</td>
<td>0.31*</td>
<td>0.36*</td>
<td>0.31*</td>
<td>0.29*</td>
<td>0.40*</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.06)</td>
<td>(0.06)</td>
<td>(0.06)</td>
<td>(0.06)</td>
<td>(0.06)</td>
<td>(0.06)</td>
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<tr>
<td></td>
<td>$\alpha_2$</td>
<td>0.11</td>
<td>0.15*</td>
<td>0.09</td>
<td>0.14*</td>
<td>0.10</td>
<td>0.17*</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.06)</td>
<td>(0.06)</td>
<td>(0.06)</td>
<td>(0.06)</td>
<td>(0.06)</td>
<td>(0.06)</td>
</tr>
<tr>
<td>Autocorrelation in residuals</td>
<td>$\rho_1$</td>
<td>-0.00</td>
<td>-0.00</td>
<td>-0.00</td>
<td>-0.01</td>
<td>-0.01</td>
<td>-0.02</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.06)</td>
<td>(0.06)</td>
<td>(0.06)</td>
<td>(0.06)</td>
<td>(0.06)</td>
<td>(0.06)</td>
</tr>
<tr>
<td></td>
<td>$Q_{10}$</td>
<td>5.69</td>
<td>17.79</td>
<td>10.40</td>
<td>8.08</td>
<td>12.98</td>
<td>8.76</td>
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<td></td>
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<td>[0.84]</td>
<td>[0.06]</td>
<td>[0.41]</td>
<td>[0.62]</td>
<td>[0.22]</td>
<td>[0.56]</td>
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Standard errors in parentheses and p-values in square brackets; * is significant at 5% level. The results are the least squares estimates of AR(2) model (12). Definitions of $\rho_1$ and $Q_{10}$: see notes of table 1.
<table>
<thead>
<tr>
<th>Risk measure</th>
<th>Horizon</th>
<th>Can</th>
<th>Fra</th>
<th>Ger</th>
<th>Ita</th>
<th>Jap</th>
<th>U.K.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Moving variance</td>
<td>1 month</td>
<td>-0.12</td>
<td>-0.03</td>
<td>-0.13</td>
<td>-0.08</td>
<td>-0.21</td>
<td>-0.05</td>
</tr>
<tr>
<td></td>
<td>1 year</td>
<td>-1.20</td>
<td>-0.33</td>
<td>-0.88</td>
<td>-0.65</td>
<td>-1.55</td>
<td>-0.76</td>
</tr>
<tr>
<td>GARCH</td>
<td>1 month</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>1 year</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>AR</td>
<td>1 month</td>
<td>0.07</td>
<td>0.15</td>
<td>0.16</td>
<td>0.14</td>
<td>0.11</td>
<td>0.26</td>
</tr>
<tr>
<td></td>
<td>1 year</td>
<td>0.02</td>
<td>0.09</td>
<td>0.07</td>
<td>0.05</td>
<td>0.06</td>
<td>0.11</td>
</tr>
</tbody>
</table>

Explanatory power is defined by $1 - V\{v_{t-l+1,1} - V_{t-l}\{s_t\}\}/V\{v_{t-l+1,1}\}$, where $v_{t-l+1,1} = \sum_{m=1}^l v_{t-l+m}$ is the volatility over the $l$ months from $t - l + 1$ to $t$, and $V_{t-l}\{s_t\}$ is the $l$-months-ahead risk measure. The moving variance measure is (9), GARCH is (10) and AR is (13). The explanatory power of the GARCH measure is zero, because we find no evidence of serial correlation in $(s_t - s_{t-1})^2$ (see table 1), so that the GARCH risk measure is constant for all exchange rates.
Hence, we use a likelihood ratio test for \( \beta \) is equivalent to expectation (8) and risk (13)) and volatility equation (12). The results for the volatility equation are similar to the ones of table 2 and therefore not reported. We also do not report the estimates for the monthly seasonality dummies implicit in the export equation.

