



**UvA-DARE (Digital Academic Repository)**

**Comment on “Problem of light diffusion in strongly scattering media”**

van Tiggelen, B.A.; Lagendijk, A.; Tip, A.

*Published in:*  
Physical Review Letters

*DOI:*  
[10.1103/PhysRevLett.71.1284](https://doi.org/10.1103/PhysRevLett.71.1284)

[Link to publication](#)

*Citation for published version (APA):*

van Tiggelen, B. A., Lagendijk, A., & Tip, A. (1993). Comment on “Problem of light diffusion in strongly scattering media”. *Physical Review Letters*, 71(8), 1284-1284. DOI: 10.1103/PhysRevLett.71.1284

**General rights**

It is not permitted to download or to forward/distribute the text or part of it without the consent of the author(s) and/or copyright holder(s), other than for strictly personal, individual use, unless the work is under an open content license (like Creative Commons).

**Disclaimer/Complaints regulations**

If you believe that digital publication of certain material infringes any of your rights or (privacy) interests, please let the Library know, stating your reasons. In case of a legitimate complaint, the Library will make the material inaccessible and/or remove it from the website. Please Ask the Library: <http://uba.uva.nl/en/contact>, or a letter to: Library of the University of Amsterdam, Secretariat, Singel 425, 1012 WP Amsterdam, The Netherlands. You will be contacted as soon as possible.

## Comment on "Problem of Light Diffusion in Strongly Scattering Media"

The paper of Barabanenkov and Ozrin [1] (BO) claims that the transport velocity  $v_E$ , defined [2] as the velocity appearing in the diffusion constant  $D = v_E \ell / 3$ , is equal to the phase velocity  $v_p$  in the case of scalar-wave diffusion. This outcome conflicts with both calculations [2-4] and independent measurements [2, 5, 6]. Our theory predicts that  $v_E \ll v_p$  near resonances and explains the reported value  $v_E/v_p \approx 0.25$ . In this Comment we show that the conclusion of BO,  $v_E = v_p + \mathcal{O}(n^2)$ , is due to an algebraic error in their calculation. In correcting the

error in their derivation, we recover our results.

We start with the expression for  $v_E$  at frequency  $E$ ,  $1/v_E = v_p(1+a)$ ;  $c_0 = 1$ . Equation (12) of BO states that

$$a(E) \sum_{\mathbf{p}} \text{Im} G(E, \mathbf{p}) = \sum_{\mathbf{p}} \left( \frac{p^2}{E^2} - 1 \right) \text{Im} G(E, \mathbf{p}) \\ = -\frac{1}{E^2} \text{Im} \sum_{\mathbf{p}} \frac{\Sigma(E, \mathbf{p})}{E^2 - p^2 - \Sigma(E, \mathbf{p})}. \quad (1)$$

( $\sum_{\mathbf{p}} = \int d^3\mathbf{p}/(2\pi)^3$ ,  $G(E, \mathbf{p}) = [E^2 - p^2 - \Sigma(E, \mathbf{p})]^{-1}$ ,  $\Sigma(E, \mathbf{p}) = n t_{\mathbf{p}\mathbf{p}}(E) + \mathcal{O}(n^2)$  retarded Green's function and self-energy; BO uses advanced Green's functions.) For low density  $n$ , Eq. (1) reduces to ( $\mathbf{k} \equiv E \hat{\mathbf{k}}$ )

$$a(E) = \frac{4\pi}{E^3} \text{Im} \sum_{\mathbf{p}} \frac{n t_{\mathbf{p}\mathbf{p}}(E)}{E^2 - p^2 + i0} = -\frac{n}{E^2} \left[ \frac{4\pi}{E} \mathcal{P} \sum_{\mathbf{p}} \frac{-\text{Im} t_{\mathbf{p}\mathbf{p}}(E)}{E^2 - p^2} + \int \frac{d\hat{\mathbf{k}}}{4\pi} \text{Re} t_{\mathbf{k}\mathbf{k}}(E) \right]. \quad (2)$$

The principal value ( $\mathcal{P}$ ) is missed in BO's evaluation of Eq. (1). The second, on-shell term in Eq. (2) alone would indeed yield  $1+a = 1/v_p^2$  and  $v_E = v_p$ . We now show that Eq. (2) in its full extent leads to  $v_E \ll v_p$  near resonances. To this end we first prove that

$$\frac{4\pi}{E} \mathcal{P} \sum_{\mathbf{p}} \frac{-\text{Im} t_{\mathbf{p}\mathbf{p}}(E)}{E^2 - p^2} + \int \frac{d\hat{\mathbf{k}}}{4\pi} \text{Re} t_{\mathbf{k}\mathbf{k}}(E) \\ = \int \frac{d\hat{\mathbf{k}}}{4\pi} \int d\mathbf{r} V(\mathbf{r}) |\psi_{\mathbf{k}}^+(\mathbf{r})|^2 \equiv \int \frac{d\hat{\mathbf{k}}}{4\pi} \langle \psi_{\mathbf{k}}^+ | V | \psi_{\mathbf{k}}^+ \rangle. \quad (3)$$

