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Abstract

We report on our research and development work in the last decade, which has recently been presented in a PhD thesis (Heck, 2012a) and which generally aimed at improving the contribution of ICT to inquiry-oriented mathematics and science education. We present the research setting and main outcomes of the PhD study, the scope of which was limited to pedagogical and software design perspectives on the use of ICT in quantitative mathematical modeling. We discuss how the study provided on the one hand a deeper understanding of the ways in which ICT can support students in carrying out an inquiry activity and can reduce the gap between research of students and professionals, and how it led on the other hand to more insight in what it takes to develop an integrated computer environment for mathematics and science education.

Introduction

We concur with Hodson (2009) that there are at least three major goals of mathematics, science and technology (MST) education: (1) learning MST, by which we mean the familiarity and understanding of ideas and concepts inherent in these fields; (2) learning about MST, which adopts a much broader view of MST, focusing on the philosophy, history and methodology of these fields; and (3) learning to do MST, by which we mean that the learner gains the ability to engage in and develop expertise in scientific inquiry and problem solving. Our focus was (and still is) on providing students with opportunities to experience how science is enacted, i.e., with authentic science, and in particular on providing students with ICT tools that allow them to act as 'real' scientists. This is how we mainly interpret the authentic nature of practical work of students: Activities in which students do research in much the same way scientists and practitioners do and in which they use high-quality tools that are similar to professional tools, but that have been designed for educational purposes. The intention to approach MST education as a study of scientific practice is more easily stated than implemented in a nationwide MST curriculum. Luckily, the Dutch curriculum reforms of the last two decades in upper level secondary education offer opportunities: Students must build up a portfolio consisting of small practical investigation tasks (4-10 hrs) and one rather large (80 hrs) cross-disciplinary research or design experiment in order to record the progress in their learning of doing science.

Our research activities were mainly exploratory case studies on the student performance and the usability of developed tools in ICT-supported practical investigation tasks in class and in out-of-school research projects. In most of these studies and field experiments, pre-university students carried out quantitative mathematical modeling activities, i.e., they explored mathematical models based on science principles with the support of ICT tools in order to come to grips with natural phenomena and to interpret real data. By real data we mean data collected by students in experiments and secondary data originating from professional empirical research. Students used in their practical work the hard- and software environment Coach (Heck, Kędzierska, & Ellermeijer, 2009). In many activities, students could apply ICT for doing research in a way that resembles research by scientists and practitioners. The students' inquiry activities were amongst others about human locomotion (walking, skipping, running, …) and other subjects in movement science, sports science, quantitative pharmacokinetics, and analysis of digital images and video clips. In the PhD thesis (Heck, 2012a), brief overviews of ten classroom studies and of twenty field experiments and student research projects have been presented; more detailed accounts have been published in scientific journals, teachers' journals, and conference proceedings. In this paper, work on modeling bouncing gaits is used to exemplify the kind of student activities that we developed and evaluated in a research setting. We present the aims and set-up of the study, and we report on the main outcomes.

Research Setting and Methods

Driving questions in our R&D work were (and still are):

1. How can the use of ICT and in particular of an integrated computer learning environment contribute to the realization of challenging, cross-disciplinary practical work of good quality, in which pre-university students can work with real data, apply mathematical methods and techniques in concrete problem situations, improve their mathematical and scientific knowledge and skills, and increase their mathematical and scientific literacy?
2. What integrated tools should the computer learning environment provide for inquiry-oriented mathematics and science education? What are the requirements for the computer learning environment from a mathematical point of view and do they link up with requirements coming from science fields?

In other words, we aimed at (1) better understanding of how, why, and to what extent ICT tools can support students in their learning and practice of scientific inquiry; and (2) more insight in what it takes to develop an integrated computer environment for learning mathematics and science in the context of inquiry-oriented approach, the usability of which is explored within educational practice. Hence, research and development are intertwined. Focus was on students' working with real data and on the design of supportive ICT tools.

The framework for our research and development work was formed by elements of design research (Van den Akker, et al., 2006), case-based design of educational software (Kahn, 2008), frameworks on using multiple representations (Kaput, 1994; Ainsworth, 2008), frameworks on evaluating inquiry activities of students (Gott & Duggan, 1995), and of models of mathematical modeling (Blum, et al., 2007). Looking at design research aspects of our study, it can be characterized as type I design research in the typology of Richey and Nelson (1996): "The product development process used in a particular situation is described and analyzed, and the final product is evaluated." Within the distinction made by Van den Akker (1999, p. 6) between formative and reconstructive studies, the research work belonged to the first category.