The estimated increase in \( \beta_y \) for Canada from 1991 onwards (due to the Free Trade Agreement) is 0.04 (0.006). The estimated extra AR(3) parameter for Canada is 0.25 (0.08). The regressors are centered, so that the estimated \( \beta_0 \) gives real exports for an average month in the sample period.

The significance of the estimates for \( \beta_y \) is based on the cointegration between \( x_t \) and \( y_t^* \). The significance of the estimates for \( \beta_E \) is not based on t-ratios. The reason is that under the null hypothesis \( \beta_E = 0 \) we lose two instead of one degree of freedom. (After all, a Poisson lag structure with free parameters \( \beta_E \) and \( \lambda_E \) is equivalent to a lag structure with free parameters \( \beta_{E1} \) and \( \beta_{E2} \) (same sign) where \( \beta_{E3}, \beta_{E4}, \ldots \) are determined by the Poisson distribution through \( \beta_{E1} \) and \( \beta_{E2} \). Therefore, \( \beta_E = 0 \) is equivalent to \( \beta_{E1} = \beta_{E2} = 0 \), so that the hypothesis \( \beta_E = 0 \) actually imposes two restrictions.) Hence, we use a likelihood ratio test for \( \beta_E = 0 \). The likelihood ratios [p-values] are 6.95 [0.03], 16.51 [0.00], 23.63 [0.00], 19.30 [0.00], 21.77 [0.00]. We thus conclude that the estimates for \( \beta_E \) are significant.

The significance of the estimates for \( \beta_V \), however, can be based on t-ratios. After all, since \( \beta_E \neq 0 \), \( \lambda_{EV} \) is identified irrespective of the value of \( \beta_V \). This implies that \( \beta_V = 0 \) imposes only one restriction, which can be tested with the t-value.

