Optical antennas on substrates and waveguides
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Citation for published version (APA):
Bernal Arango, F. (2014). Optical antennas on substrates and waveguides

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Introduction

1.1 Light and nano antennas

Human experience and perception is largely determined by the visual observation of our external universe. This perception has been argued to be a projection of the complex external reality, projection which is itself interpreted and complemented in our mind to fit in, or extend, our own mental model of the universe [1, 2]. In all cases these visual observations are unequivocally mediated by the transmissions and reflections of light by the objects and persons in our surroundings, which makes light one of the most important mediators for the acquisition of information in our life. The projections and reflections caused by sun light has shaped our understanding of the world, and since the invention of artificial light sources and light detectors, these optical means for gathering information have allowed us to see the intricate relations between the smallest of things with microscopes [3, 4], and with photomultiplier tubes and scintillators in high energy experiments [5–7] as well as the biggest of the objects [8, 9] in the universe with telescopes.

The evident vast importance that light has had in human history becomes even more apparent when we realize that two of the most important challenges that humanity is facing nowadays also have to do in one or another way with light. First, we are currently living in a moment of human history where the effects of global warming are starting to touch everyone’s experience and perception, since according to international panels of climate scientist [10] humankind’s consumption of energy, and in particular fossil

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*This idea of perception and mental models has a long standing history. In western culture we can see a clear early example of such reasoning in the text The Republic with the “The allegory of the cave” by Plato (347bc)*
fuels, has given rise firstly to a systematic global warming and secondly to depletion of available resources. Through general societal debate about IPCC findings [10], the EU system of CO$_2$ taxation [11], or the increasing shift of subsidy and research funding to renewable energy sources as consigned in the Europe 2020 plan [12], this challenge is starting to be felt throughout western society. Projections, such as those of the IPCC argue that the entire earth eco-system with human life as a marginal contributor has enjoyed a delicate balance of energy with the continuous input of energy from the sun as well as the earth-internal radiogenic sources [13] over the last millennia. This balance is currently shifting owing to the vast rise by $\sim 365\%$ in human population [14] and per capita energy consumption in the last century. Rather than acquiring the required energy from the finite supply of fossil fuels, which accounted for $\sim 85\%$ of annual energy in the last decade [13, 15], an indefinitely sustainable energy balance would require that energy is harvested directly from the sun, in all different forms in which it can be found in nature, e.g. in form of wind energy, in form of harvesting energy stored in the earth’s hydrosphere, and through photochemical or photovoltaic conversion of light into chemical reactions or electricity.

As a second challenge, we are living in an information revolution where people all over the globe are connected, can process vast amounts of information and can transmit this information in a matter of seconds from one side of the planet to the other. This revolution has been possible thanks to the conception and development of faster and more efficient computers as well as networks of optical fibers, mobile phone antennas and satellites. Nevertheless electronic chip technology is arriving to a bandwidth and power consumption bottleneck. The recent improvements in bandwidth in electronic interconnects have only been possible at the price of increasing dramatically the circuit power consumption [16]. It is expected that by 2015 interconnections of the billions of transistors in a modern processor just cannot be miniaturized any further, as stated by the International Technology Roadmap for Semiconductors (ITRS) [17]. Even forgetting for a moment the issues that photolithography has to solve below 14 nm feature sizes [17], it is a fact that shrinking the copper wires further will be accompanied by an increase in resistance, power consumption and even breakdown of the functionalities of the circuits themselves due to electro migration and material limits which will impose strong limits to this miniaturization [17].

For both challenges possible solutions are sought in the efficient manipulation and control of light. The energy problem could be solved by increasing the absorption of photons by thin and cheap solar cells [18]. On the other hand the transmission and manipulation of information in our computers could be solved by using chips with dielectric waveguides [19] and optical devices, which transmit the information in and in between chips, as well as optically manipulate, switch, direct and control the information carried by the light in waveguide modes.