From design research we took the perspective that it involves design of an intervention or experiment in the real world, that the output of my research must have practical value to real world users, and that teachers are involved in the research. We carried out three types of case studies:

1. classroom research studies, in which students did practical investigations on the basis of specially designed instructional materials;
2. field experiments, in which ICT innovations were tried out on a small scale and not necessarily in the classroom;
3. usability studies, in which we evaluated the potential of a specific ICT tool or a set of integrated tools in a particular subject or domain, leading to a set of sample activities.

These exploratory case studies served many goals:

- They were meant to gain insight in the needs of secondary school students for doing authentic inquiry work.
- They helped specify requirements for an integrated computer learning environment from a mathematical point of view.
- They served to test the usability and scope of (prototypical) implementations of particular tools for collecting, processing, and analyzing data.
- They gave an impression of the potential of ICT regarding the realization of practical work in which secondary school students are engaged in activities such as experimenting, data collection, and data analysis in much the same way as scientists and practitioners.

We took the iterative process of design research, which includes cycles of innovation and revision. But instead of the common cyclic approach to design and exploration of new teaching and learning routes, we let the case studies be the cycles in our design research as illustrated in Figure 1.

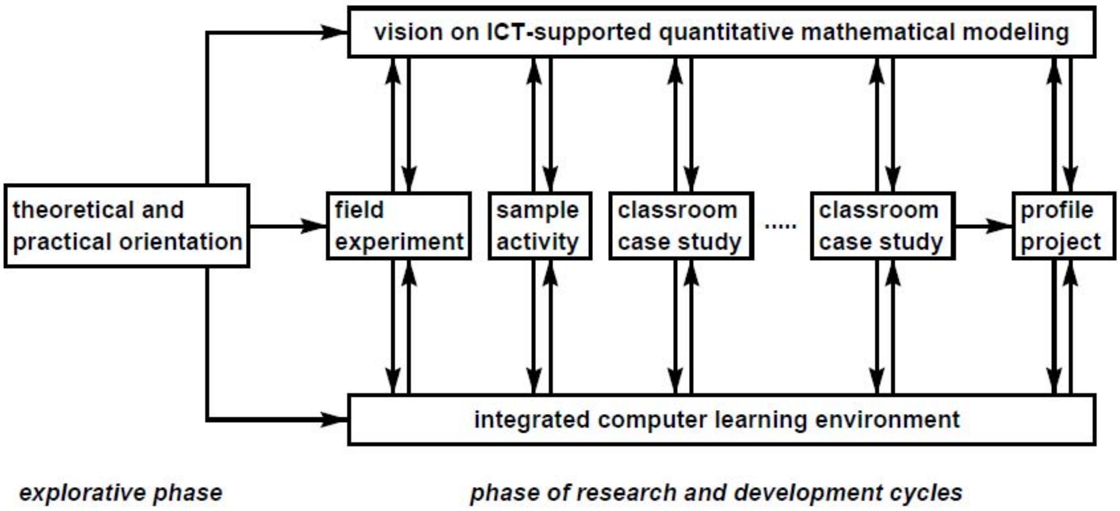

Figure 1. The set-up of the study from a design research point of view.

Classical methods for data collection in design research were applied in the classroom case studies: participatory classroom observation, teacher interviews, questionnaires for students, audio and video recordings of teacher instructions and group discussions of student teams, computer registration of student activities, and collection of worksheets and reports of students.

Tool development also had a cyclic structure, which typically went through phases of planning, development, testing, and release. This spiral model of prototype development, evaluation, and improvement resembled the following user-centered design activities taken from the ISO 9421-210 (2010) standard:

1. understand and specify the context of use;
2. specify the user and organizational requirements;
3. produce design solutions to meet user requirements (typically in the form of prototypes);
4. evaluate the designs against requirements.

When these human-centered design activities are applied to design of education software and when emphasis is put on the understanding and specification of the users, the activities and the context of use, and on the evaluation of the design by case studies in the envisioned context of use, then one speaks of a case-based design of educational software (Khan, 2008). The case-based design process can be split into three parts identified as planning, development, and testing part of the design process. Kahn (2008) linked the following five sequential phases of the design process into these parts:

1. studying pedagogical theory in preparation for the next phase;
2. making a conceptual design of the educational software;
3. constructing the program architecture;
4. developing a prototype;
5. testing of the prototype.

We adopted this framework for developing educational hard- and software. As shown in Figure 2, taken from Kahn (2008, p. 427), case studies inform phase 2 and 4, and teacher' feedback informs phase 2, 4, and 5. The use of this framework for the (re)design of video analysis tool, the data processing and analysis tools, and the graphical system-dynamics modeling tool of Coach 6 are discussed in detail in the thesis

(Heck, 2012a). These tools are in fact part of a versatile toolkit. The theoretical rationale of tool integration is that the use of multiple external representations is crucial for deep understanding of real phenomena and that this process of understanding is promoted when learners are not distracted by technical burdens that could have been avoided by the provision of tools that work well together.