### Table 4: Estimation results for U.S. exports

<table>
<thead>
<tr>
<th></th>
<th>Can</th>
<th>Fra</th>
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<tbody>
<tr>
<td><strong>Constant</strong> ( \beta_0 )</td>
<td>17.34*</td>
<td>15.38*</td>
<td>15.75*</td>
<td>14.96*</td>
<td>16.63*</td>
<td>15.95*</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.01)</td>
<td>(0.01)</td>
<td>(0.01)</td>
<td>(0.01)</td>
<td>(0.01)</td>
</tr>
<tr>
<td><strong>Foreign indust. prod.</strong> ( \beta_y )</td>
<td>1.80*</td>
<td>4.22*</td>
<td>2.27*</td>
<td>1.19*</td>
<td>1.28*</td>
<td>2.49*</td>
</tr>
<tr>
<td></td>
<td>(0.08)</td>
<td>(0.24)</td>
<td>(0.11)</td>
<td>(0.16)</td>
<td>(0.09)</td>
<td>(0.17)</td>
</tr>
<tr>
<td><strong>Lag of max. effect</strong> ( \lambda_y )</td>
<td>2.71</td>
<td>5.62</td>
<td>2.92</td>
<td>2.31</td>
<td>3.43</td>
<td>11.03</td>
</tr>
<tr>
<td></td>
<td>(0.48)</td>
<td>(0.88)</td>
<td>(0.65)</td>
<td>(0.74)</td>
<td>(1.71)</td>
<td>(1.70)</td>
</tr>
<tr>
<td><strong>Expected exc. rate</strong> ( \beta_E )</td>
<td>0.49*</td>
<td>0.50*</td>
<td>0.64*</td>
<td>0.71*</td>
<td>0.95*</td>
<td>0.61*</td>
</tr>
<tr>
<td></td>
<td>(0.18)</td>
<td>(0.10)</td>
<td>(0.06)</td>
<td>(0.11)</td>
<td>(0.10)</td>
<td>(0.11)</td>
</tr>
<tr>
<td><strong>Exchange rate risk</strong> ( \beta_{EV}/100 )</td>
<td>6.44</td>
<td>0.41</td>
<td>-0.01</td>
<td>0.27</td>
<td>0.40</td>
<td>-0.17</td>
</tr>
<tr>
<td></td>
<td>(4.39)</td>
<td>(0.35)</td>
<td>(0.27)</td>
<td>(0.33)</td>
<td>(0.49)</td>
<td>(0.18)</td>
</tr>
<tr>
<td><strong>Lag of max. effect</strong> ( \lambda_{EV} )</td>
<td>17.52</td>
<td>10.50</td>
<td>10.08</td>
<td>8.12</td>
<td>12.67</td>
<td>17.42</td>
</tr>
<tr>
<td></td>
<td>(2.23)</td>
<td>(1.78)</td>
<td>(1.18)</td>
<td>(1.56)</td>
<td>(1.02)</td>
<td>(2.05)</td>
</tr>
<tr>
<td><strong>AR(2) for error</strong> ( \theta_1 )</td>
<td>0.22*</td>
<td>0.27*</td>
<td>0.24*</td>
<td>0.35*</td>
<td>0.45*</td>
<td>0.26*</td>
</tr>
<tr>
<td></td>
<td>(0.06)</td>
<td>(0.07)</td>
<td>(0.08)</td>
<td>(0.08)</td>
<td>(0.08)</td>
<td>(0.06)</td>
</tr>
<tr>
<td><strong>Error variance</strong> 100( \sigma^2 )</td>
<td>0.30</td>
<td>0.65</td>
<td>0.50</td>
<td>0.76</td>
<td>0.36</td>
<td>0.82</td>
</tr>
<tr>
<td></td>
<td>(0.03)</td>
<td>(0.06)</td>
<td>(0.05)</td>
<td>(0.08)</td>
<td>(0.04)</td>
<td>(0.09)</td>
</tr>
<tr>
<td><strong>Log-likelihood</strong></td>
<td>-173</td>
<td>-665</td>
<td>-631</td>
<td>-764</td>
<td>-588</td>
<td>-668</td>
</tr>
</tbody>
</table>

Standard errors in parentheses; * is significantly different from zero at 5% level (the estimates for \( \lambda_y \), \( \lambda_{EV} \) and \( \sigma^2 \) have no asterisk, because the null hypotheses \( \lambda_y = 1 \), \( \lambda_{EV} = 1 \) and \( \sigma^2 = 0 \) are at the boundary of the respective parameter spaces making tests non-standard).
Table 5: Diagnostics of model assumptions and sensitivity of results to model changes