Both for harvesting photons in a solar cell, and for manipulating the emission, detection and propagation of light on a photonic chip, a major fundamental challenge is that the interaction of light with matter is weak. For instance the strong interaction of electrons and matter ensures that for an electron with an energy of 1 eV the average attenuation length through which this electron can travel in air before getting absorbed...
1.1 Light and nano antennas

by a molecule is about $6 \mu m$\(^{\dagger}\) while for a photon in the visible range this attenuation length is 30 to 190 km for violet (410 nm) and red (650 nm) photons respectively [21]. Nevertheless, since around 10 years ago the currently big field of nano-photonics and plasmonics has shown a possible path to overcome this weak interaction by using optical nano-antennas [22].

Optical nano-antennas are antennas for light. These antennas work in the optical to infrared frequency spectrum and their sizes are on the order of hundreds of nanometers [23–25]. They are capable of converting localized energy into propagating radiation in free space [23, 25] as well as in waveguides as will be seen later in this thesis. Also, these antennas can do the opposite which is converting incident radiation into sub-diffraction-limit localized foci of high energy density [23]. Fig. 1.1 shows a sketch of the operation of these sort of antennas. The materials used for optical

\[ \log(Rp) = -5.1 + 1.358x + 0.215x^2 - 0.043x^3 \]

where $x = \log(E(keV))$ and using a density for air $\rho = 0.00129 \text{ g cm}^{-3}$ (see Ref. [20])

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\(\dagger\)This distance is an approximation found by using the electron range function in (g cm\(^{-2}\)) $log(Rp) = -5.1 + 1.358x + 0.215x^2 - 0.043x^3$ where $x = log(E(keV))$ and using a density for air $\rho = 0.00129 \text{ g cm}^{-3}$ (see Ref. [20])
antennas are varied. Part of the field of optical antennas specializes in using dielectric antennas. With these dielectric materials light matter interaction can be enhanced by increasing the residence time of photons in resonant structures. Another part of the field uses metallic antennas, in a subfield called plasmonics, for which light matter interaction is increased by confining strong electromagnetic fields to volumes smaller than the diffraction limit. The light matter interaction enhancements achieved for dielectric as well as for plasmonic nanoantennas can be explained starting from the properties of the dielectric function \( \varepsilon \) for these two class of materials.

Dielectric materials present a positive permittivity \([21]\) \( \Re\{\varepsilon\} > 0 \), usually in fact \( \varepsilon > 1 \) and a \( \Im\{\varepsilon\} \approx 0 \). In case of a homogeneous \( \varepsilon \), the solutions to Maxwell’s equations are plane waves (e.g. \( \mathbf{E}(r) = E_0 e^{i\mathbf{k} \cdot \mathbf{r}} \)) of wave number \( k = |\mathbf{k}| = k_0 \sqrt{\varepsilon \mu} \), where \( k_0 = 2\pi/\lambda \) is the free space wave number and \( \lambda \) is the wavelength in free space. From the expression for the field of a plane wave and the expression for \( k \), it is immediately evident that \( \Im\{\varepsilon\} \) introduces an exponential decay, which is associated with material absorption \([21, 25]\). Due to the small values of \( \Im\{\varepsilon\} \) for dielectric materials the dissipative losses are fairly small. Because of the small losses of dielectrics they have been used to fabricate wavelength-sized objects with strong resonances, as well as ensembles of scatterers like powders and sponges that multiply scatter light \([26]\). A single scatterer can strongly scatter light when it supports a geometric resonance, for instance when a wave fits an integer number of times across the perimeter, or in the bulk of the particle. Archetypical examples are dielectric spheres, array of spheres and cylinders, nanowires and ring resonators \([27–31]\).

