Comment on ‘‘Problem of light diffusion in strongly scattering media’’

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Comment on "Problem of Light Diffusion in Strongly Scattering Media"

The paper of Barabanenkov and Ozrin [1] (BO) claims that the transport velocity v_E, defined [2] as the velocity appearing in the diffusion constant $D = v_E \ell/3$, is equal to the phase velocity v_p in the case of scalar-wave diffusion. This outcome conflicts with both calculations [2–4] and independent measurements [2, 5, 6]. Our theory predicts that $v_E \ll v_p$ near resonances and explains the reported value $v_E/v_p \approx 0.25$. In this Comment we show that the conclusion of BO, $v_E = v_p + \mathcal{O}(n^2)$, is due to an algebraic error in their calculation. In correcting the error in their derivation, we recover our results.

We start with the expression for v_E at frequency E, $1/v_E = v_p(1 + a)$; $c_0 = 1$. Equation (12) of BO states that

$$a(E) \sum_{\mathbf{p}} \operatorname{Im} G(E,p) = \sum_{\mathbf{p}} \left(\frac{p^2}{E^2} - 1\right) \operatorname{Im} G(E,p) = -\frac{1}{E^2} \operatorname{Im} \sum_{\mathbf{p}} \frac{\Sigma(E,p)}{E^2 - p^2 - \Sigma(E,p)} . \tag{1}$$

($\sum_{\mathbf{p}} = \int d^3\mathbf{p}/(2\pi)^3$, $G(E,p) = \left[E^2 - p^2 - \Sigma(E,p)\right]^{-1}$, $\Sigma(E,p) = n\, t_{\mathbf{pp}}(E) + \mathcal{O}(n^2)$ *retarded* Green's function and self-energy; BO uses *advanced* Green's functions.) For low density n, Eq. (1) reduces to ($\mathbf{k} \equiv E\,\hat{\mathbf{k}}$)

$$a(E) = \frac{4\pi}{E^3} \operatorname{Im} \sum_{\mathbf{p}} \frac{n\, t_{\mathbf{pp}}(E)}{E^2 - p^2 + i0} = -\frac{n}{E^2} \left[\frac{4\pi}{E} \mathcal{P} \sum_{\mathbf{p}} \frac{-\operatorname{Im} t_{\mathbf{pp}}(E)}{E^2 - p^2} + \int \frac{d\hat{\mathbf{k}}}{4\pi} \operatorname{Re} t_{\mathbf{kk}}(E)\right] . \tag{2}$$

The principal value ($\mathcal{P}$) is missed in BO's evaluation of Eq. (1). The second, on-shell term in Eq. (2) *alone* would indeed yield $1 + a = 1/v_p^2$ and $v_E = v_p$. We now show that Eq. (2) in its full extent leads to $v_E \ll v_p$ near resonances. To this end we first prove that

$$\frac{4\pi}{E} \mathcal{P} \sum_{\mathbf{p}} \frac{-\operatorname{Im} t_{\mathbf{pp}}(E)}{E^2 - p^2} + \int \frac{d\hat{\mathbf{k}}}{4\pi} \operatorname{Re} t_{\mathbf{kk}}(E) = \int \frac{d\hat{\mathbf{k}}}{4\pi} \int d\mathbf{r}\, V(\mathbf{r})\, |\psi_{\mathbf{k}}^+(\mathbf{r})|^2 \equiv \int \frac{d\hat{\mathbf{k}}}{4\pi} \langle \psi_{\mathbf{k}}^+ | V | \psi_{\mathbf{k}}^+ \rangle . \tag{3}$$

$|\psi_{\mathbf{k}}^+\rangle$ denotes the one-scatterer eigenfunction at eigenvalue $\mathbf{k}$, $V(\mathbf{r}) \equiv [1 - \varepsilon(\mathbf{r})]E^2$ the "potential." From scattering theory one knows that $V|\psi_{\mathbf{k}}^+\rangle = t(E)\,|\mathbf{k}\rangle$ and $|\psi_{\mathbf{k}}^+\rangle = [1 + G_0(E)\,t(E)]\,|\mathbf{k}\rangle$. It follows that

$$\langle \psi_{\mathbf{k}}^+ | V | \psi_{\mathbf{k}}^+ \rangle = \langle \psi_{\mathbf{k}}^+ | t(E) | \mathbf{k} \rangle = \sum_{\mathbf{p}} \langle \psi_{\mathbf{k}}^+ | \mathbf{p} \rangle\, t_{\mathbf{pk}}(E) = t_{\mathbf{kk}}(E) + \sum_{\mathbf{p}} \frac{|t_{\mathbf{pk}}(E)|^2}{E^2 - p^2 - i0} . \tag{4}$$

Using a generalized optical theorem [reproduced from Eq. (9) in BO for $\omega, \mathbf{q} = 0, n \to 0$], $-\operatorname{Im} t_{\mathbf{pp}}(E)/E = \int d\hat{\mathbf{k}}\, |t_{\mathbf{pk}}(E)|^2/(4\pi)^2$, the imaginary part of (4) cancels and Eq. (3) follows. Hence, by Eq. (2),

$$a(E) = n \int \frac{d\hat{\mathbf{k}}}{4\pi} \langle \psi_{\mathbf{k}}^+ | \varepsilon(\mathbf{r}) - 1 | \psi_{\mathbf{k}}^+ \rangle , \tag{5}$$

which is recognized as the potential energy of the dielectric particles. Near resonances is $1 + a \gg 1/v_p^2$ and so $v_E/v_p \ll 1$. The link of $a(E)$ with stored energy is equivalent to results obtained by Loudon [7]. It is also closely related to the *total* energy W in the particles [2]: The time needed to fill particles with energy W was seen to provide an almost exact solution. The excellent approximation $|\psi_{\mathbf{k}}^+|^2 \varepsilon(\mathbf{r}) - |\psi_{\mathbf{k}}^+|^2 \approx |\psi_{\mathbf{k}}^+|^2 \varepsilon(\mathbf{r}) - 1$ in Eq. (5) replaces $a(E)$ by a formula for the phase-delay time [8], which was used to deal with vector waves [2, 4] .

This corrected version of the BO theory thus confirms our statement that the transport velocity is considerably smaller than the phase velocity in strongly scattering media.

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