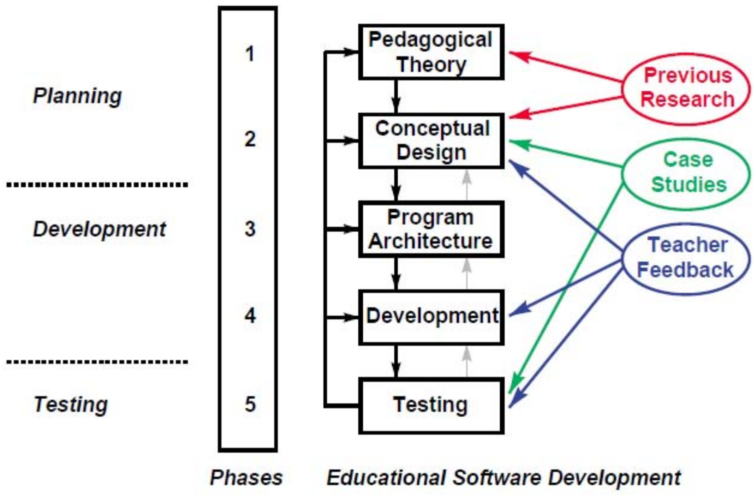

Figure 2. Phases of a case-based design process of educational software.

The Design and Analysis Framework of Student Activities

We structured and analyzed many of the quantitative mathematical modeling activities in the case studies on the basis of a modeling cycle of Blum and Leiß (2005) combined with a classical empirical inquiry cycle. The single framework, shown in Figure 3, resembles the modeling-experimental bi-cycle of Fuchs (2008) as a visual model for a form of the Scientific Method. We also used this framework for elaborating our view on quantitative mathematical modeling competency. It must be kept in mind that in reality the two cycles are not as disjoint as suggested: One can go in any order through both cycles. One can even switch half-way the theoretical modeling cycle, when a mathematical model has been derived, to an experimental cycle to verify whether the model is promising.

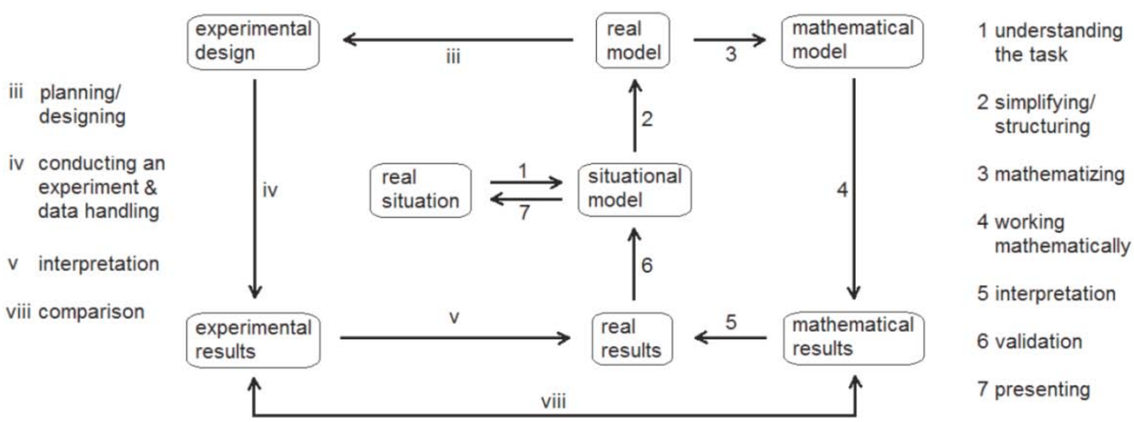

Figure 3. A Framework of quantitative mathematical model.

Overview of a Sample Investigation: Modeling Bouncing Gaits

Mathematical modeling often goes as follows: first one simplifies the situation to such an extent that a simple model can be constructed. Hereafter one evaluates this model, preferably by comparing it with experimental data, and one adapts it if necessary. In the process of evaluation, parameter estimation plays an important role, too. The complexity of finding suitable parameter values must not be underestimated. Adaptation of the model normally means that one makes the model more complicated by taking more factors that cannot really be neglected into account or by undoing some earlier simplifications. One comes into the process of simplifying first and then adding step-by-step more details to the model, with the purpose of matching the model better with reality. It is our belief that by looking at various models of one and the same phenomenon a critical attitude of students is promoted.