For the case of nano antennas made of noble metals, like gold or silver, when electric fields of light drive these plasmonic antennas, the charges in the metal are accelerated in such a way that the movement of charges will create an induced field which prevents the driving field to penetrate the metal. Indeed, evaluating the wave number for a purely real, but negative, \( \varepsilon \), shows an exponential decay length \( \delta = 1/(2k_0 \sqrt{\varepsilon}) \). This decay does not point at absorption, but points at the fact that in metals, electrons tend to rearrange to screen any incident field from penetrating further than the skin depth into the material. This effect can be included in the dielectric function of the metal. The simplest model of a metal dielectric function which only takes into account the movement of free charges is the Drude-Sommerfeld model \([21]\), which supposes the following equation of motion for any free charge in the metal:

\[
m_e \frac{\partial^2 \mathbf{r}}{\partial t^2} + m_e \Gamma \frac{\partial \mathbf{r}}{\partial t} = -e \mathbf{E}_0 e^{i\omega t}.
\]

where \( e \) is the charge of the electron, \( m_e \) is the mass of the electron, \( \mathbf{r} \) is the position of the electron, \( t \) is time, \( \Gamma \) is the damping rate for the movement of the electrons

\[\text{§}\text{Somewhat arbitrarily the field has used exclusively the term Mie resonances only for the resonances supported by dielectric scatterers like the ones just discussed, even though resonances found in Mie solutions are solutions of the electromagnetic equations for sphere and cylindrical scatterers regardless of its material, and thus any resonance in such a scatterer despite its material could be called a Mie resonance.}\]

\[\text{§The decay length or skin depth is easily found by using the intensity of a plane wave in the direction of propagation in the metal } \mathbf{E} \cdot \mathbf{E}^* \text{ when } k \text{ is purely imaginary from } k = k_0 \sqrt{\varepsilon} \text{ due to the real negative } \varepsilon\]
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(specifically $\Gamma = v_F/\ell$, where $v_F$ is the Fermi velocity, i.e. the fastest velocity of an electron in the metal, and $\ell$ is the electron mean free path) and $E_0$ is the driving harmonic field with frequency $\omega$. The effect of the movement of the driven free charges on the permittivity is included through the volume polarization density of the material $\mathbf{P} = \epsilon_0 \chi \mathbf{E} = -ne\mathbf{r}$ where $n$ is the electron charge density of the material and $\chi$ is the susceptibility of the material [32] which can also be defined as $\chi = \epsilon - 1$. Using these equations and solving Eq. (1.1) we find that for the Drude-Somerfeld model the permittivity as a function of the driving frequency $\omega$ reads:

$$\epsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2 + i\Gamma \omega}$$

(1.2)

where $\omega_p$ is the plasma frequency of the metal defined as $\sqrt{ne^2/m_e \epsilon_0}$. For $\omega < \omega_p$ the real part of $\epsilon$ is negative. For common metals $\omega_p$ is in the UV. For instance, for gold the plasma frequency is equivalent to the frequency of UV light with a wavelength of $\lambda_p = 145$ nm, for silver $\lambda_p = 129$ nm, for platinum $\lambda_p = 241$ nm and finally for aluminium $\lambda_p = 81$ nm. Any wave with a wavelength longer than these plasma frequencies will encounter a medium with a negative permittivity, while light with shorter wavelengths would encounter a dielectric material.

As it was already commented, a negative real part of epsilon impedes penetration of fields inside the material, but at the same time a negative epsilon allows for electric fields with high wave numbers $k > k_0$ to be bound to interfaces between a metal ($\text{Re}\{\epsilon\} < 0$) and a dielectric ($\text{Re}\{\epsilon\} > 0$). As example, we discuss the response of a small metal sphere driven by an incident oscillatory electric field as shown in Fig. 1.2. The electron sea easily follows the electric field, as described by Eq. (1.1) to give rise to an excess negative charge on one side of the sphere, while there is net positive charge on the other side, attributable to the ionic backbone that remains stationary. This positive charge pulls back the electrons giving rise to a resonance. Indeed, this intuition
that charge separation sets up a resonant dipole moment is evident from the so called Rayleigh polarizability,

$$\alpha(\omega) = 4\pi\epsilon_0 a^3 \frac{\epsilon(\omega) - \epsilon_{\text{env}}}{\epsilon(\omega) + 2\epsilon_{\text{env}}}.$$  \hspace{1cm} (1.3)