$|\psi_{\mathbf{k}}^+\rangle$  denotes the one-scatterer eigenfunction at eigenvalue  $\mathbf{k}$ ,  $V(\mathbf{r}) \equiv [1 - \varepsilon(\mathbf{r})]E^2$  the "potential." From scattering theory one knows that  $V |\psi_{\mathbf{k}}^+\rangle = t(E) |\mathbf{k}\rangle$  and  $|\psi_{\mathbf{k}}^+\rangle = [1 + G_0(E) t(E)] |\mathbf{k}\rangle$ . It follows that

$$\langle \psi_{\mathbf{k}}^+ | V | \psi_{\mathbf{k}}^+ \rangle = \langle \psi_{\mathbf{k}}^+ | t(E) |\mathbf{k}\rangle = \sum_{\mathbf{p}} \langle \psi_{\mathbf{k}}^+ | \mathbf{p} \rangle t_{\mathbf{p}\mathbf{k}}(E) \\ = t_{\mathbf{k}\mathbf{k}}(E) + \sum_{\mathbf{p}} \frac{|t_{\mathbf{p}\mathbf{k}}(E)|^2}{E^2 - p^2 - i0}. \quad (4)$$

Using a generalized optical theorem [reproduced from Eq. (9) in BO for  $\omega, \mathbf{q} = 0, n \rightarrow 0$ ],  $-\text{Im} t_{\mathbf{p}\mathbf{p}}(E)/E = \int d\hat{\mathbf{k}} |t_{\mathbf{p}\mathbf{k}}(E)|^2/(4\pi)^2$ , the imaginary part of (4) cancels and Eq. (3) follows. Hence, by Eq. (2),

$$a(E) = n \int \frac{d\hat{\mathbf{k}}}{4\pi} \langle \psi_{\mathbf{k}}^+ | \varepsilon(\mathbf{r}) - 1 | \psi_{\mathbf{k}}^+ \rangle, \quad (5)$$

which is recognized as the potential energy of the dielectric particles. Near resonances is  $1+a \gg 1/v_p^2$  and so  $v_E/v_p \ll 1$ . The link of  $a(E)$  with stored energy is equivalent to results obtained by Loudon [7]. It is also closely related to the total energy  $W$  in the particles [2]: The time needed to fill particles with energy  $W$  was seen to

provide an almost exact solution. The excellent approximation  $|\psi_{\mathbf{k}}^+|^2 \varepsilon(\mathbf{r}) - |\psi_{\mathbf{k}}^+|^2 \approx |\psi_{\mathbf{k}}^+|^2 \varepsilon(\mathbf{r}) - 1$  in Eq. (5) replaces  $a(E)$  by a formula for the phase-delay time [8], which was used to deal with vector waves [2, 4].

This corrected version of the BO theory thus confirms our statement that the transport velocity is considerably smaller than the phase velocity in strongly scattering media.

This work was part of the research program of FOM, and was made possible by financial support of NWO.

B. A. van Tiggelen,<sup>1</sup> A. Lagendijk,<sup>1,2</sup> and A. Tip<sup>1</sup>

<sup>1</sup>FOM-Institute for Atomic and Molecular Physics  
Kruislaan 407

1098 SJ Amsterdam, The Netherlands

<sup>2</sup>Van der Waals-Zeeman Laboratory

University of Amsterdam

Valckenierstraat 65-67

1018 XE Amsterdam, The Netherlands

Received 3 February 1993

PACS numbers: 42.25.Fx, 78.20.Dj

- [1] Yu. N. Barabanenkov and V. Ozrin, Phys. Rev. Lett. **69**, 1364 (1992).
- [2] M.P. van Albada, B.A. van Tiggelen, A. Lagendijk, and A. Tip, Phys. Rev. Lett. **66**, 3132 (1991).
- [3] B.A. van Tiggelen, A. Lagendijk, M.P. van Albada, and A. Tip, Phys. Rev. B **45**, 12 233 (1992).
- [4] E. Kogan and M. Kaveh, Phys. Rev. B **46**, 10 636 (1992).
- [5] J.F. de Boer, M.P. van Albada, and A. Lagendijk, Phys. Rev. B **45**, 658 (1992).
- [6] N. Garcia, A.Z. Genack, and A.A. Lysyanski, Phys. Rev. B **46**, 14 475 (1992).
- [7] R. Loudon, J. Phys. A **3**, 233 (1970).
- [8] J.M. Jauch, K.B. Sinha, and B.N. Misra, Helv. Phys. Acta **45**, 398 (1972).