An example of this progressive modeling approach is presented here: modeling bouncing gaits of humans such as bouncing on a jumping stick, hopping, and making kangaroo jumps. This case study, in which human body motions were explored through video analysis and computer modeling, also serves the purpose of exemplifying the kind of inquiry activities that we developed for pre-university students. Details can be found in (Heck & Uylings, 2011); here we only sketch how mathematical models using basic biomechanical principles were explored and assessed with the help of experimental data obtained from video measurement. It concerns three motions:

1. Vertical bouncing on a jumping stick;
2. Hopping upward;
3. Hopping forward like a kangaroo.

Highlight was that the model of a planar inverted spring-mass system worked qualitatively and quantitatively well for the complex motions of hopping, skipping, and running at moderate speeds, i.e., in bouncing gaits. These examples of video analysis and modeling activities give a good impression of the potential of the subject of human gait for student practical investigations or projects and as a context for applied mathematics and physics at secondary and undergraduate level. They also illustrate how close one can get to contemporary biomechanical research (cf., Geyer, 2005).

The planar inverted spring-mass system was introduced in the 2008 nationwide secondary physics computer-based examination for pre-university students. The subject was video analysis and modeling of vertical bouncing on a pneumatic jumping stick. The rather clear situation of a periodic motion of a person on a jumping stick can be described well with a model based on simple mathematics and physics: In the aerial phase, only gravity is assumed to play a role in the motion, and during ground contact Hooke's Law of elasticity is applied to the spring deformation. Then, the dynamics of the spring-mass system is determined by a second order differential equation and two initial conditions. The dynamic system can be exactly solved, but also be numerically solved via a graphical, system dynamics-based modeling tool. The left-hand window in Figure 4 shows a graphical model in Coach 6 that numerically solves the system of equations.

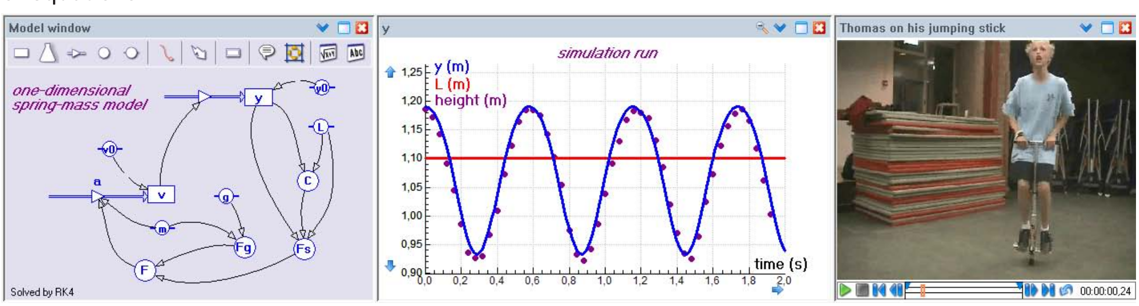

Figure 4. A graphical model implementing the one-dimensional spring-mass model and the results of a simulation run compared with data obtained from a video analysis of a video clip.

The diagram in the middle shows the graph of the computed height and the point plot of the vertical heights measured in a digital video recorded while the student was vertically bouncing on his jumping stick. Simulation results match well with the measured data for suitable parameter values. The measured data suggest that a sinusoidal regression curve would also describe the data quite well, and indeed it does from mathematical point of view. But the spring-mass model is considered better than the experimental modeling via regression because it is based on physics laws. According to this model, a sinusoidal displacement during contact phase is followed by a parabolic aerial phase. This kind of judgment of the quality of a model is what students should learn.

It turns out that the one-dimensional spring-mass model is also a good model for human hopping upward without the support of any device. For the purpose of data collection, students went to the Sports Center of the University of Amsterdam in order to hop upward and forward on a stationary and operational motorized treadmill, respectively. Motions were recorded with a high speed camera at a speed of 300 frames per second so that as many details as needed could be observed and a rather high time resolution was assured (See Figure 5).

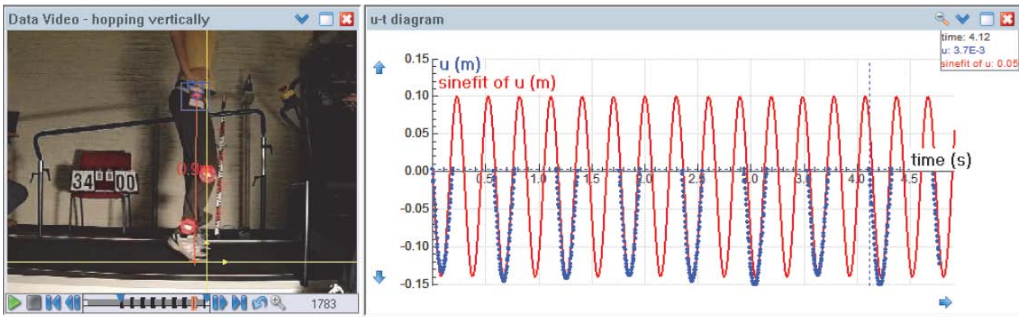

Figure 5. Video analysis of an upward hopping girl on a motorized treadmill that is not turned on.