<table>
<thead>
<tr>
<th>Change in model</th>
<th>Test</th>
<th>Can</th>
<th>Fra</th>
<th>Ger</th>
<th>Ita</th>
<th>Jap</th>
<th>U.K.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Polynomial lags</td>
<td>t-val $\beta_V$</td>
<td>1.37</td>
<td>1.64</td>
<td>-0.05</td>
<td>-0.77</td>
<td>0.37</td>
<td>-1.35</td>
</tr>
<tr>
<td>Lag of maximal</td>
<td>lik. ratio</td>
<td>0.72</td>
<td>4.06*</td>
<td>0.05</td>
<td>0.60</td>
<td>3.37</td>
<td>0.33</td>
</tr>
<tr>
<td>effect $\lambda_E \neq \lambda_V$</td>
<td>t-val $\beta_V$</td>
<td>1.47</td>
<td>1.77</td>
<td>-0.20</td>
<td>0.89</td>
<td>1.54</td>
<td>-1.08</td>
</tr>
<tr>
<td>Moving var. risk</td>
<td>t-val $\beta_V$</td>
<td>2.36*</td>
<td>2.19*</td>
<td>-0.92</td>
<td>-2.43*</td>
<td>-7.29*</td>
<td>-0.52</td>
</tr>
<tr>
<td>Autocorr. in</td>
<td>$\rho_1$</td>
<td>0.02</td>
<td>-0.04</td>
<td>-0.01</td>
<td>0.07</td>
<td>-0.02</td>
<td>-0.02</td>
</tr>
<tr>
<td>export error</td>
<td></td>
<td>(0.07)</td>
<td>(0.07)</td>
<td>(0.07)</td>
<td>(0.07)</td>
<td>(0.07)</td>
<td>(0.07)</td>
</tr>
<tr>
<td></td>
<td>$Q_{10}$</td>
<td>10.67</td>
<td>7.41</td>
<td>11.52</td>
<td>27.56*</td>
<td>7.76</td>
<td>12.07</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[0.38]</td>
<td>[0.69]</td>
<td>[0.32]</td>
<td>[0.00]</td>
<td>[0.65]</td>
<td>[0.28]</td>
</tr>
<tr>
<td>Autocorr. in</td>
<td>$\rho_1^s$</td>
<td>0.13*</td>
<td>0.04</td>
<td>0.03</td>
<td>0.02</td>
<td>-0.10</td>
<td>0.11</td>
</tr>
<tr>
<td>sqr. export error</td>
<td></td>
<td>(0.07)</td>
<td>(0.07)</td>
<td>(0.07)</td>
<td>(0.07)</td>
<td>(0.07)</td>
<td>(0.07)</td>
</tr>
<tr>
<td></td>
<td>$Q_{10}^s$</td>
<td>17.87</td>
<td>6.04</td>
<td>5.21</td>
<td>26.99*</td>
<td>18.56*</td>
<td>7.47</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[0.06]</td>
<td>[0.81]</td>
<td>[0.88]</td>
<td>[0.00]</td>
<td>[0.05]</td>
<td>[0.68]</td>
</tr>
<tr>
<td>Corr. export and</td>
<td>t-val $\sigma_{\varepsilon\varepsilon}$</td>
<td>0.71</td>
<td>1.17</td>
<td>-0.85</td>
<td>0.38</td>
<td>-0.02</td>
<td>0.54</td>
</tr>
<tr>
<td>volat. errors</td>
<td>t-val $\beta_V$</td>
<td>1.47</td>
<td>1.20</td>
<td>-0.06</td>
<td>0.74</td>
<td>0.82</td>
<td>-0.94</td>
</tr>
</tbody>
</table>

Standard errors in parentheses and p-values in square brackets; * is significant at 5% level.

For the polynomial lags, the polynomials for $\beta_{yt}$, $\beta_{El}$ and $\beta_{Vl}$ in export equation (6) are all of order three, with support \{1,...,10\} for foreign industrial production (\{1,...,20\} for the U.K.) and \{1,...,30\} for the real exchange rate expectation and risk; we restrict the begin and endpoints $\beta_{y10}, \beta_{E1}, \beta_{E30}$ to zero and use the same polynomial (apart from scale) for both exchange rate moments. Tests do not reject these assumptions. The likelihood ratios for order three versus four are 0.37, 0.15, 3.92, 0.08, 5.53 and 2.27. The likelihood ratios for the begin and endpoint restrictions are 8.17, 5.89, 3.77, 5.10, 6.76 and 2.25. Although they involve the nonstationary industrial production variable, making their distribution presumably non-standard, they do not seem to reject the respective assumptions (the $\chi^2_2$ and $\chi^2_5$ critical values that would normally apply are 5.99 and 11.07, respectively). Definitions of $\rho_1$ and $Q_{10}$: see notes of table 1; $\rho_1^s$ and $Q_{10}^s$ are similarly defined, except that they concern the squared residuals.
Figure 1: Distribution of total effect $\beta$ of regressors on exports over time according to a Poisson($\lambda$) lag structure.
Figure 2: Risk measures for Canadian dollar real exchange rate
Figure 3: Risk measures for Japanese yen real exchange rate