Where we see that in the scenario when the negative $\epsilon$, stemming from the movement of free charges, reaches values close or equal to $-2\epsilon_{\text{env}}$ this creates a strongly polarizable object. There is one very important effect that stems from the strongly bound electromagnetic mode at the interface between a metal and a dielectric, which is that the mode can be confined to very small volumes, well below the size in free space of the wavelength used [32, 33]. Because of this strong confinement the electric field per photon can be enhanced up to about 2 orders of magnitude as shown in Ref. [34]. This strong interaction of electromagnetic fields with matter thanks to plasmonic nanoantennas has been used to study surface infrared absorption enhancement SEIRA [35–39], Raman enhanced spectroscopy [38, 40–42], fluorescence enhancement [43, 44], fluorescence correlation spectroscopy [45–47], enhanced transduction in optomechanical systems [48], enhanced solar cell efficiencies [49, 50], enhanced decay rates of single quantum emitters [51], enhanced non linear frequency generation [52]. Finally many different sensing strategies have been proposed for plasmonic antennas, using the strong refractive index sensitivity due to the enhanced light matter interaction in plasmonics [53, 54].

1.2 Nano antennas as multipolar scatterers

Although the difference in $\epsilon$ between metallic and dielectric materials creates interesting almost mutually exclusive phenomena, like strong field confinement for metallic antennas [34] or lossless creation of magnetic scatterers for dielectric ones [27], the differences in the research fields of dielectric nano antennas and plasmonic nanoantennas are arguably semantic since the equations and theory behind them are the same. A general theory, with a long standing history [21, 55, 56], used to explain the scattering of any of these dielectric or metallic nanoantennas is exposed next.

An approach that is followed throughout this thesis to explain experiments and build theory, is the notion that one can decompose the response of a complex nano antenna in a handful of terms, ordered in importance according to the concept of a multipole expansion. This is easily understood starting from the general problem of scattering [57]. The solutions of Maxwell equations in free space give us all the possible modes in which electromagnetic radiation can exist in the open universe. Some solutions for these equations are readily known, as are the ubiquitous plane waves, spherical and cylindrical waves. When light in a plane wave in free space meets a very small piece of dust\(^6\) this plane wave will in general undergo a change of direction and even possibly color. In this thesis we will focus only on the first type of scattering process, i.e. linear optics, where only the direction and polarization is changed but not the color. The scattering process redistributes the light over all directions, a process

\(^6\)Size on the order of 0.1 $\mu$m for typical aerosols.
mathematically abstracted as a spherical wave centered at the dust particle, with a superimposed angle-dependent amplitude and phase function, which determines the so-called “differential scattering cross section”. To simplify this problem we start from Maxwell’s equations for homogeneous space, cast in the wave equation with $k = k_0 \sqrt{\varepsilon \mu}$

$$\nabla \times \nabla \times \mathbf{E} + k^2 \mathbf{E} = 0$$ (1.4)
$$\nabla \times \nabla \times \mathbf{H} + k^2 \mathbf{H} = 0.$$ (1.5)

Simultaneously we take into account the divergence condition,

$$\nabla \cdot \mathbf{E} = 0$$ (1.6)
$$\nabla \cdot \mathbf{H} = 0.$$ (1.7)

As shown in the book of Jackson [21] these sets of equations can be combined in one single set by using the path suggested by Bouwkamp and Casimir [58] which is based on using $\mathbf{r} \cdot \mathbf{E}$ and $\mathbf{r} \cdot \mathbf{H}$ in the Helmholtz equation, as in:

$$\nabla^2 \mathbf{r} \cdot \mathbf{E} + k^2 \mathbf{r} \cdot \mathbf{E} = 0$$ (1.8)
$$\nabla^2 \mathbf{r} \cdot \mathbf{H} + k^2 \mathbf{r} \cdot \mathbf{H} = 0,$$ (1.9)