Under the assumption that only gravitational force and spring force play a role, Newton's second law of motion and Hooke's law of elasticity lead to exactly soluble equations of motion for the height during contact and aerial phase. In the right-hand side of Figure 5, a best sinusoidal function fit for the contact phase is visible. By means of analytical solutions of the system of differential equations, initial values of parameters in the model were determined from more easily measurable quantities and then subsequently improved by regression methods or via computer modeling. As shown in Figure 6, the graphical computer model in Coach 6 can be extended to include the expressions for gravitational energy (with respect to the height taken equal to the spring-leg length), the spring energy, the kinetic energy of the system, and the total energy of the system. A student can then diagrammatically explore the different forms of energy during the motion and examine that conservation of energy holds for the model system. An animation can be used to investigate the effect of parameters.

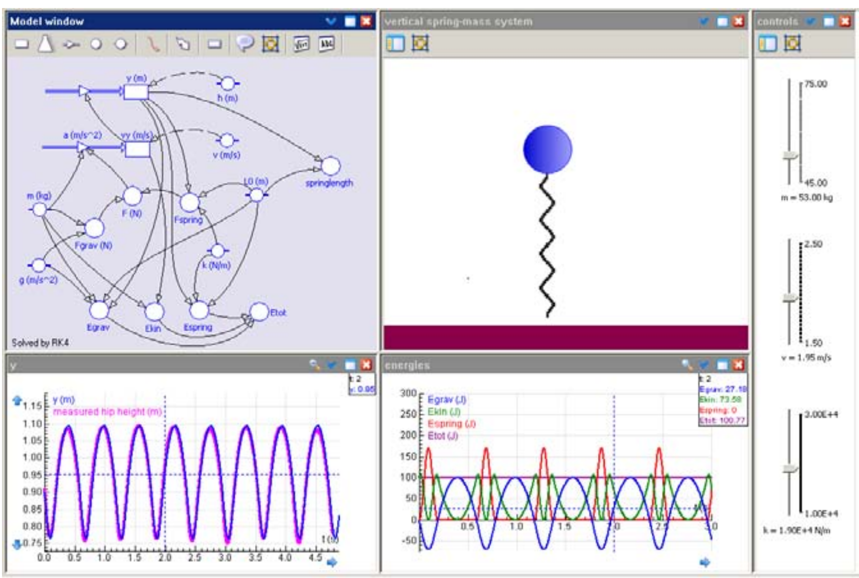

Figure 6. A Screen shot of a Coach activity consisting of a computer model and an animation of a vertical spring-mass system, in which measured hip heights are compared with computed results and energies are computed to examine forms of energy and the law of conservation of energy.

What sets the seal on the work is the application of a planar inverted spring-mass model to human double-legged forward hopping, that is, to a motion resembling kangaroo jumping. The model is now two-dimensional. In comparison with the one-dimensional spring-mass model of upward hopping, we have two new conditions: the leg angle of attack α, when the leg makes ground contact, and the angle of take-off velocity β, when the leg loses ground contact (see Figure 7). Note that $\tan\alpha = -y(0)/x(0)$ and $\tan\beta = v/u$, where u is the horizontal landing speed (equal to the speed of the motorized treadmill when the gait happens on such device) and -v is the vertical landing and take-off speed.

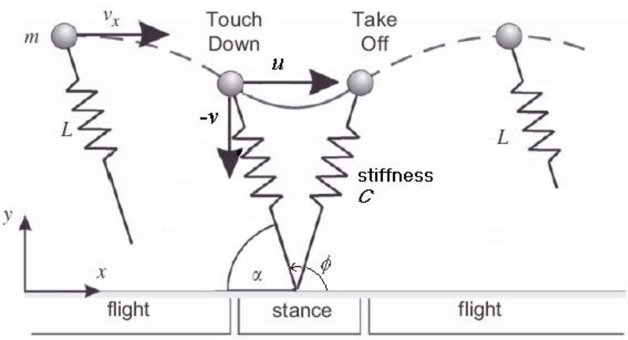

Figure 7. The planar inverted spring-mass model for forward hopping and running.

The fact that the motion is now in two dimensions makes the modeling, both from mathematical and computational point of view, much more difficult and only doable for students with a keen interest and good ability in mathematics and physics. For less gifted students the one-dimensional inverted spring-mass model is already challenging enough. To give an idea of the complexity of the two-dimensional case we mention that, in order to implement the equations of motion in a graphical modeling tool, a solution for moving the coordinate system from one stance point to the other must be found. The fact that the modeling tool of Coach 6 is designed as a hybrid system that combines a classical system dynamics approach with event-based modeling for processes that change abruptly helps solve the implementation problem of a moving frame. Nevertheless, the computer model shown in Figure 8 looks frightful and incomprehensible. This is compensated by the awesome comparison of model results with experimental data, visible in the lower-right corner of the screen shot.

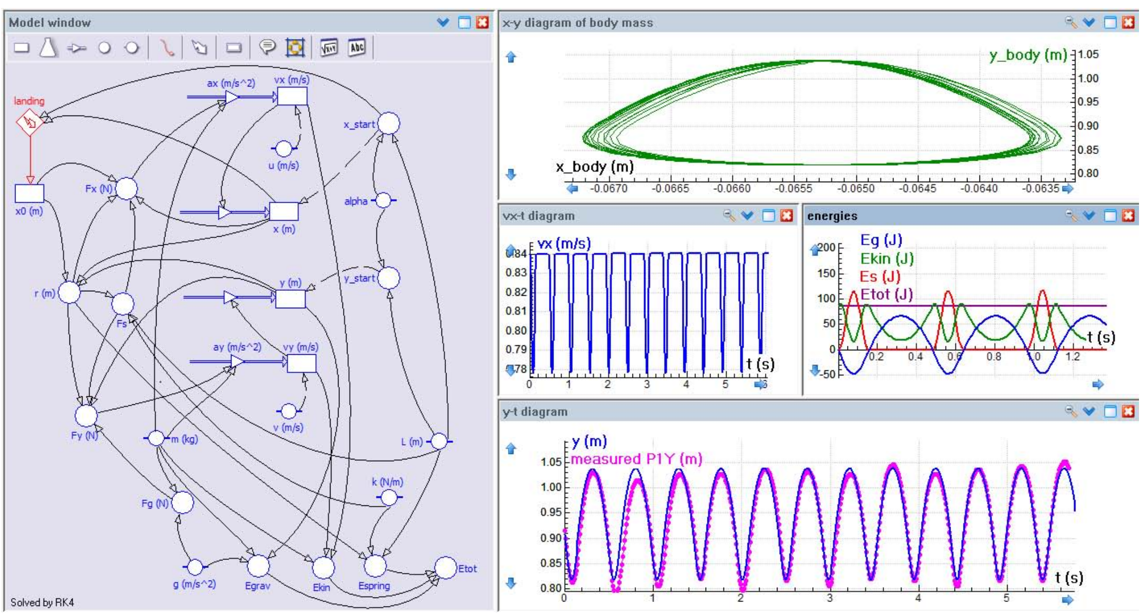

Figure 8. Screen shot of a simulation of the planar inverted spring-mass model of forward hopping like a kangaroo.

Main Outcomes of the R&D Work

What lessons did we learn from developing ICT tools that are integrated in an open, activity-based, multimedia authoring environment for MST education and from exploring their usability in specific practical investigations for upper secondary students, in sample activities, and in usability studies? What answers can we give to the two driving questions listed before? Below, we lift the veil.

Regarding the first question about realizing authentic practical work via ICT, the main role of ICT in investigative work can be summarized as the change of the computer into an instrument that allows students to collect real-time data of good quality, to construct and use computer models of dynamic systems in much the same way as scientists, engineers, and practitioners do, and to compare results from experiments, models, and theory. Furthermore, students can develop and practice through the activities their research abilities. The fact that they must apply their knowledge of mathematics and science in a meaningful way in a concrete context (hopefully) leads at the same time to deepening and consolidation of this knowledge. As a bonus, ICT can bring the real world into mathematics and science education in an attractive way.

The case studies showed that ICT tools help bridge the gap between school science on the one side and the real-world application on the other side. Especially, the motion analysis studies illustrated that upper-level pre-university students, when supported by a suitable versatile computer environment, can work directly with high-quality, real-time data about human body motion in much the same way movement scientists do. In such inquiry activities, students can practice mathematical knowledge and skills such as graph comprehension, numerical differentiation and integration, data processing and analysis, regression, and so forth. Understanding, interpretation, evaluation, and manipulation of data by means of ICT plays a large role in such activities and we found that the participants in the case studies performed surprisingly well (or at least better than expected). In the practical investigations, student can also develop the critical attitude that is necessary for successful modeling of natural phenomena. For this it is very important that the students can compare the results of computer models with real data, preferably collected in an earlier measurement activity. Confrontation of a model with reality turns modeling not only into a fun way of learning, but it also makes it exciting, challenging, and concrete work. Students all seemed to be attracted by this kind of practical work. It turned out that, as continuing work, some of them could autonomously do interesting research projects, obtain results of good quality that were comparable with results published in scientific or professional journals, and get a publication out of their work (e.g., Heck & Van Dongen, 2008; Heck, Knobbe, Nijdam, et al, 2011). Anyway, we experienced with regards to motion analysis that:

- most participants in the classroom case studies were sufficiently able to carry out sub-processes of quantitative mathematical modeling on request;
- not only upper-level pre-university students, but also students in pre-vocational secondary education were able to get an impression of what it takes to do scientific inquiry and to develop inquiry abilities by carrying out a small investigation task at their own educational level using digital video technology.
- the radius of action of video analysis, that is, the range of situations in which a person is able to activate his or her video analysis competency, seems large. Students who learned and practiced video analysis in one situation (e.g., gait analysis) seemed to have no difficulty in applying it in other situations.