It is of course very well known that Eq. (1.4) in free space gives rise to plane wave solutions of transverse electromagnetic waves. A different basis spanning all solutions to Maxwell’s equations in a homogeneous medium based on spherical waves can be derived from Eq. (1.8). When these equations are solved we obtain a general solution to the Maxwell’s equations with the form:

$$\mathbf{E}(r, \theta, \phi) = \sum_{n=1}^{\infty} \sum_{m=-n}^{n} \left[ a_{nm} \mathbf{N}_{nm}(r, \theta, \phi) + b_{nm} \mathbf{M}_{nm}(r, \theta, \phi) \right],$$ (1.10)

where $\mathbf{N}_{nm}$ and $\mathbf{M}_{nm}$ are so-called “vector spherical harmonics”. The “vector spherical harmonics” form a complete set of orthonormal basis functions and represent spherical waves: they have a radial part expressed in spherical Bessel and Hankel functions that contain asymptotic $e^{i k R}/R$ dependence. As a function of polar and azimuthal dependence they are much like the spherical harmonics known from quantum mechanics, though as expected, intricacies arise from the vector nature of light and the polarization conditions coming from Eq. (1.6) and Eq. (1.7). Each order $n$ corresponds to a multipole order, starting with the electric/magnetic dipole and quadrupole for $n=1, 2$ for the basis functions $\mathbf{N}_{nm}$ and $\mathbf{M}_{nm}$, respectively. Quantifying the physics of a single scatterer hence equates to calculating all the multipolar expansion coefficients $a_{nm}$ and $b_{nm}$ induced in response to some driving. In plasmonics and metamaterials, one often argues that only leading terms ($n=1$, maybe $n=2$) are important for scattering plane waves, while higher order multipoles become important specially as one considers the effect antennas have on emitters placed in their immediate vicinity.

While Eq. (1.10) provides the full field radiated by a scatterer, it is often insightful to think in terms of equivalent current distributions of the first few terms. Indeed
many works in literature are related to identifying such currents in full-wave near-field distributions [59]. By abstracting scatterers in this way we can represent small optical antennas by the most predominant type of currents driven in them. That is, if we have a linear antenna as in Fig. 1.3a where we can mostly excite linear oscillatory currents we could replace it in our problem by an electrical dipole and also for instance the split ring in Fig. 1.3b by a circular oscillatory current and a negative linear oscillatory current.

\[ J(r') \]

\[ \text{Figure 1.3: Sketch of: a) a silver rod and the linear currents driven in it and b) a gold split ring with the circular currents driven in it.} \]

In order to be able to find what are the fields that the induced currents in our antennas radiate we need the expression

\[ E(r) = i\omega \mu_0 \mu \int_{\Omega} \mathbf{G}(r, r') \mathbf{J}(r') \, dr' = i\omega \mu_0 \mu \int_{\Omega} \left( \mathbf{J}(r')^{\top} \mathbf{G}(r, r') \right)^{\top} \, dr' \quad (1.11) \]

where \( \mathbf{G} \) is the dyadic Green’s function of free space and \( \mathbf{J}(r') \) is the current distribution created in our antennas. Generally the dyadic Green’s function \( \mathbf{G} \) satisfies

\[ \nabla \times \nabla \times \mathbf{G} - k^2 \mathbf{G} = i\mathbf{I}\delta(r - r'), \quad (1.12) \]

where \( k = k_0 \sqrt{\mu/\varepsilon} \) is the wave number and \( \delta(r - r') \) is the distribution delta function. In practice \( \mathbf{G} \) is only known for a few cases, including free space, homogeneous environments, multilayered infinite systems, sphere and ellipsoidal cylinders. The Green’s function is extremely useful and it has been used for integral equation methods such as the method of moments and boundary element methods [60, 61]. For free space and homogeneous environments in its closed form \( \mathbf{G} \) is:

\[ \mathbf{G}(r, r') = \left[ \mathbf{I} + \frac{1}{k^2} \nabla \nabla \right] G_0(r, r') \]

\[ G_0(r, r') = \frac{e^{\pm ik|r-r'|}}{4\pi|r-r'|}, \quad (1.13) \]
The Green’s function for homogenous space as well as for a half infinite layer and a waveguiding layer will be commonly used throughout this thesis. For a deeper introduction in how to find it and use it we refer to the book of Novotny [32] as well as the book of Tai [60].