The surprisingly quick uptake of video analysis technology and motion analysis by secondary school students does not mean that there were no comments on the quality of the students' work and on the support level of the ICT tools, but at least there were no insuperable obstacles or quality issues that are difficult to improve.

Regarding the second question about a proper set of integrated tools in a computer working environment for inquiry-oriented mathematics and science education, one of the main outcomes was the progress made in the realization of the renewed STOLE concept in a particular computer environment, in our case in Coach 6 (Heck, A., Kędzierska, E. & Ellermeijer, T., 2009). STOLE is an acronym for Scientific and

Technical Open Learning Environment (Ellermeijer, 1988). Originally, it was a vision of a hard- and software environment in which tools for measuring, data processing, and modeling are integrated in a single system that supports students' learning in an inquiry-based approach of science education. Later, in the nineties, a new vision on practical work in science education arose that promoted practical investigations and research projects in which students would be engaged in activities that resemble those of scientists and practitioners (cf., Gott & Duggan, 1995; Wellington, 1998; Woolnough, 2000).The changed technological and pedagogical circumstances asked for a renewal of the STOLE concept, especially for the design phase of student-directed practical work and research projects. In this phase, a student researcher needs information: (s)he must analyze the scientific problem, simulate a model, or look up information about work of others. Thus, the computer is considered more than only a tool to collect, process, and analyze data. It must also give access to information resources and allow the display of information in various formats. The display of information and the inquiry nature of students' activities ask for multiple linked representations in multimedia-based activities. It was also envisioned that it should be possible to fine-tune the whole cycle of doing investigations and design work. This means that a teacher should be able to design a sequence of activities for a particular investigation, and to organize these activities in a project to structure the instructional materials (experiments) for the students.

This renewed STOLE concept was first implemented in Coach Junior, released in 1998. This marked the start of the PhD study and since then the quality of data processing and data analysis has been improved, more tools for video capture and measurement on digital images and video clips have been incorporated (also due to rapid advancement of ubiquitous technology at consumer level), authoring of activities and the structural organization of activities have been upgraded, and new modeling tools like graphical modeling and computer animations have been developed, amongst many other things. Highlights of the digital image and video analysis were: video capture with webcams and high speed cameras, video editing, automated point tracking, correction of perspective distortion, digital image analysis, and simultaneous video recording, sensor-based measurement, and control of experiments by sort programs. New elements of computer-based modeling, simulation, and animation were: numerical algorithms, graphical system dynamics-based modeling, an extension toward easy event-based modeling, an extension of graphical modeling with a process icon (needed, for example, in modeling chemical kinetics graphically [Heck, 2012b]), and model-based animation. Data handling was redesigned by improvements of tools for data smoothing, numerical differentiation, regression analysis, and signal analysis.

All outcomes discussed so far were positive. Although this holds in general, it's not all roses there. There are many, small and large, comments to be made on both student competency and tool design. We mention some of our findings that call for further research and field work to improve MST education. Much more can be read about this in the thesis (Heck, 2012a).

Developers of a versatile computer learning environment that offers integrated tools for mathematics, science and technology are faced with the following two difficult questions: How to deal with

1. the versatility of mathematical language and mathematical notation, and in particular, how to deal with the variability of the concept of variable in mathematics and science?
2. the differences in language between mathematics and science?

The multimedia authoring of student activities helps instructors and designers of instructional materials to make their own choices. However, the two questions remain puzzling. The interested reader is referred to (Heck, 2001; Ellermeijer & Heck, 2002) for a thorough discussion of these issues.

The students who participated in the case studies had difficulties at all levels of graph comprehension in the framework of Curcio (1987) and Shaughnessy (2007), except the first level of reading the data. At the level of reading between and beyond data, students had difficulties in interpreting and reasoning with unfamiliar graphs and graphs of derived quantities in terms of the real-world context from which the graphs originated. In video analysis activities, many a student did not autonomously use the video scrubbing technique to link graph features with motion events, as if they had forgotten about this feature of the computer learning environment. Reading behind data was difficult, but not unreachable for pre-university students.

Spreadsheet-based case studies about survival analysis and data handling of weather data illustrated this. In cases where it is less obvious that a global trend and a superimposed function can be separated, students seemed to neglect the possibility of a regression model consisting of a sum of mathematical functions. We also noted in some classroom case studies that students tended to stick to a global view in data fitting and did not autonomously consider the idea of taking a component-wise view in data fitting.

In the case studies it was often observed in class and noticed in students' written reports that the students had rather weak algebraic skills and lacked confidence in using mathematical formulas. We had the strong impression that the so-called algebraic expectation (Pierce & Stacey, 2004) of the students, which is the thinking that allows a student to monitor working with mathematical formulas, was underdeveloped, hindered them in their work, and led in some cases to a behavior of guessing a formula without giving it much thought. We also could not close our eyes for students' difficulties associated with using multiple representations. The cognitive load is definitely enlarged when multiple representations come into play and it has been reported in many research studies (cf., Ainsworth, 2008) that students find retrieving information from representations, moving between and within representations, and coming up with appropriate representations difficult. The students who participated in the classroom case studies were no exception. Lack of graph sense and representational fluency (meaning, the ability to interpret and construct various disciplinary representations, and the skills to move between representations appropriately) seemed to hinder students in extracting all information that was intrinsically available in several linked representations and in evaluating the quality of their experimental work. Yet, it is important that students learn to work effectively with multiple representations. The underlying ideas of having multiple, dynamically linked representations available in the computer learning environment for inquiry-oriented MST education are not only that it reflects scientific practice, but also that

- it illuminates the meaning of actions in one representation by exhibiting their consequences in another representation;
- the number of ways to come to a solution of a problem increases;
- understanding of a phenomenon, a problem, or a concept is refined the more representations one can interact with;
- It supports the construction of deeper understanding when students relate those representations to identify strengths and weaknesses of particular representations and shared invariant features of all representations in use.

Conclusions

The presented research and development was about the role of ICT in secondary mathematics and science education. In many school projects, students could apply ICT for doing research in a way that resembles research by scientists and practitioners. This has resulted in better understanding of the ways in which ICT can support students in carrying out a research project. In addition, more insight was obtained in what it takes to further develop an integrated computer environment for mathematics and science education. The students used in their research the software and hardware environment Coach (Heck, Kędzierska, & Ellermeijer, 2009). The student projects were amongst others about human locomotion (walking, skipping, running, …) and other subjects in movement science, quantitative pharmacokinetics, and analysis of digital images and video clips. It turned out that students could autonomously do interesting research and obtain results of good quality that are comparable with results published in scientific or professional journals. Figure 9, taken from a publication in a journal for physics teachers co-authored by a secondary school student (Heck & van Dongen, 2008), shows an experimental setting in which Coach has been used to do simultaneously a video recording, a sensor-based measurement of muscle activity via the surface EMG method, and data processing. It illustrates that ICT can help can bridge the gap between school science and real science.

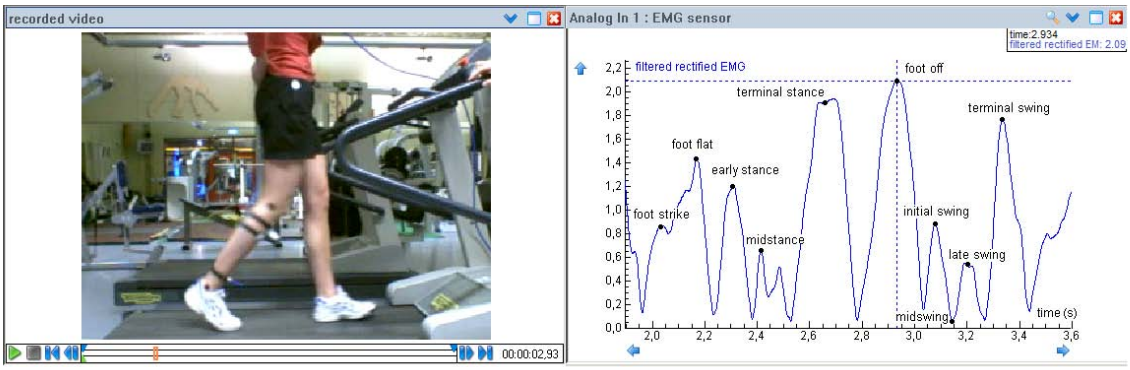

Figure 9. Screen shot of a simultaneous video recording and measurement of the EMG signal of the gastrocnemius for normal walking, followed by data processing.

It is clear that research and development never stops. An obvious continuation of the presented work is educational research on learning trajectories for introducing quantitative mathematical modeling at secondary school level. ICT tools undergo continuous improvement and extension: For example, modern computer vision techniques could be incorporated in the current educational video analysis tools.

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