Having the expression in Eq. (1.11) we can calculate the field created by a given antenna whose most predominant currents have been determined. For this purpose we need an expression for the current of the different multipolar currents. Specifically the currents $J(r')$ for the first three multipoles are [62]:

$$J_{\text{Electric Dipole}}(r) = -(i\omega)\delta(r - r')p$$  \hspace{1cm} (1.15)

$$J_{\text{Magnetic Dipole}}(r) = (1/\mu_0\mu)\delta(r - r')m \times \nabla$$  \hspace{1cm} (1.16)

$$J_{\text{Quadrupole}}(r) = -(i\omega/3!)\delta(r - r')Q \cdot \nabla$$  \hspace{1cm} (1.17)

where $p$ is a vector that defines the electric dipolar strength, $m$ is the magnetic dipolar strength vector and $Q$ is the electric quadrupolar strength tensor. The symbol $\delta(r - r')$ is the distribution delta function that implies that the distribution of these currents is localized in a point located at $r'$.

### 1.3 Polarizability tensor

In section 1.2 we have described how an antenna in which a certain current distribution has been induced can be described as a combination of basis current distributions called multipoles. We have also shown how these current distributions would radiate their power away from the antenna. We have not discussed here yet how such current distributions are created in the material of the optical antennas in the first place, which is embodied in how $a_{nm}$ and $b_{nm}$ obtain their values. The ease with which a certain charge distribution is created on a given antenna is quantified to first order by the polarizability $\alpha$ of the object (see also Eq. (1.3) for the polarizability of a small scatterer). Thus for a given antenna the electric dipolar moment $p$ (equivalent to a linear oscillatory current) created by an electric field $E$ can be described as:

$$p = \alpha E.$$  \hspace{1cm} (1.18)

In general electric fields applied along different orientations will drive currents in the antennas with different strengths and directions. Since the electric field is a vector and the current distribution is also a vector, the most general form to describe the polarizability is by a 3 by 3 second rank tensor. We will show in this thesis that this tensor may also be extended to contain the information related to the magnetic dipolar currents and their driving as well as to electric quadrupolar currents. The former can be found in Ref. [63] while the latter is part of the original work of this thesis.
1.4 Nano antennas on substrates and waveguides

In section 1.2 and 1.3 we very briefly reviewed the general approach of induced multipole moments to discuss the induced currents and radiated fields of subwavelength antennas. Indeed, if one would solely study subwavelength antennas in free space, one could envision experiments to directly measure radiation patterns, i.e., far field distributions of radiated fields, and thus map multipole expansions directly. In this thesis, we are concerned primarily with the idea that in future applications, antennas are likely not in free space. For instance, using an antenna to improve LEDs or solar cells, means that it must sit in a layered, generally high index semiconductor structure, likely close to an interface. When thinking of sensors, an antenna would likely sit on a glass substrate, or be integrated with a dielectric waveguide.

Once an antenna is integrated in a nonhomogeneous environment, such as when it is placed on a planar substrate two distinct effects become important. First, a given current distribution will have a strongly modified radiation pattern. A second, and more intricate effect, is that for a given object the ease with which an applied external field sets up a material polarization changes. We will now discuss these two facts in turn for the case of a simple electric dipole. Starting with a given current distribution of an electric dipole we may find its radiation pattern by using Eq. (1.11). If we place this current distribution in free space we obtain a radiation pattern as the one shown in Fig. 1.4a. If we change the environment of this current distribution, the reflections and scattering produced by the environment, will drastically transform the total radiation pattern as shown in Fig. 1.4b where the same dipolar current was positioned 10 nm away from a glass substrate. This figure shows how most of the radiation is directed into the substrate in a narrow angular distribution. Fig 1.4a and b thus form a clear example that for the radiation pattern the presence of the environment is as important as the current distribution in the antenna itself (this can also be seen in Eq. (1.11)).

To illustrate the second effect, i.e. how the environment changes the ease with which an applied external field sets up a material polarization, we consider the case where an electric field drives an electrically polarizable object creating a dipole moment,

$$\mathbf{p} = \alpha \cdot \mathbf{E}_{\text{total}}(\mathbf{r}')$$  \hspace{1cm} (1.19)

where \( \mathbf{p} \) is the dipole moment, \( \alpha \) is the polarizability of the object and \( \mathbf{E}_{\text{total}} \) is the total electric field at the center position of the object which is mathematically considered as a point-like dipolar current distribution centered at \( \mathbf{r}' \). If the object is not in free space but in some more complex environment, the total electric field at the scatterer is composed not only of the externally incident field \( \mathbf{E}_{\text{in}} \), but also contains any field that the scatterer itself has radiated into the environment and which subsequently returns to the scatterer, labeled here as \( \mathbf{E}_{\text{scatt}} \), that is \( \mathbf{E}_{\text{total}} = \mathbf{E}_{\text{in}} + \mathbf{E}_{\text{scatt}} \). This scattered light is exactly accounted for by the so-called “Scattered part of the Green function” where we separate \( \mathbf{G} = \mathbf{G}_0 + \mathbf{G}_{\text{scatt}} \) into a free space part, and a scattered part that quantifies the multiple scattering caused by the complex environment. For instance, if we consider a scatterer that is brought to a mirror, the scattered part of the Greens function contains the reflected field, and is proportional to the mirror reflection coefficients. Therefore
1.4 Nano antennas on substrates and waveguides

we find that our induced current equals,

\[ \mathbf{p} = \bar{\alpha} \mathbf{E}_{in} + \bar{\alpha} \cdot \omega^2 \mu_0 \mu G_{\text{scatt}}(r',r') \cdot \mathbf{p}. \] (1.20)

By using a little bit of algebra we can turn this expression into

\[ \mathbf{p} = \bar{\alpha}_{\text{corr}} \mathbf{E}_{in} \] (1.21)

where,

\[ \bar{\alpha}_{\text{corr}}^{-1} = \bar{\alpha}^{-1} - \omega^2 \mu_0 \mu G_{\text{scatt}}(r',r') \] (1.22)

This last expression shows a ‘corrected’ polarizability. With this corrected polarizability we can deal with the response of the scatterer in a complex system while taking into account self consistently all the multiple scattering between the scatterer and its environment. In general, \( G_{\text{scatt}}(r',r') \) can contribute both a real and an imaginary correction to alpha. If \( \alpha \) is a Lorentzian response function, as for a simple model for a plasmon sphere, this constitutes a change in resonance frequency (\( \text{Re} \{ \bar{\mathbf{G}} \} \)) and a change in radiation damping (\( \text{Im} \{ \bar{\mathbf{G}} \} \)). Returning to the mirror example, these can be easily understood. Suppose we place a dipole in front of a perfect mirror, so that \( G_{\text{scatt}} \) can be approximated using image charge theory. Qualitatively, the induced dipole in the scatterer will hybridize with its mirror image. Depending on the relative alignment of the mirror image and the original dipole, this will give rise either to a blue- or red shift in energy (as evident from an electrostatic energy consideration), as well as to a net increase or decrease of net radiative damping, i.e., radiated power per unit of induced dipole moment. This correction is identical to the radiative line width and resonance frequency changes experienced by spontaneous emitters, like atoms and molecules, held
in front of an interface [64], due to backaction of the radiated source field on the source itself. In fact, in order to obtain a scattering theory that satisfies energy conservation, an equation of the form of Eq. (1.22) is required to correctly define the polarizability of an object in vacuum. In such a case, one requires that the entries for Eq. (1.22) contain the electrostatic polarizability of the object $\alpha$, defined as $\alpha = \frac{3V(e-\epsilon_{\text{env}})}{(e+2\epsilon_{\text{env}})}$ (for a sphere) and $\overline{G}_{\text{scatt}}$ replaced by $\text{Im}\left\{\overline{G}_0\right\} = \frac{\omega}{\epsilon_0\mu_0}/(c6\pi)$. Here the factor $\text{Im}\left\{\overline{G}_0\right\}$ quantifies the backaction a dipole experiences in a homogeneous environment, and is equivalent to the local density of states. This correction to the polarizability is also known as a Sipe-Kranendonk correction [65], or dynamic correction. With the corrected polarizability tensor we are able to handle the inclusion of any substrate for which we have its Green’s function as long as we know what is the polarizability tensor of one such an optical antenna.

The point dipole and multipole picture is often used in the field of plasmonics to guide qualitative understanding of experimental results and full wave calculations. The main proposition of this thesis is that, when applied with care, this picture can also be pushed to give quantitative predictions of antenna physics. While it is not a replacement for all problems solved with full wave calculations, this type of simplification could mean a considerable economization of time in design and simulation tasks, and it helps to build an intuitive picture of the scattering processes. This saving of time would allow for numerical optimizations of designs of multielement scatterers for improvement of LED illumination, increased solar cell efficiencies, or also higher field confinements for waveguide integrated single molecule read out and control. In this thesis we will expose examples of how to use an extended point dipole theory corrected for the presence of non trivial environments like substrates and waveguides. We will show how this method is able to explain experimental results and how it is suitable for designs of novel antennas.

1.5 Outline of this thesis

We begin this thesis by reporting scattering experiments on single rod plasmonic nano antennas fabricated on top of a Si$_3$N$_4$ ridge waveguide (chapter 2). This study was done to assess how strongly light propagating in guided modes interact with plasmon antennas, with the ultimate goal to provide an on-chip plasmon building block for interconversion between guided modes and strongly localized fields that could, for instance, interact with active materials. For this study we use point dipole theory to unravel the performed measurements. We continue showing how by using point dipole theory we can design a multi element Yagi-Uda antenna that maximizes the incoupling of light from a single emitter, in the proximity of the antenna, into the guided mode of a waveguide (chapter 3). In this chapter we show how the system of a plasmonic antenna plus dielectric waveguide represents a very interesting technology for controlling and detecting the emission of quantum light sources, e.g. atoms, molecules or quantum dots.

To complement the experiments done in chapter 3 we used a cathodoluminescence
measurement setup to generate nanometric position controlled point sources on Yagi-Uda antennas. By using point dipole theory and a statistical analysis we explain the features appearing in the acquired spatial excitability maps. Also, we show how random changes in the size of antenna elements result in a high variability of near field high and low excitability positions which contrast with the rather robust directionality and directivity found for these antennas (chapter 4).

In (chapter 5) of this thesis we present a numerical tool with which the polarizability tensor of any optical antenna with an arbitrary geometry can be retrieved. This retrieval is done in order to be able to use point dipole theory in more general problems, involving optical antennas with virtually any shape, while not needing microscopic analytical expressions for the polarizability tensor of the antennas. We use the retrieval tool to explain the behaviour of one of the most iconic optical antennas, the split ring resonator. Besides explaining its scattering properties and exotic pseudochirality we use the multipolar expansion of this antenna to design a novel split ring based multi-element antenna with very interesting characteristics as a directive radiation source of elliptically polarized light from a localized point source (chapter 6).

Having set the stage for retrieving the polarizability tensor of an arbitrary antenna, we expand the point dipole model to include electric quadrupolar moments. We use this extended theory to analyze the behavior of two types of antennas. The first of these is the so-called “dolmen” antenna that consists of a nanorod that is a strongly dipolar scatterer coupled to a dimer of nanorods that was reported in literature to support a very strongly quadrupolar mode. In particular, we quantify the claim made in earlier reports that narrow features in the dolmen extinction spectrum known as “Fano interference” or “plasmon-induced transparency” are attributable to a quadrupolar response. The second of these antennas concerns nanopyramids made out of aluminium. Through optimization of their simultaneous electric dipole, magnetic dipole, and electric quadrupole response these allow strongly directional scattering, as well as strongly directed enhancement of emission of nearby emitters. These nano pyramids are shown to be suitable for vertical asymmetric field confinement that could be used for enhancing LED illumination and solar cells (chapter 7).

Finally we show how to use the extended point quadrupole-dipole theory in the presence of a substrate. We explain measurements of nano cylinders with strong quadrupolar moments, measured in a cathodoluminescence setup, which are capable of strongly directing light by the interference of its different multipolar moments. These “nano lighthouses” have the characteristic that they are capable of strongly directing scattered light although the antenna is composed of one single element (chapter 8).
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